Joshua Burch Algo Assignment 2

Part 1 Asymptotic Behaviour

a) $w(n) = 2w(\frac{n}{3}) + 1$

2^0 0 $w(n)$ ---- 1

2^1 1 $w(\frac{n}{3})$ $w(\frac{n}{3})$ ---- 2

2^2 2 $w(\frac{n}{9})$ $w(\frac{n}{9})$ ... ... $2^2 + 1$

⋮

2^i $w(\frac{n}{3^i})$ ---- $2^i \times 1$

$\boxed{w(n) = O(2^{\log_3 n})}$

Depth $= \log_b n$

Leaf Dominated

$\frac{n}{3^i} = 1$

$i = \log_3 n$

$\sum_{i=0}^{\log_3 n} 2^i$

$2 \cdot 2^{\log_3 n} > \sum 2^{\log_3 n}$

b) $w(n) = 5w(\frac{n}{4}) + n$

5^0 0 $w(n)$ ---- n

5^1 1 $w(\frac{n}{4})$ $w(\frac{n}{4})$ $w(\frac{n}{4})$ $w(\frac{n}{4})$ $w(\frac{n}{4})$ ---- $\frac{n}{4^1} \times 5^1$

5^i i $w(\frac{n}{4^i})$ ... ... ---- $\frac{n}{4^i} \times 5^i$

Leaf Dominated

Depth $= \log_4 n$

$\sum_{i=0}^{\log_4 n} \frac{n}{4^i} \times 5^i$

$n \cdot \sum_{i=0}^{\log_4 n} (\frac{5}{4})^i \rightarrow n \cdot \sum_{i=0}^{\log_4 n} (1 + \frac{1}{4})^i \rightarrow \log$

$\boxed{w(n) = O(n \log_4 n)}$

c) $w(n) = 7w(\frac{n}{7}) + n$

7^0 0 $w(n)$ ---- n

7^1 1 $w(\frac{n}{7})$ $w(\frac{n}{7})$ ... ---- $\frac{n}{7} \times 7^1 = n$

⋮

7^i i $w(\frac{n}{7^i})$ $\frac{n}{7^i} \times 7^i = n$

Balanced

Depth $= \log_7 n$

$n \cdot \sum_{i=0}^{\log_7 n} \frac{n}{7^i} \times 7^i$

$\boxed{w(n) = O(n \log_7 n)}$

d) $w(n) = 9w(\frac{n}{3}) + n^2$

9^0 0 $w(n) \longrightarrow n^2$

9^1 1 $(\frac{n}{3})^2$ $(\frac{n}{3})^2$ .... $\rightarrow (\frac{n}{3})^2 \times 9 = n^2$

9^i i $(\frac{n}{3^i})^2$ .... $\rightarrow (\frac{n}{3^i})^2 \times 9^i = n^2$

Balanced

Depth $= \log_3 n$

$n^2 \cdot \sum_{i=0}^{\log_3 n} (\frac{n}{3^i})^2 \times 9^i$

$\boxed{w(n) = O(n^2 \log_3 n)}$

c) $W(n) = 8W(\frac{n}{2}) + n^3$

$8^0 \quad 0 \quad W(n) \dashrightarrow n^3$

$8^1 \quad 1 \quad W(\frac{n}{2})\ W(\frac{n}{2}) \ldots \longrightarrow (\frac{n}{2})^3 \cdot 8 = n^3$

$8^i \quad i \quad W(\frac{n}{2^i})^3 \longrightarrow (\frac{n}{2^i})^3 \cdot 8^i = n^3$

Balanced

Depth $= \log_2 n$

$n^3 \cdot \sum_{i=0}^{\log_2 n} (\frac{n}{2^i})^3 \times 8^i$

$\boxed{W(n) = O(n^3 \log n)}$

f) $W(n) = 49W(\frac{n}{25}) + n^{3/2} \log n$

$49^0 \quad 0 \quad W(n) \ldots\ldots n^{3/2} \log n$

$49^1 \quad 1 \quad W(\frac{n}{25})^{3/2} \ldots \ W(\frac{n}{25})^{3/2} \ldots \ (\frac{n}{25})^{3/2} \cdot 49 \cdot \log(\frac{n}{25}) = n^{3/2} \log n$

$49^i \quad i \quad W(\frac{n}{25^i})^{3/2} \ldots\ldots\ldots (\frac{n}{25^i})^{3/2} \cdot 49^i \cdot \log(\frac{n}{25^i})$

Leaf Dominated

Depth $\log_{25} n$

$n^{3/2} \sum_{i=0}^{\log_{25} n} (\frac{49}{125})^i \cdot \log n - i \log 25$

$\boxed{W(n) = O(n^{3/2} \log n)}$

g) $W(n) = W(n-1) + 2$

$n \quad 0 \quad W(n) \longrightarrow 2$

$1 \quad W(n-1)+2 \quad W(n-1)+2 \longrightarrow 2$

$n-1 \quad i=n-1 \quad W(n-k)+2 \ldots \longrightarrow 2$

Balanced

Depth $= n-1$

$W(n) = \sum_{i=0}^{n-1} 2 = 2(n-1)$

$W(n) = 2n-2$

$\boxed{W(n) = O(n)}$

h) $W(n) = W(n-1) + n^c$, with $c \geq 1$

$n \quad 0 \quad W(n) \longrightarrow n^c$

$n-1 \quad 1 \quad (n-1)^c \quad (n-1)^c \longrightarrow (n-1)^c$

$n-i \quad i \quad (n-i)^c \ldots \quad (n-i)^c$

Depth $= n-1$

$W(n) = \sum_{(n-i)=1}^{n-1} (n-i)^c$

$W(n) = \sum_{(n-i)=1}^{n} (n-i)^c = O(n^{c+1})$

$\boxed{W(n) \ O(n^{c+1})}$

b) $w(n) = w(\sqrt{n}) + 1$

i^0 $\quad 0 \quad w(n) \longrightarrow 1$

i^1 $\quad 1 \quad w(n^{\frac{1}{2}}) \quad w(n^{\frac{1}{2}}) \longrightarrow 1$

i^i $\quad i \quad w(n^{\frac{1}{2^i} \frac{1}{\log}}) \longrightarrow n^{\frac{1}{2^i} \frac{1}{\log}} \cdot 1^i \to 1$

$\boxed{w(n) = O(\log \log n)}$

Depth

Depth $= \log_2 n$

$$\log_2 n \cdot \sum_{i=0}^{\log_2 n} n^{\frac{1}{2^i} \frac{1}{\log n}}$$

Joshua Busch

Part 2. Algorithm comparison

A. $W(N) = 5W(\frac{n}{2}) + O(n)$

5^0 0 $w(n)$ $\rightarrow n$

5^1 1 $w(\frac{n}{2})$ $w(\frac{n}{2})$ ... ... $\rightarrow \frac{n}{2} \cdot 5^1$

5^i i $w(\frac{n}{2^i})$ $\rightarrow \frac{n}{2^i} \times 5^i$

$\boxed{w(n) = O(5^{\log_2 n})}$

Depth $= \log_2 n$

$$\sum_{i=0}^{\log_2 n} \frac{n}{2^i} \times 5^i$$

$$\sum_{i=0}^{\log_2 n} \left(\frac{5}{2}\right)^i$$

B. $w(n) = 2W(n-1) + 1$

2^0 0 $w(n) \longrightarrow 1$

2^1 1 $w(n-1)$ $w(n-1)$ $\longrightarrow 2$

2^i i $w(n-i)$ $\longrightarrow 2^i$

$\boxed{w(n) = O(2^n)}$

Depth $= n-1$

$$w(n) = \sum_{i=0}^{n-1} 2^i$$

$$= \frac{2^{n-1+1} - 1}{2-1} = 2^n - 1$$

C. $w(n) = 9W(\frac{n}{3}) + O(n^2)$

9^0 0 $w(n) \longrightarrow n^2$

9^1 1 $w(\frac{n}{3})^2$ ...... $\rightarrow (\frac{n}{3})^2 \cdot 9 = n^2$

9^i i $w(\frac{n}{3^i})^2$ $\longrightarrow (\frac{n}{3^i})^2 \cdot 9^i = n^2$

$\boxed{w(n) = O(n^2 \log n)}$

Depth $= \log_3 n$

$$n^2 \cdot \sum_{i=0}^{\log_3 n} \left(\frac{n}{3^i}\right)^2 \cdot 9^i$$

Algorithm C is the most efficient Algorithm followed by A then B.