

# Joshua Burch Algo Assignment 2

## Part 1 Asymptotic Behaviour

a)  $w(n) = 2w(\frac{n}{3}) + 1$

Depth =  $\log_3 n$

Leaf Dominated

$\frac{n}{3^i} = 1$

$i = \log_3 n$

$\sum_{i=0}^{\log_3 n} 2^i$

$2 \cdot 2^{\log_3 n} > \sum_{i=0}^{\log_3 n} 2^i$

$\dots 2^i \times 1$

$w(n) = O(2^{\log_3 n})$

b)  $w(n) = 5w(\frac{n}{4}) + n$

Leaf Dominated

Depth =  $\log_4 n$

$\sum_{i=0}^{\log_4 n} \frac{n}{4^i} \times 5^i$

$n \cdot \sum_{i=0}^{\log_4 n} (\frac{5}{4})^i \rightarrow n \cdot \sum_{i=0}^{\log_4 n} (1 + \frac{1}{4})^i \rightarrow \log_4 n$

$w(n) = O(n \log n)$

c)  $w(n) = 7w(\frac{n}{7}) + n$

Balance d

Depth =  $\log_7 n$

$\sum_{i=0}^{\log_7 n} \frac{n}{7^i} \times 7^i$

$w(n) = O(n \log n)$

d)  $w(n) = 9w(\frac{n}{3}) + n^2$

Balance e

Depth =  $\log_3 n$

$\sum_{i=0}^{\log_3 n} (\frac{n}{3^i})^2 \times 9^i$

$w(n) = O(n^2 \log n)$

$(\frac{n}{3})^2 \times 9 = n^2$   
 $(\frac{n}{3^i})^2 \times 9^i = n^2$



$$c) W(n) = 8W\left(\frac{n}{2}\right) + n^3$$

$$\begin{array}{lcl} 8^0 & 0 & W(n) \longrightarrow n^3 \\ 8^1 & 1 & W\left(\frac{n}{2}\right) \longrightarrow \left(\frac{n}{2}\right)^3 \cdot 8 = n^3 \\ \vdots & \vdots & \vdots \\ 8^i & i & W\left(\frac{n}{2^i}\right)^3 \longrightarrow \left(\frac{n}{2^i}\right)^3 \cdot 8^i = n^3 \end{array}$$

Balanced  
Depth =  $\log_2 n$

$$n^3 \cdot \sum_{i=0}^{\log_2 n} \left(\frac{n}{2^i}\right)^3 \cdot 8^i$$

$$W(n) = O(n^3 \log n)$$

$$f) W(n) = 49W\left(\frac{n}{25}\right) + n^{3/2} \log n$$

$$\begin{array}{lcl} 49^0 & 0 & W(n) \longrightarrow n^{3/2} \log n \\ 49^1 & 1 & W\left(\frac{n}{25}\right)^{3/2} \cdot 49 \cdot \log\left(\frac{n}{25}\right) = n^{3/2} \log n \\ \vdots & \vdots & \vdots \\ 49^i & i & W\left(\frac{n}{25^i}\right)^{3/2} \cdot 49^i \cdot \log\left(\frac{n}{25^i}\right) = n^{3/2} \log n \end{array}$$

Leaf dominated  
Depth =  $\log_{25} n$

$$n^{3/2} \sum_{i=0}^{\log_{25} n} \left(\frac{49}{25}\right)^i \cdot \log n = i \log 25$$

$$W(n) = O(n^{3/2} \log n)$$

$$g) W(n) = W(n-1) + 2$$

$$\begin{array}{lcl} n & 0 & W(n) \longrightarrow 2 \\ & \downarrow & \downarrow \\ n-1 & 1 & W(n-1) + 2 \longrightarrow 2 \\ & \downarrow & \downarrow \\ n-2 & 2 & W(n-2) + 2 \longrightarrow 2 \\ & \vdots & \vdots \\ n-i & i & W(n-i) + 2 \longrightarrow 2 \end{array}$$

Balanced  
Depth =  $n-1$

$$W(n) = \sum_{i=0}^{n-1} 2 = 2(n-1)$$

$$W(n) = 2n - 2$$

$$W(n) = O(n)$$

$$h) W(n) = W(n-1) + n^c, \text{ with } c \geq 1$$

$$\begin{array}{lcl} n & 0 & W(n) \longrightarrow n^c \\ n-1 & 1 & (n-1)^c \longrightarrow (n-1)^c \\ \vdots & \vdots & \vdots \\ n-i & i & (n-i)^c \longrightarrow (n-i)^c \end{array}$$

Depth =  $n-1$

$$W(n) = \sum_{i=1}^{n-1} (n-i)^c$$

$$W(n) = \sum_{i=1}^{n-1} (n-i)^c = O(n^{c+1})$$

$$W(n) = O(n^{c+1})$$



$$b) w(n) = w(\sqrt{n}) + 1$$

$$\begin{array}{lcl} 1^0 & 0 & w(n) \longrightarrow 1 \\ 1^1 & 1 & w(n^{\frac{1}{2}}) \longrightarrow 1 \\ \vdots & \vdots & \vdots \\ 1^i & i & w(n^{\frac{1}{2^i}}) \longrightarrow n^{\frac{1}{2^i} \log_2 n} \cdot 1 \rightarrow 1 \end{array}$$

$$w(n) = O(\log \log n)$$

Depth

$$\text{Depth} = \log_2 n$$

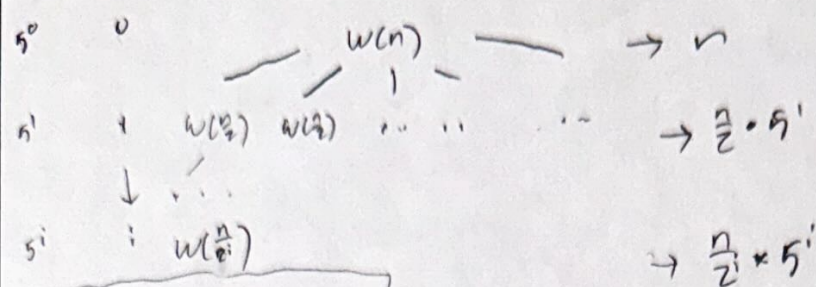
$$\log_2 n \cdot \sum_{i=0}^{\log_2 n} n^{\frac{1}{2^i} \log_2 n}$$



Joshua Busch

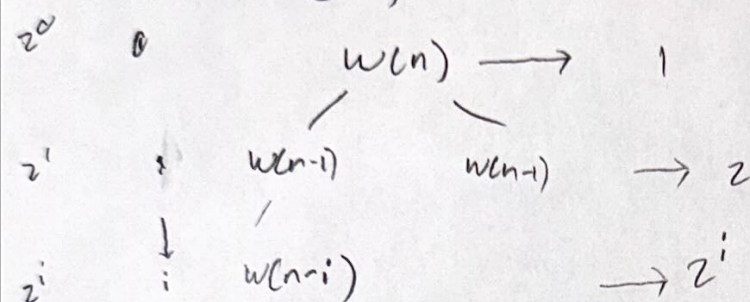
Part 2. Algorithm comparison

A.  $W(N) = 5W(\frac{N}{2}) + O(N)$



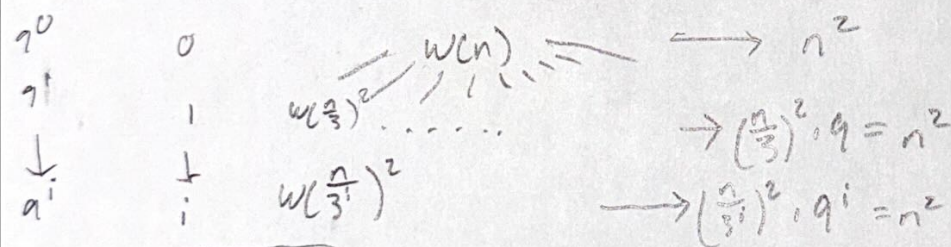
$W(N) = O(5^{\log_2 N})$

B.  $W(N) = 2W(N-1) + 1$



$W(N) = O(2^N)$

C.  $W(N) = 9W(\frac{N}{3}) + O(N^2)$



$W(N) = O(N^2 \log_3 N)$

Depth =  $\log_2 N$

$$\sum_{i=0}^{\log_2 N} \frac{N}{2^i} \times 5^i$$

$$\sum_{i=0}^{\log_2 N} \left(\frac{5}{2}\right)^i$$

Depth =  $N-1$

$$W(N) = \sum_{i=0}^{N-1} 2^i$$

$$= \frac{2^{N-1+1} - 1}{2 - 1} = 2^N - 1$$

Depth =  $\log_3 N$

$$N^2 \cdot \sum_{i=0}^{\log_3 N} \left(\frac{N}{3^i}\right)^2 \cdot 9^i$$

Algorithm C is the most efficient Algorithm followed by A then B.