Restoring Forests and Pricing Carbon Risk: A Quantitative Look at Nature's Value

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1 Introduction

The preservation of forests and natural biodiversity is a key factor in keeping the planet beautiful and thriving for generations to come. Conservationists, ecologists, and environmentalists have developed initiatives for global climate mitigation. These goals require both reduced carbon emissions and improved carbon sinks. Forest ecosystems are among the largest and most efficient natural carbon sinks. Articles such as the *Integrated global assessment of the natural forest carbon potential* [1] and documentaries like *Secrets of the Forest* [2] emphasize the importance of biodiversity over monoculture planting. This paper explores both ecological insight and a basic financial risk model involving carbon price volatility and carbon futures, bridging the gap between environmental science and quantitative finance.

2 Scientific and Media Perspectives on Forest Restoration

Forests are not tree farms—they are living, complex networks where fungi, insects, animals, and trees each play a vital role. The documentary and the article highlight how "greenwashing" occurs when companies present themselves as environmentally conscious without reintroducing full ecosystems. Instead, they often plant only a single tree species, which does not replicate the function of natural forests. These monocultures, while often tied to carbon offset credits, do not offer lasting ecological or carbon sequestration benefits. The studies emphasize that restoring degraded forests and supporting small farmers who work within these complex systems is more sustainable. They also advocate for broader social changes, including reducing meat consumption and improving land use practices. A digital map developed by researchers allows consumers to identify companies that are committed to sustainable sourcing.

3 Social Perception and Environmental Ethics

The documentary and the article reveal a deep gap in public understanding. Many of the ecological principles discussed—photosynthesis, interdependence among species, ecological cycles—are taught in elementary school. However, they are quickly forgotten or ignored. Public involvement in environmental science is limited because these foundational concepts are often overshadowed by pursuits of status, popularity, or material success. If people struggle to value each other, how can we expect them to care about trees or ecosystems? Developers often prioritize economic returns by paving green spaces, despite the ecological costs. One might ask: Is climate change accelerating because of global forces or because we continuously remove the very systems, like trees, that naturally cool our planet?

4 Carbon Risk and Financial Implications

Carbon emissions have a financial and ecological cost. Carbon pricing is designed to incentivize emissions reduction by assigning a cost to greenhouse gas output. The challenge lies in accurately measuring carbon pollution and quantifying its associated damage. Carbon credits are permits that allow the holder to emit a specified amount of CO_2 —- typically one ton per credit. These credits, also known as carbon allowances, are part of a system aimed at capping and reducing total emissions [3].

The United Nations issues credits to countries, and each country is responsible for managing its allocation and reporting. Companies are permitted to emit up to a certain threshold; emissions beyond that limit require the purchase of credits. Surplus credits can be sold on carbon exchanges. This "cap-and-trade" approach, while controversial, is active in several US states.

Carbon offsets operate within the voluntary carbon market. Here, individuals or organizations fund emissions-reducing projects and receive credits in return. These systems create a monetary incentive to reduce emissions and establish a financial penalty for inaction.

Carbon credit markets face risks, primarily due to price volatility driven by regulatory, political, and environmental uncertainty. High-integrity forest restoration projects play a vital role in mitigating this risk by providing real and verifiable carbon sequestration.

4.1 Modeling Carbon Derivatives Using Black-Scholes

Carbon prices, like other tradable assets, can be modeled using stochastic processes. The Black-Scholes model provides a way to price carbon credit options under uncertainty. Carbon asset prices can be modeled as:

$$dV_t = \mu V_t \, dt + \sigma V_t \, dW_t \tag{1}$$

where V_t is the asset value at time t, μ is the expected return, σ is the volatility, and dW_t is a Wiener process.

To price a European call option with strike price K:

$$C = \mathbb{E}[\max(V_t - K, 0)] = V_0 N(d_1) - K N(d_2) \tag{2}$$

where:

$$d_1 = \frac{\ln(\frac{V_0}{K}) + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}},$$
$$d_2 = d_1 - \sigma\sqrt{t},$$

Here, V_0 is the price today of the carbon asset, r is the risk-free interest rate, and $N(\cdot)$ is the standard normal CDF. The ratio $\lambda = (\mu - r)/\sigma$ captures the market price of risk. This model helps entities hedge against uncertain future costs associated with carbon emissions, making carbon-intensive investments more measurable in risk-adjusted terms.

4.2 Pigouvian Taxes and Subsidies

To correct for the negative externality of carbon emissions, Pigouvian taxes are imposed. These taxes align private incentives with social costs. The firm's profit maximization condition is:

$$\pi(q) = \max_q \left[p \cdot q - c(q)\right] \tag{3}$$

where p is the unit price and $c(q)$ is a convex cost function with $c'(q) > 0$ and $c''(q) > 0$.

First-order condition:

$$\pi'(q) = p - c'(q) = 0 \Rightarrow p = c'(q) \tag{4}$$

Assuming a social cost function of $SC(q) = q^2$, the marginal social cost becomes:

$$MSC(q) = 2q \tag{5}$$

The Pigouvian tax is defined as the gap between marginal social cost and marginal private cost:

$$T(q) = MSC(q) - c'(q) = 2q - c'(q) \tag{6}$$

Subsidies may also be used when marginal social benefits exceed private benefits:

$$S(q) = MPB(q) - MSB(q) \tag{7}$$

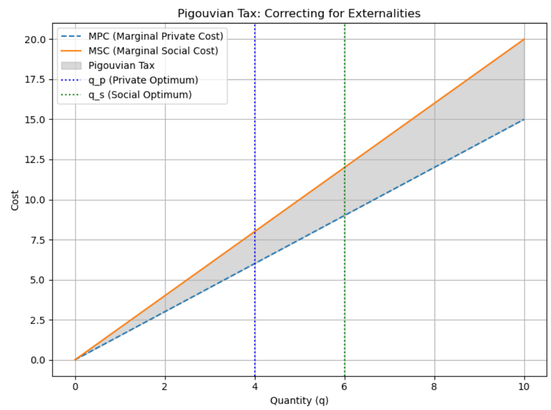

Figure 1: Offset of profit under Pigouvian tax — illustrating optimal quantity where private and social costs diverge. (Adapted from course notes by Prof. Enrique Villamor, Quantitative Environmental Risk)

Hedging strategies include forecasting carbon usage, comparing current and projected prices, and using financial instruments such as options or futures to manage cost variability.

The risk of greenwashing remains significant when low-integrity carbon credits are priced similarly to verified restoration projects.

5 Conclusion

True risk reduction comes not only from financial tools but also from ecological integrity. Bridging environmental science and financial modeling is essential for understanding and mitigating long-term carbon risk. Without shared values and accountability, even the best economic models will fall short.

References

[1] Harris, N. L., Lonneker, A., Baccini, A., et al. (2023). Integrated global assessment of the natural forest carbon potential. *Nature*, `https://www.nature.com/articles/s41586-023-06723-z`

[2] PBS (2024). Secrets of the Forest. PBS Documentaries. `https://www.pbs.org/video/secrets-of-the-forest-gijzto/`

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