Karydas, Christos; Xepapadeas, Anastasios

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Pricing climate change risks:
CAPM with rare disasters and stochastic probabilities

Christos Karydas[*]and Anastasios Xepapadeas[‡]

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Abstract

There are concerns that climate-related physical and political risks are not yet properly reflected in asset prices. To address these concerns, we develop a dynamic asset pricing framework with rare disasters related to climate change. The novelty of this paper lies in linking carbon emissions and portfolio composition with the stochastically-varying probability of these events. Using theory and simulations we study the implications of the imminent threat of climate change on different market measures and on the participation of carbon-intensive assets in the aggregate portfolio, as well as the conditions that lead to these assets becoming stranded. Our result suggest that climate change implies a positive and increasing risk premium, with the overall equity premium depending on the volatility of the stochastic process that governs climate change risk. Transition risks lower substantially the participation of carbon intensive assets in the market portfolio, which should be fully de-carbonized by the end of the century for the worst IPCC emissions scenario.

[*]Corresponding author, Center of Economic Research at ETH Zurich, Switzerland

[†]Dept. of Intern. and European Economic Studies, Athens University of Economics and Business, Greece & Dept. of Economics, University of Bologna, Italy

[‡]Email addresses: karydasc@ethz.ch (C. Karydas), xepapad@aueb.gr (A. Xepapadeas)

1   Introduction

In his path-breaking speech *Breaking the tragedy of the horizon* in 2015, the Governor of the Bank of England, Mark Carney, was the first to highlight the threat of climate change for the stability of the financial system and to identify the risks involved (Carney 2015). Although it is by now well understood that climate-related events are increasing in frequency and intensity, the full severity of climate change has not materialized yet;[1] the effects of our carbon emissions on the environment come only with a considerable time lag (Bretschger and Karydas 2018). This issue raises, therefore, the question of whether climate-related risks are properly reflected in asset pricing (Monnin 2018). The purpose of the present paper is to study the pricing of climate-related risks in the context of a dynamic CAPM approach with rare disasters. Our main objective is to explore how these risks can affect the different market measures, the participation in the aggregate portfolio of carbon-intensive assets, and the conditions under which these assets can become stranded.

There are two main types of climate-related risks relevant for market participants: physical risks and transition risks.[2] Physical risks, associated with physical damages to assets, could be event-driven (droughts, floods, storms, wildfires and crop failures) or chronic, related to long term climate shifts (e.g. sea level rise). The frequency and severity of environmental events can increase as a result of rising global temperatures (IPCC 2018, USGCRP 2018). Transition risks include among other components[3] policy risks which emerge from potential introduction of stringent carbon-pricing policies that can affect the returns of assets related with carbon-intensive technologies or processes.[4] Since investments in the carbon-based economy are mostly irreversible, stringent climate policies are likely to make the operation of carbon-intensive firms unprofitable, and, thereby leave assets stranded. Crucial to individual investors, appropriate pricing of climate-related risks, could lead to more informed, and thus efficient capital allocation decisions.

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[1]See Francis and Vavrus (2012), Cai et al. (2014) on the frequency and severity of natural disasters.

[2]See (Campiglio et al. 2018) for a discussion on the climate-related risk and their relevance for financial markets and central banks.

[3]Transition risks also include, liability risks, technology risks, market risks and reputation risks.

[4]For example Batten S. (2016) find contrasting cumulative abnormal returns experienced by a petroleum refining company and a wind turbine manufacturer the day after the announcement of the Paris agreement in December 2015; others argue that these policy risks have not yet been fully internalized by markets, e.g. Blackrock Investment Institute (2016), Sevillano and González (2018).

Relevant for central banks, climate-related damages could result in financial losses that affect the stability of the financial system through the balance sheets of insurers, banks and credit flows.[5] In combating climate change, policy has been so far predominantly fiscal and little attention has been paid to the role of central banks under climate change conditions. This attitude is, however, changing both in the context of academic research but also in the context of central banks' policy. The focus of this new line of approach is to explore the role of central banks in managing financial risks stemming from climate change. Going one step further, central banks explore whether climate change goals could be a secondary objective, along with controlling inflation and promoting macroeconomic stability (Campiglio et al. 2018, Strauss Financial Times, 05/12/2018).[6] Sitting at the heart of the global economy, and by allocating capital and risk, central banks can, therefore, promote the transition to a low-carbon economy and ensure financial stability (Prudential Regulation Authority 2018).

What constitutes an asset-pricing puzzle, is the difference between modeled and observed market measures; the equity premium and volatility puzzles have kept financial economists occupied for decades. By including the impact of rare disasters like wars and economic crises in asset pricing models the literature has come a long way in explaining investors' behavior towards risk.[7] The simple explanation of high risk premia is that investors are concerned about equity performance in rare events such as the Great Depression of the 1930s, or the two world wars of the 20th century. This is a problem with climate change, and one of the reasons why it arguably represents the greatest market failure ever seen: since its effects come only with a considerable time lag, there is not much there to see yet. With this paper we aim at offering a framework that prices the imminent risks of climate change.

With regards to our model, we consider a dynamic CAPM with rare disasters in the spirit of Barro (2009) and Wachter (2013). In these endowment economies rare disasters

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[5](Carney 2015, Battiston et al. 2017, Stolbova et al. 2018)

[6]In studying the role of Central Banks when climate changes, a more macroeconomic line of approach is to examine whether taking into account climate change in the context of dynamic stochastic general equilibrium models (DSGE) effects monetary policy when this policy is assumed to be conducted through the nominal interest rate on government bonds which follows a standard Taylor-type rule (see, e.g., Annicchiarico and Di Dio (2017), Economides and Xepapadeas (2018)).

[7]See Barro (2006), Barro and Ursúa (2008), Barro (2009), Wachter (2013), Tsai and Wachter (2015)

are captured by a Poisson process which allows for downward jumps in the return of the underlying asset. Building on Barro (2006), Wachter (2013) introduces recursive utility and stochastically-varying probability of the Poisson process, and matches the observed post-WWII equity premium and stock market volatility for the US, even for low values of relative risk aversion. To study climate-related risks, we build on the work of Wachter (2013) by introducing a second Poisson process for climate-related events; these can be natural disasters, or the introduction of a stringent climate policy. We consider a portfolio of riskless bond holdings, and two types of risky capital assets: a general one, and a brown one, the latter being responsible for CO2 emissions.[8] Macroeconomic and climate-related disasters affect both assets; climate-related policy events affect only the brown one. Using our model we can set the "correct" value on market fundamentals and calculate the participation in the aggregate portfolio of carbon-intensive assets when climate change risk is taken into account.

Our proxy for climate change is the change in global average temperature relative to the mean of a given time period, i.e. the temperature anomaly. Our methodological contribution is twofold. First, we include the observed positive relationship between the probability of climate-related events and temperature anomaly (see figure 1).[9] Second, we endogenize the effect of portfolio allocation on emissions: the higher the share of brown assets in the portfolio the higher, for a given brown-asset technology, the temperature anomaly and therefore the higher the average probability of occurrence of a climate-related disaster. The arrival of a climate-related disaster could trigger a policy jump with a certain probability. A reduction in the share of brown assets will have the opposite effects. In this way we establish a direct link between the composition of the aggregate portfolio and the arrival of climate related events.

Depending on investors information about climate change, this setup allows for two types of modelling. In the first case the representative investor considers the evolution of

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[8]We follow the definition of Prudential Regulation Authority (2015) and consider as "brown" Tier 1 & 2 assets that are directly exposed to transition risks. Tier 1 assets include coal, oil and gas extraction companies, and conventional utilities; Tier 2 assets include firms that are energy-intensive, e.g. chemicals and mining companies. Together they account for about 30% of global equity and fixed-income investments.

[9]Since the link between temperature anomaly and emissions is well established (e.g. Matthews et al. (2009), Hassler et al. (2016), Brock and Xepapadeas (2017)), this establishes the link between emissions and the probability of natural disasters.

global temperature as exogenous and not affected by his/her portfolio allocation. Exogenous temperature paths in this case could be regarded the Representative Concentration Pathways (RCPs) produced by the IPCC (van Vuuren et al. 2011); we will call this the *myopic* solution. In the second case the representative investor recognizes that his/her portfolio choices will affect the temperature path and therefore the probability of climate-related events; we will call this solution *optimal*.

The myopic simulation shows that climate change risk entails a positive premium which is increasing over time: the risk premium of climate change amounts to 0.2 percent p.a. in 2010; it can increase up to 0.5 percent p.a. by the end of the century. Most surprisingly, we find that whether this increase carries over to the overall equity premium depends on the stochastic nature of the probability of extreme environmental events. With rising emissions and volatile probability of environmental disasters, the relative importance of climate change risk for valuations increases even in times without disasters. Since, however, the risk of rare macroeconomic events makes up the largest share of the equity premium, the magnitude of the aggregate equity risk premium is only minimally affected. Compared to the benchmark without policy risk, including the transition risk of environmental policy lowers the portfolio participation of brown assets by about 10 percent already in 2010, and 12-13 percent in 2020. In the worst-case IPCC emissions scenario, all carbon-intensive assets should become stranded by the end of the century. In the optimal solution, the representative investor lowers the portfolio participation of brown assets by about 6 percent in comparison to the benchmark due to policy risk. When a Pigouvian tax is in place, there is no room for brown assets in the optimal portfolio.

To the best of our knowledge, this is the first paper to incorporate the stochastically time-varying risk of rare disasters related to climate change in a dynamic asset pricing framework, also with both types of risk exposures, namely physical and transition risks. Our contribution comes the closest to Bansal et al. (2016) who also include Poisson shocks due to climate change and, using a dataset of 39 countries over the 1970-2012 time period, find that global warming carries a positive – and increasing – risk premium and reduces asset valuations. In comparison to us, they find that climate change increases the overall risk premium in the market. We show that this result is a limiting case in our model. As explained above using our framework with volatile probabilities of extreme events, we are

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able to match observed moments in the data, even with low relative risk aversion. Moreover, our model allows for closed-form solutions, up to an indefinite integral. Additionally, with regards to data, we develop a much larger dataset of macroeconomic and climate-related disasters for 42 countries that extends over the 1911-2015 period.[10]

The paper is organized as follows. The next section formally provides intuition on how the time-varying probability of rare disasters affects risk premia. Section 3 builds the theoretical framework. We extend the model of Wachter (2013) by adding Poisson shocks due to climate change with a stochastically time-varying probability, which we build from climate science. Section 4 deals with numerical simulations. We present our methodology, calibration, and discuss our results. Section 5 concludes.

Figure 1: Number of climate-related disasters p.a. in the period 1955-2015 vs. Temperature anomaly from the mean of the 20th century. Source: EM-DAT The International Disasters Database *www.emdat.be* and *ourworldindata.org*

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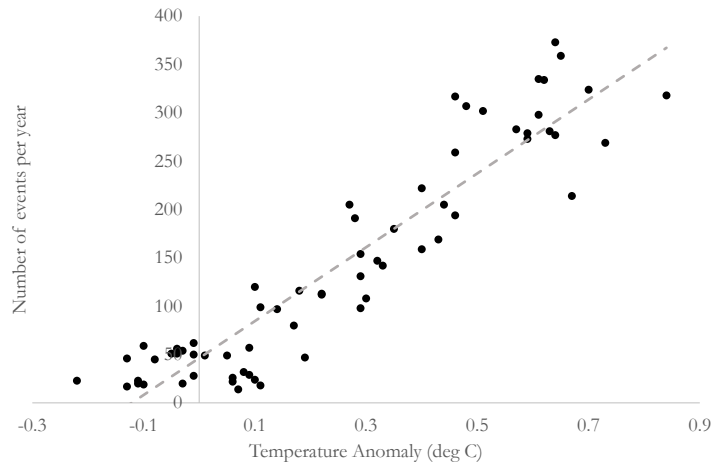
[10]In addition, Dietz et al. (2018) recently studied the effect of mitigation on consumption risk, and find that mitigation expenditure cannot be a hedge against climate change; in fact it comes with a positive beta, increasing the discount rate of projects related to emissions reduction. Other recent papers that include simple Poisson shocks in production economies to study the effects of climate change are Donadelli et al. (2017) and Bretschger and Vinogradova (2018).

2 Disaster risk premia

Risk premia arise from the covariance of marginal utility of the risk-averse investor with the price of the underlying asset or portfolio; see Cochrane (2005). In this section we show that this general result holds true both in normal times and times of rare disasters and discuss the relevance with our model in pricing climate change risk.[11] Our economy is in continuous time which we denote with the subscript t. Consider an aggregate portfolio the price of which, P, evolves stochastically over time according to the following jump-diffusion process, first introduced by Merton (1976):

$$\frac{dP_t}{P_{t^-}} = \mu_P dt + \sigma_P dW_t^T + (e^{Z_P} - 1)dY_t. \tag{1}$$

Variable W represents a vector of Wiener processes, and Y an independent Poisson jump process with intensity $\lambda_t > 0$ which can be time-varying.[12] Parameters μ_P and σ_P represent, respectively, the drift and volatility parameters of the stochastic process and can be also stochastic; σ_P is a vector. The change in $\log P$ should a rare event occur is measured by the random variable Z_P, which we assume has a time-invariant distribution that comes from the data; because prices fall when a negative shock occurs one expects this variable to be negative, i.e. $Z_P < 0$. Moreover, the process of (1) ensures that prices remain positive. How does this price process come about? Let D denote the continuous dividend stream paid by the portfolio. In equilibrium the price of this dividend claim P is determined by these cash flows and the pricing kernel m (or state price density) according to the standard asset-pricing equation, $P_t = \mathbb{E}_t \left[\int_t^\infty \frac{m_s}{m_t} D_s ds \right]$, where the symbol $\mathbb{E}$ denotes expectation. The pricing kernel can be thought as the marginal utility of the representative agent. Now let m follow a process similar to (1):

$$\frac{dm_t}{m_{t^-}} = \mu_m dt + \sigma_m dW_t^T + (e^{Z_m} - 1)dY_t. \tag{2}$$

Parameters μ_m and σ_m can be again stochastic and σ_m is a vector, as before. The change in $\log m$ should a rare event occur is given by the random variable Z_m, which has

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[11]This part has been adopted from Tsai and Wachter (2015) and is presented here for completeness.

[12]Particularly, the probability of k jumps of the Y process over the course of a short time interval Δt can be approximated by $e^{-\lambda_t \Delta t} \frac{(\lambda_t \Delta t)^k}{k!}$; we measure t in years.

also a time-invariant distribution. It is natural to assume that times of disasters are times of low consumption, and therefore high marginal utility and state prices, such that $Z_m > 0$. Random variables Z_P and Z_m can be correlated and we denote with $\mathbb{E}_z$ expectations over their joint distribution. Finally, let R denote the expected return of the portfolio, defined as the sum of the drift in the price, the dividend yield, and the expected change in the price should a rare event occur, as

$$R_t = \mu_P + \frac{D_t}{P_t} + \lambda_t \, \mathbb{E}_z[e^{Z_P} - 1]. \tag{3}$$

The equity premium arises from the comovement of the agent's marginal utility with the price process of the portfolio. Using (1) and (2), Tsai and Wachter (2015) show that given processes for state and portfolio prices, the return on equity R over the riskfree rate r, i.e. the equity premium, reads

$$R_t - r_t = -\sigma_m \sigma_P^T - \lambda_t \, \mathbb{E}_z \left[(e^{Z_m} - 1)(e^{Z_P} - 1) \right]. \tag{4}$$

The first term of equation (4), $-\sigma_m \sigma_P^T$, is responsible for compensating investors for the diffusion risk; it is the comovement of the state price density and prices in normal times. The jump-risk term exhibits the comovement of the state price density with prices during disasters.[13] Note also that, relevant to our model, when the probability of rare disasters λ_t varies stochastically over time, then this will be also shown in σ_m and σ_P. Therefore, including the (time-varying) probability of rare disasters in asset pricing models, increases risk premia to high levels observed in the market, even for low values of relative risk aversion.

Our main contribution consists of enhancing equation (4) with a second, uncorrelated, Poisson shock due to climate-related events. According to the aforementioned discussion, we expect risk premia to reflect the risk of climate change, both in times of severe environmental disasters and normal times. Moreover, in our model the time-varying arrival rate of the Poisson shock associated with climate change can either depend on exogenously given emission paths, or it can be endogenous, and dependent on investors decisions, i.e. the participation in the aggregate portfolio of carbon-intensive assets. We describe this next.

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[13]In periods without disasters the risk premium becomes equation (4) less $\lambda_t \, \mathbb{E}_z[e^{Z_P} - 1]$.

3 The model

3.1 Time-varying probability of environmental disasters

Our model features two types of disasters, macroeconomic (M) and environmental (E), each expressed by an uncorrelated Poisson jump process Y^j, with a time-varying arrival rate λ^j, $j \in \{M, E\}$. Macroeconomic disasters are events like wars and economic crises, while environmental disasters are severe events related to climate change like droughts, floods, and wildfires. With regards to the macroeconomic shocks, we follow Wachter (2013) and assume the following mean-reverting process for λ^M:

$$d\lambda_t^M = \kappa^M(\bar{\lambda}^M - \lambda_t^M)dt + \sigma_\lambda^M \sqrt{\lambda_t^M}dW_{\lambda t}^M. \tag{5}$$

Variable W_λ^M standard Brownian motion, independent of all other processes. Parameter κ^M represents the adjustment speed of the process towards its mean $\bar{\lambda}^M$; σ_λ^M is a volatility parameter. The solution to (5) leads to a Gamma stationary distribution for λ^M, provided that both κ^M and $\bar{\lambda}^M$ are positive, which we will assume (see figure 2). This process has the nice feature that λ^M can never become negative. Moreover, the square root in (5) implies that the resulting stationary distribution is highly right-skewed, while at the same time, high realizations of λ^M, make the process more volatile, and thus even higher realizations more likely, compared to a standard autoregressive process.[14]

Figure 2: The PDF of the Gamma distribution for λ_M

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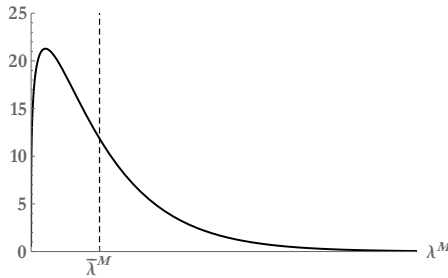
[14]See Cox et al. (1985) for more information on (5).

Our main methodological contribution lies in establishing the link between climate change and the intensity λ^E of the Poisson process related to environmental disasters Y^E. Climate shocks are triggered by emissions caused by brown capital assets K_B. Let $N > 0$ be expenditure on improving the environmental efficiency of brown capital. Accordingly, effective CO2 emissions read:

$$E_t = \phi(N_t)K_{Bt}, \tag{6}$$

with $\phi(N)$ measuring emissions intensity and $\phi'(N) < 0$. With regards to the dynamics of climate change, it is well established that cumulative emissions are linked to the change of temperature anomaly T in a rather linear fashion.[15] Following the transient climate response (TCRE) to cumulative carbon emissions a simplified continuous time representation of the global temperature anomaly dynamics can be written as in Hassler et al. (2016) and Brock and Xepapadeas (2017):

$$\dot{T}_t = \frac{1}{H}(\Lambda E_t - \delta T_t), \qquad T_0 = 0. \tag{7}$$

Parameter $H > 0$ represents the heat capacity of climate sinks, e.g. the upper ocean layer, $\Lambda > 0$ the TCRE, and $\delta \in (0,1)$ the stabilizing effect of the outgoing radiation.[16] With $\bar{E}$ a constant flow of emissions, equation (7) leads to a steady state for temperature anomaly $\bar{T} = (\Lambda/\delta)\bar{E}$. Natural disasters are increasing in frequency and intensity. Based on observations (figure 1), we will assume that the probability of natural disasters λ^E depends linearly on the temperature anomaly, i.e. $\lambda_t^E = \tilde{\lambda}^E + \xi T_t$, with $\{\tilde{\lambda}^E, \xi\}$ non-negative numbers.[17] Differentiating the last expression with respect to time, and substituting back the expression for E, yields:

$$\dot{\lambda}_t^E = \frac{\delta}{H}(\tilde{\lambda}^E - \lambda_t^E) + \frac{\xi}{H}\Lambda E_t.$$

Besides the expected positive relationship between the disaster probability and the temperature anomaly that leads to the previous expression, the arrival rate of natural

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[15]See Matthews et al. (2009, 2012), Knutti (2013), Knutti and Rogelj (2015), MacDougall et al. (2017).

[16]See also Nævdal and Oppenheimer (2007), Lemoine and Rudik (2015) and Heutel et al. (2016).

[17]Bansal et al. (2016) also use a similar linear relationship for the intensity of the Poisson shock, which however remains deterministic.

disasters can be rather stochastic. We will therefore follow a process similar to (5) according to:

$$d\lambda_t^E = \kappa^E(\bar{\lambda}_t^E - \lambda_t^E)dt + \sigma_\lambda^E\sqrt{\lambda_t^E}dW_{\lambda t}^E, \tag{8}$$

where for ease of exposition we defined $\kappa^E \equiv \delta/H$ and $\bar{\lambda}_t^E \equiv \tilde{\lambda}^E + \xi(\Lambda/\delta)E_t$, with $E_t = \phi(N_t)K_{Bt}$, and W_λ^E a standard Brownian motion, independent of all other processes. Note that, in comparison to (5), although this autoregressive process features mean reversion, the mean itself is time-varying. When emissions keep rising out of balance natural disasters are becoming more frequent in expectation but less predictable. In equilibrium, i.e. for a constant flow of net emissions (possibly zero if $K_B = 0$), the solution of (8) has also a Gamma stationary distribution, where the constant mean and variance are increasing in the equilibrium flow of emissions.[18] Note also that, in line with observations, in our calibration $\kappa^E < \kappa^M$ such that realizations of high probability of environmental disasters are more persistent than those of macroeconomic disasters.[19]

3.2 The macroeconomic environment

With regards to the economic environment, we consider the dynamic asset pricing setting of an aggregate portfolio consisting of riskless bond holdings, and risky capital assets. According to our purpose we differentiate between general capital assets K_G and brown capital assets K_B such that total capital is $K_G + K_B$ (see footnote 8). Prices of the risky assets evolve stochastically over time according to the following jump-diffusion processes:

$$\begin{aligned}\frac{dp_{Gt}}{p_{Gt^-}} &= \mu_G dt + \sigma_G dW_{Gt} + (e^{Z^M} - 1)dY_t^M + (e^{Z^E} - 1)dY_t^E, \\ \frac{dp_{Bt}}{p_{Bt^-}} &= \mu_B dt + \sigma_B dW_{Bt} + (e^{Z^M} - 1)dY_t^M + (e^{Z^E + \pi X_t} - 1)dY_t^E,\end{aligned} \tag{9}$$

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[18]Let $\bar{E}$ be an equilibrium flow of emissions. Following Cox et al. (1985), we can show that the expected value and variance of λ^E at time s, conditional on its value at time $t < s$, is given by: $\mathbb{E}[\lambda_s^E|\lambda_t^E] = \lambda_t^E e^{-(\delta/H)(s-t)} + (\tilde{\lambda}^E + \frac{\xi}{\delta}\Lambda\bar{E})\left(1 - e^{-(\delta/H)(s-t)}\right)$ and $\text{Var}[\lambda_s^E|\lambda_t^E] = \lambda_t^E \frac{(\sigma_\lambda^E)^2}{\delta/H}(e^{-(\delta/H)(s-t)} - e^{-2(\delta/H)(s-t)}) + (\tilde{\lambda}^E + \frac{\xi}{\delta}\Lambda\bar{E})\frac{(\sigma_\lambda^E)^2}{2\delta/H}(1 - e^{-(\delta/H)(s-t)})^2$. The steady state mean and variance are $\tilde{\lambda}^E + \frac{\xi}{\delta}\Lambda\bar{E}$ and $(\tilde{\lambda}^E + \frac{\xi}{\delta}\Lambda\bar{E})\frac{(\sigma_\lambda^E)^2}{2\delta/H}$, respectively, both increasing in $\bar{E}$.

[19]For exampleWachter (2013) calibrates $\kappa^M = 0.08$ while Brock and Xepapadeas (2017) set $\delta = 0.1$ PW/K and $H = 4.58$ PW y/K, giving $\kappa^E = 0.022$; see section "Calibration".

while, the return of the riskless asset is r. The subscripts used denote the asset class: G for general, and B for brown; the superscripts denote the type of shock: M for macroeconomic, E for environmental. For each asset $i \in \{G, B\}$, $\mu_i > 0$ is the drift parameter and W_i is a standard asset-specific Brownian motion with volatility σ_i; processes W_i are correlated. For each type of disaster $j \in \{M, E\}$, variable Y^j represents the Poisson jump process, with a time-varying intensity $\lambda^j > 0$; λ^M follows (5), while λ^E equation (8). The random variables $Z^j < 0$ represent the change in $\log p_i$ when a disaster of each type j occurs; their time-invariant distributions z^j are independent of all other processes. We further assume that adverse climate-related events Y^E trigger environmental policy with a certain probability $\pi \in [0, 1]$; when effective, stringent policy adds to further reduce the return of the brown asset by $X < 0$.[20] [21]

Finally, the representative agent has the continuous-time analogue of recursive Epstein-Zin preferences, as formulated by Duffie and Epstein (1992). Accordingly, we use the following recursion to define the utility function U:

$$U_t = \mathbb{E}_t \int_t^\infty f(C_v, U_v) dv, \tag{10}$$

where

$$f(C_t, U_t) = \rho(1 - \gamma)U_t \left(\log C_t - \frac{1}{1 - \gamma} \log((1 - \gamma)U_t) \right). \tag{11}$$

Parameter $\rho > 0$ is the rate of time preference, and $\gamma > 0$, measures relative risk aversion. We assume for simplicity that our utility function features a unitary elasticity of intertemporal substitution (EIS). Moreover, when $\gamma \to 1$, using the de l'Hospital rule, we can show that f reduces to the usual log-utility form. We will conventionally focus on the case of $\gamma > 1$.

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[20]One could argue that policy should follow a separate process with a positive, but not perfect, correlation with environmental shocks; another option would be to include a lag between environmental shocks and policy arrival. In the interest of tractability we abstract from both options, since they would not alter the main insights in any fundamental way.

[21]As in Bretschger and Soretz (2018), we could also assume that the effect of the policy X depends on the capital ratio K_B/K_G, such that environmental policy is more stringent in countries with an already low share of brown capital e.g. Norway.

3.3 Equilibrium

Let A represent total assets. Moreover, let $n \equiv (K_G + K_B)/A$, and $n_B \equiv K_B/(K_G + K_B)$, denote the shares of risky capital in total assets, and brown capital in capital assets, respectively. Wealth follows the process:

$$dA_t = (A_{t^-} n_t(\mu_t - r_t + \beta) + A_{t^-} r_t - C_t)\, dt + A_{t^-} n_t \left(\sigma_{At} dW_{At}^T + J_{At} dY_t^T \right), \qquad (12)$$

for some constant $\beta > 0$.[22] For ease of exposition we defined $\mu_t = n_{Bt}\mu_B + (1 - n_{Bt})\mu_G$, $\sigma_{At} = [n_{Bt}\sigma_B, (1 - n_{Bt})\sigma_G]$, $dW_{At} = [dW_{Bt}, dW_{Gt}]$, $dY_t = [dY_t^M, dY_t^E]$, and

$$J_{At} = [\underbrace{e^{Z^M} - 1}_{J_M - \text{macroeconomic shocks}} , \underbrace{e^{Z^E}(1 + n_{Bt} \overbrace{(e^{\pi X_t} - 1)}^{J_X - \text{policy effect}}) - 1}_{J_E - \text{environmental shocks}}]. \qquad (13)$$

Note that for $Z^j, X < 0$, $J_j \in (-1, 0)$, $j \in \{M, E\}$. In the event of a climate shock, wealth jumps downwards due to the pure adverse effects of climate change, and due to environmental policy which reduces further the return on brown capital. With A, λ^M and λ^E as state variables, and $V(A, \lambda^M, \lambda^E)$ the value function, using (5), (8), (12), and Itô's Lemma, optimal consumption, and portfolio choices must satisfy the following (Duffie and Epstein 1992):

$$\sup_{C, n, n_B} \{ f(C_t, V) + V_A(A_t n_t(\mu_t - r_t + \beta) + A_t r_t - C_t) + \frac{1}{2} V_{AA} A_t^2 n_t^2 \sigma_t^2$$
$$+ \sum_{j=\{M,E\}} (V_{\lambda^j} \kappa^j (\bar{\lambda}_t^j - \lambda_t^j) + \frac{1}{2} V_{\lambda^j \lambda^j} (\sigma_\lambda^j)^2 \lambda_t^j + \lambda_t^j \, \mathbb{E}_{z^j}[\tilde{V}^j - V]) \} = 0 \qquad (14)$$

with

$$\sigma_t = \sqrt{n_{Bt}^2 \sigma_B^2 + (1 - n_{Bt})^2 \sigma_G^2 + 2 n_{Bt}(1 - n_{Bt}) \sigma_{GB}}, \qquad (15)$$

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[22]To derive the dynamic budget constraint we assume that the dividend yield for each risky asset is a constant $\beta > 0$. In equilibrium $\beta = \rho$, the intertemporal discount rate of the representative agent. This is the case due to the unitary EIS in the utility function (Weil 1990, Wachter 2013) A constant price-dividend ratio for the price of consumption claims is the case also in Barro (2006), and Tsai and Wachter (2015) when EIS $\rightarrow 1$.

$\sigma_{GB} \equiv \sigma_G \sigma_B \text{cov}[dW_G, dW_B]$, $\tilde{V}^j \equiv V(A(1 + nJ_j), \lambda^M, \lambda^E)$, for $j \in \{M, E\}$, the value function after the arrival of either disasters (or both together since they are uncorrelated), $J_j \in (-1, 0)$ defined in (13), $\bar{\lambda}_t^E = \tilde{\lambda}^E + \xi(\Lambda/\delta)\phi(N_t)n_{Bt}n_t A_t$, and $\bar{\lambda}_t^M = \bar{\lambda}^M$; V_x, V_{xx} denote, respectively, first and second derivative with respect to variable x.[23] The optimal portfolio is given by the first order conditions w.r.t to C, n, and n_B. Assuming an interior solution with both risky investments active we get:

$$f_C = V_A, \tag{16}$$

$$r = \mu + \beta + \frac{V_{\lambda^E}}{V_A}\frac{\kappa^E\xi}{\delta}\Lambda\phi(N)n_B + \frac{V_{AA}A}{V_A}n\sigma^2 + \sum_{j=\{M,E\}} \lambda^j \, \mathbb{E}_{z^j}\left[\frac{\tilde{V}_A^j}{V_A}J_j\right], \tag{17}$$

$$\mu_B + \frac{V_{AA}A}{V_A}nn_B\left(\sigma_B^2 - \sigma_{GB}\right) + \lambda^E \, \mathbb{E}_{z^E}\left[\frac{\tilde{V}_A^E}{V_A}\frac{dJ_E}{dn_B}\right] + \frac{V_{\lambda^E}}{V_A}\frac{\kappa^E\xi}{\delta}\Lambda\phi(N) =$$
$$\mu_G + \frac{V_{AA}A}{V_A}n(1 - n_B)\left(\sigma_G^2 - \sigma_{GB}\right). \tag{18}$$

Equation (16) is the usual envelope condition. When $\beta = \rho$, equation (17) is the Euler equation for the expected growth of the portfolio return; the Keynes-Ramsey rule. Note that the term $\frac{V_{\lambda^E}}{V_A}\frac{\kappa^E\xi}{\delta}\Lambda\phi(N)n_B < 0$ reflects the externality price of the portfolio, i.e. the Pigouvian tax. This externality cost is zero either in a completely green economy ($n_B = 0$), or when the *myopic* representative investor does not internalize the effects of his/her portfolio allocation on climate change. The last equation is a no-arbitrage condition between risky assets. It basically says that after adjusting for their relative risk and depreciation, each asset should yield the same marginal return. Relative risk for the brown asset includes the excess volatility $\sigma_B^2 - \sigma_{GB}$, in comparisson to the excess volatility of the general asset $\sigma_G^2 - \sigma_{GB}$, the risk of policy shocks (in dJ_E/dn_B), while when the effects of higher emissions on climate change are taken into account, the return on a marginal investment in brown assets is reduced by the pollution externality cost (again

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[23]From now on we will supress the time subscript t. However, unless otherwise indicated, all variables are time-dependent.

this is zero for the *myopic* investor).[24] We can easily verify that with equal drift and volatility parameters, and no excess policy risk for the brown asset, the myopic investor (no externality cost) would choose $n_B = 1/2$ as an interior solution.

3.4 The value function

For an analytical solution to exist we need to impose the following restrictions on the emissions intensity function $\phi(N)$ and on expenditure N: we assume that $\phi(N) = \varphi N^{-1}$ with $\varphi > 0$, and that N is a constant fraction of wealth, i.e. $N = \nu A$, $\nu \in (0,1)$.[25] In this model with $EIS = 1$ consumption is a constant fraction of wealth, $C = \beta A$, with $\beta = \rho$. Moreover, the value function takes the form

$$V(A, \lambda^M, \lambda^E) = \frac{A^{1-\gamma}}{1-\gamma} e^{a+b_M \lambda^M + b_E \lambda^E}, \tag{19}$$

with

$$a = \frac{1-\gamma}{\rho}\left(\mu - \frac{1}{2}\gamma\sigma^2 + \rho\log\rho\right) + \sum_{j\in\{M,E\}} b_j \frac{\kappa^j \bar\lambda^j}{\rho}, \tag{20}$$

and

$$b_j = \frac{\kappa^j + \rho}{(\sigma_\lambda^j)^2} - \sqrt{\left(\frac{\kappa^j+\rho}{(\sigma_\lambda^j)^2}\right)^2 - 2\frac{\mathbb{E}_{z^j}\left[(1+J_j)^{1-\gamma}-1\right]}{(\sigma_\lambda^j)^2}}. \tag{21}$$

For $\gamma > 1$, $b_j > 0$, $j \in \{M, E\}$.[26] Moreover, the fact that the quantities under the root of (21) have to be positive, places a joint restriction on the severity of disasters, the risk aversion, the rate of time preference, the volatility of disasters and the heat capacity of climate sinks. After our conjectures about $\phi(\cdot)$ and N, we have $\bar\lambda^E = \tilde\lambda^E + \xi(\Lambda/\delta)(\varphi/\nu)n_B$. Having derived the expressions for a, b_M and b_E of the value function, we can now calculate the return on the riskless asset and the market's risk premium for any n_B.

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[24] The term dJ_E/dn_B including the effect of environmental policy can be calculated as $dJ_E/dn_B = e^{Z^E}[e^{\pi X}(1 + n_B X'(n_B)\pi) - 1]$, with $X'(n_B) = 0$ if the stringency of the policy does not depend on the portfolio composition.

[25] This will indeed be the case on the BGP of a standard general equilibrium framework e.g. AK model, with an environmental externality coming from economic activity; e.g. Bretschger and Vinogradova (2018).

[26] See Wachter (2013), Tsai and Wachter (2015); proof can be also provided upon request.

3.5 The riskfree rate

Let the return on the riskless asset be the riskfree rate r.[27] In equilibrium $n = 1$. From (17), (19), and $\phi = \varphi/(\nu A)$, the rate of return on the riskless asset reads:

$$r = \underbrace{\rho + \mu - \gamma\sigma^2}_{\text{standard model}} - \underbrace{\frac{b_E}{\gamma - 1}\frac{\xi\varphi}{H\nu}\Lambda n_B}_{\text{externality cost}} + \sum_{j=\{M,E\}}\lambda^j \underbrace{\mathbb{E}_{z^j}\left[J_j(1 + J_j)^{-\gamma}\right]}_{\text{macroec. \& environ. risk}}, \tag{22}$$

with $b_E > 0$ from (21) and $J_j \in (-1, 0)$ from (13). In general, higher risk induces the representative agent to save which reduces the riskfree rate; the greater the risk aversion γ the greater is this effect. Volatility aside, the riskfree rate is decreasing in the time-varying disaster probabilities λ^j, and in the exposure to these disasters through Z^j. The effect of the general disaster risk on market fundamentals has been extensively studied in Wachter (2013). Here we concentrate on the effects of economic shocks related to climate change. Ceteris paribus, a higher share of brown capital n_B leads to greater aggregate portfolio emissions which affects the riskfree rate in the following ways: it reduces it by inducing a higher externality cost (second term), by increasing the probability of natural disasters λ^E and the exposure to environmental policy (which can be also affected if $X = X(n_B)$).

3.6 The stochastic discount factor (SDF)

Duffie and Skiadas (1994) show that the SDF – or state-price density – for preferences as given by (10) and (11) in continuous time is given by:

$$m_t = \exp\left[\int_0^t f_U(C_s, U_s)ds\right] f_C(C_t, U_t). \tag{23}$$

According to the value function (19), the Poisson jump of the SDF reads $\tilde{m}/m = (\tilde{A}/A)^{-\gamma}$. This fact and Itô's Lemma imply

$$\frac{dm}{m^-} = \mu_m dt + \sigma_m dW_m^T + J_m dY^T, \tag{24}$$

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[27]In the theoretical part we abuse terminology and use the term "riskfree" rate to refer to the return on government debt. In the numerical part, similar to Barro (2006), we will allow for the probability of government default due to severe macroeconomic disasters and use the average return on the post-WWII 3-month US Treasury bill as the relevant riskfree measure for calibration.

with

$$\sigma_m = [-\gamma n_B \sigma_B, -\gamma(1 - n_B)\sigma_G, b_M \sigma_\lambda^M \sqrt{\lambda^M}, b_E \sigma_\lambda^E \sqrt{\lambda^E}], \tag{25}$$

$$J_m = [(1 + J_M)^{-\gamma} - 1, (1 + J_E)^{-\gamma} - 1], \tag{26}$$

for $dW_m = [dW_B, dW_G, dW_\lambda^M, dW_\lambda^E]$, and $dY = [dY^M, dY^E]$. It also follows from no-arbitrage that

$$\mu_m = -r - \sum_{j \in \{M,E\}} \lambda^j \, \mathbb{E}_{z^j} \left[(1 + J_j)^{-\gamma} - 1 \right]. \tag{27}$$

Since $J_m > 0$, according to (24), in the event of a macroeconomic and environmental shock the SDF jumps upwards, increasing investor's required compensation for bearing risk. Ceteris paribus, a shift towards a greener portfolio would reduce the SDF due to the lower externality cost (inside r in (27)), but also due to the reduction in the frequency of the catastrophic events – and the subsequently triggered policy.

3.7   The aggregate market and the equity premium

Having calculated the state-price density we are now in position to calculate the risk premium for any asset with a given price process; see equation (4). Each investment with a dividend claim D, can be priced according to the usual asset pricing equation $P_t = \mathbb{E}_t \left[\int_t^\infty \frac{m_s}{m_t} D_s ds \right]$. It is somewhat standard in the literature to assume that the aggregate market pays a dividend D, being leveraged consumption, i.e. $D = C^\eta$; see for example Campbell (2003).[28] From $C = \rho A$ and Itô's Lemma it follows directly that

$$\frac{dD}{D^-} = \mu_D dt + \eta \sigma_A dW_A^T + J_D dY^T, \tag{28}$$

where

$$\mu_D = \eta \mu + \frac{1}{2} \eta(\eta - 1)\sigma^2, \tag{29}$$

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[28]Consistent with observations in the event of a negative shock, dividends fall more than consumption when $\eta > 1$ Longstaff and Piazzesi (2004).

and

$$J_D = [(1 + J_M)^\eta - 1, (1 + J_E)^\eta - 1]. \tag{30}$$

We can also show (see Seo and Wachter (2018)) that the price for D reads $P = DG(\lambda^M, \lambda^E)$ with:

$$G(\lambda_t^M, \lambda_t^E) = \int_0^\infty e^{a_\eta(\tau) + b_{\eta M}(\tau)\lambda_t^M + b_{\eta E}(\tau)\lambda_t^E} d\tau, \tag{31}$$

the market's price-dividend ratio, and $a_\eta(\tau), b_{\eta j}(\tau)$ solutions to the following system of differential equations:

$$a_\eta'(\tau) = \mu_D - \mu - \rho + (1 - \eta)\gamma\sigma^2 + \frac{b_E}{\gamma - 1}\frac{\xi\varphi}{H\nu}\Lambda n_B + \sum_{j \in \{M,E\}} \kappa^j \bar{\lambda}^j b_{\eta j}(\tau), \tag{32}$$

$$b_{\eta j}'(\tau) = \frac{1}{2}(\sigma_\lambda^j b_{\eta j}(\tau))^2 + (b_j(\sigma_\lambda^j)^2 - \kappa^j)b_{\eta j}(\tau) + \mathbb{E}_{z^j}\left[(1 + J_j)^{\eta - \gamma} - (1 + J_j)^{1 - \gamma}\right]. \tag{33}$$

Since for $\tau = 0$ an asset should pay its current dividend, boundary conditions are $a_\eta(0) = b_{\eta M}(0) = b_{\eta E}(0) = 0$. The solution to (33) with the previous boundary conditions yields $b_{\eta j}(\tau) < 0$, $j \in \{M, E\}$.[29] According to (31), this implies that, ceteris paribus, draws of high disaster risk – macroeconomic and/or environmental – reduce valuations. Itô's Lemma to $P = DG$ using (28) and (31) leads to the process for prices:

$$\frac{dP}{P^-} = \mu_P dt + \sigma_P dW_m^T + J_D dY^T, \tag{34}$$

with

$$\sigma_P = \left[\eta n_B \sigma_B, \eta(1 - n_B)\sigma_G, \frac{\partial G}{\partial \lambda^M}\frac{1}{G}\sigma_\lambda^M\sqrt{\lambda^M}, \frac{\partial G}{\partial \lambda^E}\frac{1}{G}\sigma_\lambda^E\sqrt{\lambda^E}\right]. \tag{35}$$

Variations in λ^j, $j \in \{M, E\}$ create variations in G and thus in stock prices, reflected by the third and fourth term of (35). As we discuss below, these terms will be of particular

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[29]Relevant to our results, for $\sigma_\lambda^j = 0$ the solution to (32) does not include $\bar{\lambda}^j$, i.e. valuations are not affected by the expectation on the probability of extreme events of type j.

importance in our setting with a stochastic probabilities of disasters. From (4) with (25), (26), (30), and (35), we get the equity premium for the aggregate market as:

$$R - r = \eta\gamma\sigma^2 - \underbrace{\sum_{j\in\{M,E\}} \overbrace{\frac{\partial G}{\partial \lambda^j}\frac{\lambda^j}{G}}^{\epsilon_j} b_j(\sigma_\lambda^j)^2}_{-\sigma_m\sigma_P^T} - \sum_{j\in\{M,E\}} \lambda^j\, \mathbb{E}_{z^j} \underbrace{\left[((1+J_j)^{-\gamma}-1)((1+J_j)^\eta-1)\right]}_{\left[(e^{Z_{mj}}-1)(e^{Z_{Pj}}-1)\right]},$$
(36)

with $G > 0$ and $\partial G/\partial\lambda^j < 0$, $j \in \{M, E\}$. The first two terms represent the correlated movement of the SDF with the price process in times without disasters, while the third term represents the same thing in the event of an economic shock – triggered either from rare macroeconomic disasters such as wars and financial crises, or from natural disasters; see also equation (4). The first term is the risk premium in the standard CAPM, while the second term arises from the stochastic nature of the time variation in disaster risk, and further increases the equity premium. The novelty of our model lies in establishing the link between emissions and the probability of extreme environmental disasters according to (8). The part of the sums above referring to environmental disasters is the part of the equity premium due to climate change risk. Also note that the externality cost would appear through the price-dividend ratio G, by further increasing required compensation through the second term; the more so, the higher the portfolio emissions.

Finally, of importance is the term $\epsilon_j \in [-1, 0]$ defined in (36). This term represents the risk elasticity of valuations, i.e. the variation in valuations in response to variations in macroeconomic and/or climate risk. According to (31) and (32), a change in the expectation of either risk factors $\bar{\lambda}^j$, $j \in \{M, E\}$, changes the risk elasticity of valuations for both types of risk. By setting $\sigma_\lambda^E = 0$ we get the result of Bansal et al. (2016) that increasing emissions and temperature anomaly increases the likelihood of extreme environmental events $\bar{\lambda}^E$, which unambiguously raises the equity premium on the aggregate market. We now turn to numerical simulations of our model.

4 Numerical part

4.1 Methodology

The current framework can be used for two types of modelling depending on investors information about climate change. In the first case investors take emissions as given and not affected by their portfolio choices. Exogenous carbon emission / temperature paths influence the mean arrival rate of environmental shocks $\bar{\lambda}^E$, while the participation of dirty assets in the portfolio n_B depends only on the additional policy risk these assets are exposed to (see equation (18)); the Pigouvian tax is set to zero throughout. For the different emission paths we consider the Representative Concentration Pathways (RCPs) produced by the IPCC for years 2010 to 2100 (van Vuuren et al. 2011); see figure 3. We call this the *myopic* solution. As climate changes, this solution allows us to study the possible evolution of the different market measures and the portfolio participation of brown assets. In the second case investors recognize that their portfolio choices will affect future emissions and, therefore, the probability of climate-related events. Accordingly, the portfolio allocation is fully optimized in each period and thus the arrival rate of extreme environmental events becomes endogenous. We call this solution *optimal*.

The usual methodology in asset pricing models assumes time-invariability of the system under study. This allows for a straightforward Monte-Carlo simulation using a large number of random realizations of the relevant stochastic variables. In the myopic case, however, the mean of the process for λ^E changes with emissions, such that our model is not time-invariant; see equation (8). We circumvent this issue by assuming that our model is time-invariant for each given level of emissions E_t and simulate the model for each $\bar{\lambda}_t^E = \bar{\lambda}^E(E_t)$. Figure 4 presents our methodology for the myopic case. We divide our actual time horizon (2010-2100) in decades, and run the model for 50,000 simulation years for each decade, with a monthly granularity. Figure 4 presents also the resulting stationary Gamma distribution for λ^E in each time period, which changes as expected; see footnote (18).

In the optimal solution, our target is to identify a policy function linking our control variable n_B to the states of the economy, i.e. A, λ^M, and λ^E. According to equation (18) and our conjectures regarding functional forms, however, n_B only depends on λ^E and on fixed parameters. For that we generate a sequence of λ^E's, solve (18) for each of them, and

fit a $n_B(\lambda^E)$ curve. The resulting linear $n_B(\lambda^E)$ relationship is our policy function. Carbon emissions are not exogenous anymore, but depend on chosen $n_B(\lambda^E)$, from $E_t = (\varphi/\nu)n_{Bt}$. Equation (8) now becomes time invariant and allows for a straighforward simulation. We then run the optimal solution for 50'000 simulation years at a monthly frequency.

Figure 3: IPCC projected carbon emissions
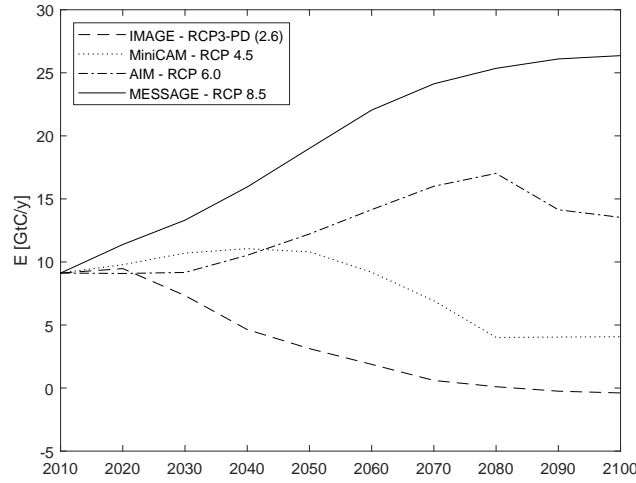
4.2   Calibration

We measure time in years and calibrate the model to match observed historical market and climate data. Our initial time period is 2010. Carbon emissions are measured in GtC (1 gigaton C= 10^9 ton C) and temperature differences in K (Kelvin); also 1 PW = 10^{15} Watts. Our benchmark is the case without policy risk, i.e. $X = 0$ and no Pigouvian tax. Below we demonstrate our benchmark calibration; Table 1 collects the chosen parameter values.

4.2.1   Environmental parameters

For most of the environmental parameters we follow Brock and Xepapadeas (2017). Accordingly, we set the stabilizing parameter of outgoing radiation to $\delta = 0.1$ PW/K and the

Figure 4: Schematic of the simulation methodology for the *myopic* case. For each actual time period, we run the model for 50,000 simulation years at a monthly frequency; $\bar{\lambda}^E$ changes with carbon emissions E_t. The figure assumes an increasing path of emissions.
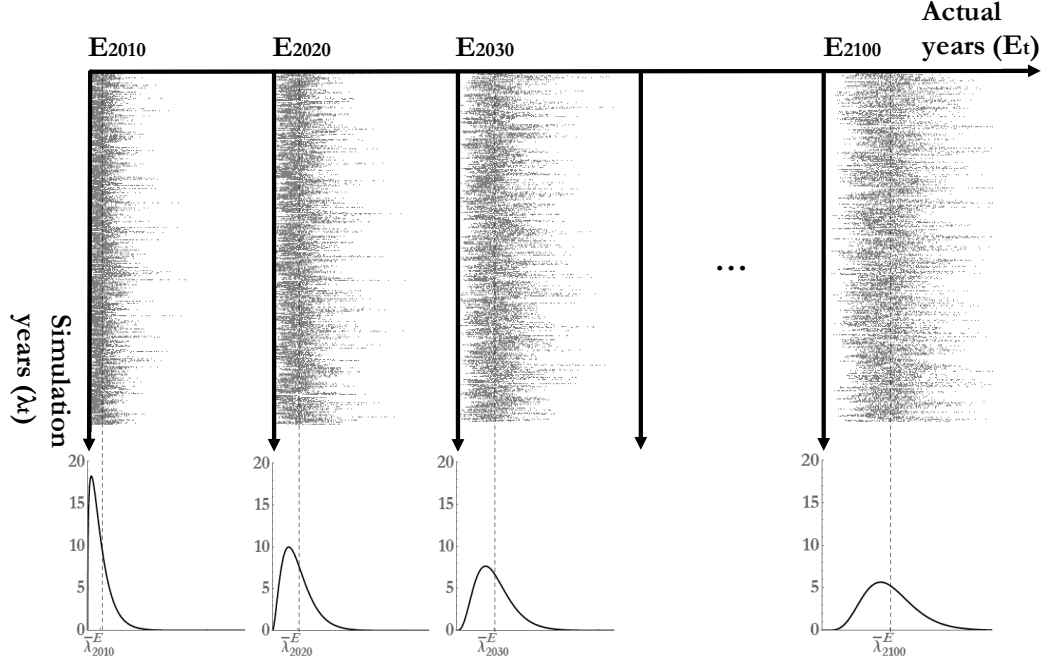
heat capacity of climate sinks to $H = 4.58$ PW y/K. The positive feedback of carbon emissions on increases in temperature anomaly is measured by the transient climate response with a mean value $\bar{\Lambda} = 0.0017$ K/GtC; see also Leduc et al. (2016). Then, the relevant climate sensitivity parameter in our model is $\Lambda \equiv \bar{\Lambda}H = 0.00779$ PW y/GtC.

4.2.2 Distributions of macroeconomic and environmental disasters

As shown in equation (13), the percentage decline in total wealth per capita J_A features both environmental and macroeconomic shocks that need to be calibrated to the data. Hence, we need to construct a separate dataset for each of the two types of shocks; from these datasets we can then calculate the distribution of percentage drops J_j, as well as the average of the Poisson intensities $\bar{\lambda}_j$, $j \in \{M, E\}$. To do so we make use of different data sources as follows.

Table 1: Parameters for the benchmark calibration

All values are in annual terms

Environmental parameters	
Outgoing radiation parameter δ (PW/K)	0.1
Heat capacity of climate sinks H (PW y/K)	4.58
Climate sensitivity Λ (PW y/GtC)	0.00779
Emissions intensity φ	1
Expenditure share for environ. efficiency of the brown asset ν	0.0329
Parameters for the stochastic processes	
Average probability of macroeconomic disasters $\bar{\lambda}^M$	0.0375
Average probability of environ. disasters for zero emissions $\tilde{\lambda}^E$	0.003
Slope of the linear $\bar{\lambda}^E(T)$ curve ξ	0.096
Mean reversion for macroeconomic disasters κ^M	0.080
Mean reversion for environmental disasters κ^E	0.0218
Volatility parameter for macroeconomic disasters σ_λ^M	0.0684
Volatility parameter for environmental disasters σ_λ^E	0.120
Drift for the gereral asset μ_G	0.025
Drift for the brown asset μ_B	0.0256
Volatility parameter for the general asset σ_G	0.0176
Volatility parameter for the brown asset σ_B	0.0259
Probality of policy reaction to extreme climatic events π	0.6
Utility parameters	
Relative risk aversion γ	3.5
Intertemporal discount rate ρ	0.011

As a first source we extend until 2015 the Barro-Ursúa dataset, Barro and Ursúa (2010), that collects consistent data on GDP per capita growth for 42 countries for the period 1911-2008.[30] For our purposes this dataset holds the real reported GDP per capita growth, i.e. after accounting of any (negative) growth effects of climate-related events. In order to calculate these growth effects of climate change we act in the following way. We first collect from the international disasters database EM-DAT (2018), all climate-related events for these 42 countries and for the 1911-2015 time period; we consider only events relevant to climate change.[31] We then follow the methodology of Loayza et al. (2012) and calculate the negative growth effects on GDP per capita of extreme environmental events (top 10% in each event category according to a severity index defined in that paper) for each country and each year; from these we keep extreme events that resulted in GDP growth damages

[30]We use percentage declines in GDP per capita, instead of consumption per capita, as a proxy for damages. Both Barro (2009) and Wachter (2013) find similar results for their CAPMs with rare disasters whether they calibrate to the consumption or GDP data. Here we follow the latter.

[31]According to the EM-DAT categories we consider meteorological events (storms/extreme temperatures), hydrological events (floods/avalanches), and climatological events (droughts/wildfires).

of more than 1%.[32] This is our first dataset including data on environmental damages. To calculate pure macroeconomic damages we add the – absolute of – environmental damages to growth entries of the extended Barro-Ursúa dataset. This yields the real GDP per capita growth had no extreme climate-related events occured. To construct our second dataset containing pure macroeconomic damages we then follow the peak-to-trough methodology for cumulative fractional declines in real GDP per capita as explained in Barro and Ursúa (2008). As in the aforementioned contribution, and in Wachter (2013), we include only peak-to-trough events that resulted in GDP drops more than 10%.[33]

Following the methodology of Barro and Ursúa (2008) the frequency of large declines in GDP per capita in our pure macroeconomic dataset yields $\bar{\lambda}^M = 0.0375$. In order to construct the linear relationship $\bar{\lambda}^E_t = \tilde{\lambda}^E + \xi(\Lambda/\delta)E_t$ for the time-varying mean of the stochastic process in (8), we divide our sample in six decades starting from the decade 1956-1965 (when the first indications of climate change became evident) and calculate $\bar{\lambda}^E$ for each decade. The fitted line, with Λ and δ from above, gives $\tilde{\lambda}^E = 0.003$ and $\xi = 0.096$, implying $\bar{\lambda}^E_{2010} = 0.071$ (global carbon emissions in 2010, amounted to $E_{2010} = 9.12$ GtC). The frequency distributions of growth damages have a mean drop size of 22% for macroeconomic events and 1.5% for environmental.[34]

4.2.3 Other parameters

We define as "brown" assets with exposure to transition risk and follow Prudential Regulation Authority (2015) to set $n_B = 0.3$ for 2010; see footnote 8.[35] We also normalize values by setting the intensity parameter $\varphi = 1$, and using $E_{2010} = \varphi n_{B,2010}/\nu$ we calculate $\nu = 0.0329$. In order to calculate the probability π that climate change policy becomes effective after an extreme climate-related event, we use the Grantham-LSE (2018) database that includes since the 1960s all laws and legislations related to climate change, covering 95% of global emissions. In this database there are in total 519 laws for our 42 countries,

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[32]Loayza et al. (2012) show that extreme climate-related events (top 10%) are always bad for economic growth and calculate the growth elasticities of different event types on different economic sectors: manufacturing; services; agriculture. Using World Bank data we calculate the sectoral shares of GDP for each country and then using the growth elasticities the country-specific climate-related damages in terms of GDP per capita growth for each year.

[33]Using the peak-to-trough methodology for macroeconomic, and not for environmental events, we implicitly make the logical assumption that macroeconomic events, such as wars or crises, have memory, while climate-related events are memory-less.

[34]The value of the mean drop size of macroeconomic events is in line with Barro and Ursúa (2008) who calculate a value of 20.7% for GDP disasters using a dataset of 36 countries in the time period 1870-2006; with their dataset they also calculate their $\lambda^M = 0.0369$. Our results in terms of macroeconomic disasters differ slightly due to the different time period, the additional countries, but also due to the dissentangling of GDP damages in macroeconomic and environmental.

[35]In addition, Oestreich and Tsiakas (2015) investigate empirically the effect of EU-ETS on German stock returns in the period 2003-2012. They divide their sample of 65 firms in clean and dirty depending on whether they received free carbon allowances or not; dirty firms occupy about 35% of this sample.

a quarter of which refers to low carbon transition laws (Nachmany et al. 2017); with 213 severe events in our dataset we calculate $\pi = 0.25 \times 519/213 \approx 0.6$.

In line with the literature, we set the coefficient of relative risk aversion to $\gamma = 3.5$.[36] The intertemporal discount rate is set to $\rho = 0.011$, such that the simulated return on government debt comes close to the post-WWII 3-month US Treasury bill. We also follow Wachter (2013) and set the aggregate drift and the volatility parameters of the market to $\mu = 0.0252$ and $\sigma = 0.02$ per annum, respectively; see equations (12) and (15). We further assume for simplicity that the two risky assets are perfectly correlated in normal times such that $\sigma_{GB} \equiv \sigma_G \sigma_B$. Then using the no-arbitrage condition (18), and the definitions of μ and σ, we choose $\mu_B = 0.0256$, $\mu_G = 0.025$, $\sigma_B = 0.0259$ and $\sigma_G = 0.0173$, that yield in the benchmark about $n_B = 0.3$, $\mu = 0.0252$, and $\sigma = 0.02$.

Finally, we need to calibrate the volatility parameters σ_λ^M and σ_λ^E for processes (5) and (8), respectively, and the mean reversion parameter of the intensity of macroeconomic disasters κ^M (the speed of adjustment for environmental disasters can be calculated as $\kappa^E = \delta/H = 0.0218$ per annum). Volatility parameters can be calculated by choosing the discriminant of (21) for both types disasters to be zero (as in Seo and Wachter (2018)). With κ^E from above this yields $\sigma_\lambda^M = 0.0684$ and $\sigma_\lambda^E = 0.120$ for $X = 0$. We calibrate $\kappa^M = 0.080$ so that the autocorrelation of the benchmark price-dividend ratio matches the value in the data (0.92).

4.3 Simulation results - myopic solution

4.3.1 Benchmark simulation

As discussed above we calibrated the model to match historical data. Our benchmark calibration involves only physical risk i.e. $X = 0$, while emission parameters are calibrated to 2010 data. Additional to the theoretical part, in line with Barro (2006) and Wachter (2013), we assume that macroeconomic disasters trigger a default on government debt with probability $q = 0.4$. Table 2 presents the results of our benchmark simulation in contrast to historical post-WWII US data. Our model and its calibration matches observed moments of interest very well: the equity premium generated (over the return on government debt) is 7.3% p.a. in comparison to 7.1% p.a. observed in the data, while simulated equity volatility is 19.1% p.a., in comparison to observed 17.8% p.a. Next we discuss the effects of climate change on model's moments.

4.3.2 The physical risk of climate change

In this part we ask ourselves about the pure effects of climate change risk on market fundamentals, with a focus on the equity premium. We simulate the model as described

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[36]Barro (2009) sets $\gamma = 4$, while Wachter (2013) $\gamma = 3$.

Table 2: Moments from simulated vs. historical data. R^b is the return on government debt; R^e denotes the gross return on equity; $AR1[P - D]$ is the first order autocorrelation of the price-dividend ratio; SR measures the Sharpe ratio.

All values are in annual terms

Moments	Simulation	US Data (1947-2010)
$\mathbb{E}[R^b]$	0.0138	0.0134 (3-month Treasury bill)
$\sigma(R^b)$	0.0313	0.0266
$\mathbb{E}[R^e - R^b]$	0.0733	0.0706
$\sigma(R^e)$	0.191	0.178
$AR1[P - D]$	0.923	0.920
SR	0.384	0.397

in figure 4. Figure 5 presents the effect of the two extreme RCP scenarios (RCP3 and RCP8.5) on the risk premium. Surprisingly, the risk premium on the aggregate market does not change much with emissions.

Is it because the effect of climate change is negligible in comparison to the effect of severe pure macroeconomic disasters? From (36) we can get the part of the equity premium that is solely due to climate change risk. As figure 6 shows, from the generated equity premium of 7.3% p.a. in the initial period, about 0.2% p.a. can be attributed to climate change risk; the remaining 7.1% is mainly due to the risk of rare macroeconomic disasters, and only a very small part is due to the standard CAPM's diffusion risk. With our calibration, this premium increases to 0.5% p.a. by the end of the century in the worst case scenario, while it naturally ceases to exist in the RCP3 scenario where emissions fall to zero in the long run. Since climate change carries a positive premium, which is also increasing with emissions, what is the reason behind the generated constant equity premium on the aggregate market?

According to (31) and (32), higher emissions, that increase $\bar{\lambda}^E$, affect the way valuations react to the different kinds of risk. With our calibration, higher emissions reduce the magnitude of the risk elasticity of valuations for macroeconomic disasters ϵ^M in (36), while they increase the one for environmental ϵ^E. Therefore, increasing emissions, change the relative importance of the two sources of risk in normal times. However, since the premium due to macroeconomic disasters is greater, this holds the overall equity premium constant; see figure (7). Setting σ_λ^j, $j \in \{M, E\}$, to zero predicts the result of Bansal et al. (2016) that increasing emissions unambiguously reduce valuations and increase the equity premium. Finally, as expected from our theory, higher emissions unambiguously decrease the return on government debt; see figure 8.

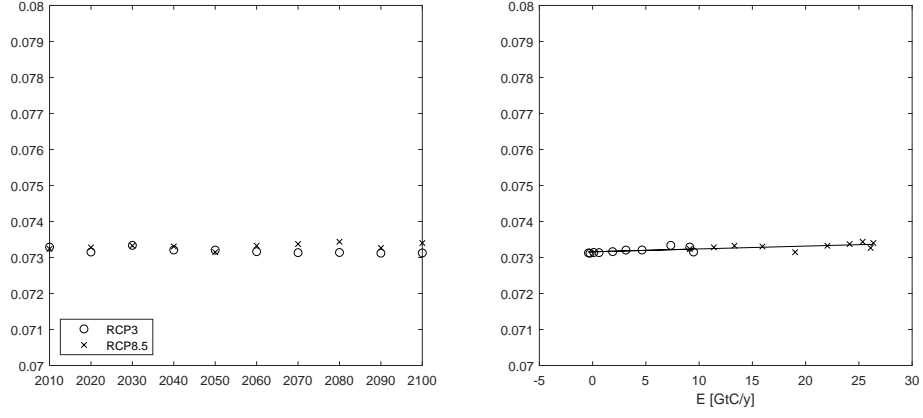Figure 5: The equity premium over the gross return on government debt (annual terms)

Figure 6: The premium of climate change risk (annual terms)
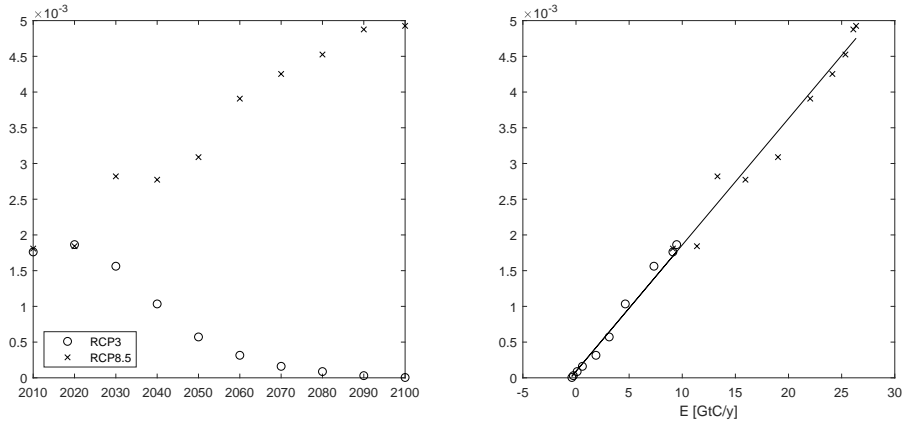

26

Figure 7: Elasticities of valuation for macroeconomic and environmental risks, $\epsilon^j = \frac{\partial G}{\partial \lambda^j} \frac{\lambda^j}{G}$
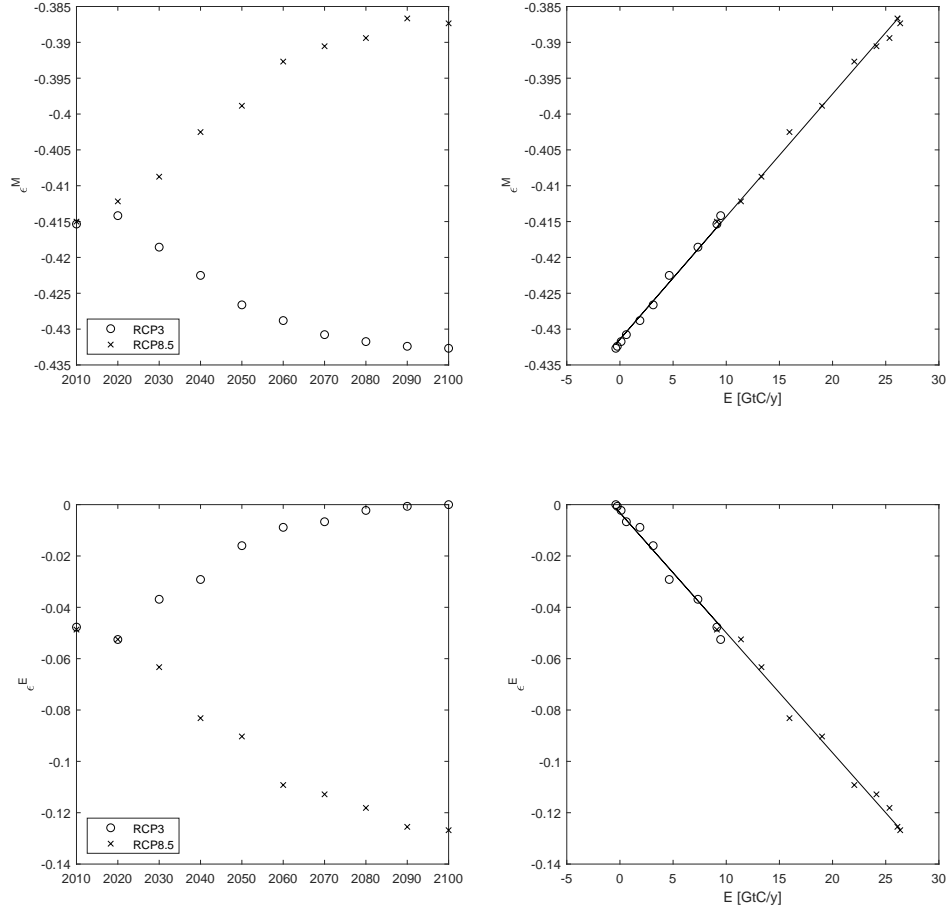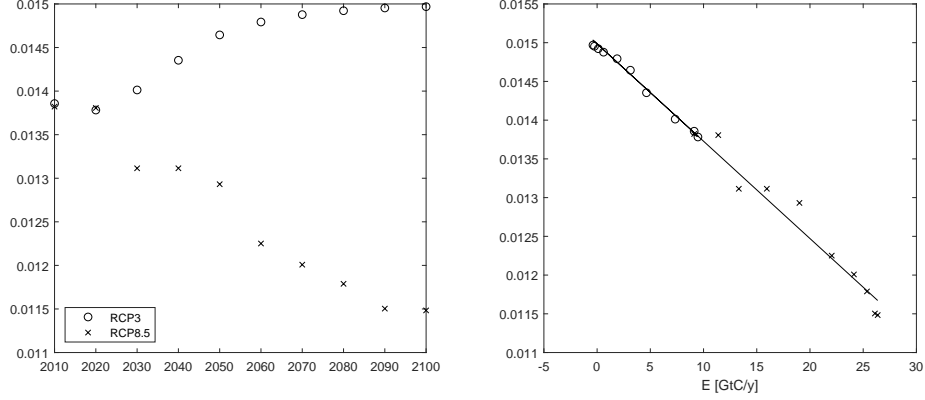
Figure 8: The effect of emissions on government bond yield (annual terms)

4.3.3 Policy risk and portfolio participation of brown assets

Our benchmark calibration assumes that there is no additional policy risk on brown assets, the share of which we calibrate to $n_B = 0.3$. In this paragraph we relax this by assuming the existence of abnormal returns on the announcement of green policies.[37] Ramiah et al. (2013) study the existence of such returns in Australia and document negative mean abnormal returns in the order of -2.5% for 10 industries, including mining, oil, gas, and real estate. Using this information and equations (9), we can calibrate our policy risk parameter with a mean drop size of $\bar{Z}^E = -1.46\%$ for environmental disasters to $X = -0.0006$.

In our myopic simulation, as the probability of extreme environmental events changes with emissions, investors optimally reallocate their portfolio by choosing n_B in each time period according to (18). Figure 9 presents the simulated portfolio participation of brown assets for the two extreme RCP scenarios, along with their "educated guesses", i.e. ones that would result if we solved (18) in each case with $\bar{\lambda}^E(E)$; we call these RCP bar. This graph holds two important pieces of information. First, according to our calibration, including policy risk leads to excess portfolio participation of brown assets of about 10% already in 2010, and 12-13% in 2020, whichever scenario we choose. In the worst case scenario, the portfolio should get decarbonized by 2100. In the best case scenario, where no global emissions are expected in the long-run, and therefore the risk of climate-related events is minimal, the portfolio participation of brown assets reaches benchmark levels. Second, there is a clear negative and linear relationship between n_B and emissions. The linearity of the $n_B(\lambda^E)$ curve will prove useful for the guess of the policy function in the optimal solution that follows.

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[37]Abnormal returns measure the performance difference between specific stocks or portfolios and the aggregate market on given dates or time periods.

Figure 9: Portfolio participation of brown assets for the two extreme emissions scenarios
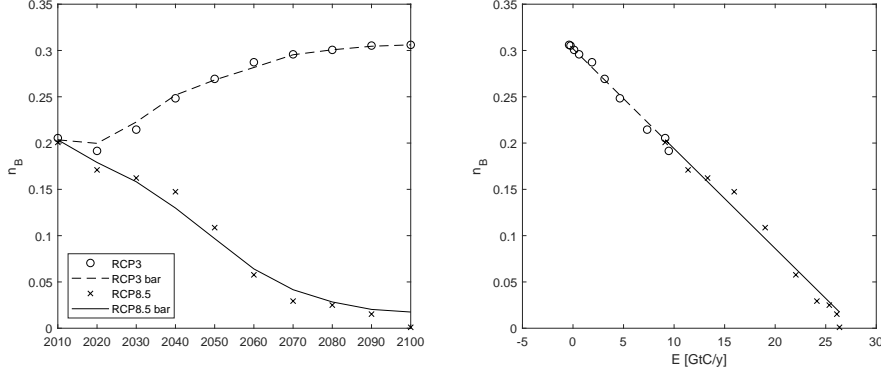
4.3.4 Simulation results - optimal solution

In the optimal solution the representative investor aknowledges the fact that higher portfolio emissions have a negative feedback on the economy, raising the risk of severe climate-related disasters and the subsequent policy they trigger. For our calibration, and with $X = -0.0006$, we generate a sequence of state variables λ^E's and solve (18) for each one. We then fit the resulting $\{\lambda^E, n_B\}$ in a linear $n_B(\lambda^E)$ curve. Using this information we run a Monte-Carlo simulation of equation (8) with $\bar{\lambda}_t^E = \tilde{\lambda}^E + \xi(\Lambda/\delta)(\varphi/\nu)n_B(\lambda_t^E)$ and solve back (18) for each of them. This results to $n_B = 0.237$, meaning that the optimal solution with policy risk (but no Pigouvian tax) suggests an overexposure of about 0.063 of brown assets in comparison to the benchmark ($n_B = 0.3$). When we include the Pigouvian tax term in (18), $n_B = 0$, i.e. including the externality cost of climate change allows for no brown assets in the optimal portfolio.[38]

5 Conclusion

There are concerns from market participants that the risk of climate change is not yet perfectly priced by capital markets. In order to price climate change risk we develop an asset pricing model with rare events and time-varying probabilities. Such models are shown to match observed equity premia for low values of relative risk aversion. In addition to the – already considered – risk of macroeconomic disasters like wars and financial crises, we include the risk of extreme adverse environmental events related to climate change. Our main contribution lies in establishing the link between carbon emissions and the stochastically-varying risk of rare climatic events. Besides the physical risk, that affects the whole market,

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[38]In fact the resulting n_B when we include the Pigouvian tax is negative. However, we do not allow for short-selling of assets in this model, and thus choose $n_B = 0$.

we include the transition risk of climate change, i.e. the risk of exposure to stringent environmental policies that lower the returns of carbon-intensive assets.

We confirm the result in the literature that climate change entails a positive and increasing risk premium. We show, however, that whether this ultimately carries over to the overall equity premium depends on the volatility of the stochastic process that governs climate change risk. When predictions of the probability of extreme environmental events are not perfect, this affects the way valuations react to both risk sources, namely macroeconomic and environmental; with increasing emissions, the relative weight shifts from macroeconomic to environmental events which keeps the aggregate equity premium relatively constant. We consider different emission projections and find that including the transition risk of environmental policies lowers substantially the participation of carbon-intensive assets in the market portfolio. In the worst case emissions scenario, the market portfolio should get fully de-carbonized by the end of the century. In the planner's solution, that includes the Pigouvian tax of the environmental externality, there is no room for any carbon-intensive assets in the optimal portfolio.

References

Annicchiarico, B. and Di Dio, F.: 2017, Ghg emissions control and monetary policy, *Environmental and Resource Economics* **67**(4), 823–851.

Bansal, R., Kiku, D. and Ochoa, M.: 2016, Price of long-run temperature shifts in capital markets, *Working paper*, National Bureau of Economic Research.

Barro, R. J.: 2006, Rare disasters and asset markets in the twentieth century, *The Quarterly Journal of Economics* **121**(3), 823–866.

Barro, R. J.: 2009, Rare disasters, asset prices, and welfare costs, *American Economic Review* **99**(1), 243–64.

Barro, R. J. and Ursúa, J. F.: 2008, Macroeconomic crises since 1870, *Brookings Papers on Economic Activity* **2008**, 255–335.

Barro, R. J. and Ursúa, J. F.: 2010, Barro-ursua macroeconomic data.
   URL: *https://scholar.harvard.edu/barro/publications/barro-ursua-macroeconomic-data*

Batten S., Sowerbutts R., T. M.: 2016, Let's talk about the weather: the impact of climate change on central banks, *Technical report*, Bank of England.

Battiston, S., Mandel, A., Monasterolo, I., Schtze, F. and Visentin, G.: 2017, A climate stress-test of the financial system, *Nature - Climate Change* **7**, 283288.

Blackrock Investment Institute: 2016, Global warming's impact on portfolios, *Technical report*.

Bretschger, L. and Karydas, C.: 2018, Optimum growth and carbon policies with lags in the climate system, *Environmental and Resource Economics* **70**(4), 781–806.

Bretschger, L. and Soretz, S.: 2018, Stranded assets: How policy uncertainty affects capital, growth, and the environment, *Technical report*, Economics Working Papers Series 18/288, ETH Zurich.

Bretschger, L. and Vinogradova, A.: 2018, Best policy response to environmental shocks: Applying a stochastic framework, *Journal of Environmental Economics and Management* .

Brock, W. and Xepapadeas, A.: 2017, Climate change policy under polar amplification, *European Economic Review* **94**.

Cai, W., Borlace, S., Lengaigne, M., Van Rensch, P., Collins, M., Vecchi, G., Timmermann, A., Santoso, A., McPhaden, M. J., Wu, L. et al.: 2014, Increasing frequency of extreme el niño events due to greenhouse warming, *Nature climate change* **4**(2), 111.

Campbell, J. Y.: 2003, Chapter 13 consumption-based asset pricing, *Financial Markets and Asset Pricing*, Vol. 1 of *Handbook of the Economics of Finance*, Elsevier, pp. 803 – 887.

Campiglio, E., Dafermos, Y., Monnin, P., Ryan-Collins, J. and Guido Schotten, M. T.: 2018, Climate change challenges for central banks and financial regulators, *Nature - Climate Change* **8**, 462468.

Carney, M.: 2015, Breaking the tragedy of the horizon - climate change and financial stability, Speech in Lloyd's of London.

Cochrane, J. H.: 2005, *Asset Pricing*, Princeton University Press.

Cox, J. C., Ingersoll, J. E. and Ross, S. A.: 1985, A theory of the term structure of interest rates, *Econometrica* **53**(2), 385–407.

Dietz, S., Gollier, C. and Kessler, L.: 2018, The climate beta, *Journal of Environmental Economics and Management* **87**, 258 – 274.

Donadelli, M., Jppner, M., Riedel, M. and Schlag, C.: 2017, Temperature shocks and welfare costs, *Journal of Economic Dynamics and Control* **82**, 331 – 355.

Duffie, D. and Epstein, L. G.: 1992, Asset pricing with stochastic differential utility, *The Review of Financial Studies* **5**(3), 411–436.

Duffie, D. and Skiadas, C.: 1994, Continuous-time security pricing: A utility gradient approach, *Journal of Mathematical Economics* **23**(2), 107 – 131.

Economides, G. and Xepapadeas, A.: 2018, Monetary policy under climate change, *Technical report*, Working Papers 247, Bank of Greece.

EM-DAT: 2018, The international disasters database.
URL: *https://www.emdat.be/*

Francis, J. A. and Vavrus, S. J.: 2012, Evidence linking arctic amplification to extreme weather in mid-latitudes, *Geophysical Research Letters* **39**(6).

Grantham-LSE: 2018, Climate change laws of the world database, *LSE - Grantham Research Institute on Climate Change and the Environment* .

Hassler, J., Krusell, P. and Smith, A.: 2016, Chapter 24 - environmental macroeconomics, Vol. 2 of *Handbook of Macroeconomics*, Elsevier, pp. 1893 – 2008.

Heutel, G., Moreno-Cruz, J. and Shayegh, S.: 2016, Climate tipping points and solar geoengineering, *Journal of Economic Behavior & Organization* **132**, 19 – 45.

IPCC: 2018, Global warming of 1.5c. an ipcc special report on the impacts of global warming of 1.5c above pre-industrial levels and related global greenhouse gas emission pathways, in the context of strengthening the global response to the threat of climate change, sustainable development, and efforts to eradicate poverty, *Technical report*, IPCC, Geneva, Switzerland.

Knutti, R.: 2013, Relationship between global emissions and global temperature rise, *UNFCCC* .
URL: *https://https://unfccc.int/sites/default/files/7_knutti.reto.3sed2.pdf*

Knutti, R. and Rogelj, J.: 2015, The legacy of our co2 emissions: a clash of scientific facts, politics and ethics, *Climate Change* **133**(3), 361–373.

Leduc, M., Damon Matthews, H. and De Ela, R.: 2016, Regional estimates of the transient climate response to cumulative co 2 emissions, *Nature - Climate Change* **6**(5), 474–478.

Lemoine, D. and Rudik, I.: 2015, Steering the climate system: Using inertia to lower the cost of policy, *Technical report*, Iowa State University.

Loayza, N. V., Olaberra, E., Rigolini, J. and Christiaensen, L.: 2012, Natural disasters and growth: Going beyond the averages, *World Development* **40**(7), 1317 – 1336.

Longstaff, F. A. and Piazzesi, M.: 2004, Corporate earnings and the equity premium, *Journal of Financial Economics* **74**(3), 401 – 421.

MacDougall, A. H., Swart, N. C. and Knutti, R.: 2017, The uncertainty in the transient climate response to cumulative co2 emissions arising from the uncertainty in physical climate parameters, *Journal of Climate* **30**(2), 813–827.

Matthews, H. D., Gillett, N., Stott, P. and Zickfeld, K.: 2009, The proportionality of global warming to cumulative carbon emissions, *Nature* **459**, 829–833.

Matthews, H., Solomon, S. and Pierrehumbert, R.: 2012, Cumulative carbon as a policy framework for achieving climate stabilization, *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences* **370**, 4365–4379.

Merton, R. C.: 1976, Option pricing when underlying stock returns are discontinuous, *Journal of Financial Economics* **3**(1), 125 – 144.

Monnin, P.: 2018, Central banks should reflect climate risks in monetary policy operations, *Policy note*, SUERF the European Money and Finance Forum.

Nachmany, M., Fankhauser, S., Setzer, J. and Averchenkova, A.: 2017, Global trends in climate change legislation and litigation, *Technical report*, LSE - Grantham Research Institute on Climate Change and the Environment.

Nævdal, E. and Oppenheimer, M.: 2007, The economics of the thermohaline circulationa problem with multiple thresholds of unknown locations, *Resource and Energy Economics* **29**(4), 262 – 283.

Oestreich, A. M. and Tsiakas, I.: 2015, Carbon emissions and stock returns: Evidence from the eu emissions trading scheme, *Journal of Banking & Finance* **58**, 294 – 308.

Prudential Regulation Authority: 2015, The impact of climate change on the uk insurance sector, *Technical report*, Bank of England.

Prudential Regulation Authority: 2018, Transition in thinking: The impact of climate change on the uk banking sector, *Technical report*, Bank of England.

Ramiah, V., Martin, B. and Moosa, I.: 2013, How does the stock market react to the announcement of green policies?, *Journal of Banking & Finance* **37**(5), 1747 – 1758.

Seo, S. B. and Wachter, J. A.: 2018, Do rare events explain cdx tranche spreads?, *The Journal of Finance* **73**(5), 2343–2383.

Sevillano, M. and González, R.: 2018, The risk of climate change for financial markets and institutions: challenges, measures adopted and international initiatives, *Technical report*, Bank of Spain Financial Stability Review.

Stolbova, V., Monasterolo, I. and Battiston, S.: 2018, A financial macro-network approach to climate policy evaluation, *Ecological Economics* **149**, 239 – 253.

Strauss, D.: Financial Times, 05/12/2018, Sustainable finance: central banks test water on climate risks, *Financial Times* .

Tsai, J. and Wachter, J. A.: 2015, Disaster risk and its implications for asset pricing, *Annual Review of Financial Economics* **7**(1), 219–252.

USGCRP: 2018, Impacts, risks, and adaptation in the united states: Fourth national climate assessment volume ii, *Technical report*, U.S. Global Change Research Program.

van Vuuren et al.: 2011, The representative concentration pathways: an overview, *Climatic Change* .

Wachter, J. A.: 2013, Can time-varying risk of rare disasters explain aggregate stock market volatility?, *The Journal of Finance* **68**(3), 987–1035.

Weil, P.: 1990, Nonexpected utility in macroeconomics, *The Quarterly Journal of Economics* **105**(1), 29–42.

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