Performance of FOPI with Error filter Based on Controllers Performance Criterion (ISE, IAE and ITAE)

Mohd Hezri Marzaki#1, Mazidah Tajjudin #2, Mohd Hezri Fazalul Rahiman#3, Ramli Adnan#4
#Frontier Materials and Industry Application, UiTM-RMI-CoRe FMIA
#Faculty of Electrical Engineering, Universiti Teknologi MARA, Shah Alam, Selangor
1#mohdhezri.marzaki.my@ieee.org

***Abstract*— Generalization of Fractional-order PID or $PI^{\lambda}D^{\mu}$ have more parameters compared to standard PID structure. In standard PID structure, it only consist of three parameters such as Proportional (P), Integral (I) and Derivative (D) term where in $PI^{\lambda}D^{\mu}$ it consists of two more adjustable parameters which are λ and μ. Consequently, it has more degree of freedom than PID and may achieve better response relatively. This paper presents performance evaluation of Fractional-order PI with error filter using three performance criteria such as Integral of Square Error (ISE), Integral of Absolute Error (IAE) and Time-weighted Integral of Absolute Error (ITAE). The controller is evaluated based on simulation and real-time study performed on a Small-Medium Industry Steam Distillation Plant (SMISD) for steam temperature control.**

***Index Terms*—Fractional-order PI Controller; Real-Time; Controller performance index.**

I. Introduction

Fractional-order calculus have been introduced in 17th century in the event of letter exchange between two most influence mathematical Leibniz and Guillaume de L'Hopital. Since then, Fractional-order calculus subject become hot topics among top notch mathematical such as Fourier, Euler, Laplace, Lagrange [1] and many more, each with their own notation and methodology which definition fit general concept of fractional calculus. The most well-known definitions are Riemann-Liouville (RL) and Grunwald-Letnikov (GL) definitions.

However from an application perspective, it fall into oblivion for many years due to lack of an acceptable geometric and physical interpretation [2]. Some shine in this area, after Podlubny purpose convincing interpretation of Riemann-Liouville definition [3]. Then later, he demonstrated the effectiveness of implementation of Fractional-order generalized $PI^{\lambda}D^{\mu}$ controller [4]. Since then, many control engineering applications were found utilizing Fractional-order system.

Current literature reviewed on control engineering applications show increment application in Fractional-order systems such as liquid level control [5], hydraulic servo system [6], robust motion control [7], hydro power plant [8], speed control of DC motor [9] and many more.

Proportional - Integral - Derivative (PID) controller is the most recognize controllers for several decades, as more than 97% from 11000 industries used PI(D) controller algorithm to control their process [10]. PID had been appreciated of its simplicity, low cost and can produce good transient response have made it favorable in most control applications [11, 12]. Owing to PID controller characteristics, continuous effort have been made to upscale their performance.

The most significant and attractive effort to enhance the PID controller performance is the introduction of Fractional-order integral and differentiation proposed by Podlubny in 1999 [4]. Later, Oustaloup had proposed the CRONE controller which is acronym for Commande Robuste d'Ordre Non Entier which means Fractional-order robust control. There are three generation of CRONE Controller such as CRONE1 [13], CRONE2 [14] and CRONE 3 [15].

The generalizations of Fractional-order PID or $PI^{\lambda}D^{\mu}$ have more parameters compared to the standard PID structure. In standard PID structure, it only consists of three parameters such as Proportional (P), Integral (I) and Derivative (D) term while in $PI^{\lambda}D^{\mu}$ there are five parameters. Such type of controller gives more flexibility in tuning towards better performance by allowing modification of system response continuously and directly [16]. However, based on literature reviewed, it shows that $PI^{\lambda}D^{\mu}$ suffer from steady state error due to missing of pole from the origin when the algorithm is approximated using filter algorithm techniques. This required additional tool such as error filter to eliminate steady state error. However there are no systematic rules that state how the parameters of error filter to be selected.

This paper presents the performance of Fractional-order PI with error filter based on three type controller performances criteria such as Integral of square error (ISE), Integral of absolute value of error (IAE) and Time-weighted Integral of absolute value of error (ITAE). The controllers are tested via simulation in Small-Medium Industry Steam Distillation Plant (SMISD) for regulating the steam temperature of the plant. The results of the best controller for each criterion will be verified via real time application for duration of 25000 seconds.

978-1-4799-7862-5/15/$31.00 © 2015 IEEE

The remaining part of the paper is organized as follows: Section II presents the overview of SMISD plant. In Section III, present tuning rule that was used in this experiment. Meanwhile, Section IV shows the Fractional-order PI configuration while Section VI and Section VII will present the results and conclusion respectively.

II. SMISD PLANT

The Small-Medium Industry Steam Distillation Plant or SMISD are used in this experiment to regulate steam temperature of a distillation process at a pre-determined set point which in this case was set at 85°C. The plant is located at Distributed Control System (DCS) Laboratory, FKE, UiTM, Shah Alam. It was used to extract essential oil from the raw materials. Previous studies had proved that high temperature could damage some chemical compounds of the essential oil and hence, compromised its quality.

The SMISD contain five main components which are controller panel, distillation tank, sensory elements, self-refilling function and condenser. Figure 1 show the Small-Medium Industry Steam Distillation Plant (SMISD) in Distribution Control System Laboratory, UiTM Shah Alam. Detailed descriptions of the plant can be found in [11, 17].

Figure 1: Pilot Scale Steam Distillation Plant

III. FOPI TUNING RULE

Performance of the PID controller are closely related to the value of Proportional (P), Integral (I) and Derivative (D). There are many PID tuning methods available and one of the most outstanding are given by John G. Ziegler and Nathaniel B. Nichols [18] namely the Ziegler-Nichols Tuning Rule. In their paper, they had proposed two tuning methods; step response method and frequency response method. It was introduced in mid-20th century and it was the earliest systematic tuning rule available for PID controller. This technique had been preferred in industry because it is simple and does not require process model except the information on dead time, time constant and gain of the process.

The parameters to calculate Ziegler-Nichols tuning rule were obtain using process reaction curve is shown in Table I. Detail descriptions on obtaining process reaction curve for the process was explained in [11] and will not be discussed here. From the process reaction curve, θ=1707.3 seconds; τ=13902.4 seconds and K=29.46.

TABLE I

Tuning Rule	PI Structure	
	Proportional Gain (K_p)	Integration Time Constant (T_i)
Ziegler-Nichols	$\frac{0.9\tau}{K\theta}$ 0.2488	3.3θ 5685.309

IV. FRACTIONAL-ORDER PI

Fractional-order calculus is basically a method used to generalize integral and differentiation operator into non-integer order operator. Generalization of integral and differentiation operator require the user to combine it into single fundamental operation, aD_t^α where a and t denote the limits of the operation and α denotes the fractional order as shown in (1):

$$aD_t^\alpha = \begin{cases} \frac{d^\alpha}{dt^\alpha} & ;\text{differentiation operator} \\ 1 & \\ \int_a^t (d\tau)^{-\alpha} & ;\text{integral operator} \end{cases} \quad (1)$$

There are several definitions of Fractional-order calculus that had been published such as Riemann-Liouville, Grunwald-Letnikov and Caputo definition. The choice of suitable definition is dependent on each application. For example Grunwald-Letnikov definition is suitable for numerical evaluation while Riemann-Liouville definition is used to find the analytical solution of simple function. Conversely, Caputo definition is suitable if real physical application is considered, such as system identification and biochemistry area.

However, when talking about application in real-time applications, it makes more sense to use less complex definition of Fractional-order since real-time application required fast response to solve the problem and a known sample of the function is required. For instance, Grunwald-Letnikov definition gives good fitting for any given function, but this definition requires one to know the sample of it function.

Hence, several filter approximations had been developed to accommodate this problem and for instance there are curve fitting method, continuous fractional expansion and Oustaloup (ORA) filter. The latter being the top notch amongst all. This technique uses recursive poles and zeros distribution to replicate the frequency response of fractional order within expected frequency range. The ORA algorithm is given in (2).

$$G(s) = K \prod_{k=-N}^{N} \frac{s+w'_k}{s+w^k} \tag{2}$$

where

$$w'_k = w_l \left(\frac{w_h}{w_l}\right)^{J^1}$$

$$w_k = w_l \left(\frac{w_h}{w_l}\right)^{J^2}$$

$$J^1 = \frac{k+N+\frac{1}{2}(1-y)}{2N+1}$$

$$J^2 = \frac{k+N+\frac{1}{2}(1+y)}{2N+1}$$

$$k = w_h^y$$

Where, y is the order of the differentiation and $2N+1$ is the order of the filter. w_h and w_l represents high and low frequency limit of the range of the frequency of interest respectively.

For this experiment, we purposely chose small frequency range due to our system operating at low frequency. So, we chose w_l =0.01 and w_h = 10000. Next is to determine the best value of *N* which determined the number of zero and pole. The effect of *N* is illustrated in Figure 2a, 2b and 2c.

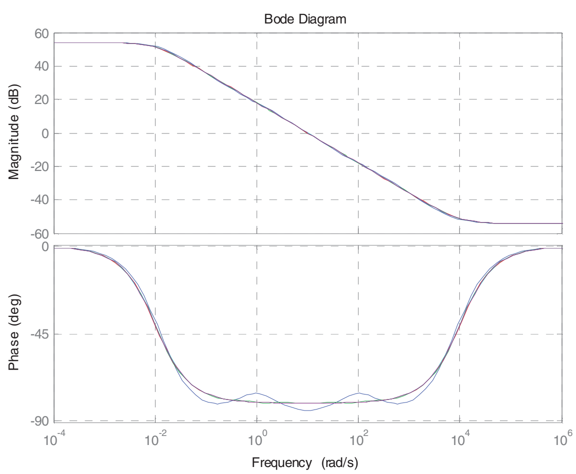

Figure 2a: Full scale figure effect of value *N*

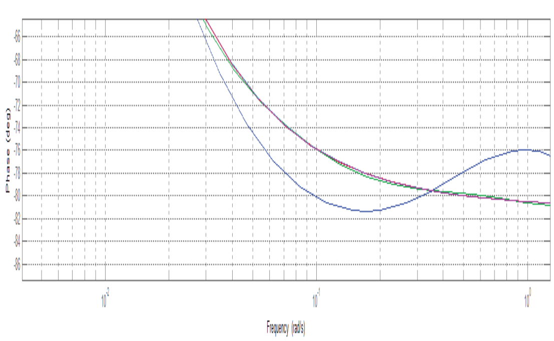

Figure 2b: Magnification scale figure effect of value *N*

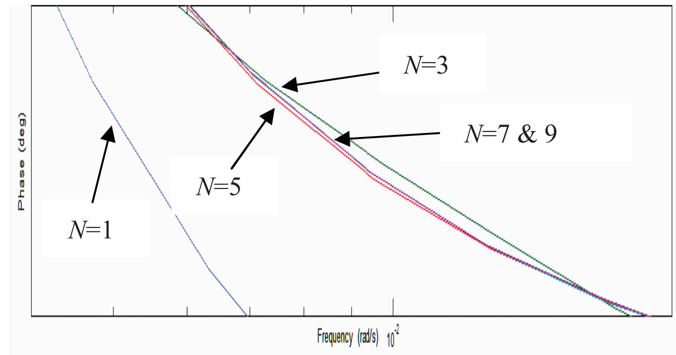

Figure 2c: 2x Magnification scale figure effect of value *N*

From Figure 2a, 2b and 2c above, it is clearly shown that for *N*=1 and *N*=3 will produced ripple in its phase response. Further magnification shows that for *N*=5, the ripple is significantly reduced. For *N*=7 and *N*=9, no significant difference is observed in the shape. Large value of *N* allows good approximation, however it required high computational power. Some tradeoff between good approximation and computation power is considered. Based on this result, we chose *N*=7, to represent our system.

Then, a series of simulation on SMISD plant have been conducted to determine the best value of λ based on ARX model in (3).

$$A(q) = 1 - 0.0001487\, q^{-1}$$
$$B(q) = 0.007103 q^{-1} \tag{3}$$

To archive better transient response gain and phase are adjusted between ±20db/decade and ±90 degree with respect to λ by adjusting the gain, *G* [19] as shown in Figure 3a and 3b. Below are the results tabulated in Table II from the experiment:

TABLE II

λ	Gain (*G*)	Steam Temperature (ºC)	Steady state error (ºC)
-0.1	0.89	80.44	4.56
-0.2	1.25	80.45	4.55
-0.3	1.39	80.46	4.54
-0.4	1.59	80.48	4.52
-0.5	1.79	80.52	4.48
-0.6	2.00	80.63	4.37
-0.7	2.24	80.88	4.12
-0.8	2.52	81.4	3.60
-0.9	2.85	82.27	2.73

**Shaded box represented the best response*

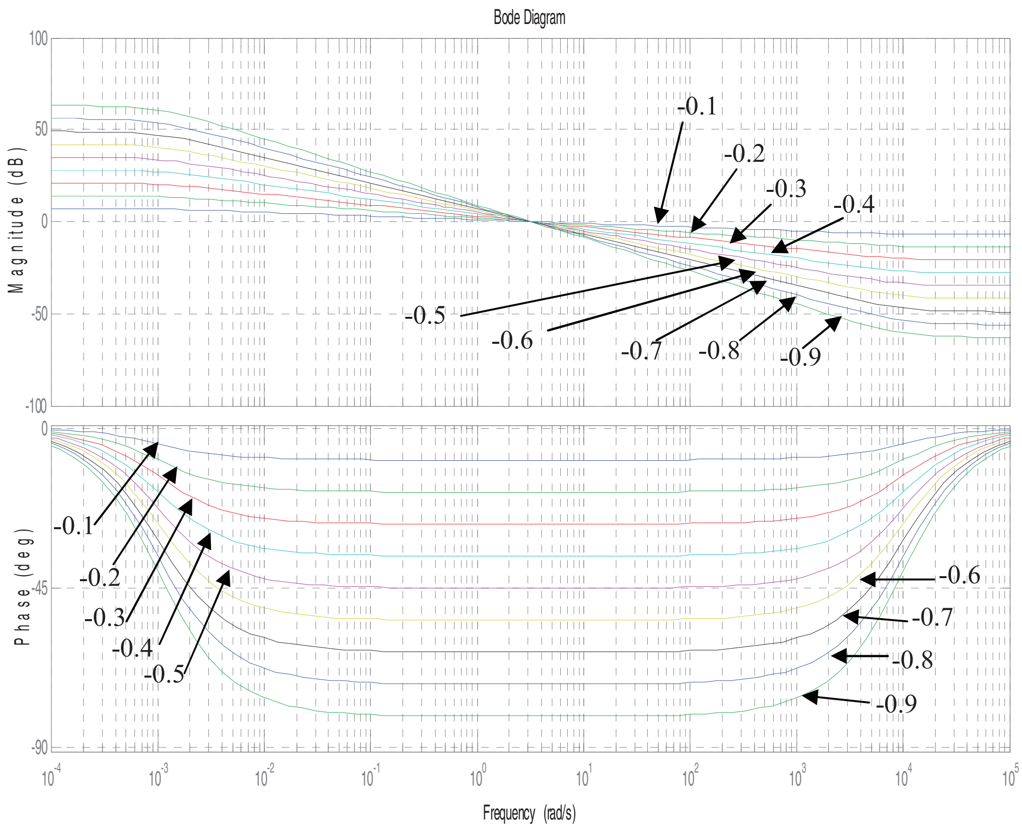

Figure 3a: Bode plot of λ before Gain, G

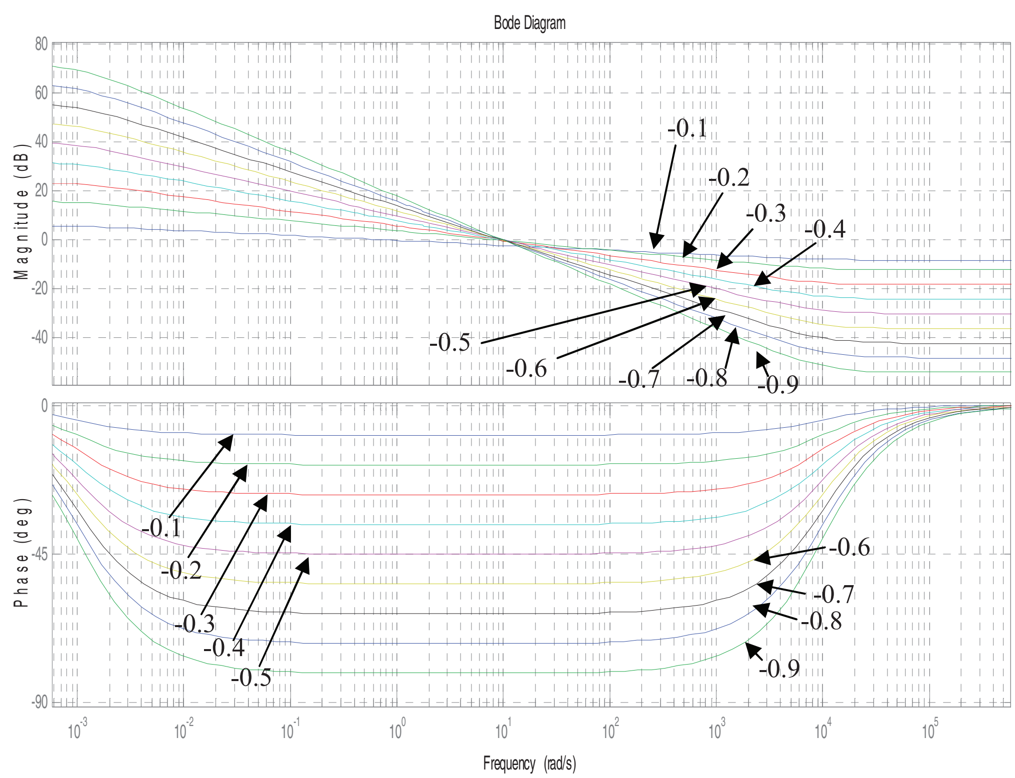

Figure 3b: Bode plot of λ after Gain, G

Based on this result λ of -0.9 is chosen because it has the lowest number of steady state error and have steam temperature closer to the predetermined set point which is 85°C.

For the result above, it is clearly shown that the persisting steady state error still occurs in all selected λ. This is due to the absence of pole at the origin when the algorithm is approximate using Oustaloup Filter Technique. Based on literature review, the result can be improved by integrating the error filter which was proposed by Feliu-Batlle *et al.* in [20]. The structure of error filter is shown as in (4).

$$\frac{s+n}{s} \tag{4}$$

The value of *n* should be small enough, so it does not drastically alter system specification. There are no systematic ways or specific rules that state how the value of *n* should be selected. We present a new technique to determine the best possible error filter using controller performance criterion. Before going further, here we discuss some controller performance criteria such as Integral of square error (ISE), Integral of absolute value of error (IAE) and Time-weighted Integral of absolute value of error (ITAE) as shown in (5),(6) and (7) respectively.

A. Integral of square error (ISE)

$$ISE = \int_0^\infty e(t)^2 dt \tag{5}$$

This criterion penalizes positive and negative errors that occur to system.

B. Integral of absolute value of error (IAE)

$$IAE = \int_0^\infty |e(t)| dt \tag{6}$$

This criterion measures the increased error in the system, resulted system that has good underdamped system.

C. Time-weighted Integral of absolute value of error (ITAE)

$$ITAE = \int_0^\infty t|e(t)| dt \tag{7}$$

This criterion is the same with IAE, but with addition weight that penalize errors that occur later.

**Note that, smaller value of performance criteria indicates the best performance.*

A series of simulations have been conducted to determine the error filter which best suited the fractional PI controller in regulation steam temperature. **Shaded box represents the best response for each class.*

TABLE III

Error Filter (*n*)	Steam Temperature (ºC)	Steady State Error (ºC)	ISE	IAE	ITAE
0.0002	85.18	-0.18	$2.338\text{x}10^6$	$8.034\text{x}10^4$	$2.849\text{x}10^8$
0.0003	85.13	-0.13	$2.408\text{x}10^6$	$9.276\text{x}10^4$	$3.712\text{x}10^8$
0.0004	85.06	-0.06	$2.489\text{x}10^6$	$9.909\text{x}10^4$	$3.87\text{x}10^8$
0.0005	85	0	$2.578\text{x}10^6$	$1.027\text{x}10^5$	$3.688\text{x}10^8$
0.0006	85	0	$2.672\text{x}10^6$	$1.05\text{x}10^5$	$3.536\text{x}10^8$
0.0007	85	0	$2.7716\text{x}10^6$	$1.067\text{x}10^5$	$3.401\text{x}10^8$
0.0008	85	0	$2.875\text{x}10^6$	$1.079\text{x}10^5$	$3.291\text{x}10^8$
0.0009	85	0	$2.982\text{x}10^6$	$1.088\text{x}10^5$	$3.202\text{x}10^8$
0.0010	85	0	$3.093\text{x}10^6$	$1.096\text{x}10^5$	$3.129\text{x}10^8$
0.0011	85	0	$3.229\text{x}10^6$	$1.132\text{x}10^5$	$3.309\text{x}10^8$
0.0012	85	0	$3.402\text{x}10^6$	$1.229\text{x}10^5$	$3.952\text{x}10^8$

As shown in Table III, both ISE and IAE have smaller indices at *n*=0.0002, however this value is not selected as best response due to it containing persisting steady state error. The same case are show in *n*=0.0003 and 0.0004.

Smaller ISE and IAE recorded with zero steady state error at *n*=0.0005 with $2.578\text{x}10^6$ and $1.027\text{x}10^5$ respectively. While ITAE have smaller error at *n*=0.0010 with $3.129\text{x}10^8$. This results of *n*=0.0005 and *n*=0.0010 will be validity in real time application of SMISD PLANT.

V. RESULT AND DISCUSSION

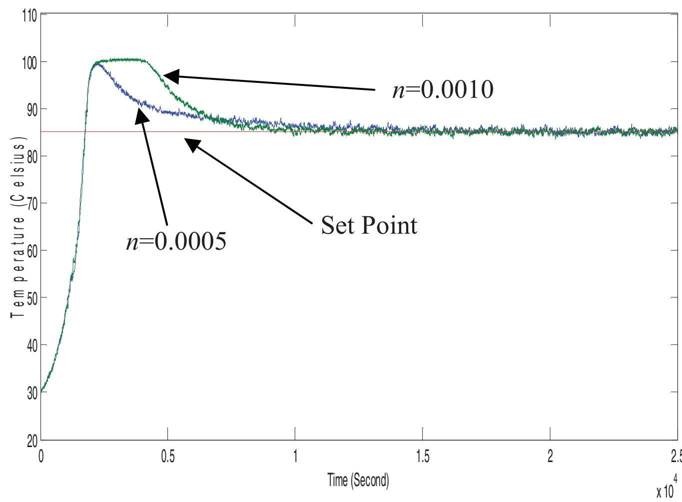

Figure 4a: Real Time result of SMISD

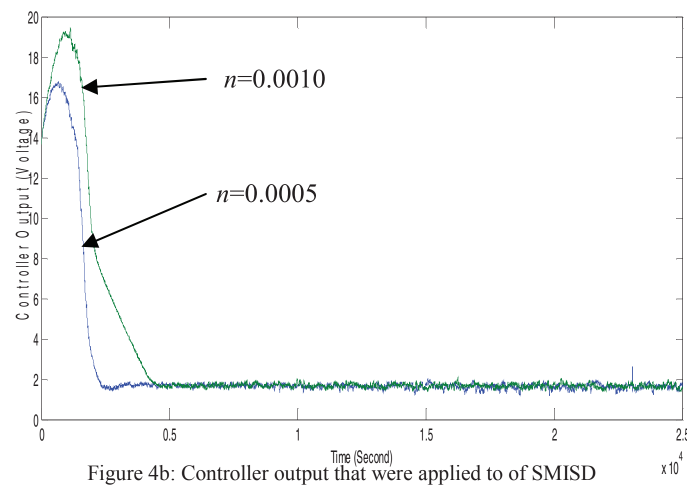

Figure 4b: Controller output that were applied to of SMISD

TABLE IV

Error Filter (*n*)	Rise Time (ºC)	Setting Time, 5% (ºC)	Percentage Overshoot (%)
0.0005	$1.6736\text{x}10^3$	$9.3071\text{x}10^3$	16.6511
0.0010	$1.6817\text{x}10^3$	$7.1923\text{x}10^3$	18.2667

**Shaded box represents the best response for each test*

Figure 4a shows result of real time application of steam temperature on SMISD plant while Figure 4b shows signal that applied to SMISD plant. Table IV presents summary of both controller performance which have similar settings except for different value of error filter, *n*. There are two different error filters that have been tested which are n=0.0005 and n=0.0010. To determine the best controller, several tests have been done on the transient performance evaluation such as test on rise time, setting time and percentage of overshoot.

Both controllers have achieved set point of 85°C. The results show that n=0.0005 have fastest rise time with 1.6736×10^3 second compared to n=0.0010 with 1.6817×10^3 second. In setting time n=0.0010 have quickest setting time with an advantage of 2115 second compare to n=0.0005. Referring to percentage overshoot, n=0.0005 have smaller overshoot with 16.65% compare with n=0.0010 with 18.27%.

From the analysis above, controller with n=0.0005 produced ISE and IAE controller performance criterion that is in favor because it has the upper hand in two of three of transient performance evaluation (rise time and percentage overshoot) while controller n=0.0010 only have advantage in setting time. With n=0.0005 chose as the best controller as indicated by the lowest ISE and IAE.

VI. Conclusion

This study has successfully implemented a Fractional-order PI with error filter in SMISD plant to regulate steam temperature. It shown that n=0.0005 with ISE and IAE controller performance criterion have better transient response in real-time compared to n=0.0010. The finding in this paper basically can be extended to another temperature control in related application by simply change the parameter of Ziegler-Nichols Tuning Rule in this paper to suit their process.

Acknowledgment

This research is supported by FRGS Fund 600-RMI/FRGS 5/3 (84/2014). The authors would like to thank the Faculty of Electrical Engineering, UiTM for providing the facilities to conduct this research.

References

[1] A. Loverro, "Fractional Calculus: History, Definitions and Applications for the Engineer," May/08/ 2004.

[2] D. Cafagna, "Fractional calculus: A mathematical tool from the past for present engineers [Past and present]," *Industrial Electronics Magazine, IEEE,* vol. 1, pp. 35-40, 2007.

[3] I. Podlubny, *Fractional Differential Equations. An Introduction to Fractional Derivatives, Fractional Differential Equations, Some Methods of Their Solution and Some of Their Applications*: Academic Press, San Diego - New York - London, 1999.

[4] I. Podlubny, "Fractional-order systems and PI/sup /spl lambda//D/sup /spl mu//-controllers," *Automatic Control, IEEE Transactions on,* vol. 44, pp. 208-214, 1999.

[5] K. Sundaravadivu, V. Jeyakumar, and K. Saravanan, "Design of Fractional Order PI controller for liquid level control of spherical tank modeled as Fractional Order System," in *Control System, Computing and Engineering (ICCSCE), 2011 IEEE International Conference on*, 2011, pp. 522-525.

[6] C. Wang, L. Wang, and M. Li, "Application research for hydraulic servo system based on fractional order proportional derivative and [proportional derivative] controllers," in *Control Conference (CCC), 2014 33rd Chinese*, 2014, pp. 4411-4416.

[7] L. Ying and C. Yang-Quan, "Fractional-order [proportional derivative] controller for robust motion control: Tuning procedure and validation," in *American Control Conference, 2009. ACC '09.*, 2009, pp. 1412-1417.

[8] M. Li and X. Dingyu, "Fractional order QFT controller for non-minimum phase hydro power plant," in *Control and Decision Conference (CCDC), 2012 24th Chinese*, 2012, pp. 2796-2801.

[9] V. Mehra, S. Srivastava, and P. Varshney, "Fractional-Order PID Controller Design for Speed Control of DC Motor," in *Emerging Trends in Engineering and Technology (ICETET), 2010 3rd International Conference on*, 2010, pp. 422-425.

[10] L. Desborough and R. Miller, "Increasing customer value of industrial control performance monitoring-Honeywell's experience," in *AIChE symposium series*, 2002, pp. 169-189.

[11] M. H. Marzaki, M. H. A. Jalil, H. Md Shariff, R. Adnan, and M. H. F. Rahiman, "Comparative study of Model Predictive Controller (MPC) and PID Controller on regulation temperature for SSISD plant," in *Control and System Graduate Research Colloquium (ICSGRC), 2014 IEEE 5th*, 2014, pp. 136-140.

[12] M. H. A. Jalil, M. H. Marzaki, N. Kasuan, M. N. Taib, and M. H. F. Rahiman, "Implementation of anti windup scheme on PID controller for regulating temperature of glycerin bleaching process," in *System Engineering and Technology (ICSET), 2013 IEEE 3rd International Conference on*, 2013, pp. 113-117.

[13] A. Oustaloup and M. Bansard, "First generation CRONE control," in *Systems, Man and Cybernetics, 1993. 'Systems Engineering in the Service of Humans', Conference Proceedings., International Conference on*, 1993, pp. 130-135 vol.2.

[14] A. Oustaloup, B. Mathieu, and P. Lanusse, "Second generation CRONE control," in *Systems, Man and Cybernetics, 1993. 'Systems Engineering in the Service of Humans', Conference Proceedings., International Conference on*, 1993, pp. 136-142 vol.2.

[15] P. Lanusse, A. Oustaloup, and B. Mathieu, "Third generation CRONE control," in *Systems, Man and Cybernetics, 1993. 'Systems Engineering in the Service of Humans', Conference Proceedings., International Conference on*, 1993, pp. 149-155 vol.2.

[16] C. Jun-yi, L. Jin, and C. Bing-Gang, "Optimization of fractional order PID controllers based on genetic algorithms," in *Machine Learning and Cybernetics, 2005. Proceedings of 2005 International Conference on*, 2005, pp. 5686-5689 Vol. 9.

[17] M. H. Marzaki, M. Tajjudin, R. Adnan, M. H. F. Rahiman, and M. H. A. Jalil, "Comparison of different model structure selection using R2, MDL and AIC criterion," in *Control and System Graduate Research Colloquium (ICSGRC), 2013 IEEE 4th*, 2013, pp. 80-85.

[18] J. G. Ziegler and N. B. Nichols, "Optimum Settings for Automatic Controllers," *Transactions of ASME,* vol. 64, pp. 759-768, // 1942.

[19] M. Tajjudin, S. F. Tahir, M. H. F. Rahiman, N. M. Arshad, and R. Adnan, "Fractional-order PI controller with relay auto-tuning method," in *Control and System Graduate Research Colloquium (ICSGRC), 2013 IEEE 4th*, 2013, pp. 121-126.

[20] V. Feliu-Batlle, R. R. Pérez, and L. S. Rodríguez, "Fractional robust control of main irrigation canals with variable dynamic parameters," *Control Engineering Practice,* vol. 15, pp. 673-686, 6// 2007.