



IBM Developer SKILLS NETWORK

Multiple Linear Regression

Estimated time needed: **15** minutes

Objectives

After completing this lab you will be able to:

- Use scikit-learn to implement Multiple Linear Regression
- Create a model, train it, test it and use the model

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Importing Needed packages

In [1]:

```
import matplotlib.pyplot as plt
import pandas as pd
import pylab as pl
import numpy as np
%matplotlib inline
```

Downloading Data

To download the data, we will use `!wget` to download it from IBM Object Storage.

In [3]:

```
!wget -O FuelConsumption.csv https://cf-courses-data.s3.us.cloud-object-storage.appdomain.cloud/IBMDeveloperSkillsNetwork-ML0101EN-SkillsNetwork/labs/Module%202/data/FuelConsumptionCo2.csv
```

```
--2021-12-22 20:50:03-- https://cf-courses-data.s3.us.cloud-object-storage.appdomain.cloud/IBMDeveloperSkillsNetwork-ML0101EN-SkillsNetwork/labs/Module%202/data/FuelConsumptionCo2.csv
```

```
Resolving cf-courses-data.s3.us.cloud-object-storage.appdomain.cloud (cf-courses-data.s3.us.cloud-object-storage.appdomain.cloud)... 169.63.118.104
Connecting to cf-courses-data.s3.us.cloud-object-storage.appdomain.cloud (cf-courses-data.s3.us.cloud-object-storage.appdomain.cloud)|169.63.118.104|:443... connected.
```

```
HTTP request sent, awaiting response... 200 OK
```

```
Length: 72629 (71K) [text/csv]
```

```
Saving to: 'FuelConsumption.csv'
```

```
FuelConsumption.csv 100%[=====>] 70.93K --.-KB/s in 0.001s
```

```
2021-12-22 20:50:03 (51.4 MB/s) - 'FuelConsumption.csv' saved [72629/72629]
```

Did you know? When it comes to Machine Learning, you will likely be working with large datasets. As a business, where can you host your data? IBM is offering a unique opportunity for businesses, with 10 Tb of IBM Cloud Object Storage: [Sign up now for free \(http://cocl.us/ML0101EN-IBM-Offer-CC\)](http://cocl.us/ML0101EN-IBM-Offer-CC)

Understanding the Data

FuelConsumption.csv :

We have downloaded a fuel consumption dataset, **FuelConsumption.csv**, which contains model-specific fuel consumption ratings and estimated carbon dioxide emissions for new light-duty vehicles for retail sale in Canada. [Dataset source \(http://open.canada.ca/data/en/dataset/98f1a129-f628-4ce4-b24d-6f16bf24dd64?utm_medium=Exinfluencer&utm_source=Exinfluencer&utm_content=000026UJ&utm_term=10006555&utm_id:SkillsNetwork-Channel-SkillsNetworkCoursesIBMDeveloperSkillsNetworkML0101ENSkillsNetwork20718538-2021-01-01\)](http://open.canada.ca/data/en/dataset/98f1a129-f628-4ce4-b24d-6f16bf24dd64?utm_medium=Exinfluencer&utm_source=Exinfluencer&utm_content=000026UJ&utm_term=10006555&utm_id:SkillsNetwork-Channel-SkillsNetworkCoursesIBMDeveloperSkillsNetworkML0101ENSkillsNetwork20718538-2021-01-01)

- **MODELYEAR** e.g. 2014
- **MAKE** e.g. Acura
- **MODEL** e.g. ILX
- **VEHICLE CLASS** e.g. SUV
- **ENGINE SIZE** e.g. 4.7
- **CYLINDERS** e.g. 6
- **TRANSMISSION** e.g. A6
- **FUELTYPE** e.g. z
- **FUEL CONSUMPTION in CITY (L/100 km)** e.g. 9.9
- **FUEL CONSUMPTION in HWY (L/100 km)** e.g. 8.9
- **FUEL CONSUMPTION COMB (L/100 km)** e.g. 9.2
- **CO2 EMISSIONS (g/km)** e.g. 182 --> low --> 0

Reading the data in

In [4]:

```
df = pd.read_csv("FuelConsumption.csv")

# take a look at the dataset
df.head()
```

Out[4]:

	MODELYEAR	MAKE	MODEL	VEHICLECLASS	ENGINE SIZE	CYLINDERS	TRANSMISSIO
0	2014	ACURA	ILX	COMPACT	2.0	4	AS
1	2014	ACURA	ILX	COMPACT	2.4	4	M
2	2014	ACURA	ILX HYBRID	COMPACT	1.5	4	AV
3	2014	ACURA	MDX 4WD	SUV - SMALL	3.5	6	AS
4	2014	ACURA	RDX AWD	SUV - SMALL	3.5	6	AS

Let's select some features that we want to use for regression.

In [5]:

```

cdf = df[['ENGINE_SIZE', 'CYLINDERS', 'FUELCONSUMPTION_CITY', 'FUELCONSUMPTION_HWY', 'FUELCONSUMPTION_COMB', 'CO2EMISSIONS']]
cdf.head(9)

```

Out[5]:

	ENGINE_SIZE	CYLINDERS	FUELCONSUMPTION_CITY	FUELCONSUMPTION_HWY	FUELCONSUMPTION_COMB
0	2.0	4	9.9	6.7	8.1
1	2.4	4	11.2	7.7	9.4
2	1.5	4	6.0	5.8	5.9
3	3.5	6	12.7	9.1	10.9
4	3.5	6	12.1	8.7	10.4
5	3.5	6	11.9	7.7	9.8
6	3.5	6	11.8	8.1	9.9
7	3.7	6	12.8	9.0	10.9
8	3.7	6	13.4	9.5	11.4

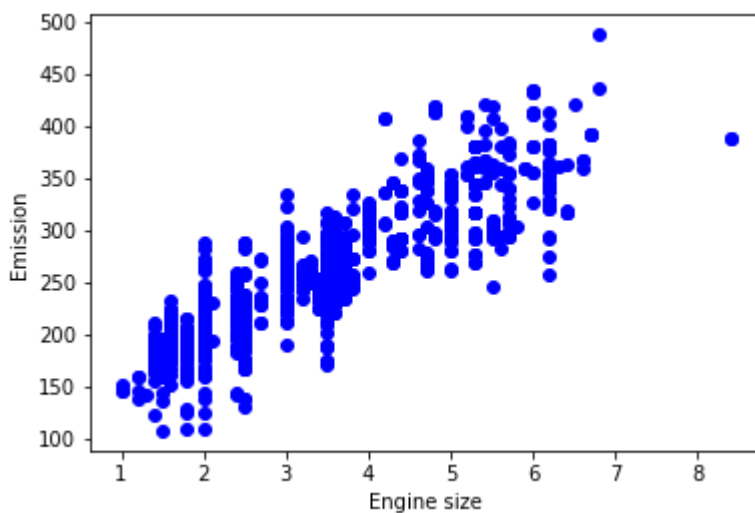
Let's plot Emission values with respect to Engine size:

In [6]:

```

plt.scatter(cdf.ENGINE_SIZE, cdf.CO2EMISSIONS, color='blue')
plt.xlabel("Engine size")
plt.ylabel("Emission")
plt.show()

```



Creating train and test dataset

Train/Test Split involves splitting the dataset into training and testing sets respectively, which are mutually exclusive. After which, you train with the training set and test with the testing set. This will provide a more accurate evaluation on out-of-sample accuracy because the testing dataset is not part of the dataset that have been used to train the model. Therefore, it gives us a better understanding of how well our model generalizes on new data.

We know the outcome of each data point in the testing dataset, making it great to test with! Since this data has not been used to train the model, the model has no knowledge of the outcome of these data points. So, in essence, it is truly an out-of-sample testing.

Let's split our dataset into train and test sets. Around 80% of the entire dataset will be used for training and 20% for testing. We create a mask to select random rows using the **np.random.rand()** function:

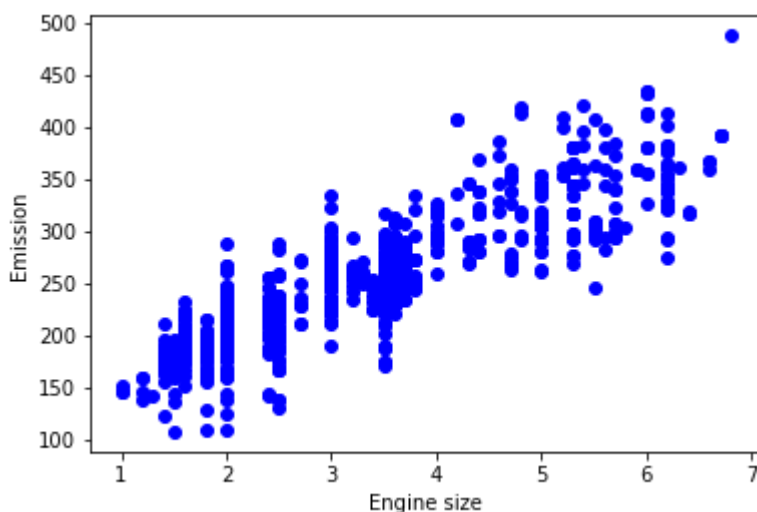
In [7]:

```
msk = np.random.rand(len(df)) < 0.8  
train = cdf[msk]  
test = cdf[~msk]
```

Train data distribution

In [8]:

```
plt.scatter(train.ENGINESIZE, train.CO2EMISSIONS, color='blue')  
plt.xlabel("Engine size")  
plt.ylabel("Emission")  
plt.show()
```



Multiple Regression Model

In reality, there are multiple variables that impact the co2emission. When more than one independent variable is present, the process is called multiple linear regression. An example of multiple linear regression is predicting co2emission using the features FUELCONSUMPTION_COMB, EngineSize and Cylinders of cars. The good thing here is that multiple linear regression model is the extension of the simple linear regression model.

In [9]:

```
from sklearn import linear_model
regr = linear_model.LinearRegression()
x = np.asanyarray(train[['ENGINE_SIZE', 'CYLINDERS', 'FUELCONSUMPTION_COMB']])
y = np.asanyarray(train[['CO2EMISSIONS']])
regr.fit(x, y)
# The coefficients
print('Coefficients: ', regr.coef_)
```

Coefficients: $\begin{bmatrix} 10.56165919 & 7.57652508 & 9.69481014 \end{bmatrix}$

```

/home/jupyterlab/conda/envs/python/lib/python3.7/site-packages/sklearn/linear_model/least_angle.py:35: DeprecationWarning: `np.float` is a deprecated alias for the builtin `float`. To silence this warning, use `float` by itself. Doing this will not modify any behavior and is safe. If you specifically wanted the numpy scalar type, use `np.float64` here.
Deprecated in NumPy 1.20; for more details and guidance: https://numpy.org/devdocs/release/1.20.0-notes.html#deprecations
    eps=np.finfo(np.float).eps,
/home/jupyterlab/conda/envs/python/lib/python3.7/site-packages/sklearn/linear_model/least_angle.py:597: DeprecationWarning: `np.float` is a deprecated alias for the builtin `float`. To silence this warning, use `float` by itself. Doing this will not modify any behavior and is safe. If you specifically wanted the numpy scalar type, use `np.float64` here.
Deprecated in NumPy 1.20; for more details and guidance: https://numpy.org/devdocs/release/1.20.0-notes.html#deprecations
    eps=np.finfo(np.float).eps, copy_X=True, fit_path=True,
/home/jupyterlab/conda/envs/python/lib/python3.7/site-packages/sklearn/linear_model/least_angle.py:836: DeprecationWarning: `np.float` is a deprecated alias for the builtin `float`. To silence this warning, use `float` by itself. Doing this will not modify any behavior and is safe. If you specifically wanted the numpy scalar type, use `np.float64` here.
Deprecated in NumPy 1.20; for more details and guidance: https://numpy.org/devdocs/release/1.20.0-notes.html#deprecations
    eps=np.finfo(np.float).eps, copy_X=True, fit_path=True,
/home/jupyterlab/conda/envs/python/lib/python3.7/site-packages/sklearn/linear_model/least_angle.py:862: DeprecationWarning: `np.float` is a deprecated alias for the builtin `float`. To silence this warning, use `float` by itself. Doing this will not modify any behavior and is safe. If you specifically wanted the numpy scalar type, use `np.float64` here.
Deprecated in NumPy 1.20; for more details and guidance: https://numpy.org/devdocs/release/1.20.0-notes.html#deprecations
    eps=np.finfo(np.float).eps, positive=False):
/home/jupyterlab/conda/envs/python/lib/python3.7/site-packages/sklearn/linear_model/least_angle.py:1097: DeprecationWarning: `np.float` is a deprecated alias for the builtin `float`. To silence this warning, use `float` by itself. Doing this will not modify any behavior and is safe. If you specifically wanted the numpy scalar type, use `np.float64` here.
Deprecated in NumPy 1.20; for more details and guidance: https://numpy.org/devdocs/release/1.20.0-notes.html#deprecations
    max_n_alphas=1000, n_jobs=None, eps=np.finfo(np.float).eps,
/home/jupyterlab/conda/envs/python/lib/python3.7/site-packages/sklearn/linear_model/least_angle.py:1344: DeprecationWarning: `np.float` is a deprecated alias for the builtin `float`. To silence this warning, use `float` by itself. Doing this will not modify any behavior and is safe. If you specifically wanted the numpy scalar type, use `np.float64` here.
Deprecated in NumPy 1.20; for more details and guidance: https://numpy.org/devdocs/release/1.20.0-notes.html#deprecations
    max_n_alphas=1000, n_jobs=None, eps=np.finfo(np.float).eps,
/home/jupyterlab/conda/envs/python/lib/python3.7/site-packages/sklearn/linear_model/least_angle.py:1480: DeprecationWarning: `np.float` is a deprecated alias for the builtin `float`. To silence this warning, use `float` by itself. Doing this will not modify any behavior and is safe. If you specifically wanted the numpy scalar type, use `np.float64` here.
Deprecated in NumPy 1.20; for more details and guidance: https://numpy.org/devdocs/release/1.20.0-notes.html#deprecations
    eps=np.finfo(np.float).eps, copy_X=True, positive=False):
/home/jupyterlab/conda/envs/python/lib/python3.7/site-packages/sklearn/linear_model/randomized_l1.py:152: DeprecationWarning: `np.float` is a deprecated alias for the builtin `float`. To silence this warning, use `float` b

```



```

y itself. Doing this will not modify any behavior and is safe. If you specifically
wanted the numpy scalar type, use `np.float64` here.
Deprecated in NumPy 1.20; for more details and guidance: https://numpy.org/devdocs/release/1.20.0-notes.html#deprecations
precompute=False, eps=np.finfo(np.float).eps,
/home/jupyterlab/conda/envs/python/lib/python3.7/site-packages/sklearn/linear_model/randomized_l1.py:320: DeprecationWarning: `np.float` is a deprecated alias for the builtin `float`. To silence this warning, use `float` by itself. Doing this will not modify any behavior and is safe. If you specifically wanted the numpy scalar type, use `np.float64` here.
Deprecated in NumPy 1.20; for more details and guidance: https://numpy.org/devdocs/release/1.20.0-notes.html#deprecations
eps=np.finfo(np.float).eps, random_state=None,
/home/jupyterlab/conda/envs/python/lib/python3.7/site-packages/sklearn/linear_model/randomized_l1.py:580: DeprecationWarning: `np.float` is a deprecated alias for the builtin `float`. To silence this warning, use `float` by itself. Doing this will not modify any behavior and is safe. If you specifically wanted the numpy scalar type, use `np.float64` here.
Deprecated in NumPy 1.20; for more details and guidance: https://numpy.org/devdocs/release/1.20.0-notes.html#deprecations
eps=4 * np.finfo(np.float).eps, n_jobs=None,

```

As mentioned before, **Coefficient** and **Intercept** are the parameters of the fitted line. Given that it is a multiple linear regression model with 3 parameters and that the parameters are the intercept and coefficients of the hyperplane, sklearn can estimate them from our data. Scikit-learn uses plain Ordinary Least Squares method to solve this problem.

Ordinary Least Squares (OLS)

OLS is a method for estimating the unknown parameters in a linear regression model. OLS chooses the parameters of a linear function of a set of explanatory variables by minimizing the sum of the squares of the differences between the target dependent variable and those predicted by the linear function. In other words, it tries to minimize the sum of squared errors (SSE) or mean squared error (MSE) between the target variable (y) and our predicted output (\hat{y}) over all samples in the dataset.

OLS can find the best parameters using the following methods:

- Solving the model parameters analytically using closed-form equations
- Using an optimization algorithm (Gradient Descent, Stochastic Gradient Descent, Newton's Method, etc.)

Prediction

In [10]:

```

y_hat= regr.predict(test[['ENGINE_SIZE', 'CYLINDERS', 'FUELCONSUMPTION_COMB']])
x = np.asanyarray(test[['ENGINE_SIZE', 'CYLINDERS', 'FUELCONSUMPTION_COMB']])
y = np.asanyarray(test[['CO2EMISSIONS']])
print("Residual sum of squares: %.2f"
      % np.mean((y_hat - y) ** 2))

# Explained variance score: 1 is perfect prediction
print('Variance score: %.2f' % regr.score(x, y))

```

Residual sum of squares: 540.11

Variance score: 0.87

Explained variance regression score: Let \hat{y} be the estimated target output, y the corresponding (correct) target output, and Var be the Variance (the square of the standard deviation). Then the explained variance is estimated as follows:

$\text{explainedVariance}(y, \hat{y}) = 1 - \frac{\text{Var}\{y - \hat{y}\}}{\text{Var}\{y\}}$ The best possible score is 1.0, the lower values are worse.

Practice

Try to use a multiple linear regression with the same dataset, but this time use FUELCONSUMPTION_CITY and FUELCONSUMPTION_HWY instead of FUELCONSUMPTION_COMB. Does it result in better accuracy?

In [12]:

```

# write your code here

regr = linear_model.LinearRegression()
x = np.asanyarray(train[['ENGINE_SIZE', 'CYLINDERS', 'FUELCONSUMPTION_CITY', 'FUELCONSUMPTION_HWY']])
y = np.asanyarray(train[['CO2EMISSIONS']])
regr.fit(x, y)
print('Coefficients: ', regr.coef_)

y_ = regr.predict(test[['ENGINE_SIZE', 'CYLINDERS', 'FUELCONSUMPTION_CITY', 'FUELCONSUMPTION_HWY']])
x = np.asanyarray(test[['ENGINE_SIZE', 'CYLINDERS', 'FUELCONSUMPTION_CITY', 'FUELCONSUMPTION_HWY']])
y = np.asanyarray(test[['CO2EMISSIONS']])
print("Residual sum of squares: %.2f" % np.mean((y_ - y) ** 2))
print('Variance score: %.2f' % regr.score(x, y))

```

Coefficients: [[10.70572286 7.18173201 6.31054221 3.04144971]]

Residual sum of squares: 541.85

Variance score: 0.87

[Click here for the solution](#)

Want to learn more?

IBM SPSS Modeler is a comprehensive analytics platform that has many machine learning algorithms. It has been designed to bring predictive intelligence to decisions made by individuals, by groups, by systems – by your enterprise as a whole. A free trial is available through this course, available here: [SPSS Modeler \(https://www.ibm.com/analytics/spss-statistics-software?utm_medium=Exinfluencer&utm_source=Exinfluencer&utm_content=000026UJ&utm_term=10006555&utm_id:SkillsNetwork-Channel-SkillsNetworkCoursesIBMDeveloperSkillsNetworkML0101ENSkillsNetwork20718538-2021-01-01\)](https://www.ibm.com/analytics/spss-statistics-software?utm_medium=Exinfluencer&utm_source=Exinfluencer&utm_content=000026UJ&utm_term=10006555&utm_id:SkillsNetwork-Channel-SkillsNetworkCoursesIBMDeveloperSkillsNetworkML0101ENSkillsNetwork20718538-2021-01-01)

Also, you can use Watson Studio to run these notebooks faster with bigger datasets. Watson Studio is IBM's leading cloud solution for data scientists, built by data scientists. With Jupyter notebooks, RStudio, Apache Spark and popular libraries pre-packaged in the cloud, Watson Studio enables data scientists to collaborate on their projects without having to install anything. Join the fast-growing community of Watson Studio users today with a free account at [Watson Studio \(https://www.ibm.com/cloud/watson-studio?utm_medium=Exinfluencer&utm_source=Exinfluencer&utm_content=000026UJ&utm_term=10006555&utm_id:SkillsNetwork-Channel-SkillsNetworkCoursesIBMDeveloperSkillsNetworkML0101ENSkillsNetwork20718538-2021-01-01\)](https://www.ibm.com/cloud/watson-studio?utm_medium=Exinfluencer&utm_source=Exinfluencer&utm_content=000026UJ&utm_term=10006555&utm_id:SkillsNetwork-Channel-SkillsNetworkCoursesIBMDeveloperSkillsNetworkML0101ENSkillsNetwork20718538-2021-01-01)

Thank you for completing this lab!

Author

Saeed Aghabozorgi

Other Contributors

[Joseph Santarcangelo \(https://www.linkedin.com/in/joseph-s-50398b136/?utm_medium=Exinfluencer&utm_source=Exinfluencer&utm_content=000026UJ&utm_term=10006555&utm_id:SkillsNetwork-Channel-SkillsNetworkCoursesIBMDeveloperSkillsNetworkML0101ENSkillsNetwork20718538-2021-01-01\)](https://www.linkedin.com/in/joseph-s-50398b136/?utm_medium=Exinfluencer&utm_source=Exinfluencer&utm_content=000026UJ&utm_term=10006555&utm_id:SkillsNetwork-Channel-SkillsNetworkCoursesIBMDeveloperSkillsNetworkML0101ENSkillsNetwork20718538-2021-01-01)

Change Log

Date (YYYY-MM-DD)	Version	Changed By	Change Description
2020-11-03	2.1	Lakshmi	Made changes in URL
2020-08-27	2.0	Lavanya	Moved lab to course repo in GitLab

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