

063-0605-00L : Computational Structural Design 1

# Computational Graphic Statics

Dr. Juney Lee & Dr. Lluis Enrique

Autumn Semester 2021

**Week 4**

Friday, October 15th

Hour 1

**09:45 – 09:50** • Recap of Exercise 2**09:50 – 10:30** • [Lecture 3 : Computational Graphic Statics](#)**10:30 – 10:45** • Break

Hour 2

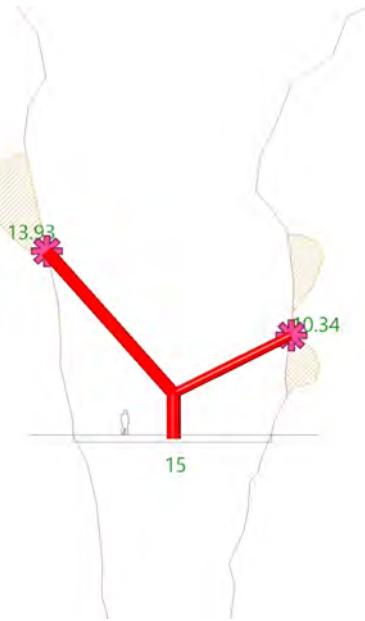
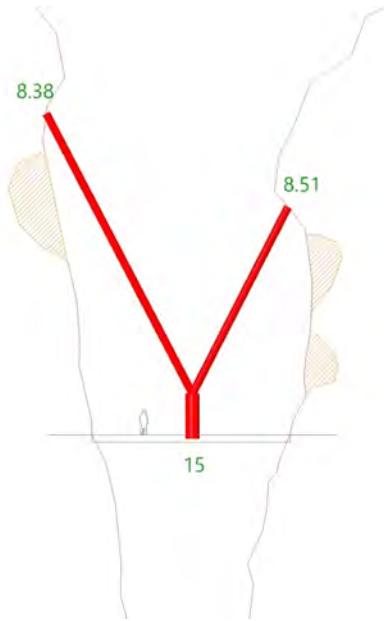
**10:45 – 11:35** • [Tutorial 3 : Procedural graphic statics 2](#)**11:30 – 11:45** • Break

Hour 3

**11:45 – 12:30** • [Tutorial 3 : Procedural graphic statics 2 \(cont'd\)](#)

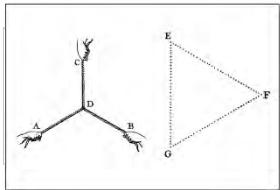
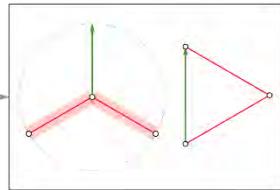
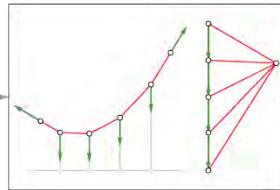
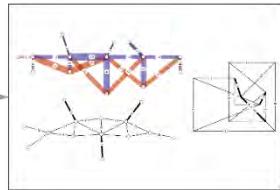
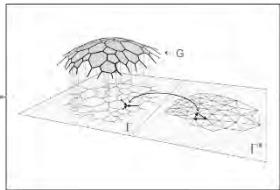
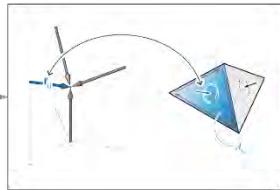
## Recap of exercise 2

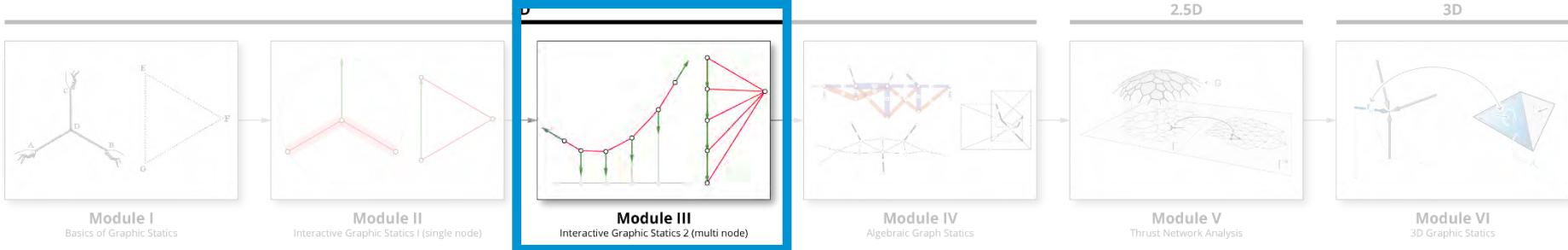
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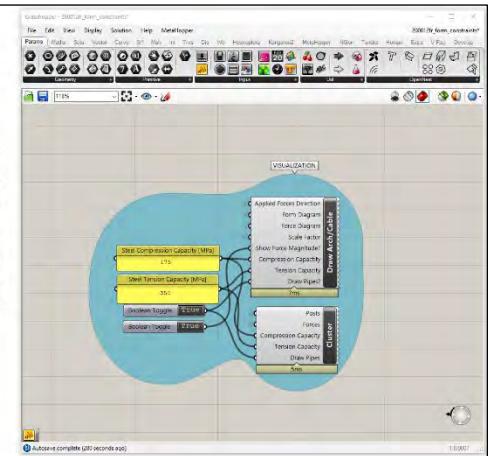
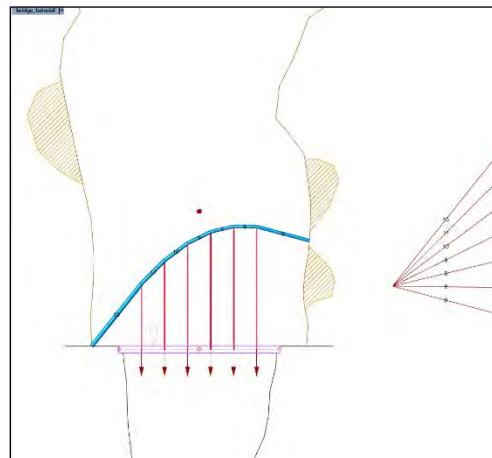
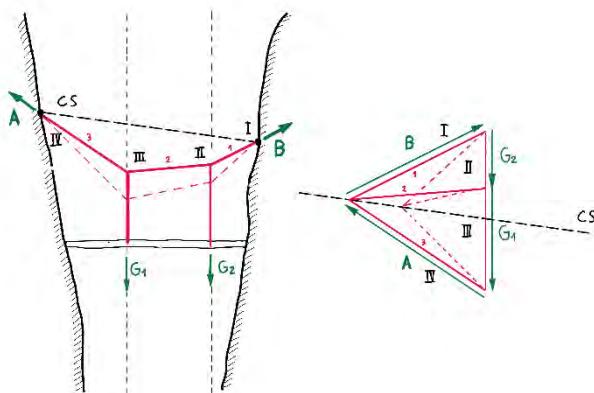
## Course schedule

Session Type/Week	Topic	Week	Day	Time	Lecturer
1. planar structures	cables & arches 2D GS (procedural)	Week 1 (9/24)	1	Lecture 1 <b>Introduction to graphic statics</b>	Dr. Juney Lee
		Week 2 (10/1)	1	Tutorial 1 Basics of graphic statics on "blackboard" Introduction to Rhino + Grasshopper (GH)	Dr. Lluís Enrique
		Week 2 (10/1)	2	Work session 1 <b>Exercise 1: Reciprocal diagrams in Rhino + GH basics</b>	Lotte Aldinger Instructors
		Week 3 (10/8)	1	Quick recap of Exercise 1	Lotte Aldinger
		Week 3 (10/8)	1	Lecture 2 <b>Algorithmic design &amp; thinking</b>	Dr. Juney Lee
		Week 3 (10/8)	2	Tutorial 2 Single node bridge in GH (form to force)	Lotte Aldinger
		Week 3 (10/8)	3	Work session 2 <b>Exercise 2: Single node bridge with GH</b>	Lotte Aldinger + Instructors
		Week 4 (10/15)	1	Quick recap of Exercise 2	Lotte Aldinger
		Week 4 (10/15)	1	Lecture 3 <b>Computational graphic statics</b>	Dr. Juney Lee
		Week 4 (10/15)	2	Tutorial 3 Multi-node bridge in Grasshopper	Dr. Lluís Enrique
		Week 5 (10/22)	1	Work session 3 <b>Exercise 3: Multi-node bridge with GH</b>	Ricardo Avelino + Instructors
trusses	2D GS (AGS)	Week 6			seminar week
		Week 7 (11/5)	1	Quick recap of Exercise 3	Dr. Lluís Enrique
		Week 7 (11/5)	1	Lecture 4 <b>Algebraic graph statics (AGS)</b>	Dr. Juney Lee
		Week 7 (11/5)	2	Tutorial 4 compas_aggs + IGS (Interactive Graphic Statics) plugin	Ricardo Avelino
		Week 8 (11/12)	1	Work session 4 <b>Exercise 4: Truss problems with IGS</b>	Ricardo Avelino + Instructors
2. surface structures	shells 2.5 GS (TNA)	Week 9 (11/19)	1	Quick recap of Exercise 4	Ricardo Avelino
		Week 9 (11/19)	1	Lecture 5 <b>Thrust Network Analysis (TNA)</b>	Dr. Juney Lee
		Week 9 (11/19)	2	Tutorial 5 compas_tna + RV2 (RhinoVIAU.T 2) plugin	Dr. Juney Lee
		Week 10 (11/26)	1	Work session 5 <b>Exercise 5: Shell design exercises with RV2</b>	Dr. Juney Lee + instructors
		Week 10 (11/26)	2	Quick recap of Exercise 5	Dr. Juney Lee
3. spatial structures	polyhedral structures 3D GS (polyhedral)	Week 11 (12/3)	1	Lecture 6 <b>3D graphic statics (3GS)</b>	Dr. Juney Lee
		Week 11 (12/3)	2	Tutorial 6 compas_3gs + 3GS (3D graphic statics) plugin	Dr. Juney Lee
		Week 11 (12/3)	3	Work session 6 <b>Exercise 6: Spatial structures with 3GS</b>	Dr. Juney Lee
		Week 12 (12/10)	1	Quick recap of Exercise 6	Dr. Juney Lee
		Week 12 (12/10)	1	Lecture 7 <b>Pt I. Outlook on computational graphic statics</b>	Dr. Juney Lee
		Week 12 (12/10)	2	Pt II. Guest Lecture	TBD
		Week 12 (12/10)	2	Q/A	Discussions, feedback, evaluations, etc.

**2D****Module I**  
Basics of Graphic Statics**Module II**  
Interactive Graphic Statics I (single node)**Module III**  
Interactive Graphic Statics 2 (multi node)**Module IV**  
Algebraic Graph Statics**2.5D****Module V**  
Thrust Network Analysis**3D****Module VI**  
3D Graphic Statics



## Module III · 2D procedural graphic statics 2



063-0605-00L : Computational Structural Design 1  
Computational Graphic Statics

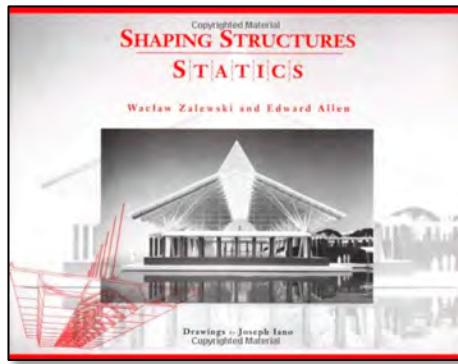
# Lecture 3

# Computational Graphic Statics

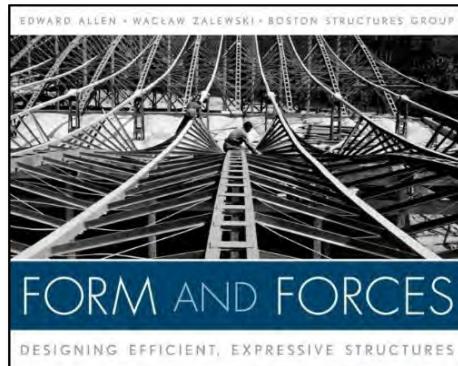
Friday, October 15th, 2021

Dr. Juney Lee

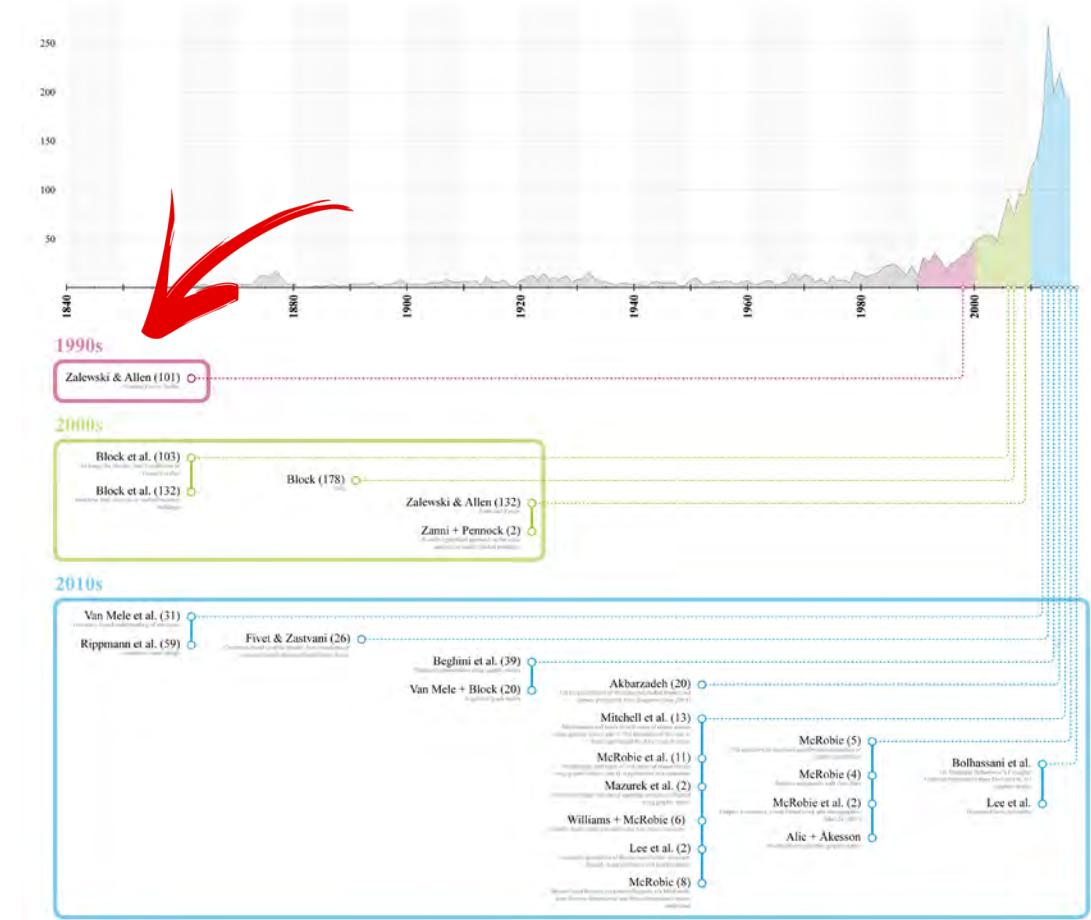
# Graphic Statics

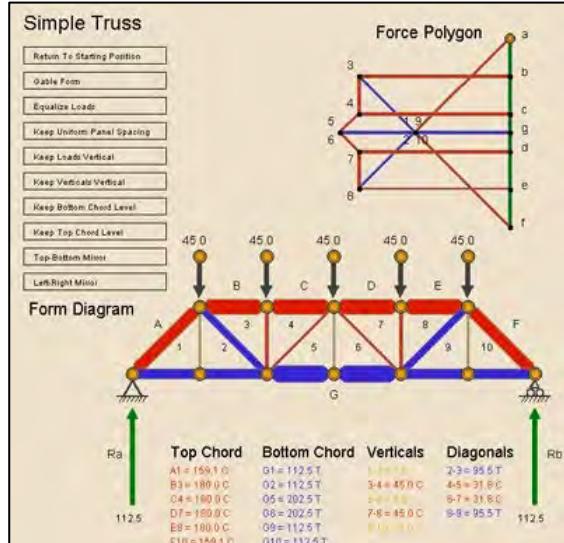


**Shaping Structures**  
Zalewski and Allen (1997)



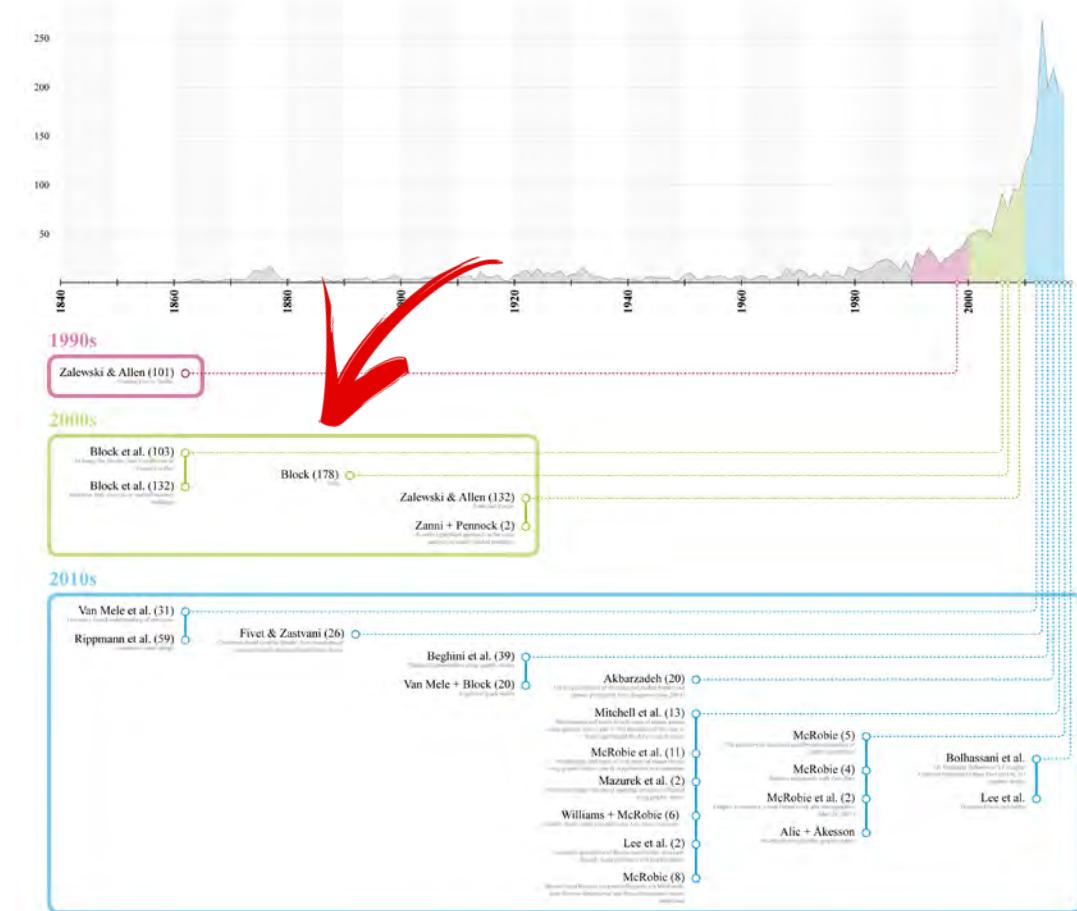
**Form and Forces**  
(Allen and Zalewski (2010))

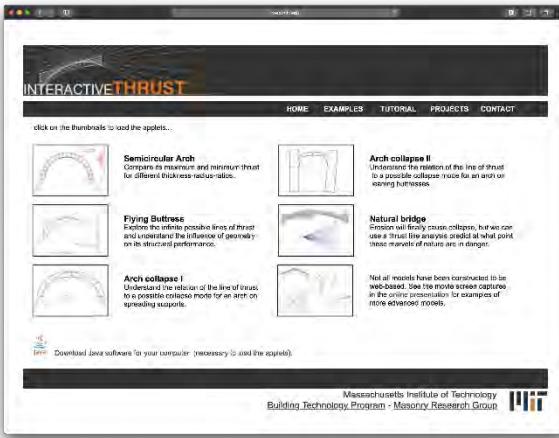




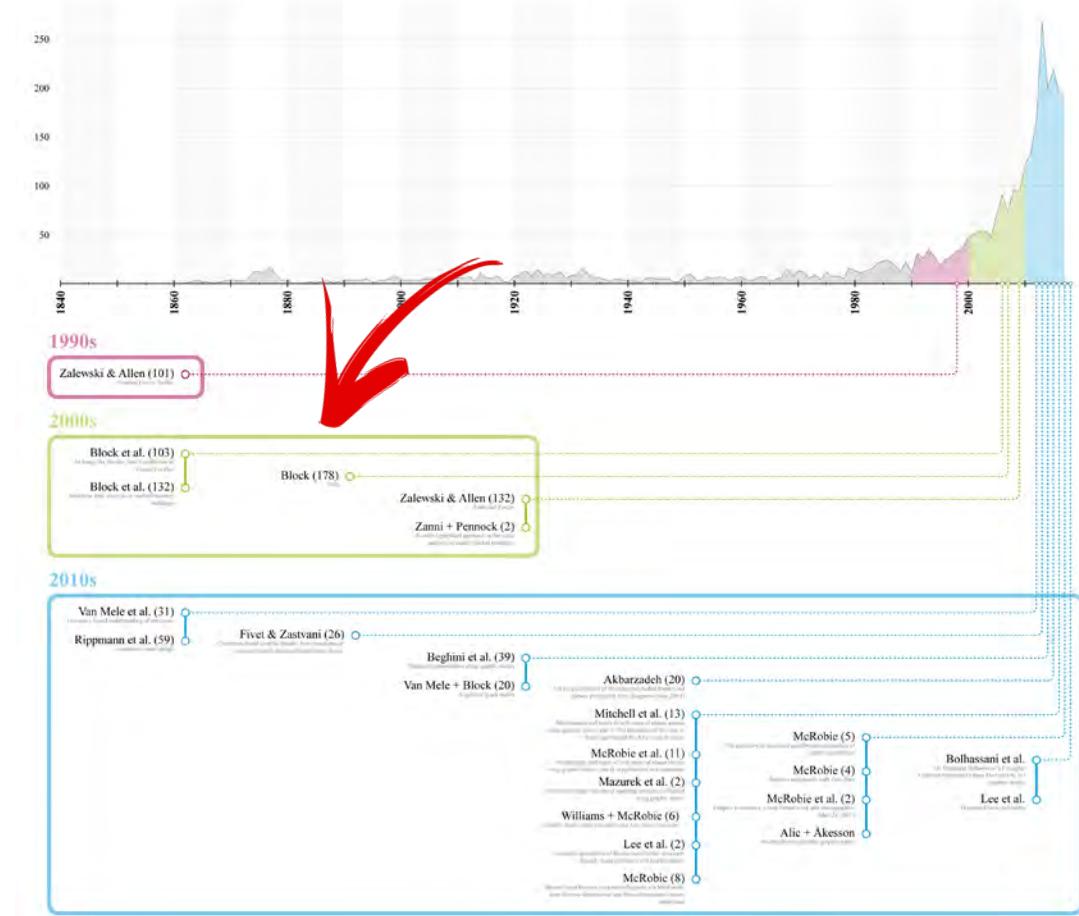
### Active Statics

Greenwald and Allen (2003)





**InteractiveTHRUST**  
Block and Ochsendorf (2005)





eEQUILIBRIUM

an interactive environment for graphic statics-based structural design

my navigation ▾

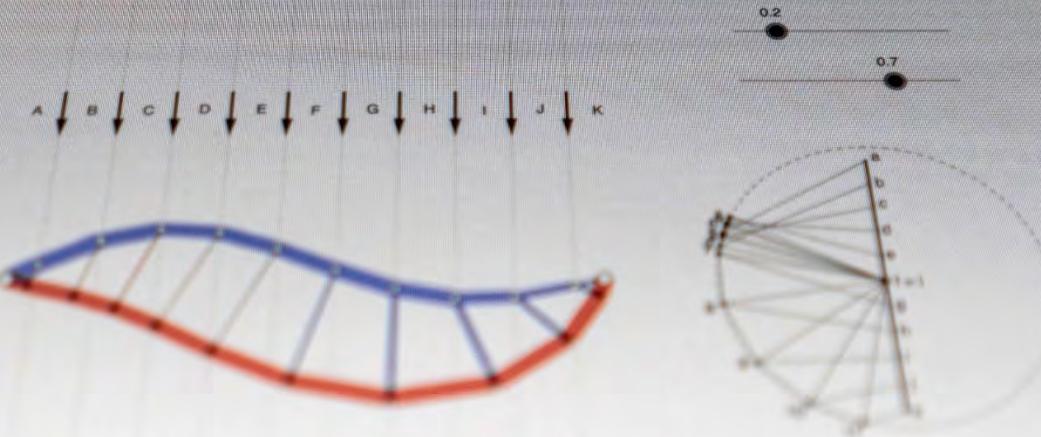
examples

case studies

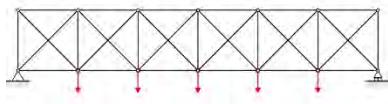
drawings

courses

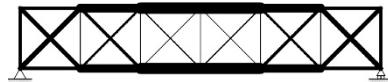
exams



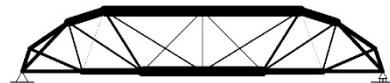
## Form analysis



Pre-determined Shape



Size



Shape / Geometry

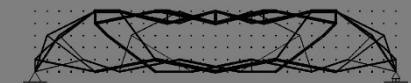
## Form generation



Unknown Shape



Topology (Continuum)



Topology (Discrete)

## Form analysis

### Interactive (parametric)

Analysis

Constraint-driven design

### Computational graphic statics

Algebraic graph(ic) statics (AGS)

Bi-directional AGS

### Optimization

## Form generation

### Form finding

Thrust network analysis (TNA)

3D graphic statics

### Generative design

Subdivision graphic statics

Shape grammars

## Visualisation

...

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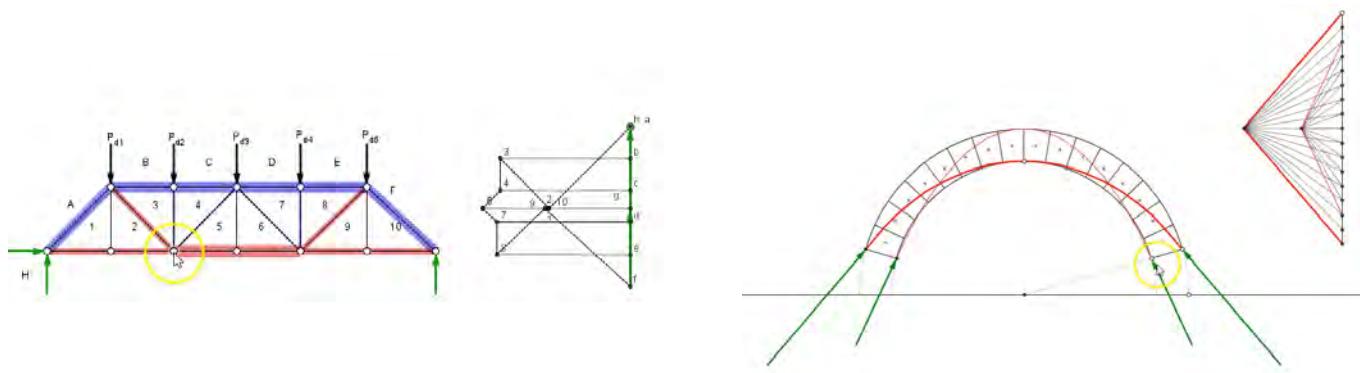
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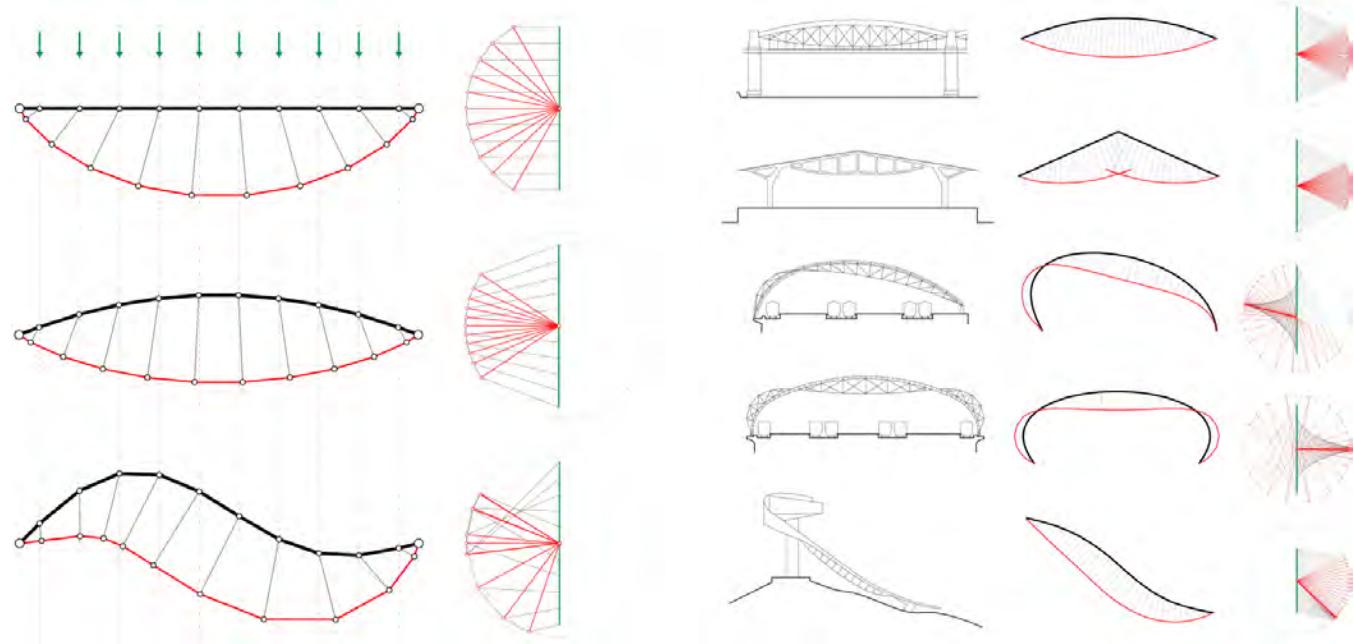
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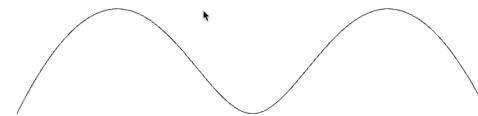
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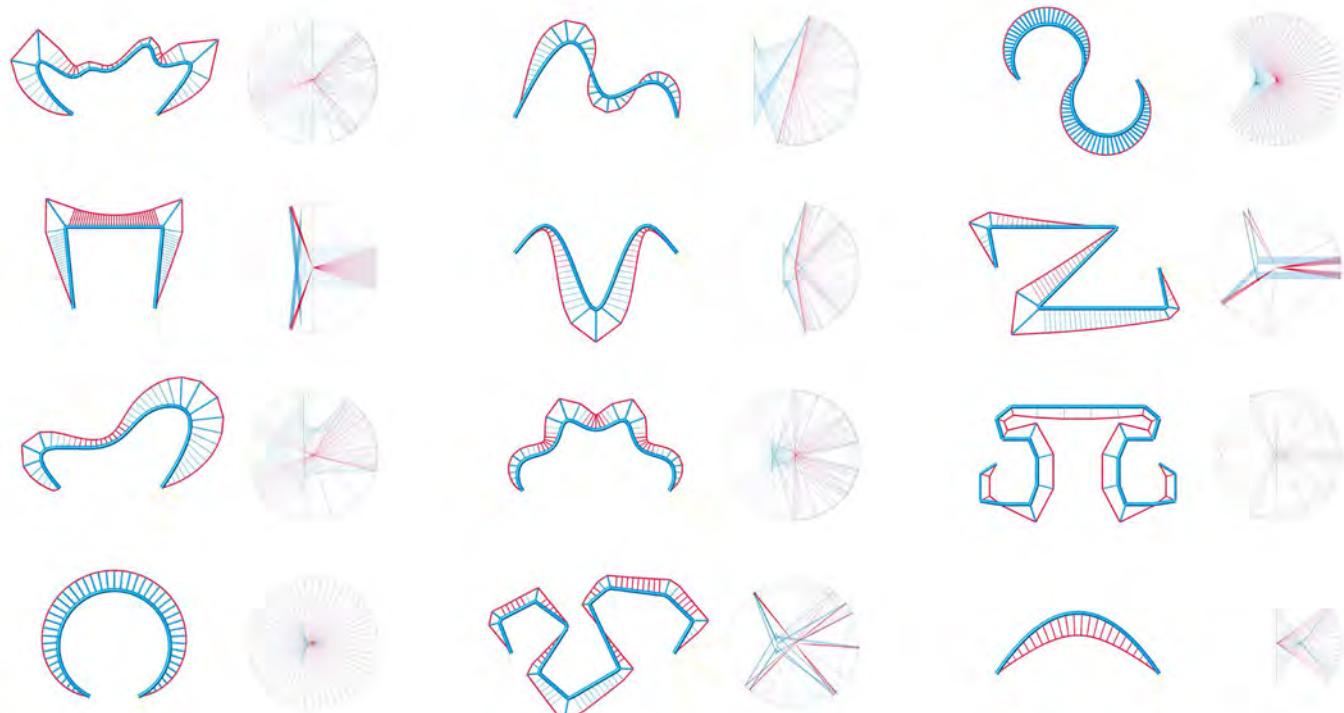
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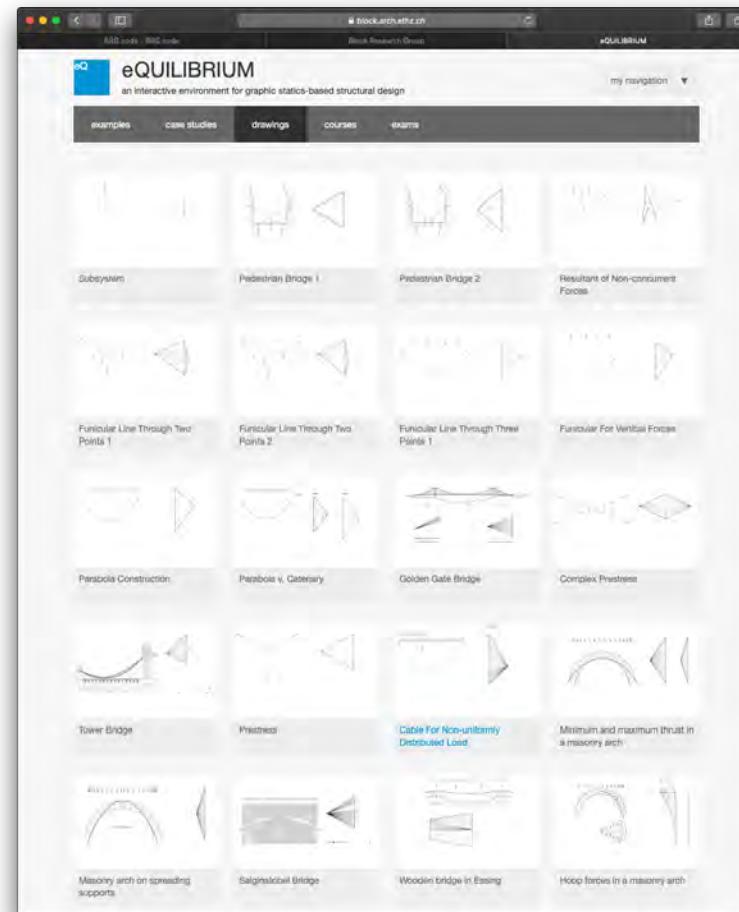
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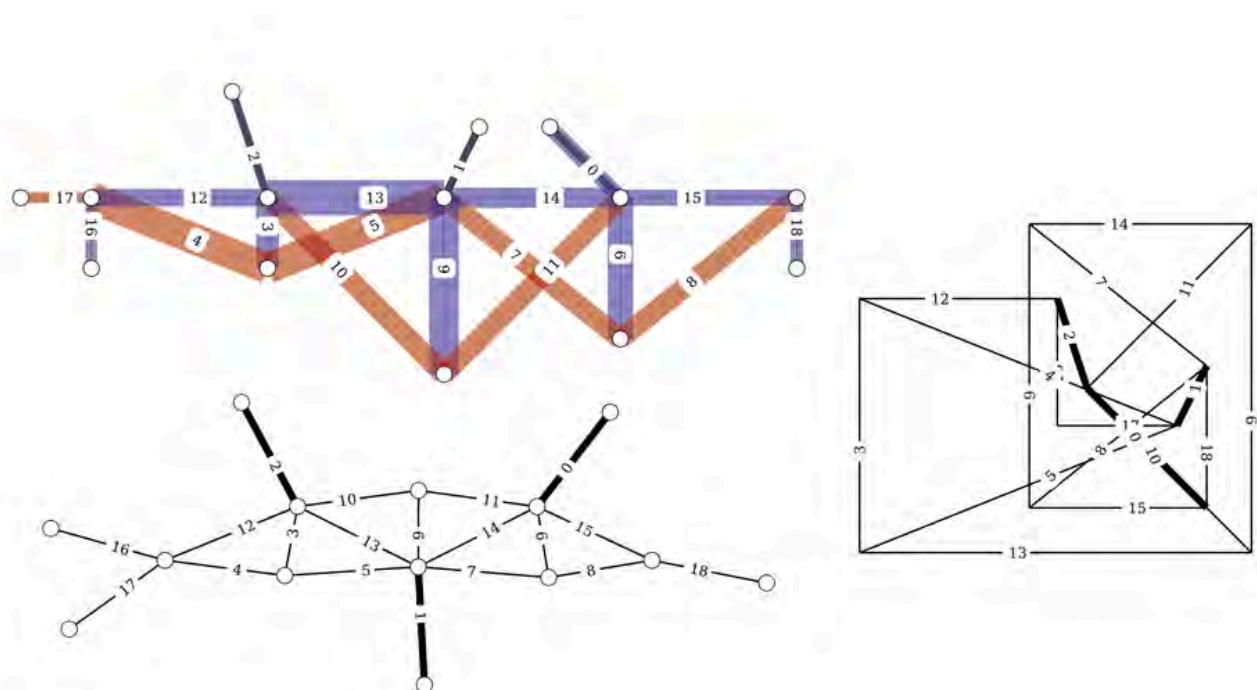
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**Algebraic graph(ic) statics (AGS)**  
Bi-directional AGS

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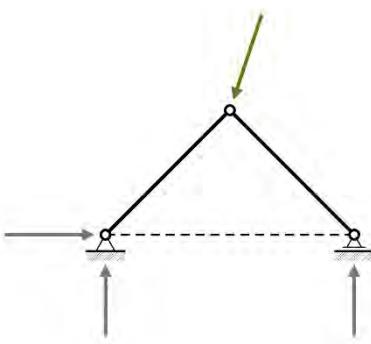
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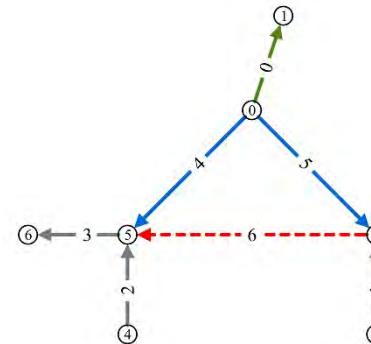
### Generative design

Subdivision graphic statics  
Shape grammars

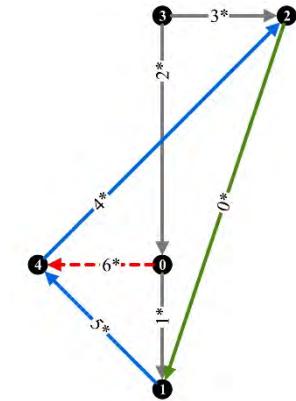
## Visualisation



**Form diagram**



**Form diagram as a graph**



**Force diagram**

Get topology and connectivity

Solve via equilibrium matrix

## Form analysis

### Interactive (parametric)

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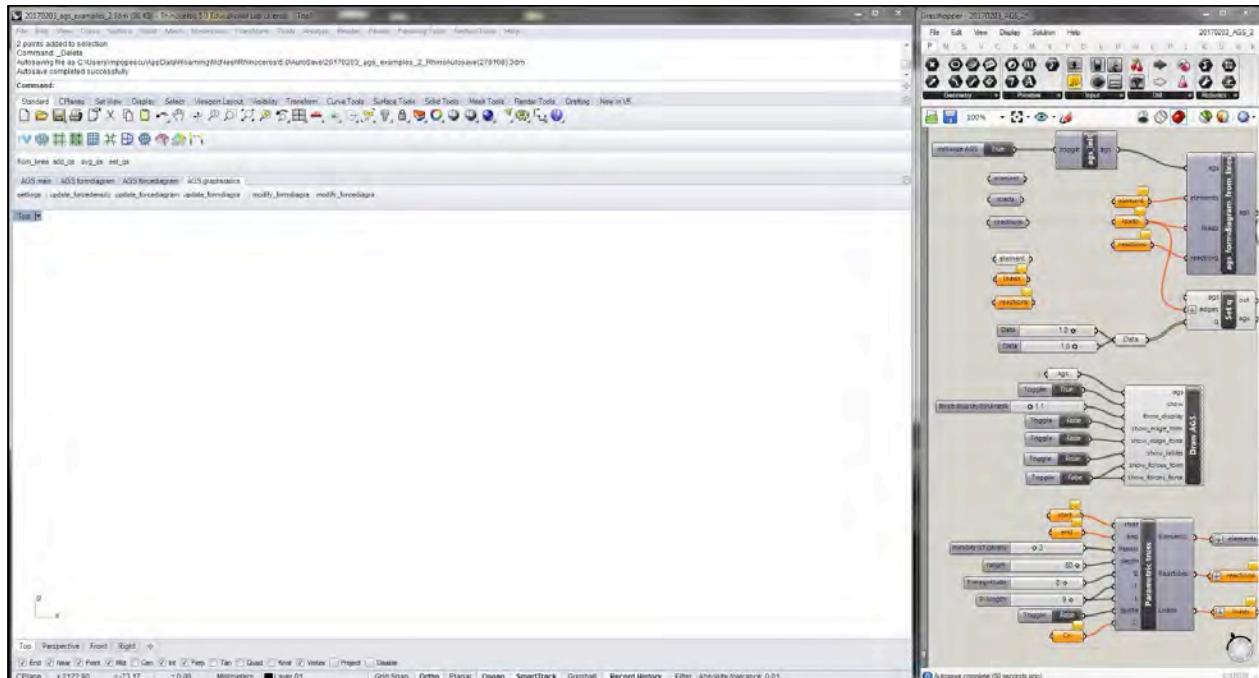
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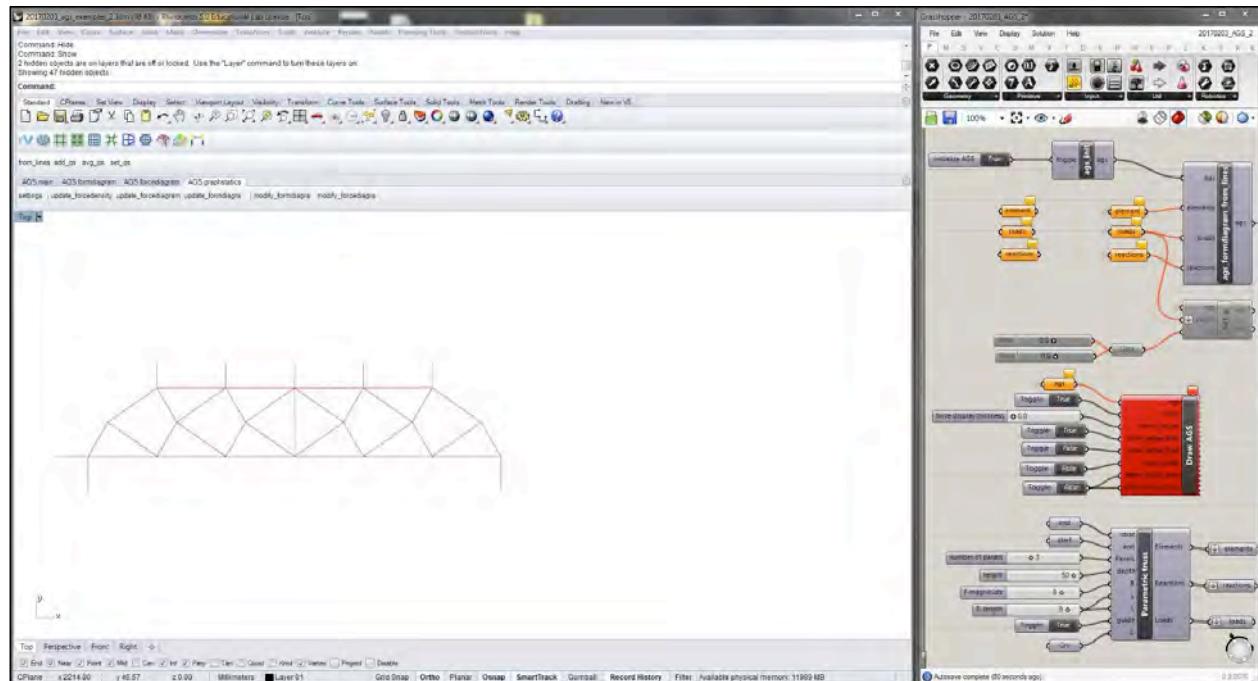
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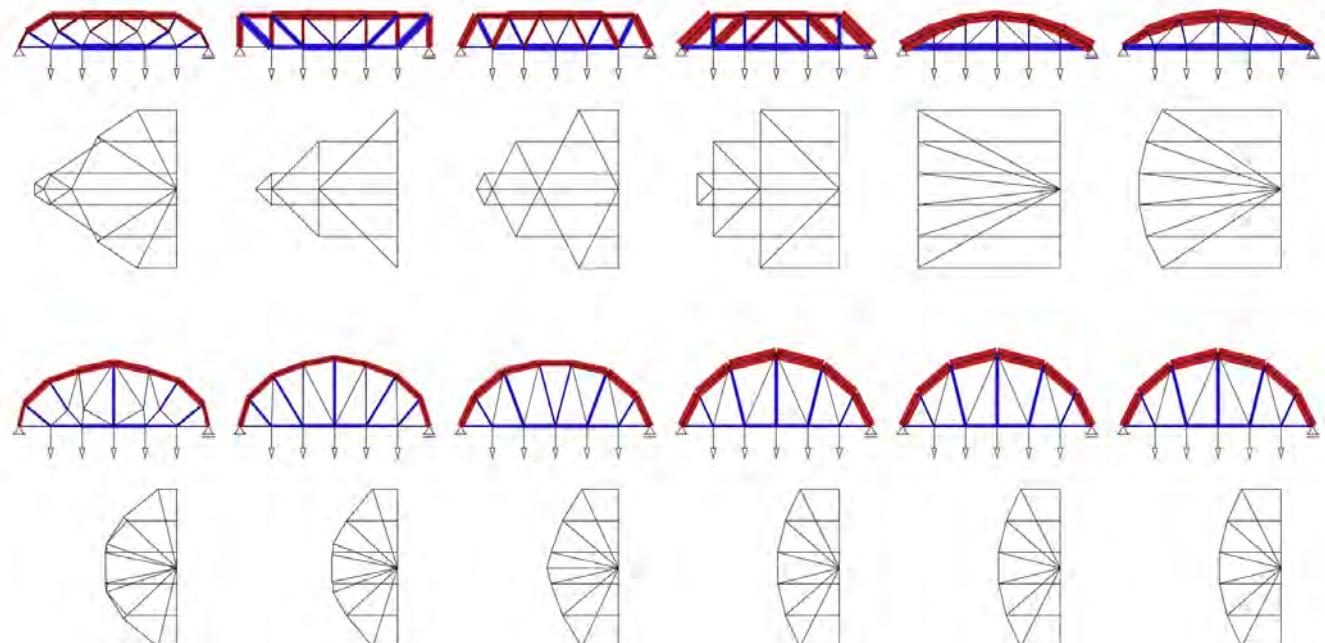
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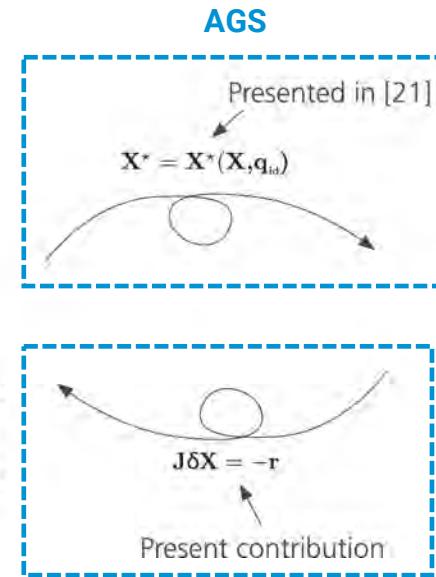
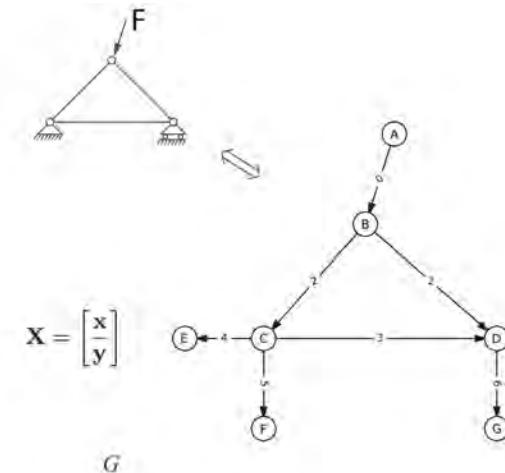
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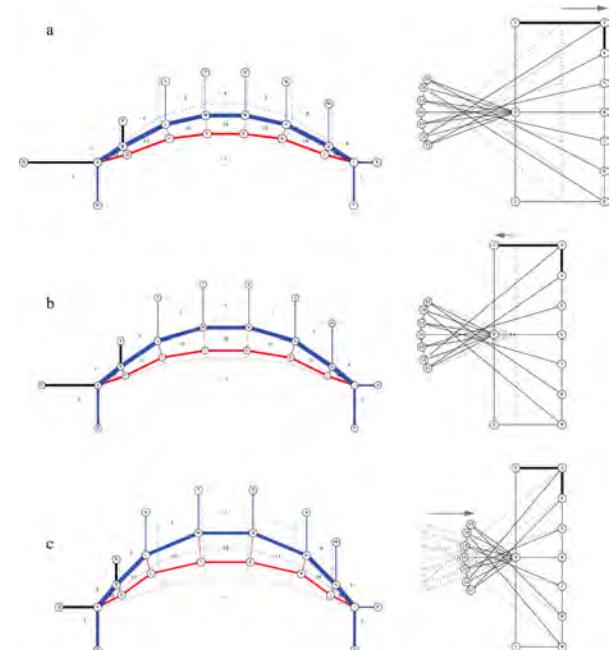
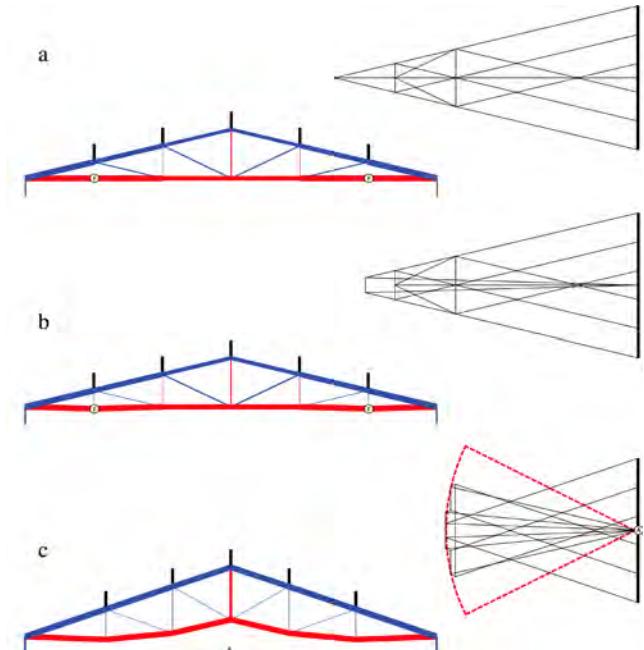
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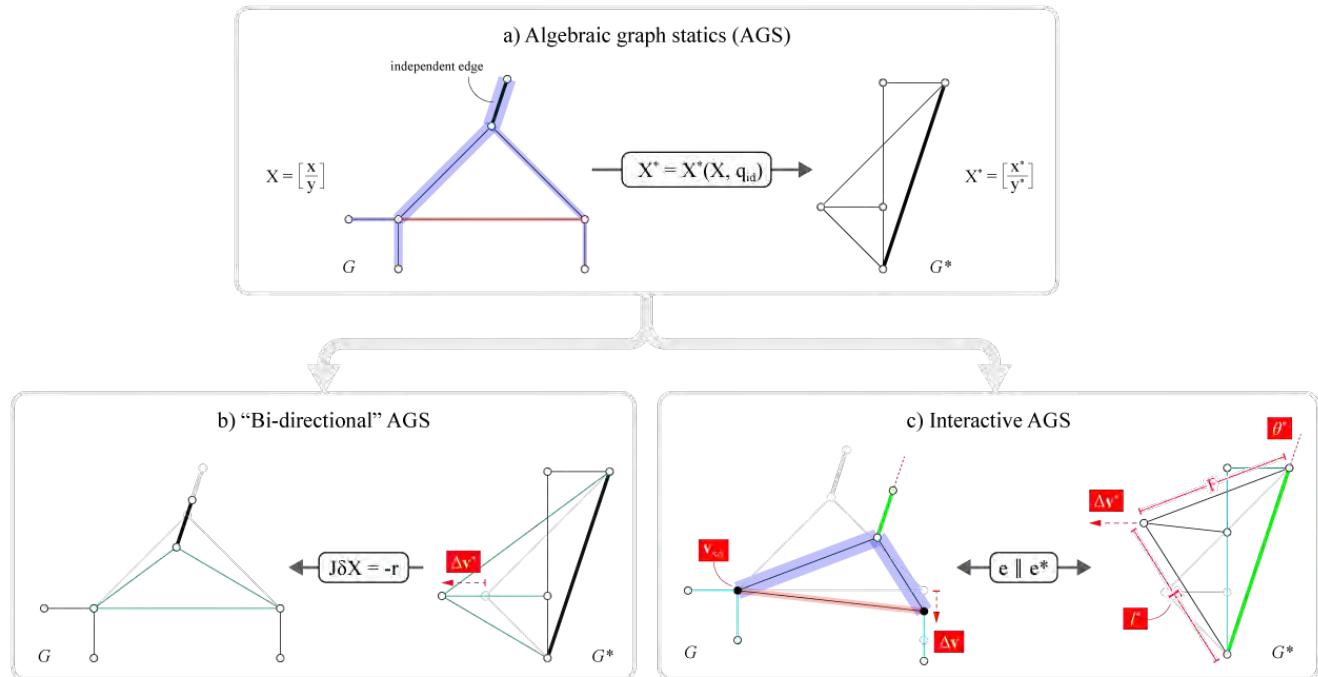
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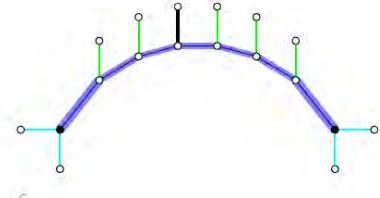
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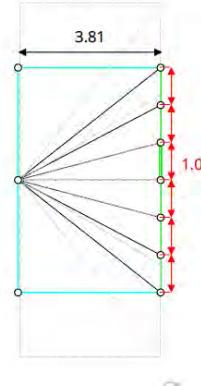
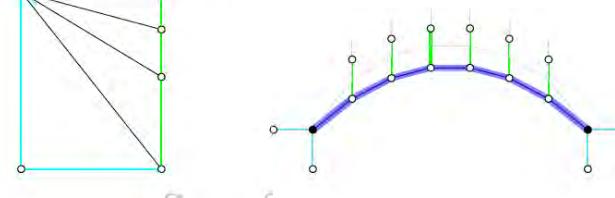
a)



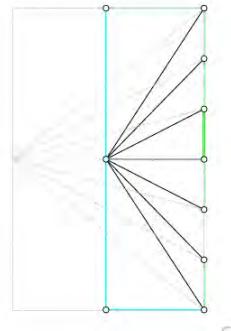
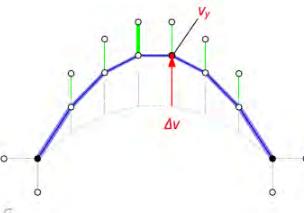
3.75

1.0

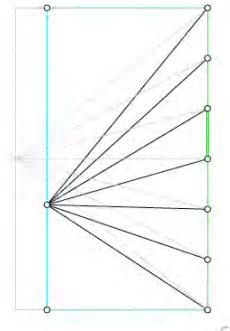
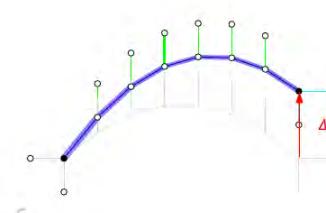
b)



a)



b)



## Form analysis

### Interactive (parametric)

- Analysis
- Constraint-driven design

### Computational graphic statics

- Algebraic graph(ic) statics (AGS)
- Bi-directional AGS

### Optimization

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- Thrust network analysis (TNA)
- 3D graphic statics

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- Subdivision graphic statics
- Shape grammars

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...

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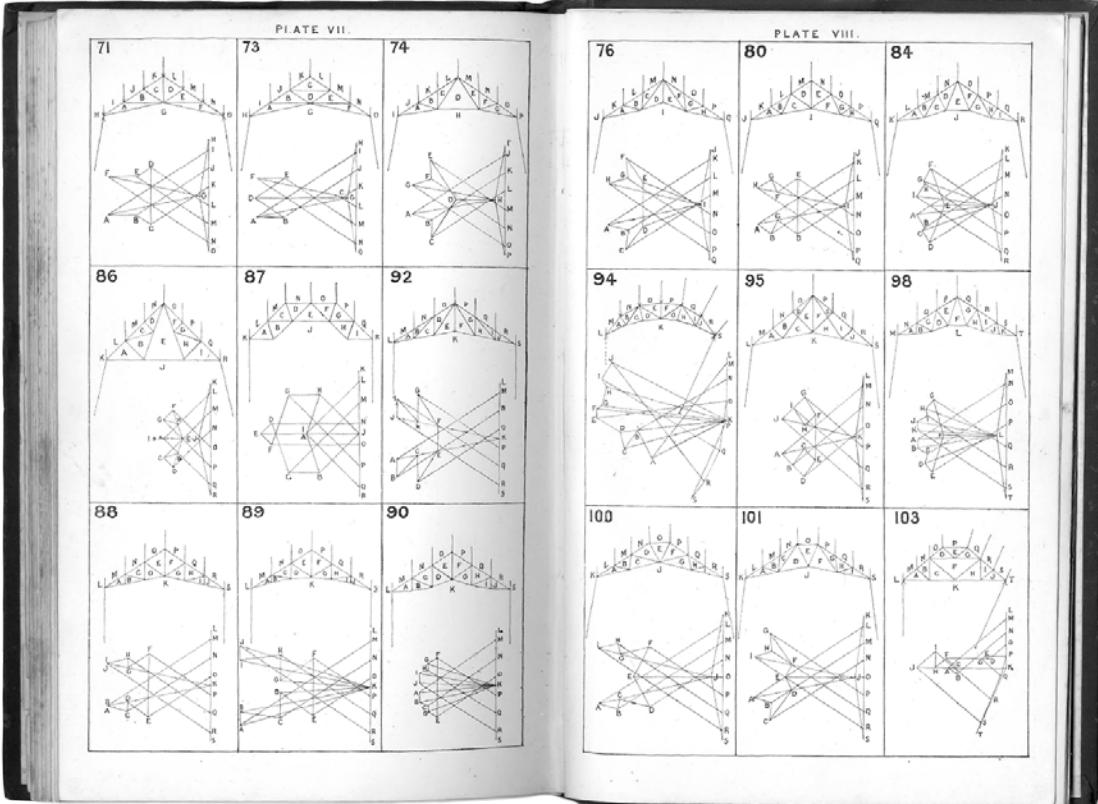
3D graphic statics

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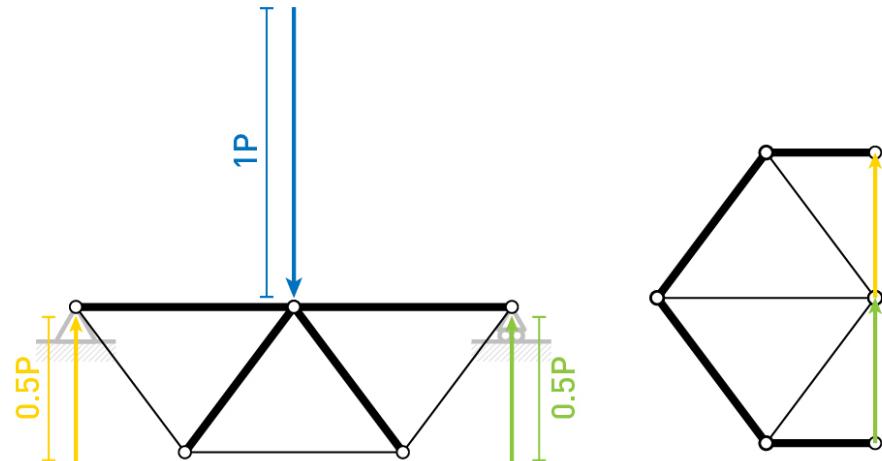
### Generative design

- Subdivision graphic statics
- Shape grammars

## Visualisation

Minimum Volume = Minimum Load Path

$$\min V = \min \frac{1}{\sigma} \sum |F_i| \cdot L_i$$



$$\begin{aligned}\Sigma F_L &= 1.0 \\ z_{FL} &= 1.0\end{aligned}$$

## Form analysis

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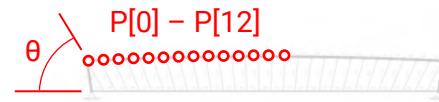
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- Shape grammars

## Visualisation

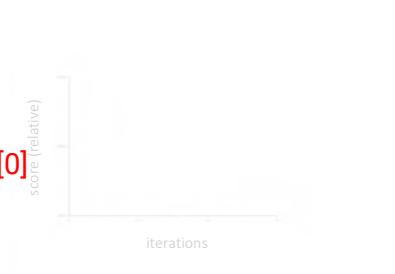
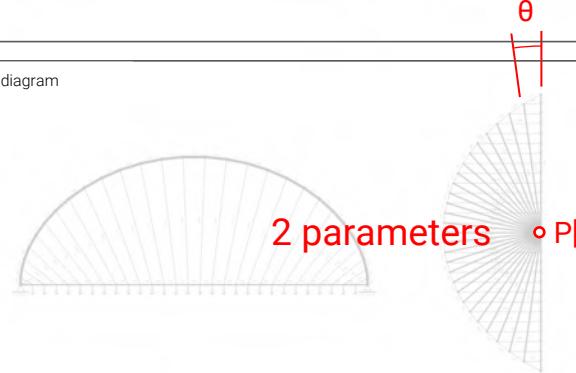
Optimization using numerical methods

14 parameters



Optimization using force diagram

2 parameters



## Form analysis

### Interactive (parametric)

- Analysis
- Constraint-driven design

### Computational graphic statics

- Algebraic graph(ic) statics (AGS)
- Bi-directional AGS

### Optimization

## Form generation

### Form finding

- Thrust network analysis (TNA)
- 3D graphic statics

### Generative design

- Subdivision graphic statics
- Shape grammars

## Visualisation



Bill Baker of Skidmore, Owings and Merrill (SOM)

## Form analysis

### Interactive (parametric) analysis

Analysis

Constraint-driven design

### Computational graphic statics

Algebraic graph(ic) statics (AGS)

Bi-directional AGS

### Optimization

## Form generation

### Form finding

Thrust network analysis (TNA)

3D graphic statics

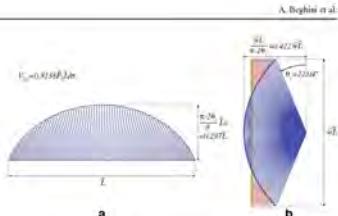
### Generative design

Subdivision graphic statics

Shape grammars

## Visualisation

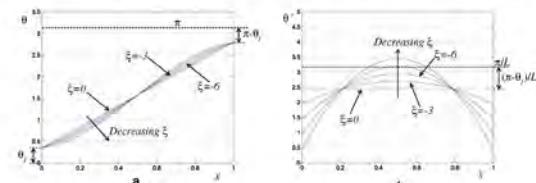
**Fig. 14** Half-space pin-pin bridge truss problem:  
a geometry of the resulting optimum and b corresponding force diagram  
 $V_{tot} = 0.8138 P_F L \beta / 3\pi e$



point (see Appendix for a proof). Using the approximation in (17) we obtain:

$$H = \frac{L}{4} \tan\left(\frac{\pi}{4} + \frac{\theta_1}{2}\right) \quad (36)$$

Therefore, the total volume of the truss in (33) using (17) is still described by only one parameter, the initial angle,  $\theta_1$ . The volume of the truss has been solved numerically and the final geometry and force diagrams are indicated in Fig. 14. The optimum is found for  $\theta_1 = 22.04^\circ$ , which corresponds to  $H = 0.371\bar{a}L$ ,  $\phi = 36.58^\circ$  and a volume  $V_{tot} = 0.8138 P_F L / \beta$ . The depth of the truss and its dual based on (24) and (26) are  $y_m = 0.2966L$  and  $y_h = 0.4215\bar{a}L$ . The force diagram Fig. 14(b) shows that the force in the bridge horizontal tie is zero at the quarter points, consistent with the results presented by McConnel (1974).



**Fig. 15** a Plot of the expanded approximation for function  $\theta(x)$  and b its derivative

Springer

Layout of a least weight single span structure with uniform load

**Table 3** Optimal bridge truss with finite number of bars (small red circles indicate the numerical divisions from Beghini et al. 2013a)

Number of Bars	Force Diagram using (17)	Force Diagram using (17)	$\sigma V_{tot} / P_F L$ from Beghini et al. (2013a)	$\sigma V_{tot} / P_F L$ using (17)	$\sigma V_{tot} / P_F L$ using (17)
6			1.176	1.175	$\theta_1 = 0.0^\circ$ ; $\bar{a} = 1.25$
8			1.149	1.149	$\theta_1 = 0.0^\circ$ ; $\bar{a} = 0.91$
12			1.072	1.070	$\theta_1 = 0.0^\circ$ ; $\bar{a} = 1.3$
16			1.081	1.079	$\theta_1 = 0.0^\circ$ ; $\bar{a} = 1.21$
24			1.090	1.079	$\theta_1 = -4.0^\circ$ ; $\theta_2 = -1.8^\circ$ ; $\bar{a} = 1.27$

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## Form analysis

### Interactive (parametric)

Analysis

Constraint-driven design

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## Visualisation

Maxwell's reciprocal diagrams and discrete Michell frames

**Fig. 5** The solution to the discrete Michell cantilever (Chan 1990; Mazzurek et al. 2011) is self-represented as a form diagram. Below is a geometric representation of the frame and force diagrams are the same

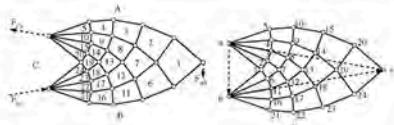


diagram in Fig. 2 is also the geometry of a second discrete optimal truss with its own loading is explored in Fig. 3. The geometry of the truss in Fig. 3 (the form diagram) is taken from the force diagram in Fig. 2.

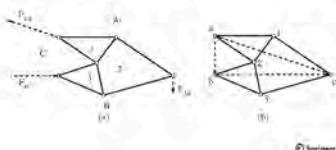
Now that two optimal truss geometries have been found, the remaining task is to find the external loads that correspond to a truss represented by the geometry of the force diagram. The external forces for this second truss are found by closing the 'open' polygons of the original form diagram in a continuous manner. The vector formed by closing an open polygon in the original form diagram is the external force that would be applied to the node for a force diagram that contains all the internal open polygons. Closing the open polygons provides the proper external forces for the dual structure because it closes the force polygon at the node which corresponds to the open polygon in the original form diagram.

It is clear that the form and force reciprocal diagrams in Figs. 2 and 3 are interchangeable. In order to make the relationships more clear, Bow's notation in Fig. 5 is reviewed.

For the problem in (1), the lengths are taken from the truss on the left and the internal forces from the truss on the right, though the reverse (i.e. internal forces from the left diagram, lengths from the right diagram) would result in the same optimal solution. The advantage to using Graphic Statics for this class of optimal problems is that it provides all of the information about the loads and the paths in a graphical manner.

Another example of paired or dual discrete trusses is given in Fig. 4, where the discrete truss on the left (form diagram) is the minimum load path structure for a given set of

**Fig. 6** The solution to the centrally-loaded beam (Michell 1904; Hemp 1973; Mazzurek et al. 2011) is self-represented as a form diagram & force diagram



Maxwell's reciprocal diagrams and discrete Michell frames

### Appendix

The following table presents a summary of the reciprocal relationships between the form and force diagrams of several optimal structures

	Form diagram	Force diagram	Dual truss
I			
II			
III			
IV			

I. Classical discrete Michell solution for the centrally-loaded beam (Michell 1904; Hemp 1973; Mazzurek et al. 2011)

II. Two discrete Michell solutions for semi-infinite space (Michell 1904; Hemp 1973)

III. Self-reciprocal cantilever with two points of support (Chan 1990; Mazzurek et al. 2011)

IV. Optimal shear bracing solution (Hemp 1973; Sironen et al. 2012). Note: Overlaying lines have been removed

## Form analysis

### Interactive (parametric)

- Analysis
- Constraint-driven design

### Computational graphic statics

- Algebraic graph(ic) statics (AGS)
- Bi-directional AGS

### Optimization

## Form generation

### Form finding

- Thrust network analysis (TNA)
- 3D graphic statics

### Generative design

- Subdivision graphic statics
- Shape grammars

## Visualisation



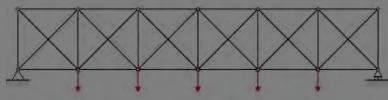
**Table 2** Optimization of the roof truss

Description	Form Diagram	Force Diagram	Normalized Volume
(a) Initial truss connectivity			1.000
(b) Benchmark truss; top chord, cantilevers and web members unconstrained			0.552
(c) Closed profiles constrained for architectural and functional reasons			0.629
(d) Depth constrained, straight web members (X-diagonals)			0.852
(e) Depth constrained			0.669

loads or the supports can change as the optimal solution is determined.

The authors are currently exploring other applications where the design variables are in the force domain.

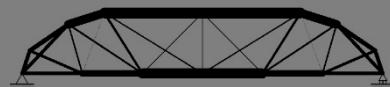
## Form analysis



Pre-determined Shape



Size



Shape / Geometry

## Form generation



Unknown Shape



Topology (Continuum)



Topology (Discrete)

## Form analysis

### Interactive (parametric)

- Analysis
- Constraint-driven design

### Computational graphic statics

- Algebraic graph(ic) statics (AGS)
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...

## Form analysis

### Interactive (parametric)

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Constraint-driven design

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## Form generation

### Form finding

Thrust network analysis (TNA)

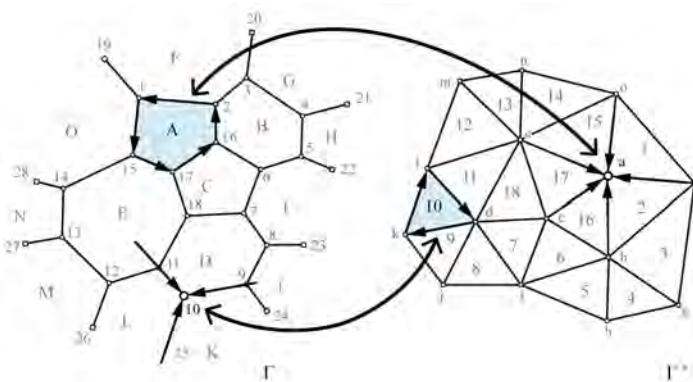
3D graphic statics

### Generative design

Subdivision graphic statics

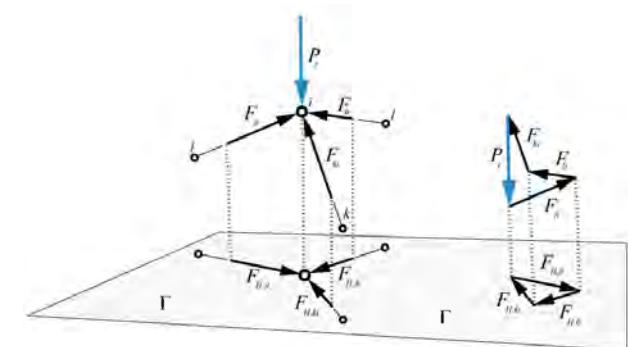
Shape grammars

## Visualisation



Form Diagram

Force Diagram



## Form analysis

### Interactive (parametric)

Analysis  
Constraint-driven design

### Computational graphic statics

Algebraic graph(ic) statics (AGS)  
Bi-directional AGS

### Optimization

## Form generation

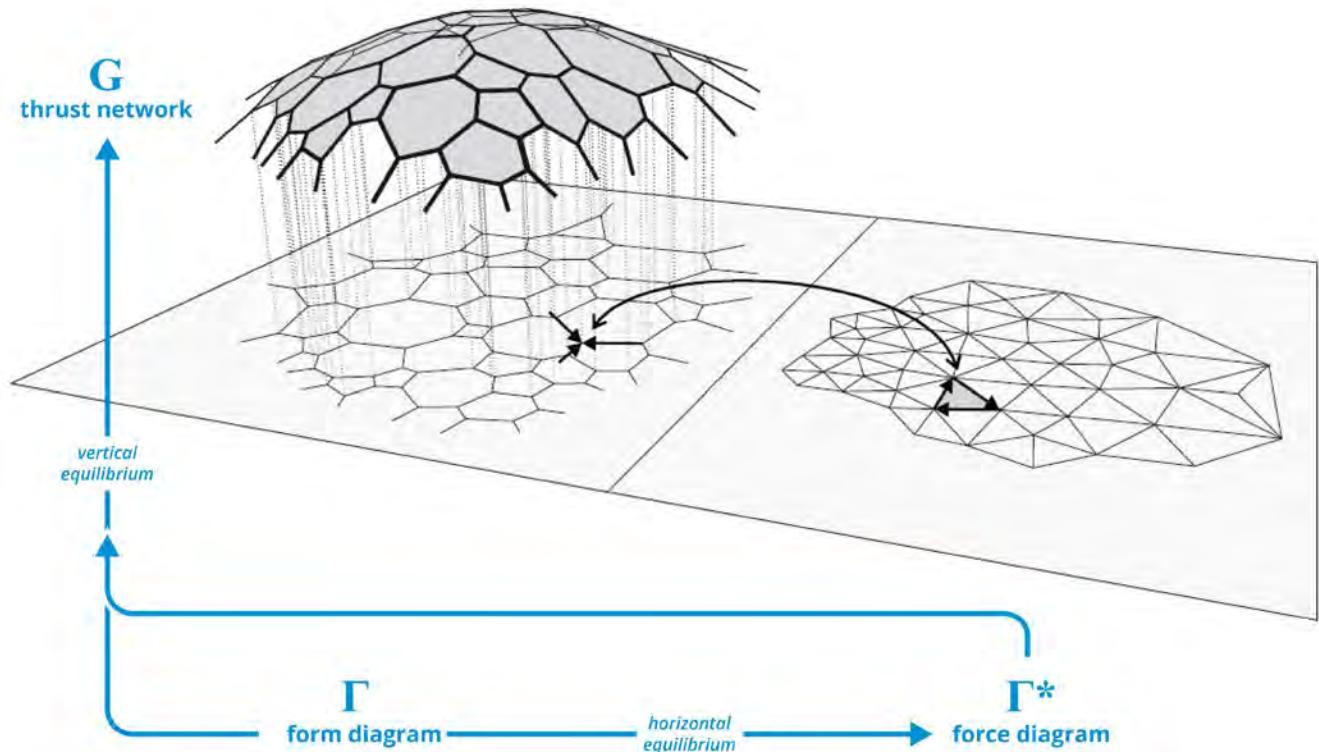
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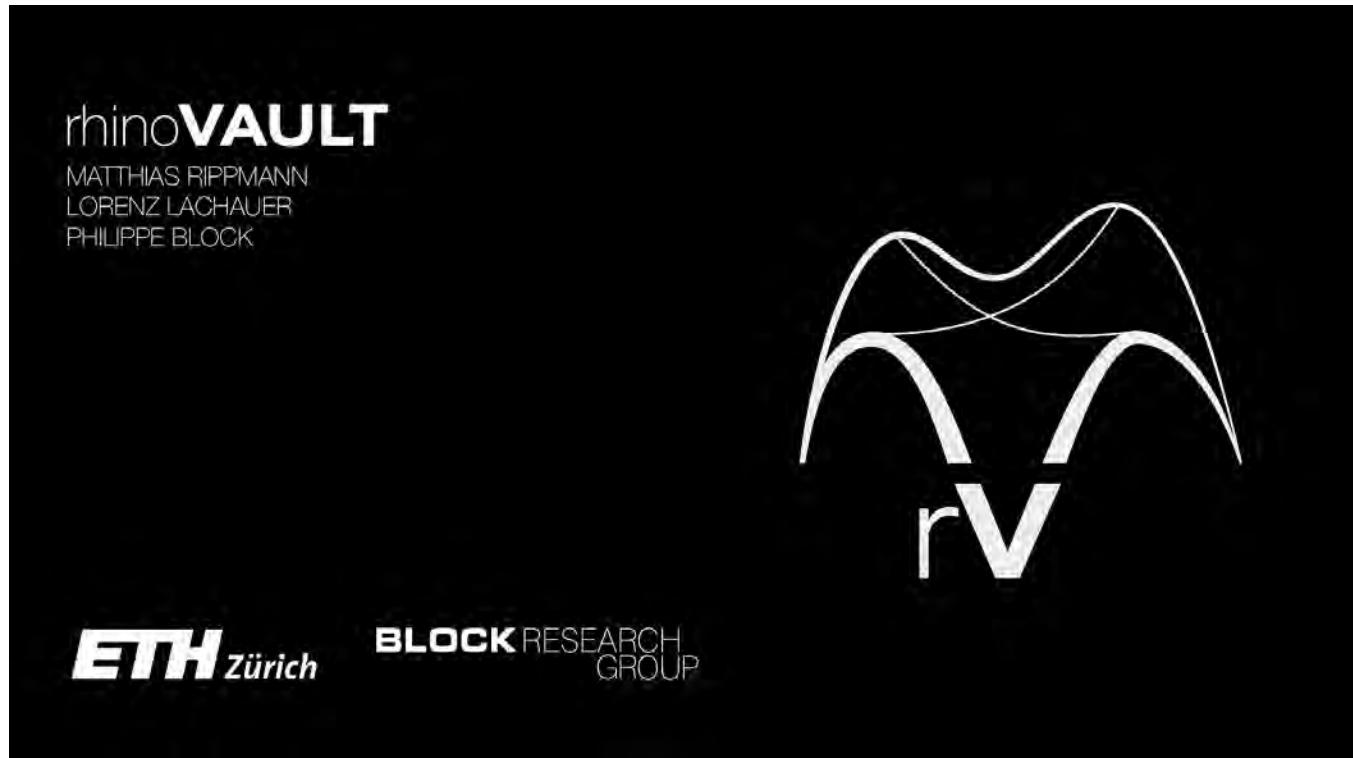
3D graphic statics

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## Form analysis

### Interactive (parametric)

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Algebraic graph(ic) statics (AGS)  
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### Optimization

## Form generation

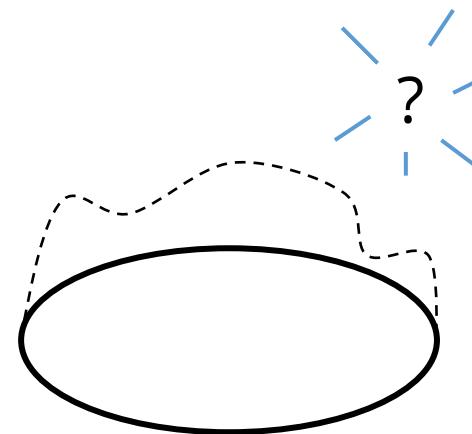
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Thrust network analysis (TNA)  
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## Form analysis

### Interactive (parametric)

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- Bi-directional AGS

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## Form generation

### Form finding

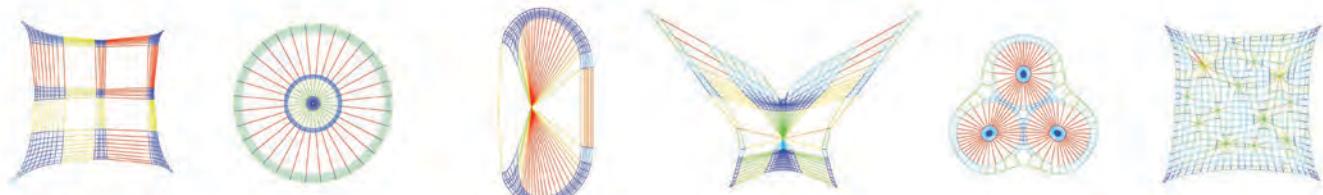
#### Thrust network analysis (TNA)

- 3D graphic statics

### Generative design

- Subdivision graphic statics
- Shape grammars

### Visualisation



## Form analysis

### Interactive (parametric)

- Analysis
- Constraint-driven design



### Computational graphic statics

- Algebraic graph(ic) statics (AGS)
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### Optimization



## Form generation

### Form finding

- [Thrust network analysis \(TNA\)](#)

- 3D graphic statics



### Generative design

- Subdivision graphic statics
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### Optimization

## Form generation

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Thrust network analysis (TNA)

[3D graphic statics](#)

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Subdivision graphic statics

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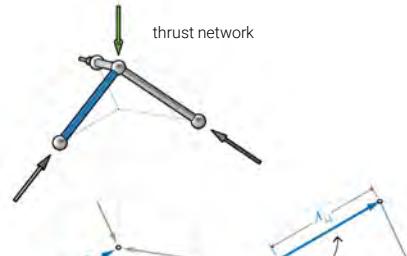


form diagram



force diagram

**2D**

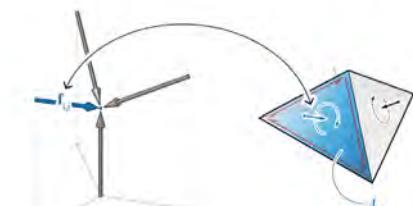


form diagram

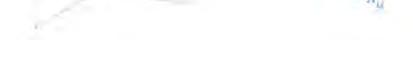


force diagram

**2.5D**



form diagram



force diagram

**3D**

## Visualisation

## Form analysis

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### Computational graphic statics

Algebraic graph(ic) statics (AGS)

Bi-directional AGS

### Optimization

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Thrust network analysis (TNA)

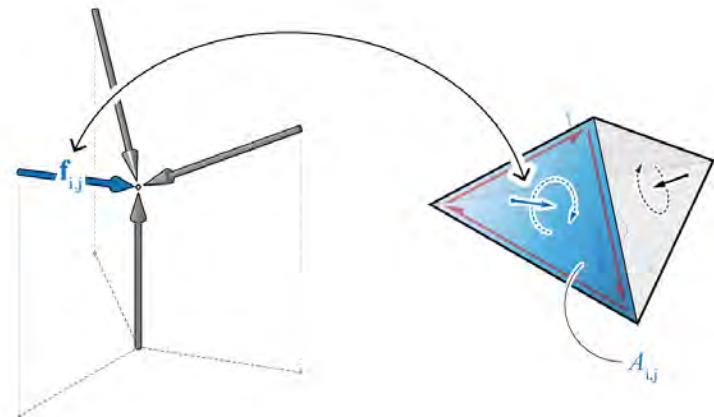
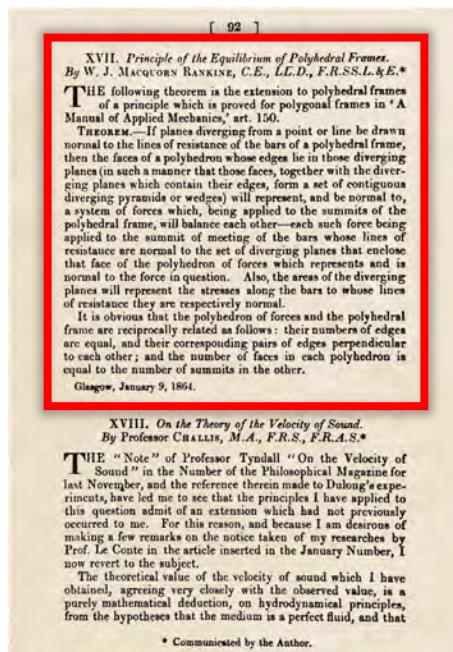
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### Form finding

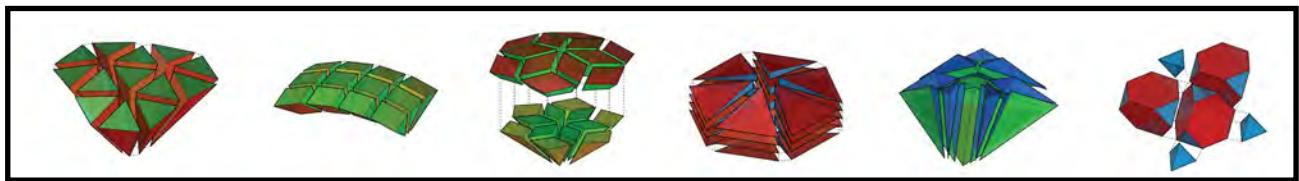
Thrust network analysis (TNA)  
[3D graphic statics](#)

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Subdivision graphic statics  
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## Visualisation

### Polyhedral force diagram



### Polyhedral form diagram

...

## Form analysis

### Interactive (parametric)

- Analysis
- Constraint-driven design

### Computational graphic statics

- Algebraic graph(ic) statics (AGS)
- Bi-directional AGS

### Optimization

## Form generation

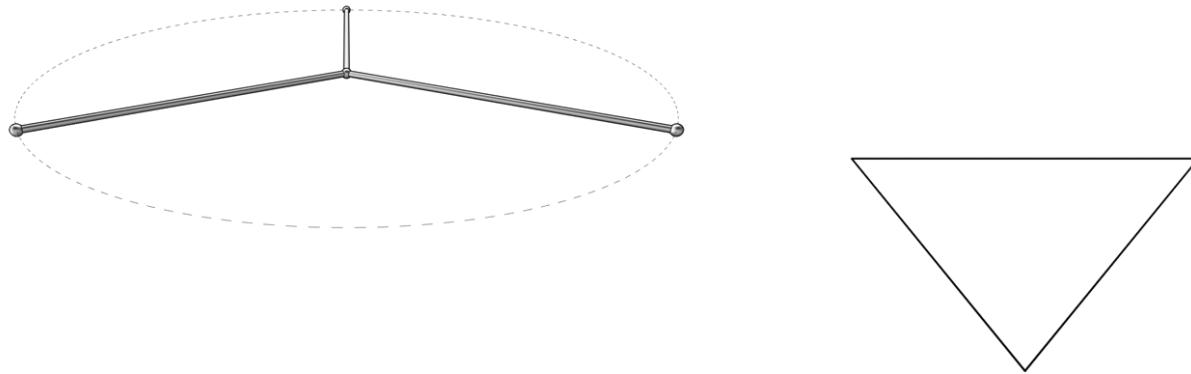
### Form finding

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- [3D graphic statics](#)

### Generative design

- Subdivision graphic statics
- Shape grammars

## Visualisation



## Form analysis

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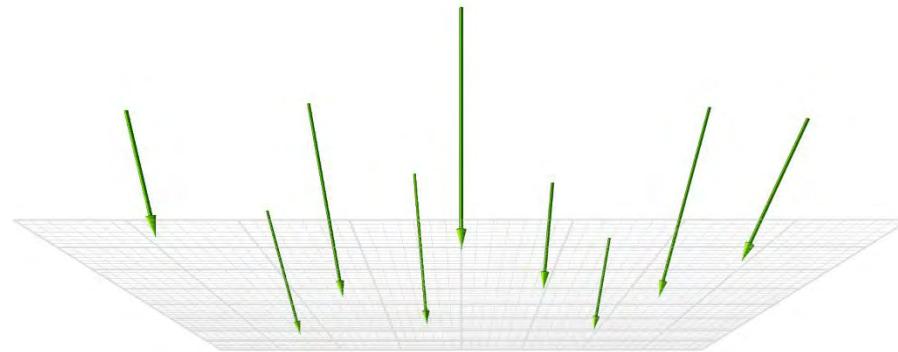
### Interactive (parametric)

Analysis  
Constraint-driven design

### Computational graphic statics

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### Optimization



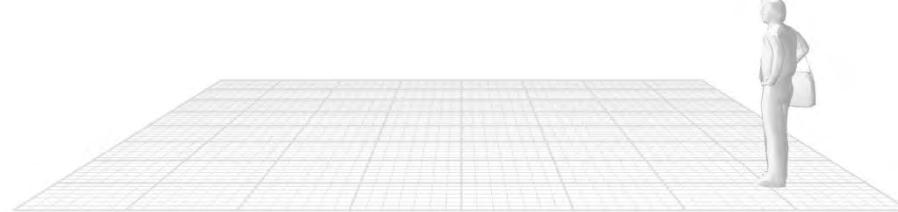
## Form generation

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## Visualisation

...

## Form analysis

### Interactive (parametric)

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### Optimization

## Form generation

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Thrust network analysis (TNA)

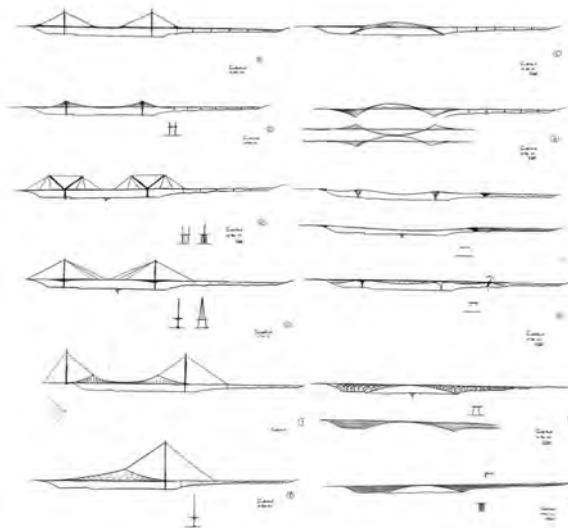
3D graphic statics

### Generative design

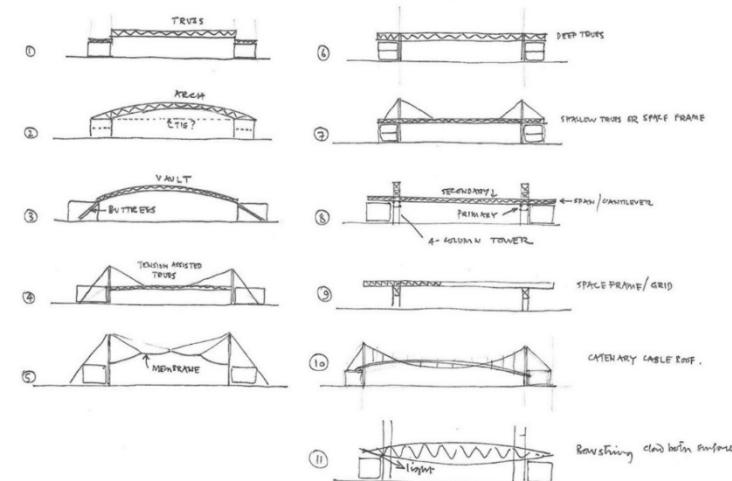
Subdivision graphic statics

Shape grammars

## Visualisation



Possible structural sections



## Form analysis

### Interactive (parametric)

Analysis

Constraint-driven design

### Computational graphic statics

Algebraic graph(ic) statics (AGS)

Bi-directional AGS

### Optimization

## Form generation

### Form finding

Thrust network analysis (TNA)

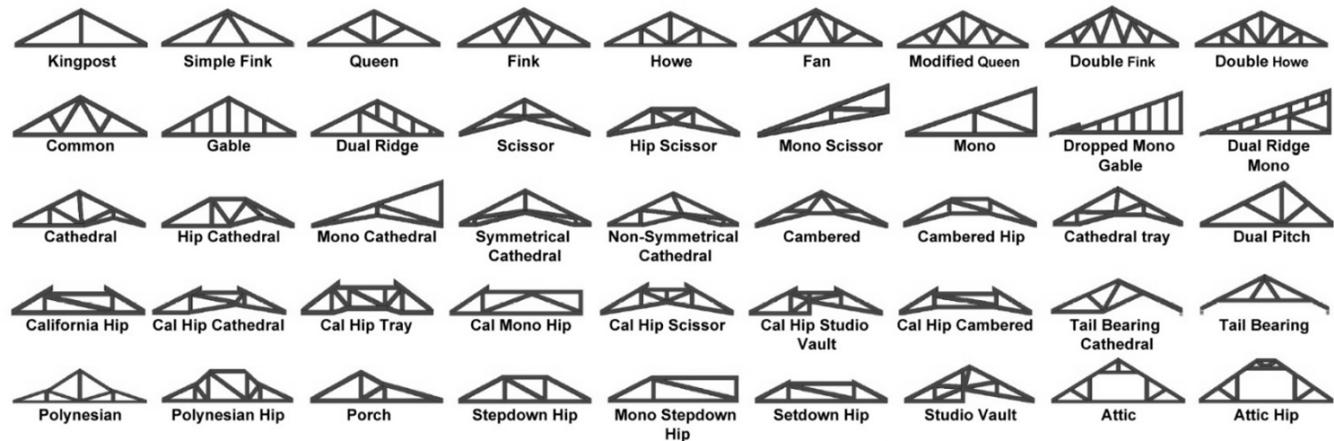
3D graphic statics

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## Visualisation



*"In order to arrive at new forms and concepts,  
we have to free ourselves from such pre-defined typologies."*

## Form analysis

### Interactive (parametric)

- Analysis
- Constraint-driven design

### Computational graphic statics

- Algebraic graph(ic) statics (AGS)
- Bi-directional AGS

### Optimization

## Form generation

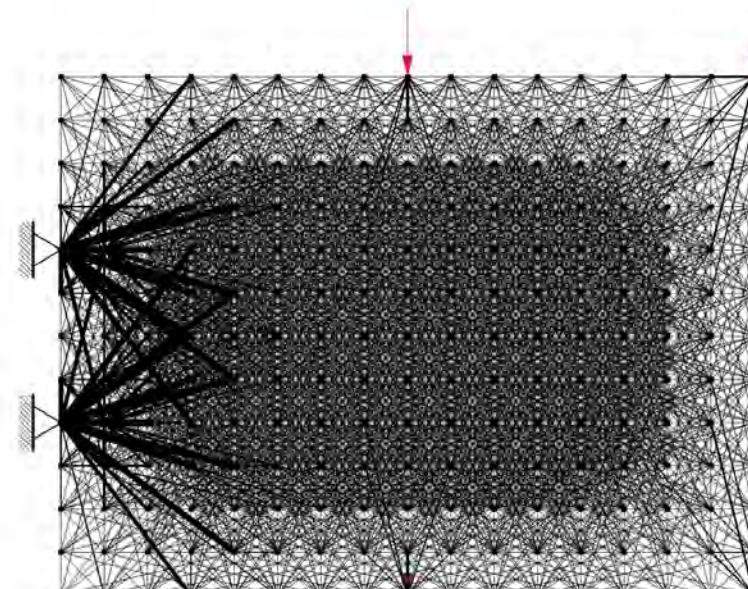
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- Thrust network analysis (TNA)
- 3D graphic statics

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## Form analysis

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Thrust network analysis (TNA)

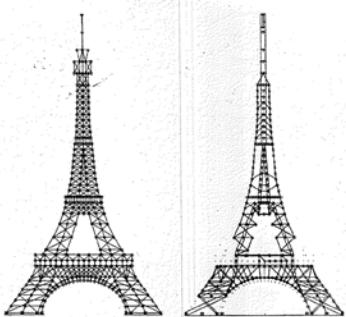
3D graphic statics

### Generative design

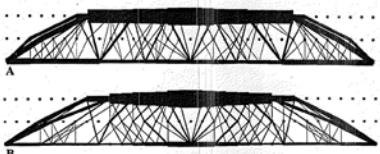
Subdivision graphic statics

Shape grammars

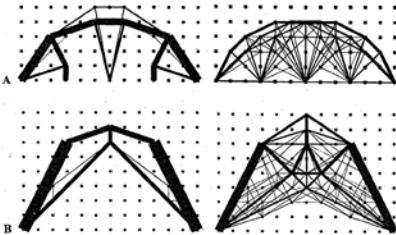
## Visualisation



**Fig. 4.5.** The flexibility in choice of ground structure. Optimal design of a well-known structure. Left hand picture shows the ground structure and the right hand picture the optimal topology for a single downward load at the top of the structure. The example shows that it is reasonable to consider multiple load cases for realistic structures. By courtesy of M. Kocvara and J. Zowe.



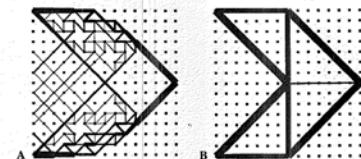
**Fig. 4.6.** The difference (and similarity) between multiple load case treated in the weighted average formulation (equal weights) (*A*) and treated in the worst case min-max formulation (*B*). Optimal truss topologies for transmitting three vertical forces to two fixed supports as in Fig. 4.7A (cf. Fig. 4.2), but for a long slender rectangular ground structure of aspect ratio 16 (like a long span bridge), with 53 by 3 equidistant nodes and all 2818 possible non-overlapping connections. In the figures, the vertical scale has been distorted in order to being able to show the results. Bendsoe, Ben-Tal and Zowe, 1993.



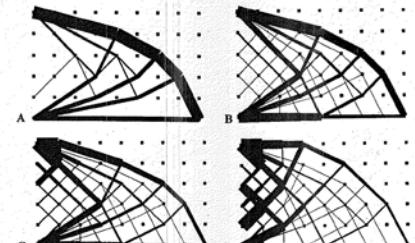
**Fig. 4.7.** The difference between multiple load and single load case problems. Optimal truss topologies for transmitting three vertical forces to two fixed supports. Two positions of the loads are considered and the nodes are optimised with the loads treated as a single load as well as three individual load cases for a more general situation. The ground structure consists of all 7744 possible non-overlapping connections between the nodal points of a regular 13 by 13 mesh in a square domain. The loads are vertical unit loads at three equidistant nodes along the lower line of nodes (*A*) or across the middle of the ground structure (*B*) (see also Fig. 4.2). The left hand column shows the single load results, the left hand column the multiple load, worst case results. We see that there are spurious rows of nodes, as these are not part of the optimal structure. A slight asymmetry of the ground structure is reflected in the optimal truss topologies. Bendsoe, Ben-Tal and Zowe, 1993.



**Fig. 4.8.** A detailed study situation of Fig. 4.7A, but with only the mid-point load applied. The ground structure is restricted to a rectangular domain of aspect ratio 2, and with the number of nodes increased to a 21 by 11 layout, with 16290 possible non-overlapping connections. Note that the supports have been moved in by two nodes from each vertical side, in order to identify an eventual restriction of having the supports at the extreme points of the ground-structure. Bendsoe, Ben-Tal and Zowe, 1993.



**Fig. 4.9.** A case where the introduction of multiple loads simplify the optimal layout. Also an example of the optimal topology for a ground structure with only neighbouring nodes in a square, regular 15 by 15 lay-out being connected (see Fig. 4.1B); this results in only 785 potential loads. All nodes at the left hand side are potential supports (see Fig. 4.2). (*A*): The optimal topology for a single vertical load at the mid-right hand node. (*B*): The optimal topology for three load cases including the load of the single load problem. The load of the single load example is twice as large as the other two loads, which is reflected in the mid-right hand node while the last load is a vertical load at the mid-left of the ground structure. This is for the weight-average formulation with equal weights. The compliance for the load case number 1 increases by only 2.6%, as compared to (*A*), which is optimal for this load only. Bendsoe, Ben-Tal and Zowe, 1993.



**Fig. 4.10.** The effect of self-weight loads. Optimal truss topologies for transmitting a single vertical force to a vertical line of supports (see Fig. 4.2). The figures show the variation for increasing specific self-weight loads, corresponding to increasing real lengths of the structures. In (*A*) self-weight is one third of the total load, in (*B*) it is one half, increased by 2 times to the design (*C*). These designs are obtained for a 9 by 6 equidistant nodal layout in a rectangular domain of aspect ratio 1.5, and all 919 possible non-overlapping connections. If all 1431 possible connections are used, the design (*C*) is modified to the design (*D*). Bendsoe, Ben-Tal and Zowe, 1993.

## Form analysis

### Interactive (parametric)

- Analysis
- Constraint-driven design

### Computational graphic statics

- Algebraic graph(ic) statics (AGS)
- Bi-directional AGS

### Optimization

## Form generation

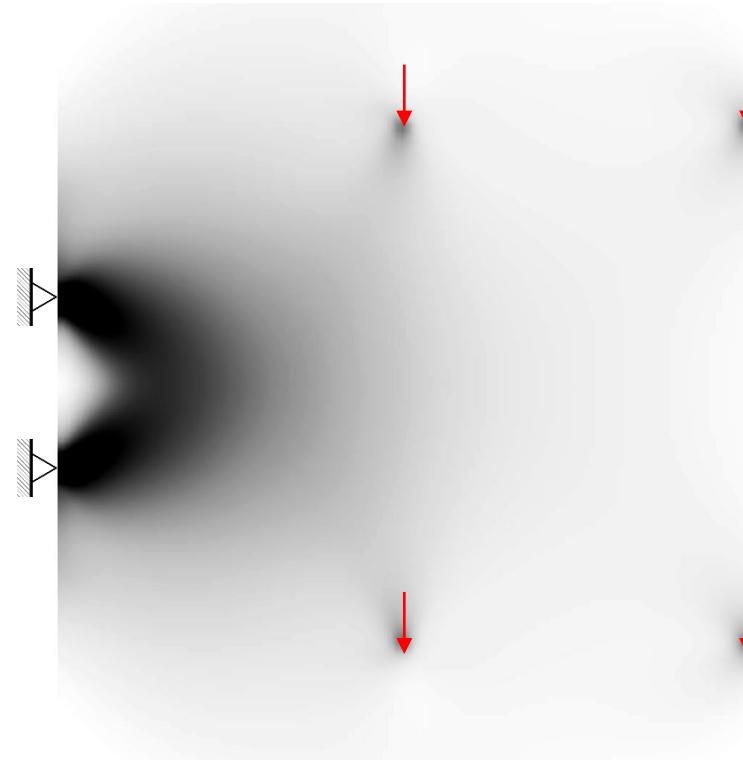
### Form finding

- Thrust network analysis (TNA)
- 3D graphic statics

### Generative design

- Subdivision graphic statics
- Shape grammars

## Visualisation



## Form analysis

### Interactive (parametric)

Analysis

Constraint-driven design

### Computational graphic statics

Algebraic graph(ic) statics (AGS)

Bi-directional AGS

### Optimization

## Form generation

### Form finding

Thrust network analysis (TNA)

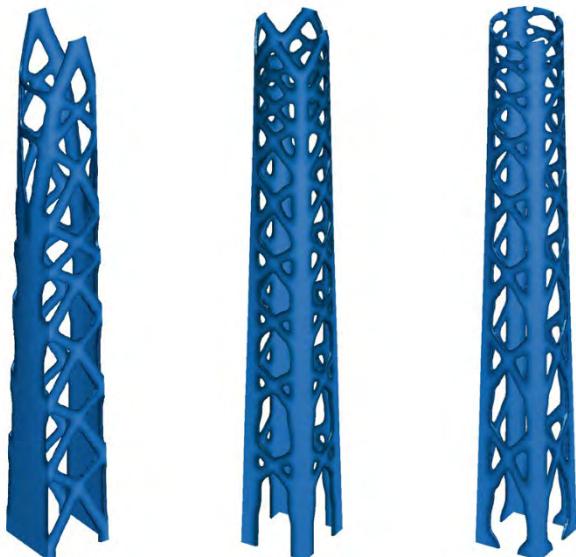
3D graphic statics

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Subdivision graphic statics

Shape grammars

## Visualisation



### SCALE

Unrealistic member thickness / stiffness !



### CONTINUITY

Fabrication & constructability issues !



### PIXELATION

Unclear location of intersections (joints)

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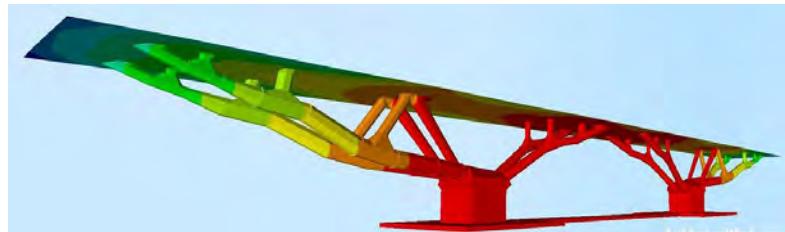
3D graphic statics

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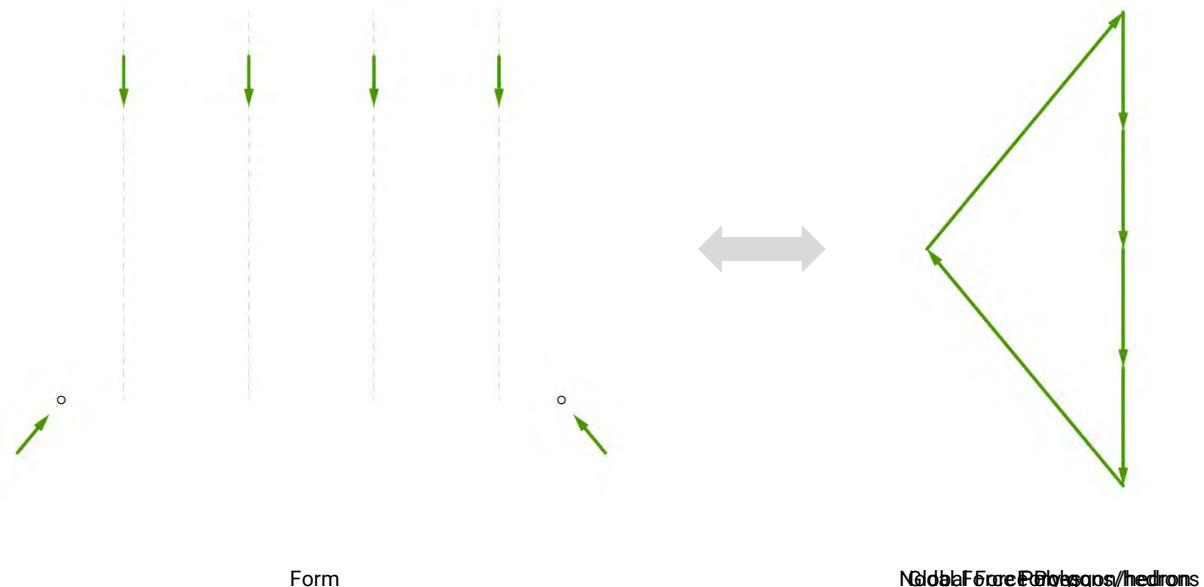
### Form finding

- Thrust network analysis (TNA)
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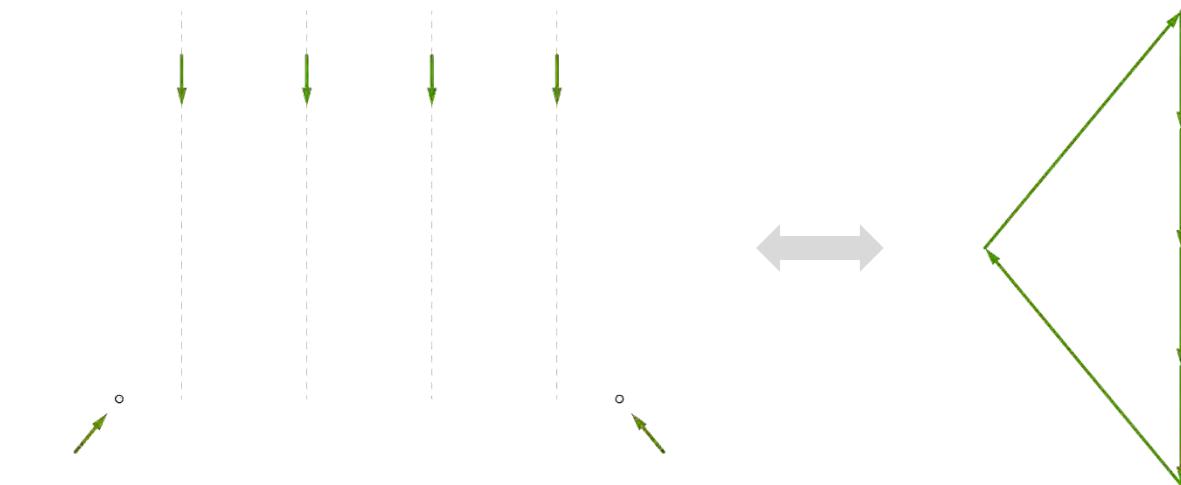
### Form finding

- Thrust network analysis (TNA)
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- Subdivision graphic statics**
- Shape grammars

## Visualisation



Subdivision of Global Force Polyhedron

Equilibrated Structure

## Form analysis

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Bi-directional AGS

### Optimization

## Form generation

### Form finding

Thrust network analysis (TNA)

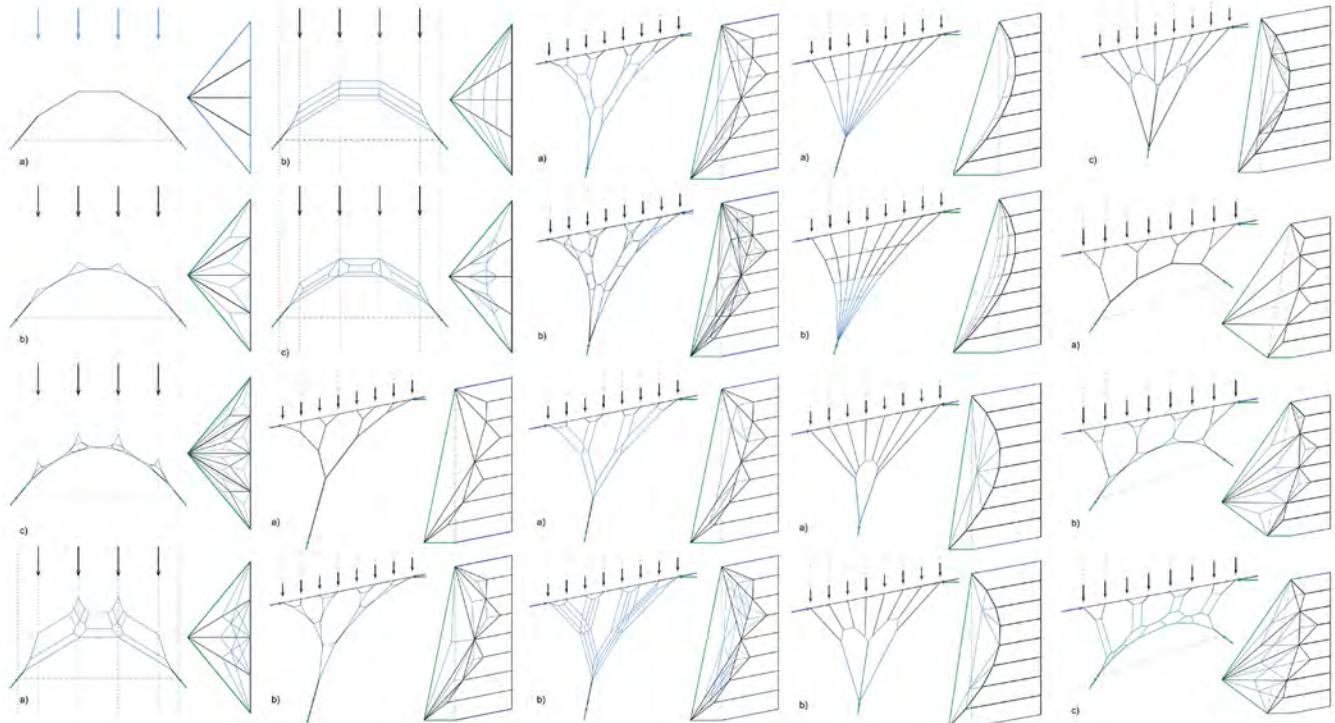
3D graphic statics

### Generative design

#### Subdivision graphic statics

Shape grammars

## Visualisation



## Form analysis

### Interactive (parametric)

Analysis  
Constraint-driven design

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Algebraic graph(ic) statics (AGS)  
Bi-directional AGS

### Optimization



## Form generation

### Form finding

Thrust network analysis (TNA)  
3D graphic statics

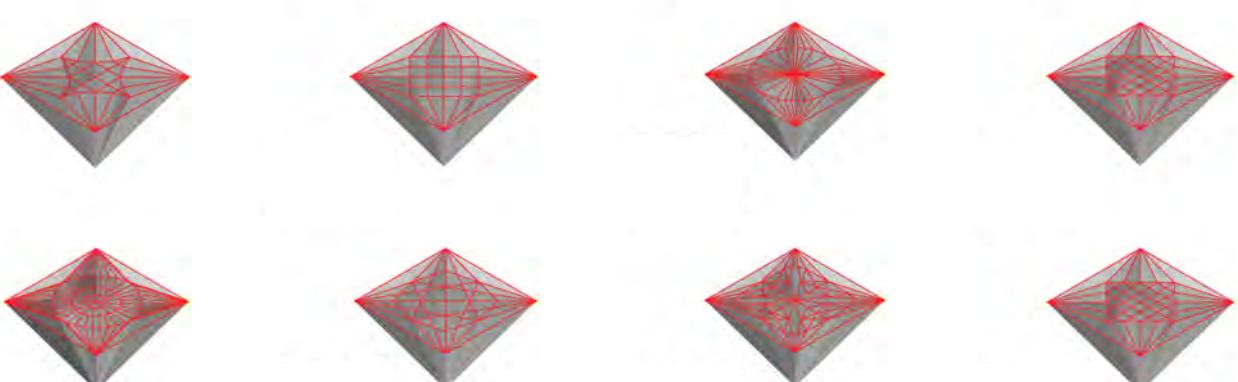
### Generative design

#### Subdivision graphic statics

Shape grammars



## Visualisation



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### Interactive (parametric)

Analysis

Constraint-driven design

### Computational graphic statics

Algebraic graph(ic) statics (AGS)

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Thrust network analysis (TNA)

3D graphic statics

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#### Subdivision graphic statics

Shape grammars

## Visualisation



Subdivision of a Face

Subdivision of Cells

## Form analysis

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Constraint-driven design

### Computational graphic statics

Algebraic graph(ic) statics (AGS)

Bi-directional AGS

### Optimization

## Form generation

### Form finding

Thrust network analysis (TNA)

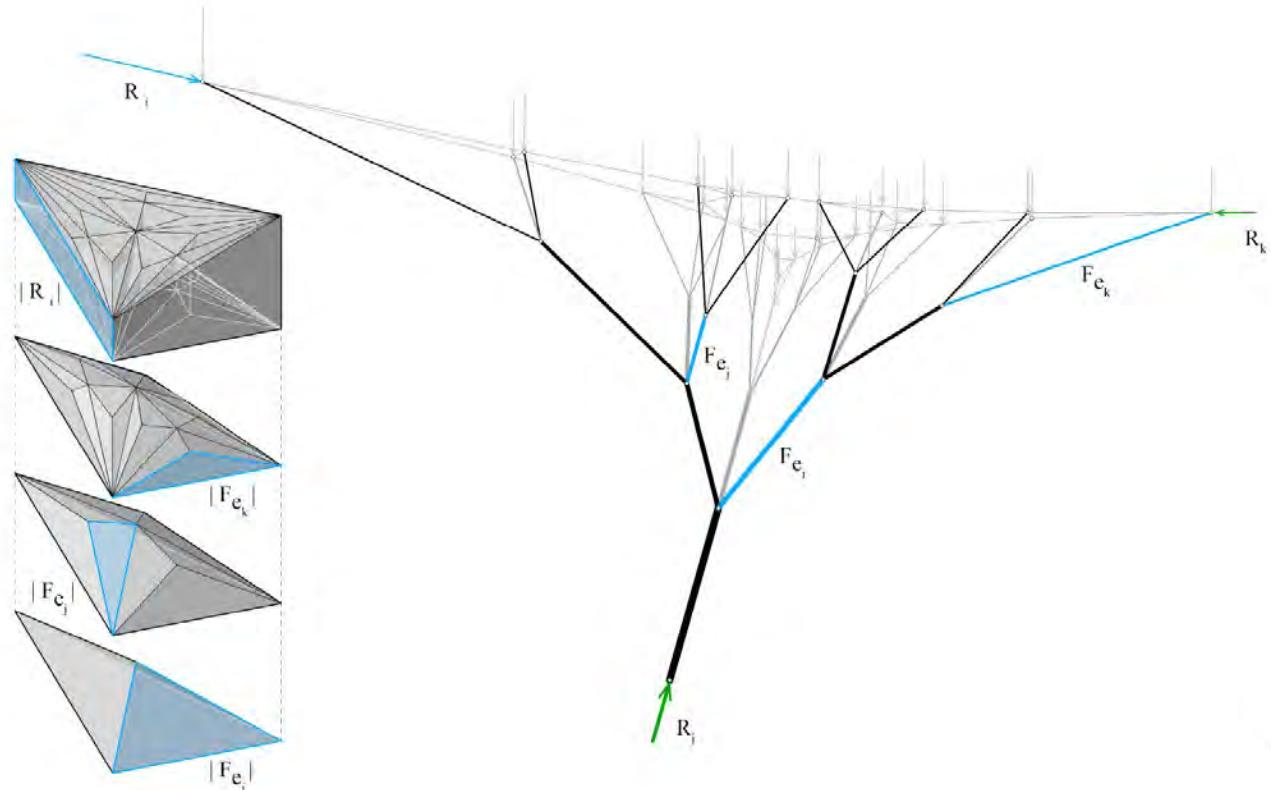
3D graphic statics

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#### Subdivision graphic statics

Shape grammars

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## Form analysis

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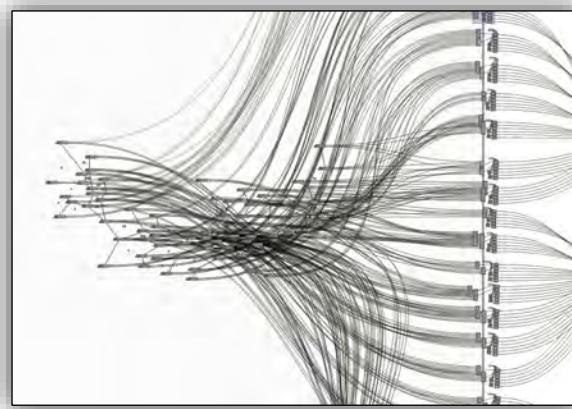
### Form finding

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- Shape grammars**

### Visualisation

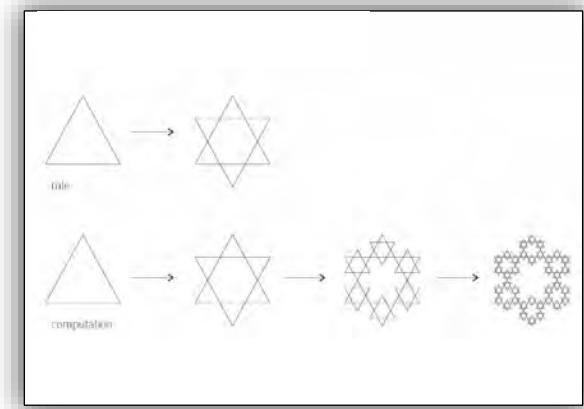


### Parametric Design Space

NUMERICAL PARAMETERS are defined ...

to

... DESCRIBE & MODIFY geometries.



### Grammatical Design Space

ACTIONS & OPERATIONS (RULES) are defined ...

to

... GENERATE geometries.

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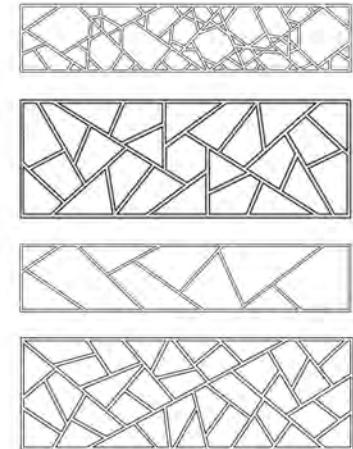
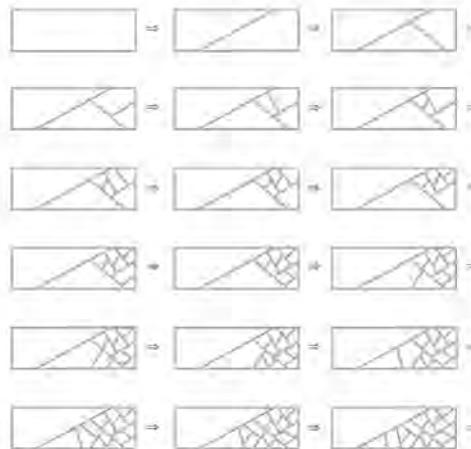
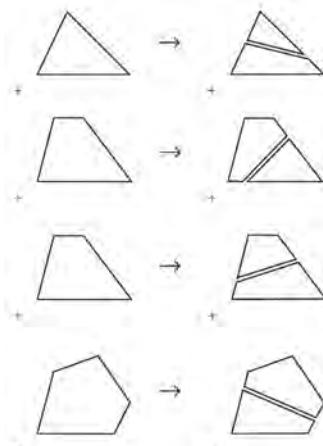
Thrust network analysis (TNA)

3D graphic statics

### Generative design

Subdivision graphic statics

**Shape grammars**



## Visualisation

...

## Form analysis

### Interactive (parametric)

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Constraint-driven design

### Computational graphic statics

Algebraic graph(ic) statics (AGS)

Bi-directional AGS

### Optimization

## Form generation

### Form finding

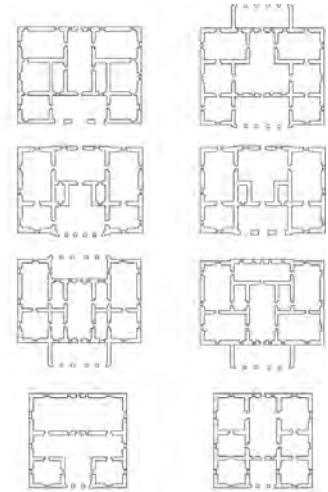
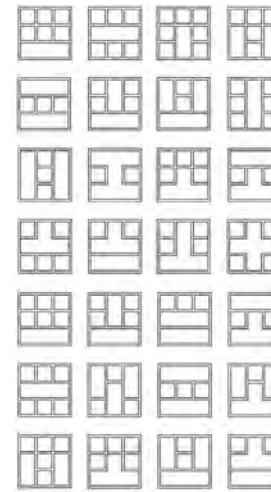
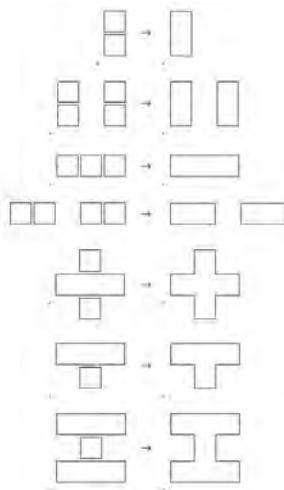
Thrust network analysis (TNA)

3D graphic statics

### Generative design

Subdivision graphic statics

**Shape grammars**



## Visualisation

...

## Form analysis

### Interactive (parametric)

Analysis

Constraint-driven design

### Computational graphic statics

Algebraic graph(ic) statics (AGS)

Bi-directional AGS

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Thrust network analysis (TNA)

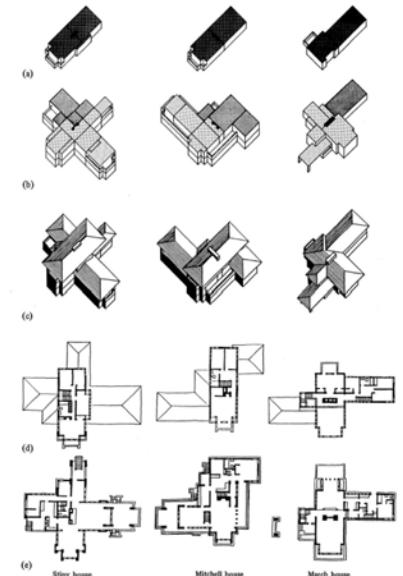
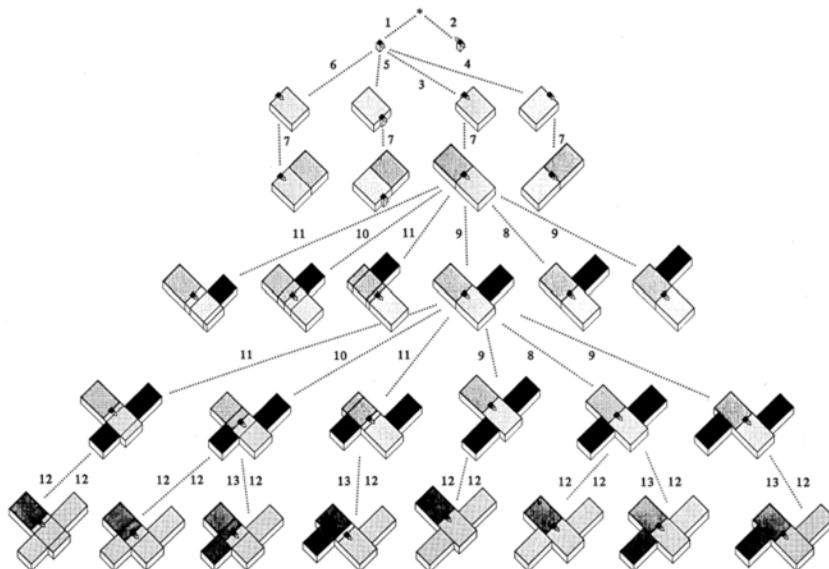
3D graphic statics

### Generative design

Subdivision graphic statics

**Shape grammars**

## Visualisation



## Form analysis

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Analysis

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Thrust network analysis (TNA)

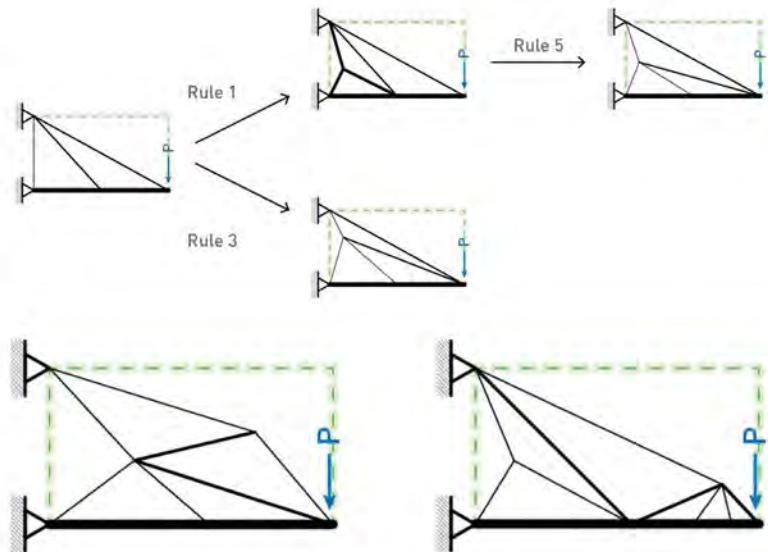
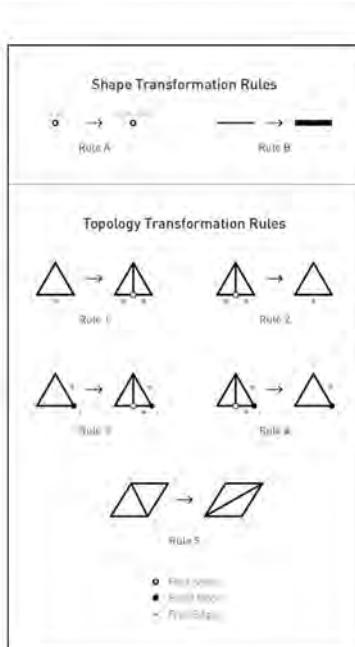
3D graphic statics

### Generative design

Subdivision graphic statics

**Shape grammars**

## Visualisation



Original Work by Shea & Cagan (1999) • Reprinted & Adapted

## Form analysis

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Bi-directional AGS

### Optimization

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Thrust network analysis (TNA)

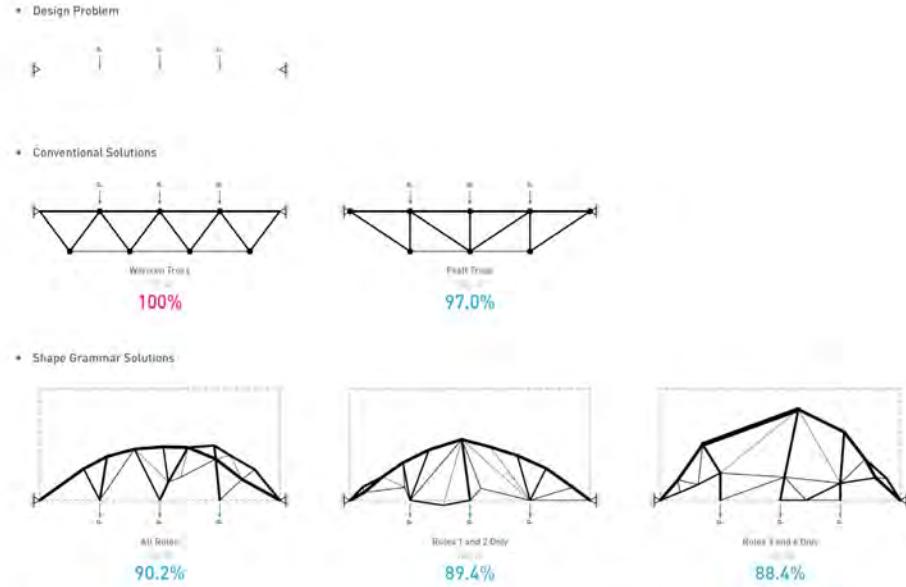
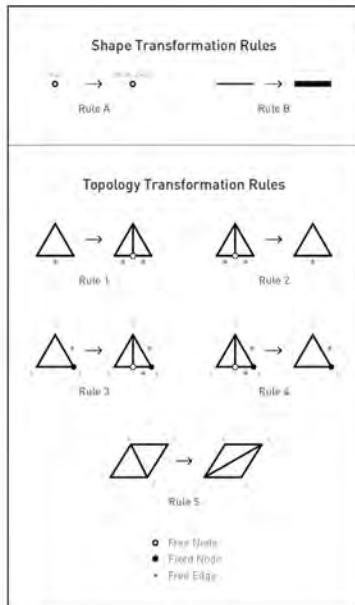
3D graphic statics

### Generative design

Subdivision graphic statics

**Shape grammars**

## Visualisation



## Form analysis

### Interactive (parametric)

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- Constraint-driven design

### Computational graphic statics

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### Optimization

## Form generation

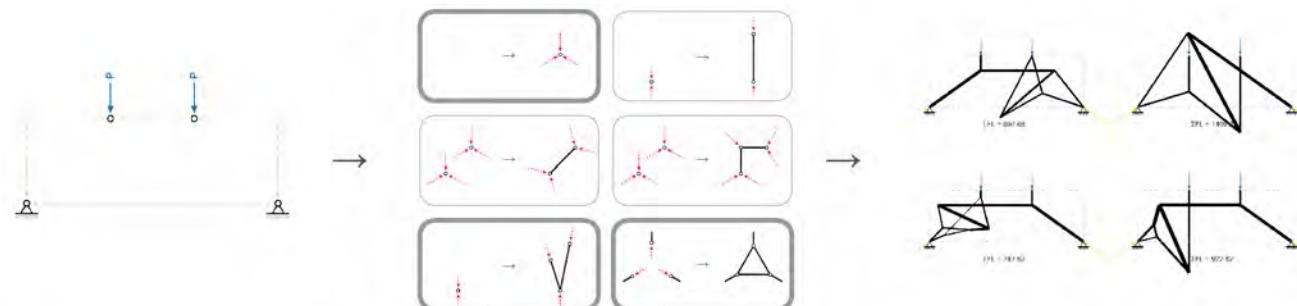
### Form finding

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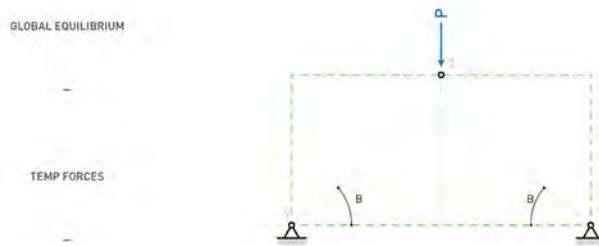
### Form finding

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## Visualisation



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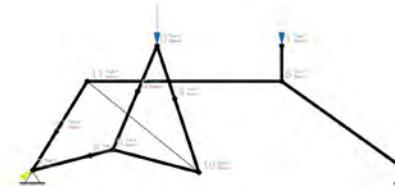
Shape grammars

## Visualisation

### Force Diagrams of Every Node



### The Design



### Evaluation Metric

ID : 0004E  
 $\Sigma F_L = 673.61$   
 $\Sigma L = 126.38$   
 $\Sigma \# = 13$

Rule 1: 25%  
 Rule 2: 25%  
 Rule 3: 25%  
 Rule 4: 25%  
 Rule 5: 25%

### RULE HISTORY

- 1 . "Rule 0: START" applied at Node 2
- 2 . "Rule 1: Connect via Projection" applied at Node 1
- 3 . "Rule 1: Connect via Projection" applied at Node 4
- 4 . "Rule 1: Connect via Projection" applied at Node 6
- 5 . "Rule 3: Sync Node" applied at Node 3
- 6 . "Rule 1: Connect via Projection" applied at Node 8
- 7 . "Rule 6: System Close" applied at Node 4
- 8 . "Rule 6: System Close" applied at Node 8
- 9 . "Rule 6: System Close" applied at Node 6

### Rule History

## Form analysis

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3D graphic statics

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Shape grammars

## Visualisation

The screenshot shows a user interface for sentiment analysis. At the top, a header reads "Sentiment Analysis". Below it, three colored emoji icons represent different sentiment levels: a yellow smiley face for positive, a neutral grey face in the middle, and a red angry face on the right. To the left of the emojis, there is sample text: "My experience so far has been fantastic!" followed by a green button labeled "POSITIVE". In the center, another sample text block says "The product is ok I guess". To the right, a third sample text block says "Your support team is". Below these examples, a large input field contains the text "This is the best sentiment analysis tool ever!!!". A blue button at the bottom of this field is labeled "Classify Text". To the right of the input field, a section titled "Results" displays a table with one row. The table has two columns: "TAG" and "CONFIDENCE". The single entry in the table is "Positive" with a confidence of "100.0%".

TAG	CONFIDENCE
Positive	100.0%

## Form analysis

### Interactive (parametric)

Analysis

Constraint-driven design

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Algebraic graph(ic) statics (AGS)

Bi-directional AGS

### Optimization

## Form generation

### Form finding

Thrust network analysis (TNA)

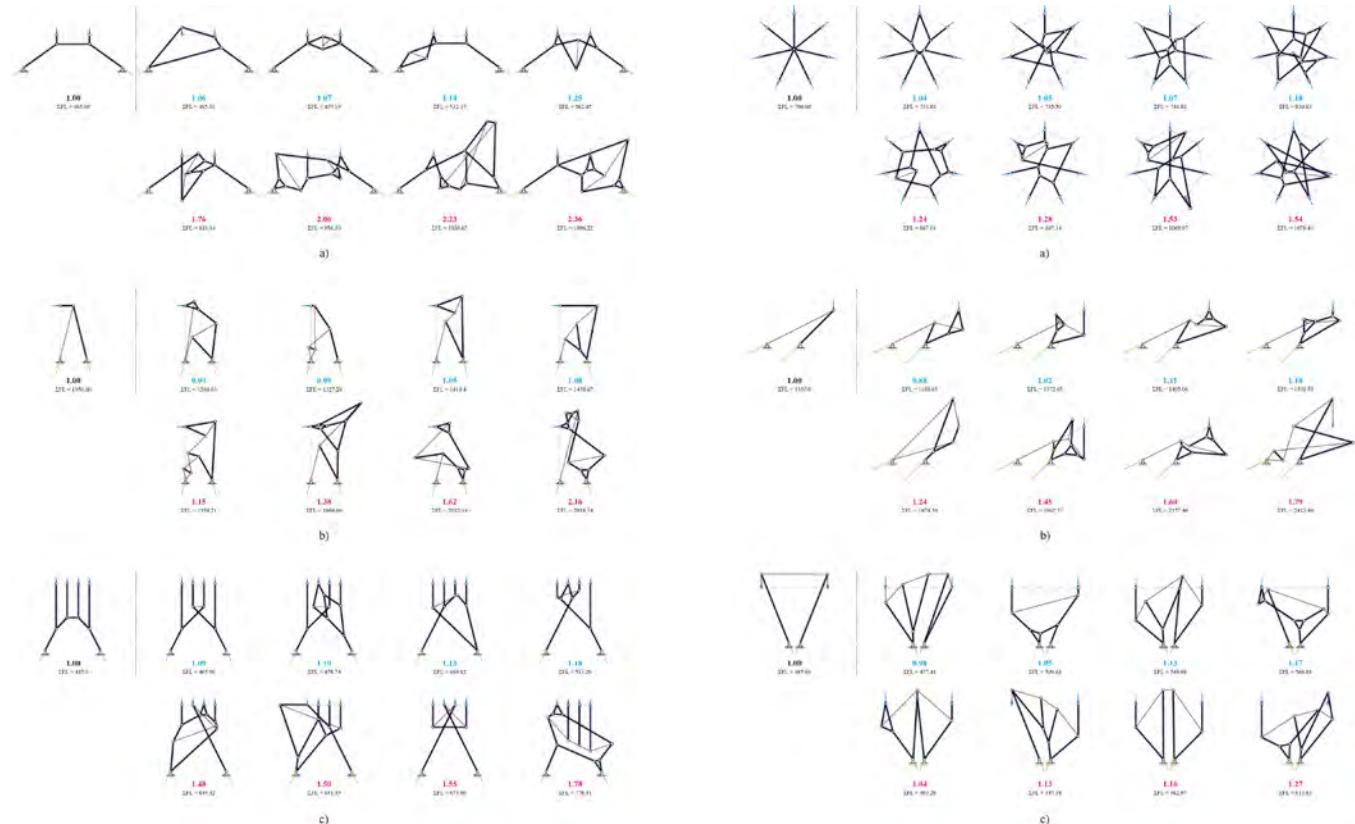
3D graphic statics

### Generative design

Subdivision graphic statics

**Shape grammars**

## Visualisation



## Form analysis

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Bi-directional AGS

### Optimization

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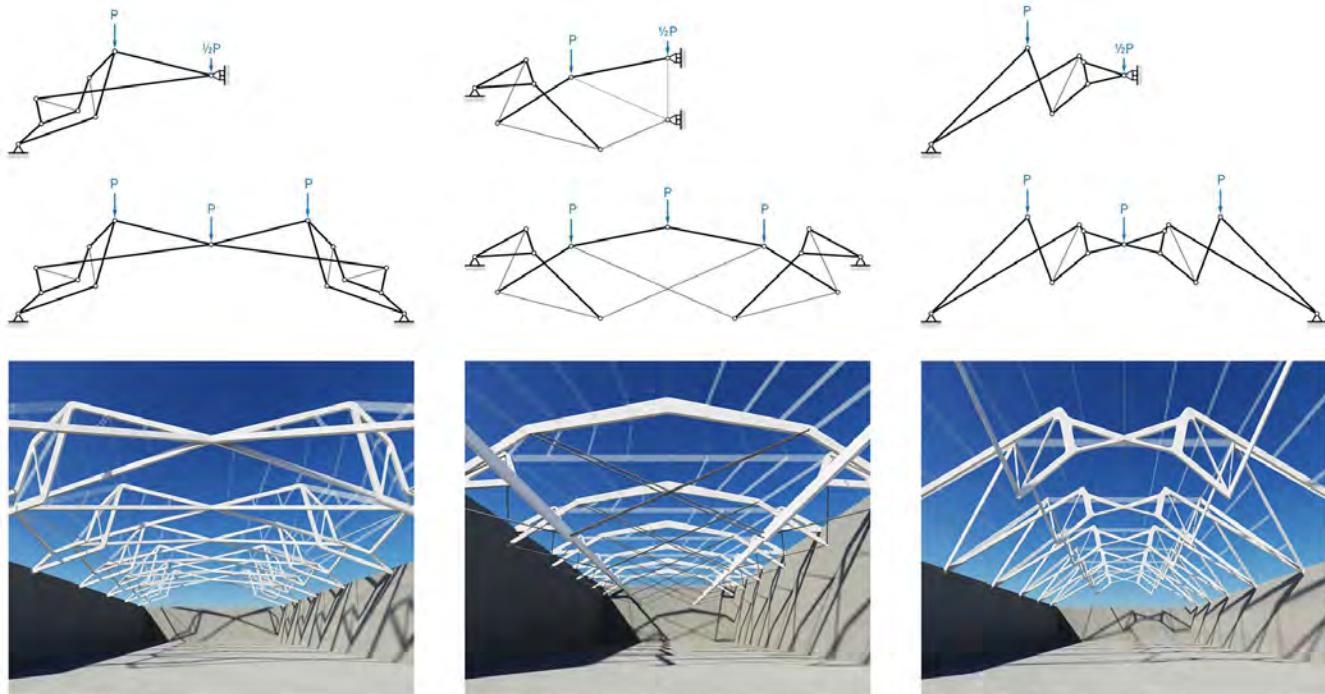
### Form finding

Thrust network analysis (TNA)  
3D graphic statics

### Generative design

Subdivision graphic statics  
**Shape grammars**

### Visualisation



## Form analysis

### Interactive (parametric)

- Analysis
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### Computational graphic statics

- Algebraic graph(ic) statics (AGS)
- Bi-directional AGS

### Optimization

## Form generation

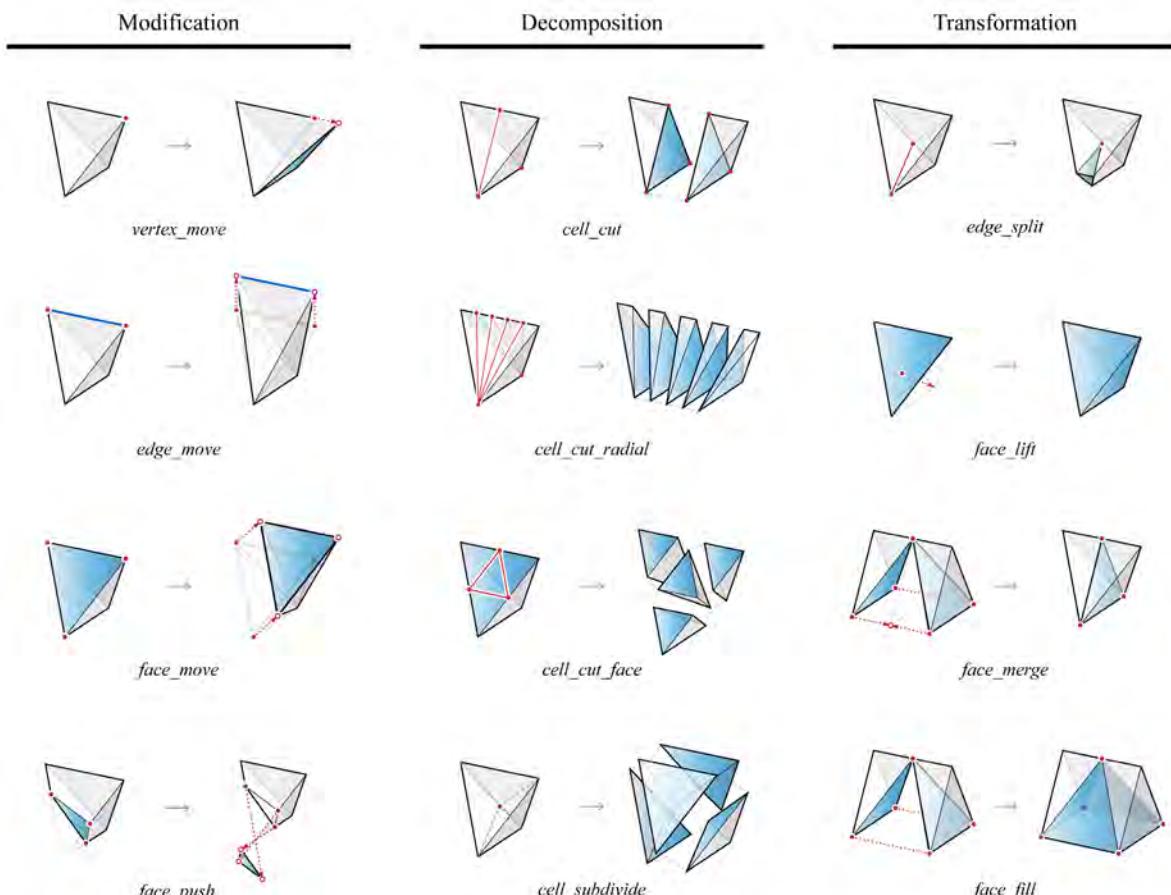
### Form finding

- Thrust network analysis (TNA)
- 3D graphic statics

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## Visualisation



## Form analysis

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### Computational graphic statics

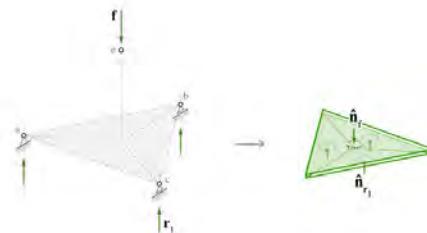
- Algebraic graph(ic) statics (AGS)
- Bi-directional AGS

### Optimization

## Form generation

### Form finding

- Thrust network analysis (TNA)
- 3D graphic statics



### Generative design

- Subdivision graphic statics
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## Visualisation

...

## Form analysis

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### Computational graphic statics

Algebraic graph(ic) statics (AGS)

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Thrust network analysis (TNA)

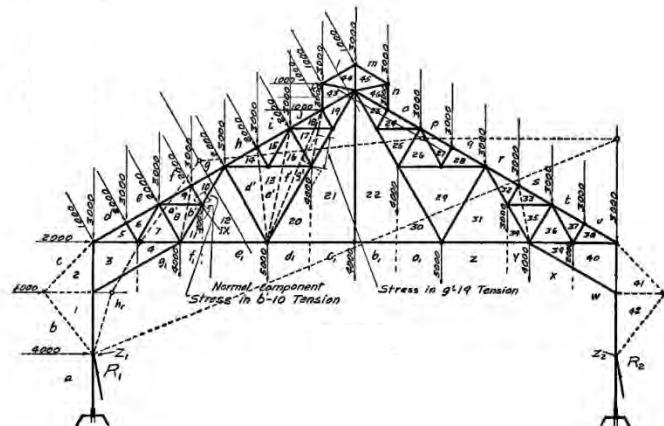
3D graphic statics

### Generative design

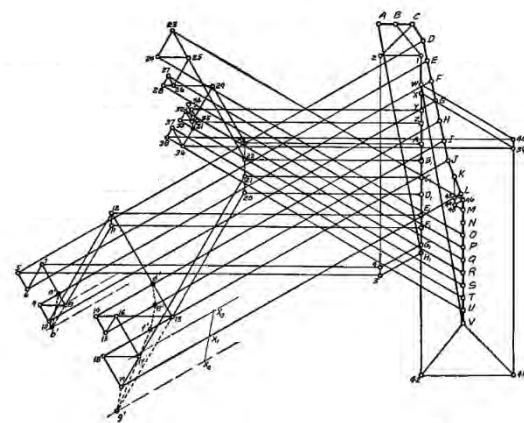
Subdivision graphic statics

Shape grammars

## Visualisation



Form Diagram



Force Diagram

## Form analysis

### Interactive (parametric)

- Analysis
- Constraint-driven design

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### Optimization

## Form generation

### Form finding

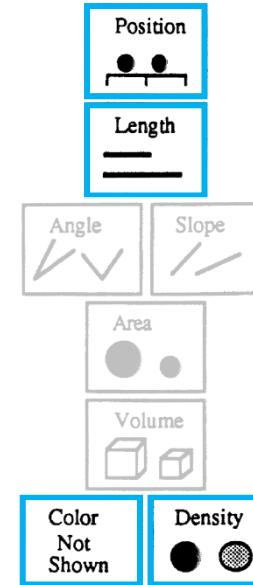
- Thrust network analysis (TNA)
- 3D graphic statics

### Generative design

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## Visualisation

More accurate



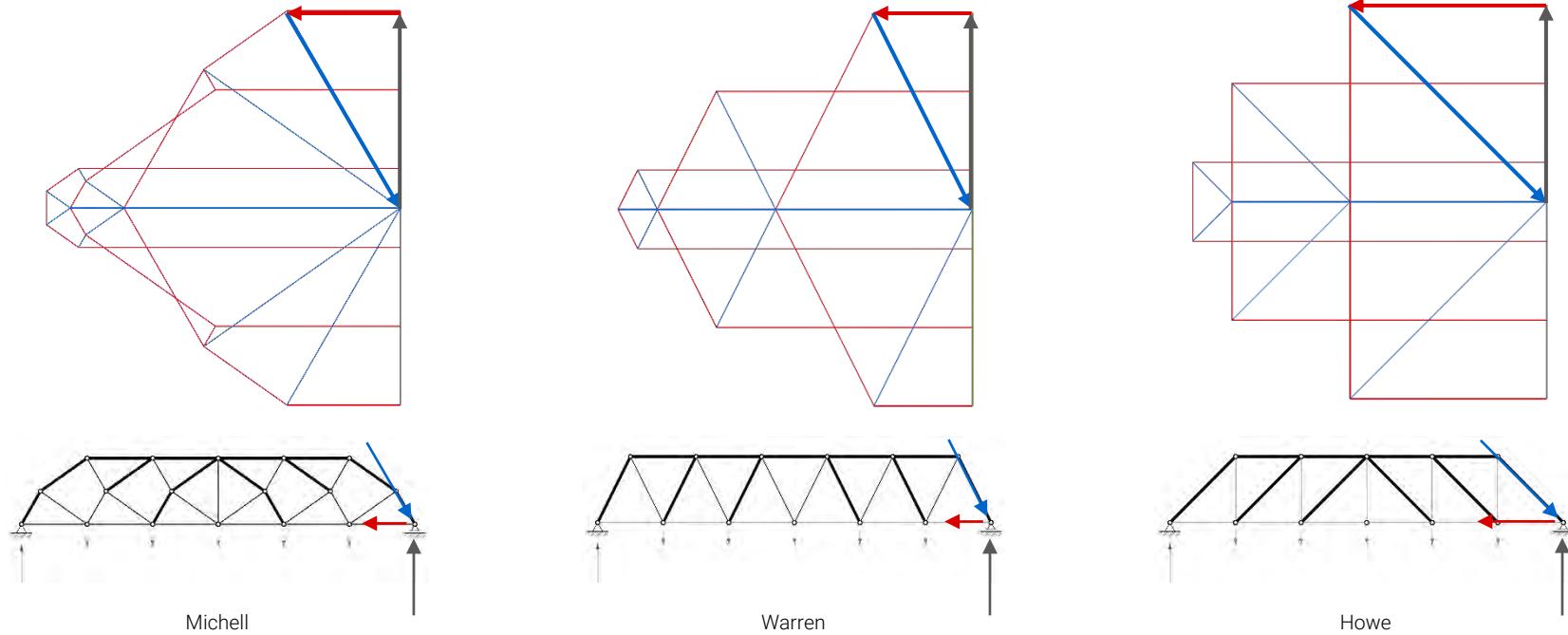
Less accurate

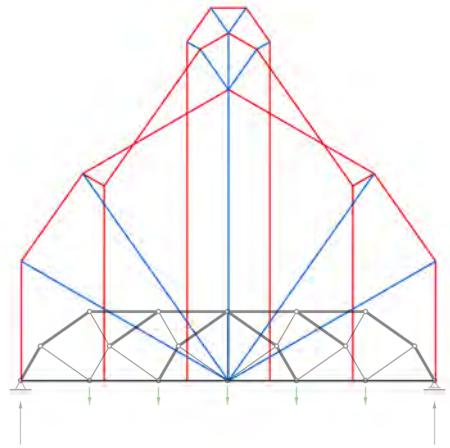
2D Graphic Statics

FE Analysis

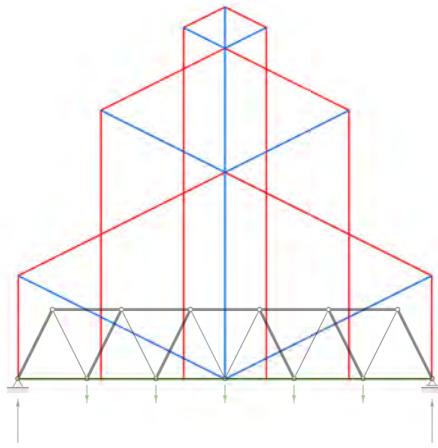
Accuracy and Effectiveness of Human Retinal Perception

## "Márcenho Daág Diagráme (parallel orientation)

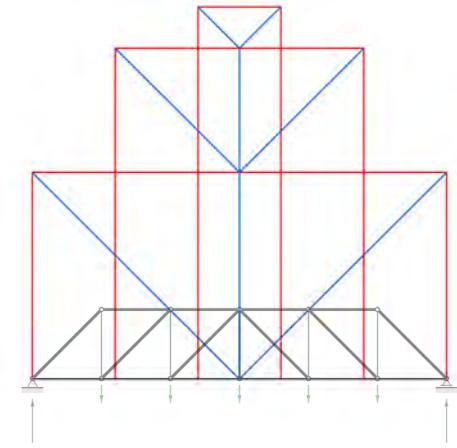




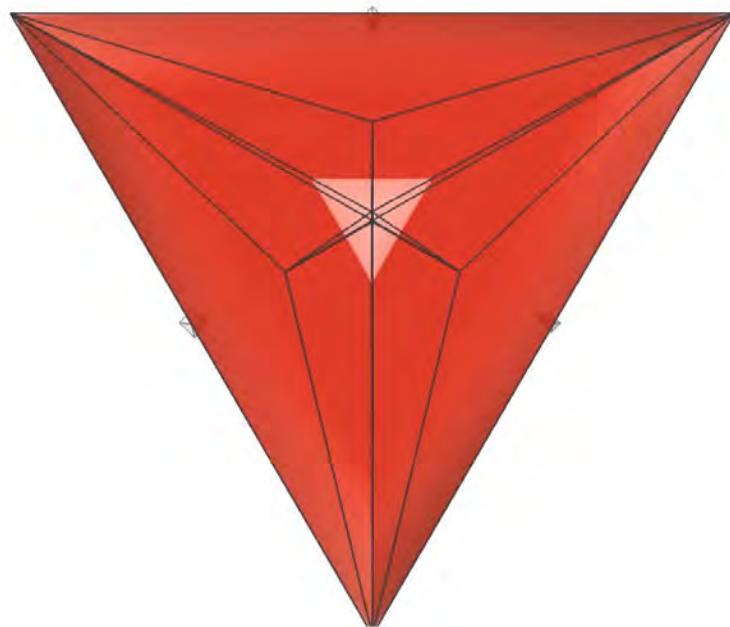
Michell



Warren

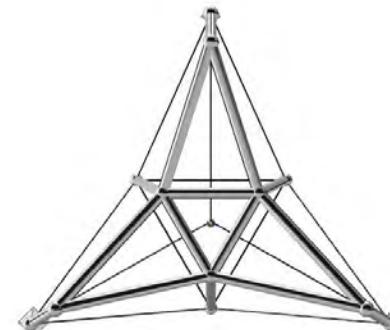


Howe

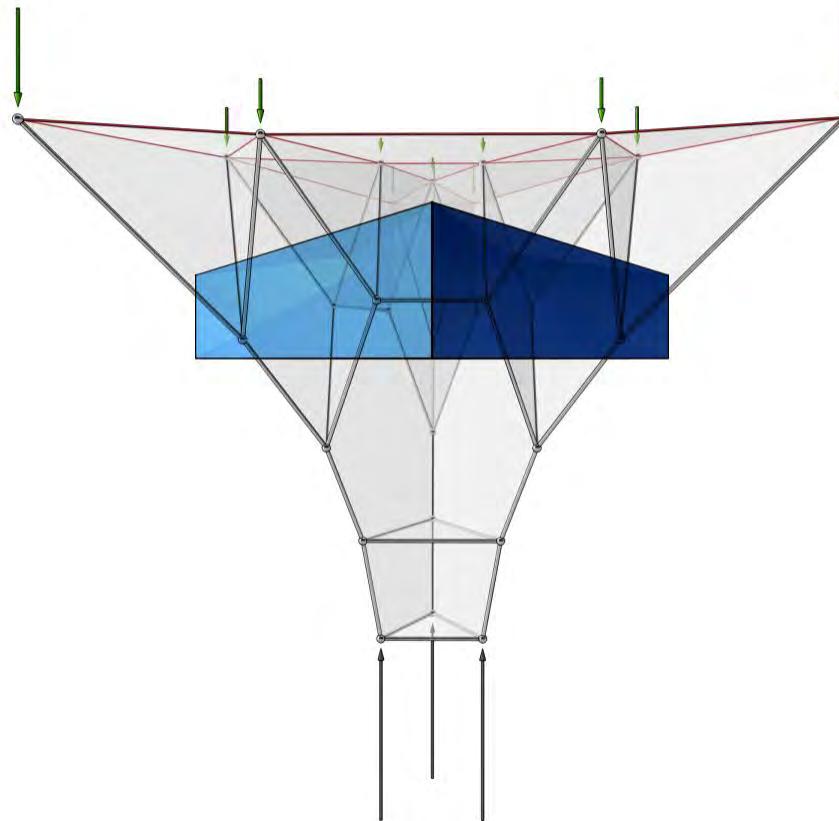


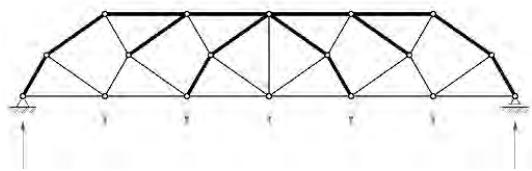
Interactive Polyhedral Force Diagram

← Synchronized Interface →

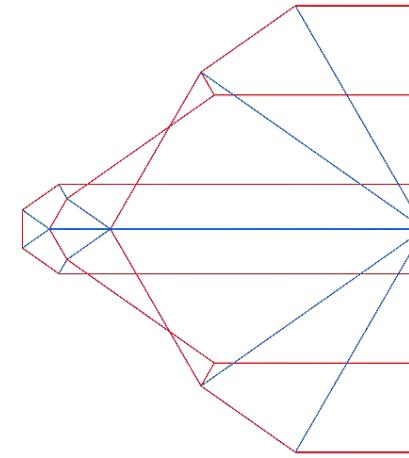


Form Diagram



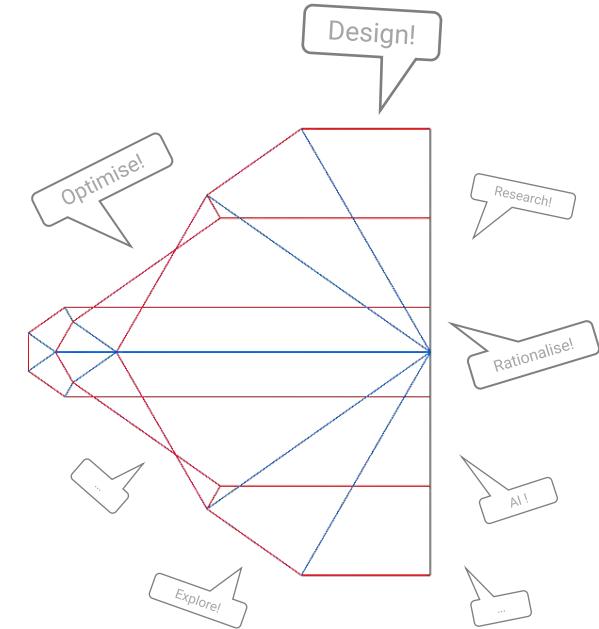


Geometry of form



Analyse

?



Geometry of forces

063-0605-00L : Computational Structural Design 1  
Computational Graphic Statics

<http://block.arch.ethz.ch>



@blockresearchgroup

**ETH** zürich DARCH

**BRG**