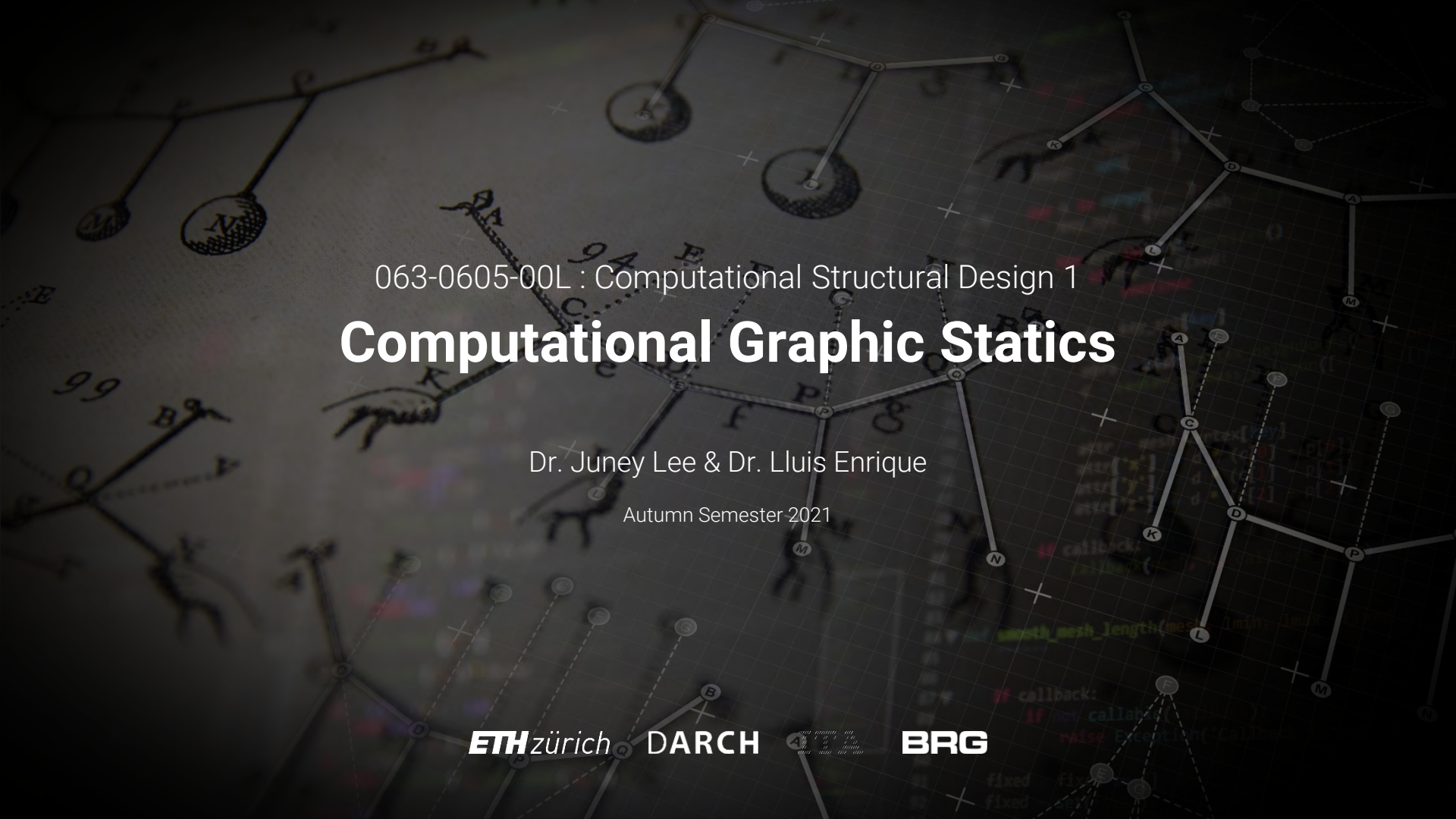


063-0605-00L : Computational Structural Design 1

Computational Graphic Statics

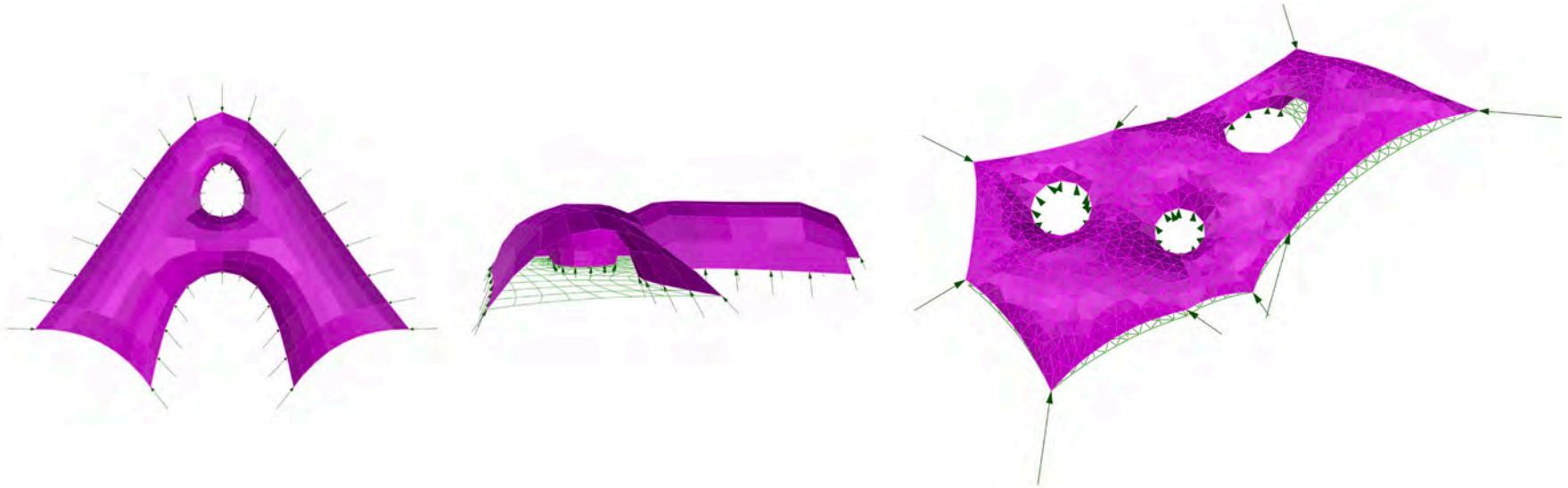
Dr. Juney Lee & Dr. Lluis Enrique

Autumn Semester 2021

ETH zürich DARCH BRG

Week 9 Friday, December 3rd

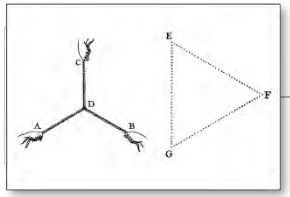
Hour 1	**09:45 – 09:50**	· Recap of Exercise 5
	09:50 – 10:30	· **Lecture 6 : 3D graphic statics**
	10:30 – 10:45	· Break
Hour 2	**10:45 – 11:35**	· **Tutorial 6 : Polyhedral 3D Graphic Statics (3GS)**
	11:30 – 11:45	· Break
Hour 3	**11:45 – 12:30**	· **Exercise 6 : Polyhedral 3D Graphic Statics (3GS)**



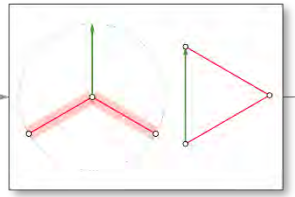
Course schedule

Structural Typology	Topic	Workflow	Week	Nr	Topic	Instructor
1. planar structures	cables & arches	Introduction I	Week 1 (9/24)	1	**Lecture 1**	**Introduction to graphic statics**	**Dr. Juney Lee**
				2	**Tutorial 1**	Basics of graphic statics on "blackboard"	Dr. Lluis Enrique
						Introduction to Rhino + Grasshopper (GH)	Lotte Aldinger
				3	**Work session 1**	***Exercise 1**: Reciprocal diagrams in Rhino + GH basics*	Instructors
		2D GS (procedural) II	Week 2 (10/1)			Quick recap of Exercise 1	Lotte Aldinger
				1	**Lecture 2**	**Algorithmic design & thinking**	**Dr. Juney Lee**
				2	**Tutorial 2**	Single node bridge in GH (form to force)	Lotte Aldinger
				3			
			Week 3 (10/8)	1	**Work session 2**	***Exercise 2**: Single node bridge with GH*	Lotte Aldinger + Instructors
				2			
				3			
		III				Quick recap of Exercise 2	Lotte Aldinger
			Week 4 (10/15)	1	**Lecture 3**	**Computational graphic statics**	**Dr. Juney Lee**
				2	**Tutorial 3**	Multi-node bridge in Grasshopper	Dr. Lluis Enrique
				3			
			Week 5 (10/22)	1	**Work session 3**	***Exercise 3**: Multi-node bridge with GH*	Dr. Lluis Enrique + Instructors
				2			
				3			
			Week 6			seminar week	
	trusses	2D GS (AGS) IV	Week 7 (11/5)			Quick recap of Exercise 3	Dr. Lluis Enrique
				1	**Lecture 4**	**Algebraic graph statics (AGS)**	**Dr. Juney Lee**
				2	**Tutorial 4**	compas_ags + IGS (Interactive Graphic Statics) plugin	Ricardo Avelino
				3			
			Week 8 (11/12)	1	**Work session 4**	***Exercise 4**: Truss problems with IGS*	Ricardo Avelino + Instructors
				2			
				3			
2. surface structures	shells	2.5 GS (TNA) V	Week 9 (11/19)			Quick recap of Exercise 4	Ricardo Avelino
				1	**Lecture 5**	**Thrust Network Analysis (TNA)**	**Dr. Juney Lee**
				2	**Tutorial 5**	compas_tna + RV2 (RhinoVAULT 2) plugin	Dr. Juney Lee
				3			
			Week 10 (11/26)	1	**Work session 5**	***Exercise 5**: Shell design exercises with RV2*	Dr. Juney Lee + Instructors
				2			
3. spatial structures	polyhedral structures	3D GS (polyhedral) VI	Week 11 (12/3)			Quick recap of Exercise 5	Dr. Juney Lee
				1	**Lecture 6**	**3D graphic statics (3GS)**	**Dr. Juney Lee**
				2	**Tutorial 6**	compas_3gs + 3GS (3D graphic statics) plugin	Dr. Juney Lee
				3	**Work session 6**	***Exercise 6**: Spatial structures with 3GS*	
			Week 12 (12/10)	1	**Lecture 7**	**Pt I. Outlook on computational graphic statics**	**Dr. Juney Lee**
						Pt II. Guest Lecture	**TBD**
				2	Q/A	Discussions, feedback, evaluations, etc.	

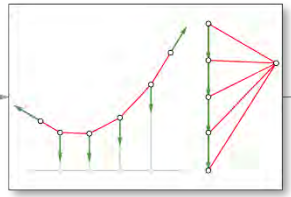
2D | **2.5D** | **3D**



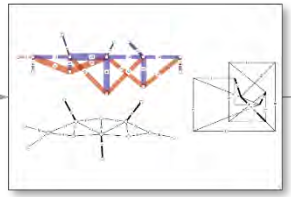
Module I
Basics of Graphic Statics



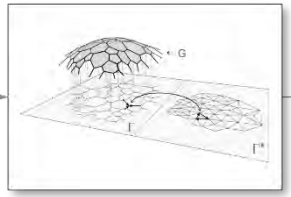
Module II
Interactive Graphic Statics I (single node)



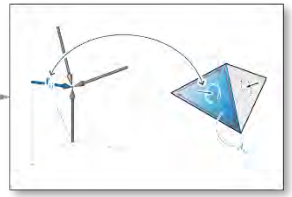
Module III
Interactive Graphic Statics 2 (multi node)



Module IV
Algebraic Graph Statics

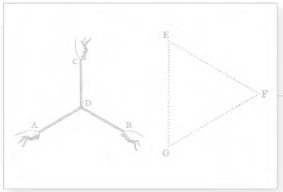


Module V
Thrust Network Analysis



Module VI
3D Graphic Statics

2D

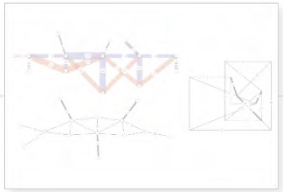


Module I
Basics of Graphic Statics

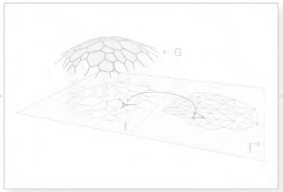
Module II
Interactive Graphic Statics I (single node)

Module III
Interactive Graphic Statics 2 (multi node)

2.5D

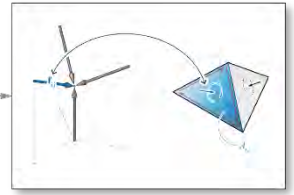


Module IV
Algebraic Graph Statics



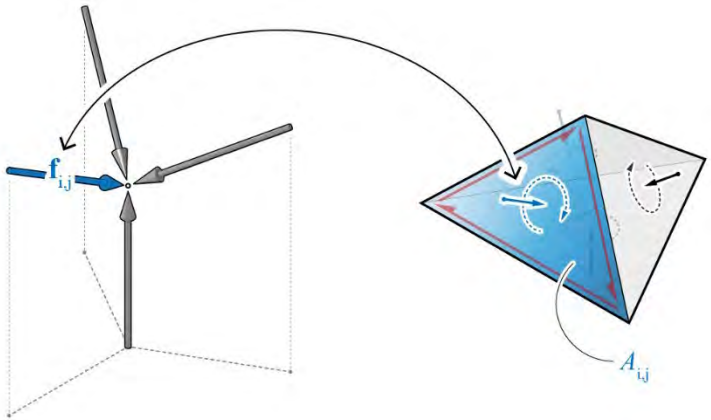
Module V
Thrust Network Analysis

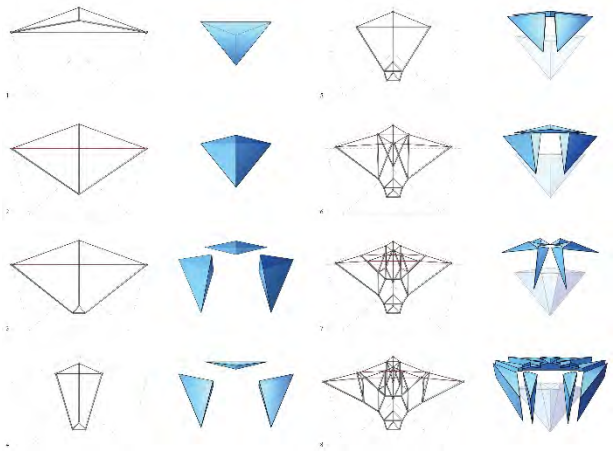
3D



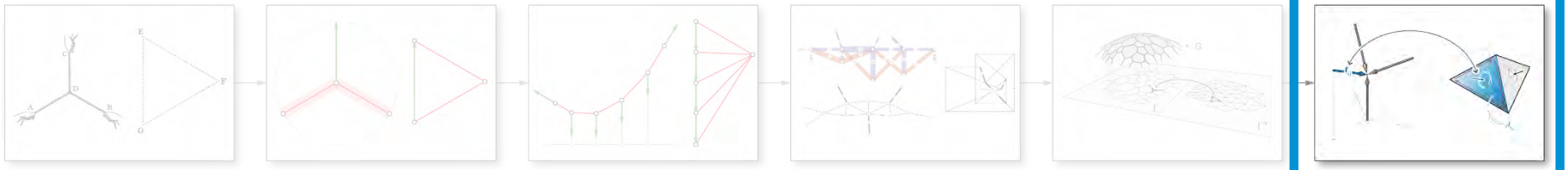
Module VI
3D Graphic Statics

Module VI · 3D graphic statics





2D 2.5D 3D



Module I
Basics of Graphic Statics

Module II
Interactive Graphic Statics I (single node)

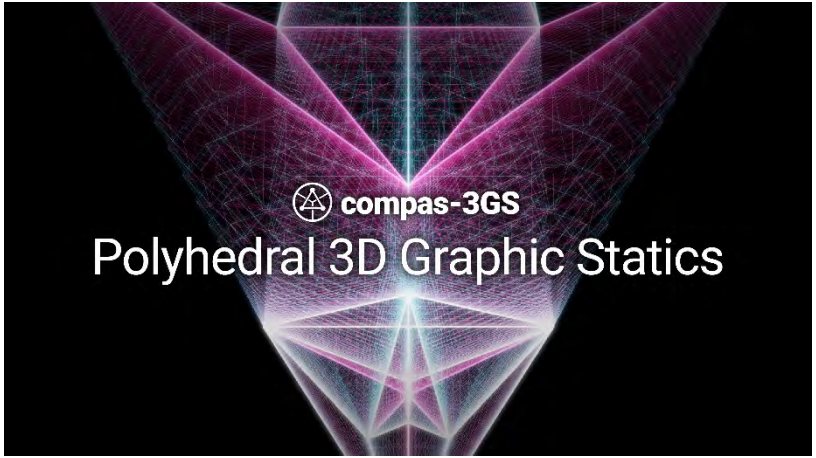
Module III
Interactive Graphic Statics 2 (multi node)

Module IV
Algebraic Graph Statics

Module V
Thrust Network Analysis

Module VI
3D Graphic Statics

Module VI · 3D graphic statics



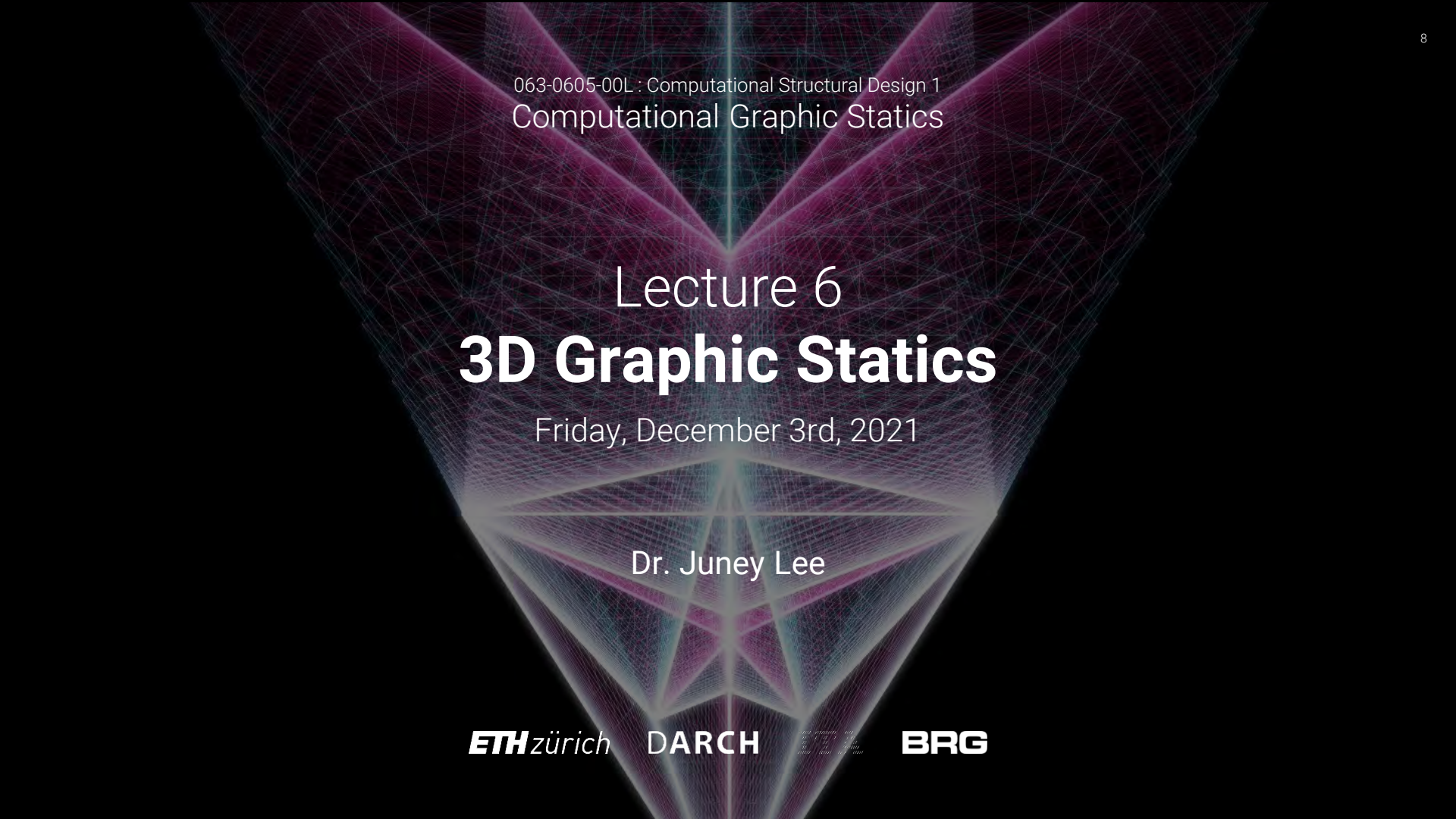
063-0605-00L : Computational Structural Design 1

Computational Graphic Statics

Lecture 6
3D Graphic Statics

Friday, December 3rd, 2021

Dr. Juney Lee



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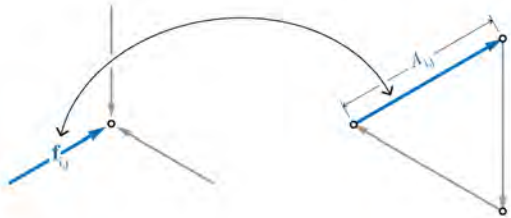
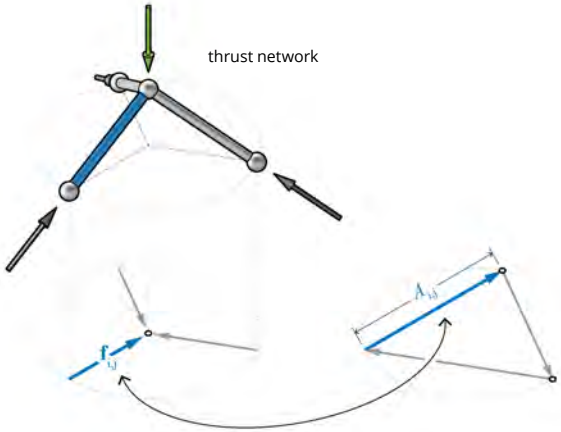


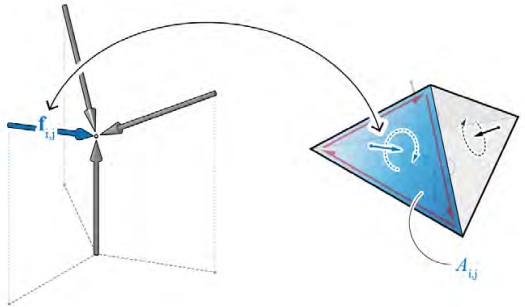
form diagram force diagram

2 D



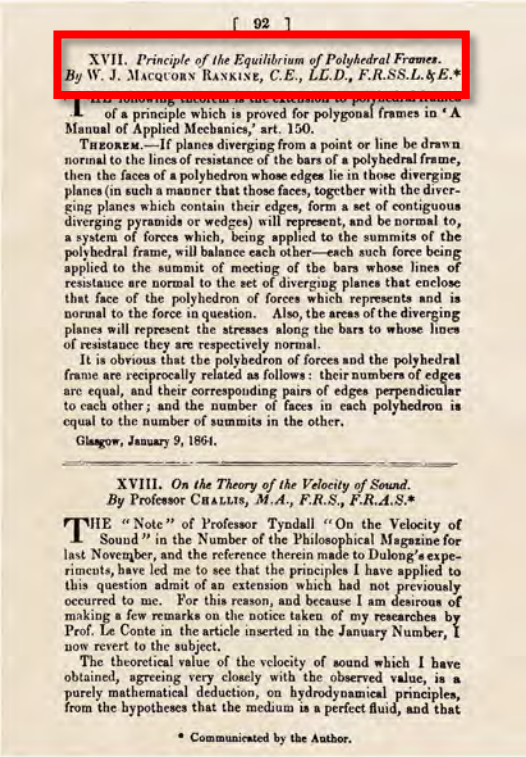
form diagram force diagram

2.5 D



form diagram force diagram

3 D



[92]

XVII. *Principle of the Equilibrium of Polyhedral Frames.* By W. J. Macquorn Rankine, C.E., LL.D., F.R.SS.L.&E.*

[illegible] of a principle which is proved for polygonal frames in 'A Manual of Applied Mechanics,' art. 150.

Theorem.—If planes diverging from a point or line be drawn normal to the lines of resistance of the bars of a polyhedral frame, then the faces of a polyhedron whose edges lie in those diverging planes (in such a manner that those faces, together with the diverging planes which contain their edges, form a set of contiguous diverging pyramids or wedges) will represent, and be normal to, a system of forces which, being applied to the summits of the polyhedral frame, will balance each other—each such force being applied to the summit of meeting of the bars whose lines of resistance are normal to the set of diverging planes that enclose that face of the polyhedron of forces which represents and is normal to the force in question. Also, the areas of the diverging planes will represent the stresses along the bars to whose lines of resistance they are respectively normal.

It is obvious that the polyhedron of forces and the polyhedral frame are reciprocally related as follows: their numbers of edges are equal, and their corresponding pairs of edges perpendicular to each other; and the number of faces in each polyhedron is equal to the number of summits in the other.

Glasgow, January 9, 1864.

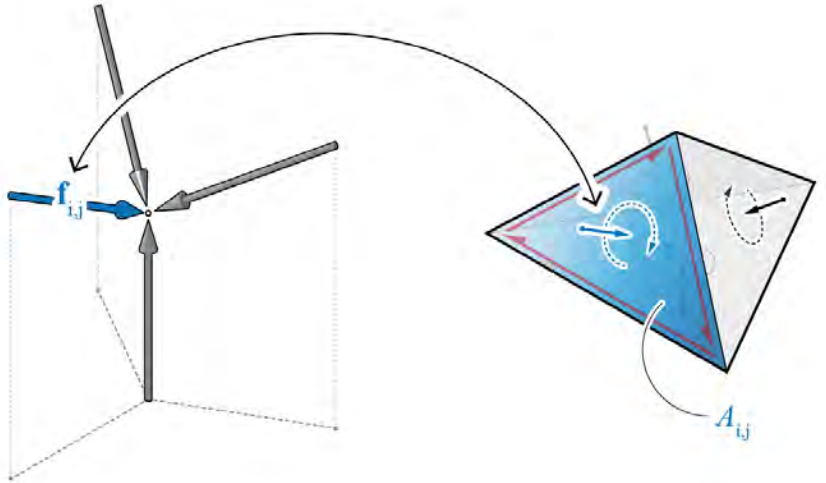
XVIII. *On the Theory of the Velocity of Sound.* By Professor Challis, M.A., F.R.S., F.R.A.S.*

THE "Note" of Professor Tyndall "On the Velocity of Sound" in the Number of the Philosophical Magazine for last November, and the reference therein made to Dulong's experiments, have led me to see that the principles I have applied to this question admit of an extension which had not previously occurred to me. For this reason, and because I am desirous of making a few remarks on the notice taken of my researches by Prof. Le Conte in the article inserted in the January Number, I now revert to the subject.

The theoretical value of the velocity of sound which I have obtained, agreeing very closely with the observed value, is a purely mathematical deduction, on hydrodynamical principles, from the hypotheses that the medium is a perfect fluid, and that

* Communicated by the Author.

"Principle of the Equilibrium of Polyhedral Frames"
Rankine (1864)



"On the equilibrium of funicular polyhedral frames and convex polyhedral force diagrams"
Akbarzadeh et al. (2015)

Contents lists available at

Computer-Aided Design

journal homepage:

On the equilibrium of funicular polyhedral frames and convex polyhedral force diagrams

Masoud Akbarzadeh, Tom Van Mele, Philippe Block

ETH Zurich, Institute of Technology in Architecture, ETH Zurich, Switzerland

HIGHLIGHTS

- Three-dimensional extension of graphic statics using polyhedral form and force diagrams.
- Defining the topological and geometrical relationships of 3D reciprocal diagrams.
- Design of compression and tension-only spatial structures with externally applied loads.
- Designing complex funicular spatial forms by aggregating convex force polyhedral cells.
- CAD implementation to manipulate the geometry of the force and explore its effects on the forms.

ARTICLE INFO

Article history:
Received 18 June 2014
Accepted 13 January 2015

Keywords:
Three-dimensional graphic statics
Reciprocal diagrams
Spatial systems of forces
Convex force polyhedra
Rankine's principle of equilibrium of polyhedral frames

ABSTRACT

This paper presents a three-dimensional extension of graphic statics using polyhedral form and force diagrams for the design of compression-only and tension-only spatial structures with externally applied loads. It explains the concept of 3D reciprocity based on Rankine's reciprocal proportion for the equilibrium of spatial frames. It provides a definition for polyhedral reciprocal form and force diagrams that allows unloading external forces and discusses their geometrical and topological characteristics. This paper furthermore provides a geometrical procedure for constructing a pair of reciprocal polyhedral diagrams from a given polyhedron representing either the form or force diagram of a structural system. Using this method, this paper furthermore suggests a design strategy for finding complex funicular spatial forms in pure compression (or tension), based on the construction of force diagrams through the aggregation of convex polyhedral cells. Finally, it discusses the effect of changes in the geometry of the force diagram on the geometry of the form diagram and the distribution of forces in it.

1. Introduction

Graphic statics is a method for structural form finding that originates in the pre-digital era [1–4], but continues to be used and developed even today [5–7].

In graphic statics, the geometry and equilibrium of forces of a structural system are represented by two reciprocal diagrams: the *form* and the *force* diagram. Since the geometrical relationship between these diagrams provides explicit control over both form and forces of a structure simultaneously, graphic statics is considered as an intuitive technique for structural design, relevant to architects, engineers, researchers, and students.

Despite its clear strengths and advantages, existing method of graphic statics has some limitations. The most important one is that a designer can only design three-dimensional structures by reducing them to a combination of two-dimensional sub-systems, for example, by using projections [8,9].

This paper therefore presents a three-dimensional version of graphic statics using reciprocal polyhedral form and force diagrams for the design and analysis of spatial frames with externally applied loads.

1.1. Previous work

Reciprocal diagrams are the basis of the conventional graphical methods of structural form finding. This section provides a brief summary of the use of reciprocal diagrams and other geometric techniques in structural design.

2D reciprocal diagrams

The concept of investigating static equilibrium through geometric constructions involving polygons of forces first appeared

This paper has been recommended for acceptance by Helmut Pottmann.

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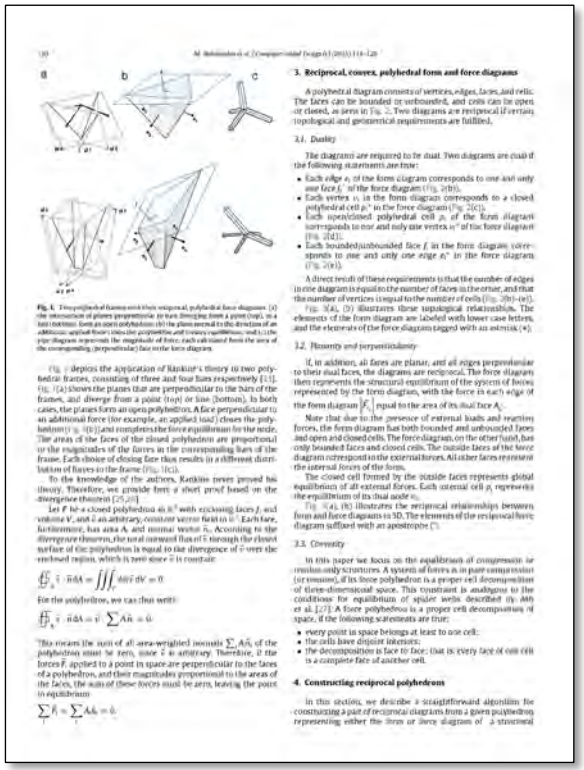


Fig. 1. Two polyhedral frames and their reciprocal, polyhedral force diagrams: (a) the intersection of planes perpendicular to bars diverging from a point (top), in a bar (bottom) form an open polyhedron; (b) the plane normal to the direction of an additional applied force (dashed) completes and closes the equilibrium; and (c) the force diagram represents the magnitude of force, each calculated from the area of the corresponding (perpendicular) face in the form diagram.

Fig. 1 depicts the application of Rankine's theory to two polyhedral frames, consisting of three and four bars respectively [1,3]. Fig. 1(a) shows the planes that are perpendicular to the bars of the frames, and diverge from a point (top) or line (bottom). In both cases, the planes form an open polyhedron. A face perpendicular to an additional force (for example, an applied load) closes the polyhedron (Fig. 1(b)) and completes the force equilibrium for the node. The areas of the faces of the closed polyhedron are proportional to the magnitudes of the forces in the corresponding bars of the frame. Each choice of closing face thus results in a different distribution of forces in the frame (Fig. 1(c)).

To the knowledge of the authors, Rankine never proved his theory. Therefore, we provide here a short proof based on the divergence theorem [25,26].

Let F be a closed polyhedron in $\mathbb{R}^3$ with enclosing faces f_i and volume V, and $\vec{n}$ an arbitrary, constant vector field in $\mathbb{R}^3$. Each face, furthermore, has area A_i and normal vector $\vec{n}_i$. According to the divergence theorem, the total outward flux of $\vec{n}$ through the closed surface of the polyhedron is equal to the divergence of $\vec{n}$ over the enclosed region, which is zero since $\vec{n}$ is constant:

$$\oint_S \vec{n} \cdot \vec{n}\, dA = \iiint_V \vec{n} \cdot \vec{0}\, dV = 0.$$

For the polyhedron, we can thus write:

$$\oint_S \vec{n} \cdot \vec{n}\, dA = \vec{n} \cdot \sum_i A_i \vec{n}_i = 0.$$

This means the sum of all area-weighted normals $\sum_i A_i \vec{n}_i$ of the polyhedron must be zero, since $\vec{n}$ is arbitrary. Therefore, if the forces $\vec{F}_i$ applied to a point in space are perpendicular to the faces of a polyhedron, and their magnitudes proportional to the areas of the faces, the sum of these forces must be zero, leaving the point in equilibrium:

$$\sum_i \vec{F}_i = \sum_i A_i \vec{n}_i = \vec{0}.$$

3. Reciprocal, convex, polyhedral form and force diagrams

A polyhedral diagram consists of vertices, edges, faces, and cells. The faces can be bounded or unbounded, and cells can be open or closed, as seen in Fig. 2. Two diagrams are reciprocal if certain topological and geometrical requirements are fulfilled.

3.1. Duality

The diagrams are required to be dual. Two diagrams are dual if the following statements are true:

- Each edge e_i of the form diagram corresponds to one and only one face f_i^* of the force diagram (Fig. 2(b)).
- Each vertex v_i in the form diagram corresponds to a closed polyhedral cell p_i^* in the force diagram (Fig. 2(c)).
- Each open/closed polyhedral cell p_i of the form diagram corresponds to one and only one vertex v_i^* of the force diagram (Fig. 2(d)).
- Each bounded/unbounded face f_i in the form diagram corresponds to one and only one edge e_i^* in the force diagram (Fig. 2(e)).

A direct result of these requirements is that the number of edges in one diagram is equal to the number of faces in the other, and that the number of vertices is equal to the number of cells (Fig. 2(b)–(e)). Fig. 3(a), (b) illustrates these topological relationships. The elements of the form diagram are labeled with lower case letters, and the elements of the force diagrams tagged with an asterisk (*).

3.2. Planarity and perpendicularity

If, in addition, all faces are planar, and all edges perpendicular to their dual faces, the diagrams are reciprocal. The force diagram then represents the structural equilibrium of the system of forces represented by the form diagram, with the force in each edge of the form diagram $\vec{F}_i$ equal to the area of its dual face A_i^*.

Note that due to the presence of external loads and reaction forces, the form diagram has both bounded and unbounded faces and open and closed cells. The force diagram, on the other hand, has only bounded faces and closed cells. The outside faces of the force diagram correspond to the external forces. All other faces represent the internal forces of the form.

The closed cell formed by the outside faces represents global equilibrium of all external forces. Each internal cell p_i represents the equilibrium of its dual node v_i.

Fig. 3(a), (b) illustrates the reciprocal relationships between form and force diagrams in 3D. The elements of the reciprocal force diagram suffixed with an apostrophe (*).

3.3. Convexity

In this paper we focus on the equilibrium of compression or tension-only structures. A system of forces is in pure compression (or tension), if its force polyhedron is a proper cell decomposition of three-dimensional space. This constraint is analogous to the conditions for equilibrium of spider webs described by Ash et al. [27]: A force polyhedron is a proper cell decomposition of space, if the following statements are true:

- every point in space belongs at least to one cell;
- the cells have disjoint interiors;
- the decomposition is face to face; that is: every face of one cell is a complete face of another cell.

4. Constructing reciprocal polyhedrons

In this section, we describe a straightforward algorithm for constructing a pair of reciprocal diagrams from a given polyhedron representing either the form or force diagram of a structural

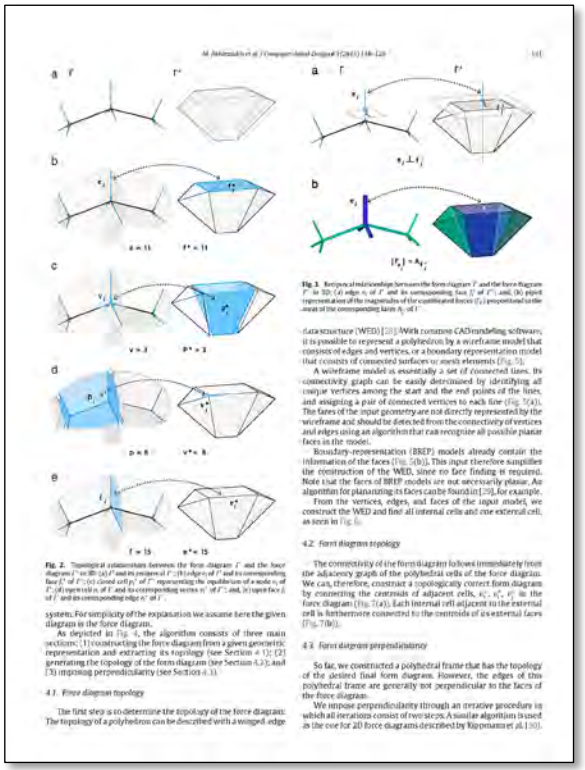


Fig. 2. Topological relationships between the form diagram Γ and the force diagram Γ^* in 3D: (a) edges e_i of Γ and its corresponding face f_i^* of Γ^*; and, (b) equivalent representation of the magnitudes of the coordinated forces ($\vec{F}_i$) proportional to the area of the corresponding face A_i^* of Γ^*.

Fig. 3. Reciprocal relationships between the form diagram Γ and the force diagram Γ^* in 3D: (a) edges e_i of Γ and its corresponding face f_i^* of Γ^*; and, (b) equivalent representation of the magnitudes of the coordinated forces ($\vec{F}_i$) proportional to the area of the corresponding face A_i^* of Γ^*.

datastructure (WED) [28]. With common CAD modelling software, it is possible to represent a polyhedron by a wireframe model that consists of edges and vertices, or a boundary representation model that consists of connected surfaces or meshs elements (Fig. 5).

A wireframe model is essentially a set of connected lines. Its connectivity graph can be easily determined by identifying all unique vertices among the start and the end points of the lines, and assigning a pair of connected vertices to each line (Fig. 5(a)). The faces of the input geometry are not directly represented by the wireframe and should be detected from the connectivity of vertices and edges using an algorithm that can recognize all possible planar faces in the model.

Boundary-representation (BREP) models already contain the information of the faces (Fig. 5(b)). This input therefore simplifies the construction of the WED, since no face finding is required. Note that the faces of BREP models are not necessarily planar. An algorithm for planarizing its faces can be found in [29], for example.

From the vertices, edges, and faces of the input model, we construct the WED and find all internal cells and one external cell, as seen in Fig. 6.

4.2. Form diagram topology

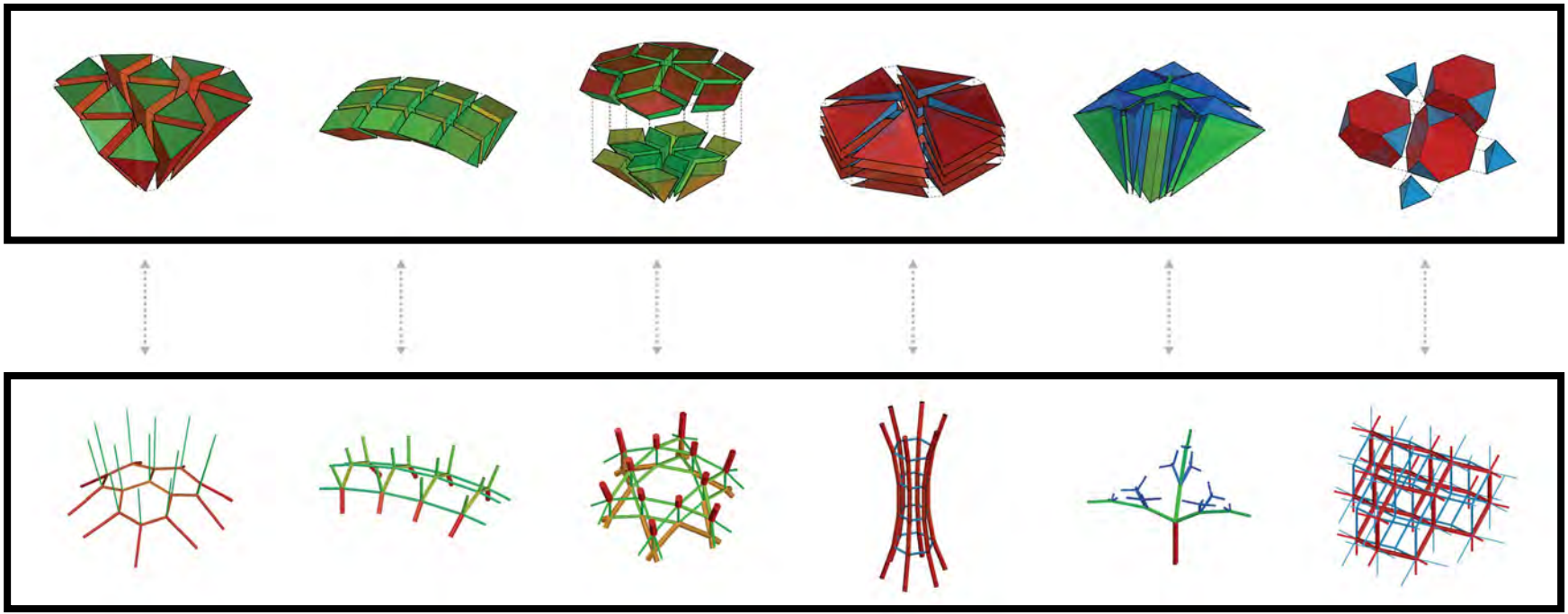
The connectivity of the form diagram follows immediately from the adjacency graph of the polyhedral cells of the force diagram. We can, therefore, construct a topologically correct form diagram by connecting the centroids of adjacent cells, c_i^*, c_j^*, c_k^* in the force diagram (Fig. 7(a)). Each internal cell adjacent to the external cell is furthermore connected to the centroids of its external faces (Fig. 7(b)).

4.3. Form diagram perpendicularity

So far, we constructed a polyhedral frame that has the topology of the desired final form diagram. However, the edges of this polyhedral frame are generally not perpendicular to the faces of the force diagram.

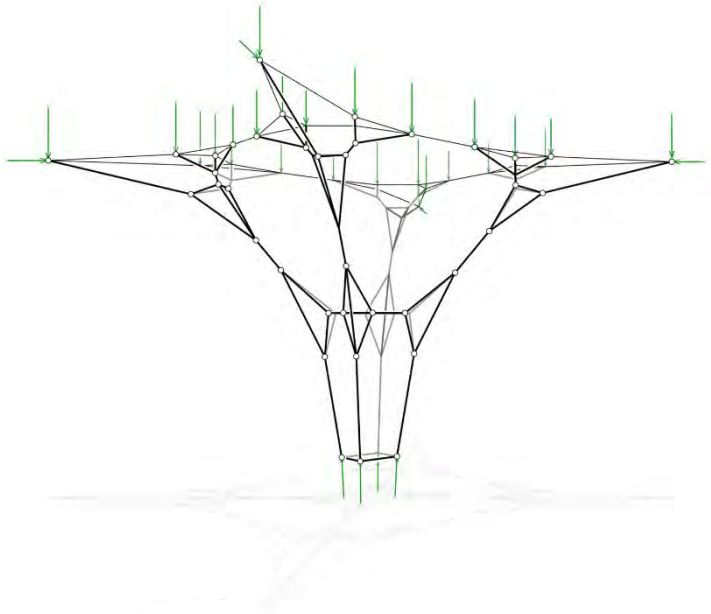
We impose perpendicularity through an iterative procedure in which all iterations consist of two steps. A similar algorithm based as the one for 2D force diagrams described by Rippmann et al. [30].

Polyhedral force diagram

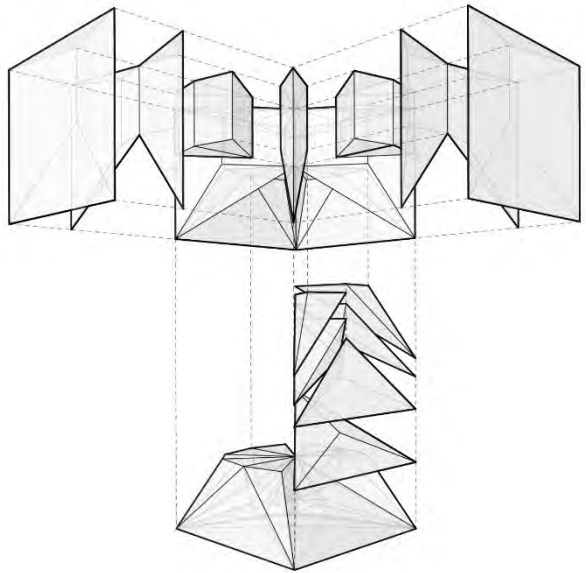


Polyhedral form diagram

"On the equilibrium of funicular polyhedral frames and convex polyhedral force diagrams" | Akbarzadeh et al. (2015)

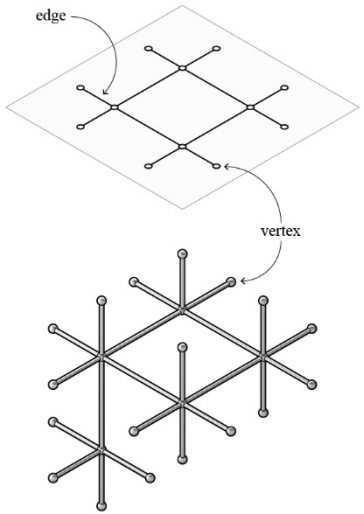


Polyhedral Form Diagram



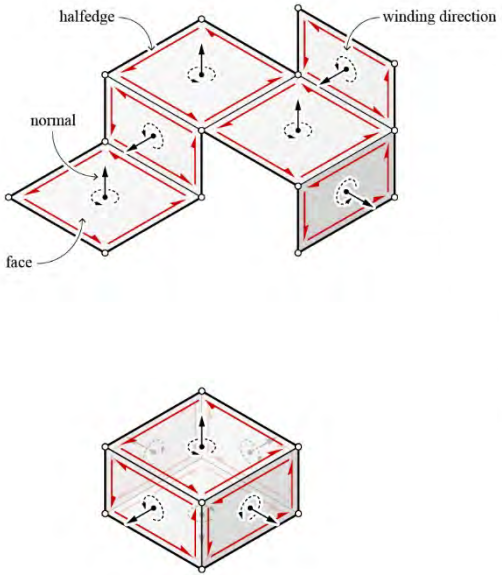
Polyhedral Force Diagram

"On the equilibrium of funicular polyhedral frames and convex polyhedral force diagrams" | Akbarzadeh et al. (2015)



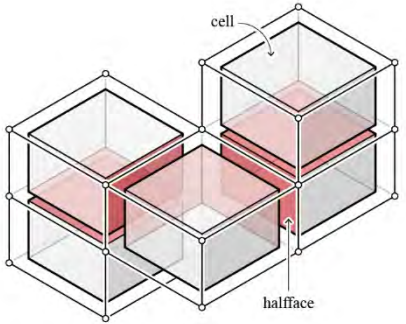
network

network of vertices



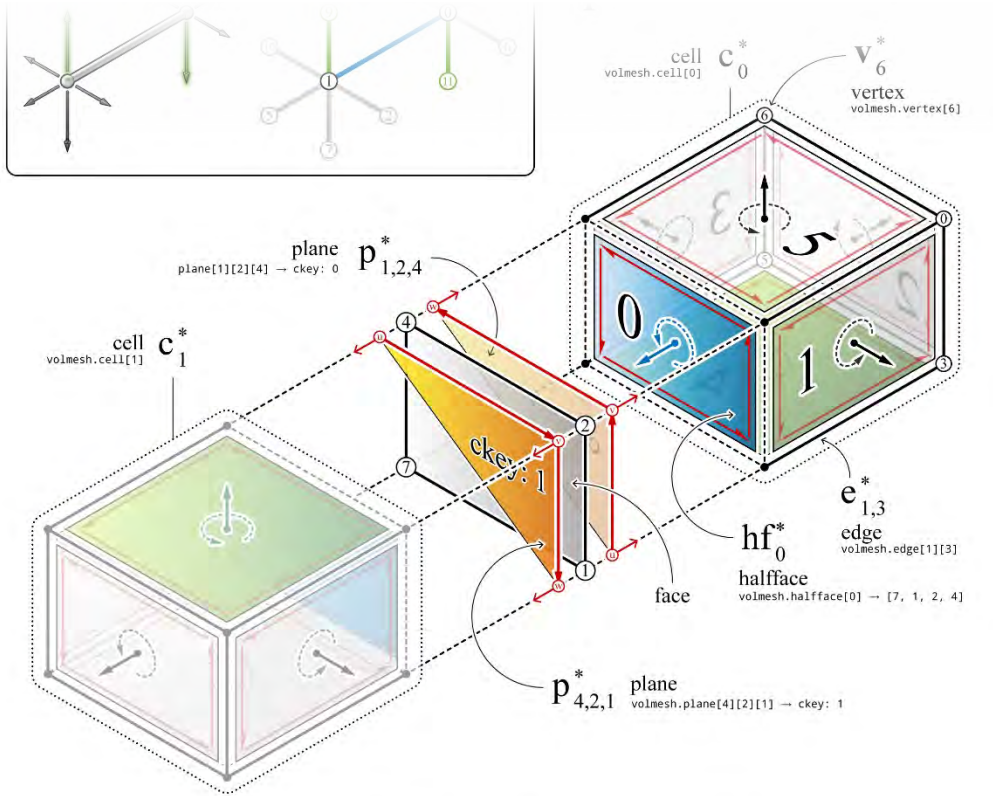
mesh

network of faces



volmesh

network of cells



vertex

vkey	attr
0	'x': 10.0 'y': 10.0 'z': 7.0
1	'x': 10.0 'y': 0.0 'z': 0.0

edge

u	v : attr
0	2: {} 3: {}
1	2: {} 3: {}
2	4: {}
3	0: {} 5: {}

halfface

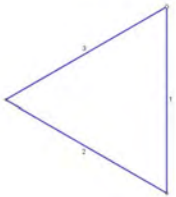
hfkey	vkeys
0	7 1 2 4
1	0 2

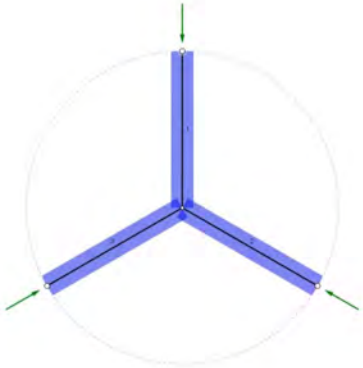
cell

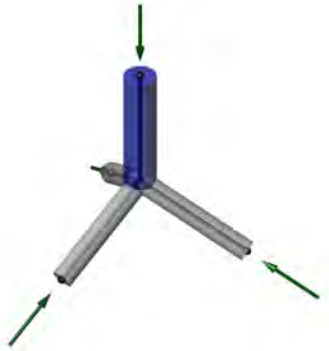
ckey	u	v : hfkey
0	0	2: 1 3: 2 6: 5
1		7: 4 2: 0 3: 1

plane

u	v	w : ckey	w : ckey
0	2	1: 0	4: 1
	3	1: 1	5: 0
	6	4: 0	5: 1
1	7	4: 1	5: 0
	2	0: 1	4: 0







2D graphic statics

Control lengths of force polygons → Control force magnitudes

3D graphic statics

Control areas of force polyhedra → Control force magnitudes

NOT as straight forward or intuitive

17

GEOMETRY-BASED UNDERSTANDING OF STRUCTURES

Tom VAN MELE[1], Lorenz LACHAUER[2], Matthias RIPPMANN[3] and Philippe BLOCK[4]

Computer-Aided Design

Algebraic graph statics*

T. Van Mele*, P. Block

Rankine Reciprocals with Zero Bars

Allan McRobie

Cambridge University Engineering Department

Trumpington St, Cambridge, CB2 1PZ, UK. ian290@cam.ac.uk

April 11, 2016

Downloaded from http://rsos.royalsocietypublishing.org/ on March 23, 2017

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The geometry of structural equilibrium

Allan McRobie

Cambridge University Engineering Department, Trumpington Street, Cambridge CB2 1PZ, UK

Research

Cite this article: McRobie A. 2017 The geometry of structural equilibrium. R. Soc. open sci. 4: 160759.

Structural optimization using graphic statics

Lauren L. Beghini · Juan Carrion · Alessandro Beghini · Arkadiusz Mazurek · William F. Baker

RESEARCH PAPER

Computer-Aided Design

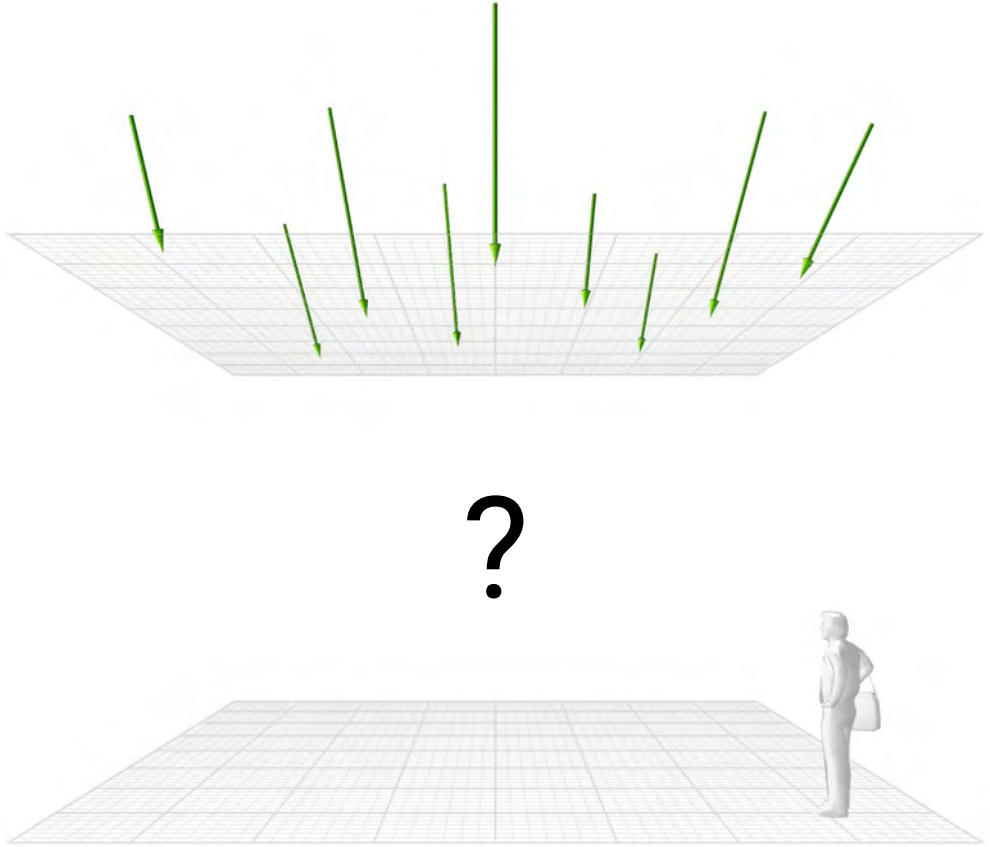
On the equilibrium of funicular polyhedral frames and convex polyhedral force diagrams*

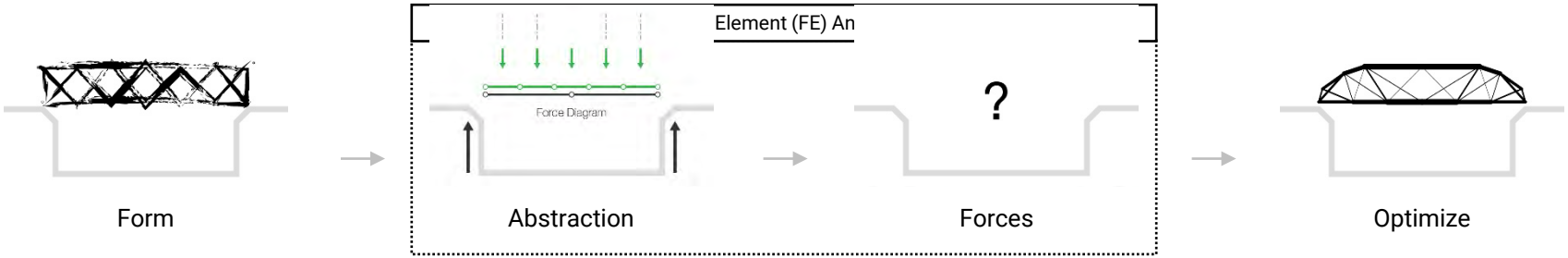
Masoud Akbarzadeh, Tom Van Mele, Philippe Block*

HIGHLIGHTS

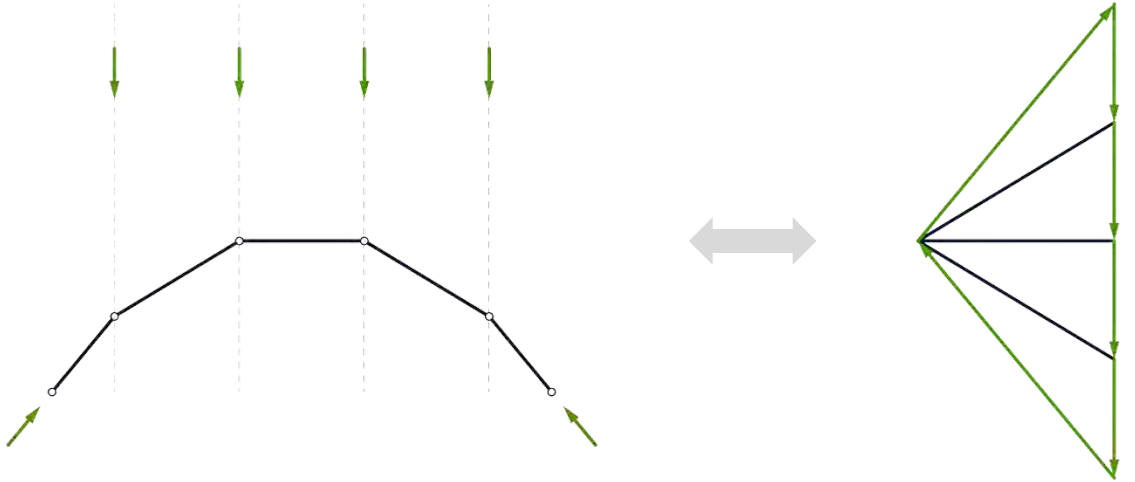
- Three-dimensional extension of graphic statics using
- Defining the topological and geometrical
- Design of compression and
- Designing complex
- CAD


Computational Design Framework
for
3D Graphic Statics
Juney Lee
ETH Zurich • PhD Dissertation No. 25526



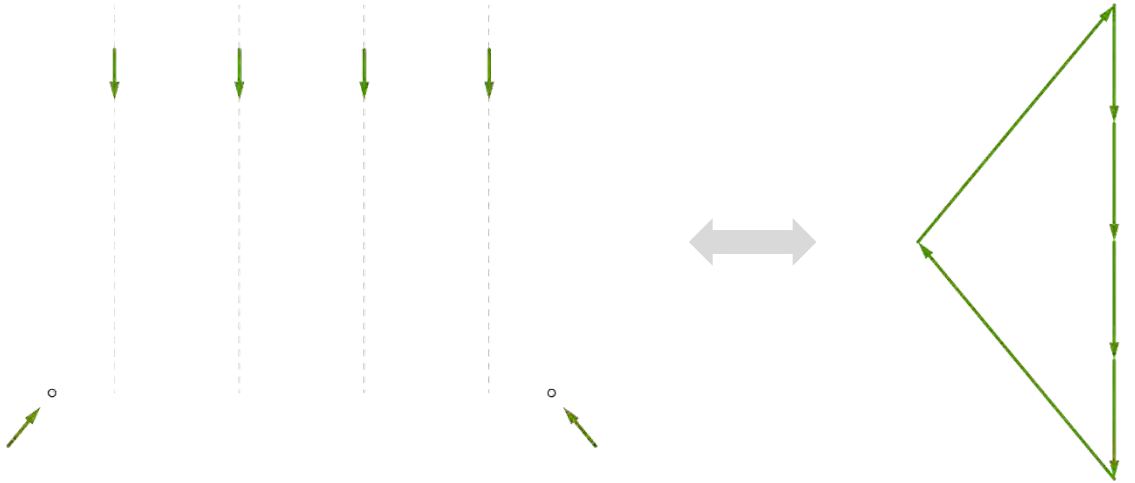


Proposed Force-driven Design



Geometry of Form

Geometry of Forces



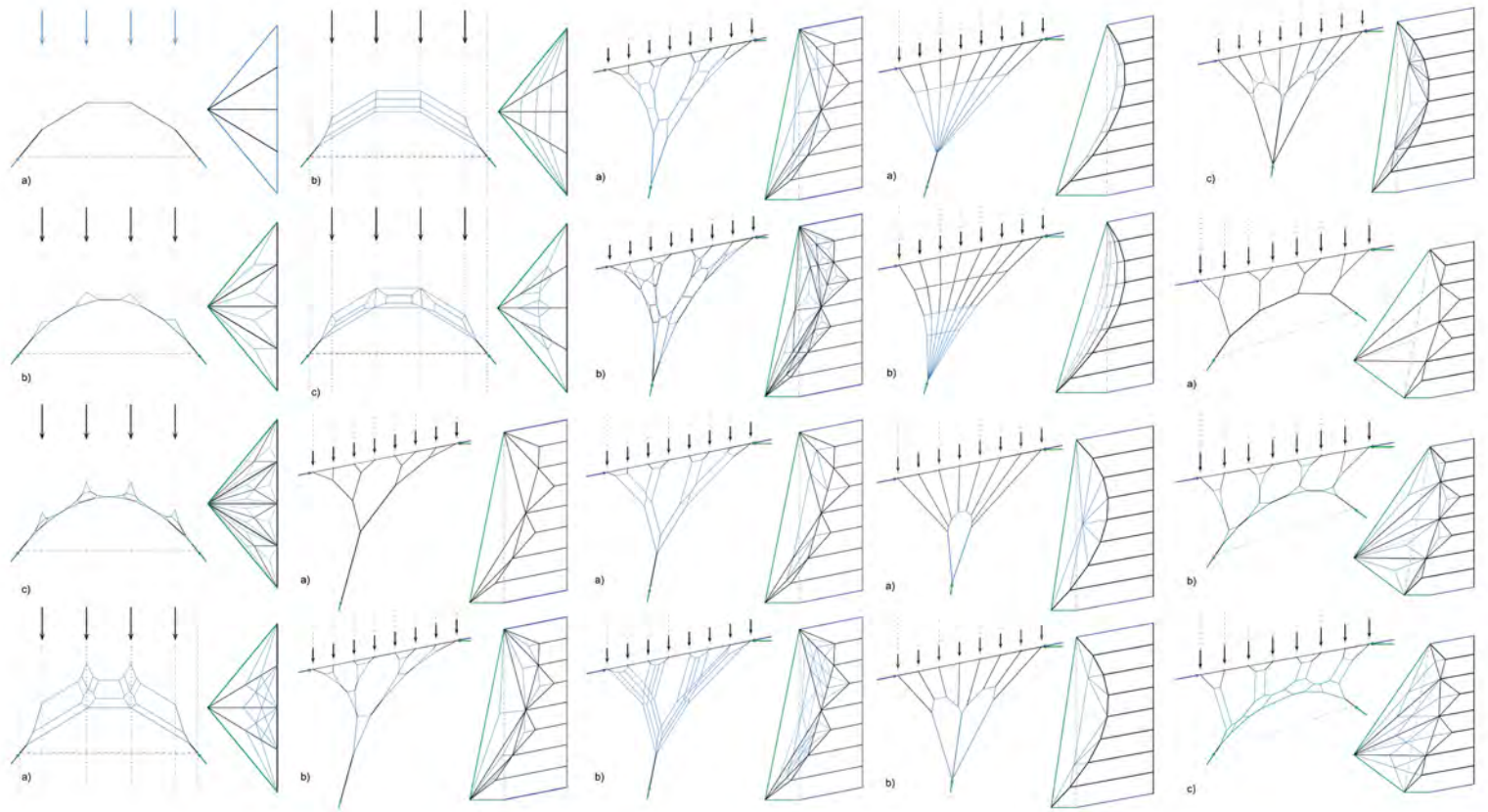
Design the Geometry of Forces

Geometry of Form

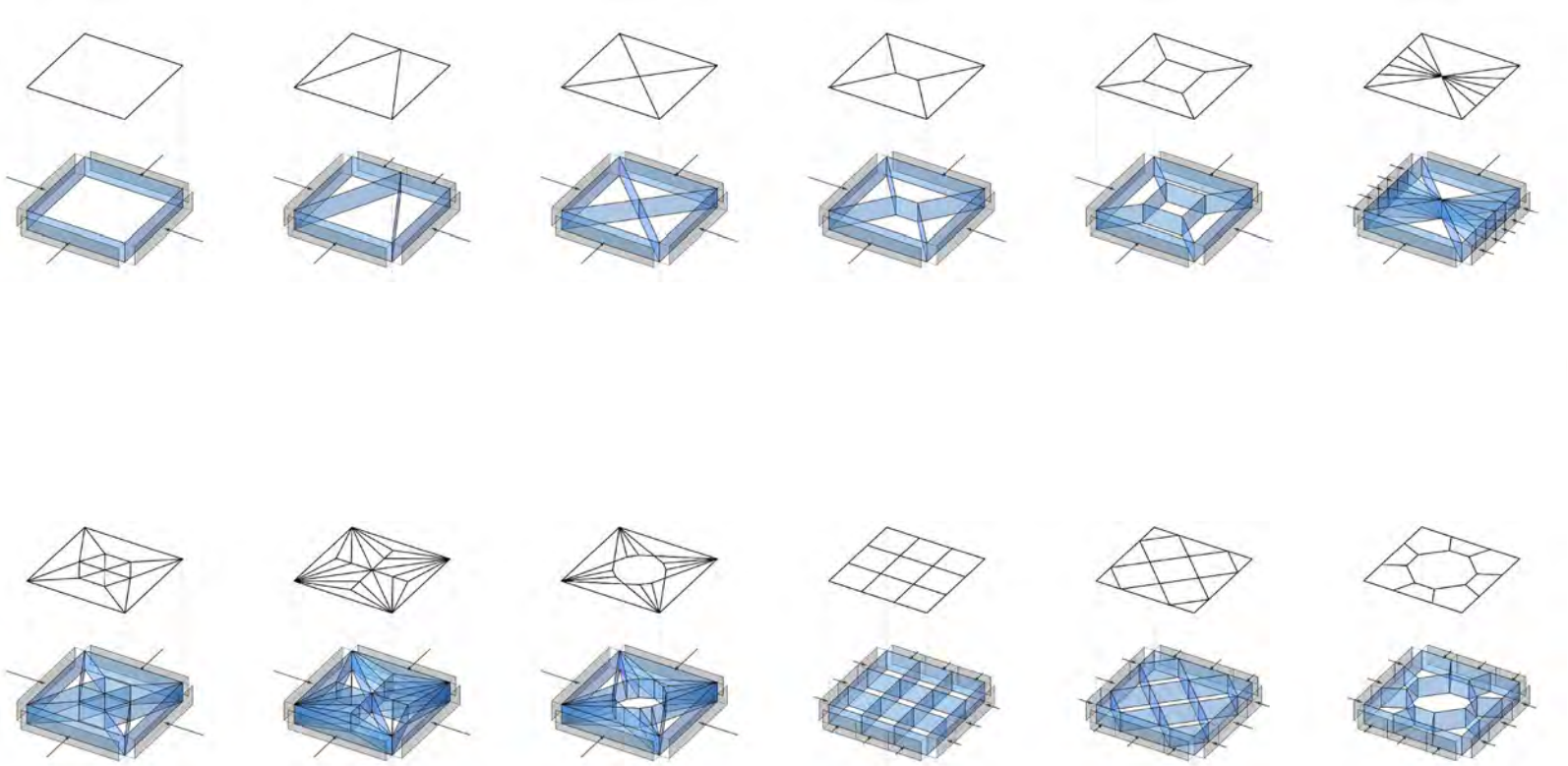
Discovery of New Structural Forms

Geometry of Forces

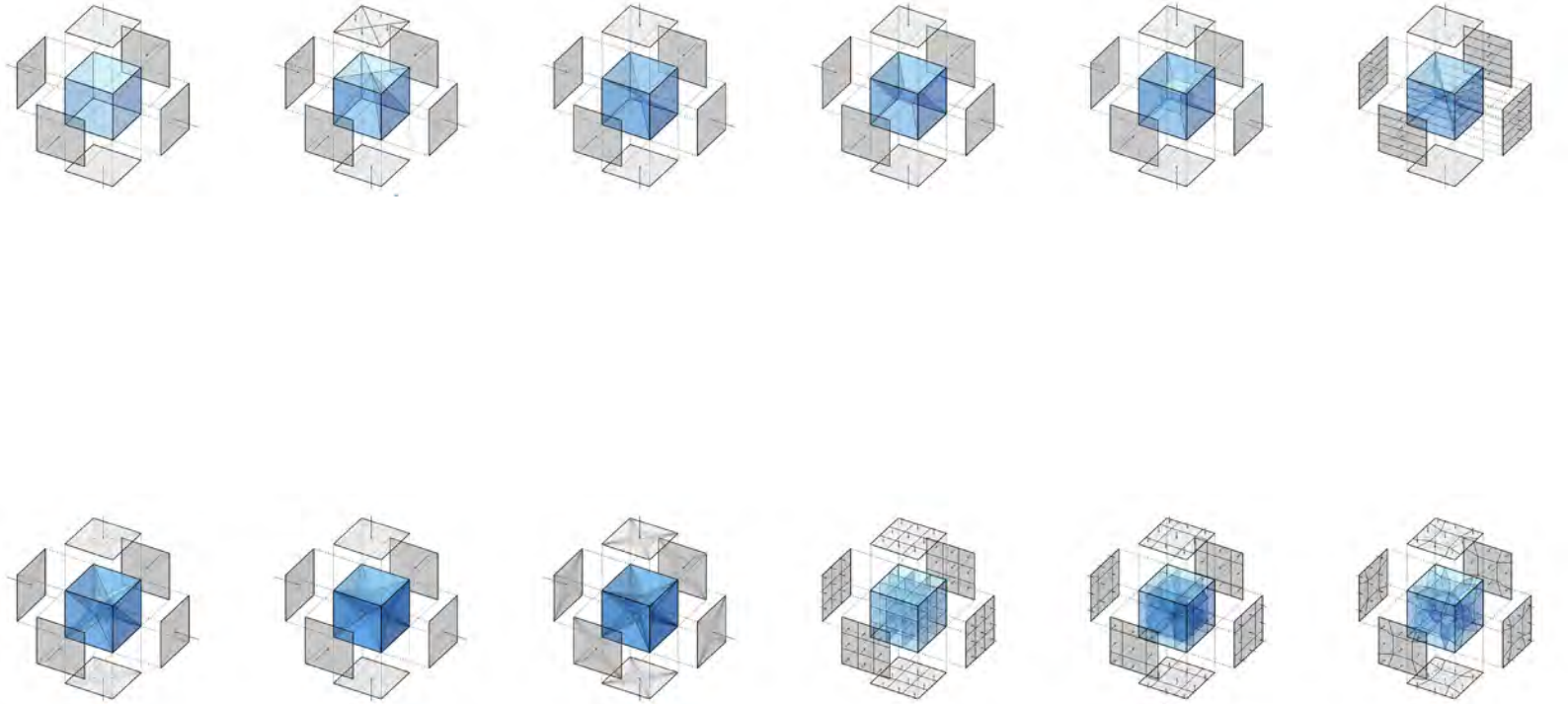
After, "Compression-only Form finding through Finite Subdivision of the Force Polygon" | Akbarzadeh et al. (2014)



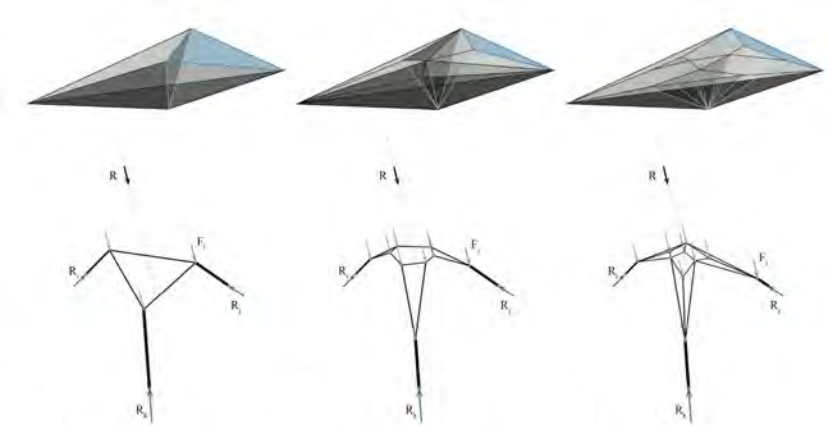
"Compression-only Form finding through Finite Subdivision of the Force Polygon" | Akbarzadeh et al. (2014)



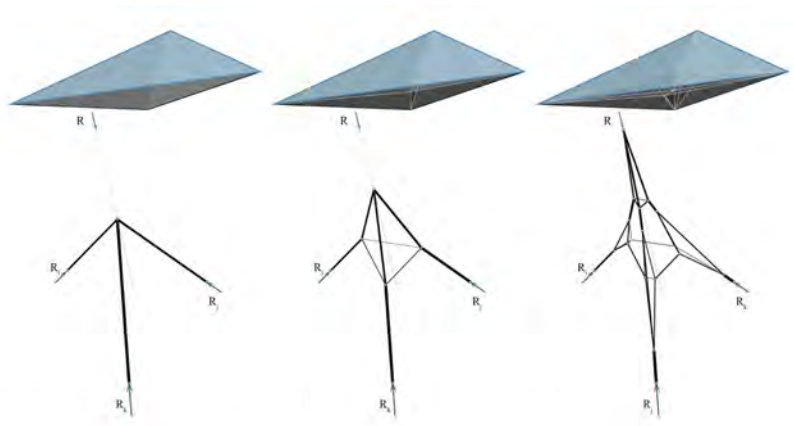
"Computational Design Framework for 3D Graphic Statics," PhD dissertation | Lee (2018)



"Computational Design Framework for 3D Graphic Statics," PhD dissertation | Lee (2018)

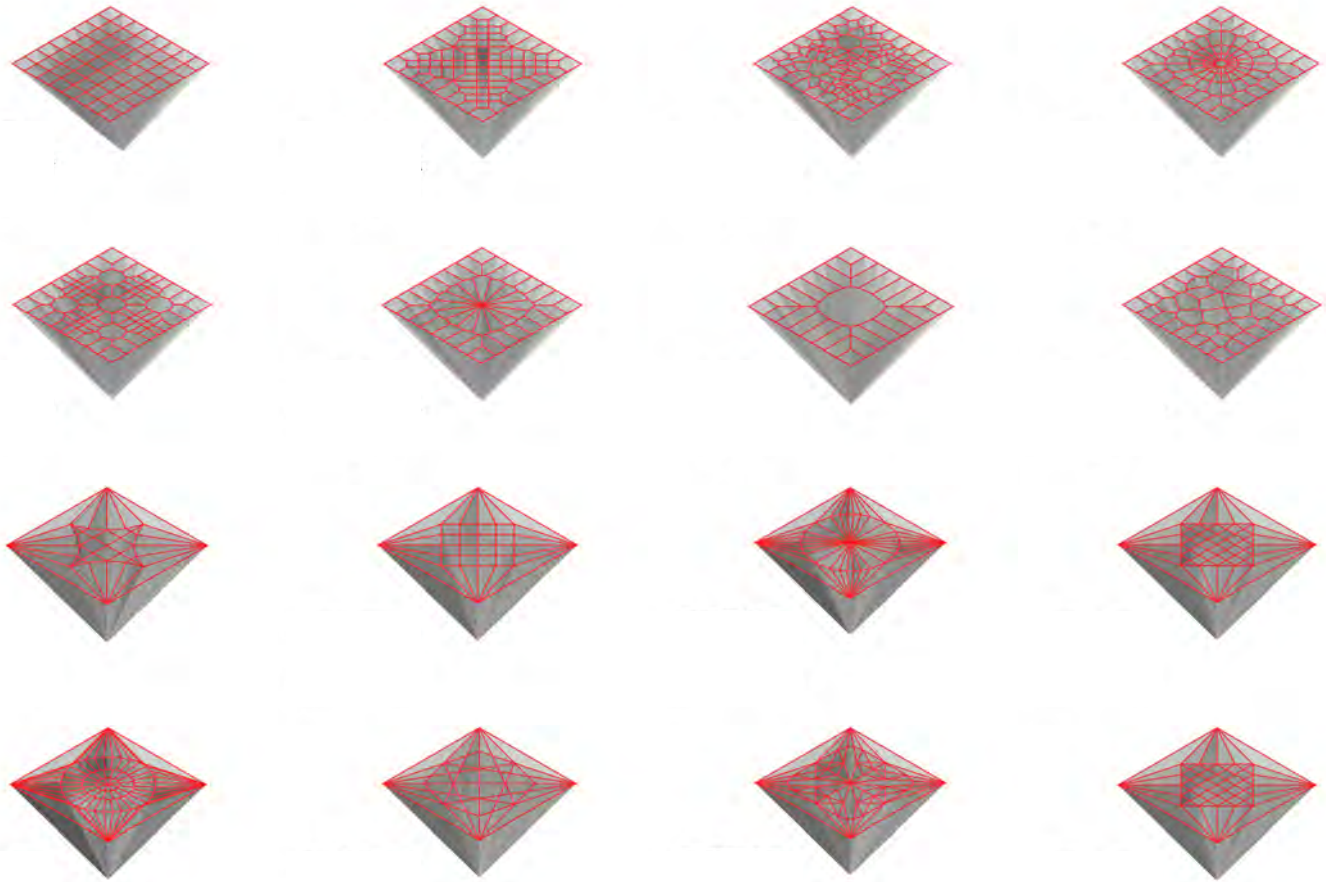


Subdivision of outer faces → changes boundary conditions

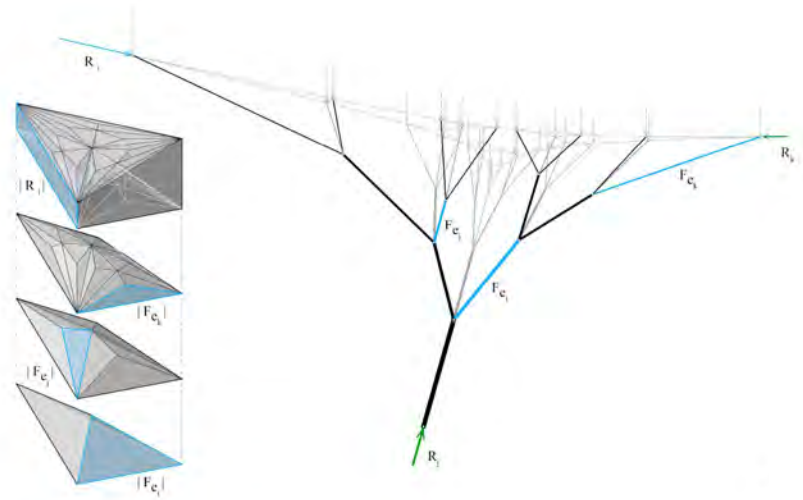


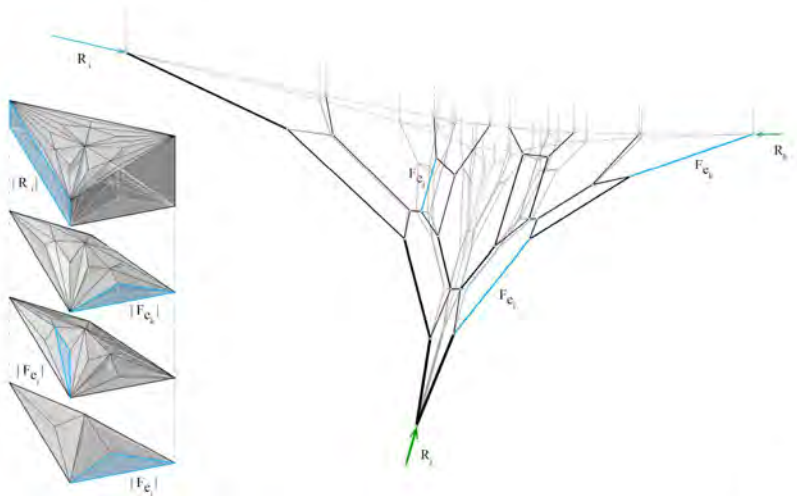
Subdivision of cells → refinement of predefined form

"Three-dimensional Compression Form Finding through Subdivision" | Akbarzadeh et al. (2015)

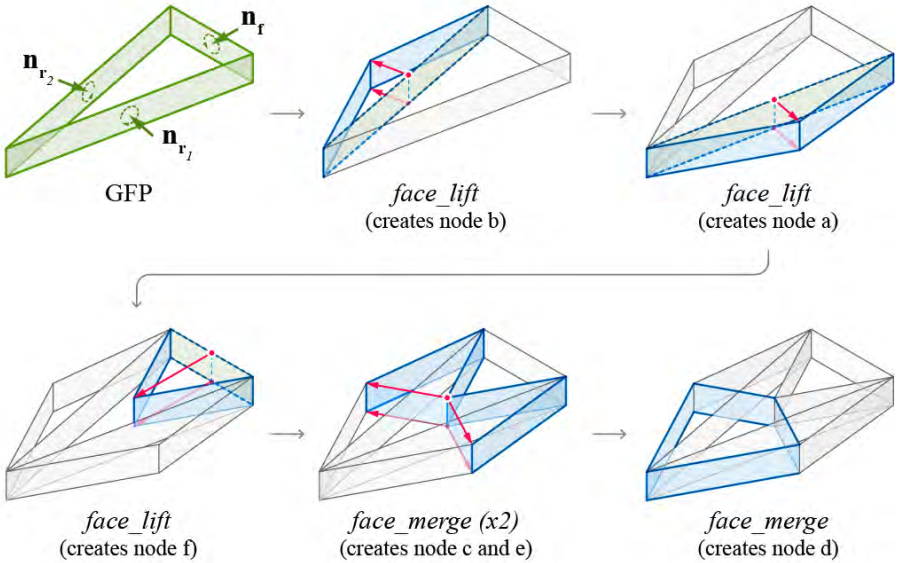


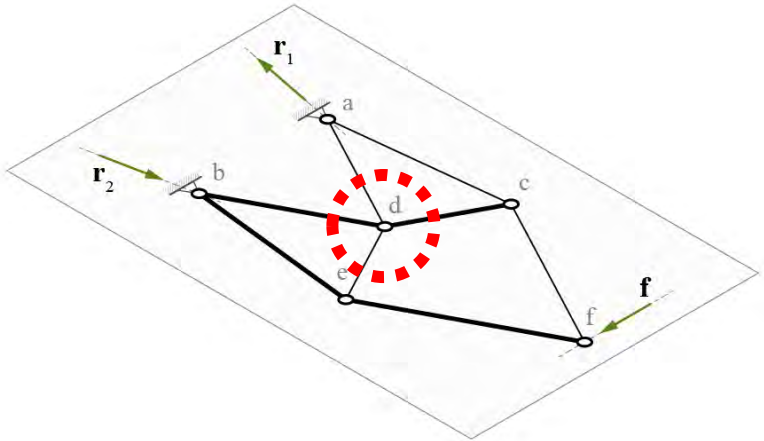
"Computational Design Framework for 3D Graphic Statics," PhD dissertation | Lee (2018)

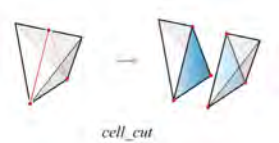


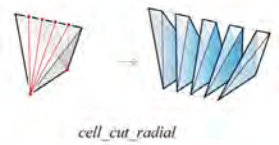


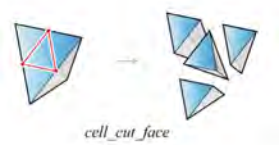
"Three-dimensional Graphical Statics using Polyhedral Reciprocal Diagrams," PhD Dissertation | Akbarzadeh (2016)

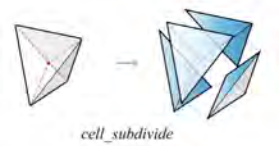

$\mathbf{n}_f$
$\mathbf{n}_{r_2}$
$\mathbf{n}_{r_1}$
GFP
face_lift
(creates node b)
face_lift
(creates node a)
face_lift
(creates node f)
face_merge (x2)
(creates node c and e)
face_merge
(creates node d)


$\mathbf{r}_1$
$\mathbf{r}_2$
a
b
c
d
e
f
f

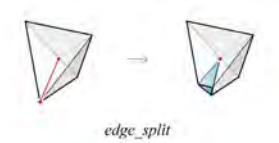


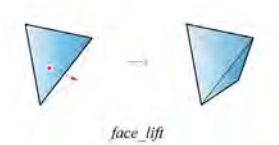


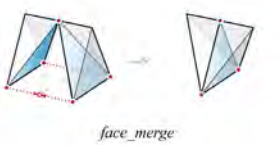


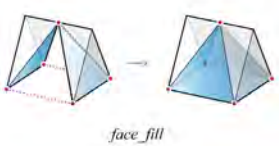


Subdivision (decomposition)

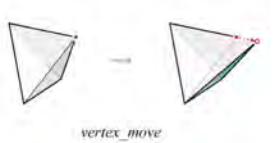


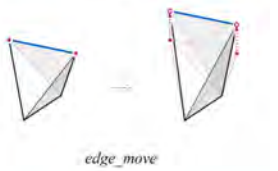


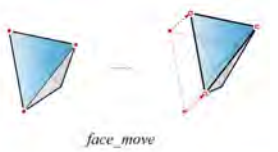


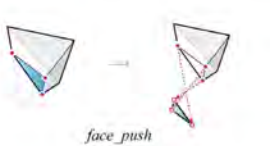


Transformation (additive)



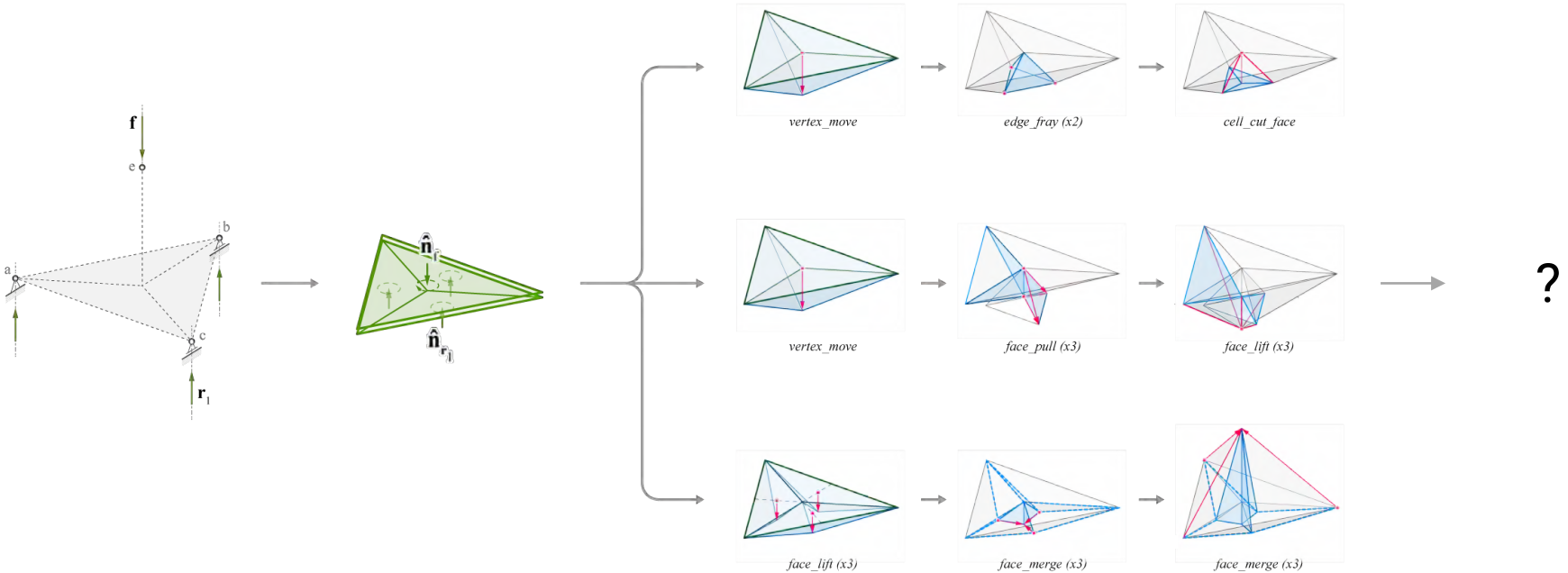




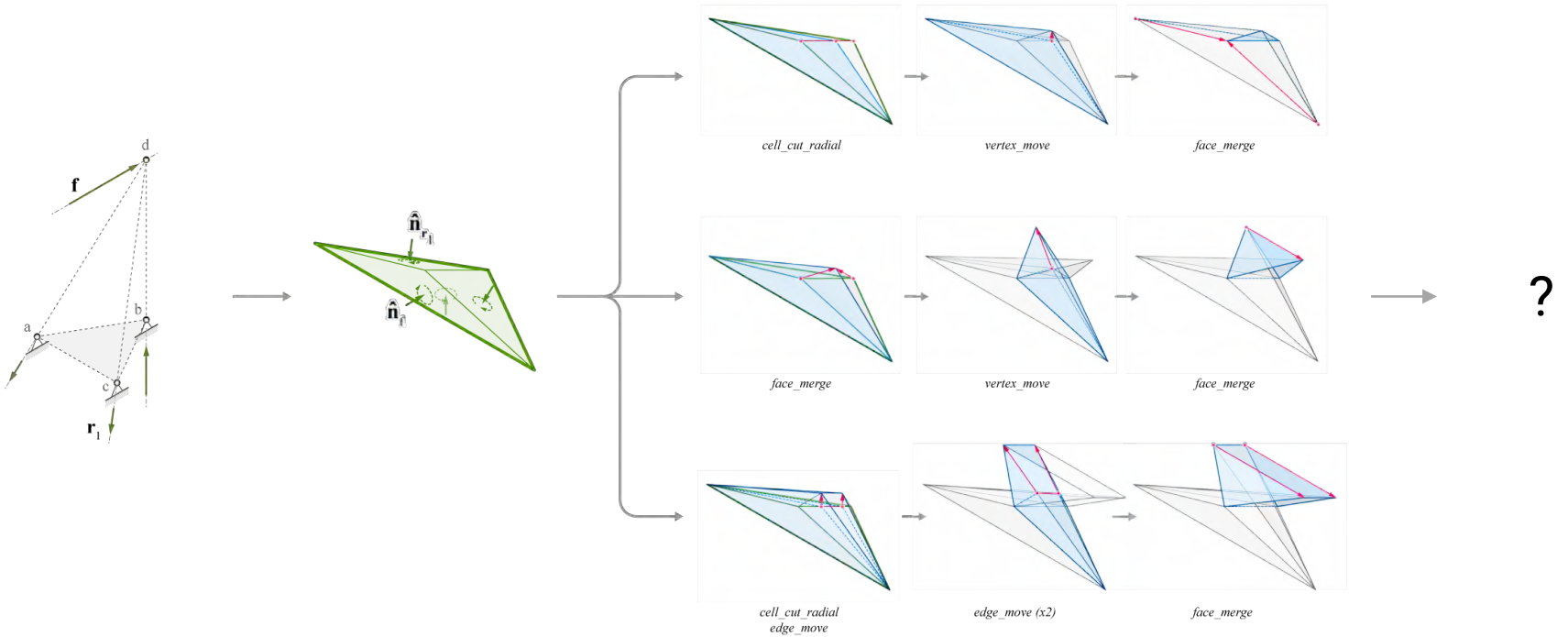


Modification

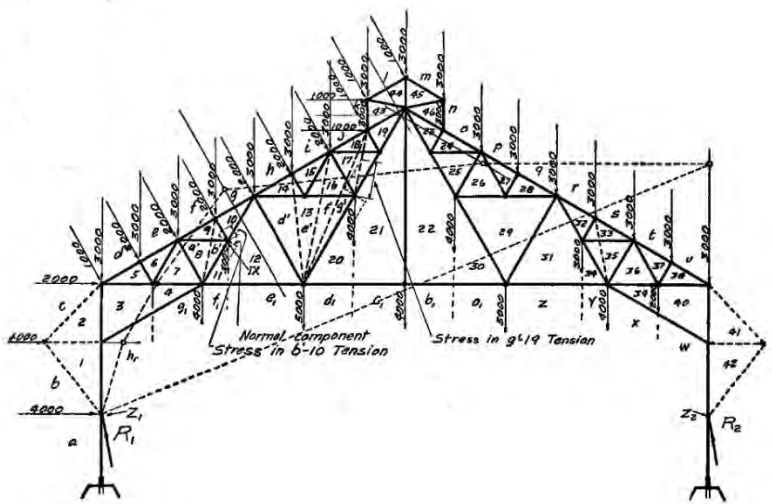
"Form-finding explorations through geometric transformations and modifications of force polyhedrons" | Lee, Van Mele & Block (2017)



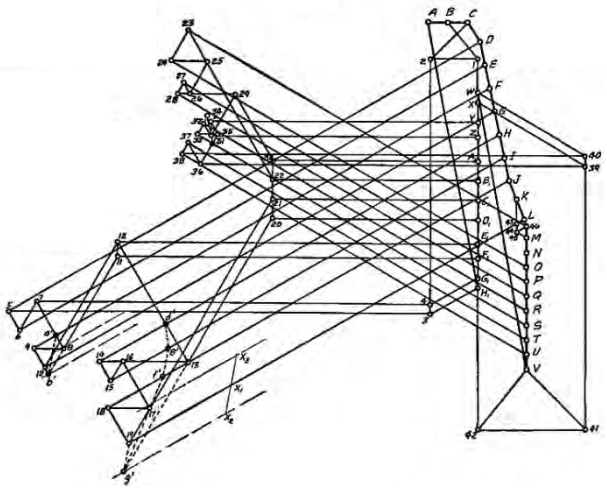
"Form-finding explorations through geometric transformations and modifications of force polyhedrons" | Lee, Van Mele & Block (2017)



"Form-finding explorations through geometric transformations and modifications of force polyhedrons" | Lee, Van Mele & Block (2017)

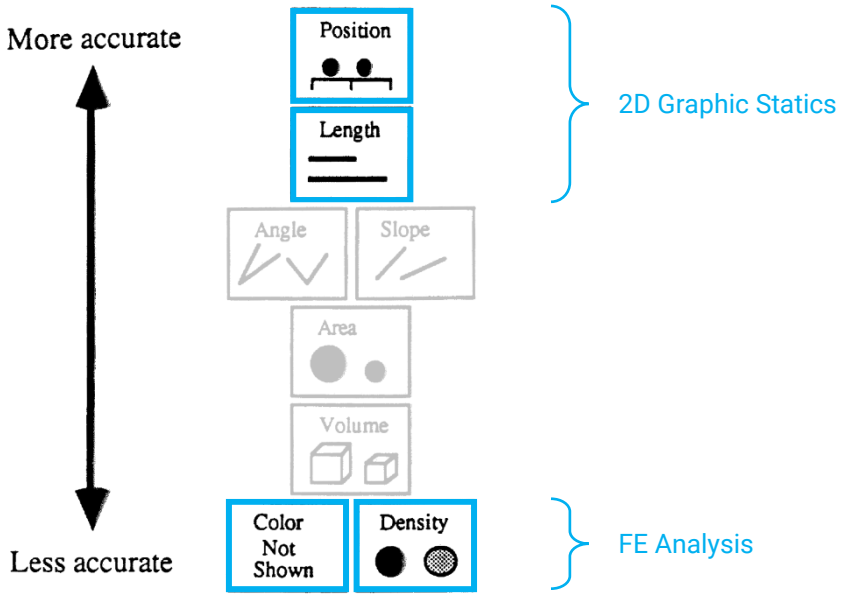


Form Diagram



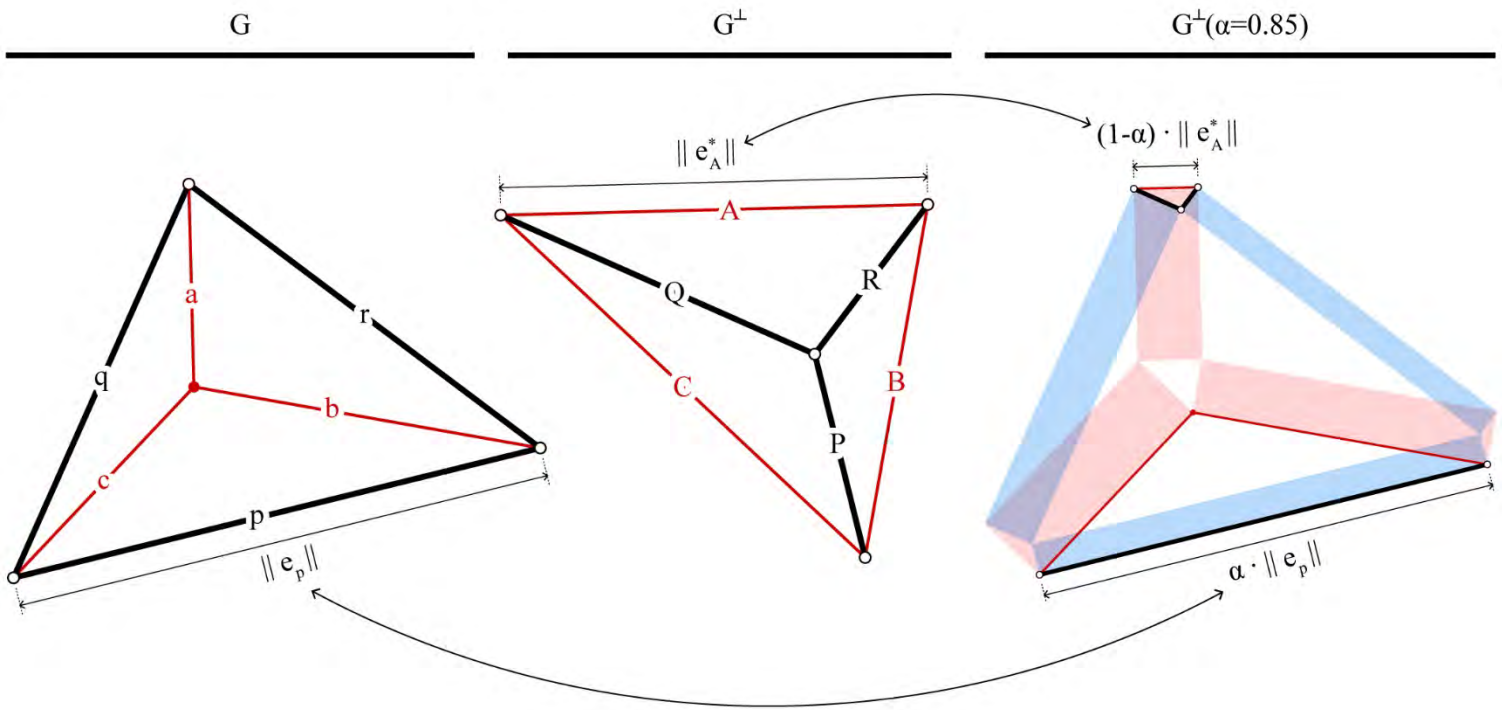
Force Diagram

"Form-finding explorations through geometric transformations and modifications of force polyhedrons" | Lee, Van Mele & Block (2016)

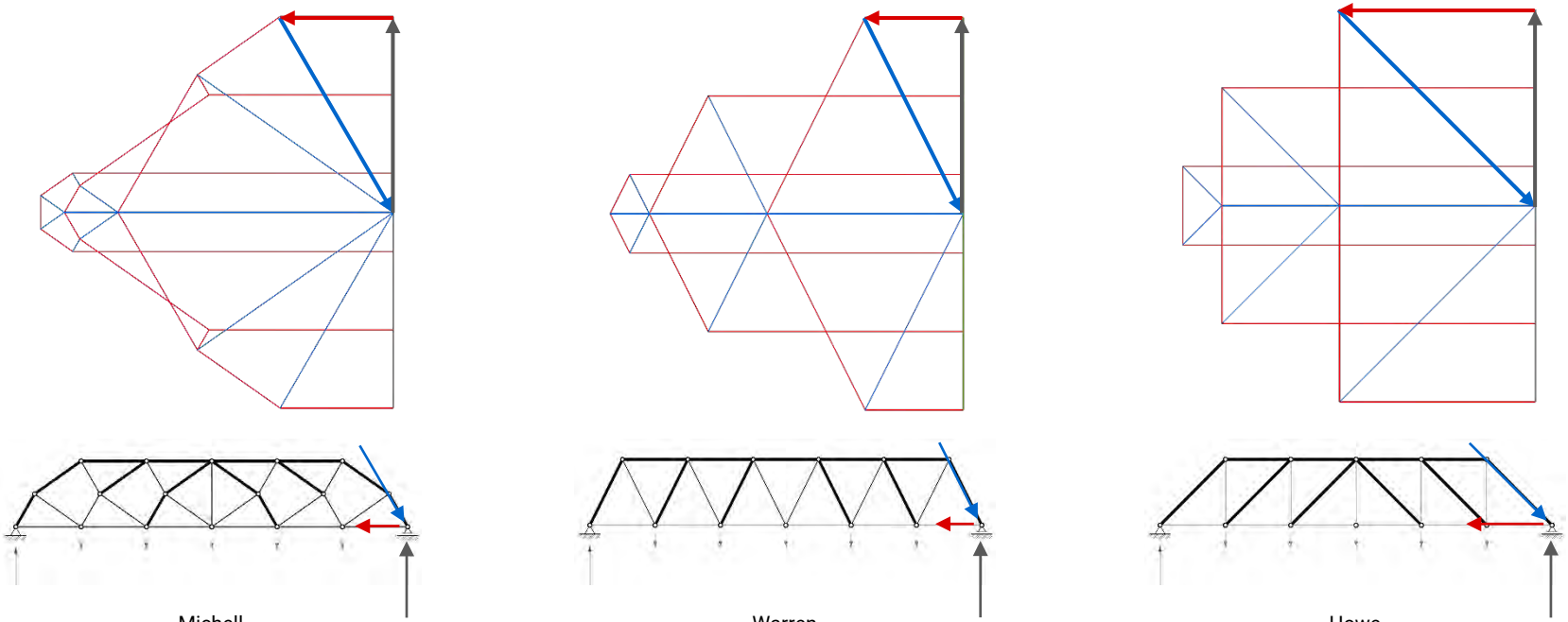


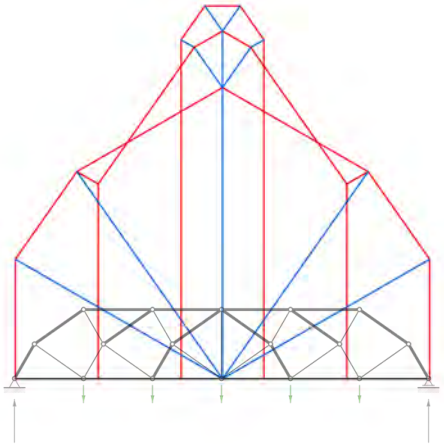
Accuracy and Effectiveness of Human Retinal Perception

"Automating the Design of Graphical Presentation of Relational Information" | Mackinlay (1986)

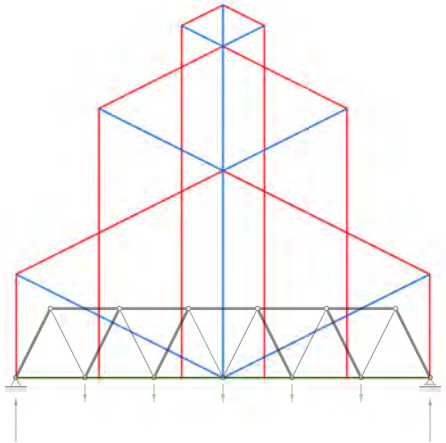

G
G⊥
G⊥(α=0.85)
|| e*A ||
(1-α) · || e*A ||
a
r
q
b
c
p
|| ep ||
A
Q
R
C
B
P
α · || ep ||

“Maxwell” Diagram (perpendicular orientation)

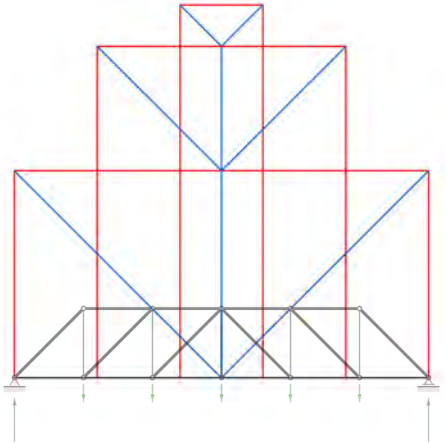




Michell

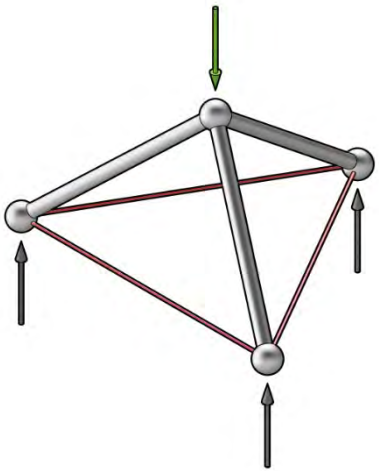


Warren

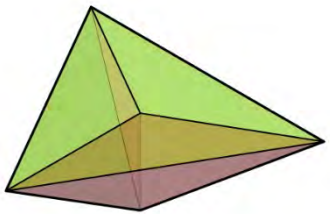


Howe

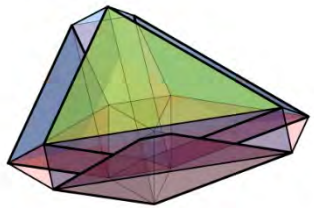
a) Γ



b) $\Gamma^{\perp}(\alpha)$



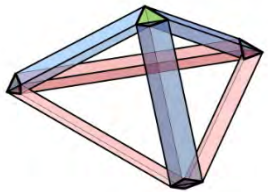
$\alpha = 0$



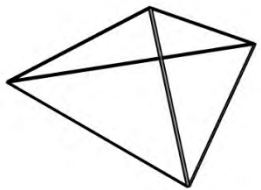
$\alpha = 0.25$

$\alpha = 0.5$

$\alpha = 0.75$



$\alpha = 0.90$



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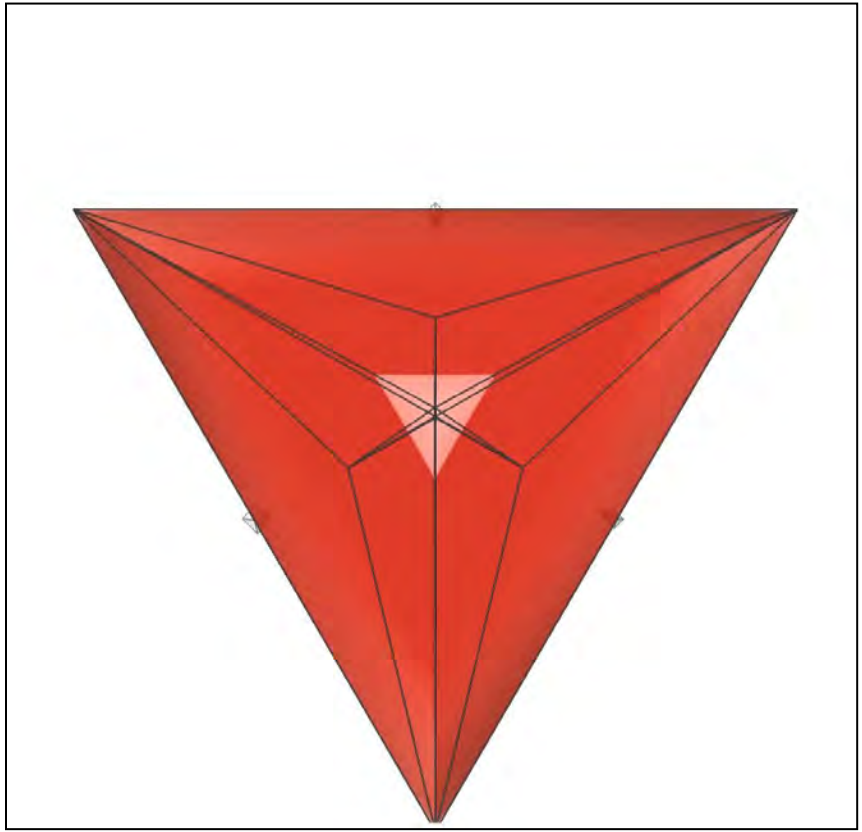
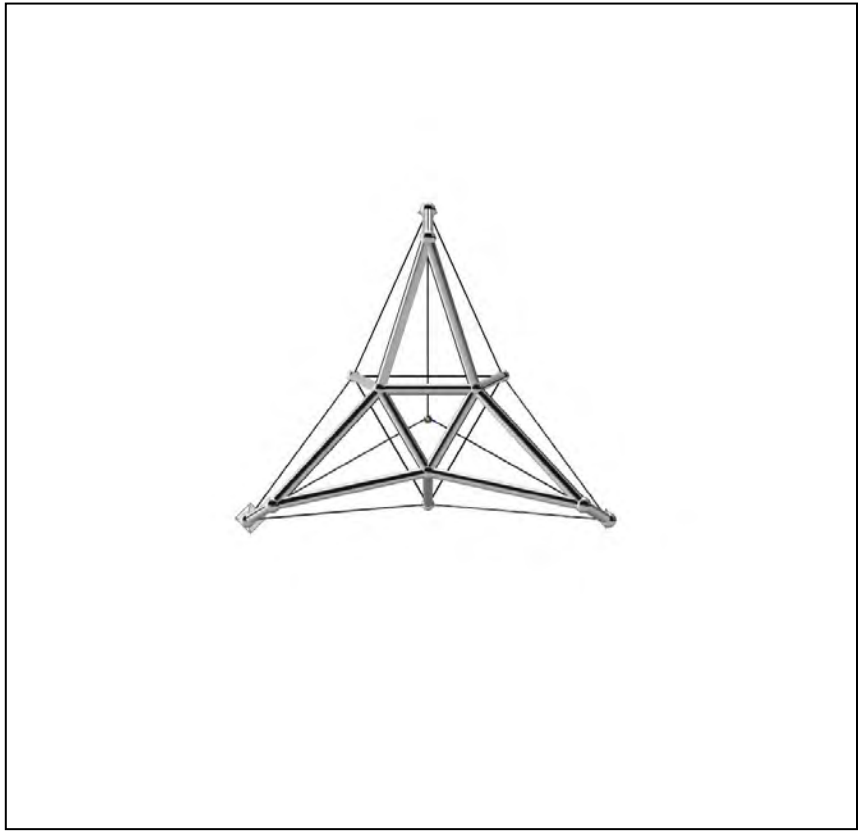




Interactive Polyhedral Force Diagram ← Synchronized Interface → **Form Diagram**

"Computational Design Framework for 3D Graphic Statics," PhD dissertation | Lee (2018)

2017 Seoul Biennale of Architecture and Urbanism

MycoTree

Beyond Mining - Urban Growth

Sustainable Construction
KIT Karlsruhe

Karsten Schlesier
Felix Heisel
Dirk Hebel

Block Research Group
ETH Zürich

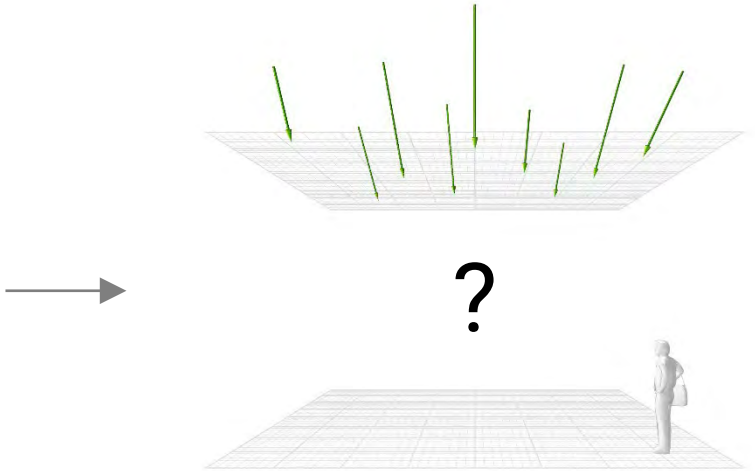
Juney Lee
Matthias Rippmann
Tomás Méndez Echenagucia
Andrew Liew
Noelle Paulson
Tom Van Mele
Philippe Block

Future Cities Laboratory
Singapore-ETH

Nazanin Saeidi
Alireza Javadian
Adi Reza Nugroho
Robbi Zidna Ilman
Erlambang Adjidarma
Ronaldiaz Hartantyo
Hokie Christian
Orion Tan
Sheng Yu
Kelly Cooper

Photo by Carlina Teteris


BP AUTOBAHN CENTER



Compression-only in full 3D?

mushroom
(fruit, the “plant”)

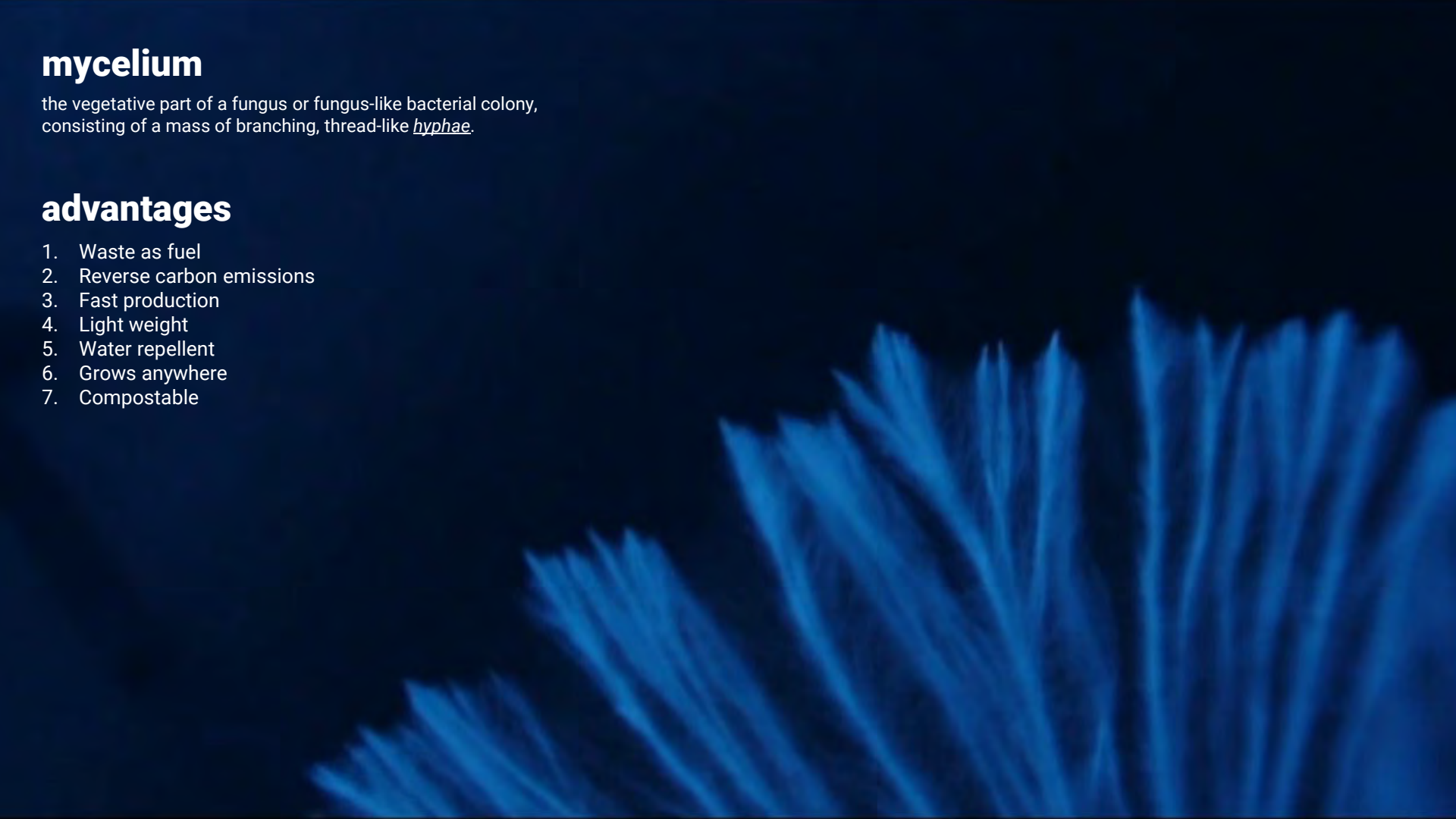
mycelium
(roots, the “animal”)

mycelium

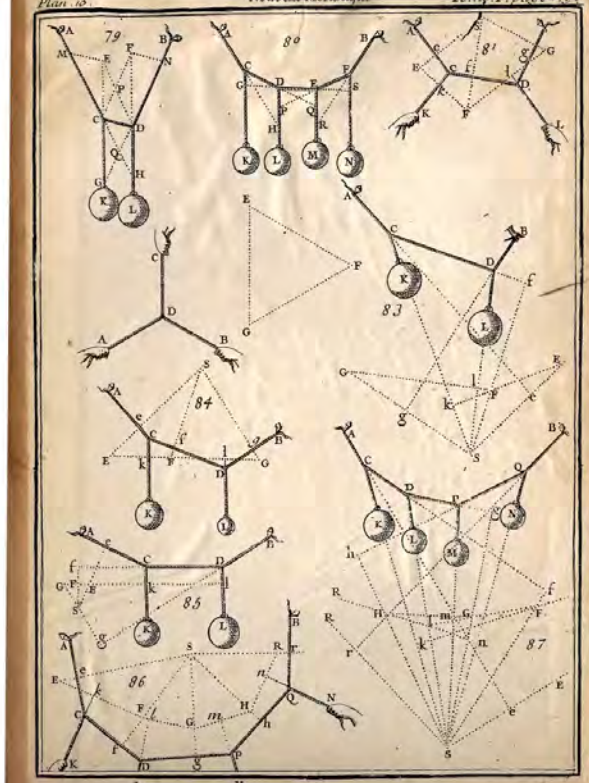
the vegetative part of a fungus or fungus-like bacterial colony, consisting of a mass of branching, thread-like *hyphae*.

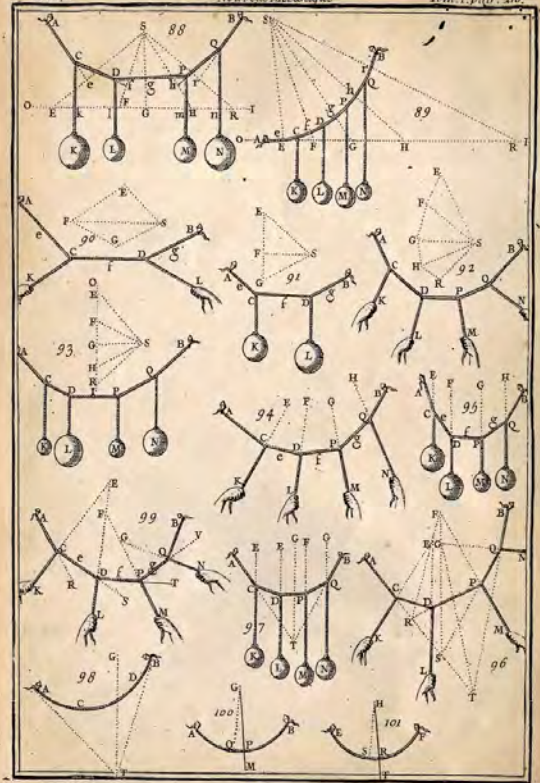
advantages

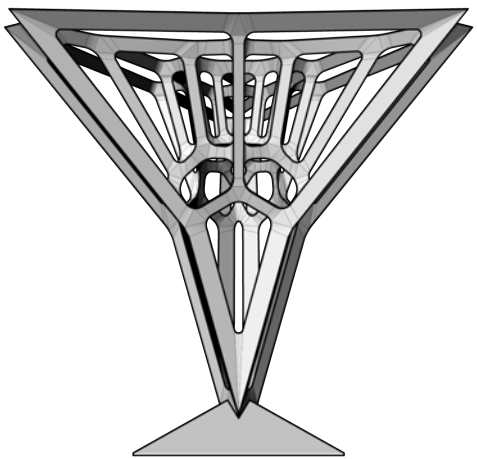
1. Waste as fuel
2. Reverse carbon emissions
3. Fast production
4. Light weight
5. Water repellent
6. Grows anywhere
7. Compostable

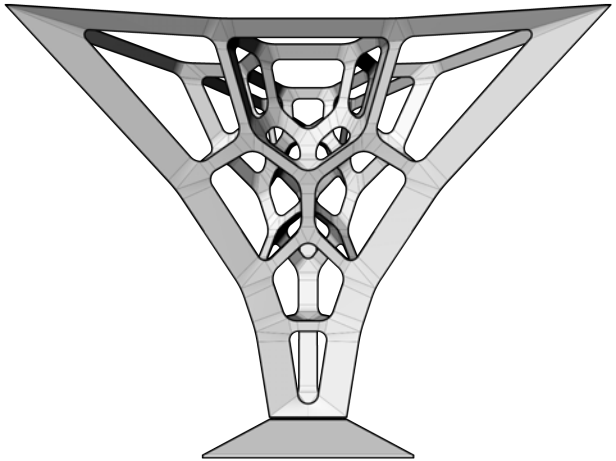


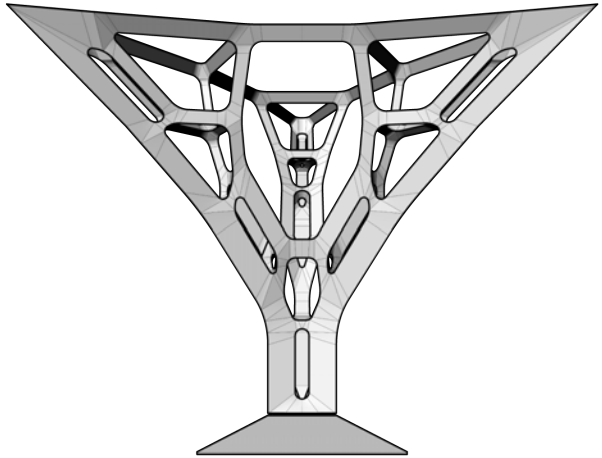

0.15 – 0.4 MPa

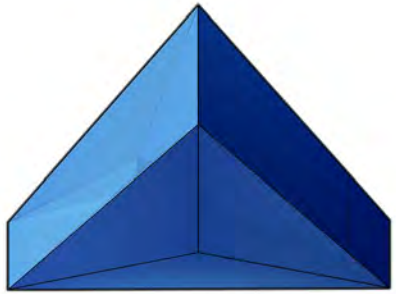
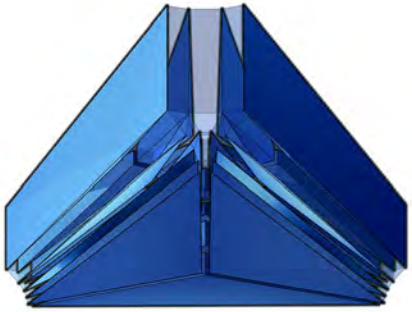
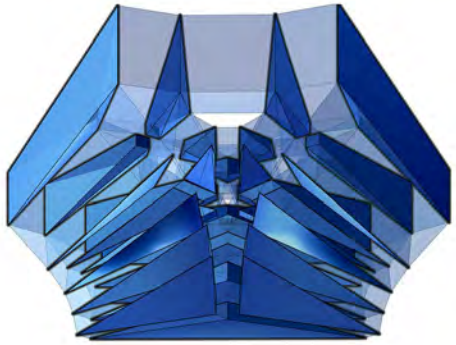
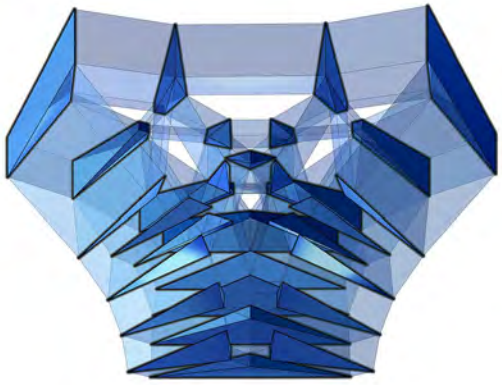
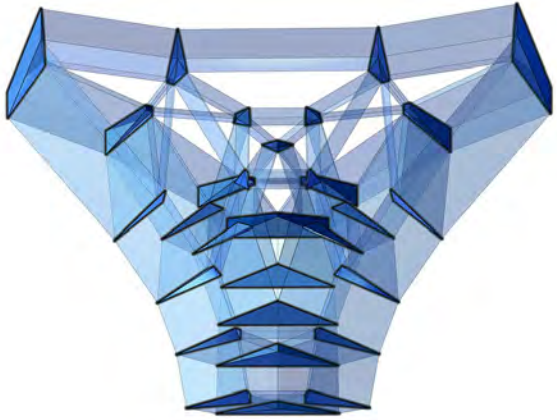
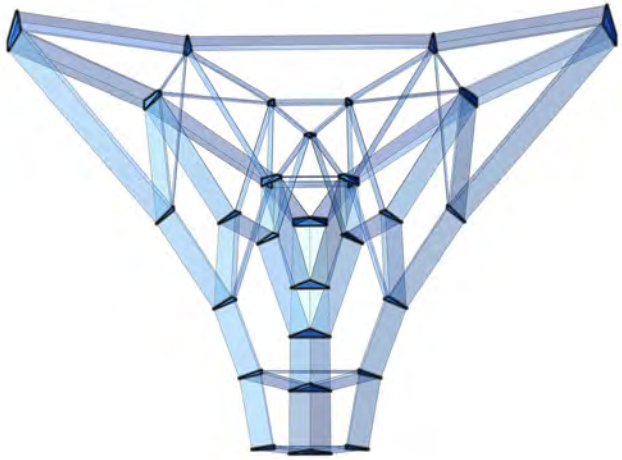


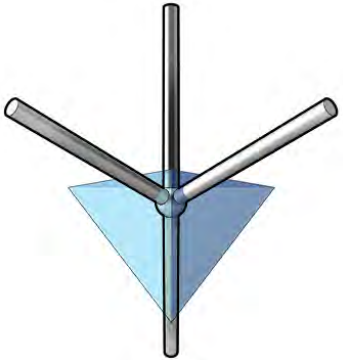




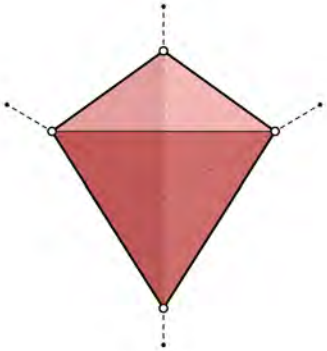




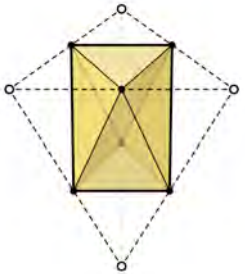









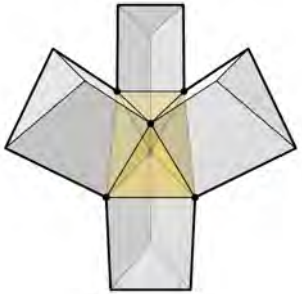
1. force polyhedron



2. EGI



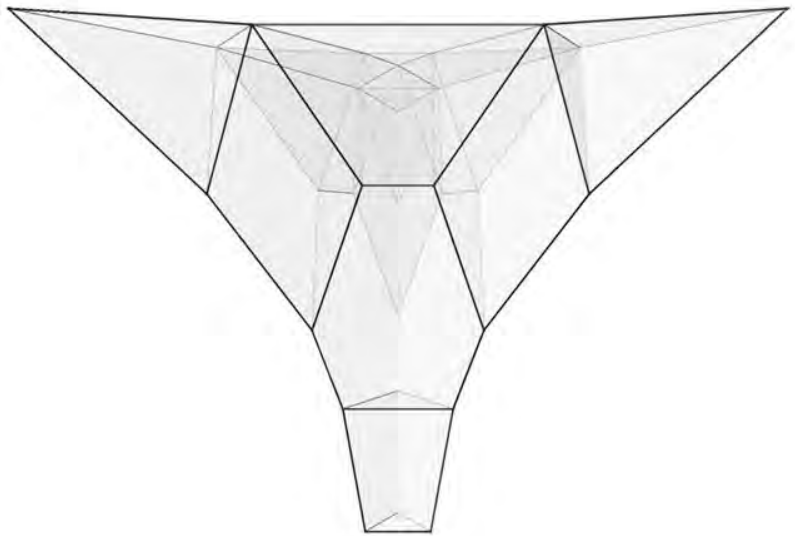
3. EGI dual

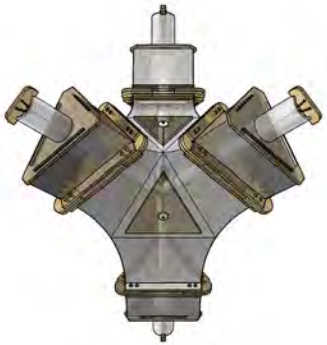


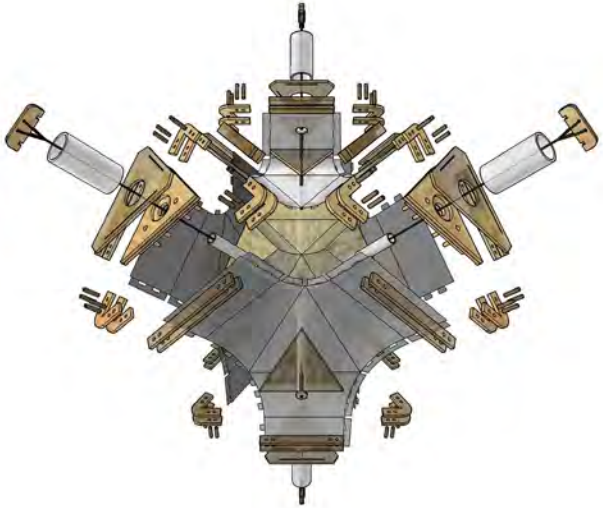
4. massing

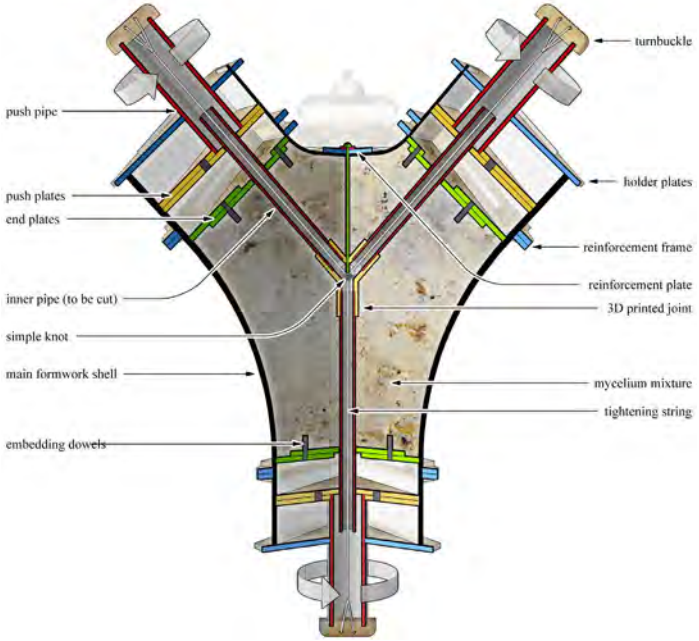
5. target geometry

6. mould geometry



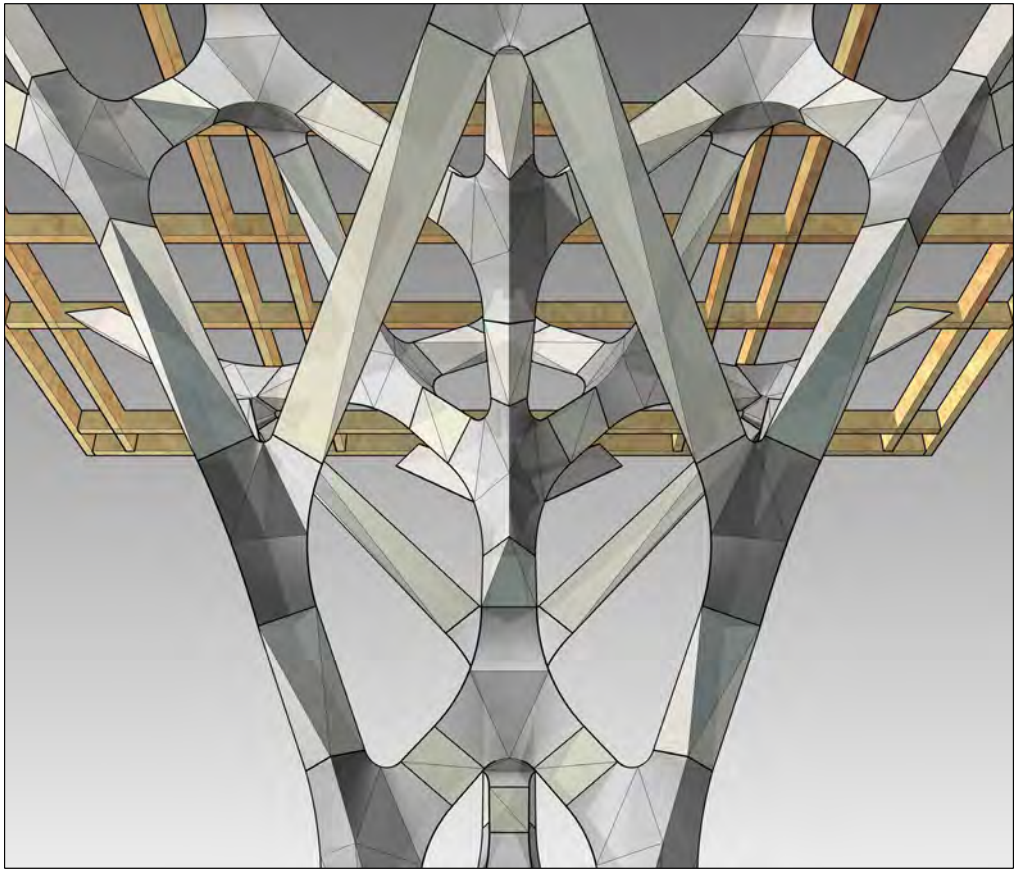


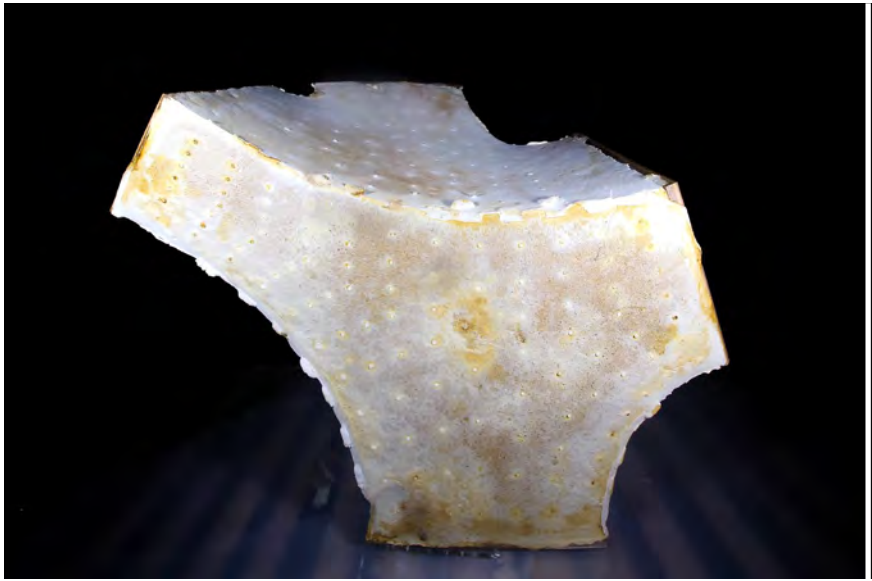


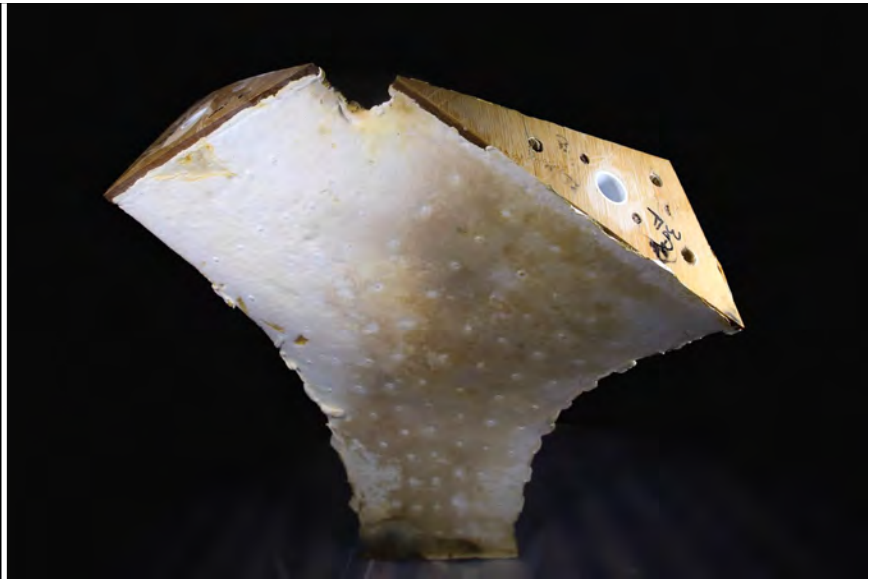

turnbuckle
push pipe
holder plates
push plates
end plates
reinforcement frame
reinforcement plate
inner pipe (to be cut)
3D printed joint
simple knot
main formwork shell
mycelium mixture
tightening string
embedding dowels

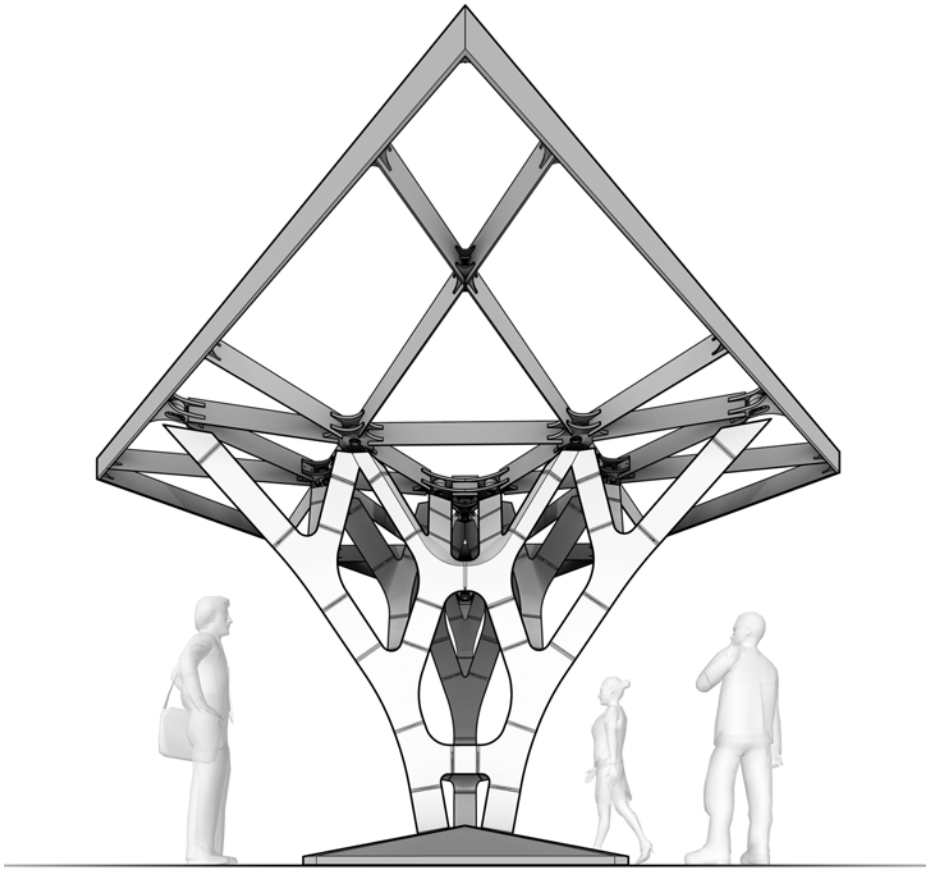
From polyhedral force diagram

From cell network

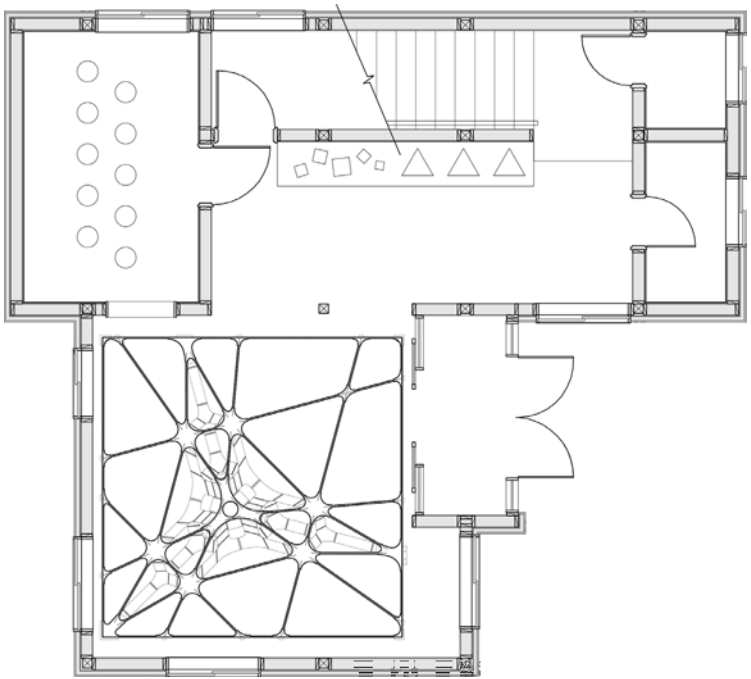


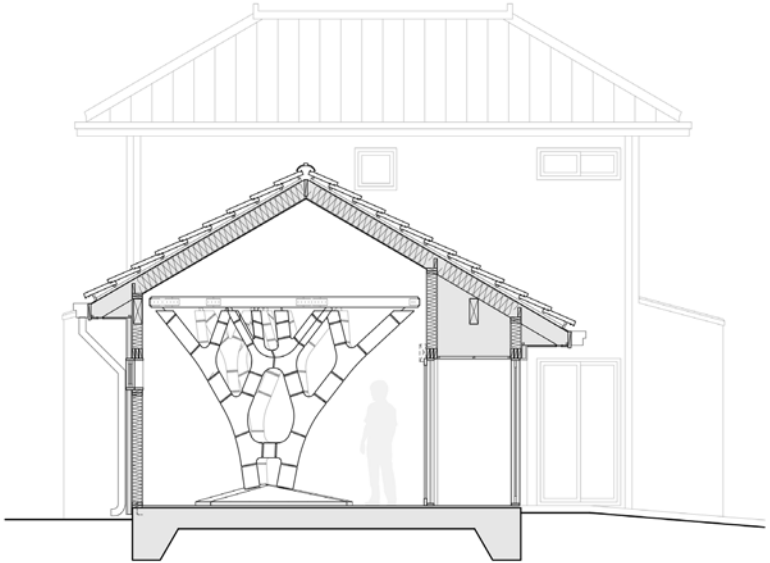






- 4m x 4m x 3m (height)
- Weight 315.9 kg
  - Grid 133.7 kg
  - Tree (mycelium) 182.2 kg
- 36 linear mycelium members, ranging from 17cm to 60cm in length
- 15 total nodes (5 types), ranging from 5.94 kg to 9.78.8kg
- Mycelium strength: 0.2 Mpa
- Design + development : 6 months
- Construction time : **1 week**





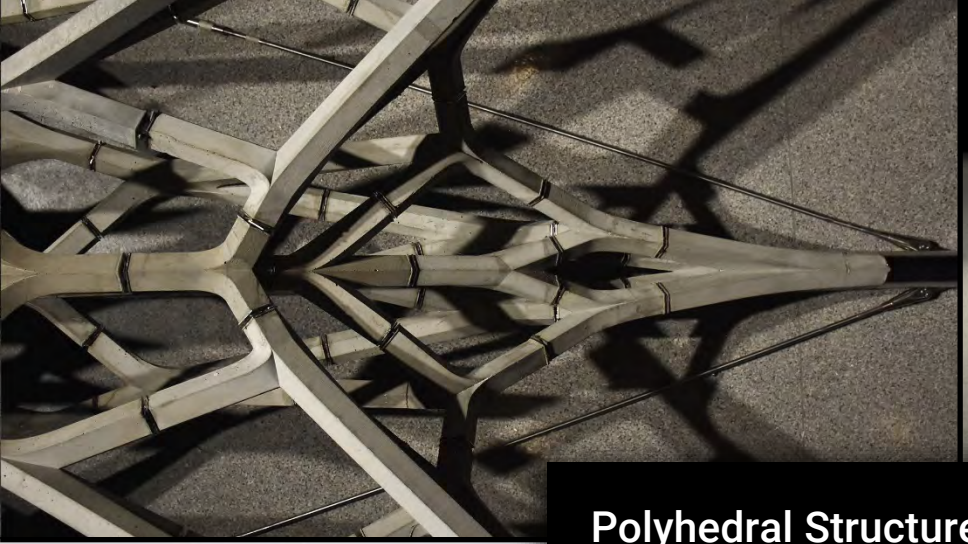
61

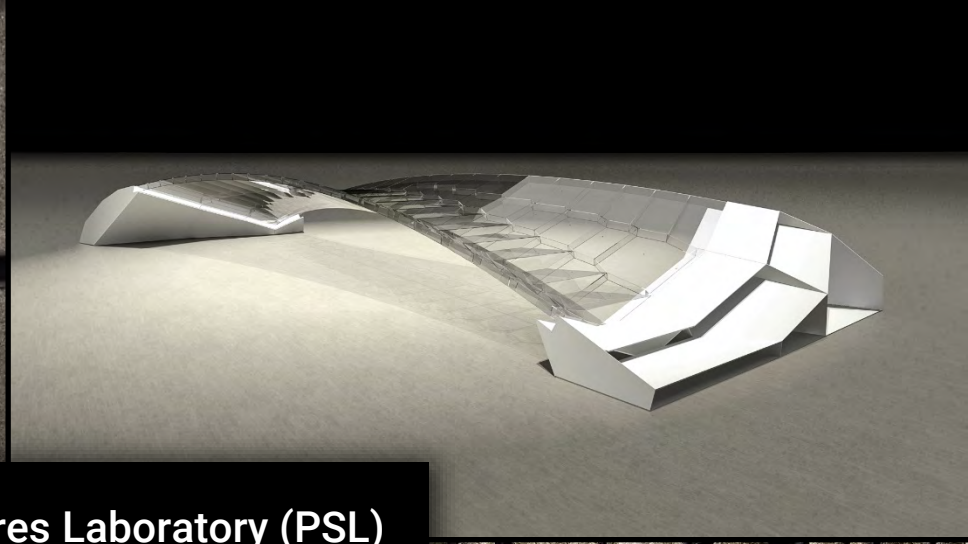
Photo: Carlina Tetris

Photo: Carlina Tetris

Photo: Carlina Tetris

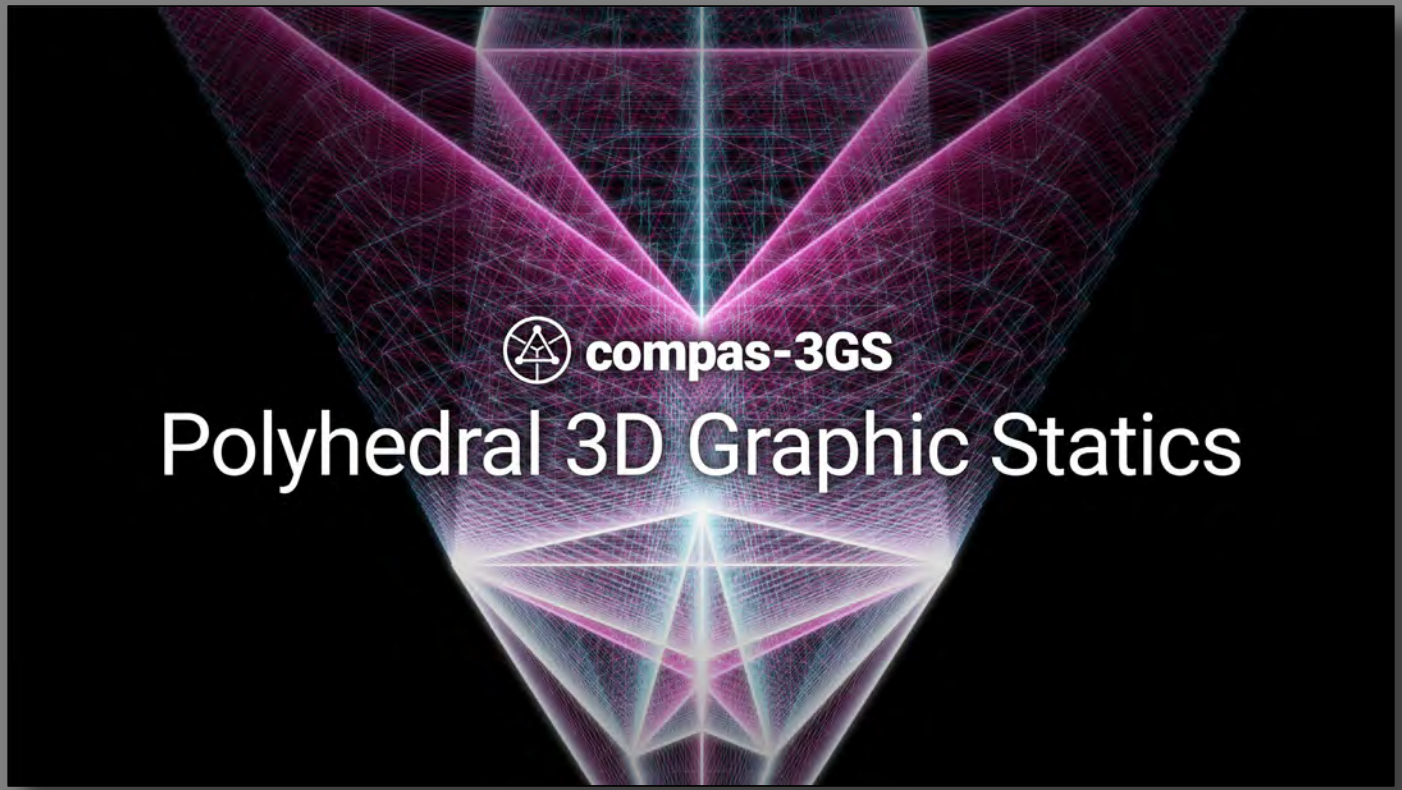
Photo: Carlina Tetris





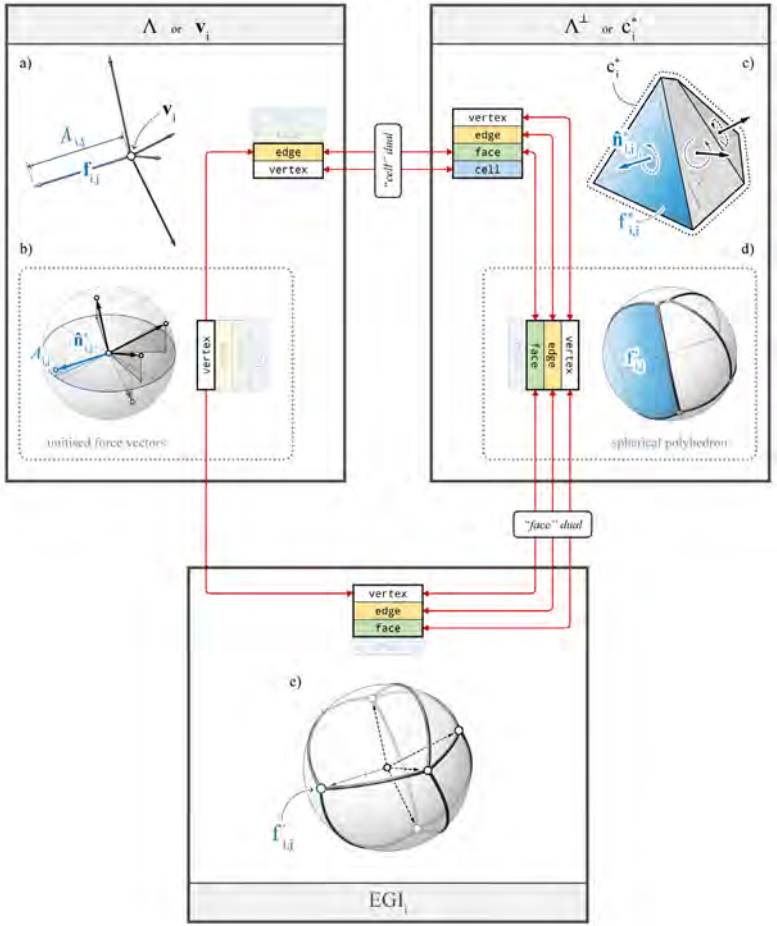
Polyhedral Structures Laboratory (PSL)

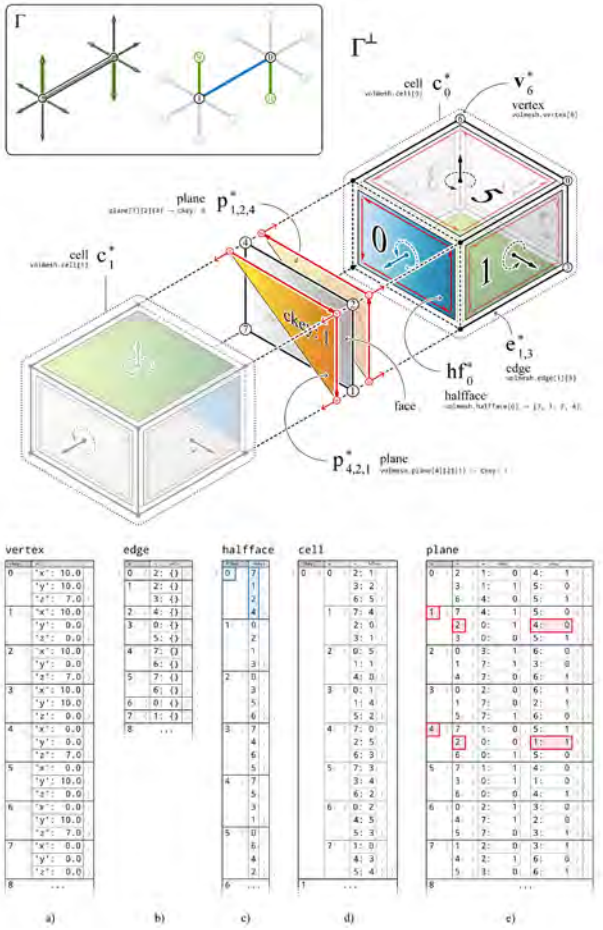
Prof. Dr. Masoud Akbarzadeh • University of Pennsylvania

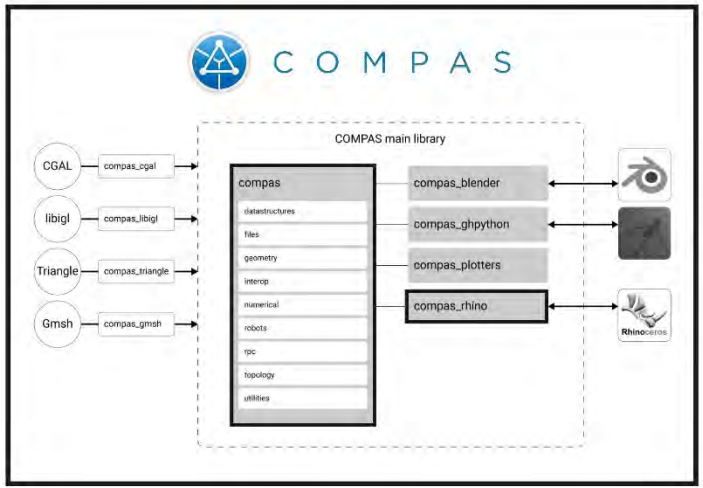

compas-3GS
Polyhedral 3D Graphic Statics

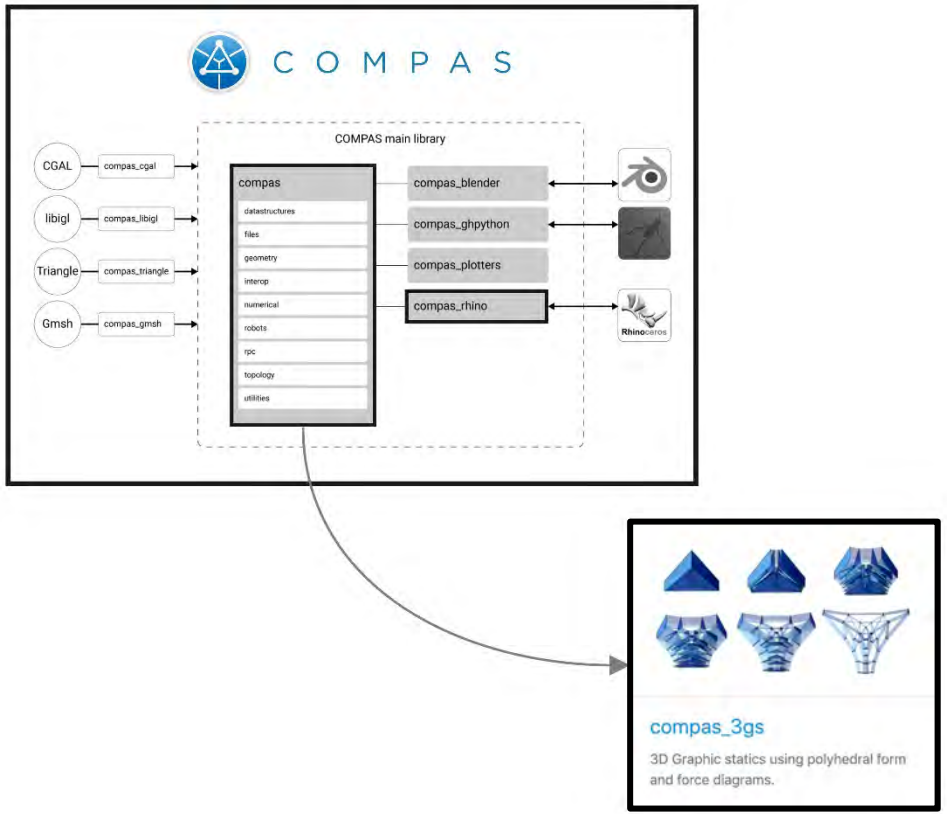
COMPAS

Open-source, Python-based framework for computational research and collaboration in architecture, engineering and digital fabrication


Λ or v_i
Λ⊥ or c*_i
a)
b)
c)
d)
e)
edge
vertex
"cell" dual
vertex
edge
face
cell
unitised force vectors
spherical polyhedron
"face" dual
vertex
edge
face
EGI_i


Γ
Γ⊥
cell c*_0
v*_6
vertex
plane p*_1,2,4
cell c*_1
face
e*_1,3
edge
hf*_0
halfface
p*_4,2,1 plane
vertex
edge
halfface
cell
plane
a)
b)
c)
d)
e)


COMPAS
COMPAS main library
CGAL
compas_cgal
libigl
compas_libigl
Triangle
compas_triangle
Gmsh
compas_gmsh
compas
datastructures
files
geometry
intercop
numerical
robots
rpc
topology
utilities
compas_blender
compas_ghpython
compas_plotters
compas_rhino
Rhinoceros


COMPAS
COMPAS main library
CGAL
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Triangle
compas_triangle
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datastructures
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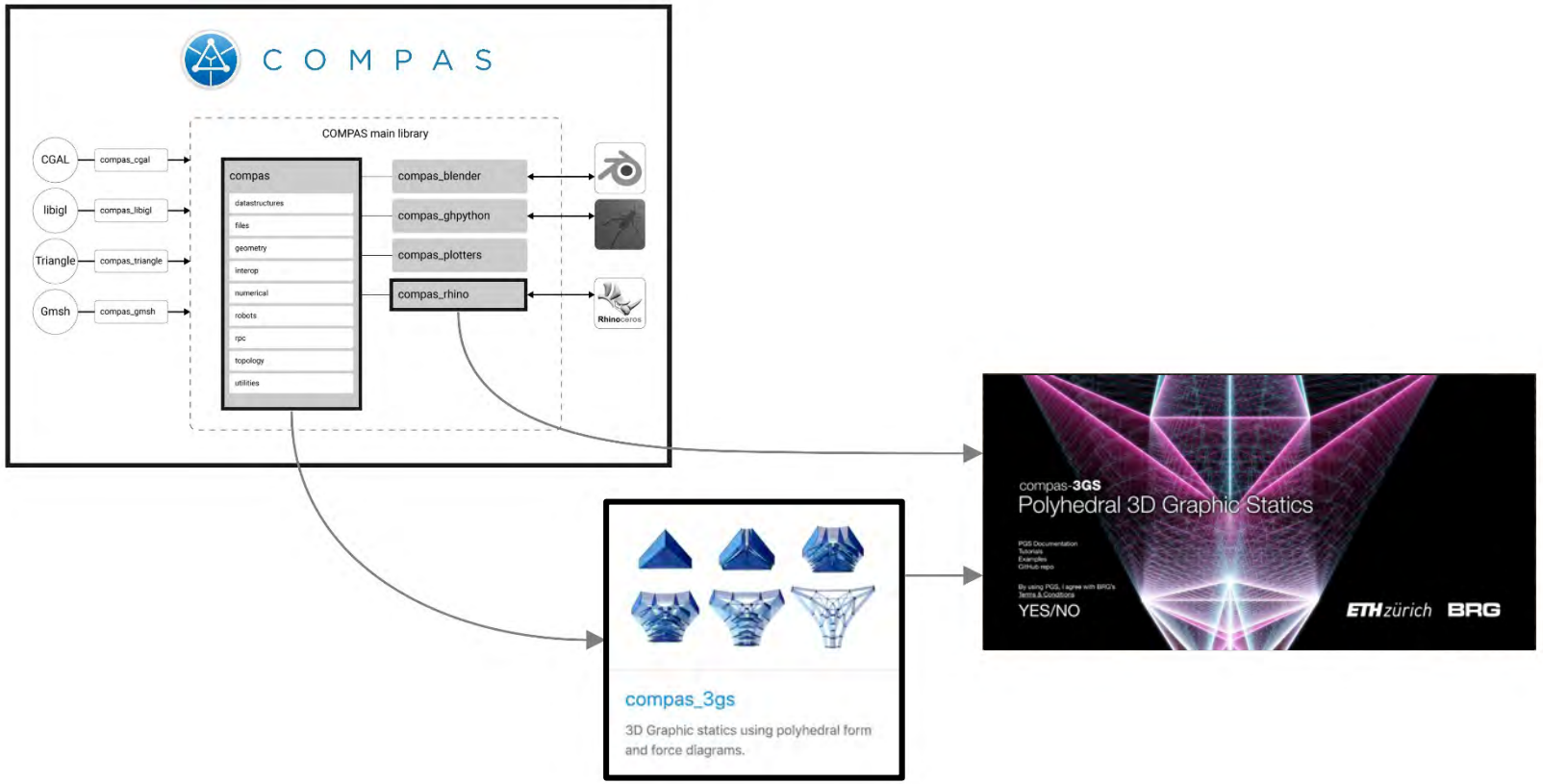
utilities
compas_blender
compas_ghpython
compas_plotters
compas_rhino
Rhinoceros
compas_3gs
3D Graphic statics using polyhedral form and force diagrams.


COMPAS
COMPAS main library
CGAL
compas_cgal
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compas_libigl
Triangle
compas_triangle
Gmsh
compas_gmsh
compas
datastructures
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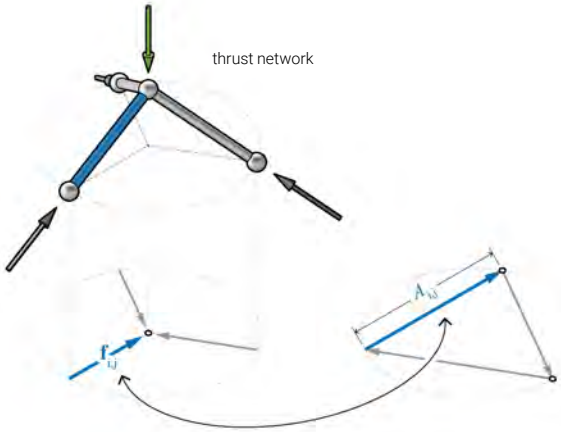
rpc
topology
utilities
compas_blender
compas_ghpython
compas_plotters
compas_rhino
Rhinoceros
compas_3gs
3D Graphic statics using polyhedral form and force diagrams.
compas-3GS
Polyhedral 3D Graphic Statics
YES/NO
ETH zürich
BRG

2D graphic statics

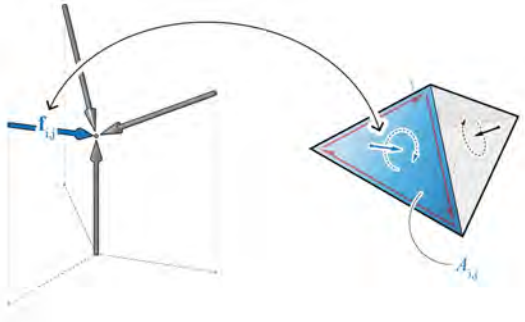
form diagram force diagram

2.5D graphic statics

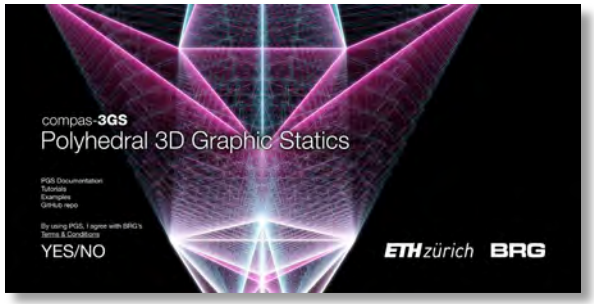


form diagram force diagram

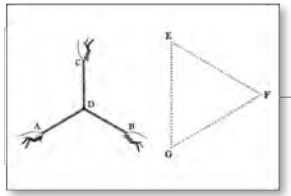
3D graphic statics



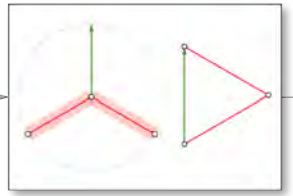
form diagram force diagram



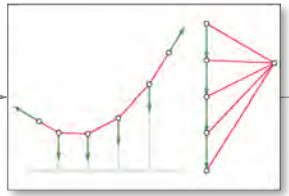
2D



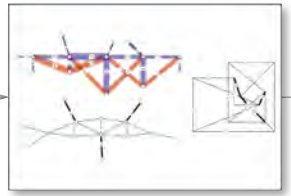
Module I
Basics of Graphic Statics



Module II
Interactive Graphic Statics I (single node)

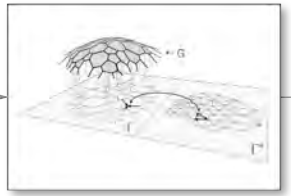


Module III
Interactive Graphic Statics 2 (multi node)



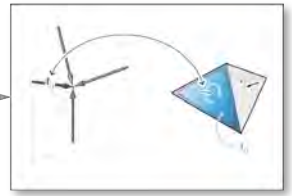
Module IV
Algebraic Graph Statics

2.5D

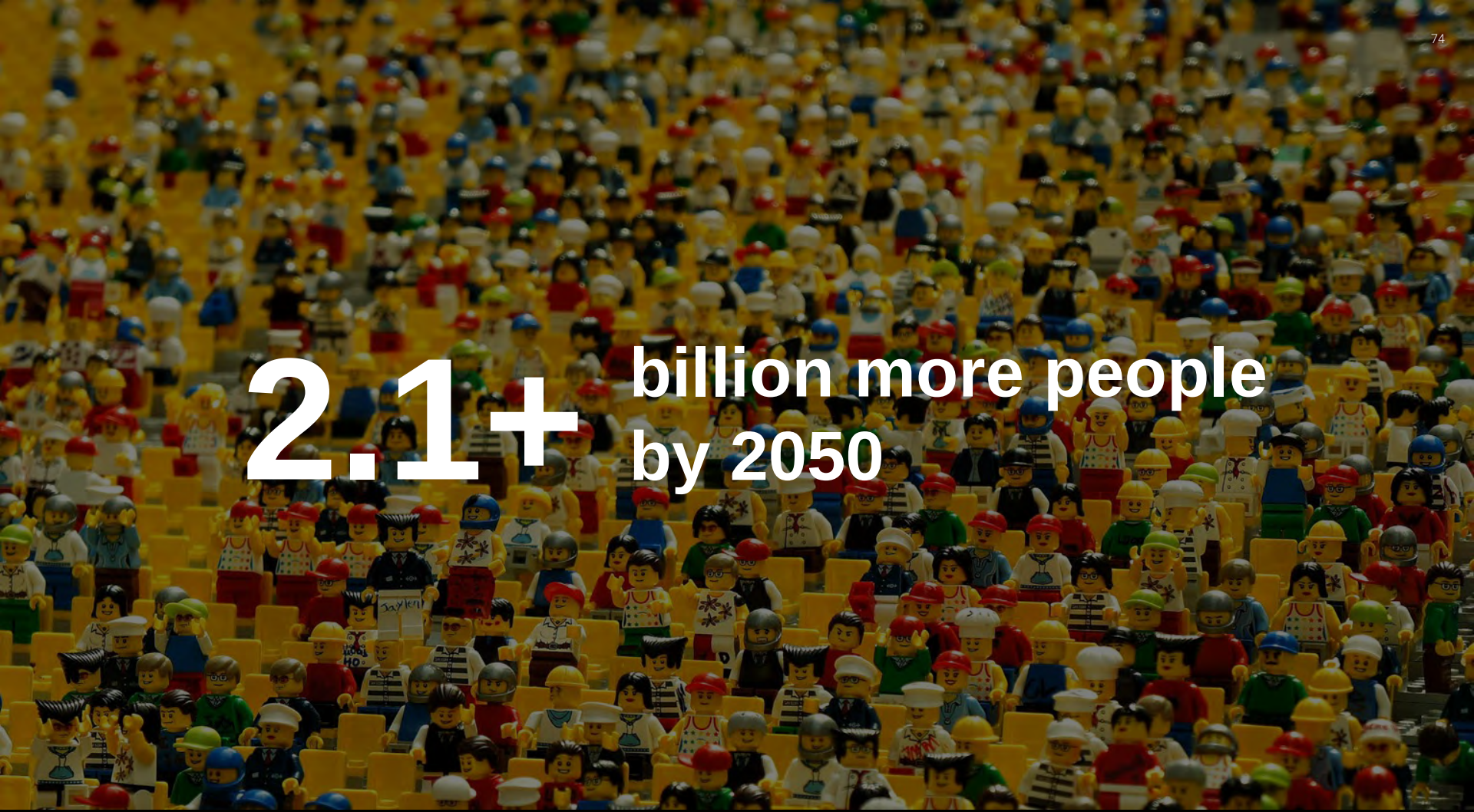


Module V
Thrust Network Analysis

3D



Module VI
3D Graphic Statics



Source: UN Department of Economic and Social Affairs (2018)

Photo: policy.friendsoftheearth.uk

Source: Bill and Melinda Gates Foundation (2019)

Environmental Impact of AEC Industry

Emissions

40%

Improving the rate of resource productivity

–doing more with less–

faster than the economic growth rate
is the notion behind “decoupling”…

That goal, however, demands an **urgent rethink** of the links between resource use and economic prosperity, buttressed by a massive investment in **technological, financial and social innovation**.

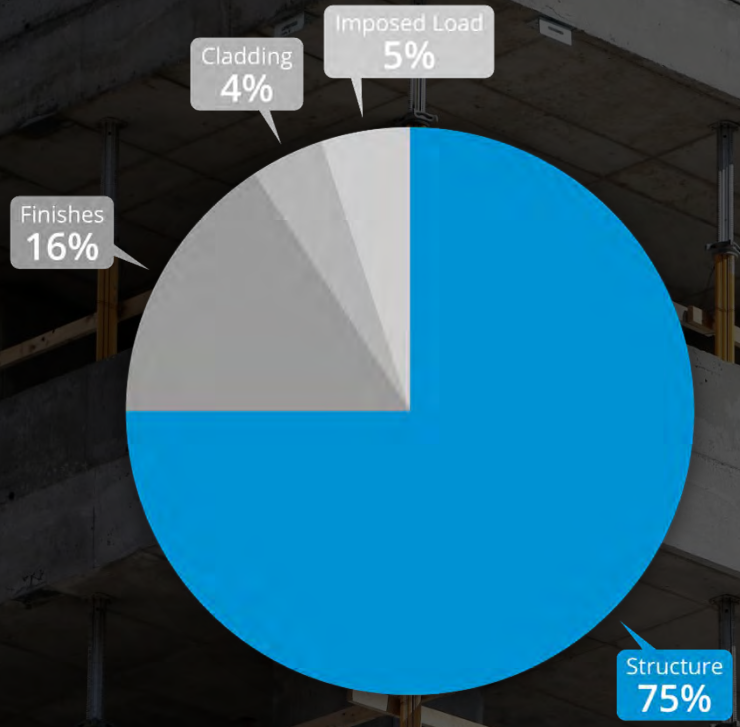
Energy

≈ **40%**

“Decoupling Natural Resource Use and Environmental Impacts from Economic Growth”
United Nations Environmental Programme (2011)

Source: Global Status Report 2018 Source: Chatham House Source: Construction Dive Source: World Energy Outlook 2019

Superstructure of a Typical Mid-rise Building



Source: Skidmore, Owings & Merrill

Photo: news.mit.edu


We need to change the way we **design** structures
We need to change the way we **build** structures

Towards Robotically Fabricated Composite Building Structures

R&D for the BUGA Fibre Pavilion 2019 and the livMatS Pavilion 2021

Serban Bodea, Block Research Group

063-0605-00L : Computational Structural Design 1

Computational Graphic Statics

Dr. Juney Lee & Dr. Lluis Enrique

Autumn Semester 2021

ETH zürich DARCH BRG

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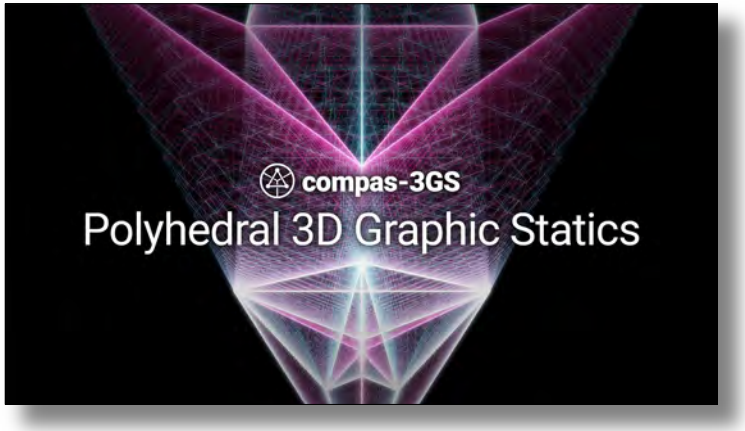

compas dashboard

v0.7.1

3GS

V0.2.0