Big Bang Nucleosynthesis

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1 Introduction

The very early universe was hot and dense enough to enable fusion of elements beyond hydrogen. Bear Protons and Neutrons where combined to form everything from Deuterium to Lithium-7 in non-negligible quantities. The actual mass fractions[1] produced, where a mass fraction for a particular isotope is defined to be,

$$X_i = \frac{A_i n_i}{\rho_b N_A} \tag{1}$$

where A_i is the mass number, n_i the number density, N_A Avogadro's number, and ρ_b the baryon density, are very sensitive to the temperature of the system, in our case the universe. Cosmology allows us to to predict the evolution of the temperature of the universe. Because the temperature of the universe is defined by the photons that fill it, and the photons that fill it are those that emerge from the time of recombination or last scattering i.e the Cosmic Microwave Background (CMB). We, to a very good approximation, need to track just these photons to determine the temperature of the universe as a function of time.

If we approximate the CMB to be a blackbody we can relate its energy density to its temperature via Stefan Boltzmann's Law,

$$\epsilon_{rad} = \sigma T^4/c \quad \text{Plugging in for } \epsilon_{rad} \tag{2}$$

$$\frac{\epsilon_{rad,0}}{a^4(t)} = \sigma T^4/c \quad \text{whch implies} \tag{3}$$

$$T \propto \frac{1}{a(t)} \quad \text{Given that } a(t = today) = a(t_0 = 1) \tag{4}$$

$$T = \frac{T_0}{a(t)} \quad \text{Where } T_0 \text{ is the temperature today.} \tag{5}$$

Thus we merely need to solve for the scale factor, $a(t)$, as a function of time to know the evolution

[1] We note that by definintion the SUm of all mass fractions for all isotopes must add up to one.

of the temperature for all time. To do this we turn to the Freedman equation,

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2\left(\frac{\Omega_{M,0}}{a^3} + \frac{\Omega_{R,0}}{a^4} + \Omega_{\Lambda,0} + \frac{\Omega_{K,0}}{a^2}\right) \tag{6}$$

and solve for $a(t)$. In general this equation cannot be solved analytically but under the very good approximation that during the time of interest[2] the universe was radiation dominated, the differential equation becomes,

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2\frac{\Omega_{R,0}}{a^4} \tag{7}$$

Giving us,

$$a(t) = \sqrt{2H_0 t\sqrt{\Omega_{R,0}}} \tag{8}$$

Where H_0 is the Hubble Constant, and $\Omega_{R,0}$ is the radiation density parameter as measured today.

For ourAnalysis we take $\Omega_{R,0} = 0.04$, $Omega_{r,0} = 8 \times 10^{-5}$, $T_0 = 2.7[K]$ and $H_0 = 10[km/s \cdot Mpc]$. These values are relisted in table 1 for convenience. We reference *On the Synthesis Of Elements at very HIgh Temperatures* by Wagoner et all for the reaction rates of interest and determine the Mass Fraction evolution vs time for Protons, Neutrons, Deuterium, He^3, He^4, tritium. We finally vary $Omega_{B,0}$ to calculate the manner in which the freeze out values change vs this parameter.

$\boldsymbol{\Omega_{B,0}}$	$\boldsymbol{\Omega_{R,0}}$	$\boldsymbol{H_0}$	$\boldsymbol{T_0}$
0.04	8×10^{-5}	$70[\frac{km}{s \cdot Mpc}]$	2.7

Table 1: *Cosmological parameters used for the calcualtions.*

1.1 The initial Condiitons

For convenience we take our staring time as the time after the big bang such that the following conditions are met:

[2]Alll of nucleosynteshis took place within a few thousands of seconds from the big bang.

1. The universe was hot enough to enable thermal equilibrium between the particles present but not so hot that the particles are relativistic (i.e. enforcing $k_B T << m_p c^2$).

2. The only nucleons present at this time are bare Neutrons and Protons

Under the thermal equilibrium requirement the number of Neutrons to Protons is given by a Maxwell Boltzmann distribution,

$$\frac{N_n}{N_p} = \left(\frac{m_n}{m_p}\right)^{3/2} e^{-\frac{(m_n - m_p)c^2}{k_B T}} \tag{9}$$

At a temperature of $k_B T \approx 0.8 MeV$ the mass difference between the proton and neutron becomes non negligible and the two are no longer in thermal equilibrium. Their relative abundance is well approximated by equation 9 and is given by $\approx \frac{1}{5}$.

From this time forward neutrons are free to decay until it is cool enough that neutrons can bind with other nuclei to form more stable isotopes such as D, He_3 and He_4. This moment corresponds to a temperature of $\approx$ 0.06[MeV] which in turn corresponds to a time from $k_B T \approx 0.8 MeV$ of 340$[s]$, refer to equation 8 and equation 5. Given that the neutron half life is 614$[s]$ the initial neutron to proton ratio we are left with is,

$$\frac{N_n}{N_p} = \frac{1}{5} \times e^{-\frac{340[s] \times ln(2)}{614[s]}} \approx \frac{1}{7.3} \tag{10}$$

2 Strong and electromagnetic Reactions

The relevant reactions where taken from *On the Synthesis Of Elements at Very High Temperatures* by Wagoner et all. They are shown on figure 4 and figure 5, in appendix A. All reactions except those involving 3 isotopes on the left or right hand side where taken into consideration.

Referring to figure 4 and figure 5, the quantities under the reactions are the rates the first quantity is the forward rate R_{fow} and the backward rate R_{back}. Denoting the elements on the left hand side as E_{LHS} and their corresponding mass fractions as E_{LHS}; and the elements on the right hand side as E_{RHS} and their corresponding mass fractions as E_{RHS}, the evolution of the mass fraction for the i^{th} element in the reaction is given by,

$$\frac{1}{A_i}\frac{dX_i}{dt} = -\sum 1/A_j X_{LHS,j} * R_{fow} + \sum 1/A_k X_{LHS,k} * R_{back} \quad \text{if } E_i \text{ is on the Left Hand Side} \tag{11}$$
$$\frac{1}{A_i}\frac{dX_i}{dt} = \sum 1/A_j X_{LHS,j} * R_{fow} - \sum 1/A_k X_{LHS,k} * R_{back} \quad \text{if } E_i \text{ is on the Right Hand Side} \tag{12}$$

As more reactions are considered we simply add terms to the Right Hand Side of the equations above.

Mathematica's NDSolve function was used to solve the six coupled first order differential equations for Protons, Neutrons, Deuterium, He^3, He^4, and Tritium with the following initial conditions at time equal to $340s$,

$$X_{P,0} + X_{N,0} = 1 \tag{13}$$
$$\frac{X_{P,0}}{X_{N,0}} = 7.3 \tag{14}$$
$$X_{D,0} = X_{He_3,0} = X_{He_4,0} = X_{T,0} = 0 \tag{15}$$

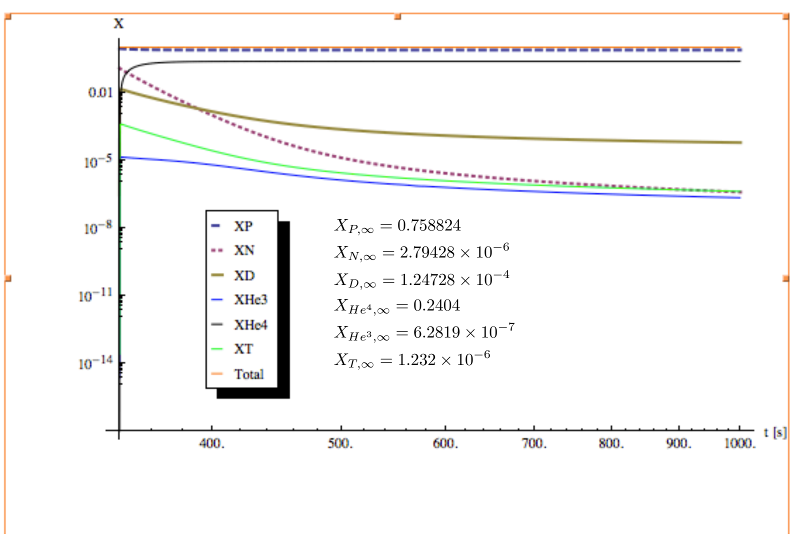

Figure 1: *The evolution of the mass fraction vs ttime fopr a unvirese with the properties listed in table 1. Listed also are the final freese out mass frations for all the isotopes involved*

We also included the weak neutron decay by approximating its lifetime to be $882[s]$.

The results from this calculation is shown on figure 1.

3 Varying $Omega_{B,0}$

We also varied the value of $Omega_{B,0}$ with values of 0.001 to 0.1 and computed the freeze out values at each. The results are plotted on figure 2 and figure 3.

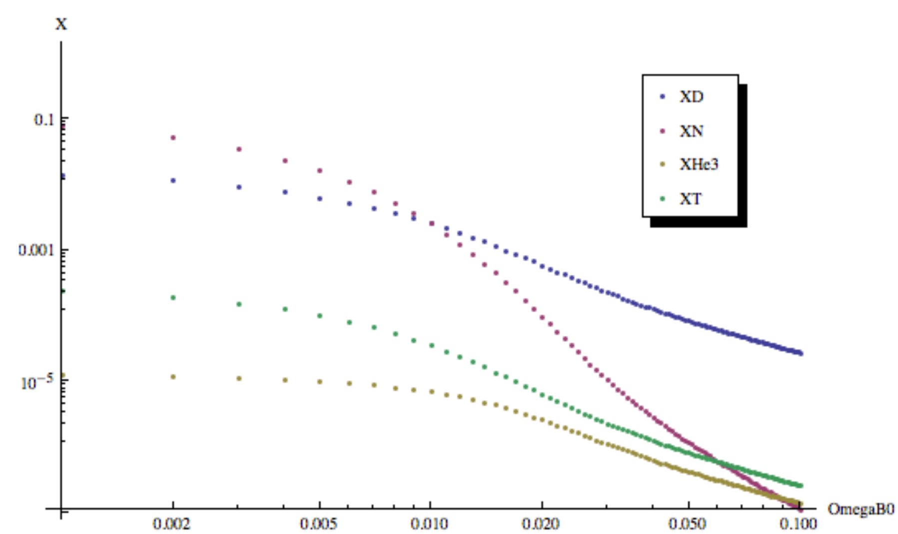

Figure 2: *The evolution of the mass fraction vs time for a unvirese with the properties listed in table 1. Listed also are the final freese out mass frations for all the isotopes involved*

Values of the final mass fractions where also sampled and portrayed in table 2.

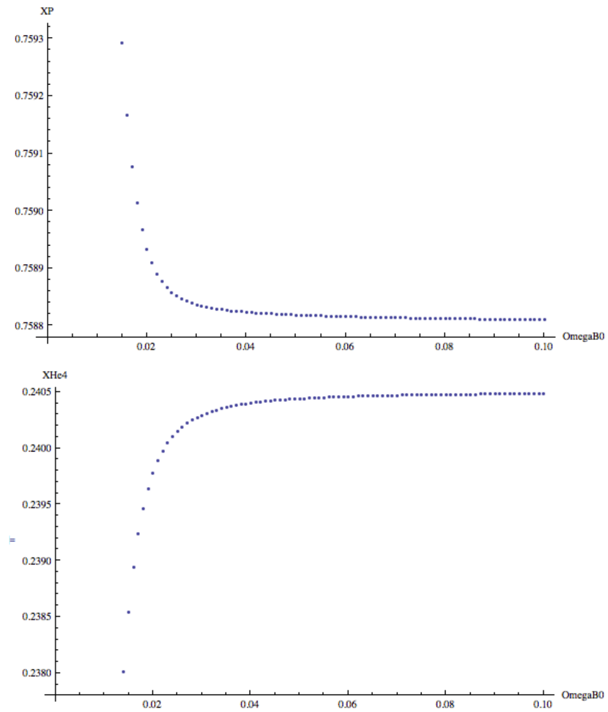

Figure 3: *The evolution of the mass fraction vs time for a unvirese with the properties listed in table 1.*

$\Omega_{B,0}$	$X_{P,\infty}$	$X_{N,\infty}$	$X_{D,\infty}$	$X_{He_3,\infty}$	$X_{He_4,\infty}$	$X_{T,\infty}$
0.001	0.836334	0.0772533	0.0139512,	0.0000122267	0.0721095	0.000242501
0.01	0.761369	0.00249106	0.00253585	6.93674E-6	0.23306	0.0000352829
0.02	0.758934	0.0000961728	0.000581908	2.58356E-6	0.23978	6.31472E-6
0.03	0.758837	0.0000105211	0.000225343	1.1056E-6	0.240293	2.27072E-6
0.04	0.758824	2.79428E-6	0.000124728	6.2819E-7	0.2404	1.232E-6
0.05	0.758819	1.15776E-6	0.0000828317	4.20119E-7	0.24044	8.11364E-7
0.06	0.758816	6.00474E-7	0.0000606641	3.09628E-7	0.240459	5.90366E-7
0.07	0.758814	3.56085E-7	0.0000472095	2.41924E-7	0.24047	4.56767E-7
0.08	0.75881	2.29286E-7	0.0000381675	1.96233E-7	0.240476	3.6721E-7
0.09	0.758811	1.56893E-7	0.0000317518	1.63716E-7	0.240481	3.03765E-7
0.1	0.75881	1.12166E-7	0.0000269685	1.39424E-7	0.240484	2.56523E-7

Table 2: *The mass fractions sampled at values of* $\Omega_{B,0}$ *between* 0.001 *and* 0.1.

4 Discusion

It is clear that the final elemental abundances are heavily dependent values of cosmological parameters, since the scale factor is and the temperature is in turn dependent on the scale factor. Of particular importance in big bang nucleosynthesis is $\Omega_{B,0}$. Thus by measuring elemental abundances in primordial gas that has been relatively unaltered since the end of big bang nucleosynthesis one can find via t the theory the appropriate value of $\Omega_{B,0}$. This is commonly done by measuring deuterium abundances from quasar light as it passes through primordial gas. From the spectrum of this light one determines relative abundance by the strength of lines corresponding to deuterium with respect to other lines. These measurements suggest that $\Omega_{B,0} \approx 0.04$. Given that this is the case and that $\sum \Omega_{i,0} = 1$ where is the rest of the mass density of the universe made of.

Given that CMB data along with other independent studies strongly suggest that we live in a flat universe, most of the universe must be made up of things we cannot detect and have thus dubbed "Dark." It is now believed that the universe is nearly entirely made up of dark matter and dark energy.

A Reactions

(b) Strong and Electromagnetic

(1) $p + n \rightleftarrows D + \gamma$ 2 225

$[pn] = 2.5 \times 10^4 \rho_b$

$\lambda_\gamma(D) = 4.68 \times 10^9 [pn] \rho_b^{-1} T_9^{3/2} \exp(-25.82 T_9^{-1})$

(2) $p + D \rightleftarrows He^3 + \gamma$ 5 494

$[pD] = 2.23 \times 10^3 \rho_b T_9^{-2/3} \exp(-3.72 T_9^{-1/3})(1 + 0.112 T_9^{1/3}$
$+ 3.38 T_9^{2/3} + 2.65 T_9)$

$\lambda_\gamma(He^3) = 1.63 \times 10^{10} [pD] \rho_b^{-1} T_9^{3/2} \exp(-63.75 T_9^{-1})$

(3) $n + D \rightleftarrows T + \gamma$ 6 257

$[nD] = \rho_b (75.5 + 1250 T_9)$

$\lambda_\gamma(T) = 1.63 \times 10^{10} [nD] \rho_b^{-1} T_9^{3/2} \exp(-72.62 T_9^{-1})$

(4) $n + He^3 \rightleftarrows p + T$ 0 7638

$[nHe^3]_p = 7.06 \times 10^8 \rho_b$

$[pT]_n = [nHe^3]_p \exp(-8.864 T_9^{-1})$

(5) $p + T \rightleftarrows He^4 + \gamma$ 19 81

$[pT]_\gamma = 2.87 \times 10^4 \rho_b T_9^{-2/3} \exp(-3.87 T_9^{-1/3}) (1 + 0.108 T_9^{1/3}$
$+ 0.466 T_9^{2/3} + 0.352 T_9 + 0.300 T_9^{4/3} + 0.576 T_9^{5/3})$

$\lambda_\gamma(He^4)_p = 2.59 \times 10^{10} [pT]_\gamma \rho_b^{-1} T_9^{3/2} \exp(-229.9 T_9^{-1})$

(6) $n + He^3 \rightleftarrows He^4 + \gamma$ 20 58

$[nHe^3]_\gamma = 6.0 \times 10^3 \rho_b T_9$

$\lambda_\gamma(He^4)_n = 2.60 \times 10^{10} [nHe^3]_\gamma \rho_b^{-1} T_9^{3/2} \exp(-238.8 T_9^{-1})$

(7) $D + D \rightleftarrows n + He^3$ 3 269

$[DD]_n = 3.9 \times 10^8 \rho_b T_9^{-2/3} \exp(-4.26 T_9^{-1/3}) (1 + 0.0979 T_9^{1/3}$
$+ 0.642 T_9^{2/3} + 0.440 T_9)$

$[nHe^3]_D = 1.73 [DD]_n \exp(-37.94 T_9^{-1})$

(8) $D + D \rightleftarrows p + T$ 4.033

$[DD]_p = [DD]_n$

$[pT]_D = 1.73 [DD]_p \exp(-46.80 T_9^{-1})$

11

Figure 4: *Strong and electromagnetic Reactions from* On the Synthesis Of Elements at very HIgh Temperatures *by Wagoner et a.*

Q(MeV)

(9) $D + D \rightleftarrows He^4 + \gamma$ 23 85

$[DD]_\gamma = 24.1\ \rho_b\ T_9^{-2/3} \exp(-4.26\ T_9^{-1/3})\ (T_9^{2/3} + 0.685\ T_9 + 0.152\ T_9^{4/3} + 0.265\ T_9^{5/3})$

$\lambda_\gamma(He^4)_D = 4.50 \times 10^{10}\ [DD]_\gamma\ \rho_b^{-1}\ T_9^{3/2} \exp(-276.7\ T_9^{-1})$

(10) $D + He^3 \rightleftarrows He^4 + p$ 18 35

$[DHe^3] = 2.60 \times 10^9\ \rho_b\ T_9^{-3/2} \exp(-2.99\ T_9^{-1})$

$[He^4 p] = 5.50\ [DHe^3] \exp(-213.0\ T_9^{-1})$

(11) $D + T \rightleftarrows He^4 + n$ 17 59

$[DT] = 1.38 \times 10^9\ \rho_b\ T_9^{-3/2} \exp(-0.745\ T_9^{-1})$

$[He^4 n] = 5.50\ [DT] \exp(-204.1\ T_9^{-1})$

(12) $He^3 + He^3 \rightleftarrows He^4 + p + p$ 12 86

$[He^3He^3] = 1.19 \times 10^{10}\ \rho_b\ T_9^{-2/3} \exp(-12.25\ T_9^{-1/3})(1 + 0.0340\ T_9^{1/3})$

$[He^4 pp] = 3.37 \times 10^{-10}\ [He^3He^3]\ \rho_b\ T_9^{-3/2} \exp(-149.2\ T_9^{-1})$

(13) $T + T \rightleftarrows He^4 + n + n$ 11 33

$[TT] = 1.10 \times 10^9\ \rho_b\ T_9^{-2/3} \exp(-4.87\ T_9^{-1/3})\ (1 + 0.0857\ T_9^{1/3})$

$[He^4 nn] = 3.37 \times 10^{-10}\ [TT]\ \rho_b\ T_9^{-3/2} \exp(-131.5\ T_9^{-1})$

(14) $He^3 + T \rightleftarrows He^4 + p + n$ 12 10

$[He^3T]_{pn} = 5.60 \times 10^9\ \rho_b\ T_9^{-2/3} \exp(-7.72\ T_9^{-1/3})\ (1 + 0.0540\ T_9^{1/3})$

$[He^4 pn] = 3.37 \times 10^{-10}\ [He^3T]_{pn}\ \rho_b\ T_9^{-3/2} \exp(-140.4\ T_9^{-1})$

(15) $He^3 + T \rightleftarrows He^4 + D$ 14 32

$[He^3T]_D = 3.88 \times 10^9\ \rho_b\ T_9^{-2/3} \exp(-7.72\ T_9^{-1/3})\ (1 + 0.0540\ T_9^{1/3})$

$[He^4D] = 1.59\ [He^3T]_D \exp(-166.2\ T_9^{-1})$

Figure 5: *Strong and electromagnetic Reactions from* On the Synthesis Of Elements at very HIgh Temperatures *by Wagoner et a.*

Here T_g is the temperature of the universe, and rho_b is the baryon density which is related to the baryon density parameter in the following manner,

$$\rho_b(t) = \frac{3H_0}{8\pi G}\Omega_{b,0} \tag{16}$$

The rates depend on these parameters since the the greater the baryon density the the more likely particles are to interact and the greater the temperature the more likely they are to overcome the coulomb barrier. This is an overtly simplified explanation, the exact form of the rates takes into account statistical and quantum mechanical nature of interactions in an effort to compute interaction cross-ssections. A rate is given by,

$$R_i = \rho_b N_A \langle \sigma v \rangle_i \tag{17}$$

Where σ is the interaction cross-section and v is the velocity of the particle.