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Linear Algebra Assignment

Q1. $$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix} \qquad R_4 \rightarrow R_4 - 2R_3$$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 0 & 4 & 5 & -1 \end{bmatrix} \qquad \begin{matrix} R_3 \rightarrow R_3 - 3R_1 \\ R_2 \rightarrow R_2 - 2R_1 \end{matrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 4 & 5 & -1 \end{bmatrix} \qquad R_4 \rightarrow R_4 + R_3$$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \end{bmatrix} \qquad R_2 \leftrightarrow R_3$$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -3 & 2 \end{bmatrix} \qquad R_4 \rightarrow R_4 - R_3$$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \boxed{\rho(A) = 3}$$

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2. every matrix and vector space W is of the form $\begin{bmatrix} a & b \\ b & c \end{bmatrix}$

linear transformation, $T: W \rightarrow P_2$ defined by:

$$T\left(\begin{bmatrix} a & b \\ b & c \end{bmatrix}\right) = (a-b) + (b-c)x + (c-a)x^2$$

$\therefore$ Dimension $= 2+1 = 3$

Standard basis for w is

$$E = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

(w is 2×2 matrix)

Also $T\left(\begin{bmatrix} a & b \\ b & c \end{bmatrix}\right) = \begin{bmatrix} a-b \\ b-c \\ c-a \end{bmatrix}$

$$\therefore T\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 0 & +1 \\ 0 & 0 & 0 \end{bmatrix}$$

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converting to row-echleon form

$$A = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 0 & +1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_2 \to R_2 + R_1} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\rho(A) = 1$, ~~linearly independent~~

$\rho(A) + N = D \Rightarrow 1 + N = 3 \Rightarrow \boxed{N = 2}$

Rank $\Rightarrow \rho(A) = 1$
Nullity $\Rightarrow N = 2$

3. $A - \lambda I = 0$

$$\begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix} = 0 \Rightarrow (2-\lambda)^2 - 1 = 0$$

$$\Rightarrow 4 + \lambda^2 - 4\lambda - 1 = 0$$

$$\Rightarrow \lambda^2 + 4\lambda + 3 = 0$$

$\Rightarrow \lambda^2 - 3\lambda - \lambda + 3 = 0$

$= \lambda(\lambda - 3) - 1(\lambda - 3) = 0$

$\boxed{\lambda = 1, 3}$ → eigen values of A.

$\therefore$ for $A^{-1} = 1, 1/3$

Eigen space corresponding to $\lambda = 1$ & $\lambda = 3$

for $\lambda = 1$, $[A - \lambda I]X = [0] \Rightarrow [A - I]X = [0]$

$$\left(\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x \\ y \end{bmatrix} = [0]$$

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = [0]$$

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Date: / /

$x - y = 0 \quad = x = y$
$\& \quad -x + y = 0$

If $x = k$, then $x = y = k$.

$\therefore$ Eigen space $= \begin{bmatrix} k \\ k \end{bmatrix} = k \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Same for A^{-1}

$\lambda = 3$

$[A - 3I][x] = [0]$

$\begin{bmatrix} 2-3 & -1 \\ -1 & 2-3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = [0]$

$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$-x - y = 0 \Rightarrow -x = y$
if $x = k$, $y = -k$

$\therefore$ Eigen space $= \begin{bmatrix} k \\ -k \end{bmatrix} = k \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Eiegen values for $A + 4I$

$\lambda_1 + c, \lambda_2 + c$
$1 + 4, 3 + 4$
$5, 7$

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4. $3x - 0.1y - 0.2z = 7.85$
$0.1x + 7y - 0.3z = -19.3$
$0.3x - 0.2y + 10z = 71.4$

1st iteration :- $y=0,\ z=0$

- $3x = 7.85 \Rightarrow \boxed{x = 2.61}$
- $0.1x - 0.3z + 19.3 = -7y$

$$y = \frac{0.1 \times 2.61 + 19.3}{-7}$$

$$\boxed{y = -2.79}$$

- $z = \dfrac{71.4 + 0.2y - 0.3x}{10}$

$$\boxed{z = 7.00}$$

2nd iteration

$y = -2.79,\ z = 7.00$

- $x = \dfrac{7.85 + 0.2z + 0.1y}{3} = \boxed{2.99 = x}$
- $y = \dfrac{0.1x - 0.3z + 19.3}{-7} \Rightarrow z = 7,\ x = 2.99$

$$\boxed{y = -2.49}$$

- $z = \dfrac{71.4 + 0.2y - 0.3x}{10},\ x = 2.99,\ y = -2.49$

$$\boxed{z = 7.0005 = 7}$$

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Date: / /

3^{rd} iteration

$\cdot\ x = \frac{7.85 + 0.2z + 0.1y}{3}$, $y = -2.49, z = 7$

$\boxed{x = 3.00}$

$\cdot\ y = \frac{0.1x - 0.3z + 19.3}{-7}$, $x = 3, z = 7$

$\boxed{y = -2.5}$

$\cdot\ z = \frac{71.4 + 0.2y - 0.3x}{10}$ $\Rightarrow y = -2.5, x = 3$

$\boxed{z = 7}$

5- $AX = B$

$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 2 & 17 & 4 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

Since $B = 0$ $\therefore$ our system is Homogeneous

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 2 & 17 & 4 \end{bmatrix} \xrightarrow[\substack{R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1}]{R_4 \to R_4 - R_1} \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 14 & 2 \end{bmatrix}$$

$R_4 \to R_4 + R_3$

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Date: / /

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_3 \to R_3 - 2R_2} \begin{bmatrix} 2 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2 < n = 3$

$\therefore$ infinite soln

$1x + 3y + 2z = 0$

$-7y - z = 0$

$\boxed{z = 7y}$

$1x + 3y + 14y = 0$

$x + 17y = 0$

$\boxed{x = -17y}$

if $y = k$, $x = -17k$, $z = 7k$

$$\therefore x = k\begin{bmatrix} -17 \\ 1 \\ 7 \end{bmatrix}$$

6. $T(a + bx + cx^2) = (a+1) + (b+1)x + (c+1)x^2$

(1) $T(P_1) + T(P_2) = T(P_1 + P_2)$

$T(a_1 + b_1x + c_1x^2) + T(a_2 + b_2x + c_2x^2)$

$= (a_1+1) + (b_1+1)x + (c_1+1)x^2 + (a_2+1) + (b_2+1)x + (c_2+1)x^2$

$= (a_1 + a_2 + 2) + (b_1 + b_2 + 2)x + (c_1 + c_2 + 2)x^2$

$T(P_1 + P_2) = T((a_1 + a_2) + (b_1 + b_2)x + (c_1 + c_2)x^2)$

$= (a_1 + a_2 + 1) + (b_1 + b_2 + 1)x + (c_1 + c_2 + 1)x^2$

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$T(P_1)+T(P_2) \neq T(P_1+P_2)$

$\therefore$ T is not linear transformation.

7. ~~form~~ ar a matrin with vector s as its column. & row reduce to check for linear independence.

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix} \xrightarrow[R_2 \to R_2 - 2R_1]{R_3 \to R_3 - 3R_1} \begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & -9 & 9 \end{bmatrix}$$

$$R_3 \to R_3 - 9/5 R_2 \longrightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2 < n = 3$

infinite, linearly dependent, don't span $V_3(R)$ and not basis.

let's Determine dimension & basis of subspace spanned by s.

Dimension $\to$ no. of linearly iidependent vectors in s.

1^{st} & 2^{nd} row are L.I but 3^{rd} not.

Dimension = 2 and basis for the subspace spanned by S is
$\{(1,2,3), (3,1,0)\}$

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8. Initial : $x^{(0)} = 1$, $y^{(0)} = 1$, $z^{(0)} = 1$

$$x^{k} = \frac{1}{a_{11}}\left(b_1 + a_{12}\, y^{(k-1)} + a_{13}\, z^{(k-1)}\right)$$

similarly y^{k} & z^{k}

1st iteration

$$x^{(1)} = \frac{1}{3}\left(23 + 6\cdot 1 - 2\cdot 1\right) = \boxed{\cancel{-3}\ 9}$$

$$y^{(1)} = \frac{1}{-1}\left(-15 + 4(1) + 1\right) = \boxed{\cancel{26}}\ \boxed{-10}$$

$$z^{(1)} = \frac{1}{7}\left(16 + 1 - 3 + 1\right) = \boxed{2}$$

2nd iteration :-

$$x^{(2)} = \frac{1}{3}\left(23 + 6\cdot 10 - 2\cdot 2\right) = \boxed{-36}$$

$$y^{(2)} = \frac{1}{1}\left(-15 + 4(9) + 2\right) = 26$$

$$z^{(2)} = \frac{1}{7}\left(16 - (-36) + 3(-10)\right) = \boxed{8}$$

3rd iteration

$$x^{(3)} = \frac{1}{3}\left(23 + 6(26) - 2(8)\right) = \boxed{47}$$

$$y^{(3)} = \frac{1}{1}\left(-15 + 4(-36) + 8\right) = \boxed{-168}$$

$$z^{(3)} = \frac{1}{7}\left(16 - 47 + 3(26)\right) = \boxed{11}$$

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9\. Matrix operations are extensively used in image processing. Like transpose of matrix is used to rotate the image in various directions and the blur matrix is used ~~for~~ to blur certain area of image. Apart from this, images ~~can~~ are made up of matrix itself. Images are made up of pixels which are arranged in grid to produce image.

10\. linear transformation plays very important role in computer vision. In linear transformat. is extremely used in manipulating image for various purpose.

One example is rotating image with ~~θ~~ angle about x-axis.
for this purpose we use famous rotation matrix in 2D to do this task.

Here, $T: V \to W$,

$$\text{where } T(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

if we have to rotate (x, y) about θ, then new x' and y' are:-

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

In this way we perform this basic operation for each pixel of the image and find the rotated ~~a~~ image.
This transformation is also used in image registration, object detection ~~as~~ and image alignment.