Modular neural nets

In the previous exercise, we computed the loss and gradient for a two-layer neural network in a single monolithic function. This isn't very difficult for a small two-layer network, but would be tedious and error-prone for larger networks. Ideally we want to build networks using a more modular design so that we can snap together different types of layers and loss functions in order to quickly experiment with different architectures.

In this exercise we will implement this approach, and develop a number of different layer types in isolation that can then be easily plugged together. For each layer we will implement `forward` and `backward` functions. The `forward` function will receive data, weights, and other parameters, and will return both an output and a `cache` object that stores data needed for the backward pass. The `backward` function will recieve upstream derivatives and the cache object, and will return gradients with respect to the data and all of the weights. This will allow us to write code that looks like this:

```
def two_layer_net(X, W1, b1, W2, b2, reg):
    # Forward pass; compute scores
    s1, fc1_cache = affine_forward(X, W1, b1)
    a1, relu_cache = relu_forward(s1)
    scores, fc2_cache = affine_forward(a1, W2, b2)

    # Loss functions return data loss and gradients on scores
    data_loss, dscores = svm_loss(scores, y)

    # Compute backward pass
    da1, dW2, db2 = affine_backward(dscores, fc2_cache)
    ds1 = relu_backward(da1, relu_cache)
    dX, dW1, db1 = affine_backward(ds1, fc1_cache)

    # A real network would add regularization here

    # Return loss and gradients
    return loss, dW1, db1, dW2, db2
```

In [1]:
```
# As usual, a bit of setup

import numpy as np
import matplotlib.pyplot as plt
from cs231n.gradient_check import eval_numerical_gradient_array, eval_numerical_gradient
from cs231n.layers import *

%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# for auto-reloading external modules
# see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-ipython
%load_ext autoreload
%autoreload 2

def rel_error(x, y):
  """ returns relative error """
  return np.max(np.abs(x - y) / (np.maximum(1e-8, np.abs(x) + np.abs(y))))
```

Affine layer: forward

Open the file `cs231n/layers.py` and implement the `affine_forward` function.

Once you are done we will test your can test your implementation by running the following:

In [2]:
```
# Test the affine_forward function

num_inputs = 2
input_shape = (4, 5, 6)
output_dim = 3

input_size = num_inputs * np.prod(input_shape)
weight_size = output_dim * np.prod(input_shape)

x = np.linspace(-0.1, 0.5, num=input_size).reshape(num_inputs, *input_shape)
w = np.linspace(-0.2, 0.3, num=weight_size).reshape(np.prod(input_shape), output_dim)
b = np.linspace(-0.3, 0.1, num=output_dim)

out, _ = affine_forward(x, w, b)
correct_out = np.array([[ 1.49834967,  1.70660132,  1.91485297],
                        [ 3.25553199,  3.5141327,   3.77273342]])

# Compare your output with ours. The error should be around 1e-9.
print 'Testing affine_forward function:'
print 'difference: ', rel_error(out, correct_out)
```

```
Testing affine_forward function:
difference:  9.76984772881e-10
```

Affine layer: backward

Now implement the `affine_backward` function. You can test your implementation using numeric gradient checking.

In [3]:
```
# Test the affine_backward function

x = np.random.randn(10, 2, 3)
w = np.random.randn(6, 5)
b = np.random.randn(5)
dout = np.random.randn(10, 5)

dx_num = eval_numerical_gradient_array(lambda x: affine_forward(x, w, b)[0], x, dout)
dw_num = eval_numerical_gradient_array(lambda w: affine_forward(x, w, b)[0], w, dout)
db_num = eval_numerical_gradient_array(lambda b: affine_forward(x, w, b)[0], b, dout)

_, cache = affine_forward(x, w, b)
dx, dw, db = affine_backward(dout, cache)

# The error should be less than 1e-10
print 'Testing affine_backward function:'
print 'dx error: ', rel_error(dx_num, dx)
print 'dw error: ', rel_error(dw_num, dw)
print 'db error: ', rel_error(db_num, db)
```

```
Testing affine_backward function:
dx error:  4.56445941839e-10
dw error:  4.29739110725e-11
db error:  7.63583030011e-12
```

ReLU layer: forward

Implement the `relu_forward` function and test your implementation by running the following:

In [4]:
```
# Test the relu_forward function

x = np.linspace(-0.5, 0.5, num=12).reshape(3, 4)

out, _ = relu_forward(x)
correct_out = np.array([[ 0.,          0.,          0.,          0.,         ],
                        [ 0.,          0.,          0.04545455,  0.13636364,],
                        [ 0.22727273,  0.31818182,  0.40909091,  0.5,       ]])

# Compare your output with ours. The error should be around 1e-8
print 'Testing relu_forward function:'
print 'difference: ', rel_error(out, correct_out)
```

```
Testing relu_forward function:
difference:  4.99999979802e-08
```

ReLU layer: backward

Implement the `relu_backward` function and test your implementation using numeric gradient checking:

In [5]:
```
x = np.random.randn(10, 10)
dout = np.random.randn(*x.shape)

dx_num = eval_numerical_gradient_array(lambda x: relu_forward(x)[0], x, dout)

_, cache = relu_forward(x)
dx = relu_backward(dout, cache)

# The error should be around 1e-12
print 'Testing relu_backward function:'
print 'dx error: ', rel_error(dx_num, dx)
```

```
Testing relu_backward function:
dx error:  3.27561078887e-12
```

Loss layers: Softmax and SVM

You implemented these loss functions in the last assignment, so we'll give them to you for free here. It's still a good idea to test them to make sure they work correctly.

In [6]:
```
num_classes, num_inputs = 10, 50
x = 0.001 * np.random.randn(num_inputs, num_classes)
y = np.random.randint(num_classes, size=num_inputs)

dx_num = eval_numerical_gradient(lambda x: svm_loss(x, y)[0], x, verbose=False)
loss, dx = svm_loss(x, y)

# Test svm_loss function. Loss should be around 9 and dx error should be 1e-9
print 'Testing svm_loss:'
print 'loss: ', loss
print 'dx error: ', rel_error(dx_num, dx)

dx_num = eval_numerical_gradient(lambda x: softmax_loss(x, y)[0], x, verbose=False)
loss, dx = softmax_loss(x, y)

# Test softmax_loss function. Loss should be 2.3 and dx error should be 1e-8
print '\nTesting softmax_loss:'
print 'loss: ', loss
print 'dx error: ', rel_error(dx_num, dx)
```

```
Testing svm_loss:
loss:  9.0017283099
dx error:  1.40215660067e-09

Testing softmax_loss:
loss:  2.30275835517
dx error:  7.78128193239e-09
```

Convolution layer: forward naive

We are now ready to implement the forward pass for a convolutional layer. Implement the function `conv_forward_naive` in the file `cs231n/layers.py`.

You don't have to worry too much about efficiency at this point; just write the code in whatever way you find most clear.

You can test your implementation by running the following:

In [7]:
```
x_shape = (2, 3, 4, 4)
w_shape = (3, 3, 4, 4)
x = np.linspace(-0.1, 0.5, num=np.prod(x_shape)).reshape(x_shape)
w = np.linspace(-0.2, 0.3, num=np.prod(w_shape)).reshape(w_shape)
b = np.linspace(-0.1, 0.2, num=3)

conv_param = {'stride': 2, 'pad': 1}
out, _ = conv_forward_naive(x, w, b, conv_param)
correct_out = np.array([[[[[-0.08759809, -0.10987781],
                           [-0.18387192, -0.2109216 ]],
                          [[ 0.21027089,  0.21661097],
                           [ 0.22847626,  0.23004637]],
                          [[ 0.50813986,  0.54309974],
                           [ 0.64082444,  0.67101435]]],
                         [[[-0.98053589, -1.03143541],
                           [-1.19128892, -1.24695841]],
                          [[ 0.69108355,  0.66880383],
                           [ 0.59480972,  0.56776003]],
                          [[ 2.36270298,  2.36904306],
                           [ 2.38090835,  2.38247847]]]]])

# Compare your output to ours; difference should be around 1e-8
print 'Testing conv_forward_naive'
print 'difference: ', rel_error(out, correct_out)
```

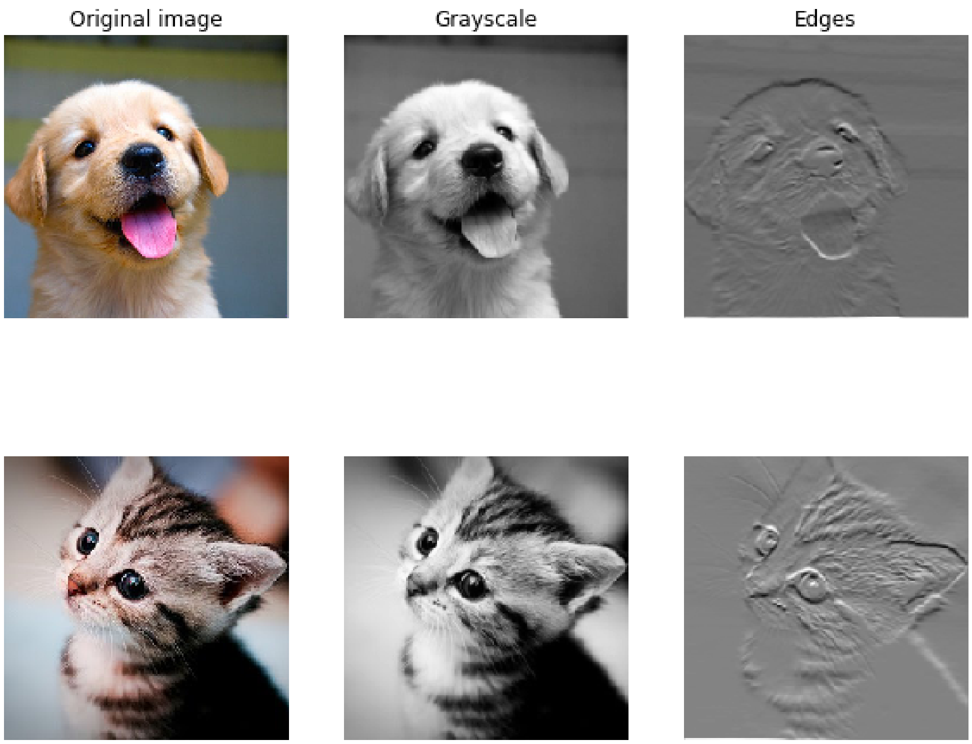
```
Testing conv_forward_naive
difference:  2.21214765759e-08
```

Aside: Image processing via convolutions

As fun way to both check your implementation and gain a better understanding of the type of operation that convolutional layers can perform, we will set up an input containing two images and manually set up filters that perform common image processing operations (grayscale conversion and edge detection). The convolution forward pass will apply these operations to each of the input images. We can then visualize the results as a sanity check.

In [8]:

```
from scipy.misc import imread, imresize

kitten, puppy = imread('kitten.jpg'), imread('puppy.jpg')
# kitten is wide, and puppy is already square
d = kitten.shape[1] - kitten.shape[0]
kitten_cropped = kitten[:, d/2:-d/2, :]

img_size = 200   # Make this smaller if it runs too slow
x = np.zeros((2, 3, img_size, img_size))
x[0, :, :, :] = imresize(puppy, (img_size, img_size)).transpose((2, 0, 1))
x[1, :, :, :] = imresize(kitten_cropped, (img_size, img_size)).transpose((2, 0, 1))

# Set up a convolutional weights holding 2 filters, each 3x3
w = np.zeros((2, 3, 3, 3))

# The first filter converts the image to grayscale.
# Set up the red, green, and blue channels of the filter.
w[0, 0, :, :] = [[0, 0, 0], [0, 0.3, 0], [0, 0, 0]]
w[0, 1, :, :] = [[0, 0, 0], [0, 0.6, 0], [0, 0, 0]]
w[0, 2, :, :] = [[0, 0, 0], [0, 0.1, 0], [0, 0, 0]]

# Second filter detects horizontal edges in the blue channel.
w[1, 2, :, :] = [[1, 2, 1], [0, 0, 0], [-1, -2, -1]]

# Vector of biases. We don't need any bias for the grayscale
# filter, but for the edge detection filter we want to add 128
# to each output so that nothing is negative.
b = np.array([0, 128])

# Compute the result of convolving each input in x with each filter in w,
# offsetting by b, and storing the results in out.
out, _ = conv_forward_naive(x, w, b, {'stride': 1, 'pad': 1})

def imshow_noax(img, normalize=True):
    """ Tiny helper to show images as uint8 and remove axis labels """
    if normalize:
        img_max, img_min = np.max(img), np.min(img)
        img = 255.0 * (img - img_min) / (img_max - img_min)
    plt.imshow(img.astype('uint8'))
    plt.gca().axis('off')

# Show the original images and the results of the conv operation
plt.subplot(2, 3, 1)
imshow_noax(puppy, normalize=False)
plt.title('Original image')
plt.subplot(2, 3, 2)
imshow_noax(out[0, 0])
plt.title('Grayscale')
plt.subplot(2, 3, 3)
imshow_noax(out[0, 1])
plt.title('Edges')
plt.subplot(2, 3, 4)
imshow_noax(kitten_cropped, normalize=False)
plt.subplot(2, 3, 5)
imshow_noax(out[1, 0])
plt.subplot(2, 3, 6)
imshow_noax(out[1, 1])
plt.show()
```



Convolution layer: backward naive

Next you need to implement the function `conv_backward_naive` in the file `cs231n/layers.py`. As usual, we will check your implementation with numeric gradient checking.

In [9]:
```
x = np.random.randn(4, 3, 5, 5)
w = np.random.randn(2, 3, 3, 3)
b = np.random.randn(2,)
dout = np.random.randn(4, 2, 5, 5)
conv_param = {'stride': 1, 'pad': 1}

dx_num = eval_numerical_gradient_array(lambda x: conv_forward_naive(x, w, b, conv_param)[0], x, dout
dw_num = eval_numerical_gradient_array(lambda w: conv_forward_naive(x, w, b, conv_param)[0], w, dout
db_num = eval_numerical_gradient_array(lambda b: conv_forward_naive(x, w, b, conv_param)[0], b, dout

out, cache = conv_forward_naive(x, w, b, conv_param)
dx, dw, db = conv_backward_naive(dout, cache)


# Your errors should be around 1e-9'
print 'Testing conv_backward_naive function'
print 'dx error: ', rel_error(dx, dx_num)
print 'dw error: ', rel_error(dw, dw_num)
print 'db error: ', rel_error(db, db_num)
```

```
Testing conv_backward_naive function
dx error:  6.70599676817e-10
dw error:  5.52552082967e-09
db error:  2.76230950545e-11
```

Max pooling layer: forward naive

The last layer we need for a basic convolutional neural network is the max pooling layer. First implement the forward pass in the function `max_pool_forward_naive` in the file `cs231n/layers.py`.

In [10]:
```
x_shape = (2, 3, 4, 4)
x = np.linspace(-0.3, 0.4, num=np.prod(x_shape)).reshape(x_shape)
pool_param = {'pool_width': 2, 'pool_height': 2, 'stride': 2}

out, _ = max_pool_forward_naive(x, pool_param)

correct_out = np.array([[[[-0.26315789, -0.24842105],
                          [-0.20421053, -0.18947368]],
                         [[-0.14526316, -0.13052632],
                          [-0.08631579, -0.07157895]],
                         [[-0.02736842, -0.01263158],
                          [ 0.03157895,  0.04631579]]],
                        [[[ 0.09052632,  0.10526316],
                          [ 0.14947368,  0.16421053]],
                         [[ 0.20842105,  0.22315789],
                          [ 0.26736842,  0.28210526]],
                         [[ 0.32631579,  0.34105263],
                          [ 0.38526316,  0.4       ]]]])

# Compare your output with ours. Difference should be around 1e-8.
print 'Testing max_pool_forward_naive function:'
print 'difference: ', rel_error(out, correct_out)
```

```
Testing max_pool_forward_naive function:
difference:  4.16666651573e-08
```

Max pooling layer: backward naive

Implement the backward pass for a max pooling layer in the function `max_pool_backward_naive` in the file `cs231n/layers.py`. As always we check the correctness of the backward pass using numerical gradient checking.

In [11]:
```
x = np.random.randn(3, 2, 8, 8)
dout = np.random.randn(3, 2, 4, 4)
pool_param = {'pool_height': 2, 'pool_width': 2, 'stride': 2}

dx_num = eval_numerical_gradient_array(lambda x: max_pool_forward_naive(x, pool_param)[0], x, dout)

out, cache = max_pool_forward_naive(x, pool_param)
dx = max_pool_backward_naive(dout, cache)

# Your error should be around 1e-12
print 'Testing max_pool_backward_naive function:'
print 'dx error: ', rel_error(dx, dx_num)
```

```
Testing max_pool_backward_naive function:
dx error:  3.27562632354e-12
```

Fast layers

Making convolution and pooling layers fast can be challenging. To spare you the pain, we've provided fast implementations of the forward and backward passes for convolution and pooling layers in the file `cs231n/fast_layers.py`.

The fast convolution implementation depends on a Cython extension; to compile it you need to run the following from the `cs231n` directory:

```
python setup.py build_ext --inplace
```

The API for the fast versions of the convolution and pooling layers is exactly the same as the naive versions that you implemented above: the forward pass receives data, weights, and parameters and produces outputs and a cache object; the backward pass recieves upstream derivatives and the cache object and produces gradients with respect to the data and weights.

NOTE: The fast implementation for pooling will only perform optimally if the pooling regions are non-overlapping and tile the input. If these conditions are not met then the fast pooling implementation will not be much faster than the naive implementation.

You can compare the performance of the naive and fast versions of these layers by running the following:

In [12]:
```python
from cs231n.fast_layers import conv_forward_fast, conv_backward_fast
from time import time

x = np.random.randn(100, 3, 31, 31)
w = np.random.randn(25, 3, 3, 3)
b = np.random.randn(25,)
dout = np.random.randn(100, 25, 16, 16)
conv_param = {'stride': 2, 'pad': 1}

t0 = time()
out_naive, cache_naive = conv_forward_naive(x, w, b, conv_param)
t1 = time()
out_fast, cache_fast = conv_forward_fast(x, w, b, conv_param)
t2 = time()

print 'Testing conv_forward_fast:'
print 'Naive: %fs' % (t1 - t0)
print 'Fast: %fs' % (t2 - t1)
print 'Speedup: %fx' % ((t1 - t0) / (t2 - t1))
print 'Difference: ', rel_error(out_naive, out_fast)

t0 = time()
dx_naive, dw_naive, db_naive = conv_backward_naive(dout, cache_naive)
t1 = time()
dx_fast, dw_fast, db_fast = conv_backward_fast(dout, cache_fast)
t2 = time()

print '\nTesting conv_backward_fast:'
print 'Naive: %fs' % (t1 - t0)
print 'Fast: %fs' % (t2 - t1)
print 'Speedup: %fx' % ((t1 - t0) / (t2 - t1))
print 'dx difference: ', rel_error(dx_naive, dx_fast)
print 'dw difference: ', rel_error(dw_naive, dw_fast)
print 'db difference: ', rel_error(db_naive, db_fast)
```

```
Testing conv_forward_fast:
Naive: 0.025628s
Fast: 0.016615s
Speedup: 1.542475x
Difference:  8.25927664985e-14

Testing conv_backward_fast:
Naive: 0.154112s
Fast: 0.012026s
Speedup: 12.814813x
dx difference:  8.11102958604e-12
dw difference:  2.09635293053e-13
db difference:  1.80618072326e-14
```

In [14]:
```
from cs231n.fast_layers import max_pool_forward_fast, max_pool_backward_fast

x = np.random.randn(100, 3, 32, 32)
dout = np.random.randn(100, 3, 16, 16)
pool_param = {'pool_height': 2, 'pool_width': 2, 'stride': 2}

t0 = time()
out_naive, cache_naive = max_pool_forward_naive(x, pool_param)
t1 = time()
out_fast, cache_fast = max_pool_forward_fast(x, pool_param)
t2 = time()

print 'Testing pool_forward_fast:'
print 'Naive: %fs' % (t1 - t0)
print 'fast: %fs' % (t2 - t1)
print 'speedup: %fx' % ((t1 - t0) / (t2 - t1))
print 'difference: ', rel_error(out_naive, out_fast)

t0 = time()
dx_naive = max_pool_backward_naive(dout, cache_naive)
t1 = time()
dx_fast = max_pool_backward_fast(dout, cache_fast)
t2 = time()

print '\nTesting pool_backward_fast:'
print 'Naive: %fs' % (t1 - t0)
print 'fast: %fs' % (t2 - t1)
print 'speedup: %fx' % ((t1 - t0) / (t2 - t1))
print 'dx difference: ', rel_error(dx_naive, dx_fast)
```

```
Testing pool_forward_fast:
Naive: 0.002711s
fast: 0.002161s
speedup: 1.254662x
difference:  0.0

Testing pool_backward_fast:
Naive: 0.013489s
fast: 0.010477s
speedup: 1.287480x
dx difference:  0.0
```

Sandwich layers

There are a couple common layer "sandwiches" that frequently appear in ConvNets. For example convolutional layers are frequently followed by ReLU and pooling, and affine layers are frequently followed by ReLU. To make it more convenient to use these common patterns, we have defined several convenience layers in the file `cs231n/layer_utils.py`. Lets grad-check them to make sure that they work correctly:

In [16]:
```
from cs231n.layer_utils import conv_relu_pool_forward, conv_relu_pool_backward

x = np.random.randn(2, 3, 16, 16)
w = np.random.randn(3, 3, 3, 3)
b = np.random.randn(3,)
dout = np.random.randn(2, 3, 8, 8)
conv_param = {'stride': 1, 'pad': 1}
pool_param = {'pool_height': 2, 'pool_width': 2, 'stride': 2}

out, cache = conv_relu_pool_forward(x, w, b, conv_param, pool_param)
dx, dw, db = conv_relu_pool_backward(dout, cache)

dx_num = eval_numerical_gradient_array(lambda x: conv_relu_pool_forward(x, w, b, conv_param, pool_pa
dw_num = eval_numerical_gradient_array(lambda w: conv_relu_pool_forward(x, w, b, conv_param, pool_pa
db_num = eval_numerical_gradient_array(lambda b: conv_relu_pool_forward(x, w, b, conv_param, pool_pa

print 'Testing conv_relu_pool_forward:'
print 'dx error: ', rel_error(dx_num, dx)
print 'dw error: ', rel_error(dw_num, dw)
print 'db error: ', rel_error(db_num, db)
```

```
Testing conv_relu_pool_forward:
dx error:  1.03064720587e-07
dw error:  8.21981169382e-10
db error:  1.16775205696e-09
```

In [17]:
```
from cs231n.layer_utils import conv_relu_forward, conv_relu_backward

x = np.random.randn(2, 3, 8, 8)
w = np.random.randn(3, 3, 3, 3)
b = np.random.randn(3,)
dout = np.random.randn(2, 3, 8, 8)
conv_param = {'stride': 1, 'pad': 1}

out, cache = conv_relu_forward(x, w, b, conv_param)
dx, dw, db = conv_relu_backward(dout, cache)

dx_num = eval_numerical_gradient_array(lambda x: conv_relu_forward(x, w, b, conv_param)[0], x, dout)
dw_num = eval_numerical_gradient_array(lambda w: conv_relu_forward(x, w, b, conv_param)[0], w, dout)
db_num = eval_numerical_gradient_array(lambda b: conv_relu_forward(x, w, b, conv_param)[0], b, dout)

print 'Testing conv_relu_forward:'
print 'dx error: ', rel_error(dx_num, dx)
print 'dw error: ', rel_error(dw_num, dw)
print 'db error: ', rel_error(db_num, db)
```

```
Testing conv_relu_forward:
dx error:  3.02435282585e-09
dw error:  3.15794230901e-09
db error:  2.14416051723e-10
```

In [18]:
```
from cs231n.layer_utils import affine_relu_forward, affine_relu_backward

x = np.random.randn(2, 3, 4)
w = np.random.randn(12, 10)
b = np.random.randn(10)
dout = np.random.randn(2, 10)

out, cache = affine_relu_forward(x, w, b)
dx, dw, db = affine_relu_backward(dout, cache)

dx_num = eval_numerical_gradient_array(lambda x: affine_relu_forward(x, w, b)[0], x, dout)
dw_num = eval_numerical_gradient_array(lambda w: affine_relu_forward(x, w, b)[0], w, dout)
db_num = eval_numerical_gradient_array(lambda b: affine_relu_forward(x, w, b)[0], b, dout)

print 'Testing affine_relu_forward:'
print 'dx error: ', rel_error(dx_num, dx)
print 'dw error: ', rel_error(dw_num, dw)
print 'db error: ', rel_error(db_num, db)
```

```
Testing affine_relu_forward:
dx error:  4.06811319877e-10
dw error:  1.13396650837e-09
db error:  1.89289199544e-11
```

In [ ]: