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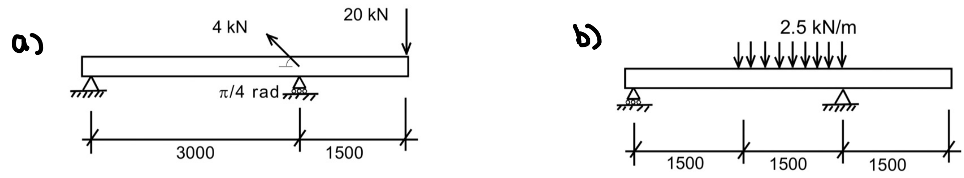
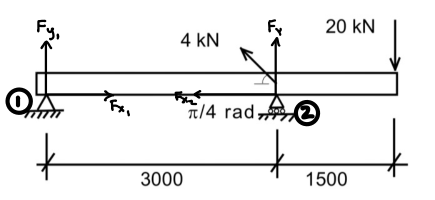
Problem Set #4
CIV 102
October 3, 2023

$\Sigma H=0 \quad \Sigma V=0 \quad \Sigma M=0$

1. Calculate the support reactions for the following beams (All dimensions are in mm):





$$F_y = 4\sin\left(\tfrac{\pi}{4}\right) = 4\tfrac{\sqrt{2}}{2} = 2\sqrt{2}$$

$$F_x = 4\cos\left(\tfrac{\pi}{4}\right) = 4\tfrac{\sqrt{2}}{2} = 2\sqrt{2}$$

Vertical Forces

$\Sigma F_y = 0 \quad 4\sin\left(\tfrac{\pi}{4}\right) - 20 + F_{y_1} + F_{y_2} = 0$

$F_{y_1} + F_{y_2} = 20 - 2\sqrt{2}$

Horizontal Forces

$\Sigma F_x = 0 \quad F_{x_1} - 4\cos\left(\tfrac{\pi}{4}\right) = 0 \quad \therefore F_{x_1} = 2\sqrt{2} = 2.83\text{ kN}$

Moments

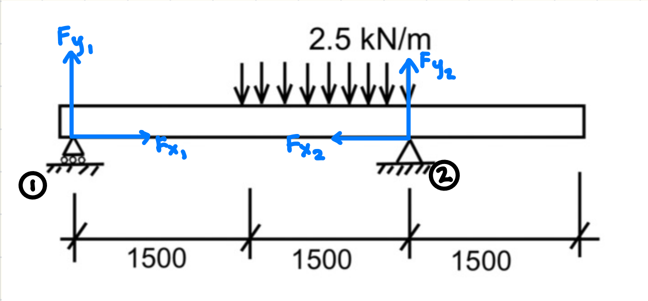
$\Sigma M_1 = 0 \quad F_{y_2}(3000) + 4\sin\left(\tfrac{\pi}{4}\right)(3000) - 20(4500) = 0$

$F_{y_2} = 27.17\text{ kN}$

$\Sigma M_2 = 0 \quad -F_{y_1}(3000) - 20(1500) = 0$

$F_{y_1} = -10\text{ kN}$

$\boxed{\therefore \text{ support 1: 10 kN downwards, 2.83 kN right} \quad \text{support 2: 27.2 kN up}}$



$$\sum M_1^{\circlearrowleft +} = 0 \qquad F_{y_2}(3) - \int_{1.5}^{3.0} 2.5\, r\, dr = 0$$

$$F_{y_2}(3) - \left[\frac{2.5r^2}{2}\right]_{1.5}^{3.0} = 0$$

$$F_{y_2}(3) - \left[8.4375\right] = 0$$

$$F_{y_2} = 2.81 \text{ kN}$$

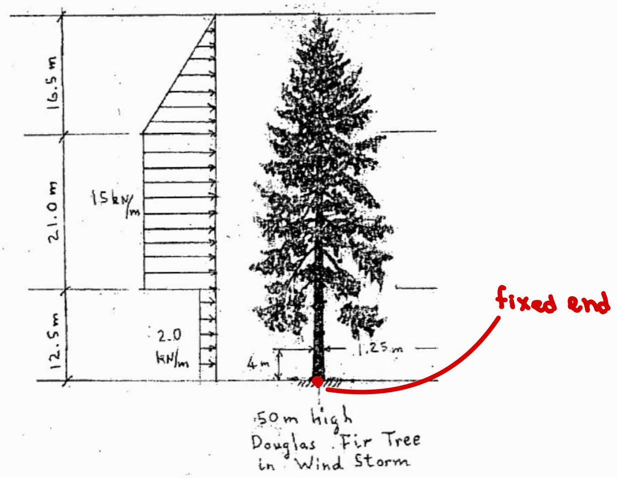
$$\sum M_2^{\circlearrowleft +} = 0 \qquad \int_{0}^{1.5} 2.5\, r\, dr - F_{y_1}(3) = 0$$

$$F_{y_1} = \frac{\left[\frac{2.5r^2}{2}\right]_0^{1.5}}{3} = 9.38 \times 10^{-1} \text{ kN}$$

↳ No horizontal forces ($F_{x_1} = F_{x_2} = 0$).

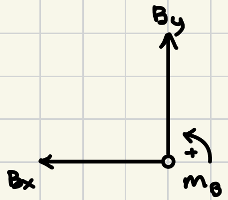
∴ support 1: 2.81 kN up   support 2: 0.938 kN up

2. During an extreme wind storm a 50 m high Douglas Fir tree is subjected to a horizontal wind pressure of about 1.5 kN/m^2 on the frontal area of the tree. The resulting horizontal loads which must be resisted by the trunk of the tree and carried to the ground can be approximated as shown in the diagram below. Calculate the reaction forces which must be resisted at the base of the tree. The tree acts as a vertical "cantilever" which is attached to the ground by a "fixed end" which can resist horizontal, vertical and rotational displacements.



Reaction Forces

$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma M^{\circlearrowleft +} = 0$



① $\Sigma F_x = 0$

$(12.5\text{ m})(2.0 \frac{kN}{m}) + (21\text{ m})(15 \frac{kN}{m}) + \left(\frac{16.5}{2}\text{ m}\right)\left(15 \frac{kN}{m}\right) - B_x = 0$

$B_x = 463.75 \text{ kN} \approx 464 \text{ kN}$

$\boxed{\therefore \text{ the support reaction } B_x \text{ is } 464 \text{ kN (left).}}$

② $\Sigma F_y = 0 \qquad B_y = 0$

$\boxed{\therefore \text{ the support reaction vertically is } 0 \text{ kN.}}$

$15r + 502.5 - \frac{15}{16.5}r^2 - \frac{15 \cdot 33.5}{16.5}r$

$502.5 - \frac{255}{16.5}r$

$$M_{WIND} = \int_0^{12.5} 2.0r\,dr + \int_{12.5}^{33.5} 15r\,dr + \int_0^{16.5} \left(15 - \frac{15}{16.5}r\right)(r+33.5)\,dr$$

$$= \left[\frac{2.0r^2}{2}\right]_0^{12.5} + \left[\frac{15r^2}{2}\right]_{12.5}^{33.5} + \left[502.5r - \frac{255r^2}{33} - \frac{15r^3}{49.5}\right]_0^{16.5}$$

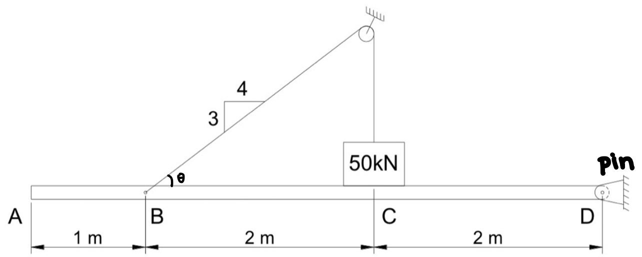
$$= 156.25 + 7245 + 4826.25$$

$$= 12230 \text{ kN}\cdot\text{m}$$

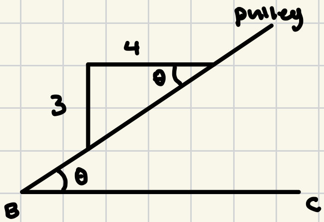
$\therefore$ the reaction force that must be resisted is 12230 kN·m in rotation (counterclockwise).

3. In the structure shown below, a cable is attached to the 50 kN weight and to the beam A-D at point B. If the horizontal uniform beam weighs 8 kN/m, determine the following:

a) The horizontal and vertical component of the force that the pin at D exerts on the beam A-D.
b) The force in the cable.
c) The normal force exerted by the beam A-D on the 50-kN weight at point C.



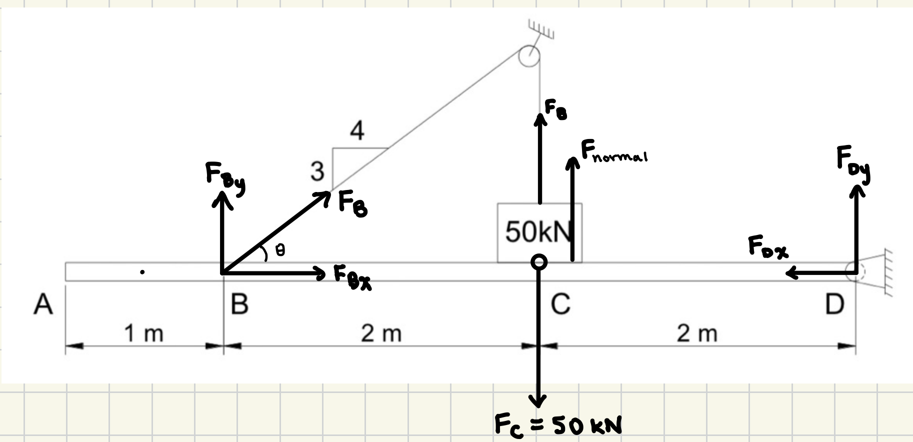
a) ① Solve for θ (angle between pulley and point C).



$$\tan\theta = \frac{3}{4}$$

$$\theta = \tan^{-1}\left(\frac{3}{4}\right)$$

$$= 36.89^{\circ}$$



Take D as a center of rotation.

$\sum M_D = 0$

$$\int_0^5 8r\,dr = \frac{8r^2}{2}\Big|_0^5 = 100\,[\text{kN}\cdot\text{m}]$$

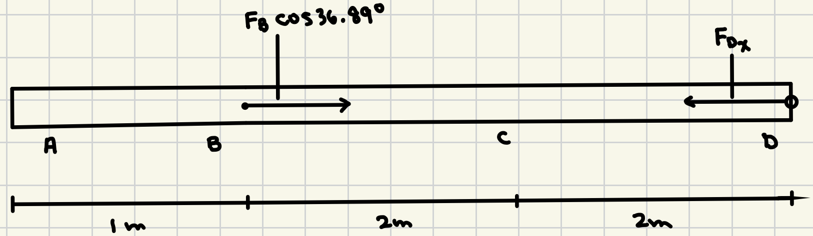
$$100 + 50(2) - F_B \sin 36.89^\circ (4) - F_B(2) = 0$$

$$4F_B \sin 36.89^\circ + 2F_B = 200$$

$$F_B(4\sin 36.89^\circ + 2) = 200$$

$$F_B = \frac{200}{4\sin 36.89^\circ + 2} = 45.4 \text{ kN}$$

Show horizontal forces.



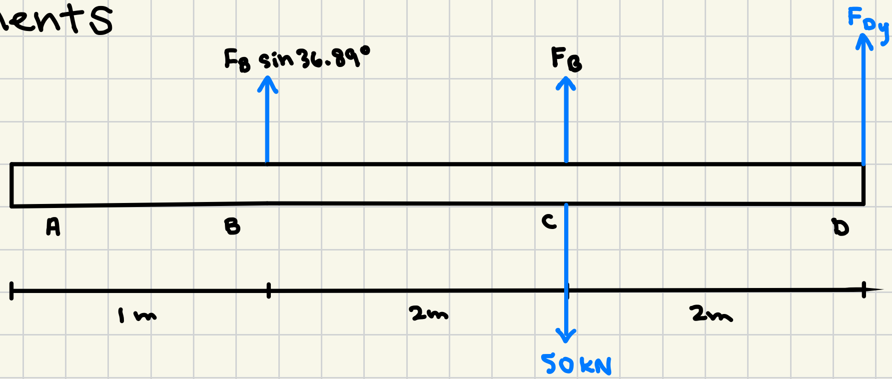
$\sum F_x = 0 \quad F_{Dx} = F_B \cos 36.89^\circ$

$= 45.4 \cos 36.89^\circ$

$= 36.3$ kN

$\boxed{\therefore \text{ the horizontal force at D is 36.3 kN (into the beam).}}$

Moments



Take center of rotation about C.

$\Sigma M_c = 0$

From A to C:

$$\int_0^3 8r\,dr = \left.\frac{8r^2}{2}\right|_0^3 = 36 \text{ kN·m}$$

From C to D:

$$\int_0^2 8r\,dr = \left.\frac{8r^2}{2}\right|_0^2 = 16 \text{ kN·m}$$

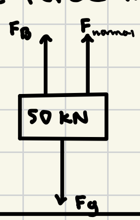
$$F_{Dy}(2) - 16 - F_B \sin 36.89°(2) + 36 = 0$$
$$2F_{Dy} - 2F_B \sin 36.89° + 20 = 0$$
$$F_{Dy} - F_B \sin 36.89° + 10 = 0$$

$$\boxed{\begin{aligned} F_{Dy} &= 45.4 \sin 36.89° - 10 \\ &= 17.25 \text{ kN} \end{aligned}}$$

a) The horizontal force at D is 36.3 kN (into the beam) and the vertical force at D is 17.25 kN up.

b) The force in the cable is 45.4 kN.

c)



$\Sigma F_y = 0$

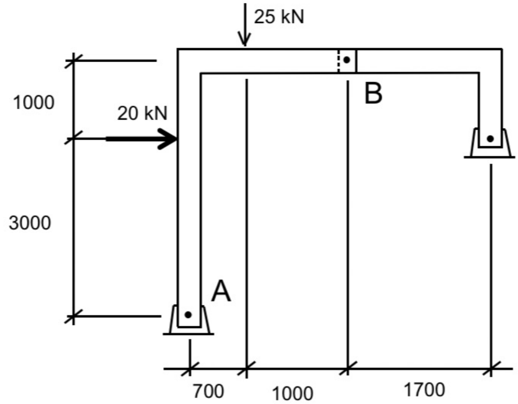
$50 = F_B + F_{normal}$

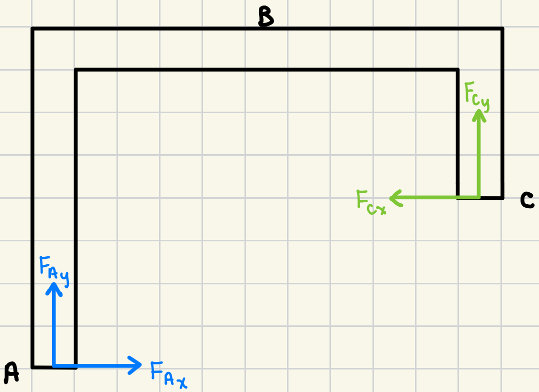
$50 = 45.4 + F_{normal}$

$F_{normal} = 4.6$ kN

∴ the normal force is 4.6 kN.

4. A three-pinned arch is subjected to the loading shown. Pin supports are located at point A and point C. A hinge is located at point B. Calculate the reaction forces at point A and point C. In addition, calculate the force carried through the hinge at point B.





A and C are pins.

↳ Unable to resist moments

↳ 2 support forces (x and y)

↳ 4 unknowns

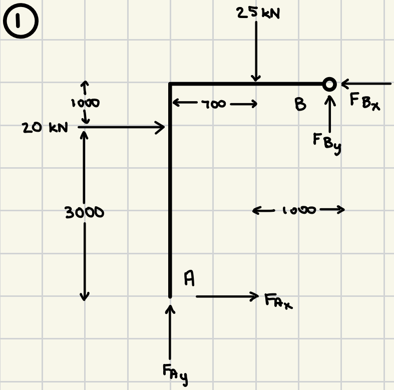
B is an internal hinge.

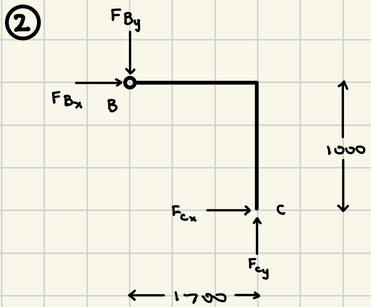
↳ unable to resist moment

↳ Reduces indeterminacy to three

Cut the free body diagrams at B (internal hinge).

Each free body diagram is in equilibrium.





Take right and up to be positive.

① $\sum F_x = 0 \qquad 20 + F_{Ax} - F_{Bx} = 0$

$\sum F_y = 0 \qquad F_{Ay} + F_{By} - 25 = 0$

$\sum M_A^{\circlearrowleft +} = 0 \qquad -(20)(3000) - 25(700) + F_{Bx}(4000) + F_{By}(1700) = 0$

$4000\,F_{Bx} + 1700\,F_{By} - 77500 = 0$

② $\sum F_x = 0 \qquad F_{Bx} + F_{Cx} = 0$

$\sum F_y = 0 \qquad F_{Cy} - F_{By} = 0$

$\sum M_C^{\circlearrowleft +} = 0 \qquad F_{By}(1700) - F_{Bx}(1000) = 0$

Solve for F_{Bx} and F_{By}.

$$\begin{array}{rl} & 4000\,F_{Bx} + 1700\,F_{By} - 77500 = 0 \\ - & -1000\,F_{Bx} + 1700\,F_{By} = 0 \\ \hline & 5000\,F_{Bx} - 77500 = 0 \end{array}$$

$$F_{Bx} = \frac{77500}{5000} = 15.5 \text{ kN}$$

$$-1000(15.5) + 1700\,F_{By} = 0$$

$$F_{By} = \frac{15\,500}{1700} = 9.12 \text{ kN}$$

$\boxed{\therefore \text{ the force carried through the hinge is } 15.5 \text{ kN horizontally and } 9.12 \text{ kN vertically.}}$

Solve for F_{Ax} and F_{Ay}.

$20 + F_{Ax} - F_{Bx} = 0$

$20 + F_{Ax} - 15.5 = 0$

$F_{Ax} = -4.5 \text{ kN}$

$F_{Ay} + F_{By} - 25 = 0$

$F_{Ay} + 9.12 - 25 = 0$

$F_{Ay} = 15.88 \text{ kN}$

$\boxed{\therefore \text{ the support reactions at A are } -4.5 \text{ kN horizontally and } 15.88 \text{ kN vertically.}}$

Solve for F_{cx} and F_{cy}.

$$F_{Bx} + F_{cx} = 0$$
$$F_{cx} = -F_{Bx}$$
$$F_{cx} = -15.5 \text{ kN}$$

$$F_{cy} - F_{By} = 0$$
$$F_{cy} = F_{By}$$
$$F_{cy} = 9.12 \text{ kN}$$

∴ the support reactions at C are −15.5 kN horizontally and 9.12 kN vertically.