INEL4206
Microprocessors

Lectures 1 & 2
Overview of
Number Systems, Boolean Algebra

Representation of Data

- Integers are written using a positional numbering system, where each digit represents the coefficient in a power series:

$$N = a_{n-1}b^{n-1} + a_{n-2}b^{n-2} + \ldots + a_1 b^1 + a_0 b^0$$

where n is the number of digits, b is the base, and a_i are the coefficients where each is an integer in the range $0 < a_i < b$.

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Numbering Systems

- Number systems
  - Three main characteristics:
    - Number of independent digits
      - Base or radix (b)
    - Place values of different digits
      - $b^{n-1} \ldots b^2 b^1 b^0$ with 0 known as the least significant digit and n – 1 the most significant digit
      - Fractional part (if present) is represented as $b^{-1}b^{-2}b^{-3}\ldots$
    - Maximum number of values that can be represented given a fixed number of digits (n)
      - b^n

Base 10 system (decimal)

- 10 different digit values
  - 0,1,2,3,4,5,6,7,8,9
- Example:
  - 9138.504

    $9138 = 8x10^0 + 3x10^1 + 1x10^2 + 9x10^3$

    $504 = 5x10^{-1} + 0x10^{-2} + 4x10^{-3}$
- With 10 digits, what are maximum number of values we can represent?
  - 10^{10} ranging from 0 to 10^{10} - 1

Base 2 system (binary)

- 2 different digits known as binary digits or “bits”
  - 0,1
- Example: (first 16 binary numbers, 4 bits)

  0000, 0001, 0010, 0011, 0100, 0101, 0110, 0111,

  1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111
- With 10 digits, what are maximum number of values we can represent?
  - 2^{10} =1024 values ranging from 0000000000 to 1111111111

Binary System Advantages

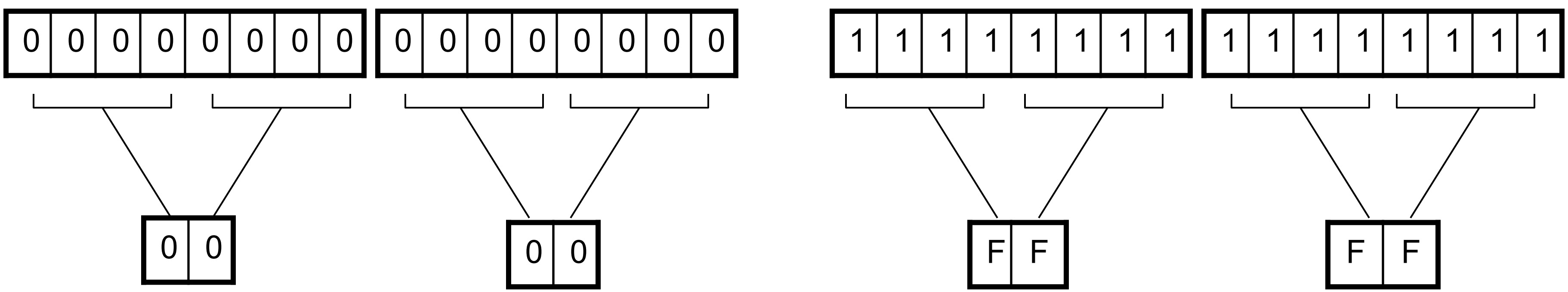
- George Boole (1854) mathematics of logic (Boolean Algebra)
  - truth values (false, true) represented by 0,1
- All data can be represented as a sequence of 1s and 0s
- Claude Shannon (1937) use of boolean algebra and binary arithmetic to create logic gates that simplified electromechanical relays used in the phone system switches
  - Foundation of digital circuit design
- Basic electronic devices can be operated in 2 different modes: (e.g. BJTs cut-off / saturation)
- Circuit implementation of 0s & 1s arithmetic easy to implement

Other popular base systems

- Base 8 (Octal)
  - Similar to decimal, just remove digits 8 & 9
- Base 16 (Hexadecimal or hex)
  - 16 different digits
  - 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F
  - Short form to represent large binary numbers
    - Bits are arranged in group of 8 (byte)
    - Working with large binary numbers consume a large amount of digits, (e.g. memory addresses)

Large binary to Hex

- 64K memories have up to 2^{16} = 65,536 different values



- In general 4 binary digits = 1 hex digit
  – 3 binary digits = 1 hex digit

Common Powers

$2^{-3} = 0.125$
$2^{-2} = 0.25$
$2^{-1} = 0.5$
$2^{0} = 1$
$2^{1} = 2$
$2^{2} = 4$
$2^{3} = 8$
$2^{4} = 16$
$2^{5} = 32$
$2^{6} = 64$
$2^{7} = 128$
$2^{8} = 256$
$2^{9} = 512$
$2^{10} = 1024$
$2^{11} = 2048$
$2^{12} = 4096$

$16^{0} = 1 = 2^{0}$
$16^{1} = 16 = 2^{4}$
$16^{2} = 256 = 2^{8}$
$16^{3} = 4096 = 2^{12}$

$2^{10} = 1024 = 1$ K
$2^{20} = 1048576 = 1$ M (1 Mega) $= 1024$ K $= 2^{10} * 2^{10}$
$2^{30} = 1073741824 = 1$ G (1 Giga)

Binary to decimal conversion

- 1001.0101

  $1001 = 1x2^0 + 0x2^1 + 0x2^2 + 1x2^3$

  $= 1 + 0 + 0 + 8 = 9$

  $.0101 = 0x2^{-1} + 1x2^{-2} + 0x2^{-3} + 1x2^{-4}$

  $= 0 + .25 + 0 + .0625 = .3125$

  $1001.0101 = 9.3125$

Decimal to binary conversion

- 13.375

13 :

$$\frac{13}{2} = 6 \ \ R = 1$$

$$\frac{6}{2} = 3 \ \ R = 0$$

$$\frac{3}{2} = 1 \ \ R = 1$$

$$\frac{1}{2} = 0 \ \ R = 1 \ \ (\text{MSB})$$

.375

$$0.375 \times 2 = 0.75 \ \ C0$$

$$0.75 \times 2 = 0.5 \ \ C1$$

$$0.5 \times 2 = 0 \ \ C1$$

Thus $13.375_{10} = 1101.011_2$

Base Conversion

Convert 53 to binary

53/2 = 26, R. = 1 ← Least Significant Digit

26/2 = 13, R. = 0

13/2 = 6 , R. = 1

6 /2 = 3, R. = 0

3/2 = 1, R. = 1

1/2 = 0, R. = 1 ← Most Significant Digit

$$53 = 0b\ 110101$$
$$= 1*2^5 + 1*2^4 + 0*2^3 + 1*2^2 + 0*2^1 + 1*2^0$$
$$= 32 + 16 + 0 + 4 + 0 + 1 = 53$$

Base Conversion (2)

Convert 53 to Hex

$$53/16 = 3, \qquad R = 5$$

$$3/16 = 0, \qquad R = 3$$

$$53 = 0x35$$

$$= 3 * 16^1 + 5 * 16^0$$

$$= 48 + 5 = 53$$

Binary arithmetic

- Addition
  - Basic rules:

$$0 + 0 = 0$$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

$1 + 1 = 0$ carry ‘1’ to the next more significant bit

$1 + 1 + 1 = 1$ carry ‘1’ to the next more significant bit

Binary arithmetic

- Subtraction
  - Basic rules

$$0 - 0 = 0$$

$$1 - 0 = 1$$

$$1 - 1 = 0$$

$$0 - 1 = 1$$ borrow ‘1’ form the next most significant bit

Binary Arithmetic

addition

$$
\begin{array}{r}
0111 \\
1001\ 0101 \\
+\ \underline{0100\ 0011} \\
1101\ 1000
\end{array}
$$

subtraction

$$
\begin{array}{cccccccccc}
 & 0\,2 & & & & & 0\,2 & & \\
 & \cancel{1}\,0 & 0 & 1 & & 0 & \cancel{1}\,0 & 1 \\
- & \underline{0\,1} & \underline{0} & \underline{0} & & \underline{0} & \underline{0\,1} & \underline{1} \\
 & 0\,1 & 0 & 1 & & 0 & 0\,1 & 0
\end{array}
$$

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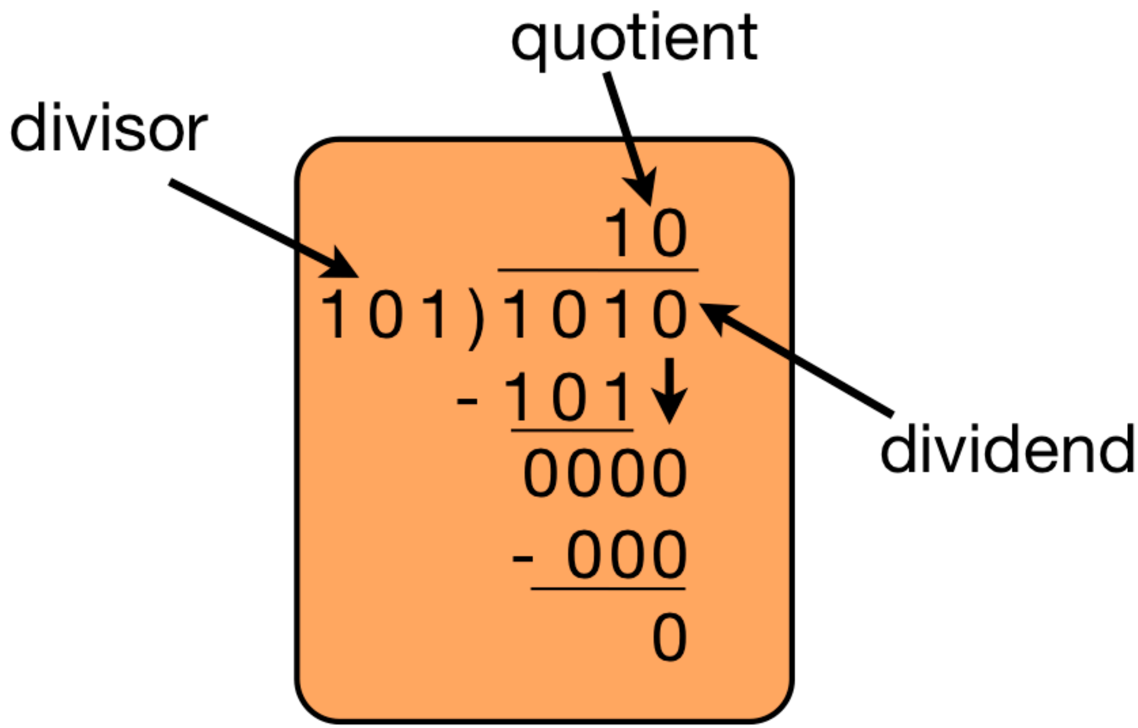
Binary Arithmetic

multiplication

```
     1001 0101
x           10
     0000 0000
+ 1 0010 101
  1 0010 1010
```

division



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Complements

Binary	1’s	2’s	10010110	01101001	01101010
Decimal	9’s	10’s	2496	7503	7504
Octal	7’s	8’s	562	215	216
Hex	15’s	16’s	3BF	C40	C41

- Binary 1’s and 2s complements are important because they allow for easy arithmetic logic implementation.
- In 2s complement notation, the positive value is the same binary and the negative is the 2s complement of the positive value

  +9 : 00001001

  –9 : 11110111

- In 8-bit binary MSB provides sign, rest of the bits are the number representation, possible values range: $+(2^{(n-1)} - 1)$ to $-(2^{(n-1)} - 1)$

Addition using 2’s complement

- Final carry obtained while adding MSBs should be disregarded
  - Consider –18 and –37 addition:

2’s –18: 11101110

2’s –37: 11011011

Sum: 11001001 : 2’s –55

Subtraction using 2’s complement

- Similar to addition, add 2’s complement of subtrahend and disregard carry:
  - Consider +24 – +14

2’s +24:	00011000		00011000
2’s +14:	00001110	2’s:	<u>11110010</u>
		Sum:	00001010