INEL 4206. First EXAM. February 20, 2013

Problem 1. Answer the following questions:

1.1. What is an embedded system? **ANSWER: An embedded system contains tightly coupled hardware and software components to perform a single function and forms part of a larger system**

1.2. Enumerate five attributes of an embedded system
ANSWER: (a) It forms part of a larger system; (b) is not intended to be in- dependently programmable by the user, (c) it is expected to work with minimal or no human interaction; (d) It is reactive; (e) heavily constrained; (f) Mostly operating in real time

1.3. Which are the two sets of components that constitute the structure of an embedded system?
ANSWER: The hardware system and the software system.

1.4. What is the role of each of the components mentioned in the previous question?
ANSWER:
Hardware System: Comprise the Physical (electronics) components necessary to perform the requested function.
Software System: The set of programs necessary to give functionality to the hardware components.

1.5. What are the three defining categories used for classifying embedded systems?
ANSWER: Small, Distributed, and High Performance.

1.6. Which are the five steps in the life cycle of embedded systems?
ANSWER: Birth, Design, Growth, Maturity, and Decline.

1.7. What is a bit?
ANSWER: a 0 or a 1.

1.8. Define an n-bit word?
ANSWER: An order sequence (a permutation) of n bits.

1.9. Define the most significant and least significant bits of a word.
ANSWER:
Most significant bit: the leftmost bit in the word.
Least Significant Bit: The rightmost bit in the word.

1.10. Define a nibble, a byte, a double word and a quad.
ANSWER: Nibble is a 4-bit word; Byte is an 8-bit word; Double Word is a 32-bit word; Quad is a 64-bit word.

1.11. Write the following as a power of 2: 8K, 512M, 64G, 4T
ANSWER: 8k=$2^3 \times 2^{10} = 2^{13}$**; 512M =** $2^9 \times 2^{20} = 2^{29}$**;**
64G = $2^6 \times 2^{30} = 2^{36}$**; and 4T =** $2^2 \times 2^{40} = 2^{42}$

1.12. A number is expressed in a positional system base r as $a_4a_3a_2a_1a_0.a_{-1}a_{-2}$. Write the power expansion defined by this notation.
ANSWER:

$$a_4r^4 + a_3r^3 + a_2r^2 + a_1r + a_0.a_{-1}r^{-1} + a_{-2}r^{-2}$$

1.13. How many different words can be written using N bits?
ANSWER:
2^N **words**.

1.14. How do you write 16^6 in hexadecimal base?
ANSWER: 1000000h

1.15. In a word with n bits, $a_{n-1}a_{n-2}\ldots a_1a_0$, what is the series power that describes the decimal equivalent in two's complement convention?
ANSWER: $-2^{n-1}a_{n-1} + 2^{n-2}a_{n-2}\ldots + 2a_1 + a_0$

1.16. Two signed numbers are represented using two's complement convention with 15 bits. For P, the representation is 52AFh, while for Q the representation is 3109h. Tell the sign for each one.
ANSWER: P is negative (because its binary is 101001010101111) and Q is positive (0110...11)

1.17. For real numbers in format F(m.p), what is the step?
ANSWER: 2^{-p}

1.18. Write the rule for BCD addition
ANSWER: Apply in each digit column: If the addition (hex or binary equivalent) is greater than 9, add 6.

1.19. What is wrong with the following reasoning?
"In binary systems, to subtract 6 (110B) from 9 (1001B) using the method of addition of two's complement, we take the two's complement of 6 by complementing the bits and adding one (001B+1 = 010B) and then add it to 9 (1001B). "
ANSWER: Both operands must have the same number of digits. Hence, 6 must be expressed as 0110, which yields a two's complement 1010.

1.20. What is the range of integers covered with n bits in unsigned normal binary convention and in two's complement convention?
ANSWER: For unsigned numbers: 0 to $2^n - 1$
For signed numbers -2^{n-1} **to** $2^{n-1} - 1$

1.21. What is the range of unsigned and signed real numbers covered with n bits in fixed point format F(p.q)?

ANSWER: For unsigned numbers: 0 to $(2^n - 1) \times 2^{-p}$
For signed numbers $-(2^{n-1}) \times 2^{-p}$ **to** $(2^{n-1} - 1) \times 2^{-p}$

1.22. Define "sign extension".

ANSWER: Given an n-bit word representing a signed number, we may repeat the sign bit to the left as many times as necessary to convert the word to an m-bit word, m≥n.

1.23. What is called the "sign bit" in an n-bit word.

ANSWER: The most significant bit.

1.24. When does overflow happen in the addition of two signed binary numbers?

ANSWER: When the addition of two numbers of the same sign results in a number of different sign. (Fixed number of bits)

1.25. Represent 1985 in packed and in unpacked BCD formats.

ANSWER: Packed: 00011001 10000101 => 19 85. Unpacked: 01h 09h 08h 05h.

Problem 2. Answer the following questions:

2.1. If the word $b_3b_2b_1b_0$ represents the unsigned decimal number M using normal binary representation, what does the following represent in terms of M?

- $b_3b_2b_1b_00$ **ANSWER:** 2M
- $b_3b_2b_1b_01$ **ANSWER:** 2M+1
- $0b_3b_2b_1$ **ANSWER:** Quotient of M/2

Development:
If $b_3b_2b_1b_0 \to M = 2^3b_3 + 2^2b_2 + 2^1b_1 + b_0$, then

$$b_3b_2b_1b_00 \to 2^4b_3 + 2^3b_2 + 2^2b_1 + 2b_0 + 0 = 2(2^3b_3 + 2^2b_2 + 2^1b_1 + b_0) = 2M$$
$$b_3b_2b_1b_01 \to 2^4b_3 + 2^3b_2 + 2^2b_1 + 2b_0 + 1 = 2(2^3b_3 + 2^2b_2 + 2^1b_1 + b_0) + 1 = 2M + 1$$
$$0b_3b_2b_1 \to 2^2b_3 + 2^b_1 + b_1$$

and

$$\frac{M}{2} = \overbrace{2^2b_3 + 2^b_1 + b_1}^{Quotient} + \frac{b_0}{2}(\text{ residue: } b_0$$

2.2. The Analog-to-Digital converter in the MSP430 has control registers to configure the way it functions. In one of these registers, the ADC12CTL1, the group of bits 4-3 are used to select the clock source with the following code:
00 - The ADC oscillator; 01 - the ACLK clock;
10 - the MCLK clock; 11 - the SMCLK clock.

On the other hand, bits 7-5 indicate that the clock frequency should be divided by N, where N is expressed in a bias -1 notation. (**Originally an error: bias 1**)
In a given configuration for the ADC converter, what is the clock selected and by what number is its frequency divided if the register ADC12CTL1 contents is expressed as 2A35h?

ANSWER: *We expand the word (not necessarily complete) and concentrate on reading the fields of bits 7-5 and 4-3:*

0	0	1	0	1	0	1	0	**<u>0 0 1</u>**	**<u>1 0</u>**	1	0	0
b_{15}	b_{14}	b_{13}	b_{12}	b_{11}	b_{10}	b_8	b_8	$b_7 - b_5$	$b_4\, b_3$	b_2	b_1	b_0

From field 4-3, the selected source is ***MCLK****. The decimal equivalent for field 7-5 is* 001b = 1 *and the bias* -1. *Therefore* N = 1 -(-1)=2. *The frequency is divided by 2. .*

2.3. A 32-bit code X consists of three fields: the mantissa, the exponent, and the sign. The exponent is encoded in the field of bits 23-30 (**error en original: 23-31**) in an unsigned 127 biased form, while the sign is the most significant bit, equal to 1 if the number is negative. What is the sign and what is the exponent for a number encoded as 9B32DF4Eh.

ANSWER: *Again, we expand the word, but only the first bits to concentrate on reading the fields of bits 30-23 and 4-3:*

1	**0**	**0**	**1**	**1**	**0**	**1**	**1**	**0**	0	1	$\cdots$
b_{31}	b_{30}	b_{29}	b_{28}	b_{27}	b_{26}	b_{25}	b_{24}	b_{23}	b_{22}	b_{21}	$\cdots$

In decimal terms, the code is 00110110b = 54. *Therefore, since the bias is* 127, *we have* Exponent = 54-127 = -73.

2.4. Represent -7.34 in format F(m.4), where m is such that m+4 is the minimum number of bits to represent the data in this format. Then, represent -7.34 in format F(8.4), expressing the result in hex notation.

ANSWER: *The question can be restated in terms of the range covered with format* F(m.4) *in the following terms: What is the minimum value of* N=m+4 *such that* -7.34 *is in the interval* $[-(2^{N-1}/2^4),(2^{N-1}-1)/2^4]$?. *This means*

$$-\frac{2^{N-1}}{16} \leq -7.34 \quad 2^{N-1} \geq 117.44$$

which yields N=8 and m=4. *Therefore, we look to encode* -7.34 *in format* F(4.4).

We can work +7.34 *as follows:*

$7 \rightarrow 0111$

$0.34\times2=0.68$; $0.68\times2=1.36$; $0.36\times2=0.72$; and $0.72\times2=1.44$

Hence $+\ 7.34 \rightarrow 0111.0101$. *Taking two's complement the code becomes*

-7.34 $\rightarrow$ 10001011 = 8Bh.

Problem 3. The byte DAh is stored in memory. What does it mean if the convention being used is

3.1. unsigned normal binary notation? **ANSWER:** $13 \times 16 + 10 = 218$

3.2. signed two's complement notation? **ANSWER:** 1. $2^8 - 218 = 38$ so DA represents -38. (OR) 2. Complement for DAh is 26h and 2x16+6 = 38, so DA represents -38 (OR .. try others)

3.3. ASCII? (ASCII table below) **ANSWER:** 'Z'. (Bit 7 is irrelevant: 101 1010 = 5A)

3.4. Biased 127 notation? **ANSWER:** 218 - 127 = **91**

3.5. unsigned format F(6.2) notation? **ANSWER:** $218/2^2$=**54.5**

3.6. unsigned format F(4.4) notation? **ANSWER:** $218/2^4$=**13.625**

3.7. signed format F(6.2) notation? **ANSWER:** $-38/2^2$=**-9.5**

3.8. signed format F(4.4) notation? **ANSWER:** $-38/2^4$=**-2.375**

3.9. a real number in the continuous interval [-7,7] encoded in regular subintervals with eight bits? **ANSWER:** *The number encoded is the level* Q_{218} *calculated as* $Q_{218} = -7 + (218)\Delta$, *where* Δ *is the quantization resolution* $\Delta = (7-(-7))/2^8 = 0.0546875$. *Therefore, the encoded realnumber is* ***4.921875***.

Table 1: ASCII Code Chart

		Most Significant Digit							
		0	**1**	**2**	**3**	**4**	**5**	**6**	**7**
	0	NUL	DLE	SP	0	@	P	‘	p
	1	SOH	DC	!	1	A	Q	a	q
	2	STX	DC2	”	2	B	R	b	r
	3	ETX	DC3	#	3	C	S	c	s
	4	EOT	DC4	$	4	D	T	d	t
	5	ENQ	NAK	%	5	E	U	e	u
Least	**6**	ACK	SYN	&	6	F	V	f	v
	7	BEL	ETB	’	7	G	W	g	w
Significant	**8**	BS	CAN	(	8	H	X	h	x
	9	HT	EM	)	9	I	Y	i	y
Digit	**A**	LF	SUB	*	:	J	Z	j	z
	B	VT	ESC	+	;	K	[	k	{
	C	FF	FS	,	<	L	\	l	|
	D	CR	GS	-	=	M	]	m	}
	E	SO	RS	.	>	N	ˆ	n	˜
	F	SI	US	/	?	O	_	o	DEL

Problem 4. The continuous interval [-6,6] is encoded with regular subintervals using 6 bits.

4.1. What is the resolution? **ANSWER: 0.1875**

4.2. What is the code for -3.78 and what the quantization error?**ANSWER: Code 001011 = 0B; Error = 0.1575**

4.3. What is the minimum number of bits required for a quantization error less than 0.05 using regular subintervals?**ANSWER:**

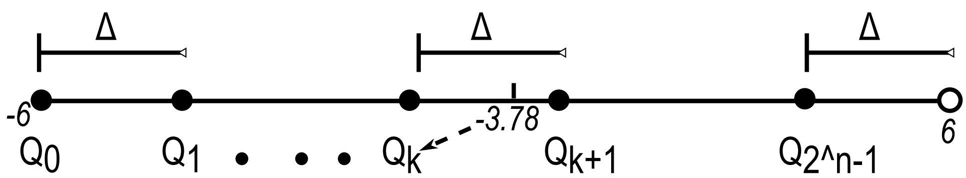
4.4. Encode again -3.78 with the new result. **ANSWER:**

Development for answers:

(4.1) For the interval [a,b] we use the definition:

$$\Delta = \frac{b-a}{2^n} = \frac{6-(-6)}{2^6} = 0.1875$$

(4.2) Working with regular subintervals, what we are looking for is the binary equivalent for k (see figure below) such that -3.78 is in the interval [Q_k, Q_{k+1}), where Q_k= -6 + $k\Delta$.



We have then

$$k = \ \text{int}\left[\frac{-3.78-(-6)}{\Delta}\right] \ = \ \text{int}\left[2^6\frac{-3.78-(-6)}{6-(-6)}\right] = \text{int}[11.84] = 11.$$

and

$$Q_{11} = -6 + 11\Delta = -3.9375$$

The error is -3.78 - Q_{11} = 0.1575.

(4.3) Since the upper bound for error is Δ, the number of bits n must satisfy

$$\Delta = \frac{12}{2^n} < 0.05$$

Problem 5. BCD (Binary Coded Decimal) or (Binary Coded Digits) coding system for unsigned integers consists in coding separately each digit. Using nibbles, the most popular one is the normal BCD code in which digits are coded by a normal binary representation. In this system, 389 is coded as 0011 1000 1001. But this is not the only possibility. Others are to use biased nibbles or different weights.

Normal BCD is also known as 8-4-2-1 BCD, because in the nibble $b_3b_2b_1b_0$, the weights assigned in the power expansion are 8, 4, 2 and 1, respectively, as in normal binary. In this code, for example, 0110 represents digit 6, since

$$0 \times 8 + 1 \times 4 + 1 \times 2 + 0 \times 1 = 6$$

. But in an 8-4-(-2)-(-1), or 84(-2)(-1) BCD coding, weights are 8, 4, -2 and -1, respectively, so 0110 represents 2 because

$$0 \times 8 + 1 \times 4 + 1 \times (-2) + 0 \times (-1) = 2$$

5.1. (4 primera columna, 6 cada otra columna) Using the weighting and bias principles for BCD codes, complete the following table:

The table has been completed. The bold characters stand for the answer:

Decimal Value	8-4-2-1 Code	84(-2)(-1) Code	Bias 3 Code
0	**0000**	**0000**	**0011**
1	**0001**	**0111**	**0100**
2	**0010**	0110	**0101**
3	**0011**	**0101**	0110
4	**0100**	**0100**	**0111**
5	**0101**	**1011**	**1000**
6	0110	**1010**	**1001**
7	**0111**	**1001**	**1010**
8	**1000**	**1000**	**1011**
9	**1001**	**1111**	**1100**

5.2. Represent 6825 in the different BCD (packed) codes mentioned in the table. Express your result both in binary and in hex notation.

Answer: BCD encoding consists in writing the code for digits individually. Packed means that we use two encoded digits (by nibbles) per byte. With this

CODE 8421 (normal): 01101000 00100101 → 6825h

CODE 84(-2)(-1) : 10101000 01101011 → A86Bh

CODE Bias 3: 10011011 01011000 → 9B58h