

Simplex Solver

February 3, 2026

Problem

Given the following linear system and objective function, find the optimal solution.

$$\begin{aligned} & \max 2x_1 + x_2 \\ \left\{ \begin{array}{l} x_1 + 6004874563154451/2251799813685248x_2 \leq 4 \\ x_1 + x_2 \leq 2 \\ 2x_1 \leq 3 \end{array} \right. \end{aligned}$$

Solution

Add slack variables to turn all inequalities to equalities.

$$\left\{ \begin{array}{l} x_1 + 6004874563154451/2251799813685248x_2 + s_1 = 4 \\ x_1 + x_2 + s_2 = 2 \\ 2x_1 + s_3 = 3 \end{array} \right.$$

Create the initial tableau of the new linear system.

$$\left[\begin{array}{ccccc|c} x_1 & x_2 & s_1 & s_2 & s_3 & b \\ \hline 1 & 6004874563154451/2251799813685248 & 1 & 0 & 0 & 4 \\ 1 & 1 & 0 & 1 & 0 & 2 \\ 2 & 0 & 0 & 0 & 1 & 3 \\ \hline -2 & -1 & 0 & 0 & 0 & 0 \end{array} \right] \begin{matrix} s_1 \\ s_2 \\ s_3 \end{matrix}$$

There are negative elements in the bottom row, so the current solution is not optimal. Thus, pivot to improve the current solution. The entering variable is x_1 and the departing variable is s_3 .

Perform elementary row operations until the pivot element is 1 and all other elements in the entering column are 0.

$$\left[\begin{array}{ccccc|c} x_1 & x_2 & s_1 & s_2 & s_3 & b \\ \hline 0 & 6004874563154451/2251799813685248 & 1 & 0 & -1/2 & 5/2 \\ 0 & 1 & 0 & 1 & -1/2 & 1/2 \\ 1 & 0 & 0 & 0 & 1/2 & 3/2 \\ \hline 0 & -1 & 0 & 0 & 1 & 3 \end{array} \right] \begin{matrix} s_1 \\ s_2 \\ x_1 \end{matrix}$$

There are negative elements in the bottom row, so the current solution is not optimal. Thus, pivot to improve the current solution. The entering variable is x_2 and the departing variable is s_2 .

Perform elementary row operations until the pivot element is 1 and all other elements in the entering column are 0.

x_1	x_2	s_1	s_2	s_3	
0	0	1	$-6004874563154451/2251799813685248$	$3753074749469203/4503599627370496$	5254124
0	1	0	1		$-1/2$
1	0	0	0		$1/2$
0	0	0	1		$1/2$

There are no negative elements in the bottom row, so we know the solution is optimal. Thus, the solution is:

$$s_1 = \frac{5254124505271789}{4503599627370496}, s_2 = 0, s_3 = 0, x_1 = \frac{3}{2}, x_2 = \frac{1}{2}, z = \frac{7}{2}$$