PLAN · LECTURE LIKELIHOOD LANDSCAPE
BAYESIAN
METROPOLIS-HASTINGS
PS11

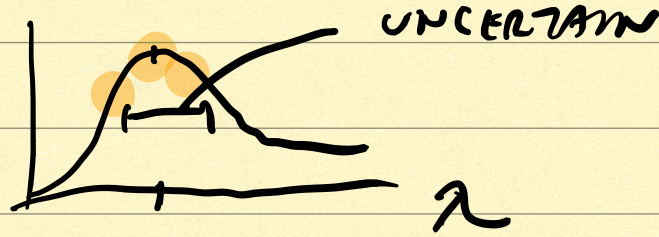
THE ENTIRE LIKELIHOOD LANDSCAPE

EX $T \sim p_T(t) = \lambda e^{-\lambda t} \quad t>0$

$= \frac{1}{\tau} e^{-t/\tau} \quad t>0$

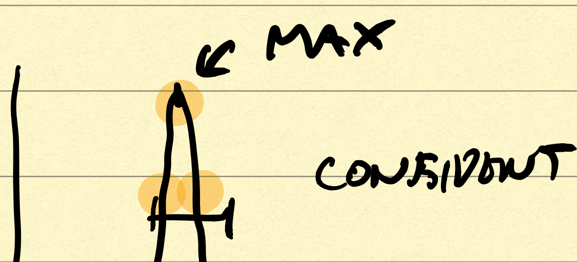
DATA: T_1

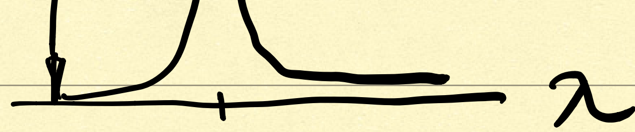
LIKELIHOOD $L(\lambda) = \lambda e^{-\lambda T_1}$



DATA: $T_1 \ldots T_N$

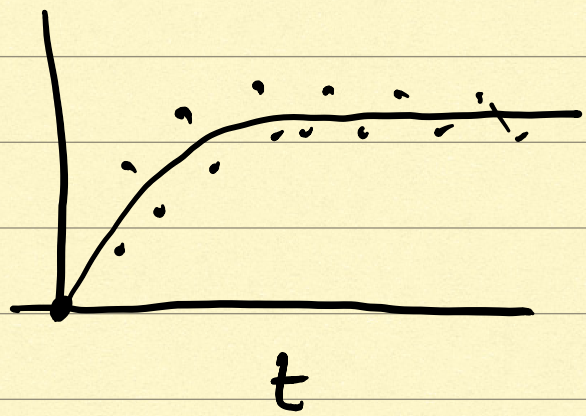
LIKELIHOOD $L(\lambda) = \lambda^N e^{-\lambda \sum T_i}$





EX $\quad \frac{dX}{dt} = k_{on} - k_{off} X \qquad X(0)=0$

$Y = X + \epsilon \qquad \epsilon \sim \text{iid NORMAL}(\sigma)$

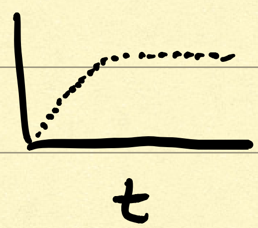


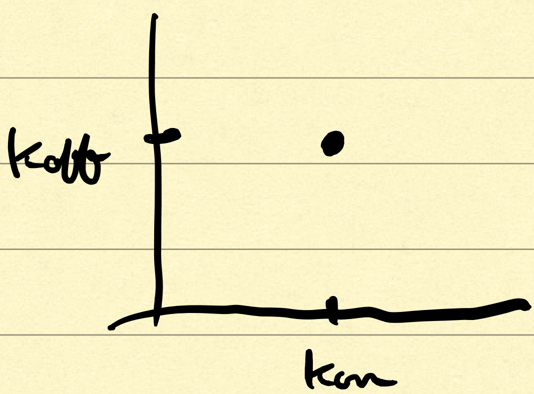
$X_{ss} = \frac{k_{on}}{k_{off}}$

$X(t) = \frac{k_{on}}{k_{off}}\left(1 - e^{-k_{off} t}\right)$

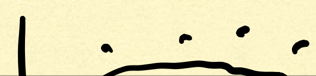
LOTS OF DATA

SMALL σ

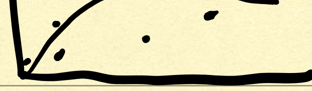


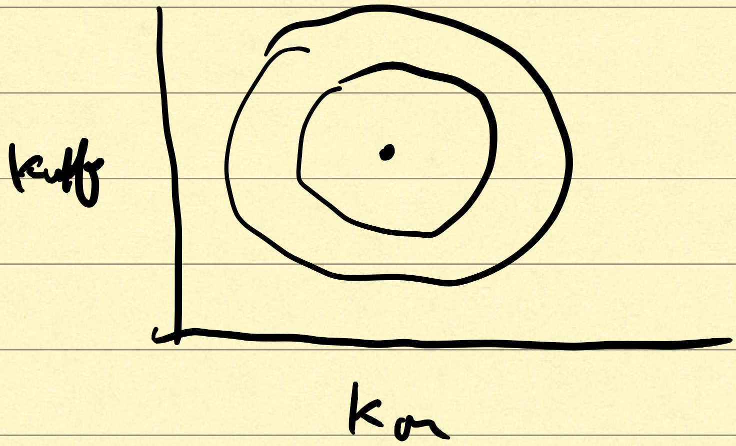


LITTLE DATA,

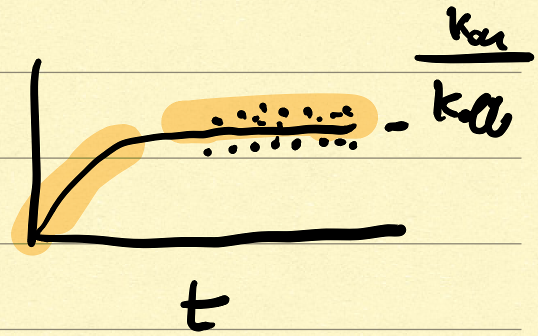


BIG σ





DATA ONLY AT LATE TIMES



UNDERCONSTRAINED!

- $X_{ss} = \frac{k_{on}}{k_{off}}$

CONFIDENCE

ISSUE: LIKELIHOOD LANDSCAPES ARE NOT PROBABILITY DISTRIBUTIONS

- NO MEAN, STANDARD DEVIATION
- NO 90% CONFIDENCE INTERVALS

POSTERIOR

$L(\Theta)$

PRIOR

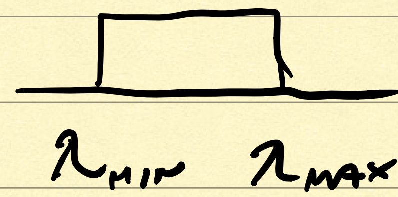
$$p(\theta|x) = \frac{p(x|\theta)\cdot p(\theta)}{p(x)}$$

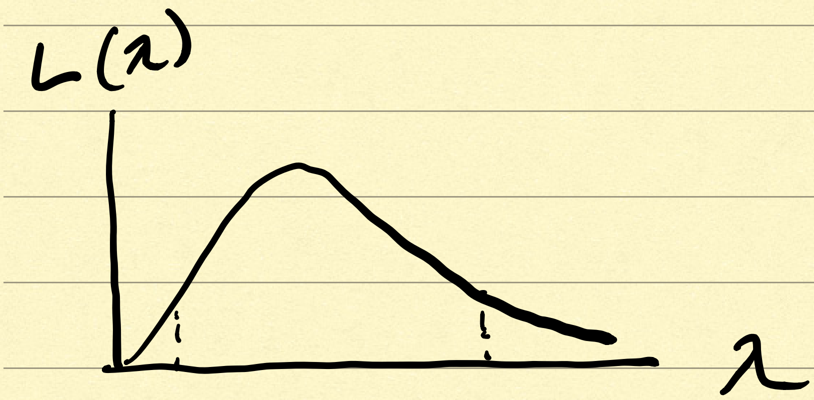
↑ KNOWN SINCE

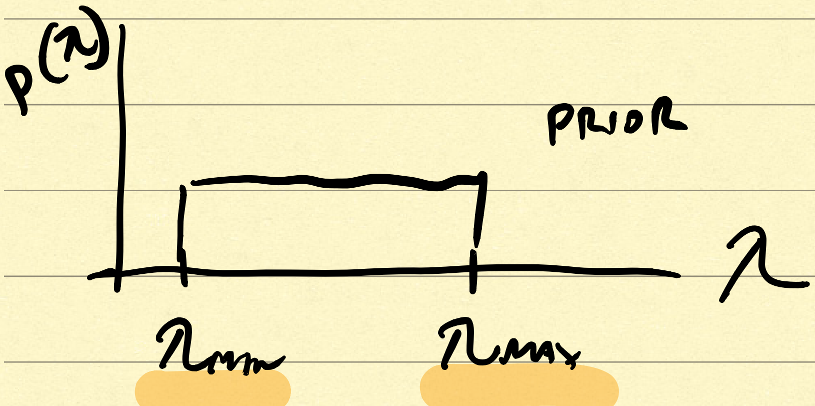
$$\int p(\theta|x)\,d\theta = 1$$

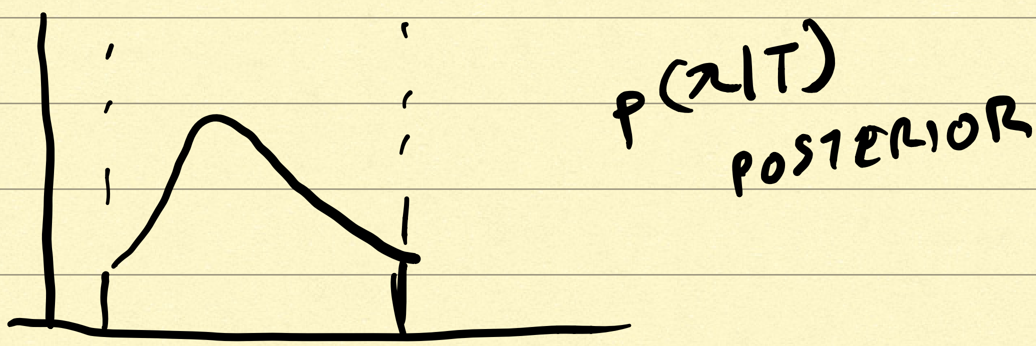
EX EXPONENTIAL $T \sim p_T(t) = \lambda e^{-\lambda t}$ $t>0$

PRIOR $\lambda \sim$ UNIFORM$(\lambda_{MIN}, \lambda_{MAX})$









NOTES

1) AS $\lambda_{min} \to 0$, $\lambda_{max} \to \infty$

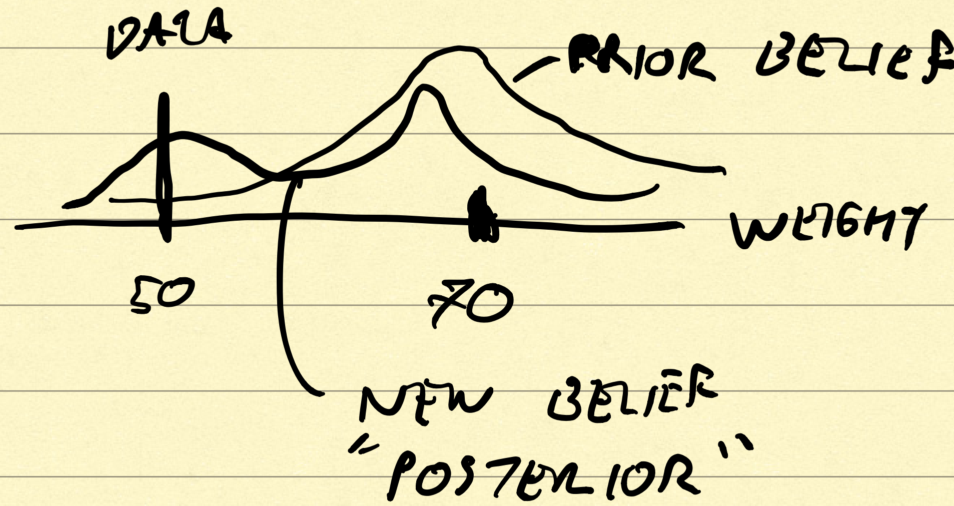
$$p(\lambda | T) \to \frac{\lambda e^{-\lambda T}}{\left(1/T^2\right)}$$

POSTERIOR IS WELL-DEFINED BUT
PRIOR UNIF$(0, \infty)$ IS NOT!

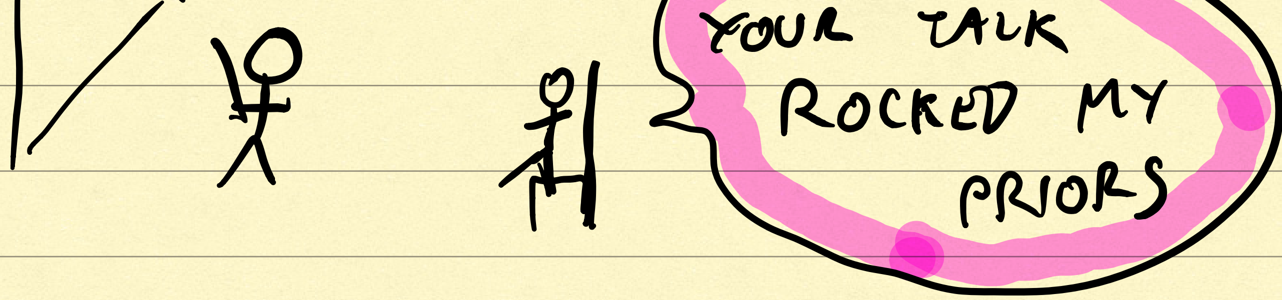
↖ IRREGULAR PRIOR

2a) NON-UNIFORM PRIOR

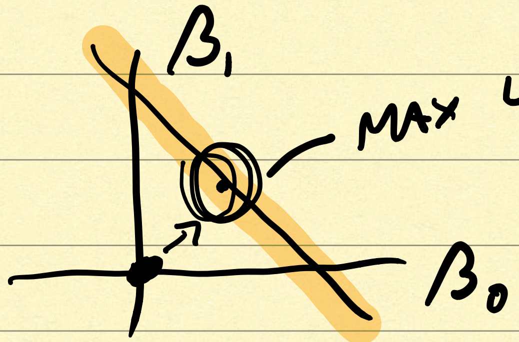
MY DOG WEIGHS 70 lbs
VET'S SCALE: 50 lbs



BAYESIAN THINKING



2b)



PRIOR: $p(\beta_0, \beta_1) = e^{-\left(\frac{\beta_0^2 + \beta_1^2}{\lambda}\right)}$

RIDGE

PRIOR: $p(\beta_0, \beta_1) = \sum e^{-\frac{|\beta_i|}{\lambda}}$

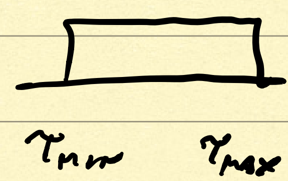
LASSO

NOTE 3 EXPONENTIAL T

$$p_\tau(t) = \frac{1}{\tau} e^{-t/\tau} \quad t > 0$$

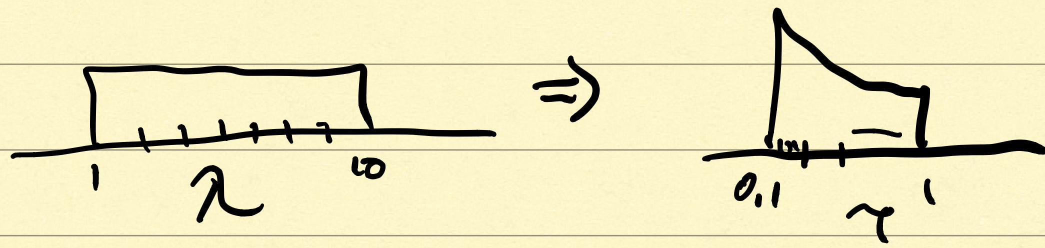
$$L(\tau) = \frac{1}{\tau} e^{-T/\tau} \quad t > 0$$

PRIOR: $p_\tau(\tau) = \begin{cases} \frac{1}{\tau_{max} - \tau_{min}} \\ 0 \end{cases}$



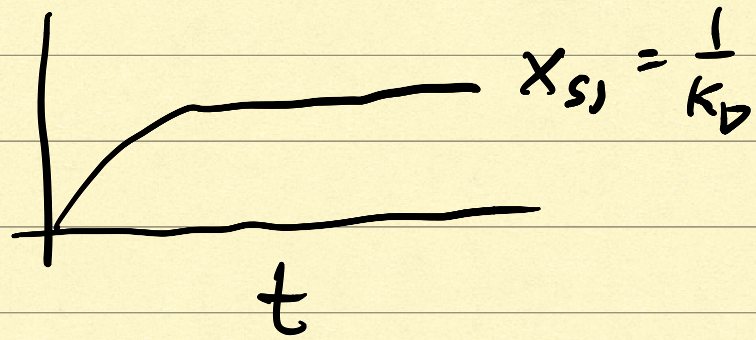
RECALL $\lambda \sim \mathrm{UNIF}(\lambda_{min}, \lambda_{max})$

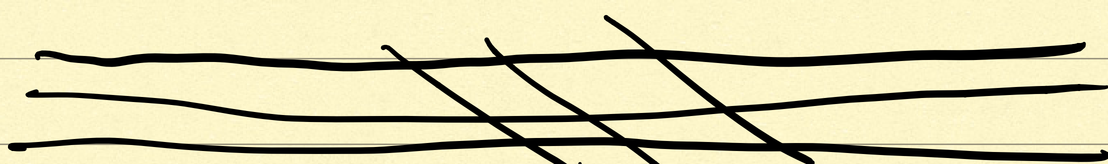
$$\tau = \frac{1}{\lambda} \Rightarrow \tau \sim p_\tau(\tau) = \left(\frac{1}{\frac{1}{\lambda_{min}} - \frac{1}{\lambda_{max}}}\right)\frac{1}{\tau^2}$$



THERE IS NO SUCH THING AS A FLAT PRIOR (?)

EX $$\frac{dX}{dt} = \frac{1}{\tau}\left(\frac{1}{K_D} - X\right) \qquad X(0) = 0$$





HOW DO WE COMPUTE THE POSTERIOR?

METROPOLIS - HASTINGS

(TYPE OF MONTE CARLO MARKOV CHAIN)

METROPOLIS - HASTINGS

PICK θ_0

FOR $i = 1 \dots i_{MAX}$

$\theta_{i+1}^{PROPOSE} = \theta_i + RAND$

IF $L(\theta_{i+1}^{PROPOSE}) > L(\theta_i)$

$\theta_{i+1} = \theta_{i+1}^{PROPOSE}$

ELSE

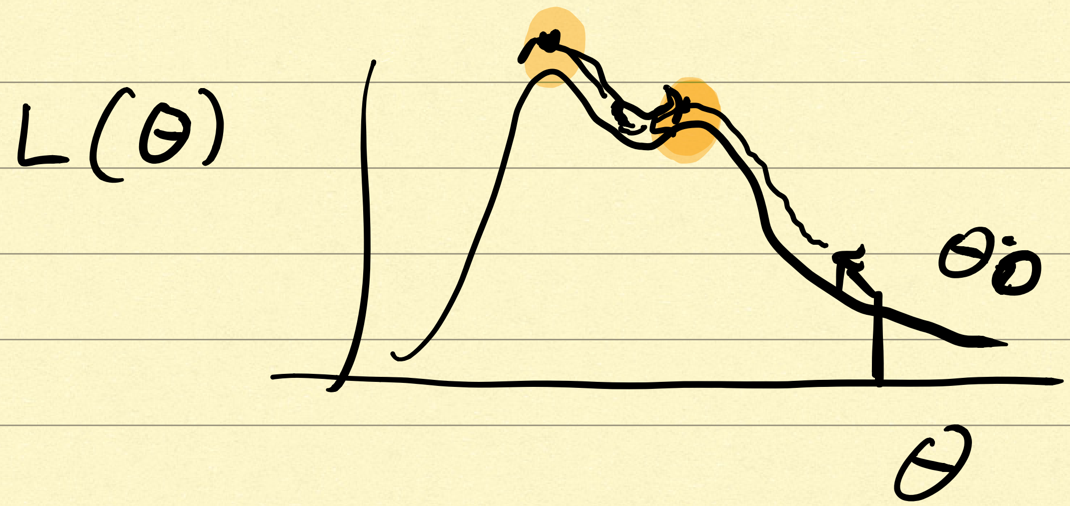
WITH PROBABILITY $L(\theta^{PROPOSE}) / L(\theta_i)$

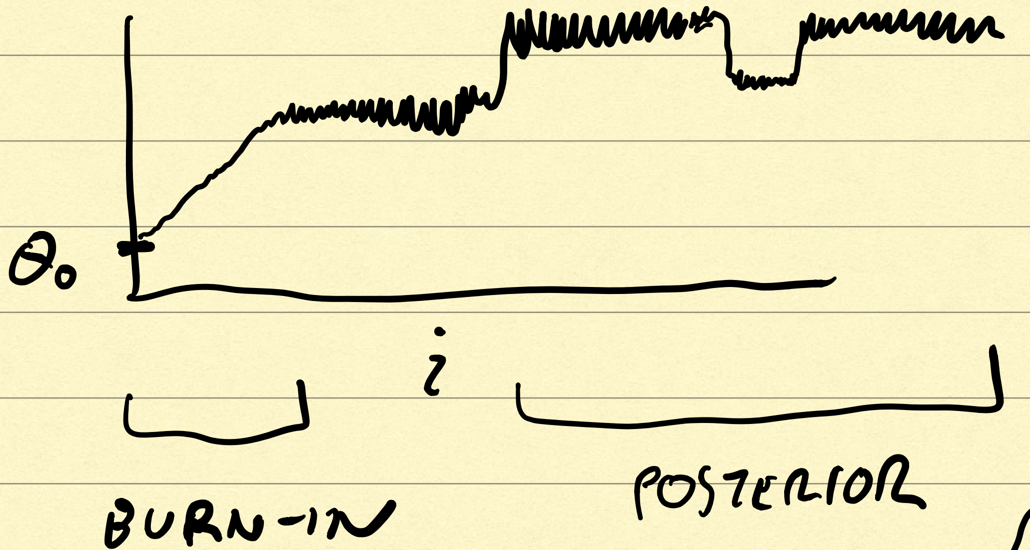
$\theta_{i+1} = \theta_{i+1}^{PROPOSE}$

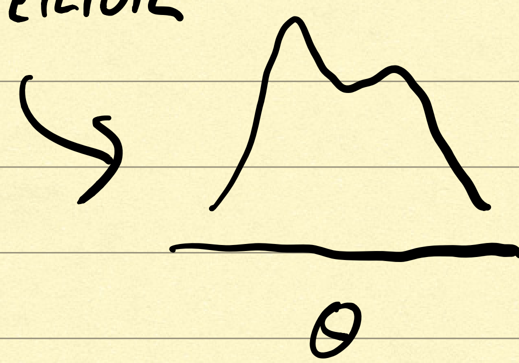
ELSE

$\theta_{i+1} = \theta_i$

END







FOR NUMERICAL HEALTH,

$$\ell(\theta) = -\log L(\theta)$$

PROBABILITY $e^{-(\mathcal{L}(\theta_i) - \mathcal{L}(\theta_{i+1}))}$