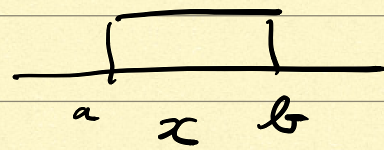
Hi!

FAMOUS CONTINUOUS RANDOM VARIABLES

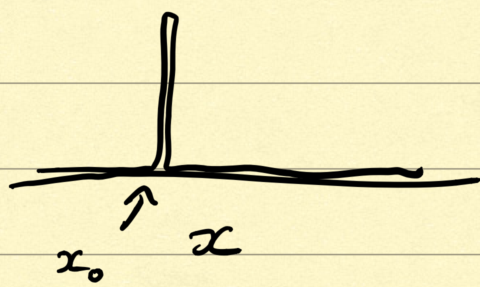
- UNIFORM $\quad X \sim \text{UNIF}(a, b)$

$$p_X(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{ELSE} \end{cases}$$



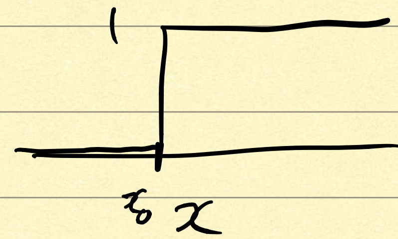
$$F_X(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a < x < b \\ 1 & \end{cases}$$

- DELTA DISTRIBUTION



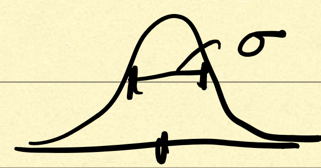
$$p_X(x) = \delta(x - x_0)$$

$$F_X(x) = \begin{cases} 0 & x < x_0 \\ 1 & x > x_0 \end{cases}$$



- GAUSSIAN / NORMAL $\quad X \sim \text{GAUSSIAN}(\mu, \sigma)$

$$p_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

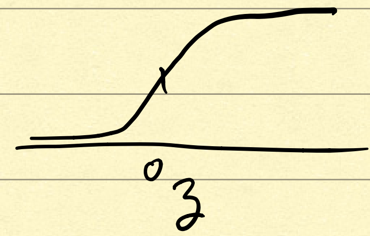


μ $\quad$ x

STANDARD NORMAL $\quad Z \sim$ STDNORM

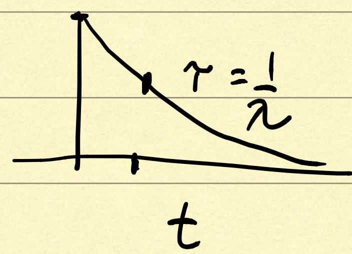
$$p_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

$$F_Z(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{\tilde{z}^2}{2}} \, d\tilde{z} = \mathrm{erf}(z)$$



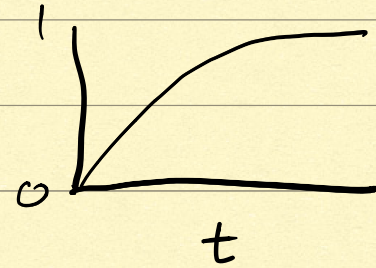
• EXPONENTIAL $\quad T \sim \mathrm{EXP}(\lambda)$

$$p_T(t) = \lambda e^{-\lambda t} \quad t > 0$$



$$p_T(t) = \frac{1}{\tau} e^{-t/\tau} \quad t > 0$$

$$F_T(t) = 1 - e^{-\lambda t}$$



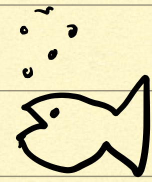
λ - RATE $\quad$ UNITS $= \frac{1}{\mathrm{min}}$

$$E[T] = \tau = \frac{1}{\lambda}$$

λ

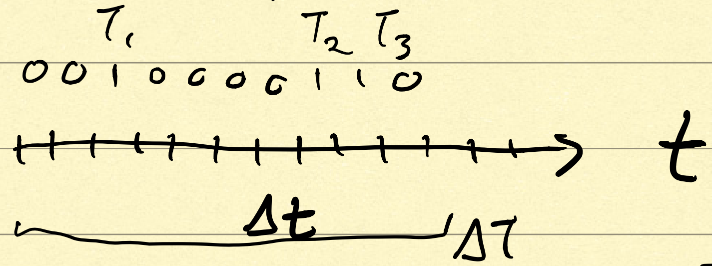
$$\sigma_T = \tau = \frac{1}{\lambda}$$

THE POISSON PROCESS



RECALL DISCRETE TIME BERNOULLI SERIES

PROBABILITY p OF SUCCESS AT EVERY TIME



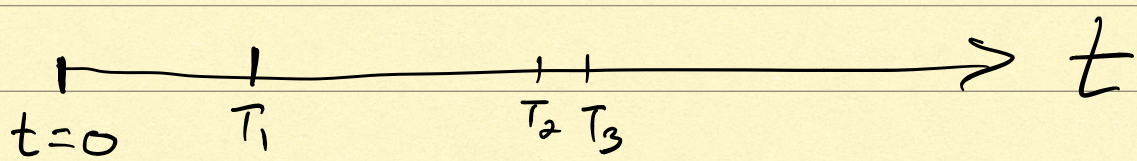
- TIME TO FIRST EVENT $T_1 \sim \text{GEOM}(p)$
- TIME BETWEEN EVENTS $T_2 - T_1 \sim \text{GEOM}(p)$
- IN A TIME INTERVAL ΔT, THE NUMBER OF EVENTS IS

$$N \sim \text{BINOMIAL}(\Delta T, p)$$

POISSON PROCESS DEFINITION #1:

A CONTINUOUS-TIME STOCHASTIC PROCESS THAT IS $p = \lambda \Delta t$

WITH $\Delta t \to 0$



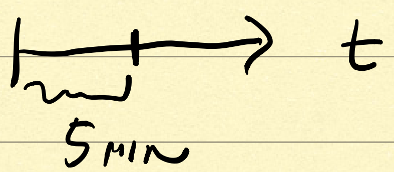
PROPERTIES

1) TIME TO FIRST EVENT $T_1 \sim EXP(\lambda)$
TIME BETWEEN EVENTS $T_2 - T_1 \sim EXP(\lambda)$

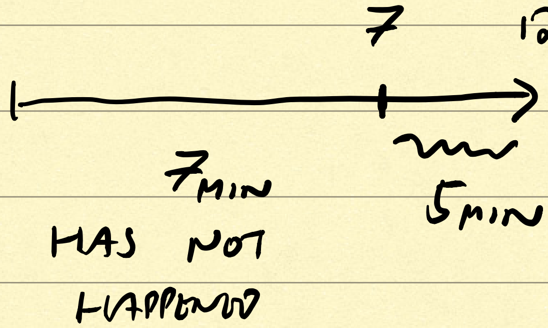
2) IN AN INTERVAL ΔT, THE NUMBER OF EVENTS N IS

$$N \sim p_N(n) = \frac{(\lambda \Delta T)^n e^{\lambda \Delta T}}{n!}$$

POISSON DISTRIBUTION

3) 

$$\mathbb{P}(T_1 < 5_{min}) = 1 - e^{-\lambda(5_{min})}$$



$$\mathbb{P}(T_1 < 12_{min} \mid T_1 > 7_{min})$$

$$= \frac{\mathbb{P}(T_1 < 12_{min} \cap T_1 > 7_{min})}{P(T_1 > 7_{min})}$$

$$= \int_{7_{min}}^{12_{min}} \lambda e^{-\lambda t} dt = -e^{-\lambda(12_{min})} + e^{-\lambda(7_{min})}$$

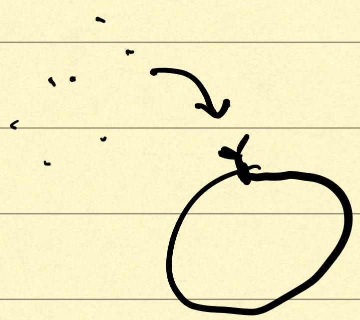
$$e^{-\lambda(7min)} \qquad\qquad e^{-\lambda 7min}$$

$$= 1 - e^{-\lambda 5min} = \mathbb{P}(T_1 < 5min)$$

MEMORYLESS PROPERTY (CONTINUOUS TIME)

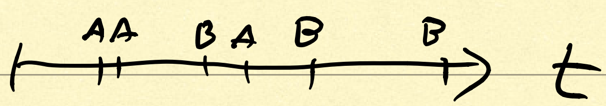
POISSON PROCESS DEFINITION #2

POISSON PROCESS IS THE UNIQUE CONTINUOUS TIME STOCHASTIC PROCESS THAT IS MEMORYLESS (UNIQUE UP TO λ).



MUTATIONS

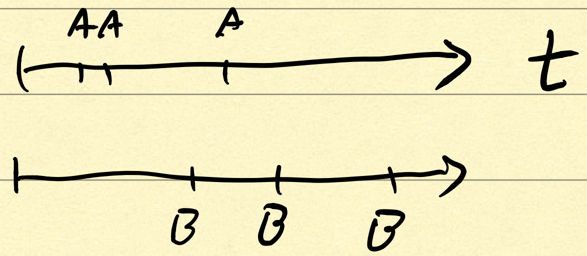
4) IF EACH EVENT IS "TYPE A" OR "TYPE B" WITH PROBABILITY p_A AND $p_B = 1 - p_A$ THEN



λ

TYPE A EVENTS ARE POISSON WITH RATE $p_A \lambda$

TYPE B EVENTS ARE POISSON WITH RATE $p_B \lambda$



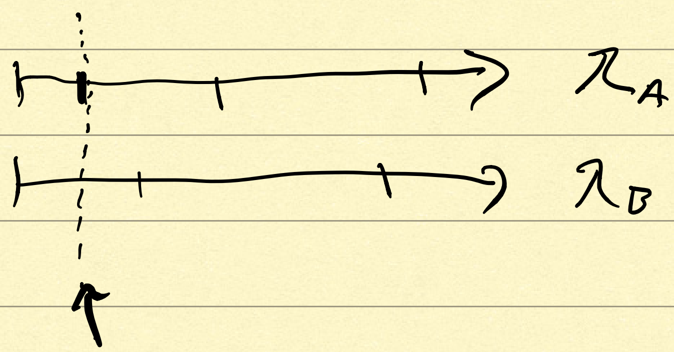
$p_A \lambda$

$p_B \lambda$

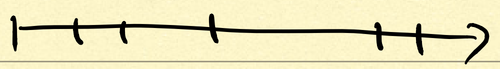
POISSON THINNING

5) TWO POISSON PROCESSES WITH RATES λ_A , λ_B

THE FIRST TO OCCUR $T_1 = \min(T_{1A}, T_{1B})$

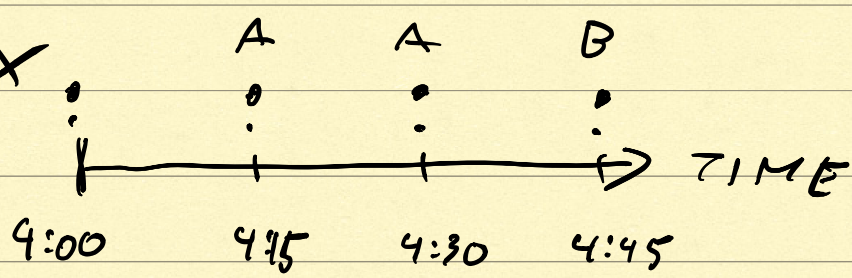


IS POISSON WITH RATE $(\lambda_A + \lambda_B)$



POISSON RACING PROPERTY

EX.



PROBLEM SET

- SUPPOSE RATE OF MUTATION $\lambda = 1\,yr^{-1}$

  WE NEED 1 OF 2 POSSIBLE MUTATIONS

  TIME TO WAIT: $T = \min(T_A, T_B)$

  $$E[T] = \frac{1}{\lambda+\lambda} = \frac{1}{2}\frac{1}{\lambda}$$

- WE NEED BOTH MUTATIONS

  TIME TO WAIT $T = \max(T_A, T_B)$

  WHAT IS $E[T]$?

MULTIPLE CHOICE

1. 6x $\frac{1}{\lambda}$
2. 5x $\frac{2}{\lambda}$
3. 0x $1.5\,\frac{1}{\lambda}$
4. 4x $\left(\frac{1}{\lambda}\right)^2$

OTHER 2x