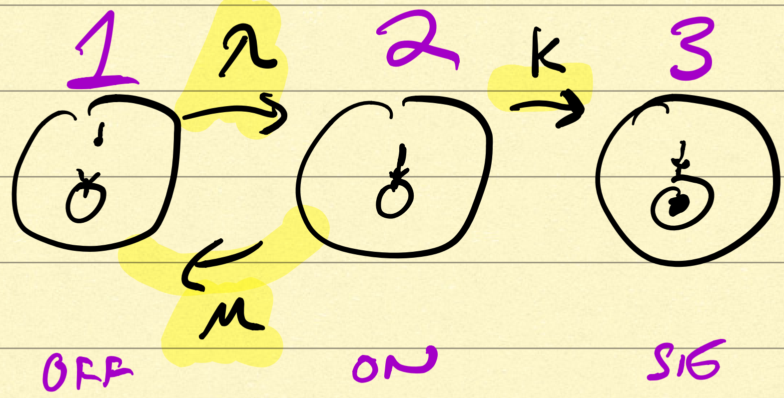
Hi!

PS 5 B ii



$$\frac{d\vec{\mathbb{P}}}{dt} = M \cdot \vec{\mathbb{P}}$$

$$M = \begin{bmatrix} -\lambda & +\mu & 0 \\ +\lambda & -(\mu + k) & 0 \\ & +k & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -k & 0 \\ 0 & +k & 0 \end{bmatrix}$$

$$\downarrow$$

$T_{2\to 3} = ?$

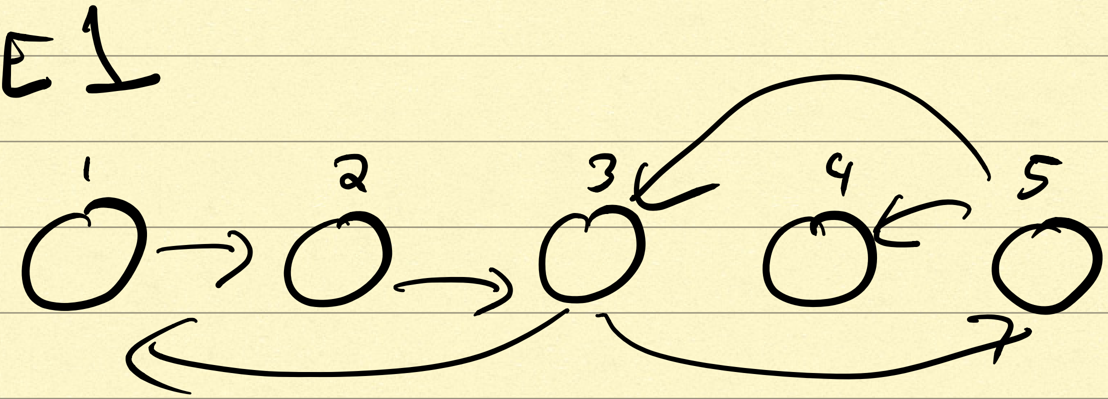
$$M_{-3} = \begin{bmatrix} -\lambda & +\mu \\ +\lambda & -(\mu+k) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -k \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -\lambda & +\mu \\ +\lambda & -(\mu+k) \end{bmatrix} \cdot \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \quad \vdots$$

$\ldots \quad T_1 , \; T_2$

$$T_{2\to 3} = T_1 + T_2 = \boxed{\frac{1}{k}\left(1 + \frac{\mu}{\lambda}\right)}$$

ASIDE 1



$$T_{3 \to 5}$$

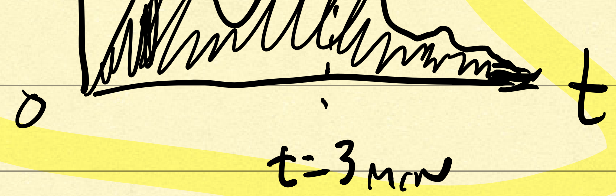
$$M_{-5} = \begin{bmatrix} & \\ & \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} = M_{-5} \cdot \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix}$$

$$T_{3 \to 5} = T_1 + T_2 + T_3 + T_4$$

ASIDE 2

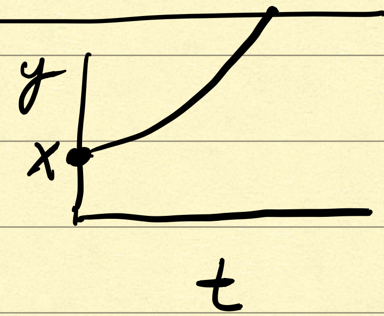
$\mathbb{P}_i(t)$



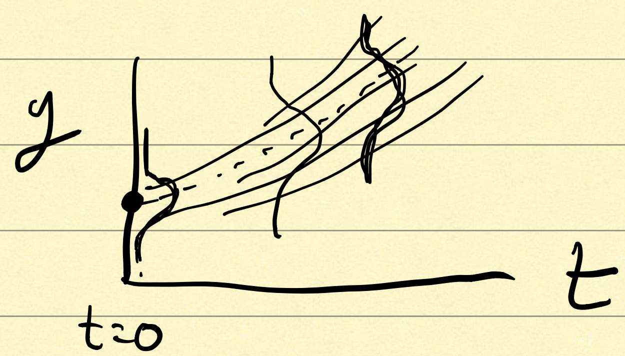
$$\int_0^\infty \mathbb{P}_i(t)\,dt$$

PARAMETRIC NOISE ~ HETEROGENEITY

EX $\quad y' = Ay \quad y(0) = X$



SUPPOSE $X \sim p_X(x)$



$y(t) = Xe^{At}$

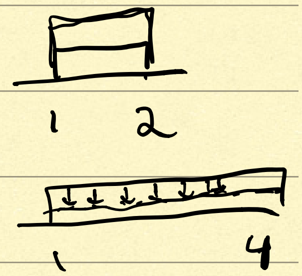
WHAT IS $y \sim p_y(y)$?

GIVEN A RANDOM VARIABLE $X \sim p_X(x)$,

SUPPOSE $Y = g(X)$, THEN WHAT IS $p_Y(y)$?

SUPPOSE $Y = cX$

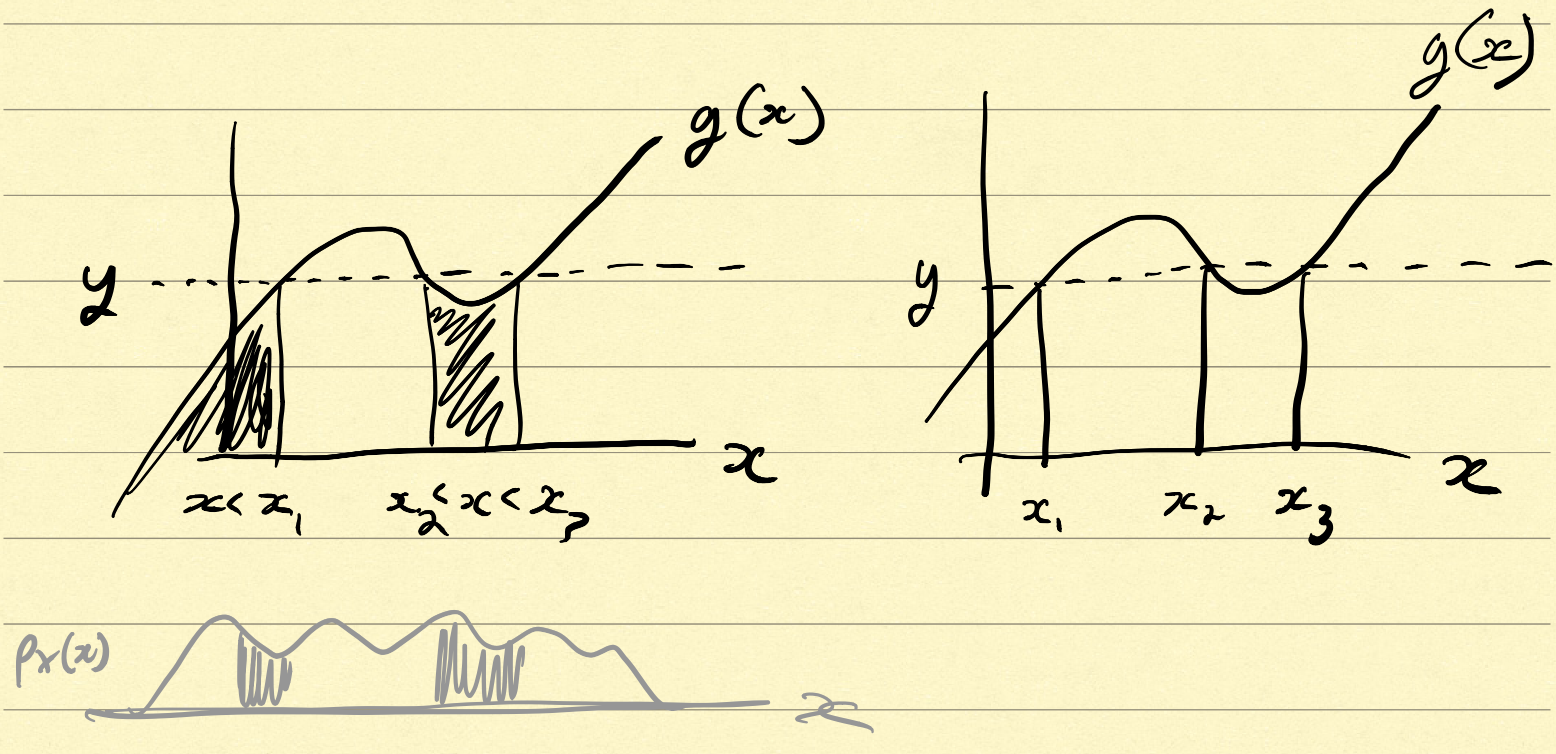
~~NOT~~ $p_Y(y) = c \cdot p_X(x)$



FOR A GENERAL FUNCTION $g(x)$,
DEFINE

$$I_y = \{x : g(x) \le y\}$$ INDEX SET OF y

$$\{x_k\} = \{x : g(x) = y\}$$ PRE-IMAGE OF y

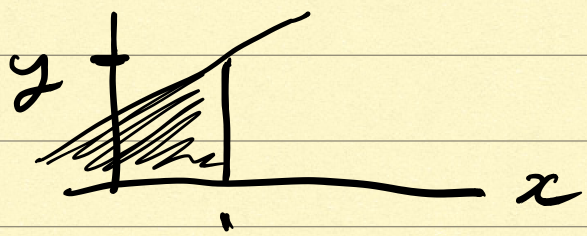


$$F_Y(y) = \mathbb{P}(Y \leq y)$$

$$= \mathbb{P}(x \in I_y) = \int_{I_y} p_x(x)\,dx$$

EX $\quad y = ax + b$

$a > 0$



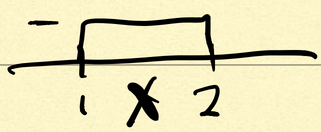
$x = \dfrac{y-b}{a}$

$$I_y = \left\{ x \leq \frac{y-b}{a} \right\}$$

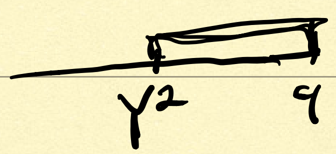
GIVEN $\quad F_x(x)$ THEN

$$F_Y(y) = F_x\left(\frac{y-b}{a}\right)$$

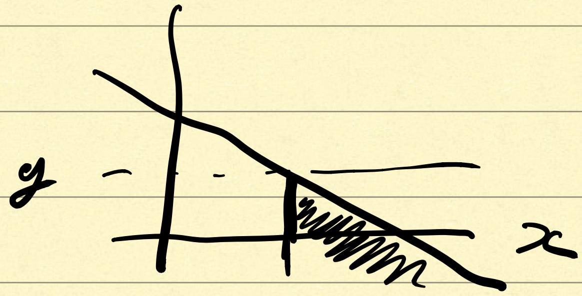
$$p_Y(y) = \frac{d}{dy} F_Y(y) = \frac{1}{a}\, p_x\left(\frac{y-b}{a}\right)$$



$Y = 2X$



EX $a < 0$



$x = \frac{y-b}{a}$

$$I_y = \left\{ x \geq \frac{y-b}{a} \right\}$$

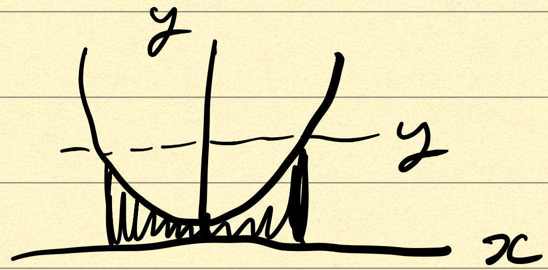
$$F_Y(y) = 1 - F_X\left(\frac{y-b}{a}\right)$$

$$p_Y(y) = \frac{-1}{a}\, p_X\left(\frac{y-b}{a}\right)$$

$$p_Y(y) = \left|\frac{1}{a}\right| p_X\left(\frac{y-b}{a}\right) \qquad a \in \mathbb{R}$$

EX $Y = X^2$

$g(x) = x^2$



$-\sqrt{y}$ $\quad$ $+\sqrt{y}$

$$I_y = \begin{cases} (-\sqrt{y}, +\sqrt{y}) & y \geq 0 \\ \emptyset & y < 0 \end{cases}$$

$$F_Y(y) = \begin{cases} F_X(\sqrt{y}) - F_X(-\sqrt{y}) & y \geq 0 \\ 0 & y < 0 \end{cases}$$

$$p_Y(y) = \begin{cases} p_X(\sqrt{y}) \cdot \frac{1}{2\sqrt{y}} + p_X(-\sqrt{y}) \frac{1}{2\sqrt{y}} & y \geq 0 \\ 0 & y < 0 \end{cases}$$

GENERAL FORMULA

LET $X \sim p_X(x)$ AND $Y = g(X)$

$$p_Y(y) = \sum_k p_X(x_k(y)) \cdot \left| \frac{dx_k}{dy} \right|$$

WHERE $\{x_k\}$ IS THE PRE-IMAGE OF g

IF $\frac{dg}{dx} = 0$,

$$p_Y(y) = \sum_k p_X(g_k^{-1}(y)) \cdot \left(\left| \frac{dg}{dx} \right| \right)^{-1}$$