

MEMORYLESS PROPERTY

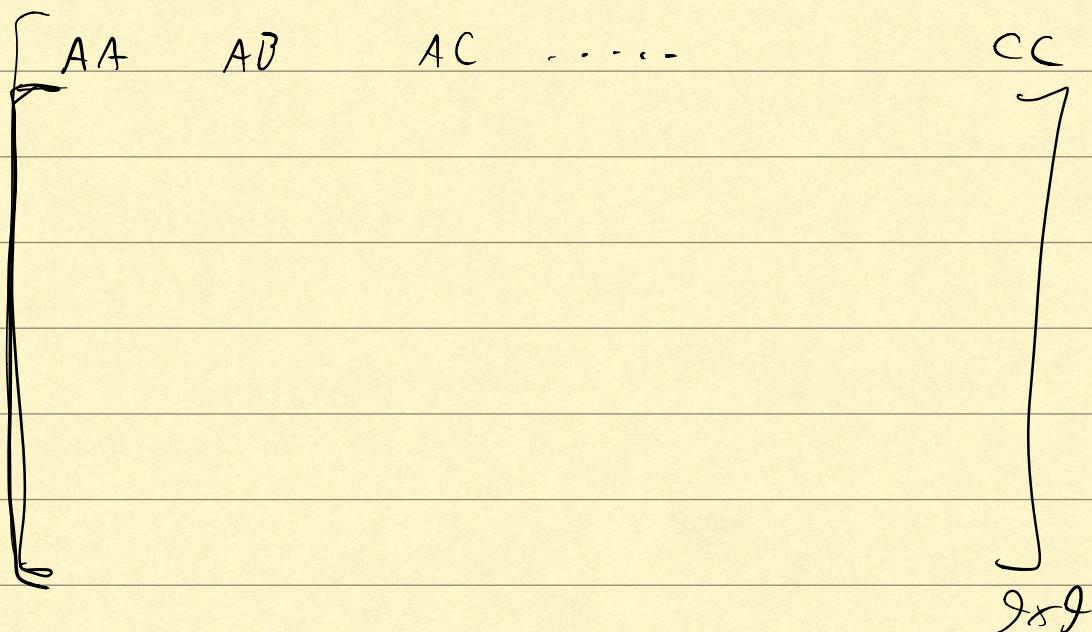
$$P(X_{n+1} | X_n, X_{n-1}, X_{n-2}, \dots) = P(X_{n+1} | X_n)$$



EX 3-STATE THAT HAD "HISTORY" OF 2

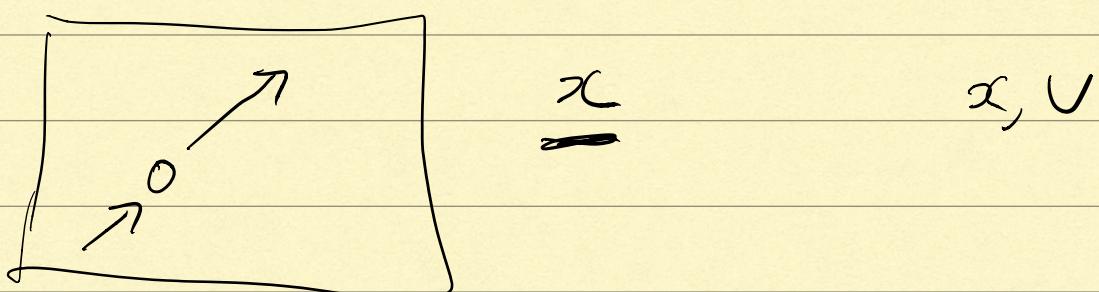
$$P(X_{n+1} | X_n, X_{n-1})$$

DEFINE 9-STATE



~~REMARK~~

EX. BILLIARD TABLE



EX. MORE MEMORYLESS THAN MARKOV

H H H T T T T T T


S = (# UMBRELLAS AT LAB, # AT HOME)

S = (# UMBRELLAS WITH BIOLOGIST) 

CONTINUOUS RANDOM VARIABLES

$X \in S$

\mathbb{P}, P

Ex $S \in [0, 1]$

$S \in (-\infty, \infty)$

$S \in [0, \infty)$

has units

DENSITY FUNCTION $p_x(x)$

such that $\mathbb{P}(A) = \int p_x(x) dx$

↑
EVENT
A

CUMULATIVE DISTRIBUTION

$$\mathbb{P}(X \leq x) = F_x(x)$$

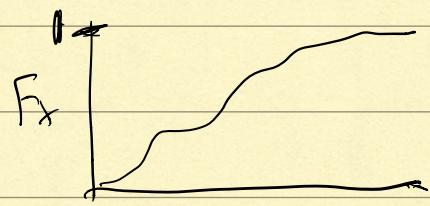
FROM Axioms

$$\int_{x \in S} p_x(x) dx = 1$$

$$F_x(x) = 0 \quad \text{AS } x \rightarrow -\infty$$

$$F_x(x) = 1 \quad \text{AS } x \rightarrow +\infty$$

F_x is non-decreasing

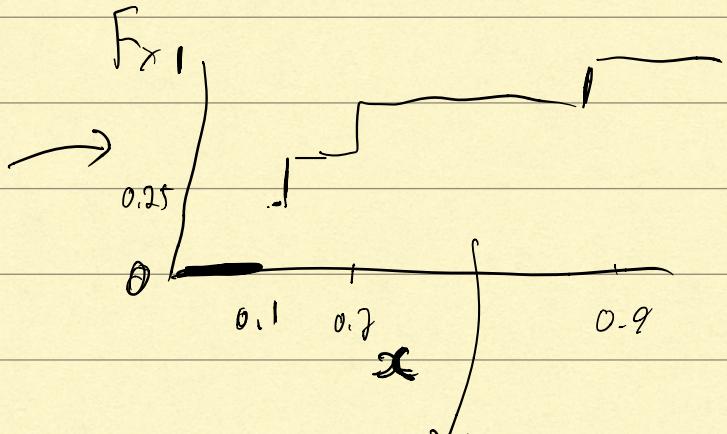


$$p_x(x) = \frac{d}{dx} F_x(x)$$

$$F_x(x) = \int_{-\infty}^x p_x(\tilde{x}) d\tilde{x}$$

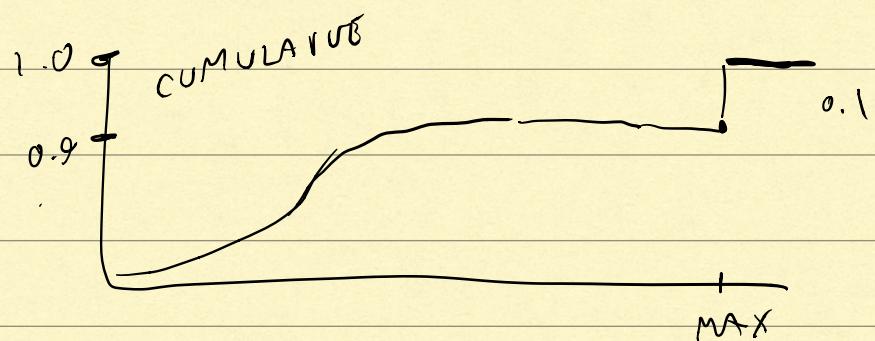
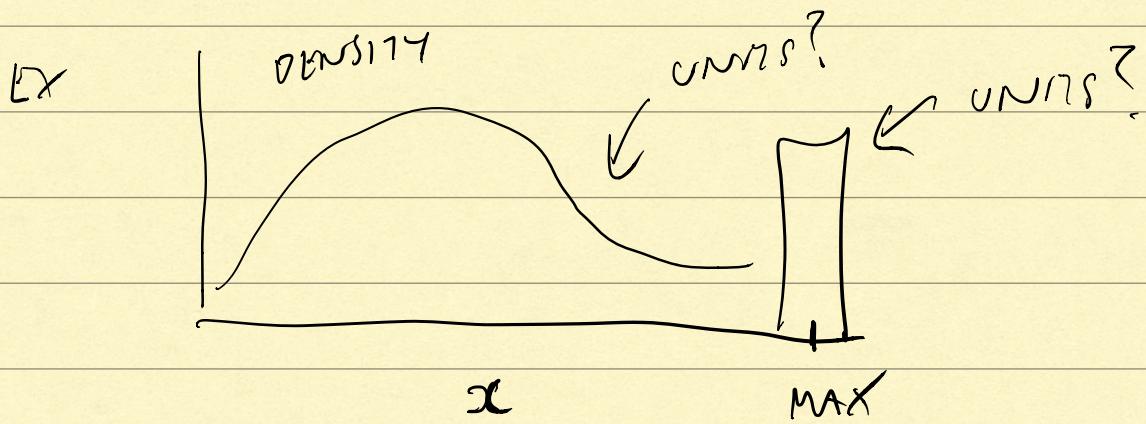
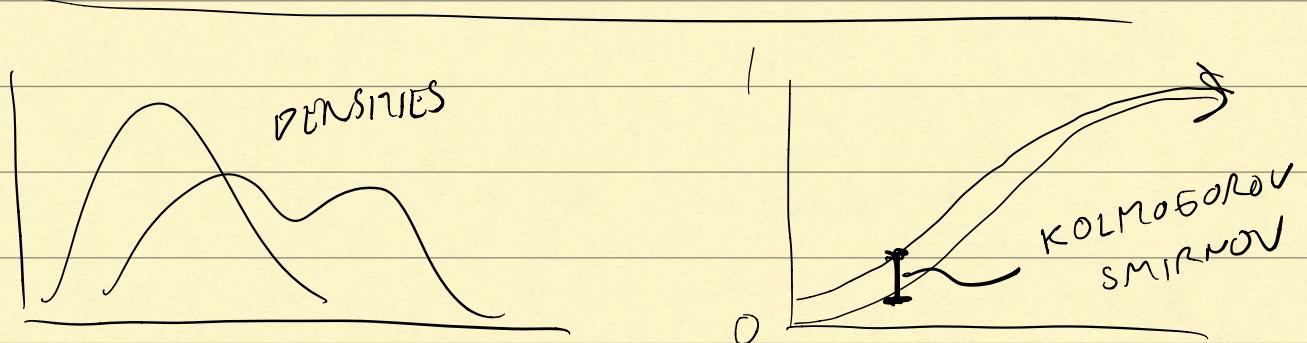
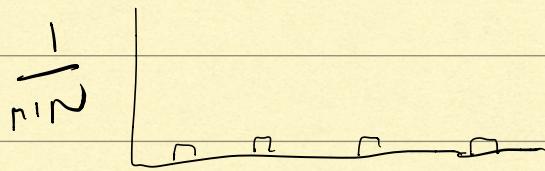
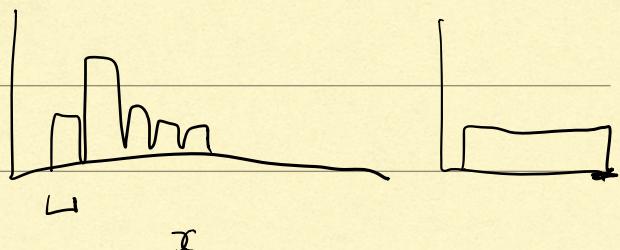
CUMULATIVE

DATA $X = \begin{bmatrix} 0.1 \\ 0.11 \\ 0.9 \\ 0.2 \end{bmatrix}$



EMPIRICAL

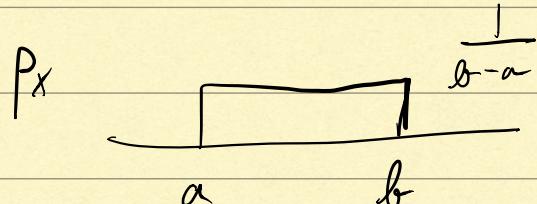
DATA $X \rightarrow$ DENSITY



FAMOUS CONTINUOUS RANDOM VARIABLES

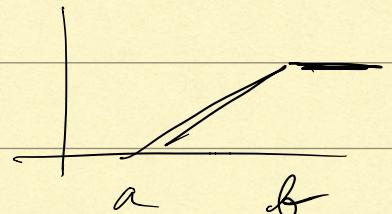
- UNIFORM

$$X \sim \text{UNIF}(a, b)$$

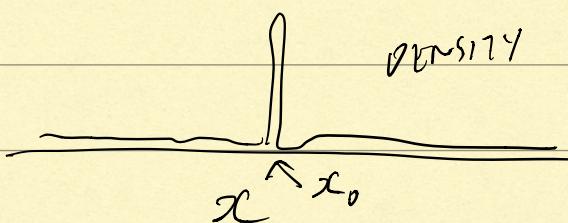


$$P_X(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

$$F_X(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \leq x < b \\ 1 & b \leq x \end{cases}$$

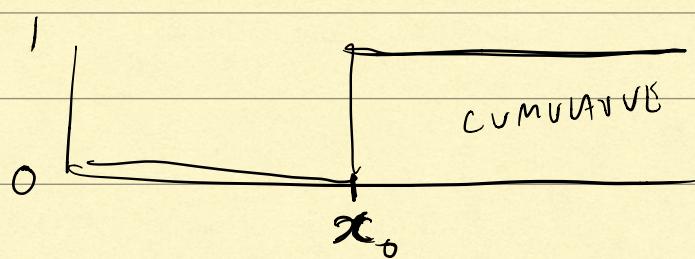


- DELTA DISTRIBUTION



$$P_X(x) = \delta(x - x_0)$$

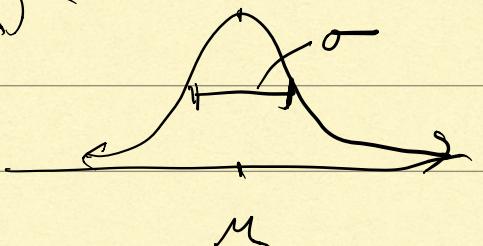
$$F_X(x) = \begin{cases} 0 & x < x_0 \\ 1 & x \leq x_0 \end{cases}$$



- GAUSSIAN / NORMAL

$$X \sim \text{GAUSSIAN}(\mu, \sigma)$$

$$p_x(x) = \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



$Z \sim \text{STANDARD NORMAL } \mu=0, \sigma=1$

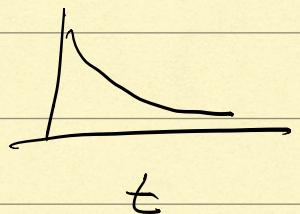
$$F_x(x) =$$

$$= \operatorname{erf}(x)$$

• EXPONENTIAL

$$T \sim \text{EXP}(\lambda)$$

$$p_T(t) = \lambda e^{-\lambda t} \quad t > 0$$



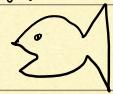
$$p_T(t) = \frac{1}{\gamma} e^{-t/\gamma} \quad \lambda = \frac{1}{\gamma}$$

$$F_T(t) = 1 - e^{-\lambda t}$$

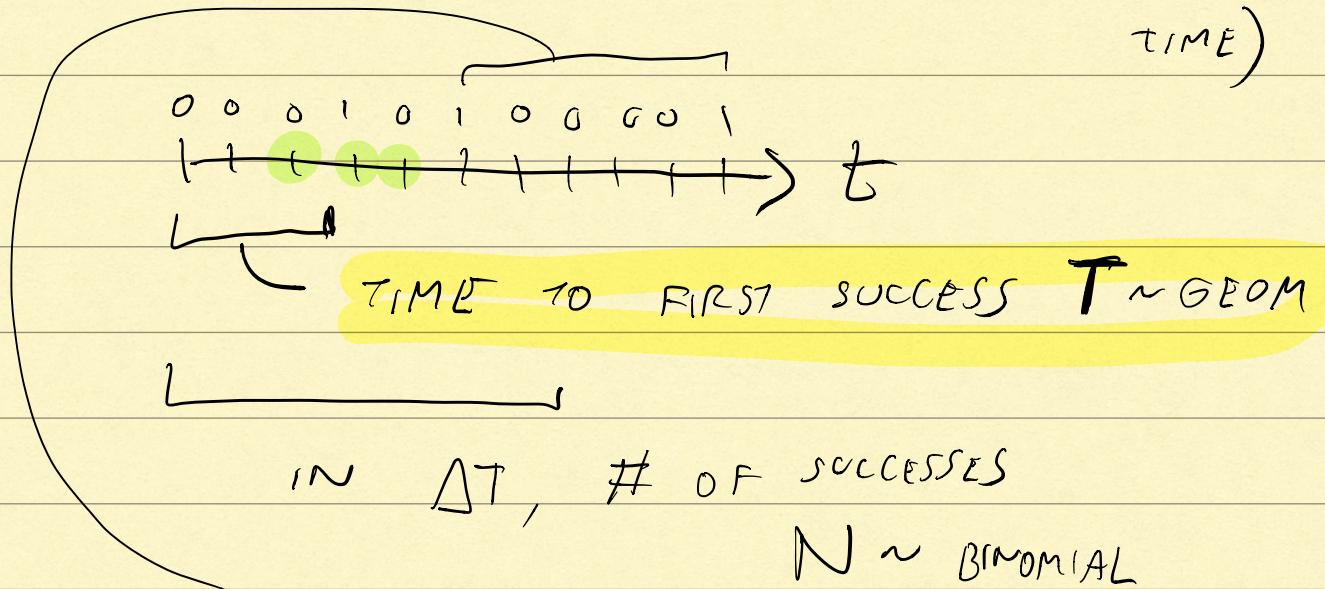
↓
RATE

$$E[T] = \gamma = \frac{1}{\lambda} \quad \gamma - \text{MEAN}$$

POISSON PROCESS



RECALL A BERNoulli TRIAL SERIES (DISCRETE TIME)



$N \sim \text{BINOMIAL}$

TIME BETWEEN SUCCESSES $T \sim \text{GEOM}$

A POISSON PROCESS IS

A CONTINUOUS TIME STOCHASTIC PROCESS

THAT IS THE LIMIT OF A

BERNOULLI TRIAL SERIES WITH

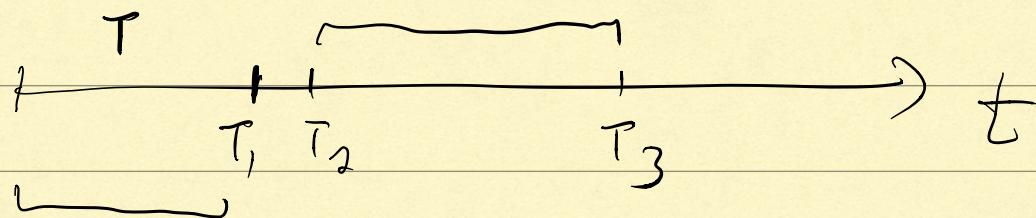
TIME $\Delta t \rightarrow 0$ AND

$$\cancel{p\Delta t = \lambda \text{ constant}}$$

$$p = \lambda \Delta t$$

↓ CONSTANT

λ UNITS $\sqrt{\text{TIME}}$



TIME TO FIRST EVENT $T \sim \text{Exp}(\lambda)$

TIME BETWEEN EVENTS $T \sim \text{Exp}(\lambda)$

IF EVENTS IN A PERIOD ΔT

IS $N \sim \text{Poisson}(\lambda \Delta T)$

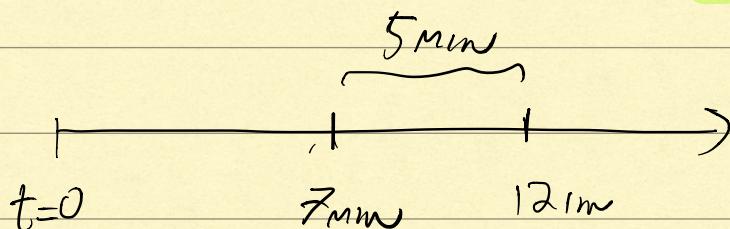
$$p_N(i) = \frac{(\lambda \Delta T)^i e^{-\lambda \Delta T}}{i!} \quad i \geq 0$$

PROPERTIES

- WAITING FOR EVENT

$P(T_1 < 5_{\text{min}})$

$$= 1 - e^{-\lambda \cdot 5_{\text{min}}}$$



$$P(T_1 < 12_{\text{min}} \mid T_1 > 7_{\text{min}})$$

$$= \frac{\mathbb{P}(T_1 < 12_{\min} \cap T_1 > 7_{\min})}{\mathbb{P}(T_1 > 7_{\min})}$$

$$= \frac{\int_{7_{\min}}^{12_{\min}} \lambda e^{-\lambda t} dt}{1 - (1 - e^{-\lambda \cdot 7_{\min}})}$$

$$= 1 - e^{-\lambda \cdot 5_{\min}} = \mathbb{P}(T_1 < 5_{\min})$$

MEMORYLESS PROPERTY (CONTINUOUS TIME)

A POISSON PROCESS IS THE UNIQUE
MEMORYLESS CONTINUOUS TIME STOCHASTIC
PROCESS

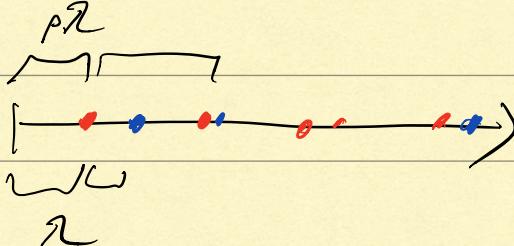
* IF EVENT A IS POISSON WITH RATE λ_A
EVENT B IS POISSON WITH RATE λ_B

THEN TIME TO FIRST EVENT IS POISSON
WITH RATE $\lambda = \lambda_A + \lambda_B$

RACING PROPERTY

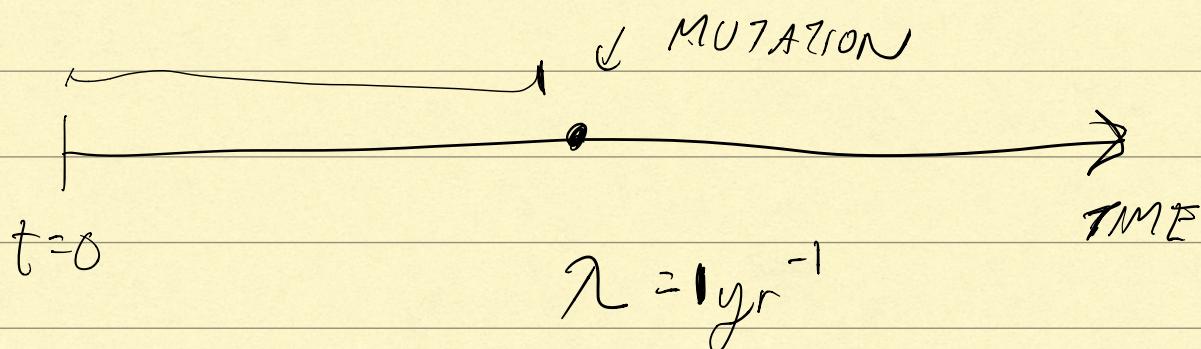
• ONE POISSON PROCESS WITH RATE λ
 WITH EACH EVENT TYPE A, OR TYPE B
 WITH PROBABILITY p , $(1-p)$, INDEPENDENT,
 THEN

TYPE A IS POISSON WITH RATE $p\lambda$
 TYPE B IS POISSON WITH RATE $(1-p)\lambda$



POISSON
THINNING
PROPERTY

MUTATION



CASE 1: SUPPOSE WE WAIT FOR 1 OR
 2 POSSIBLE MUTATIONS

$$E[\text{FIRST MUTANTS}] = ?$$

THIS IS POISSON WITH RATE 2λ

$$E[\text{FIRST MUTANTS}] = \frac{1}{2\lambda}$$

CASE 2: SUPPOSE WE WAIT FOR BOTH
MUTATIONS