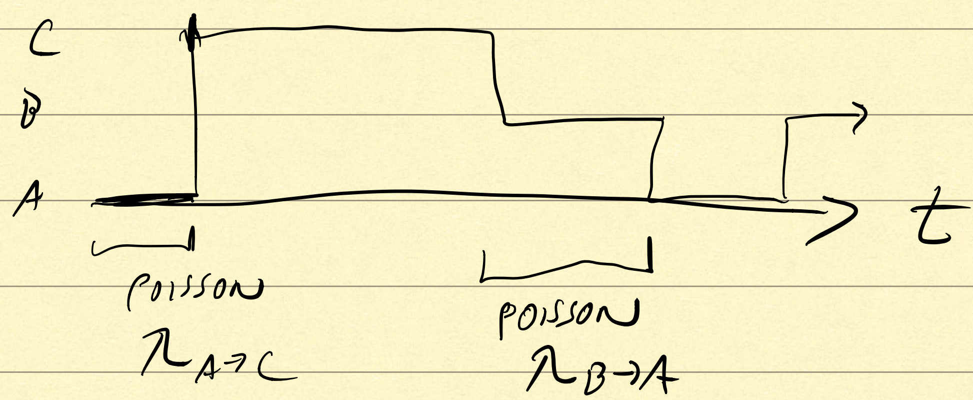
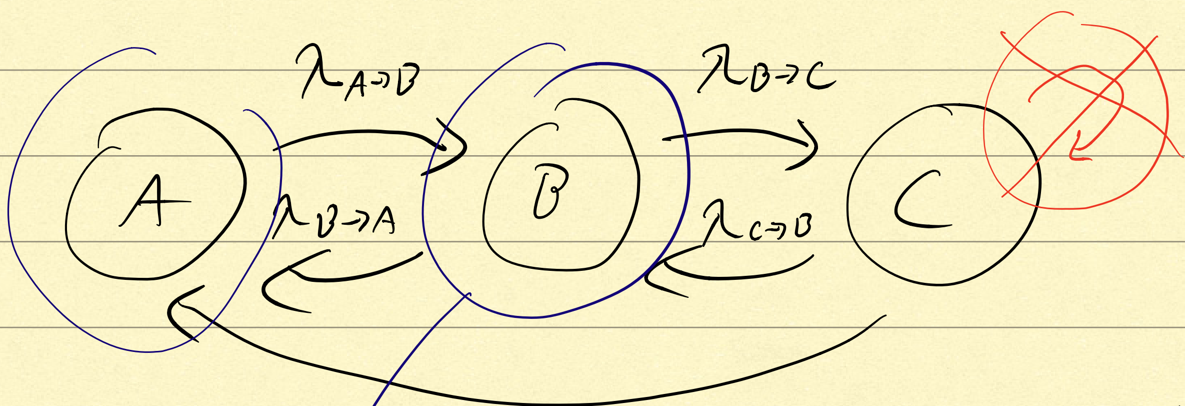
# CONTINUOUS TIME MARKOV CHAINS

CONTINUOUS TIME STOCHASTIC PROCESS WHERE STATE SPACE HAS $N$ DISCRETE STATES, AND TRANSITIONS BETWEEN STATES ARE POISSON.




$\lambda_{C \to A} \longrightarrow$ UNITS $(s^{-1})$

$B \to C \quad \lambda_{B \to C}$

$B \to A \quad \lambda_{B \to A}$

NEXT EVENT $B \to X$ POISSON RATE $\lambda_{B \to C}$

$$+\lambda_{B\to A}$$

PROBABILITY $B \to C$ : $\dfrac{\lambda_{B\to C}}{\lambda_{B\to C} + \lambda_{B\to A}}$

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$\mathbb{P}_i(t)$ - PROBABILITY OF BEING IN STATE $i$ AT TIME $t$

LET $\vec{\mathbb{P}}(t) = \begin{bmatrix} \mathbb{P}_1(t) \\ \vdots \\ \mathbb{P}_N(t) \end{bmatrix}$ THEN

$$\frac{d}{dt}\vec{\mathbb{P}}(t) = \begin{bmatrix} -\sum_i \lambda_{1\to i} & & \lambda_{N\to 1} \\ \lambda_{1\to 2} & \ddots & \\ \vdots & & \\ \lambda_{1\to N} & & -\sum_i \lambda_{N\to i} \end{bmatrix} \cdot \vec{\mathbb{P}}(t)$$

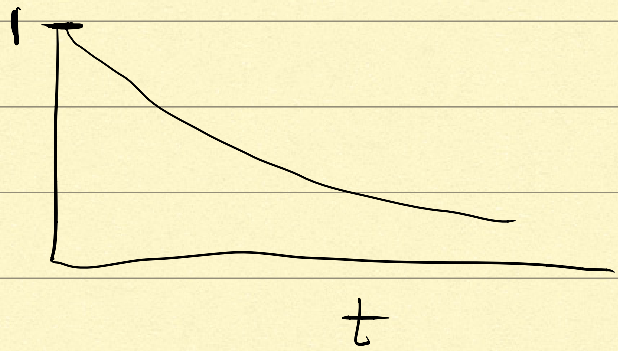
EX $N(t)$ - # OF POISSON EVENTS AT TIME $t$

$$\text{MATRIX} = \begin{bmatrix} -\lambda & 0 & 0 & \ddots \\ \lambda & -\lambda & 0 & \\ 0 & \lambda & -\lambda & \end{bmatrix}$$

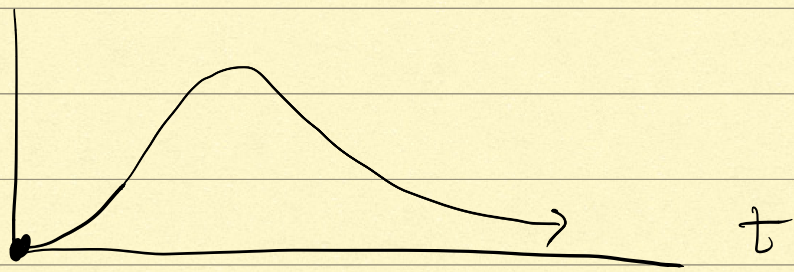
$$\begin{bmatrix} 0 & 0 & \lambda & -\lambda & \\ 0 & 0 & 0 & \lambda & \ddots \end{bmatrix}$$

$$\vec{\mathbb{P}}(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

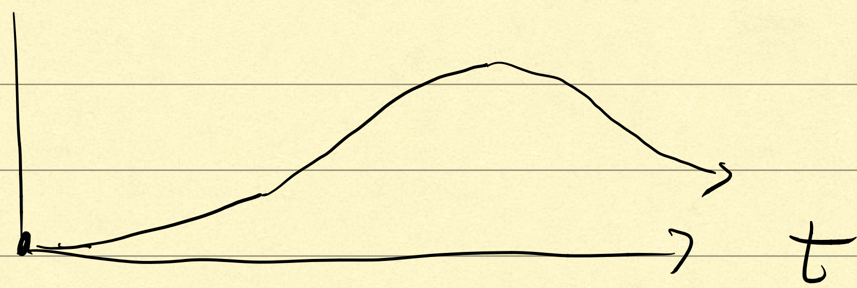
$\mathbb{P}_0$



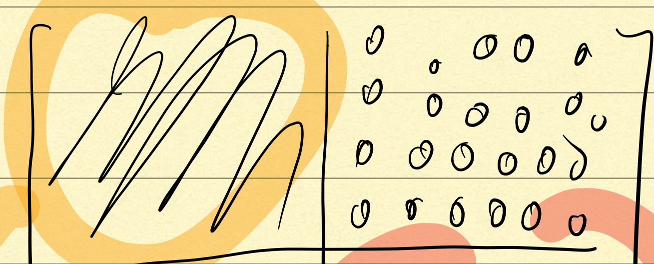

$\mathbb{P}_1$

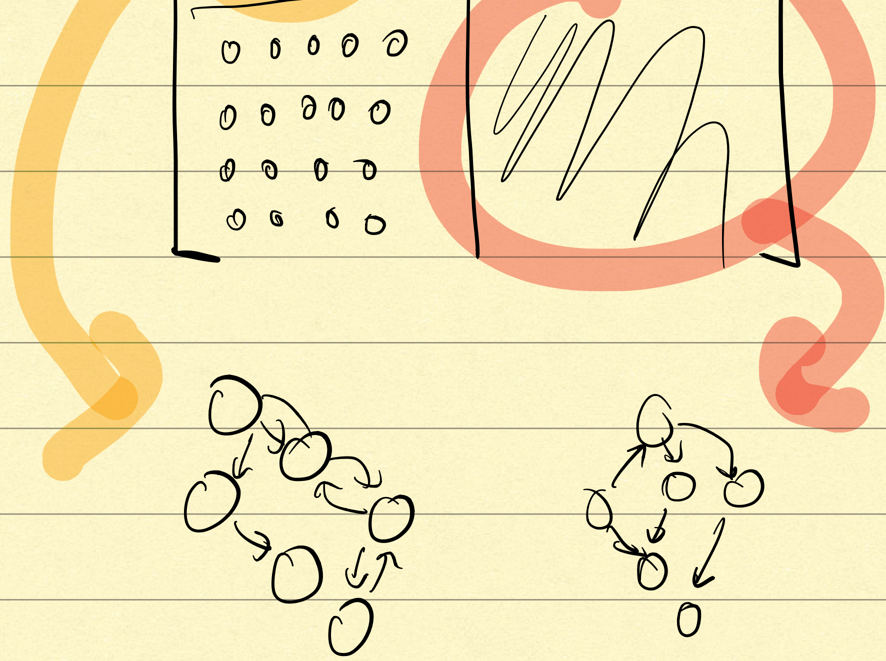


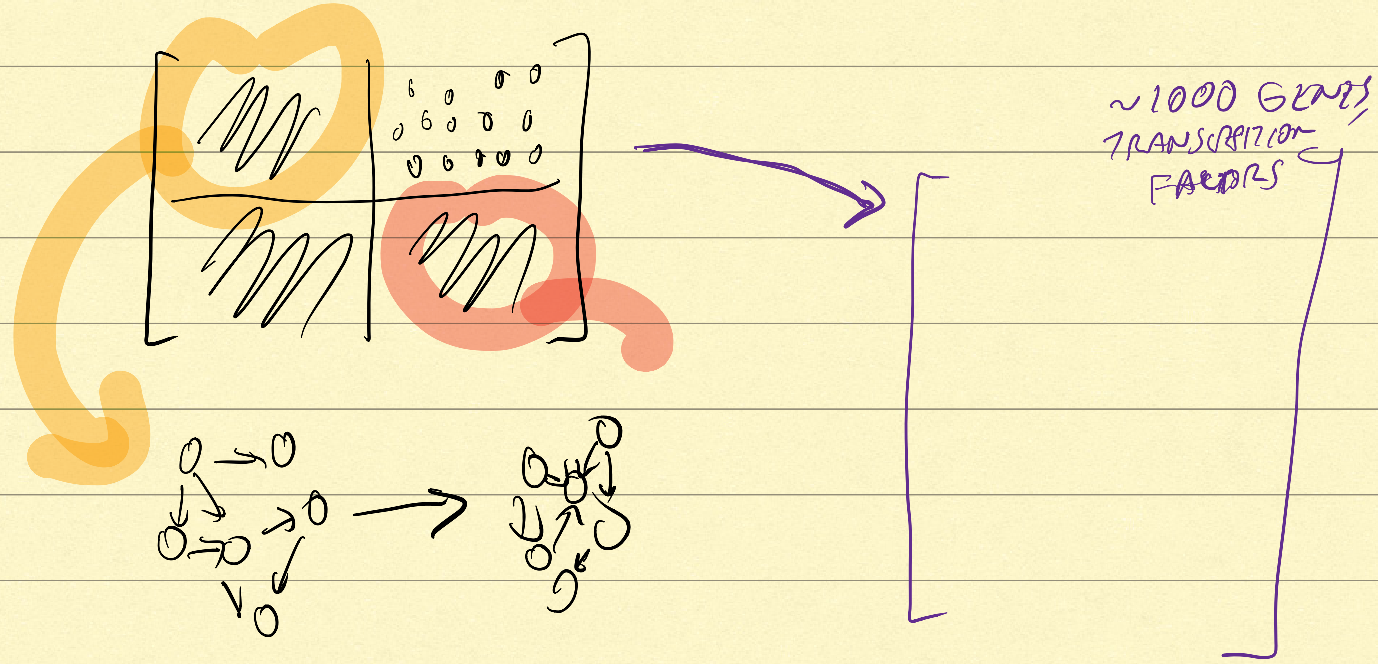

$\mathbb{P}_2$



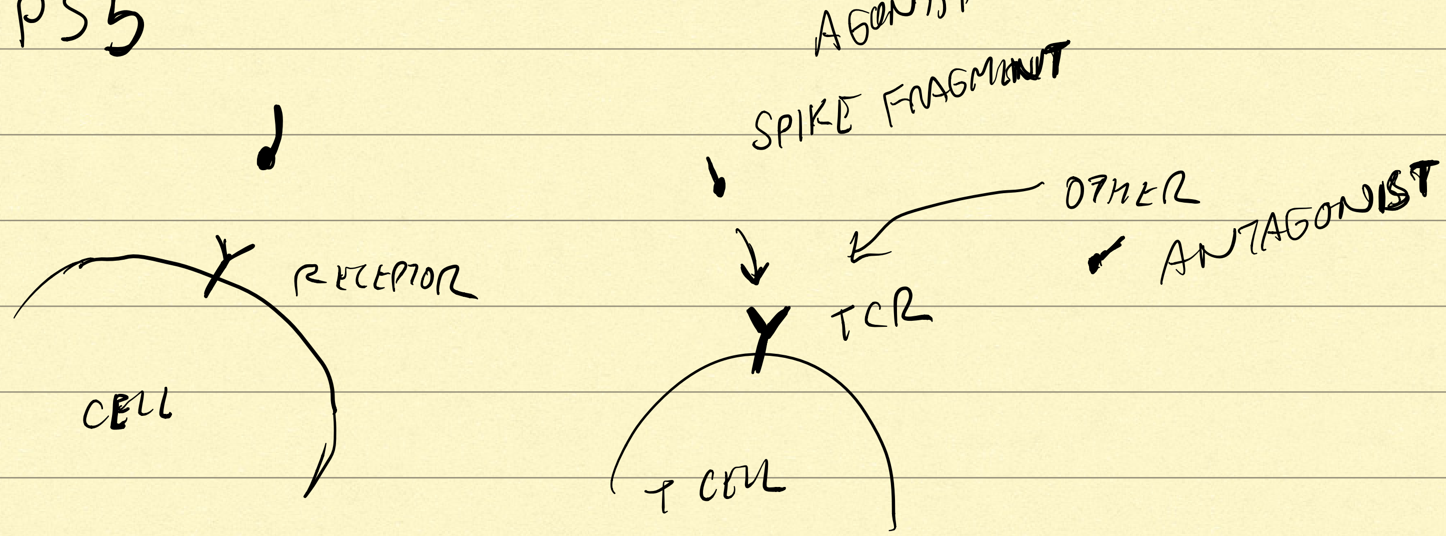

EX.

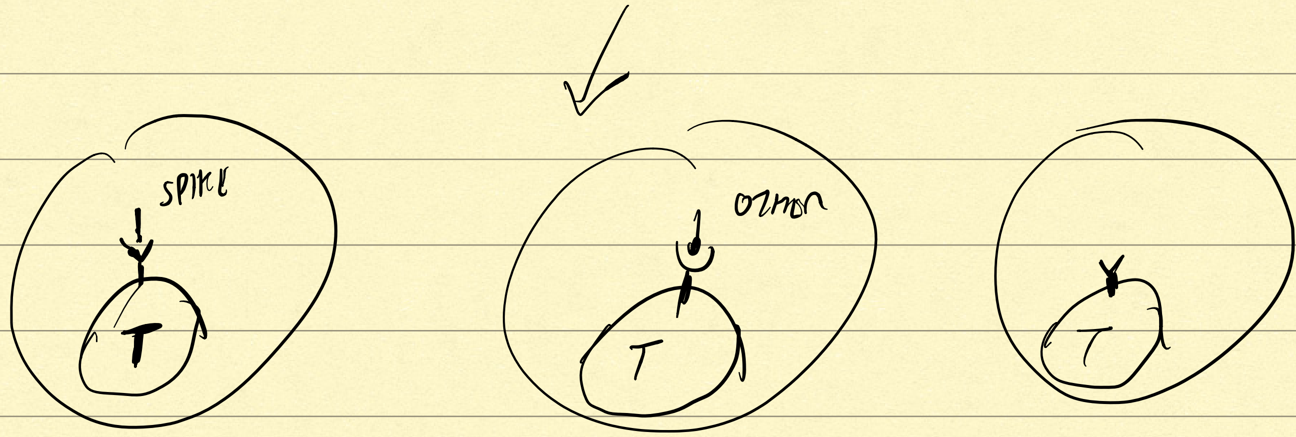


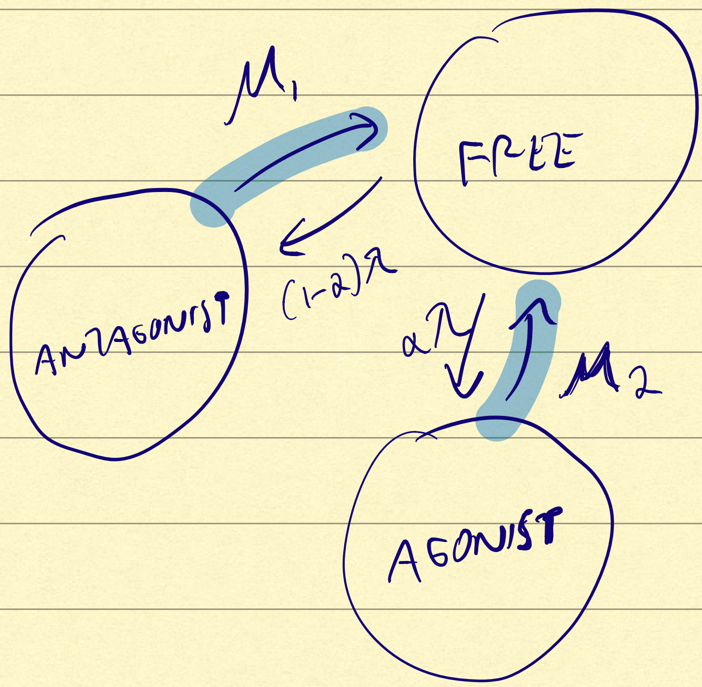



~1000 GENES,
TRANSCRIPTION FACTORS

# PS5


AGONIST
SPIKE FRAGMENT
OTHER
ANTAGONIST
RECEPTOR
CELL
TCR
T CELL




$\cancel{1-\mu}$

$$
\begin{array}{cccc}
\text{ANTAG.} & \text{AG.} & \text{FREE} & \\
\left[\begin{array}{c} -\mu_1 \\ 0 \\ \mu_1 \end{array}\right. & \begin{array}{c} 0 \\ -\mu_2 \\ \mu_2 \end{array} & \left.\begin{array}{c} (1-\alpha)\lambda \\ \alpha\lambda \\ -\lambda \end{array}\right] & \begin{array}{l} \text{ANTAG} \\ \text{AG} \\ \text{FREE} \end{array}
\end{array}
$$

$$\frac{d\mathbb{P}_{FREE}}{dt} = +\mu_1 \mathbb{P}_{ANTAG} + \mu_2 \mathbb{P}_{AG} - \lambda \mathbb{P}_{FREE}$$

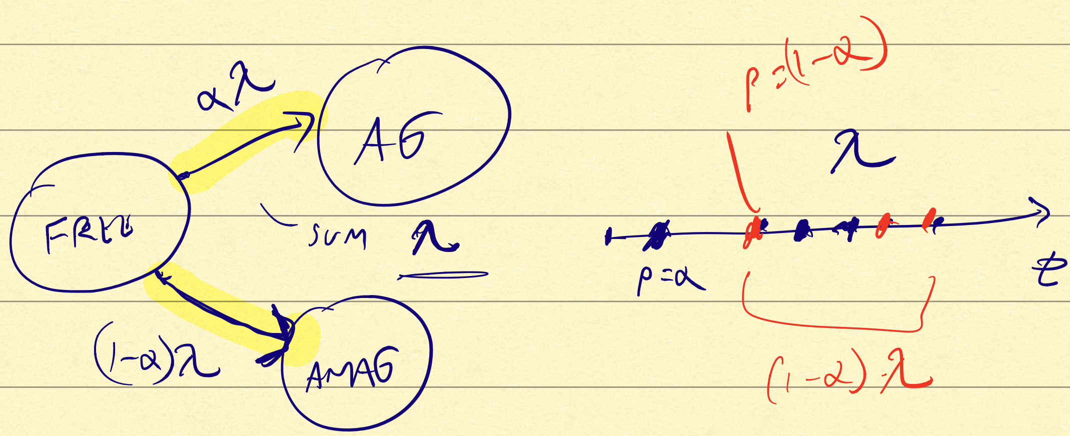
E[TIME OCCUPIED WITH AG] =

FRACTION OF TIME WITH AG: $< \alpha$

FRACTION OF TIME WITH ANTAG: $< 1-\alpha$

$$\begin{array}{cc} & \begin{array}{ccc} \text{AG} & \text{ANTAG} & \text{FREE} \end{array} \\ \begin{array}{c} \text{AG} \\ \text{ANTAG} \\ \text{FREE} \end{array} & \begin{bmatrix} & 0 & \\ 0 & & \\ & & \end{bmatrix} \end{array}$$

$$\begin{array}{cc} & \begin{array}{ccc} \text{FREE} & \text{AG} & \text{ANTAG} \end{array} \\ \begin{array}{c} \text{FREE} \\ \text{AG} \\ \text{ANTAG} \end{array} & \begin{bmatrix} -\lambda & \mu_2 & \mu_1 \\ \alpha\lambda & -\mu_2 & 0 \\ (1-\alpha)\lambda & 0 & -\mu_1 \end{bmatrix} \end{array}$$



ASSUME $\mu_1, \mu_2$ ARE RATES

$\mathbb{P}(\text{AG})$
$\mathbb{P}(\text{ANTAG})$
$\mathbb{P}(\text{FREE})$

CHECKS: IF $\alpha \to 0$, THEN $\mathbb{P}(\text{AG}) \to 0$

IF $\mu_1 \to \infty$, THEN $\mathbb{P}(\text{ANTAG}) \to 0$

IF $\mu_1 \to 0$, THEN $\mathbb{P}(\text{ANTAG}) \to 1$

IF $\alpha = \frac{1}{2}$, AND $\mu_1 = \mu_2$, THEN

$$\mathbb{P}(\text{AG}) = \mathbb{P}(\text{ANTAG})$$

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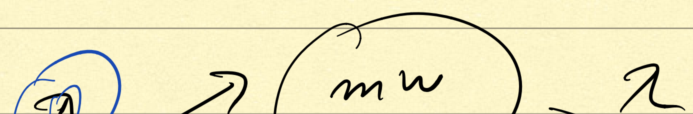
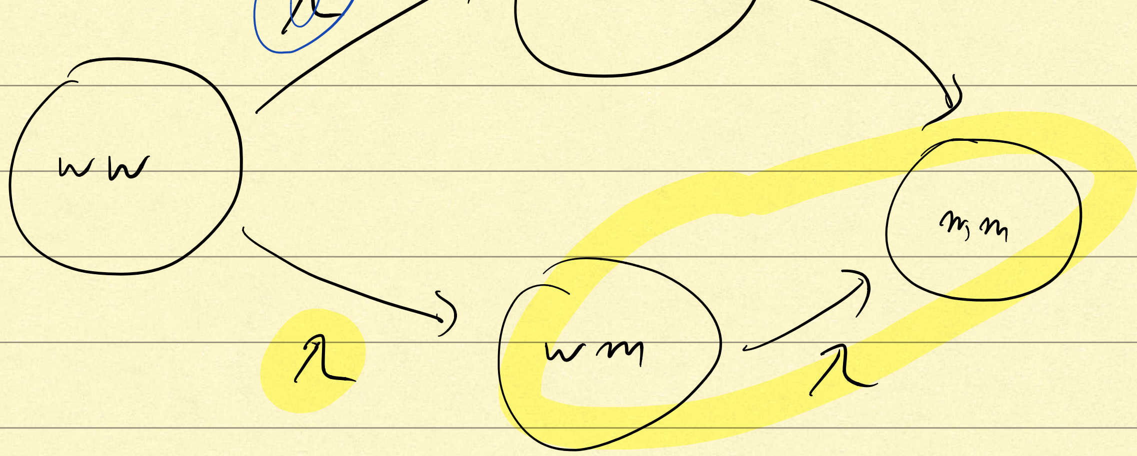
**EX** DOUBLE MUTANT

$$w \xrightarrow{\lambda} m$$

$$w \xrightarrow{\lambda} m$$

$$E[T_{0\to 2}] = 1.5\,\frac{1}{\lambda}$$






$$
\begin{array}{cccc}
ww & wm & mw & mm \\
\end{array}
$$
$$
\begin{bmatrix}
-2\lambda & 0 & 0 & 0 \\
\lambda & -\lambda & 0 & 0 \\
\lambda & 0 & -\lambda & 0 \\
0 & \lambda & \lambda & 0
\end{bmatrix}
$$

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MEAN FIRST PASSAGE TIME

FROM STATE $i$ TO $j$ IS

LET $M_{-j}$ - TRANSITION MATRIX REMOVED $j$TH COLUMN & ROW

$j$TH ROW $\rightarrow$
$$
\begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \end{bmatrix} = M_{-j} \cdot \vec{T}_k \qquad T_k = \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_{N-1} \end{bmatrix}
$$

$$\boxed{E[T_{i \to j}] = \sum_k T_k}$$