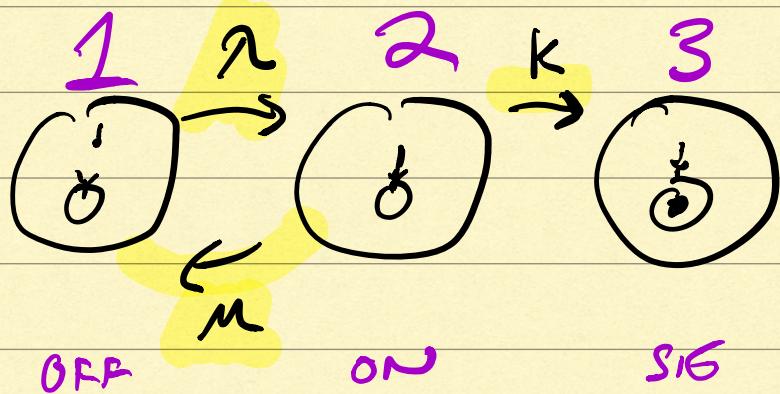


Hi!

PS5 Bi



$$\frac{d\vec{P}}{dt} = M \cdot \vec{P}$$

$$M = \begin{bmatrix} -\lambda & +\mu & 0 \\ +\lambda & -(\mu+k) & 0 \\ +k & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -k & 0 \\ 0 & +k & 0 \end{bmatrix}$$

$$T_{2 \rightarrow 3} = ?$$

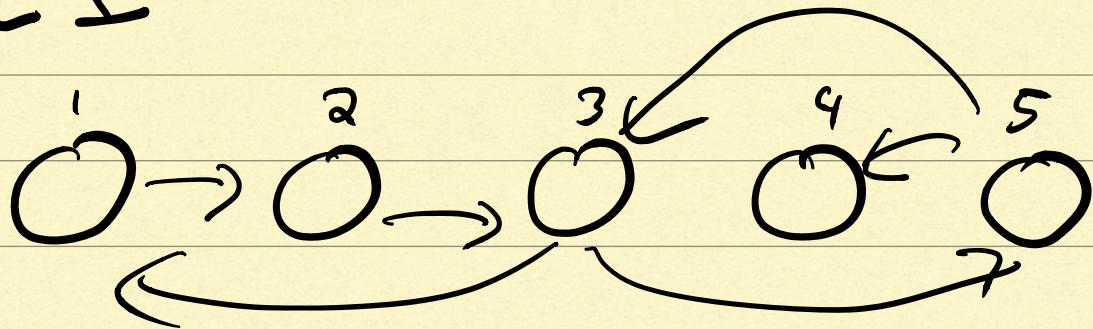
$$M_{-3} = \begin{bmatrix} -\lambda & +\mu \\ +\lambda & -(\mu+k) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -k \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -\lambda & +\mu \\ +\lambda & -(\mu+k) \end{bmatrix} \cdot \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}$$

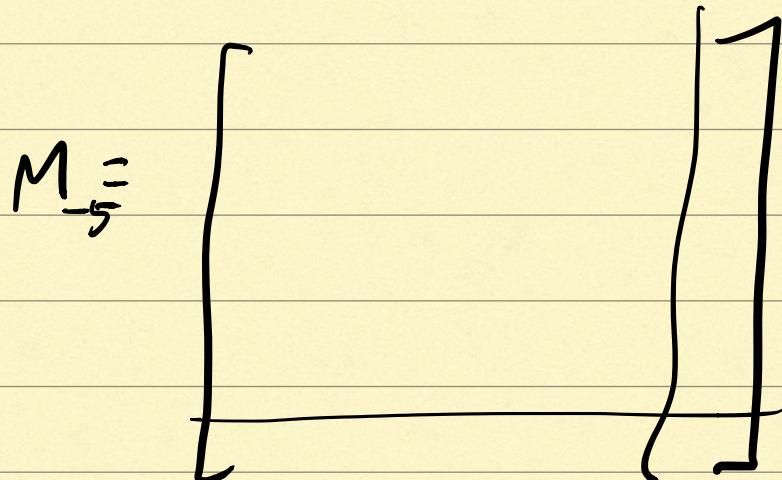
... T_1, T_2

$$T_{2 \rightarrow 3} = T_1 + T_2 = \boxed{\frac{1}{k} \left(1 + \frac{\mu}{\lambda} \right)}$$

ASIDE 1



$T_{3 \rightarrow 5}$



$$\begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} = M_{-5} \cdot \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix}$$

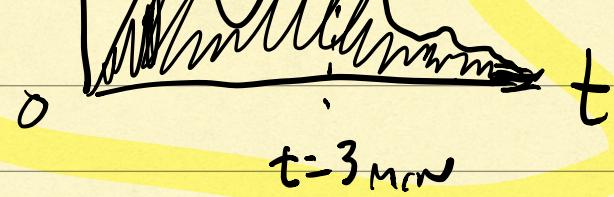
$$T_{3 \rightarrow 5} = T_1 + T_2 + T_3 + T_4$$



ASIDE 2



$P_i(t)$

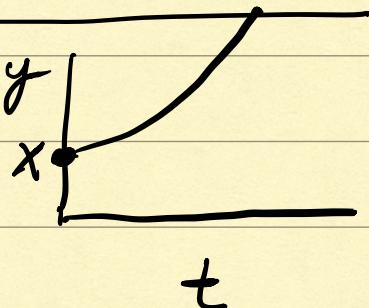


$$\int_0^\infty P_i(t) dt$$

PARAMETRIC NOISE \sim HETEROGENEITY

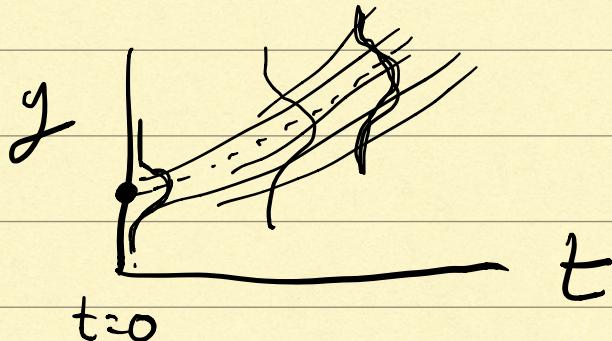
ex

$$y' = Ay \quad y(0) = x$$



$$\text{suppose } X \sim p_x(x)$$

$$y(t) = x e^{At}$$



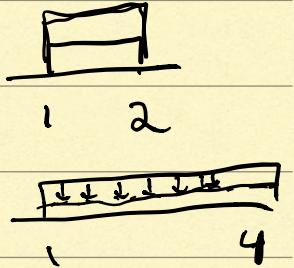
WHAT IS $y \sim p_y(y)$?

GIVEN A RANDOM VARIABLE $X \sim p_x(x)$,

SUPPOSE $Y = g(X)$, THEN WHAT IS $p_y(y)$?

SUPPOSE $Y = cX$

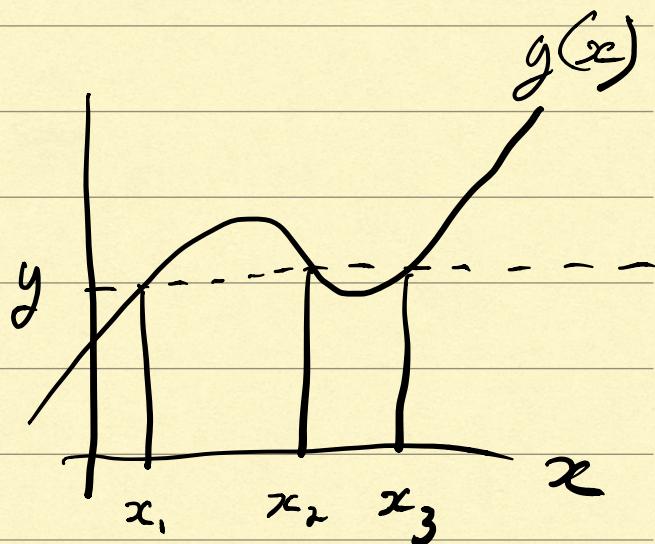
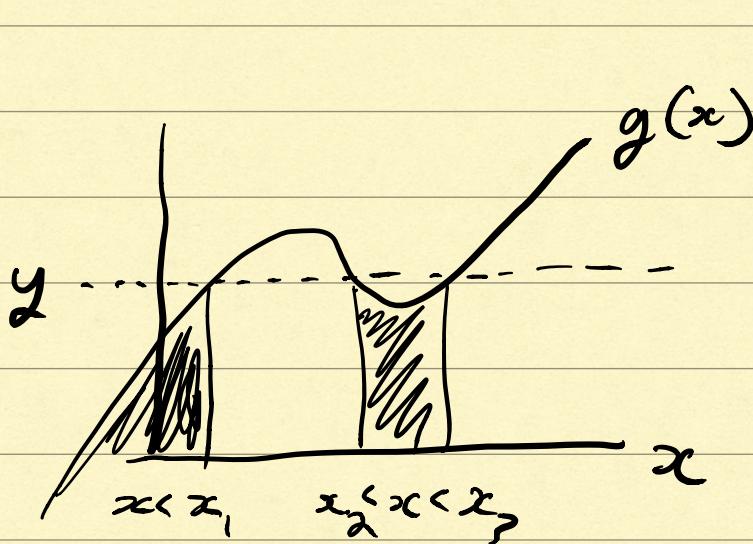
$$\text{NOT } \rho_Y(y) = c \cdot \rho_X(x)$$



FOR A GENERAL FUNCTION $g(x)$,
DEFINE

$$I_y = \{x : g(x) \leq y\} \quad \text{INDEX SET OF } y$$

$$\{x_k\} = \{x : g(x) = y\} \quad \text{PRE-IMAGE of } y$$

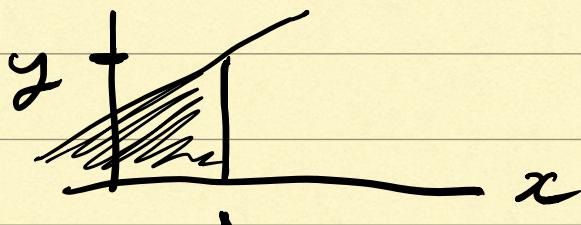


$$F_Y(y) = P(Y \leq y)$$

$$= P(x \in I_y) = \int_{I_y} p_x(x) dx$$

$$\text{Ex } y = ax + b$$

$$a > 0$$

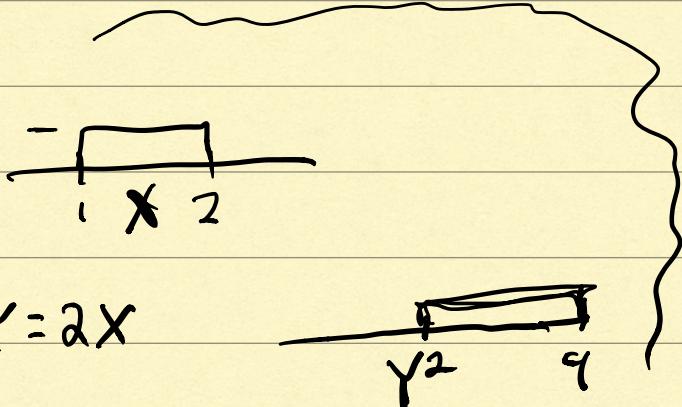


$$I_y = \left\{ x \mid \frac{y-b}{a} \right\}$$

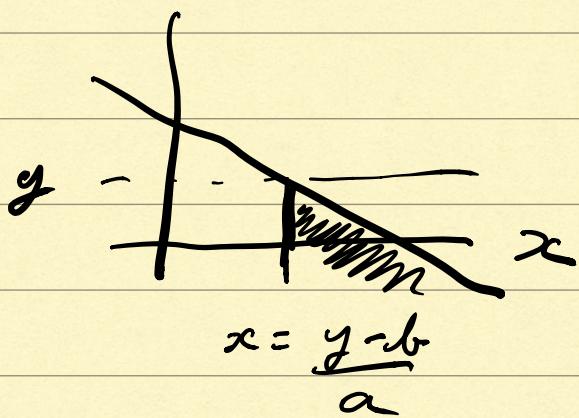
GIVEN $F_x(x)$, THEN

$$F_Y(y) = F_x\left(\frac{y-b}{a}\right)$$

$$p_Y(y) = \frac{d}{dy} F_Y(y) = \frac{1}{a} p_x\left(\frac{y-b}{a}\right)$$



EX $a < 0$



$$I_y = \left\{ x \geq \frac{y-b}{a} \right\}$$

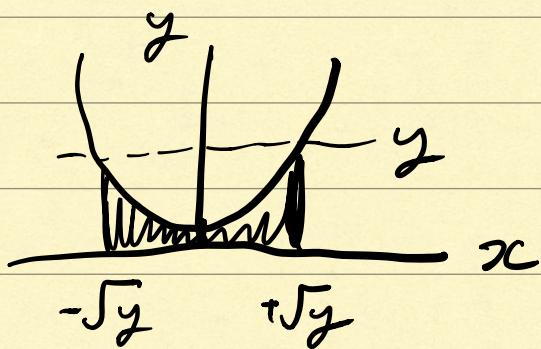
$$F_Y(y) = 1 - F_X\left(\frac{y-b}{a}\right)$$

$$\rho_Y(y) = -\frac{1}{a} \rho_X\left(\frac{y-b}{a}\right)$$

$$\rho_Y(y) = \frac{1}{|a|} \rho_X\left(\frac{y-b}{a}\right) \quad a \in \mathbb{R}$$

EX $Y = X^2$

$$g(x) = x^2$$



$$I_y = \begin{cases} (-\sqrt{y}, +\sqrt{y}) & y \geq 0 \\ \emptyset & y < 0 \end{cases}$$

$$F_Y(y) = \begin{cases} F_X(\sqrt{y}) - F_X(-\sqrt{y}) & y \geq 0 \\ 0 & y < 0 \end{cases}$$

$$p_Y(y) = \begin{cases} p_X(\sqrt{y}) \cdot \frac{1}{2\sqrt{y}} & y \geq 0 \\ 0 & y < 0 \end{cases}$$

GENERAL FORMULA

LET $X \sim p_X(x)$ AND $Y = g(x)$

$$p_Y(y) = \sum_k p_X(x_k(y)) \cdot \left| \frac{dx_k}{dy} \right|$$

WHERE $\{x_k\}$ IS THE PRE-IMAGE OF y

IF $\frac{dg}{dx} \neq 0$, THIS SIMPLIFIES TO

$$P_y(y) = \sum_k P_x(g_k^{-1}(y)) \cdot \left(\left| \frac{dg}{dx} \right| \right)^{-1}$$