IF STATE SPACE IS DISCRETE, X IS A DISCRETE RANDOM VARIABLE

$$p_X(x) = \mathbb{P}(X = x)$$

NOTATION $X \sim p_X(x)$

MOMENTS

$$E[X^n] = \sum_{i \in S} i^n \, p_X(i)$$

STATE SPACE

ZEROTH MOMENT

$$E[X^0] = \sum_{i \in S} p_X(i) = 1$$

FIRST MOMENT

$$E[X] = \sum_{i \in S} i \, p_X(i) = \text{MEAN}$$

SYMBOL μ_X

SECOND MOMENT

$$E[X^2] = \sum_{i \in S} i^2 \, p_X(i) =$$

$$E[X^2] - \mu_x^2 = E[(x-\mu)^2]$$

$= \text{VARIANCE}$

SYMBOL σ_x^2

$\sigma_x = \sqrt{\sigma_x^2}$ STANDARD DEVIATION

FAMOUS DISCRETE RANDOM VARIABLES

- UNIFORM BETWEEN a, AND b

- BERNOULLI $X=0$ $\mathbb{P}(X=0) = p_x(0) = 1-p$
  $X=1$ $\mathbb{P}(X=1) = p_x(1) = p$

$$E[X] = p$$

STOCHASTIC PROCESS OF INDEPENDENT BERNOULLI TRIALS

$$X_t = \{\underbrace{0\ 0\ 0\ 1}_{}\ 1\ \overbrace{0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0}^{n} \ldots\}$$

1

- GEOMETRIC

$$p_x(k) = (1-p)^{k-1} \cdot p \qquad k = 0, 1, 2 \ldots$$

ATTEMPTS IN A SERIES OF BERNOULLI TRIALS UNTIL FIRST

SUCCESS $X=1$

$$E[X] = \frac{1}{p}$$

- BINOMIAL

"CHOOSE"

$$p_X(k) = \binom{n}{k} p^k (1-k)^{n-k}$$

$\boxed{\text{\# OF SUCCESSES } (X=1) \text{ IN A SERIES OF } n \text{ BERNOULLI TRIALS}}$

RECAP

$$\begin{bmatrix} p_A(t+1) \\ p_B(t+1) \\ p_C(t+1) \end{bmatrix} = \underbrace{\begin{bmatrix} p_{A\to A} & p_{B\to A} & p_{C\to A} \\ p_{A\to B} & p_{B\to B} & p_{C\to B} \\ p_{A\to C} & p_{B\to C} & p_{C\to C} \end{bmatrix}}_{\underline{M}} \cdot \begin{bmatrix} p_A(t) \\ p_B(t) \\ p_C(t) \end{bmatrix}$$

COLUMNS MUST SUM TO UNITY

ASIDE REDNER

$$M \vec{p}^{\,t+1} = \vec{p}^{\,t}$$

TRANSPOSE!

$$\vec{p}^{\,t+1} = M \cdot \vec{p}^{\,t}$$

PS2

$$
\begin{array}{c|c}
\begin{array}{cccccc} S & E1 & E2 & E3 & I & END \end{array} & \\
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & \boxed{\frac{20000}{20011}} & \boxed{} & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & \boxed{\frac{10}{20011}} & \boxed{} & 0 \\
0 & 0 & 0 & \boxed{\frac{1}{20011}} & 0 & 1
\end{bmatrix}
&
\begin{array}{c} S \\ E1 \\ E2 \\ E3 \\ I \\ END \end{array}
\end{array}
$$

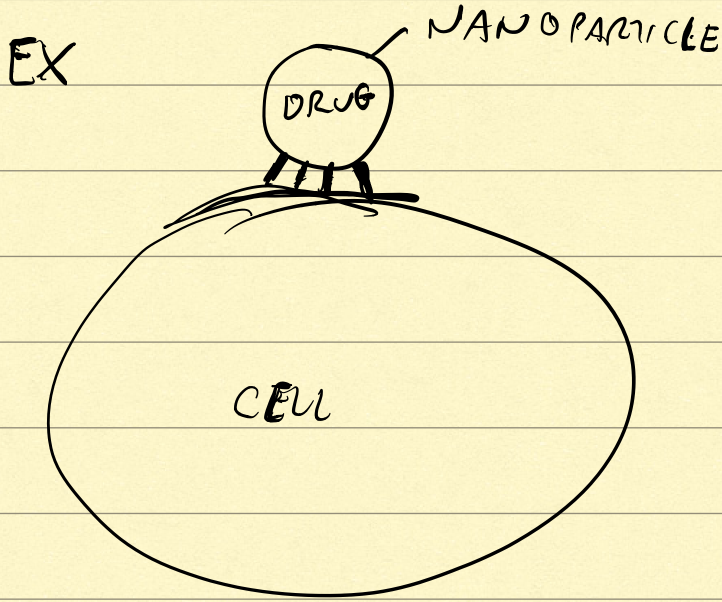
$$
\begin{bmatrix} 0 \\ \frac{19999}{20000} \\ 0 \\ 0 \\ \frac{10}{20000} \\ \frac{1}{20000} \end{bmatrix}
$$

$$
\begin{bmatrix} 0 \\ \frac{19999}{20010} \\ 0 \\ 0 \\ ? \\ ? \end{bmatrix}
$$

2x

MEAN FIRST PASSAGE TIME

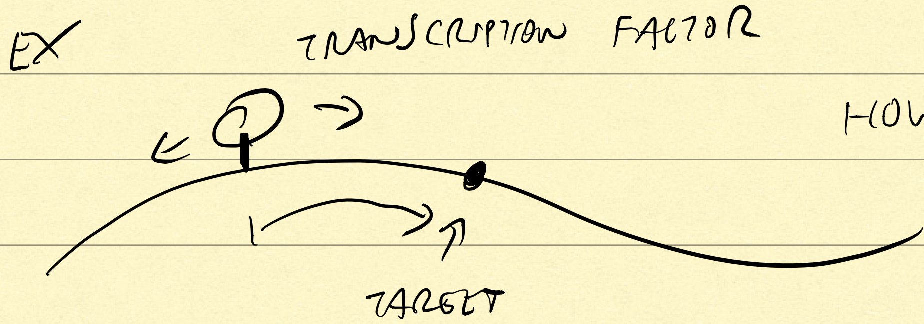
EX



IF CURRENTLY 3 "LEGS" ATTACHED, HOW LONG UNTIL 0 LEGS ATTACHED?

$$3 \to 2 \to 3 \to 4 \to 3 \to 2 \to 1 \to 0$$

EX



HOW LONG UNTIL FIRST PASS AT TARGET?

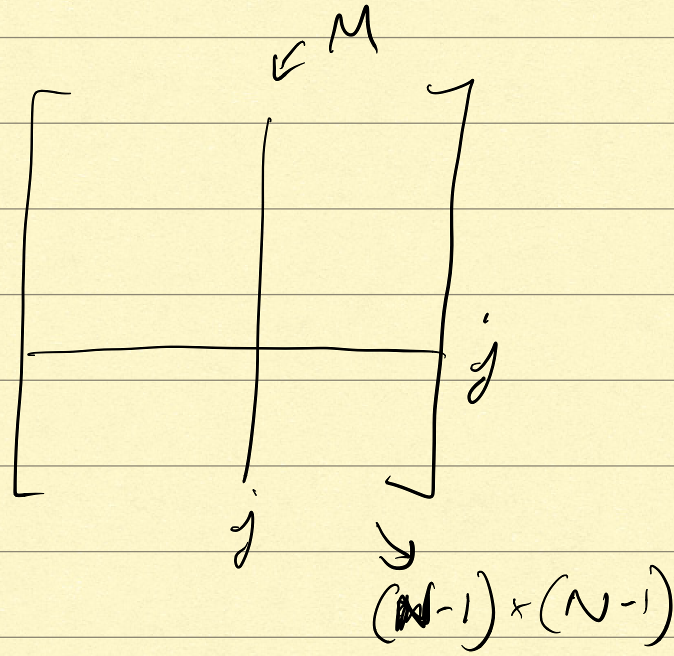
FOR A MARKOV CHAIN

$$\vec{P}_{t+1} = \underbrace{\underline{\underline{M}}}_{N \times N} \cdot \vec{P}_t \qquad \vec{P}_t = \begin{bmatrix} P_1 \\ \vdots \\ P_N \end{bmatrix}$$

ON AVERAGE HOW MANY STEPS DOES IT TAKE FROM STATE k TO STATE j?

THEOREM

DEFINE M_{-j} =



AS M, WITH j^{TH} COLUMN AND j^{TH} ROW REMOVED.

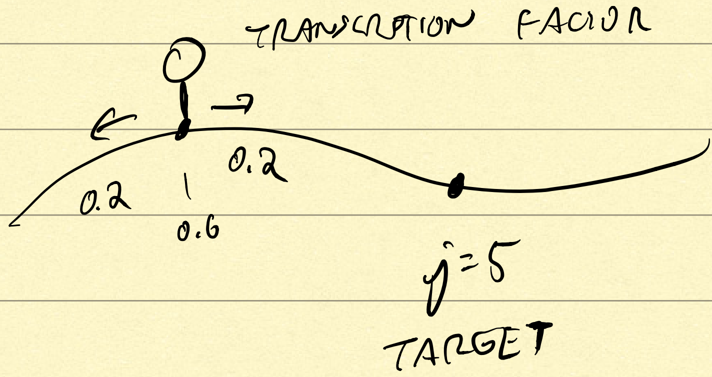
LET T_{kj} BE THE MEAN TIME FROM k TO j.

$$\vec{T}_j = \underbrace{\begin{bmatrix} T_{1j} \\ \vdots \\ T_{kj} \\ \vdots \\ T_{N-1\,j} \end{bmatrix}}_{(N-1)}$$

WITH NO j^{TH} ELEMENT

$$\boxed{\text{THEN} \quad \underbrace{\begin{bmatrix} -1 \\ -1 \\ \vdots \\ -1 \end{bmatrix}}_{(N-1)} = \left(M_{-j} - \overset{\text{IDENTITY}}{I}\right) \cdot \underbrace{\vec{T}_j}_{\text{ANSWER!}}}$$

EX & SKETCH OF PROOF



$$M = \begin{bmatrix} 0.6 & 0.2 & & & & 0.2 \\ 0.2 & 0.6 & 0.2 & & & \\ & 0.2 & 0.6 & & & \\ & & 0.2 & \ddots & & \\ & & & & & 0.2 \\ 0.2 & & & & & 0.6 \end{bmatrix}$$

$$T_{kj} = E[T_j \mid X_0 = k]$$

$$\begin{aligned} E[T_j \mid X_0 = k] &= E[T_j \mid X_0 = k, \text{LEFT}]\ 0.2 \quad (\text{LEFT}) \\ &+ E[T_j \mid X_0 = k, \text{STAY}]\ 0.6 \quad (\text{STAY}) \\ &+ E[T_j \mid X_0 = k, \text{RIGHT}]\ 0.2 \quad (\text{RIGHT}) \end{aligned}$$

$$\begin{aligned} T_{kj} &= (T_{k-1,j} + 1)\, 0.2 \\ &+ (T_{k,j} + 1)\, 0.6 \\ &+ (T_{k-1,j} + 1)\, 0.2 \end{aligned}$$

$$-1 = 0.2\, T_{k-1,j} + (0.6 - 1)\, T_{k,j} + 0.2\, T_{k-1,j}$$

$\downarrow$

$$\begin{bmatrix} -1 \\ -1 \\ \vdots \end{bmatrix} = \left(\begin{bmatrix} 0.6 & 0.2 & \\ 0.2 & 0.6 & \\ & & \ddots \end{bmatrix} - \begin{bmatrix} 1 & & \\ & 1 & \\ & & \ddots \end{bmatrix} \right) \vec{T}_j$$

$$\begin{bmatrix} \vdots \\ -1 \\ -1 \end{bmatrix} \quad \left(\begin{bmatrix} 0.2 & 0.6 & 0.2 & \\ & 0.2 & 0.6 \end{bmatrix} \quad \begin{bmatrix} - & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \\ & & & & 1 \end{bmatrix} \right)$$

$$\begin{bmatrix} -1 \\ -1 \\ -1 \\ \vdots \end{bmatrix} = \left(M_{-j} - I \right) \cdot \vec{T}_j$$