MEMORYLESS PROPERTY

$$\mathbb{P}(X_{n+1} \mid X_n, X_{n-1}, X_{n-2} \ldots) = \mathbb{P}(X_{n+1} \mid X_n)$$

EX 3-STATE THAT HAD "HISTORY" OF 2

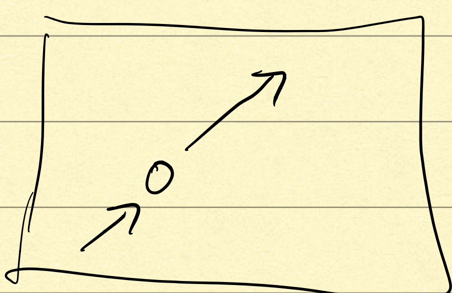
$$\mathbb{P}(X_{n+1} \mid X_n, X_{n-1})$$

DEFINE 9-STATE

$$\begin{array}{ccccc} AA & AB & AC & \cdots & CC \end{array}$$

$$\begin{bmatrix} \; & & \\ & & \\ & & \end{bmatrix}_{9\times 9}$$

EX. BILLIARD TABLE



$\underline{x}$ $\qquad$ x, v

EX. MORE MEMORYLESS THAN MARKOV

HHHTTTTTT

$S = (\#\text{UMBRELLAS AT LAB}, \#\text{ AT HOME})$

$S = (\#\text{UMBRELLAS WITH BIOLOGIST})$

CONTINUOUS RANDOM VARIABLES

$X \in S$ $\qquad\qquad\qquad\qquad \mathbb{P}, \rho$

EX $\quad S \in [0,1]$

$S \in (-\infty, \infty)$

$S \in [0, \infty)$

HAS UNITS

DENSITY FUNCTION $\rho_X(x)$

SUCH THAT $\mathbb{P}(A) = \int \rho_X(x)\, dx$

$\quad A$

EVENT

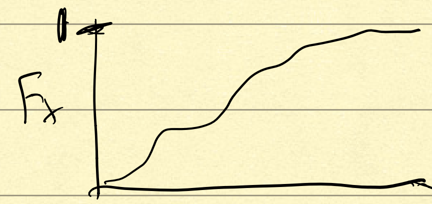
CUMULATIVE DISTRIBUTION

$$\mathbb{P}(X \leq x) = F_X(x)$$

FROM AXIOMS $\quad \displaystyle\int_{x \in S} p_X(x)\, dx = 1$

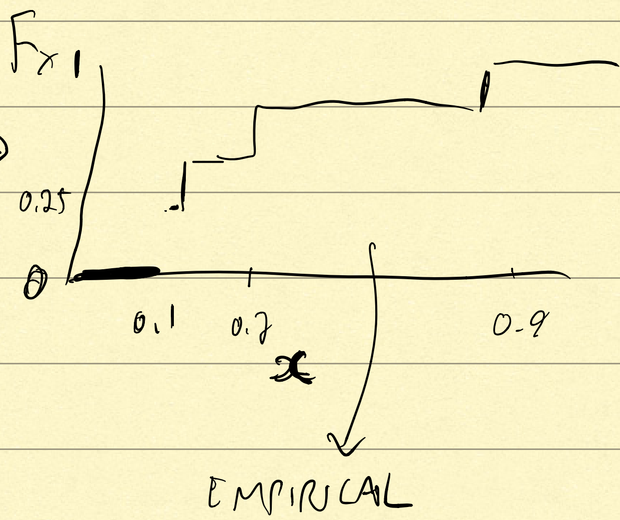
$$F_X(x) = 0 \quad \text{AS} \quad x \to -\infty$$
$$F_X(x) = 1 \quad \text{AS} \quad x \to +\infty$$

F_X IS NON-DECREASING

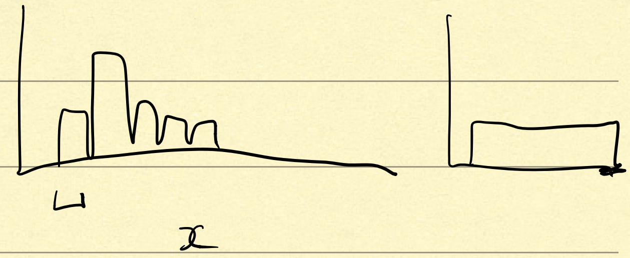


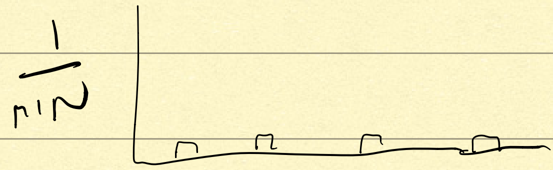
$$p_X(x) = \frac{d}{dx} F_X(x) \qquad F_X(x) = \int_{-\infty}^{x} p_X(\tilde{x})\, d\tilde{x}$$

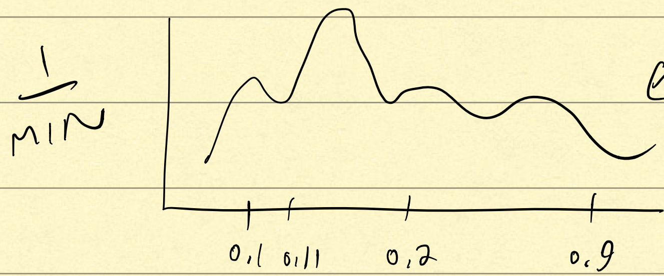
CUMULATIVE DATA $\quad X = \begin{bmatrix} 0.1 \\ 0.11 \\ 0.9 \\ 0.2 \end{bmatrix} \longrightarrow$



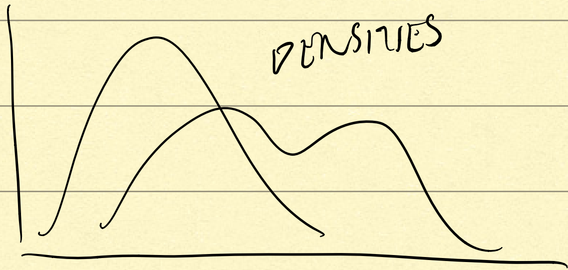
DATA X → DENSITY

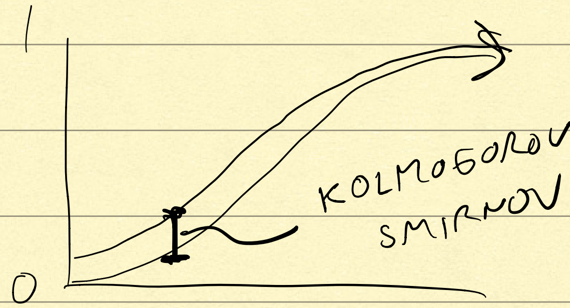




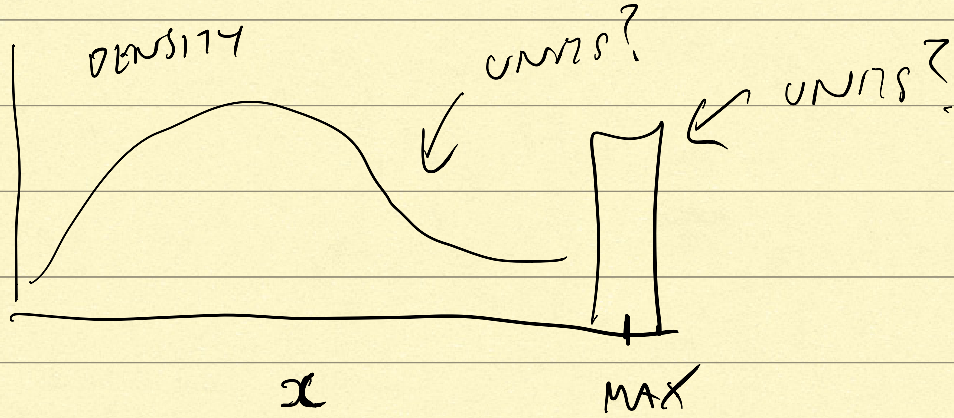


← KERNEL



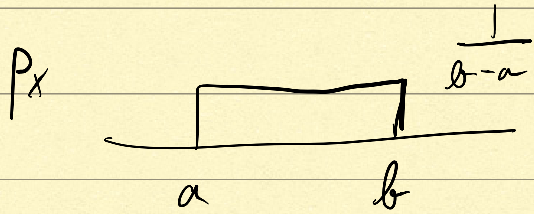


EX



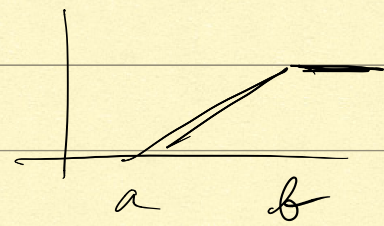
FAMOUS CONTINUOUS RANDOM VARIABLES

- UNIFORM $\quad X \sim \text{UNIF}(a, b)$

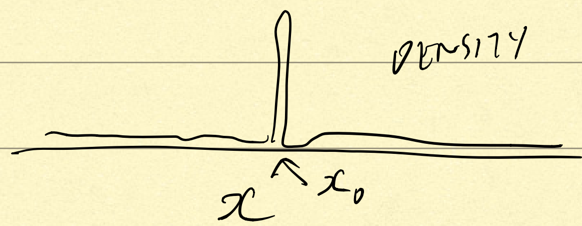


$$p_X(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{OTHERWISE} \end{cases}$$

$$F_X(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \leq x < b \\ 1 & b \leq x \end{cases}$$

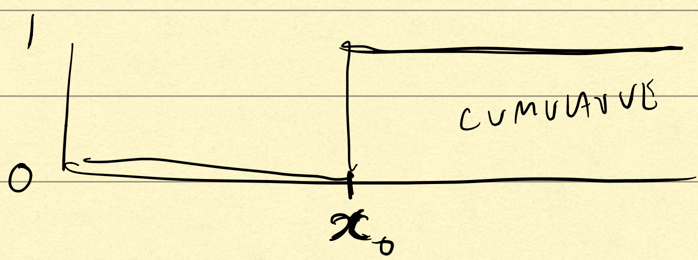


- DELTA DISTRIBUTION



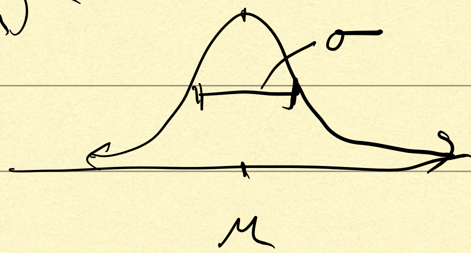
$$p_X(x) = \delta(x - x_0)$$

$$F_X(x) = \begin{cases} 0 & x < x_0 \\ 1 & x_0 \leq x \end{cases}$$



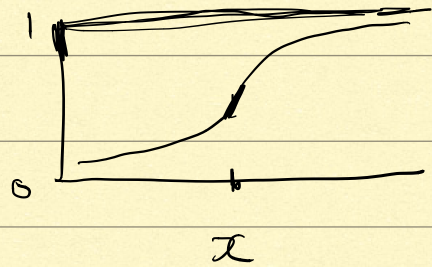
- GAUSSIAN / NORMAL $\quad X \sim \text{GAUSSIAN}(\mu, \sigma)$

$$p_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)}{2\sigma^2}}$$



$Z \sim$ STANDARD NORMAL $\mu = 0, \ \sigma = 1$

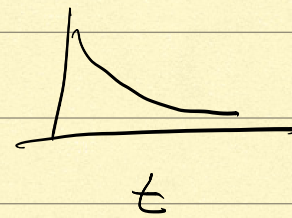
$F_X(x) =$



$= \mathrm{erf}(x)$

- EXPONENTIAL $\quad T \sim \mathrm{EXP}(\lambda)$

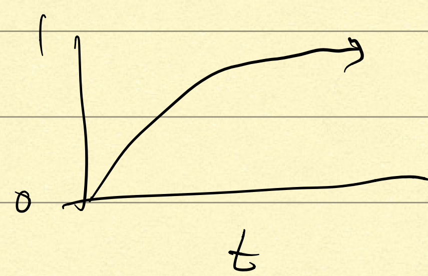
$p_T(t) = \lambda e^{-\lambda t} \quad t > 0$



$p_T(t) = \frac{1}{\tau} e^{-t/\tau} \qquad \lambda = \frac{1}{\tau}$

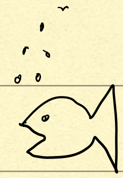
$F_T(t) = 1 - e^{\overset{-\lambda t}{\downarrow}}$

RATE

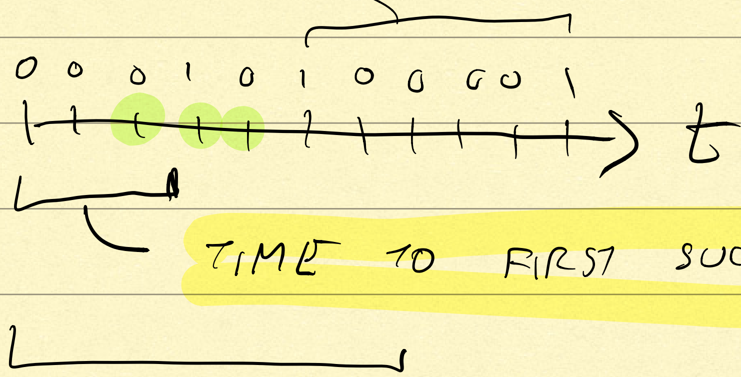


$E[T] = \tau = \frac{1}{\lambda} \qquad \tau$ - MEAN

POISSON PROCESS



RECALL A BERNOULLI TRIAL SERIES (DISCRETE TIME)



TIME TO FIRST SUCCESS $T \sim$ GEOM

IN ΔT, # OF SUCCESSES

$N \sim$ BINOMIAL

TIME BETWEEN SUCCESSES $T \sim$ GEOM

A POISSON PROCESS IS

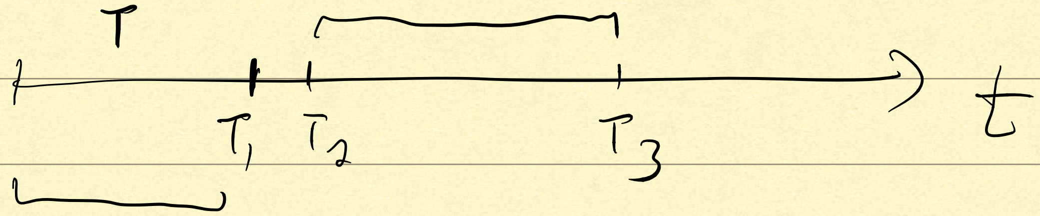
A CONTINUOUS TIME STOCHASTIC PROCESS THAT IS THE LIMIT OF A BERNOULLI TRIAL SERIES WITH TIME $\Delta t \to 0$ AND

~~$p\Delta t = \lambda$ CONSTANT~~

$p = \lambda \Delta t$

CONSTANT

λ UNITS $1/\text{TIME}$



TIME TO FIRST EVENT $T \sim \exp(\lambda)$

TIME BETWEEN EVENTS $T \sim \exp(\lambda)$

EVENTS IN A PERIOD ΔT

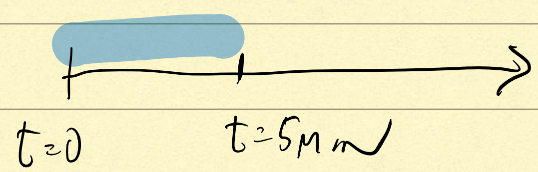
IS $N \sim \text{POISSON}(\lambda, \Delta T)$

$$p_N(i) = \frac{(\lambda \Delta T)^i \, e^{-\lambda \Delta T}}{i!} \qquad i \geq 0$$

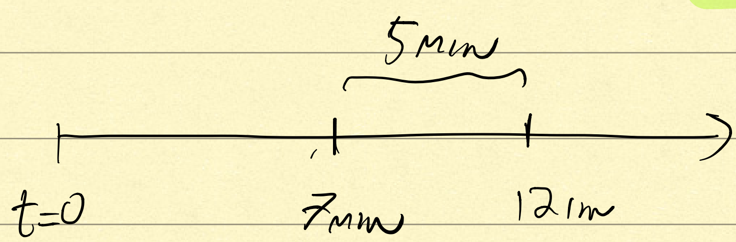
DISCRETE

PROPERTIES

- WAITING FOR EVENT



$$\mathbb{P}(T_1 < 5_{min})$$
$$= 1 - e^{-\lambda \cdot 5_{min}}$$



$$\mathbb{P}(T_1 < 12_{min} \mid T_1 > 7_{min})$$

$$= \frac{\mathbb{P}\left(T_1 < 12_{\text{MIN}} \cap T_1 > 7_{\text{MIN}}\right)}{\mathbb{P}\left(T_1 > 7_{\text{MIN}}\right)}$$

$$= \frac{\int_{7_{\text{MIN}}}^{12_{\text{MIN}}} \lambda e^{-\lambda t}\, dt}{1 - \left(1 - e^{-\lambda \cdot 7_{\text{MIN}}}\right)} \quad \ldots$$

$$= 1 - e^{-\lambda \cdot 5_{\text{MIN}}} = \mathbb{P}\left(T_1 < 5_{\text{MIN}}\right)$$

MEMORYLESS PROPERTY (CONTINUOUS TIME)

A POISSON PROCESS IS THE UNIQUE MEMORYLESS CONTINUOUS TIME STOCHASTIC PROCESS.

- IF EVENT A IS POISSON WITH RATE λ_A
  EVENT B IS POISSON WITH RATE λ_B

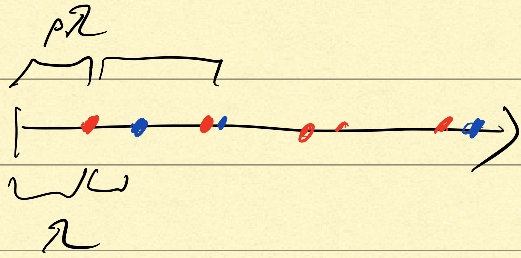
THEN TIME TO FIRST EVENT IS POISSON WITH RATE $\lambda = \lambda_A + \lambda_B$

RACING PROPERTY

- ONE POISSON PROCESS WITH RATE λ WITH EACH EVENT TYPE A, OR TYPE B WITH PROBABILITY p, $(1-p)$, INDEPENDENT, THEN

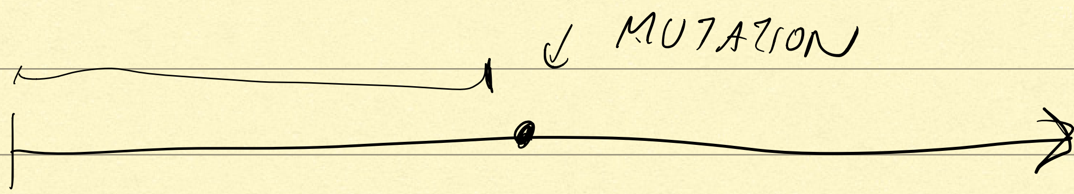
TYPE A AS POISSON WITH RATE $p\lambda$

TYPE B IS POISSON WITH RATE $(1-p\lambda)$



POISSON THINNING PROPERTY

MUTATION



$\lambda = 1\,yr^{-1}$

CASE 1: SUPPOSE WE WAIT FOR 1 OR 2 POSSIBLE MUTATIONS

$E[\text{FIRST MUTANTS}] = ?$

THIS IS POISSON WITH RATE 2λ

$$E[\text{FIRST MUTANTS}] = \frac{1}{2\lambda}$$

CASE 2: SUPPOSE WE WAIT FOR BOTH MUTATIONS