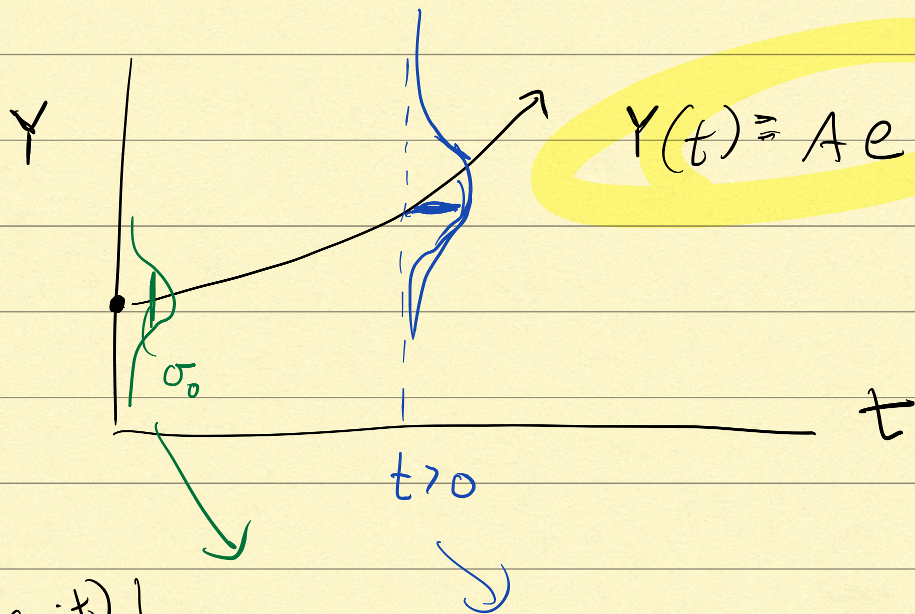
EX

$$\frac{dY}{dt} = (R-1)\,Y \qquad R = 2.5$$

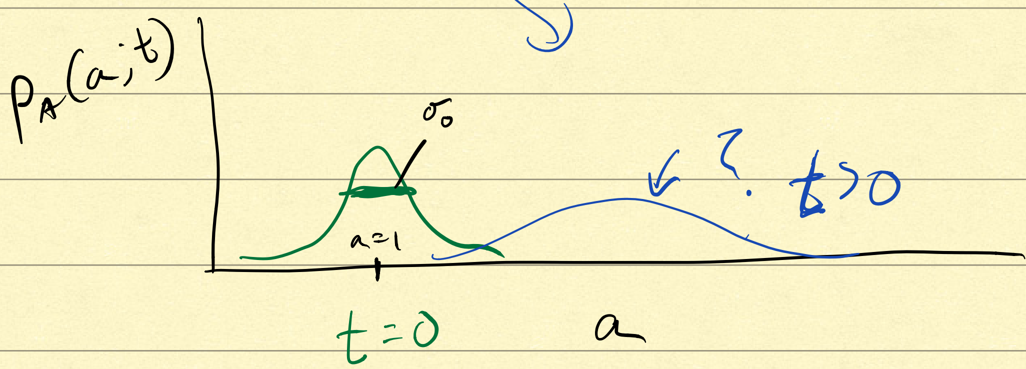
$$Y(0) = A$$

SUPPOSE $A \sim p_A(a)$

$$p_A(a) = \frac{1}{\sqrt{2\pi}\,\sigma_0}\, e^{-\frac{(a-1)^3}{2\sigma_0^2}}$$



$$Y(t) = A e^{1.5t}$$



$$Y = g(A) \qquad g(a) = a e^{1.5t}$$

$$g^{-1}(y) = \frac{y}{e^{1.5t}} = y e^{-1.5t}$$

$$y = a e^{1.5t}$$

$$\frac{dg}{da} = e^{1.5t}$$

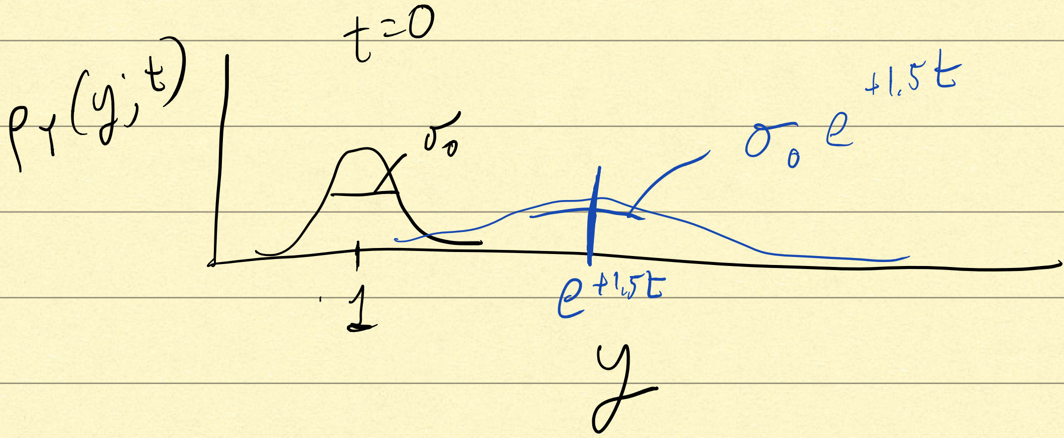
$$\left.\frac{dg}{da}\right|_{a=g^{-1}(y)} = e^{1.5t}$$

$$X = g(a)$$

$$p_X(x) = p_A(g^{-1}(x)) \left|\left.\frac{dg}{da}\right|_{a=g^{-1}(x)}\right|^{-1}$$

$$p_Y(y;t) = \frac{1}{\sqrt{2\pi}\,\sigma_0} \exp\left(-\frac{(ye^{-1.5t}-1)^2}{2\sigma_0^2}\right) \cdot e^{-1.5t}$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma_0 e^{+1.5t}} \exp\left(-\frac{(y-e^{+1.5t})^2}{2\sigma_0^2(e^{+1.5t})^2}\right)$$



$$\frac{(ye^{-1.5t}-1)^2}{2\sigma_0^2}$$

$$\frac{\left(e^{+1.5t}\right)^2\left(ye^{-1.5t}-1\right)^2}{\left(e^{+1.5t}\right)^2 2\sigma_0^2}$$

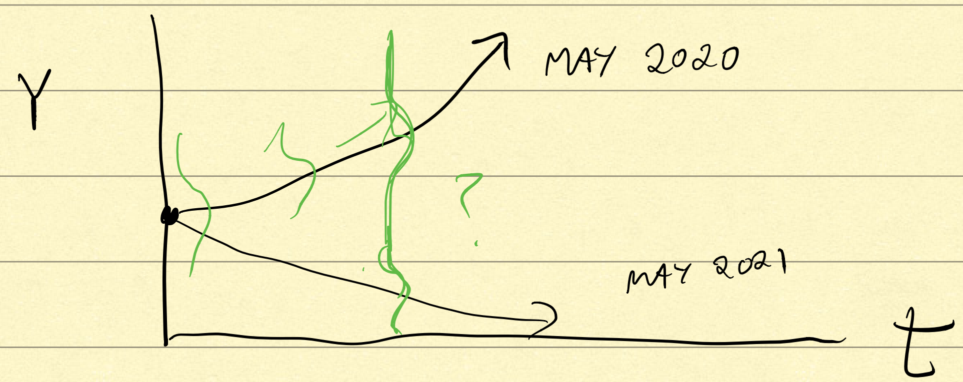
PS6

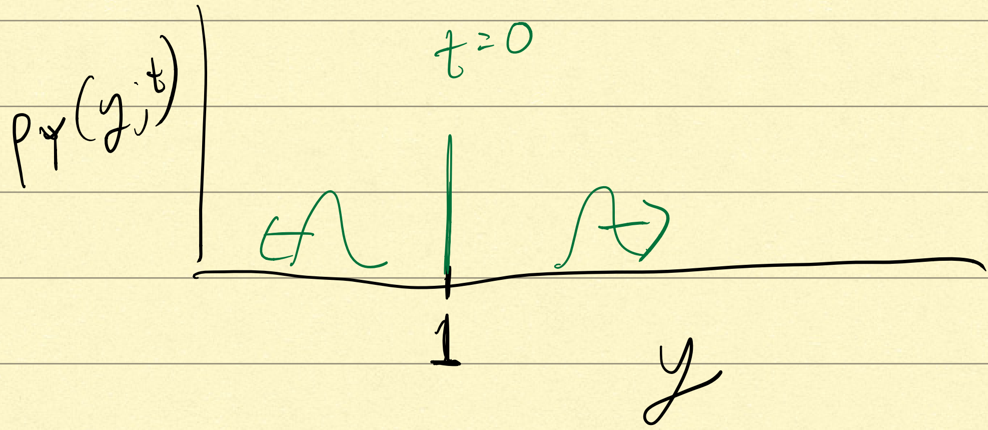
$$\frac{dY}{dt} = (R-1)\,Y$$

$$Y(0) = 1$$

$$R \sim p_R(r) =$$

WHAT IS $p_Y(y;t)$?





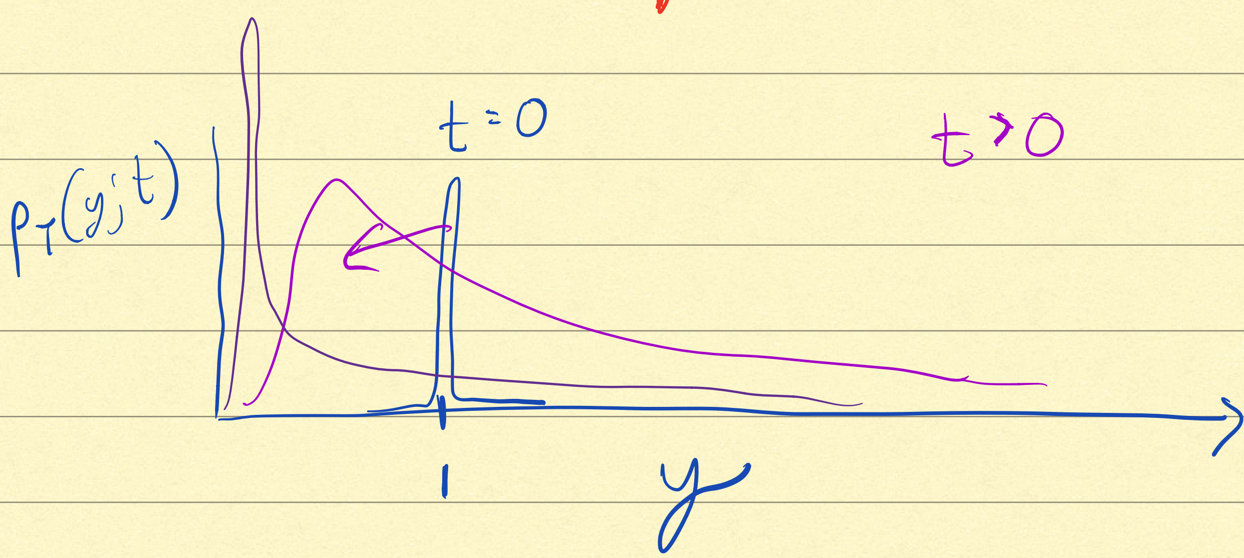
ROOM 2 CONJECTURE ASSUMING $y > 0$

$$p_Y(y;t) = \frac{1}{\sqrt{2\pi}\,\sigma_0 t\, y} \exp\left(- \frac{(\ln y)^2}{2(\sigma_0 t)^2} \right)$$

ROOM 1 CONJECTURE AGREES WITH ROOM 2!

ROOM 3 AGREES!

ROOM 4 AGREES!

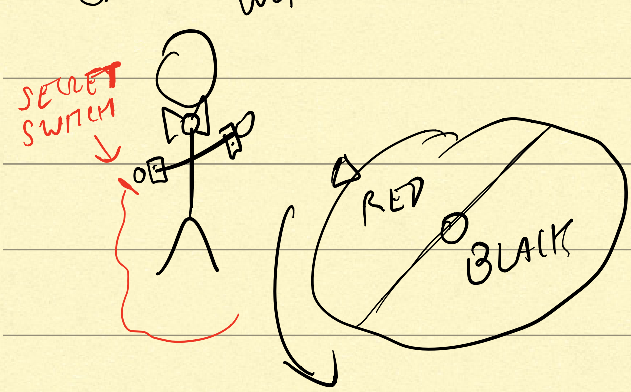


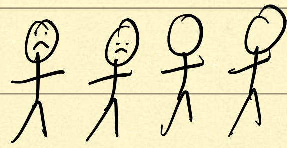
$$\mathbb{P}(Y > 1) = \int_1^\infty p_Y(y;t)\, dy$$

$$\mathbb{P}(Y < 1) = \int_0^1 p_Y(y;t)\, dy$$

PART II

CASINO WORKER





SPIN #1 SPIN #2 SPIN #3

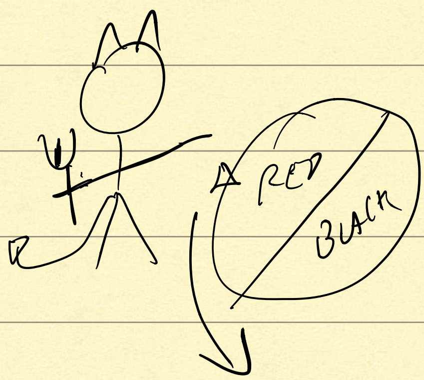
LOSS L L

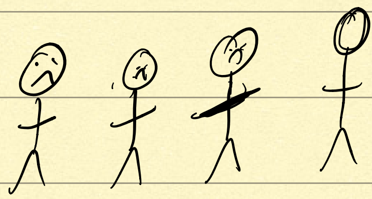
$\mathbb{P}(\text{SPIN } \#4 = L)$?

0.5 ← MAJORITY

$\mathbb{P} > 0.5$

PROFESSOR HAHAHA!





SPIN #1 SPIN #2 SPIN #3

L L L

$\mathbb{P}(\text{SPIN } \#4 = L) = ?$

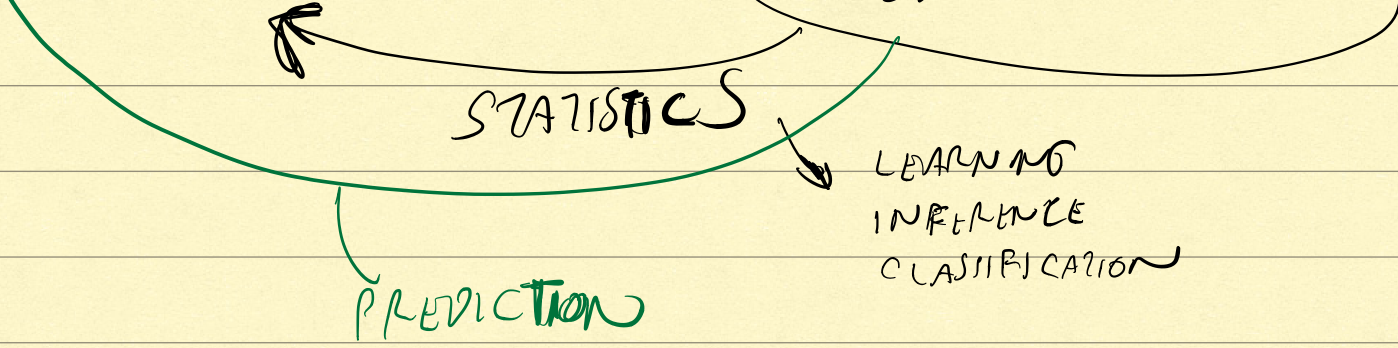
0.5 ← MAJORITY

PROBABILITY

MODEL → CONSEQUENCES

EX i.i.d.

OBSERVATION



RECALL

PARAMETER

PROBABILITY DENSITY EX $p_T(t) = \lambda e^{-\lambda t} = p_T(t; \lambda)$

PROBABILITY FUNCTION EX. $p_N(k) = \dfrac{\lambda^k e^{-\lambda}}{k!} = p_N(k; \lambda$

THE <u>LIKELIHOOD FUNCTION</u> IS THE PROBABILITY DENSITY (CONTINUOUS) OR PROBABILITY FUNCTION (DISCRETE), VIEWED AS A FUNCTION OF THE PARAMETERS

EX $$L(\lambda) = \lambda e^{-\lambda t}$$

NOTE $$\int L(\lambda)\, d\lambda \neq 1$$

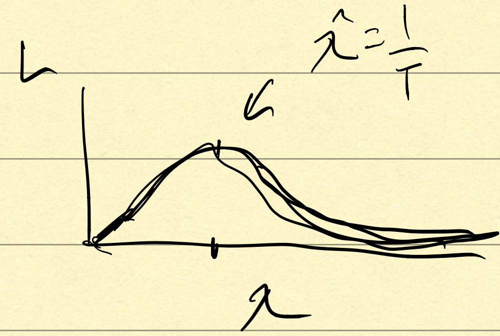
STRATEGY: TO FIND A PARAMETER θ FROM AN OBSERVATION X, TAKE

THE $\theta = \hat{\theta}$ THAT MAXIMIZES

$$L(\theta; X)$$

↳ MAXIMUM LIKELIHOOD ESTIMATION

EX T - TIME



$$L(\lambda) = \lambda e^{-\lambda t}$$

$$L(\lambda; T) = \lambda e^{-\lambda t}$$

MAXIMIZE: $\dfrac{\partial L}{\partial \lambda} = 0 \quad \dots \quad \hat{\lambda} = \dfrac{1}{T}$

EX SUPPOSE THERE ARE N OBSERVATIONS,
ASSUME i.i.d.

$$L(\theta) = \prod_{i=1}^{N} p(x_i; \theta)$$

EX N NORMAL RANDOM VARIABLES

$$L(\mu, \sigma) = \prod^{N} \frac{1}{\quad} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

$$\prod_{i=1} \ \sqrt{2\pi}\,\sigma \qquad 2\sigma$$

$$\frac{\partial L}{\partial \mu} = 0, \quad \frac{\partial L}{\partial \sigma} = 0 \quad \ldots$$

$$\hat{\mu} = \frac{1}{N} \sum_{i=1}^{N} X_i$$

$$\hat{\sigma} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (X_i - \hat{\mu})^2}$$

EX UNIFORM RANDOM VARIABLES

$$p_X(x; a, b) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{ELSE} \end{cases}$$