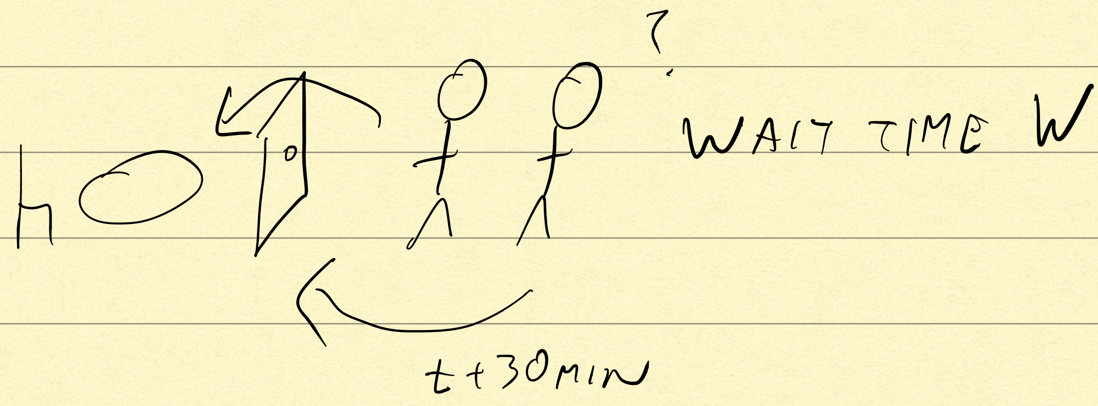
PS 4A



$$E[W] = E[W \mid A]\,\mathbb{P}(A) + E[W \mid B]\,\mathbb{P}(B)$$

A, B PARTITION S

OPTION 1:

FIRST MEETING DURATION

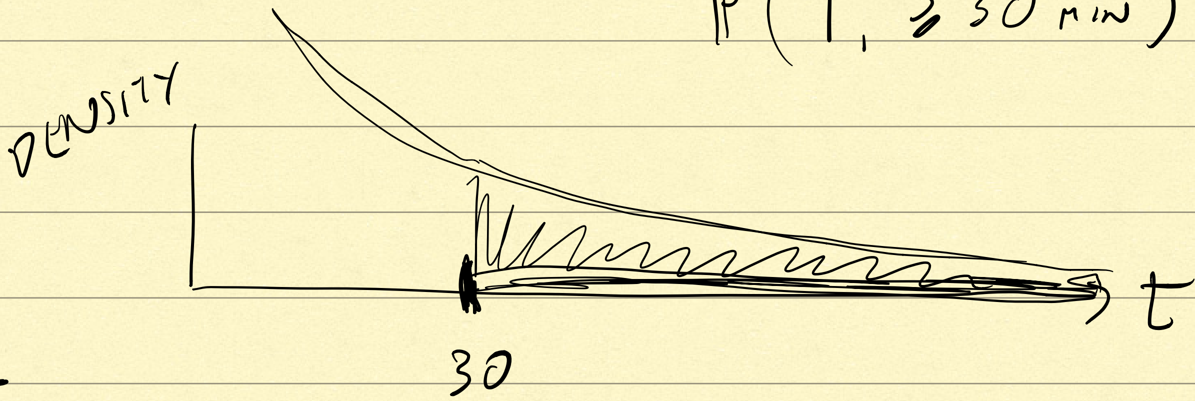
$$E[W] = E[W \mid T_1 < 30\text{min}]\,\mathbb{P}(T_1 < 30\text{min}) + E[W \mid T_1 \geq 30\text{min}] \cdot \mathbb{P}(T_1 \geq 30\text{min})$$

OPTION 2: K - # OF MEETINGS ENDING IN FIRST 30 MIN

$K \sim$ POISSON

$$E[W] = E[W \mid K \geq 1]\,\mathbb{P}(K \geq 1) + E[W \mid K = 0]\,\mathbb{P}(K = 0)$$

$$\mathbb{P}(T_1 \geq 30\,\text{min})$$



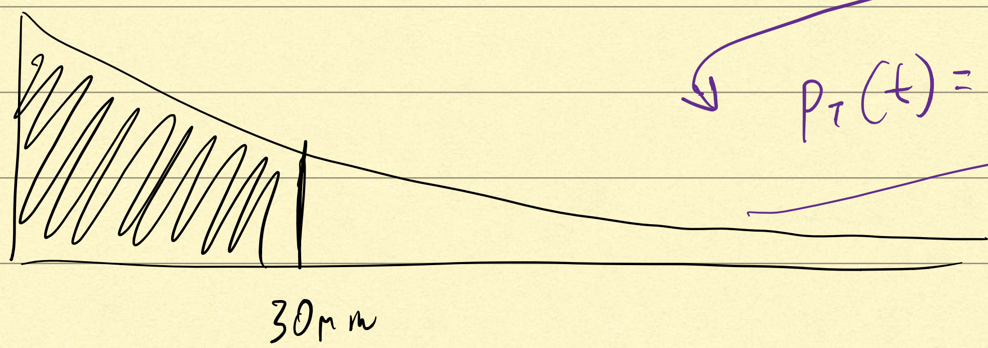
I

$p_T(t)$

$$\mathbb{P}(T_1 \geq 30\,\text{min}) = \int_{30\,\text{min}}^{\infty} p_T(t)\,dt$$

II

$$p_T(t) = \lambda e^{-\lambda t}$$



$$\mathbb{P}(T_1 \geq 30\,\text{min}) = 1 - \mathbb{P}(T_1 < 30)$$

$$= 1 - \int_0^{30} p_T(t)\,dt$$

III

$$\mathbb{P}(T_1 \leq t) = F_T(t)$$

$$\mathbb{P}(T_1 \geq 30\,\text{min}) = 1 - \mathbb{P}(T_1 < 30\,\text{min})$$

$$= 1 - \mathbb{P}(T_1 \leq 30\,\text{min})$$

$= 1 - F_T(T)$

$F_T(t) = 1 - e^{-\lambda t}$

PS4A

W ?

$W = \frac{30 \text{ min}}{e} \approx 11 \text{ min}$

$W = \frac{60 \text{ min}}{e} \cdot \frac{1}{e} \approx 8 \text{ min}$

$$E[W] = \underbrace{E[W \mid T_1 > 30 \text{ min}]}_{K=0} \cdot \underbrace{\mathbb{P}(T_1 > 30 \text{ min})}_{K=0}$$

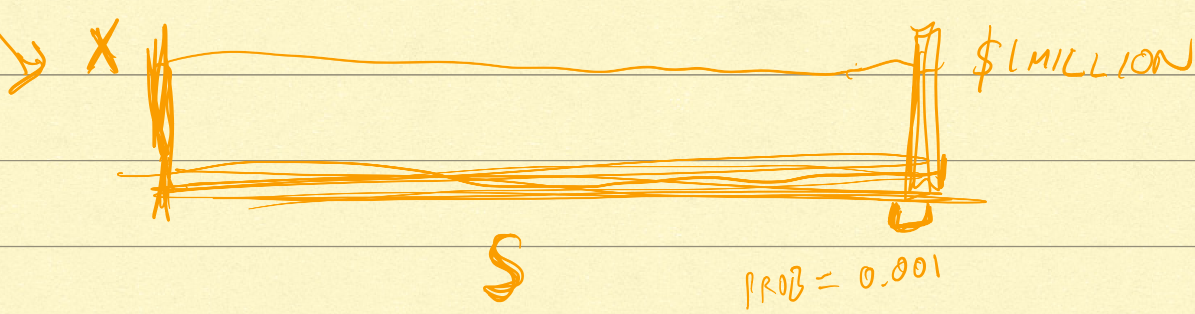
$\cdot \left(\frac{1}{e}\right)$

T_1 - DURATION OF FIRST MEETING

$$\boxed{E[T_1 \mid T_1 \in A] \overset{?}{=} \frac{\int_A t\, \rho_T(t)\, dt}{\int_A \rho_T(t)\, dt}}$$

$$E[T_1 \mid T_1 \in A] = \frac{\int_{30\,min}^{\infty} t\, \lambda e^{-\lambda t}\, dt}{\int_{30\,min}^{\infty} \lambda e^{-\lambda t}\, dt}$$

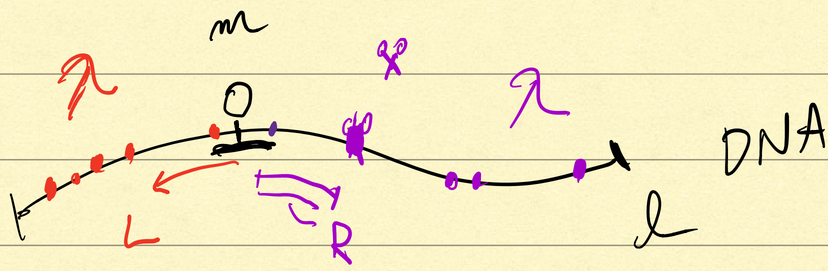
$$= \frac{\frac{60}{e}}{\frac{1}{e}} = 60\ min$$



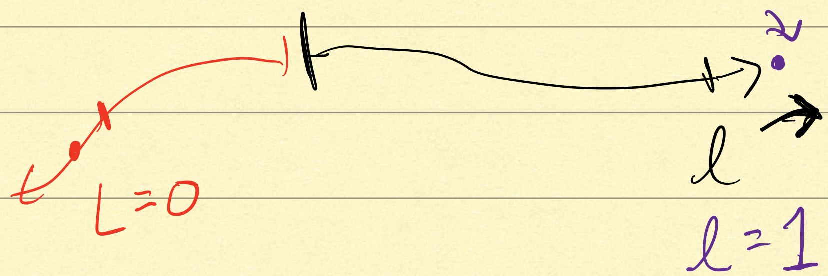
$$E[W \mid T_1 \in A] = 60\,min - 30\,min$$
$$= 30\,min$$

PS4B

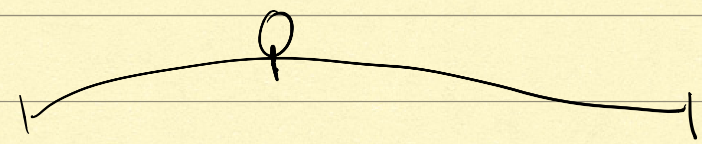
1x



$0 \le m \le \ell$



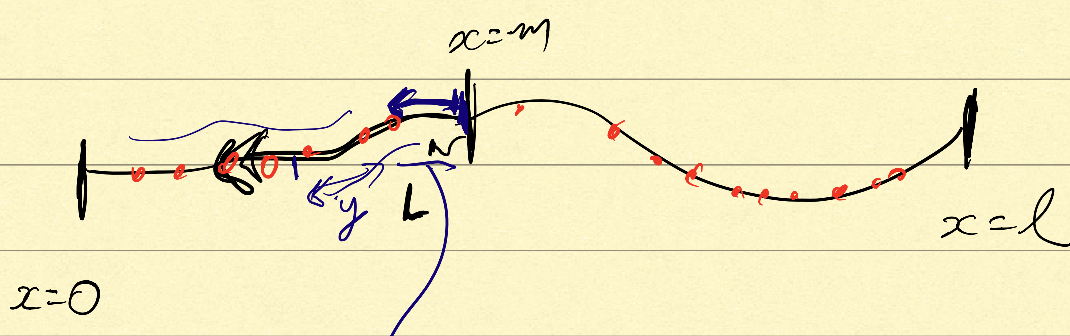
nX



AS $n \to \infty$, $E[L] \to m$

$E[R] \to m$

$$\mathbb{P}(L < x) \overset{?}{=} \left(1 - e^{-\lambda}\right)^{\frac{x}{m}}$$



$$Y = m - L$$

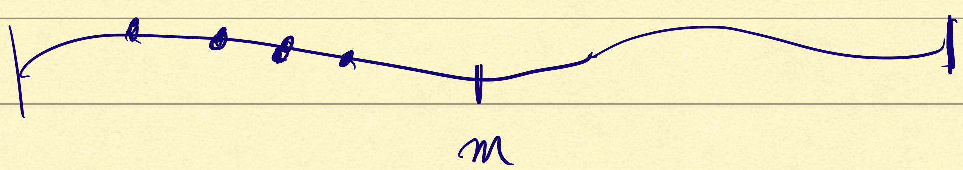
$$Y \sim \text{EXPONENTIAL}(\lambda)$$

$$P(Y \leq y) = 1 - e^{-\lambda y} \qquad 0 \leq y \leq m$$

$$P(L \geq y) = 1 - e^{-\lambda y} \qquad 0 \leq y \leq m$$

$$\downarrow$$

$$P(L \geq x) = 1 - e^{-\lambda x} \qquad 0 \leq x \leq m$$



\# OF EVENTS TO LEFT OF m

$$p(i) = \frac{(\lambda \Delta t)^i \cdot e^{-\lambda \Delta t}}{i!} \qquad i = 0 \ldots$$

$$= \frac{(\lambda m)^i e^{-\lambda m}}{i!}$$

$= e^{-\lambda m} = \mathbb{P}(\text{NO EVENTS TO LEFT})$

i) $= \mathbb{P}(L_1 = 0)$ ✓✓

iii) $p(i) = \dfrac{(\lambda(1-m))^i \, e^{-\lambda(1-m)}}{i!}$

? ? $p(i=0) = e^{-\lambda(1-m)} = \mathbb{P}(R_1 = 1)$

? $\neq$

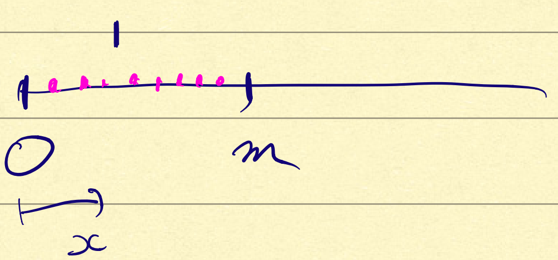
$\mathbb{P}(R_1 = 1) = 1 - e^{-\lambda m} + e^{-\lambda}$

ii) ? $\mathbb{P}(L_1 < x) = 1 - e^{-\lambda m}$?

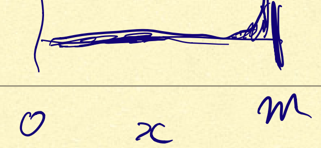
? $\mathbb{P}(L_1 < x) = 1 - e^{-\lambda x}$?

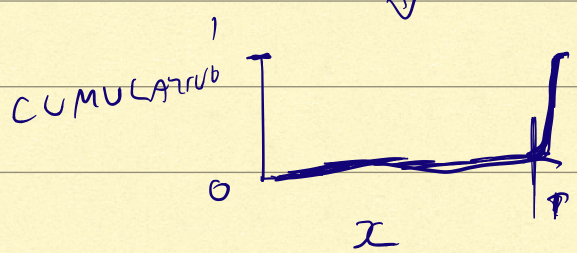
CHECK

$\lambda \to \infty$



DENSITY


0
x
m


1
CUMULATIVE
0
x