

Hi!

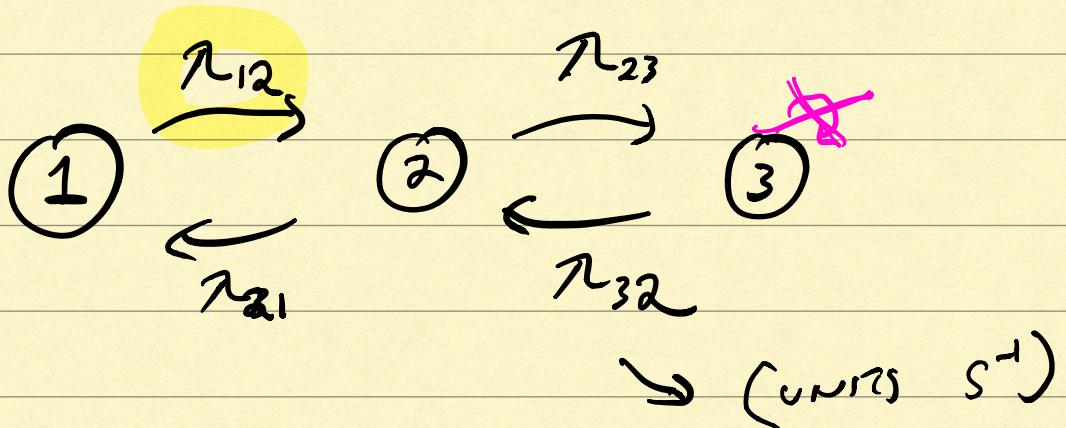
CONTINUOUS-TIME MARKOV CHAINS

CONTINUOUS-TIME STOCHASTIC PROCESS

WITH N DISCRETE STATES

TRANSITIONS BETWEEN STATES ARE POISSON

Ex



$P_i(t)$ PROBABILITY OF STATE i AT TIME t

$$\vec{P}(t) = \begin{bmatrix} P_1(t) \\ \vdots \\ P_N(t) \end{bmatrix} \quad \text{then}$$

$$\frac{d}{dt} \vec{P} = \begin{bmatrix} -\sum \pi_{i,j} & \pi_{21} & \cdots & \pi_{N1} \\ \pi_{12} & & & \\ \pi_{13} & & & \\ \vdots & & & \\ \pi_{1N} & & & \end{bmatrix} \cdot \vec{P}(t)$$

EX $N(t) - \# \text{ OF POISSON EVENTS}$

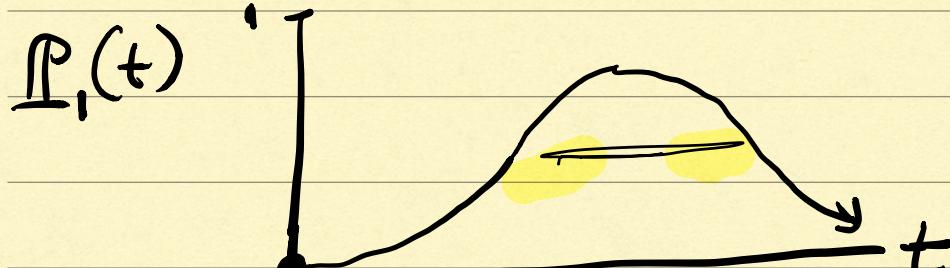
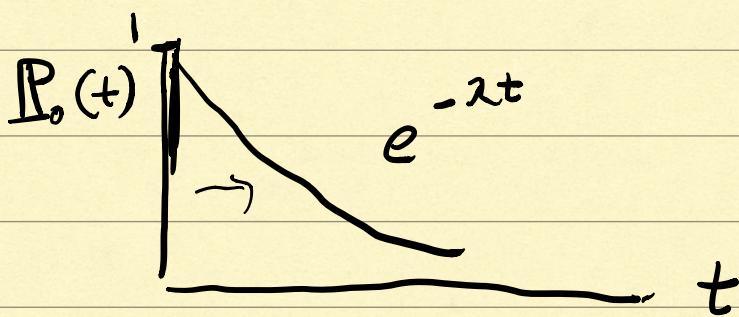
$$N(0) = 0$$

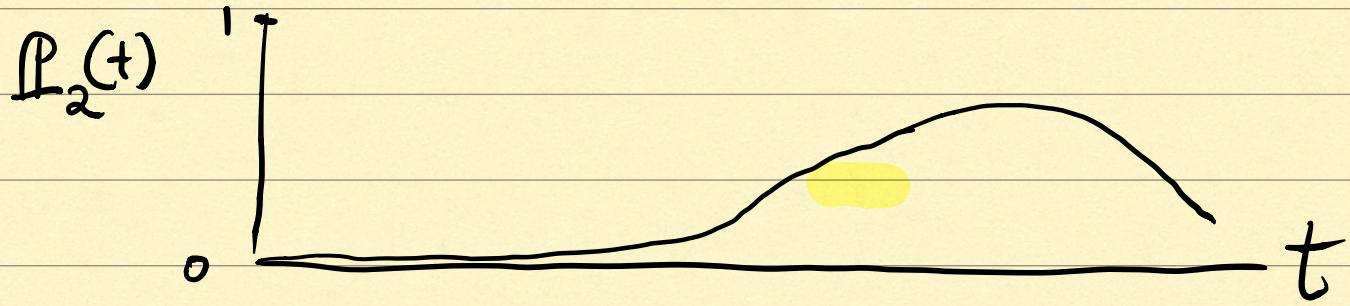
$$\frac{d}{dt} \vec{P} = \underline{M} \cdot \vec{P}$$

PURE
BIRTH
PROCESS

$$\begin{bmatrix} -\lambda & 0 & 0 \\ \lambda & -\lambda & 0 \\ 0 & \lambda & -\lambda \\ 0 & 0 & \lambda \\ 0 & \vdots & \vdots \end{bmatrix}$$

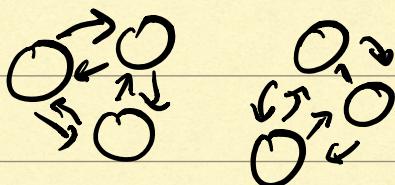
$$\vec{P}(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix}$$



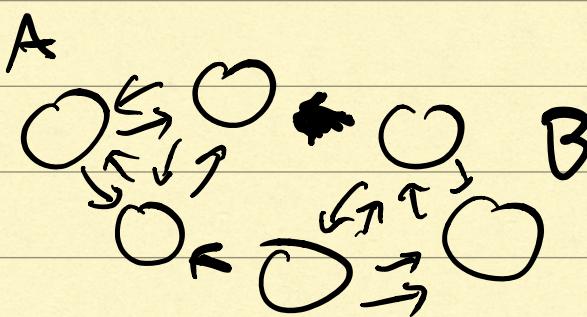


Ex

$$M = \left[\begin{array}{c|c} \text{wavy line} & \begin{matrix} \circ & \circ & \circ \\ \circ & \circ & \circ \\ \circ & \circ & \circ \end{matrix} \\ \hline \begin{matrix} \circ & \circ & \circ \\ \circ & \circ & \circ \\ \circ & \circ & \circ \end{matrix} & \text{wavy line} \end{array} \right]$$



$$M = \left[\begin{array}{c|c} \text{wavy line A} & \text{wavy line} \\ \hline \begin{matrix} \circ & \circ & \circ \\ \circ & \circ & \circ \\ \circ & \circ & \circ \end{matrix} & \text{wavy line B} \end{array} \right]$$

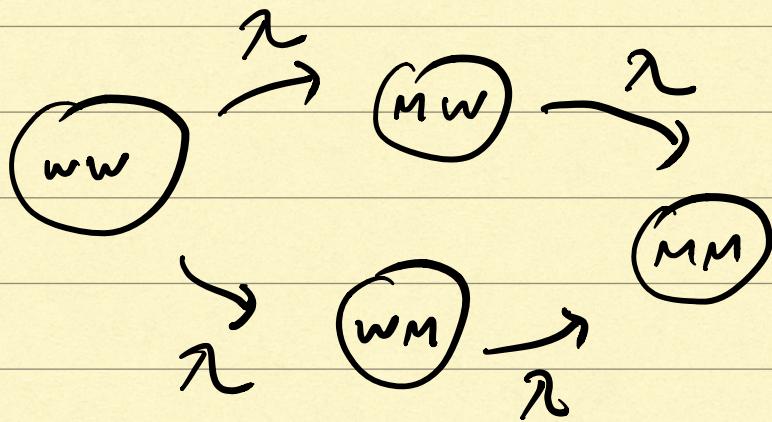


EX

WAITING FOR 2 MUTATIONS

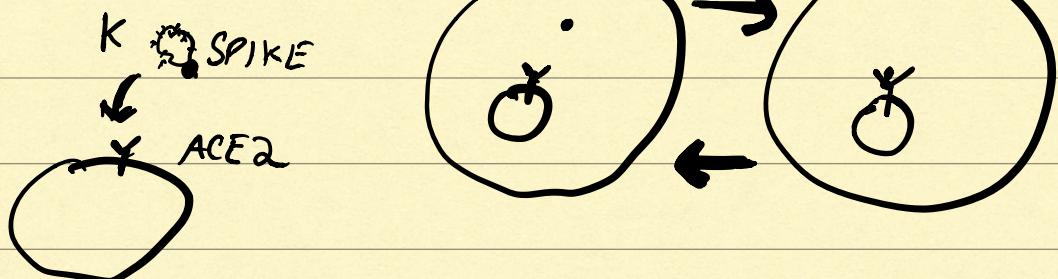
⇒ MAX TWO EVENTS

STATES: (w, w)
 (M, w)
 (w, M)
 (M, M)



$$M = \begin{bmatrix} -\lambda & 0 & 0 & 0 \\ \lambda & -\lambda & 0 & 0 \\ \lambda & 0 & -\lambda & 0 \\ 0 & \lambda & \lambda & 0 \end{bmatrix}$$

PS 5 A



$$\alpha = 0, \mu_2 = \lambda = 1$$

$$P_{\text{FREE}} = \frac{1}{2}$$

$$P_{\text{ANTAGONIST}} = \frac{1}{2}$$

$$P_{\text{AGONIST}} = 0$$

VINCENT HU : $P_{\text{FREE}} \neq \frac{\lambda}{\alpha\mu_1 + (1-\alpha)\mu_2 + \lambda}$

$$\lambda \rightarrow 0 \quad P_{\text{FREE}} \rightarrow 1 \quad ?$$

$$P_{\text{FREE}} \neq 1 - \frac{\lambda}{\alpha\mu_1 + (1-\alpha)\mu_2 + \lambda}$$

NARGES

$$P_{\text{FREE}} = \frac{\frac{1}{\lambda}}{\alpha \frac{1}{\mu_1} + (1-\alpha) \frac{1}{\mu_2}}$$

$$P_{\text{FREE}} \neq 1 - \frac{\alpha \frac{\lambda}{\mu_1} + (1-\alpha) \frac{\lambda}{\mu_2}}{\lambda}$$

VINCENT HU 2

$$\alpha = 0, \mu_1 = \mu_2 = \lambda = 1$$

$$P_{\text{FREE}} = \frac{1}{\lambda} = \frac{1}{0 + \frac{1}{\lambda} + \frac{1}{\lambda}} = \frac{1}{2}$$

$$\frac{\alpha}{\mu_1} + \frac{1-\alpha}{\mu_2} + \frac{1}{\lambda}$$

()



$$\lambda \rightarrow 0$$

$$P_{\text{FREE}} \rightarrow \frac{\frac{1}{\lambda}}{\frac{1}{\lambda}} \rightarrow 1$$

$$T_{\text{bound}} = \alpha T_{\text{bound}}^{\text{AG}} + (1-\alpha) T_{\text{bound}}^{\text{ANT}}$$

PS 5 A

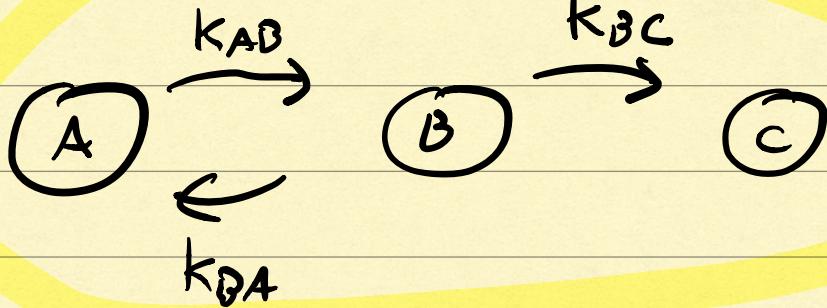
$$\frac{d}{dt} \begin{bmatrix} P_{\text{FREE}} \\ P_{\text{AG}} \\ P_{\text{ANT}} \end{bmatrix} = \begin{bmatrix} -\lambda & +\mu_1 & +\mu_2 \\ \alpha\lambda & -\mu_1 & 0 \\ (1-\alpha)\lambda & 0 & -\mu_2 \end{bmatrix} \begin{bmatrix} P_{\text{FREE}} \\ P_{\text{AG}} \\ P_{\text{ANT}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$P_{\text{FREE}} + P_{\text{AG}} + P_{\text{ANT}} = 1$$

()

MEAN FIRST PASSAGE TIMES

EX



$$M = \begin{bmatrix} -k_{AB} & k_{BA} & 0 \\ k_{AB} & -(k_{BA} + k_{BC}) & 0 \\ 0 & k_{BC} & 0 \end{bmatrix}$$

$\frac{d\vec{P}}{dt} = M \cdot \vec{P}$

C - ABSORBING
A, B - TRANSIENT

MEAN TIME IN A BEFORE LEAVING (FOREVER)

$$T_A = \frac{\int_0^\infty P_A(t) dt}{\int_0^\infty P_A(t) dt}$$

MEAN TIME TO C: $E[T_{A \rightarrow C}] = T_A + T_B$

$$\int_0^\infty \frac{dP_A}{dt} dt = -k_{AB} \int_0^\infty P_A dt + k_{BA} \int_0^\infty P_B dt$$

$$\int_0^\infty \frac{dP_B}{dt} dt = +k_{AB} \int_0^\infty P_A dt - k_{BA} \int_0^\infty P_B dt - k_{BC} \int_0^\infty P_B dt$$

$$\frac{d P_c}{dt} = \text{wavy line}$$

$$P_A(t) \Big|_0^\infty = 0 - 1 = -k_{AB} T_A + k_{BA} T_B$$

$$P_B(t) \Big|_0^\infty = 0 - 0 = +k_{AB} T_A - k_{BA} T_B - k_{BC} T_B$$

$$-1 = -k_{AB} T_A + k_{BA} T_B$$

$$0 = k_{AB} T_A - k_{BA} T_B - k_{BC} T_B$$

$$\text{THEN } E[T_{A \rightarrow C}] = T_A + T_B$$

IN GENERAL

LET M_{-j} - TRANSITION MATRIX

WITH j^{TH} ROW + COLUMN

REMOVED

$$\begin{array}{c} \text{i}^{\text{TH}} \\ \text{ROW} \end{array} \rightarrow \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 0 \end{bmatrix} = M_{-j} \cdot \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ \vdots \\ T_{n-1} \end{bmatrix}$$

$$E[T_{i \rightarrow j}] = \sum_k T_k$$

