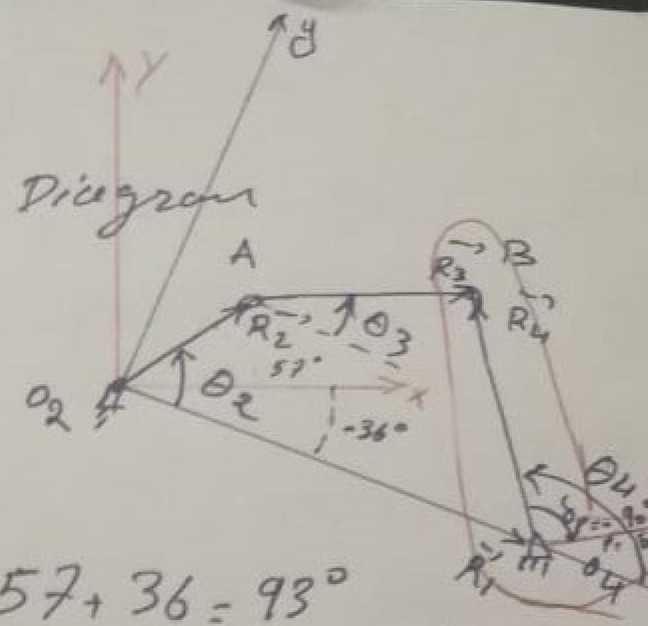
Problem 6-30: $\begin{cases} a=40 \\ b=96 \\ c=122 \\ d=162 \end{cases}$



-1/ First Draw the Kinematic Diagram

-2/ Draw the Local C.S.

-3/ Draw the vector loop

4/ calculate $\theta_{2local} = \theta_2 + \alpha = 57+36 = 93°$

5/ Apply the Formulas of the given mechanism (Position Analysis)

$K_1 = \frac{d}{a} = 4.05$ $K_2 = \frac{d}{c} = 1.3279$ $K_3 = \frac{a^2-b^2+c^2+d^2}{2ac} = 3.4336$

$$\begin{cases} A = \cos\theta_2 - K_1 - K_2\cos\theta_2 + K_3 = -0.5992 \\ B = -2\sin\theta_2 = -1.9973 \\ C = K_1 - (K_2+1)\cos\theta_2 + K_3 = 7.6054 \end{cases}$$

$\theta_{4_{1,2}} = 2\arctan\left(\frac{-B \pm \sqrt{B^2-4AC}}{2A}\right)$

$\theta_{4,1} = 2\arctan\left(\frac{1.9973+4.7136}{-1.1484}\right) = -159.75°$ Rejected

$\theta_{4,2} = 2\arctan\left(\frac{1.9973-4.7}{-1.1984}\right)$

$\boxed{\theta_4 = 132.386° \text{ Accepted}}$

$\theta_3 = \frac{-a\sin\theta_2 + c\sin\theta_4}{b} = \frac{-40\sin(93°)+122\sin(132.386°)}{96} = 0.522$

$\Rightarrow \boxed{\theta_3 = 31.5046°}$ verify on the Vector loop

6/ Velocity Analysis: Apply the formula Directly

$\omega_3 = \frac{a}{b}\omega_2 \frac{\sin(\theta_4-\theta_2)}{\sin(\theta_3-\theta_4)} = -5.385$ rad/s

$\omega_4 = \frac{a}{c}\omega_2 \frac{\sin(\theta_2-\theta_3)}{\sin(\theta_4-\theta_3)} = 5.868$ rad/s dt

$\vec{V_A} = a j \omega_2 e^{j\theta_2} = \cancel{a j \omega_2}\ a j \omega_2 (\cos\theta_2 + j \sin\theta_2) = a\omega_2(-\sin\theta_2 + j\cos\theta_2)$

$\vec{V_A} = -798.9 - 41.9j$ mm/s

$\|\vec{V_A}\| = \sqrt{798.9^2 + 41.9^2}$

$\|\vec{V_A}\| = 800$ mm/s

$\text{ang}\ \vec{V_A} = \arctan\left(\frac{-41.9}{-798.9}\right) = 3° + 180 = 183° = -177°$

should be verified

~~In the local C.S.~~

$x < 0$, $y < 0$ $\Rightarrow$ $+\pi$, $+180$

In the global C.S.:

$\|\vec{V_A}\| = 800$ mm/s (the same)

$\text{ang}\ \vec{V_A}_{global} = \text{ang}\ \vec{V_A}_{local} + \alpha = 183 + (-36°) = 147°$

8/ Added question: Find the velocity of Point P:

$\vec{R_{P/O_4}} = p e^{j(\theta_4 + \delta_4)} = p e^{j(\theta_4 - 90°)}$

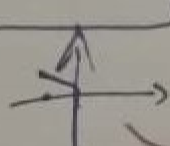
$\vec{V_P} = p j \omega_4 e^{j(\theta_4 - 90°)} = p j \omega_4 [\cos(\theta_4 - 90°) + j \sin(\theta_4 - 90°)] = p\omega_4(-\sin + j\cos$

$\vec{V_P} = 50(5.868)[-\sin(\underbrace{132.386° - 90°}_{42.386}) + j\cos(132.386 - 90°)]$

$\boxed{\vec{V_P} = -197.8 + j\,216.7 \text{ mm/s}}$

$\|\vec{V_P}\| = \sqrt{197.8^2 + 216.7^2} = 292.3$ mm/s

$\text{ang}\ V_P = \arctan\left(\frac{216.7}{-197.8}\right) = -47.6° + 180 = 132.$



verificatn

in the global C.S.:

$\|\vec{V_P}\| = 292.3$ the same

$\text{ang}\ \vec{V_P}_{global} = \text{ang}\ \vec{V_P}_{local} + \alpha = 132.4° + (-36°) = 96.4°$

Design of Machinery

Assignment

Chapters 4, 6 and 7

Position, velocity and acceleration analysis

Dr. Jaafar Hallal

$\vec{V_A} = \frac{d\vec{R_A}}{dt} = \frac{d}{dt}\left(a e^{j\theta_2}\right) = a j \omega_2 e^{j\theta_2}$

$\vec{V_A} = a\omega_2(-\sin\theta_2 + j\cos\theta_2)$ $\quad \|\vec{V_A}\| = \sqrt{1536^2+816.8^2} = 1740$ mm/s $\quad \arg\vec{V_A} = -28^\circ$

$\vec{V_A} = 1536 - 816.8j$ mm/s $\quad \arg\vec{V_A}\,glob = -28^\circ + (-25^\circ) = -53$

$\vec{V_B} = c\omega_4(-\sin\theta_4 + j\cos\theta_4)$ $\quad \|\vec{V_B}\| = 951.9$ mm/s $\quad \arg\vec{V_B} = -87.3^\circ$

$\vec{V_B} = 44.5 - 950.8$ mm/s $\quad \arg\vec{V_B}\,glob = -87^\circ + (-25^\circ) = 112^\circ$

Verified on the vector loop

Acceleration Analysis:

$\vec{A_A} = a\alpha_2(-\sin\theta_2 + j\cos\theta_2) - a\omega_2^2(\cos\theta_2 + j\sin\theta_2)$

$\vec{A_A} = -14814 - 21683j$ mm/s^2 $\quad \|\vec{A_A}\| = 26261$ mm/s^2

$\arg\vec{A_A} = -124^\circ$ $\quad \arg\vec{A_A}\,glob = -124 + (-25^\circ) = -149.3^\circ$

$\alpha_3 = \frac{CD - AF}{AE - BD}$

$\alpha_4 = \frac{CE - BF}{AE - BD}$

$\alpha_3 = 231.44$ rad/s^2

$\alpha_4 = -7.768$ rad/s^2

$A = c\sin\theta_4 = -5.145$

$B = b\sin\theta_3 = -107.57$

$D = c\cos(\theta_4) = -109.88$

$E = b\cos(\theta_3) = 9.662$

$F = a\alpha_2\cos\theta_2 - a\omega_2^2\sin\theta_2 - b\omega_3^2\sin\theta_3 + c\omega_4^2\sin\theta_4$

$= -1.38\times10^3$

~~2368 rad/s²~~

$C = a\alpha_2\sin\theta_2 + a\omega_2^2\cos\theta_2 + b\omega_3^2\cos\theta_3 - c\omega_4^2\cos\theta_4$

$= 2.49\times10^4$

$\vec{A_B} = \frac{d^2(R_{B/O_4})}{dt^2} = \frac{d^2}{dt^2}\left(c e^{j\theta_4}\right) = c\alpha_4(-\sin\theta_4 + j\cos\theta_4) - c\omega_4^2(\cos\theta_4 + j\sin\theta_4)$

$\vec{A_B} = 8189 + 1239j$ mm/s^2 $\quad \|\vec{A_B}\| = 8282$ mm/s^2

$\arg\vec{A_B} = 8.6^\circ$ $\quad \arg\vec{A_B}\,glob = 8.6^\circ + (-25^\circ) = -16.4^\circ$

1. Fourbar pin-jointed position analysis (4, 6.27, 7.21)

The angle between the X and x axes is 25-deg. Find the angular displacement of link 4 when link 2 rotates clockwise from the position shown (+37 deg) to horizontal (0 deg).

Find ω_4, V_A and V_B in the local coordinate system if ω_2=15 rad/s CW.

Find alpha2, A_A and A_B in the global coordinate system if $\alpha 2$= 25 rad/s^2 CCW.

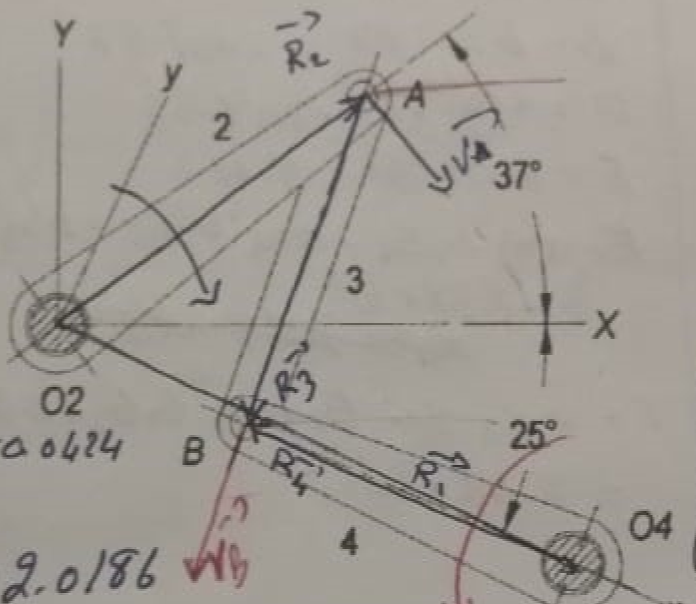
Take L2 = $\overset{a}{116}$ mm., L3 = $\overset{b}{108}$ mm., L4 = $\overset{c}{110}$ mm and L1= $\overset{d}{174}$ mm.

Position Analysis:

$K_1 = \frac{d}{a} = 1.5$

$K_2 = \frac{d}{c} = 1.5818$

$K_3 = \frac{a^2-b^2+c^2+d^2}{2ac} = 1.7307$



γ $\boxed{-25^\circ}$

global to local

$\theta_{2local} = \theta_{2global} - \gamma$

$\theta_{2local} = 37^\circ - (-25)$

$\theta_{2local} = 62^\circ$

$A = \cos(\theta_2) - K_1 - K_2\cos\theta_2 + K_3 = -0.0424$

$B = -2\sin\theta_2 = -1.7659$

$C = K_1 - (K_2+1)\cos\theta_2 + K_3 = 2.0186$

$\Rightarrow \theta_4 = 2\arctan\left(\frac{-B \pm \sqrt{B^2-4AC}}{2A}\right) = 182.681^\circ$ only the accepted.

$\theta_3 = 275.13^\circ$ $\quad b\sin\theta_3 = -a\sin\theta_2 + c\sin\theta_4$

Velocity Analysis:

$\omega_3 = \frac{a}{b}\omega_2 \frac{\sin(\theta_4-\theta_2)}{\sin(\theta_3-\theta_4)} = -13.87$ rad/s

$\omega_4 = \frac{a}{c}\omega_2 \frac{\sin(\theta_2-\theta_3)}{\sin(\theta_4-\theta_3)} = 8.65$ rad/s

2. Fourbar slider crank

For the compressor shown in the following figure, find the position of the piston with respect to O_2. Also find the linear velocity of the piston as the crank rotates clockwise at constant rate of 120 rad/s.

Put clearly the vector loop and the global coordinate system. All dimensions are in inches.

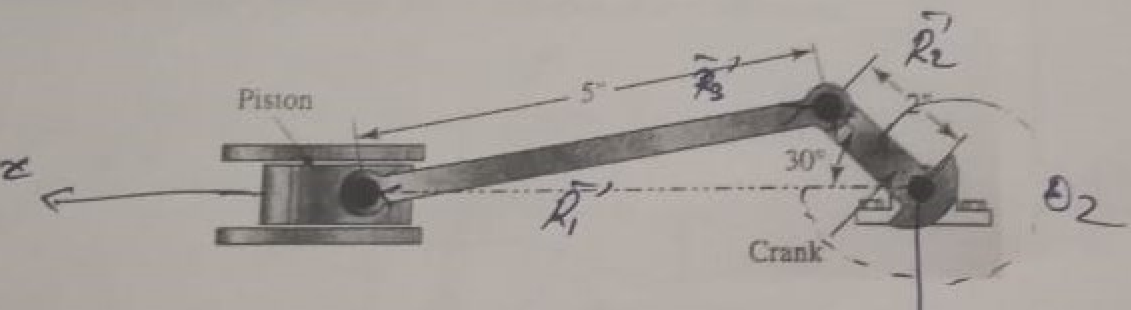
$a = 2''$

$b = 5''$

$c = 0$

$d = ?$

$\theta_2 = 330^\circ$



$\theta_{31} = \arcsin\left(\dfrac{a \sin\theta_2 - c}{b}\right) = -11.537^\circ$ Rejected

$\theta_{32} = \arcsin\left(-\dfrac{a \sin\theta_2 - c}{b}\right) + \pi = 191.537^\circ$ Accepted

$d = a\cos\theta_2 - b\cos\theta_3 = 6.32$ in

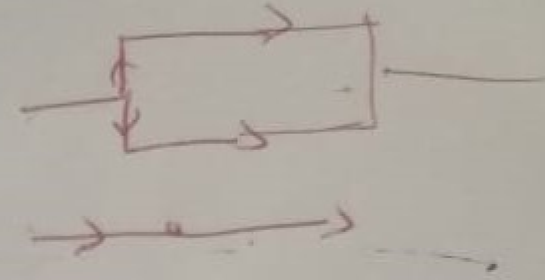
velocity analysis: $\omega_2 = -120$ rad/s

$\omega_3 = \dfrac{a}{b}\dfrac{\cos\theta_2}{\cos\theta_3}\omega_2 = 42.43$ rad/s ccw

$\dot{d} = -a\omega_2 \sin\theta_2 + b\omega_3 \sin\theta_3 = -162.4$ in/s $\rightarrow$

3. Sixbar

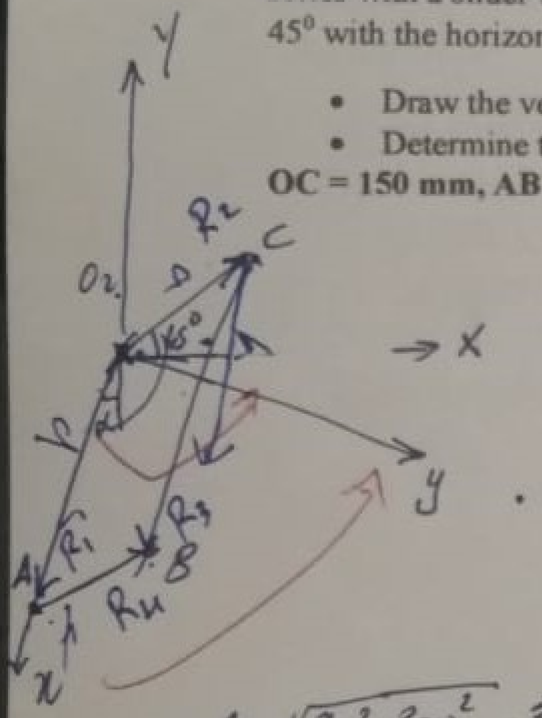
~~Question 2~~

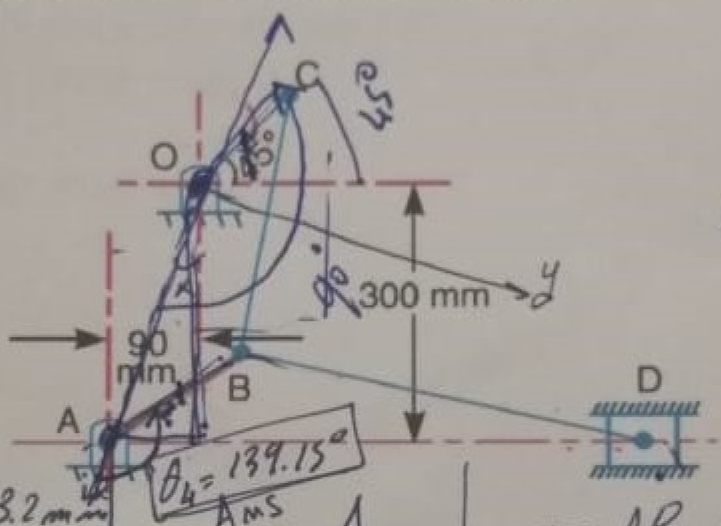


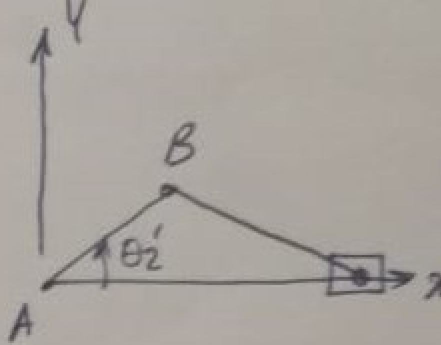
The device shown in figure below can be analyzed as a pin-jointed fourbar mechanism in series with a slider-crank mechanism. At the instant shown, the crank OC makes an angle 45^0 with the horizontal axis;

- Draw the vector loop of the two mechanisms separately
- Determine the position of the slider D with respect to A.

OC = 150 mm, AB = 200 mm, BC = 300 mm, BD = 450 mm.







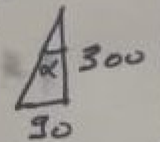
$\theta_2' = \theta_4 - 90° - \alpha$

$\boxed{\theta_2' = 32.45°}$

$OA = \sqrt{90^2 + 300^2} = 313.2\,mm$

$\alpha = \tan^{-1}\left(\frac{90}{300}\right) = 16.7°$

AMS



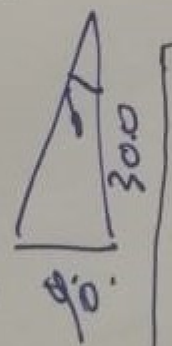
$\Rightarrow a = OC = 150\,mm$

$b = BC = 300\,mm$

$c = AB = 200\,mm$

$d = OA = 313.209\,mm$

$\theta_2 = \alpha + 90° + 45° = 151.7°$



$K_1 = \frac{d}{a} = 2.088 \quad K_2 = \frac{d}{c} = 1.566$

$K_3 = \frac{a^2 - b^2 + c^2 + d^2}{2ac} = 1.1767$

$A = -0.413 \quad B = -0.948 \quad C = 5.52$

$\theta_{41} = 139.15$ Accepted

$\theta_{42} = -157.24$ Rejected

$a = AB = 200\,mm$

$b = BD = 450\,mm$

$c = 0$

$\theta_{31} = \sin^{-1}\left(\frac{a \sin\theta_2 - c}{b}\right) = 13.74°$ Rejected

$\theta_{32} = \sin^{-1}\left(-\frac{a \sin\theta_2 - c}{b}\right) + \pi = 166.26°$ Accepted

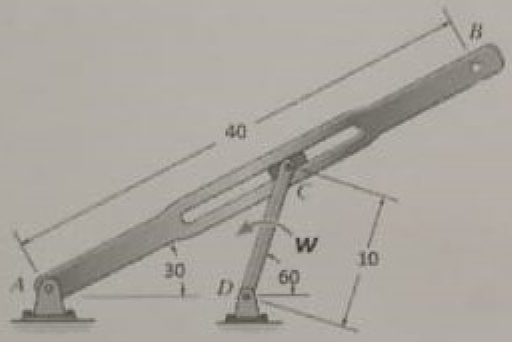
$d = a\cos\theta_2 - b\cos\theta_3 = 605.79\,mm$

4. Quick return mechanism

For the quick-return mechanism shown below, the rod *DC* rotates with a constant angular velocity of 1 rad/s CCW. For the position shown find:

a) the position of the sliding block *C* with respect to A
b) the angular velocity of member *AB* and the velocity of sliding of block *C* within the member *AB*
c) the velocity of the node B.

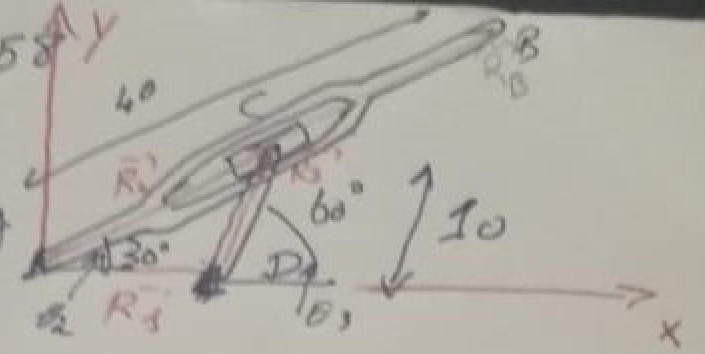
Dimensions are in inches.



→

Same from chile slide nb 58

$\omega_2 = 1$ rad/s ccw



1/ Draw the C.S. and the vector loop
try to choose the best C.S.

2/ write the vector loop equation

$$\vec{R_2} - \vec{R_3} - \vec{R_1} = \vec{0}$$

$$\boxed{(AC)e^{j\theta_2} - (DC)e^{j\theta_3} - (AD)e^{j\theta_1} = 0}$$

Note that Always $\theta_1 = 0$

$$(AC)[\cos\theta_2 + j\sin\theta_2] - (DC)[\cos\theta_3 + j\sin\theta_3] - (AD)[\cos\theta_1 + j\sin\theta_1] = 0$$

$$\begin{cases} \underline{(AC)}(\cos\underset{30}{\theta_2}) - (\underset{10}{DC})\cos(\underset{60}{\theta_3}) - \underline{(AD)} = 0 \\ \underline{(AC)}(\sin\underset{30}{\theta_2}) - (\underset{10}{DC})\sin(\underset{60}{\theta_3}) = 0 \end{cases}$$

$\Rightarrow AD = (AC)\cos\theta_2 - (DC)\cos(\theta_3)$
$\boxed{AD = 10}$

$\Rightarrow \boxed{AC = \dfrac{DC \sin\theta_3}{\sin\theta_2} = 17.32}$

or simply using the sine law

3/ Derive the vector loop equation with respect to the time

$$\Rightarrow : (\dot{AC})e^{j\theta_2} + (AC)\omega_2 j e^{j\theta_2} - (DC) j\omega_3 e^{j\theta_3} = 0$$

$$(\dot{AC})[\cos\theta_2 + j\sin\theta_2] + (AC)\omega_2[-\sin\theta_2 + j\cos\theta_2] - (DC)\omega_3[-\sin\theta_3 + j\cos\theta_3] = 0$$

$$\begin{cases} \underline{(\dot{AC})}\cos\underset{30}{\theta_2} - \underset{17.32}{(AC)}\underline{\omega_2}\sin\underset{30}{\theta_2} + \underset{10}{(DC)}\underset{1\,rd/s}{\omega_3}(\sin\underset{60}{\theta_3}) = 0 & \text{Real} \\ \underline{(\dot{AC})}\sin\underset{30}{\theta_2} + \underset{17.32}{(AC)}\underline{\omega_2}\cos\underset{30}{\theta_2} - \underset{10}{(DC)}\underset{1\,rd/s}{\omega_3}(\cos\underset{60}{\theta_3}) = 0 & \text{Imag} \end{cases}$$

2 eq 2 unknowns $\Rightarrow$ $\boxed{(\dot{AC}) = -0.05 \text{ m/s}}$ velocity of slip $\boxed{\omega_2 = 0.5 \text{ rad/s}}$

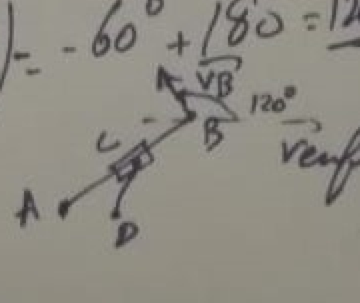
$\vec{v_B}$: $\vec{R_B} = (AB)e^{j\theta_B}$ $\theta_B = \theta_2 = 30°$ $|AB| = 40$

$$\vec{v_B} = (AB)j\omega_2 e^{j\theta_B} = (40)j(0.5)[\cos(30°) + j\sin(30°)] =$$

$$\vec{v_B} = -10 + 17.32j$$

$\|\vec{v_B}\| = 20$ m/s

angle $\vec{v_B}$ $= \tan^{-1}\left(\dfrac{17.32}{-10}\right) = -60° + 180 = 12$



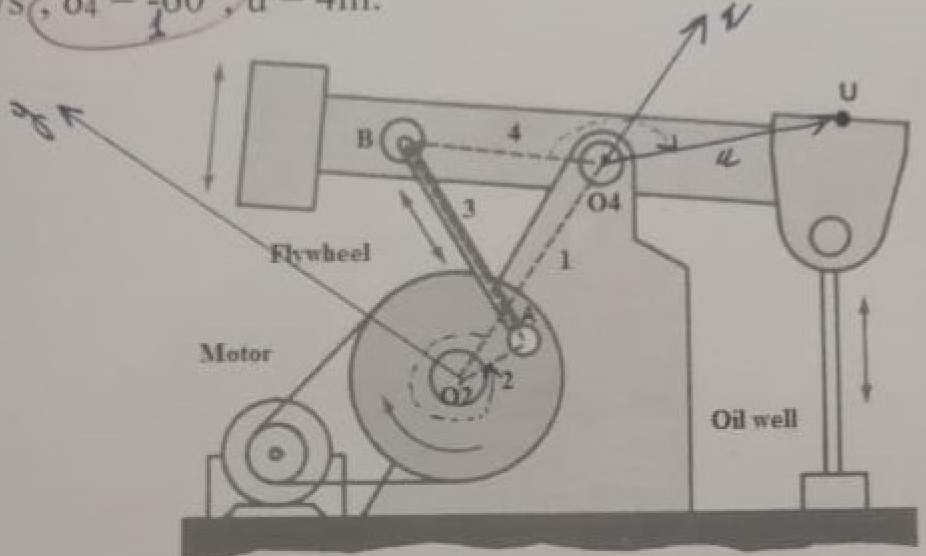
5. Fourbar pin-jointed mechanism

The device in the figure below is an oil well pump. Link number is shown on the figure.

d a b c

L1=4m; L2=1m; L3=3.5m and L4=3m

In the local coordinate systems, take θ_2=315°, ω2= 6 rad/s, α2= -1 rad/s², δ_4 = -60°, u = 4m.



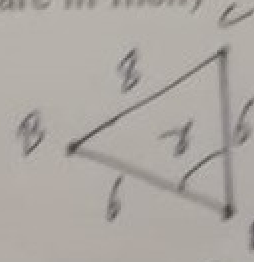
a) Draw the kinematic diagram of the mechanism and the vector loop.
b) Find the acceleration of point A, B and U

6. Sixbar mechanism

For the following six-bar drag-Link mechanism, the crank O_2A makes an angle $\theta_2 = 225°$.

a) Determine the Grashof conditions and Barker classification for the first Fourbar (O2-A-B-O4)

b) Find the position of the Slider 6. (Dimensions are in Inch)



$BC^2 = O_4B^2 + O_4C^2 - 2(O_4B)(O_4C)\cos(\gamma)$

$\Rightarrow \boxed{\gamma = 83.6°}$

$\theta_2 = 225°$

$k_1 =$

$k_2 =$

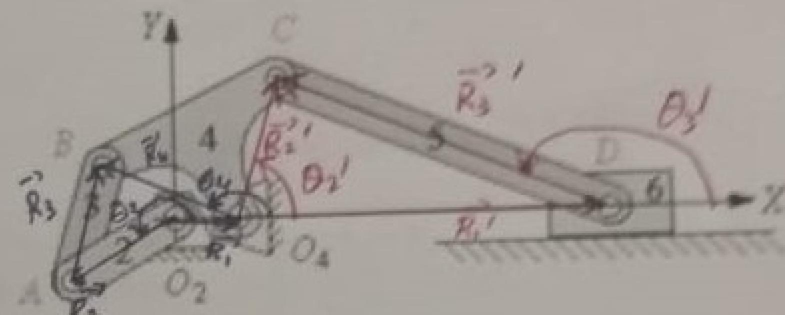
$k_3 =$

$A =$

$B =$

$C =$

$\theta_4 =$



$L_2 = 5 \quad L_3 = 5 \quad L_5 = 15 \quad BC = 8$

$O_2O_4 = 2.5 \quad O_4B = 6 \quad O_4C = 6$

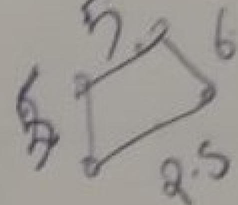
$\theta_2' = \theta_4 - \gamma$

$\Rightarrow \theta_3 =$

$d =$

the position of the slider D in the global C.S.

$x_D = O_2O_4 + d$



$9.5 \leq 10$

Grashof