Problem 3: (20 points)

An idealized incompressible flow has the proposed three-dimensional velocity distribution

$$\vec{V} = f(x)\vec{i} + zy^3\vec{j} - \frac{3}{2}z^2y^2\vec{k}$$

- Find the appropriate form of the function $f(x)$ which satisfies the continuity relation for incompressible flow.

$$v_x = u = f(x)$$

$$v_y = v = z y^3$$

$$v_z = w = \frac{-3}{2} z^2 y^2 .$$

Incompressible fluid flow

$$\Rightarrow \operatorname{div} \vec{v} = 0.$$

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

$$\frac{\partial f(x)}{\partial x} + \frac{\partial (zy^3)}{\partial y} + \frac{\partial}{\partial z}\left(-\frac{3}{2} y^2 z^2\right) = 0.$$

$$f'(x) + 3zy^2 - 3zy^2 = 0.$$

$$f'(x) = 0 \Rightarrow f(x) = \text{constant} = c.$$

Problem 4 (30 points)

The pump discharges water at *B* at 0.05 m^3 / s. Neglect the friction between the intake at *A* and the outlet at *B*. Take the power input (electric power) to the pump is 8 kW. The efficiency of the pump is $e = 0.7$.

a- Determine the output power of the pump (power received by the water)

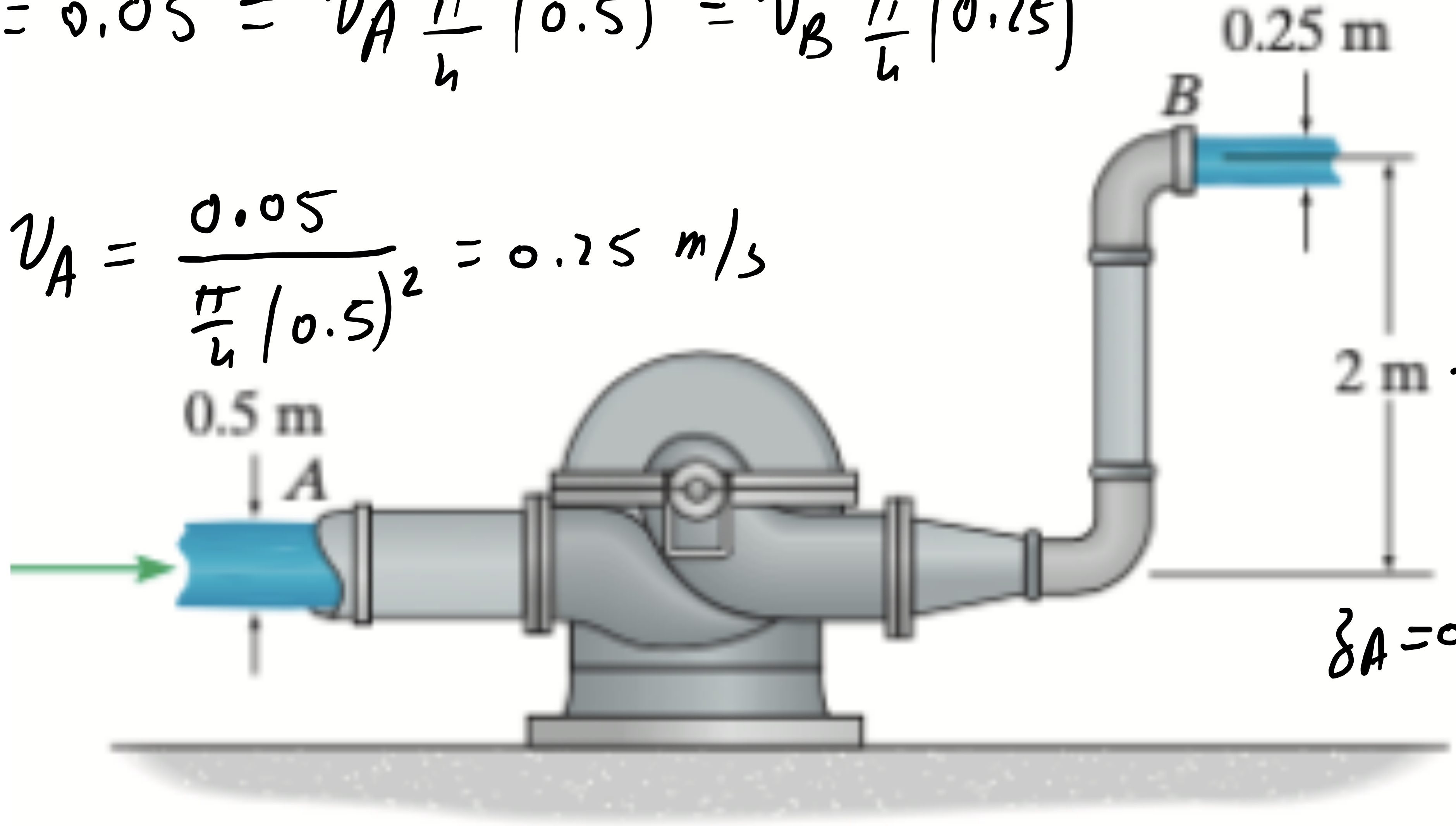
b- Determine the difference in pressure between *A* and *B* $(P_B - P_A) = ??$.

$q_v = 0.05 \ m^3/s$. $\qquad h_L = 0$.

$P_{in\ pump} = P_{elect} = 8\ Kw \qquad e = 0.7$.

$q_v = 0.05 = v_A \frac{\pi}{4}(0.5)^2 = v_B \frac{\pi}{4}(0.25)^2$

$\Rightarrow v_A = \dfrac{0.05}{\frac{\pi}{4}(0.5)^2} = 0.25 \text{ m/s}$



$2\text{ m} = z_B$

$z_A = 0.$

$v_B = \dfrac{0.05}{\pi/4\,(0.25)^2} = 1.01 \text{ m/s}$

a)

$$e = \frac{P_{out}}{P_{in}} = \frac{P_{mech}}{P_{elec}} = 0.7$$

$$P_{mech} = 0.7\, P_{elec} = 0.7(8) = 5.6\ \text{kW}.$$

$$P_{mech} = \gamma q_v h_p \Rightarrow h_p = \frac{P_{mech}}{\gamma q_v} = \frac{5600}{10^3 \times 9.81 \times 0.05}$$

$$h_p = 11.4\ \text{m}.$$

b) $P_B - P_A = ??$

work-energy equation is:

$$\frac{P_A}{\gamma} + z_A + \frac{v_A^2}{2g} + h_p = \frac{P_B}{\gamma} + z_B + \frac{v_B^2}{2g} + \cancel{h_T}^{\,0} + \cancel{h_L}^{\,0}$$

$$\left(\frac{P_B - P_A}{\gamma}\right) = z_A - z_B + \frac{v_A^2 - v_B^2}{2g} + h_p.$$

$$= |-2| + \left(\frac{0.25^2 - 1^2}{2 \times 9.81}\right) + (11.4) = 9.38\,m$$

$$P_B - P_A = \gamma(9.38) = 9810\,(9.38) = 91921 \text{ Pa}$$

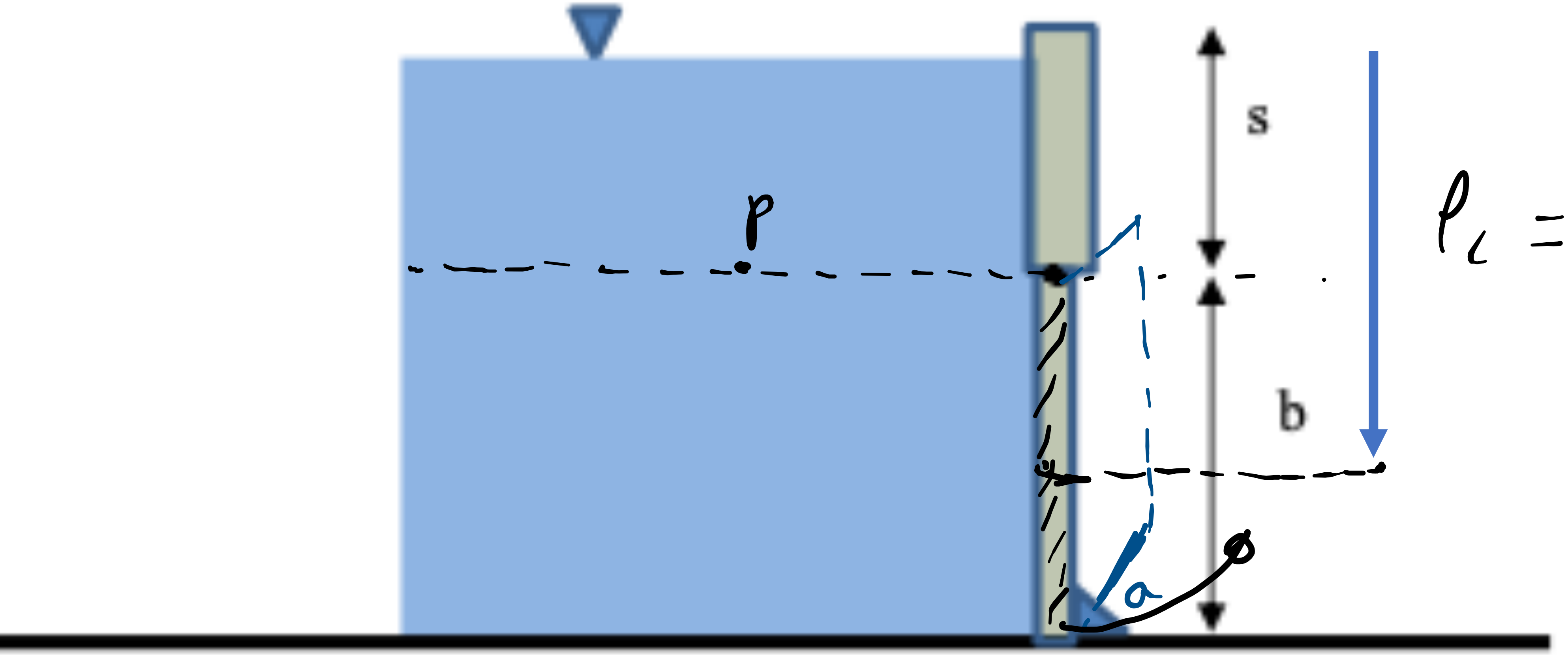
$$\approx 92 \text{ KPa.}$$

Problem 2: (25 points)

A rectangular gate of height ***b*** and width ***a*** (into the page) holds back water in a reservoir. (The gate can swing open to let some water out when necessary.) The height from the water surface to the hinge is s. take the density of water 1000 kg/m^3, $g = 9.8\ m/s^2$ and the moment of inertia of the gate $I_{xx} = \frac{a\,b^3}{12}$.

a- Determine the expression of the resultant force exerted by the water on the gate.

b- Find the location of the resultant force on the gate.



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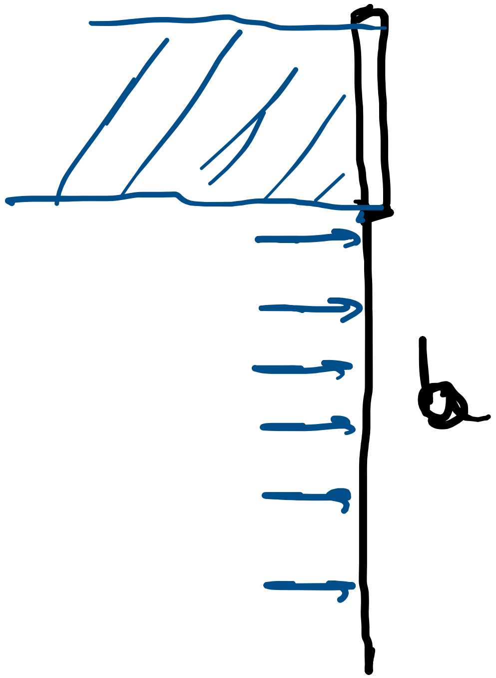
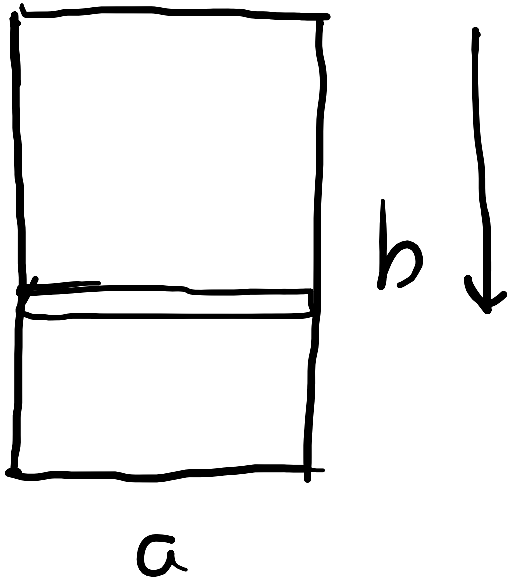
$$h_c = s + \frac{b}{2}.$$

$$A = a \times b.$$

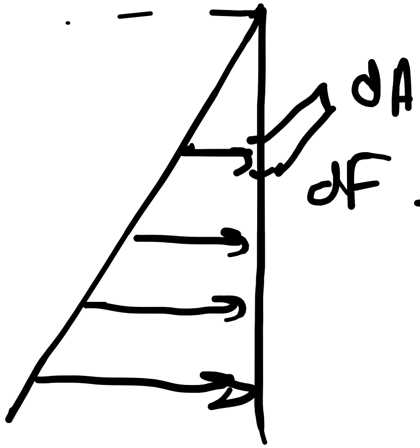




$F = ??$

$y_F = ??$

$l_{c_F} = l_{c_p}$.



$dF = P\,dA$.

$$F = \gamma\, \ell_c \times A$$

$$= \gamma\left(s+\frac{b}{2}\right)\underbrace{(a\times b)}_{A}.$$

$$= 9.81\times10^3\left(s+\frac{b}{2}\right)(a\times b)$$

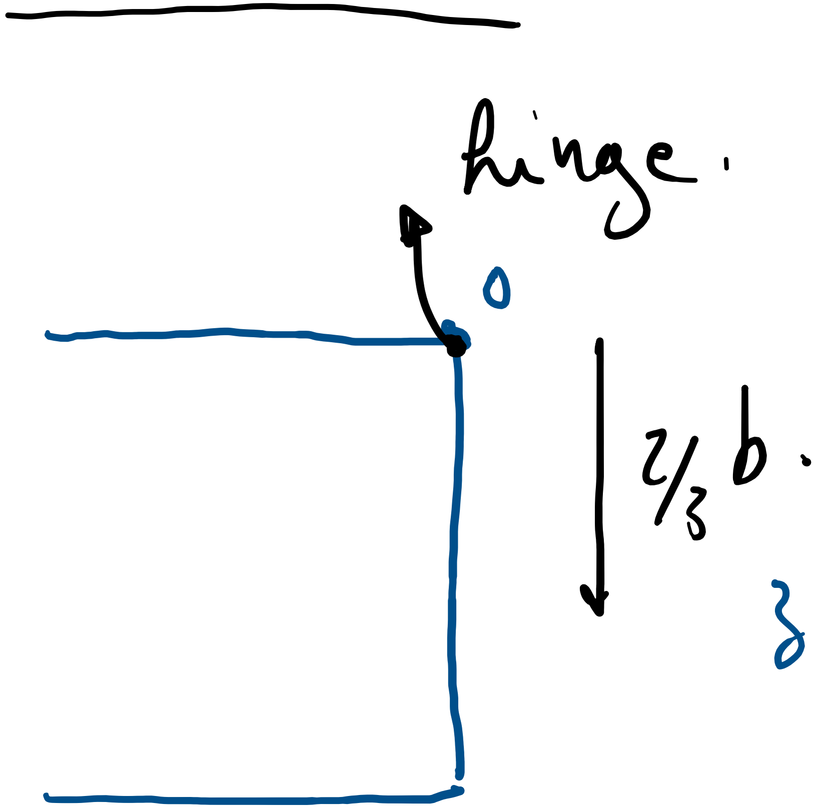
$$F = \underbrace{\gamma\, s\, A} + \gamma\frac{b}{2}A.$$

$A = a \times b.$

area of the gate.

$\ell_c =$ position of centroide.

center of area (gate).

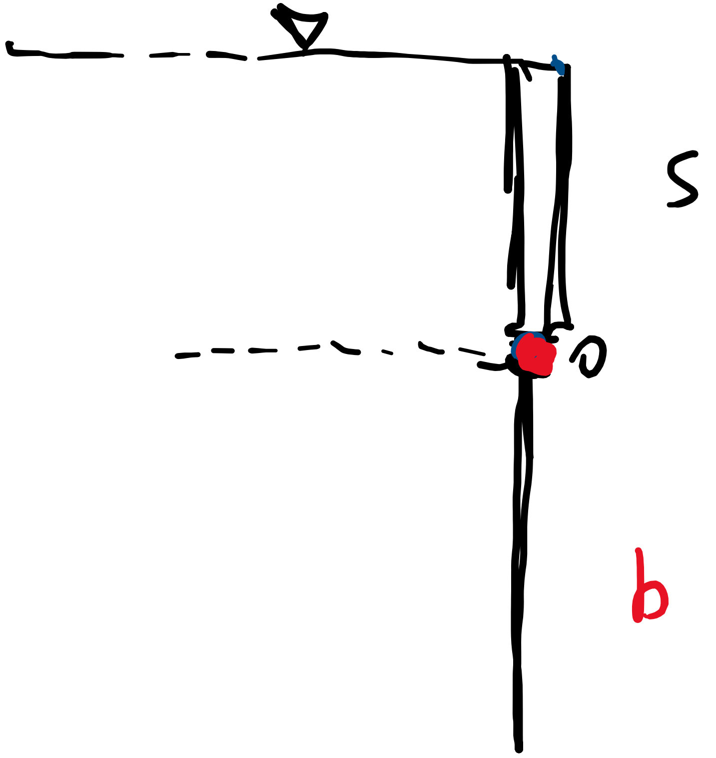


$$z = \frac{2}{3} b.$$

position of force.

$$\ell_p = \ell_c + \frac{I_{xx}}{\ell_c A}$$

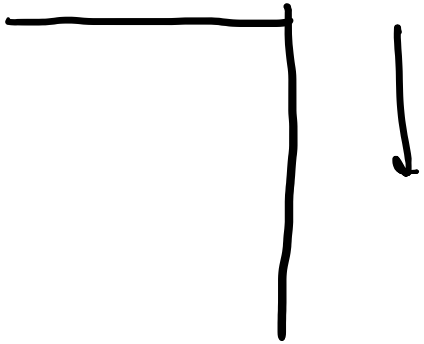
$$= \left(s + b/2\right) + \frac{\frac{\cancel{a}b^3}{12}}{\left(s+\frac{b}{2}\right)(\cancel{a}b)}$$



$$\ell_p = \left(s + \frac{b}{2}\right) + \frac{b^2}{\left(s+\frac{b}{2}\right)12}.$$

$$e_p = e_c + \frac{I}{e_c A}$$

$$= \frac{b}{2} + \frac{\frac{ab}{12}}{\frac{b}{2}(ab)}$$

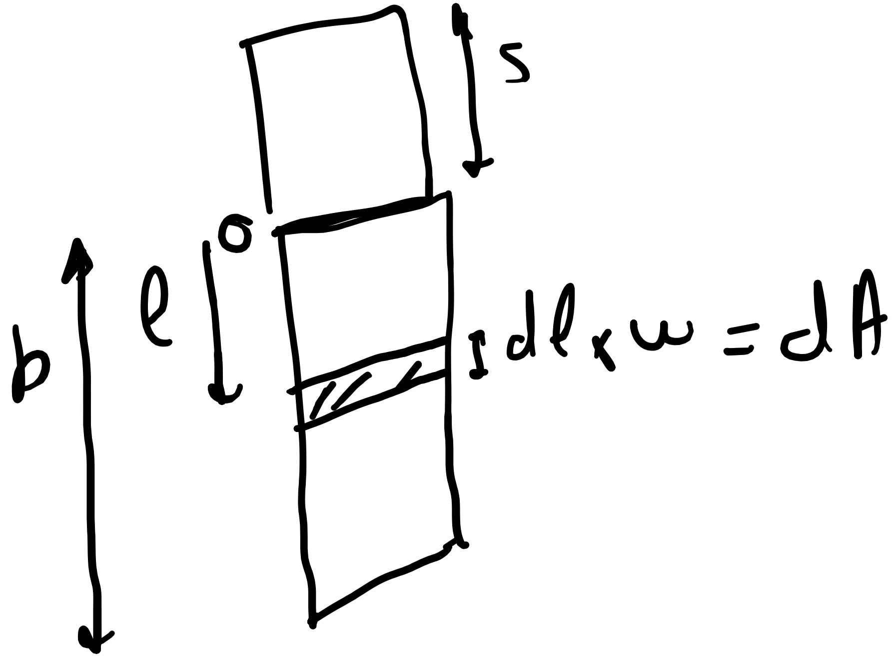


$e_c = \frac{b}{2}$.

$$\ell_F F = \int \ell \, dF \qquad dF = P \, dA$$

$$\ell_F F = \int_0^b \gamma(s+\ell)\ell w \, d\ell \qquad = \gamma(s+\ell) w \, d\ell.$$

$$0 \leq \ell \leq b$$

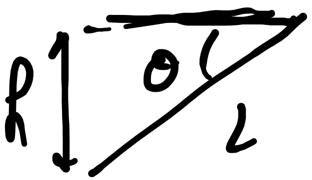


Compute the force F' required to keep the gate in equilibrium. The gate is 2 m wide and weighs 2 kN, L =2 m and ϑ = 30 degrees.

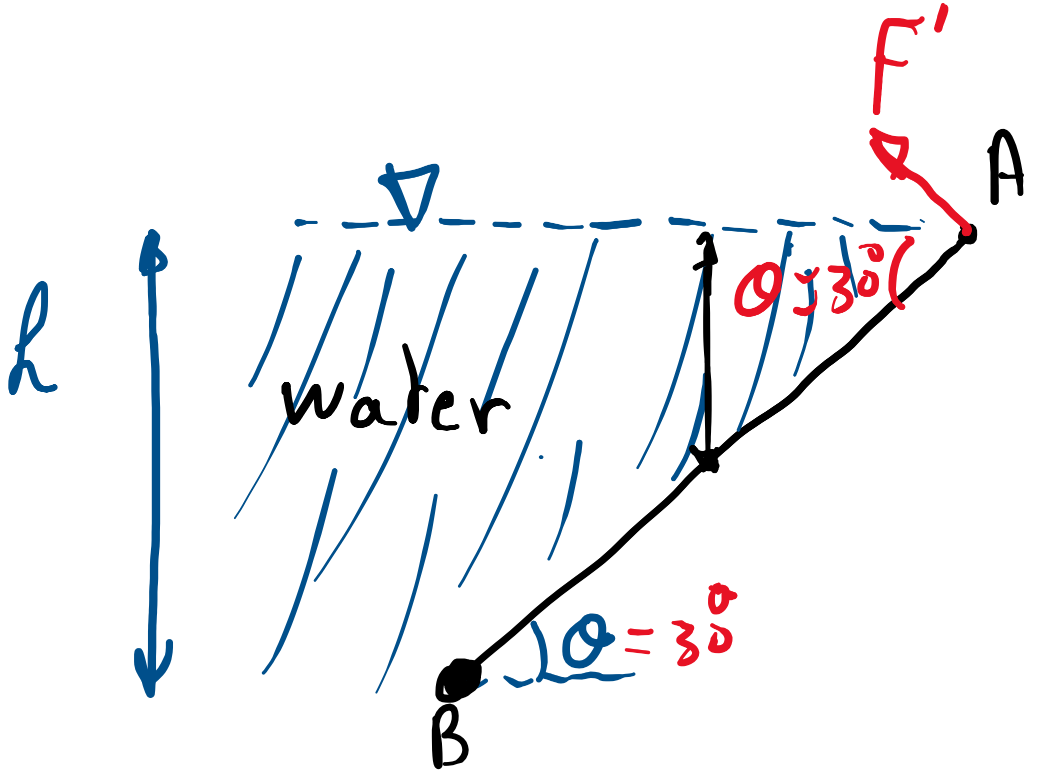
$$\sin 30^\circ = \frac{h}{L}$$

$$\Rightarrow h = L\sin\theta$$

$$= L\sin 30^\circ$$



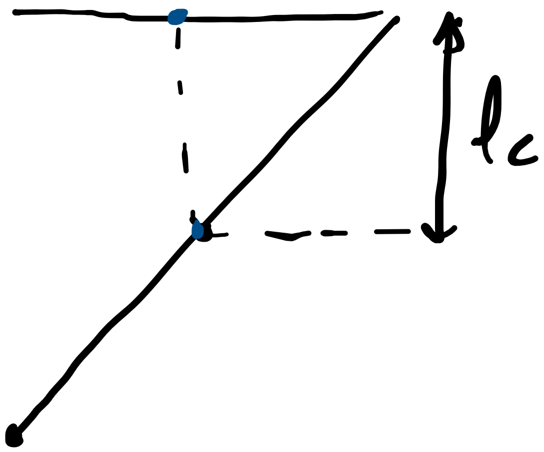
$$= 2\left(\frac{1}{2}\right) = 1\,m$$



$$F = \gamma \, \ell_c \cdot A$$

$$A = 2 \times 2 = 4 \, m^2.$$

$$\gamma_w = 9.81 \times 10^3$$



$$\sin 30^\circ = \frac{\ell_c}{L/2} \Rightarrow \ell_c = \frac{L}{2} \sin 30^\circ$$

$$= \sin 30^\circ$$

$$= \frac{1}{2} \, m.$$

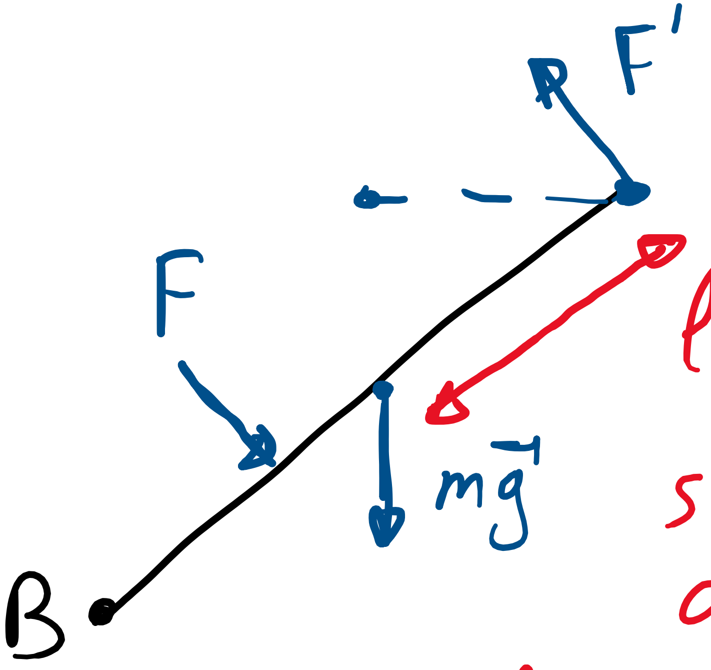
$$F = 9.81 \times 10^3 \left(\frac{1}{2}\right)(4) = 19.62 \times 10^3 \, N$$

$$= 19.62 \, KN.$$

ℓ_c' = slant distance

$$\ell_F = \underbrace{\ell_c'} + \frac{I}{\ell_c' A} =$$

$$=$$

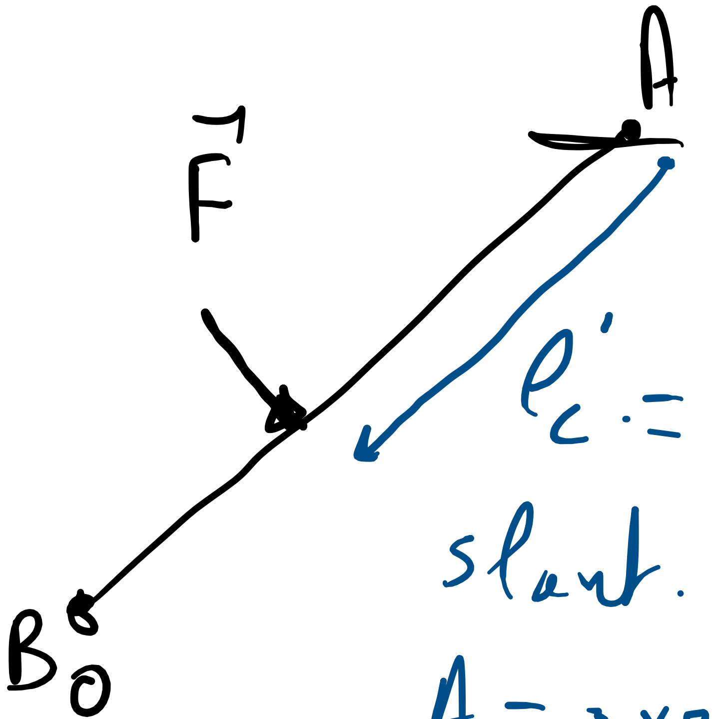


ℓ_c' = slant distance from water surface until centre of area.

$$I = \frac{bh^3}{12} = \frac{w \times L^3}{12} = \frac{2 \times 2^3}{12} = \frac{4}{3}.$$

$$\ell_P = \ell_c' + \frac{I}{\ell_c' A}$$

$$= 1 + \frac{4/3}{1(4)} = 1 + \frac{1}{3} = \frac{4}{3}.$$



$\ell_c' = \frac{L}{2} = \frac{2}{2} = 1m$

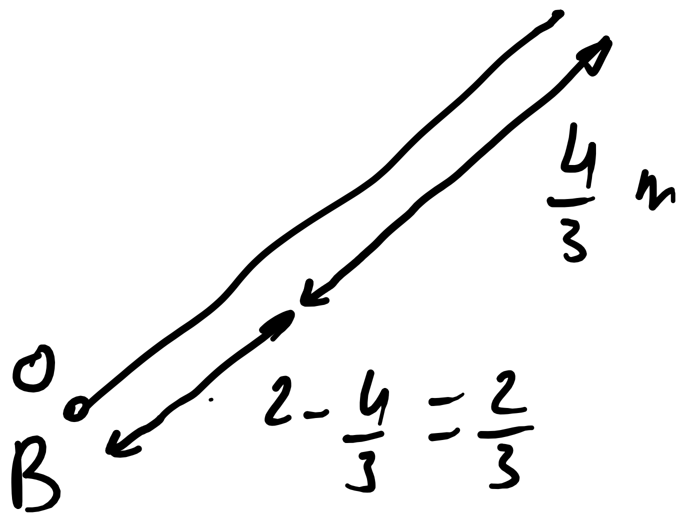
slant.

$A = 2 \times 2 = 4m^2$

$$\left.\begin{aligned} F &= 19.62\ KN. \\ r_{F/B} &= \tfrac{2}{3}\ m. \end{aligned}\right\}$$

$$\left\{\begin{aligned} w &= 2\ KN. \\ r_{w/B} &= 1\ m. \end{aligned}\right.$$

$$\left\{\begin{aligned} F' &= ?? \\ r_{F'/B} &= L = 2\ m. \end{aligned}\right.$$



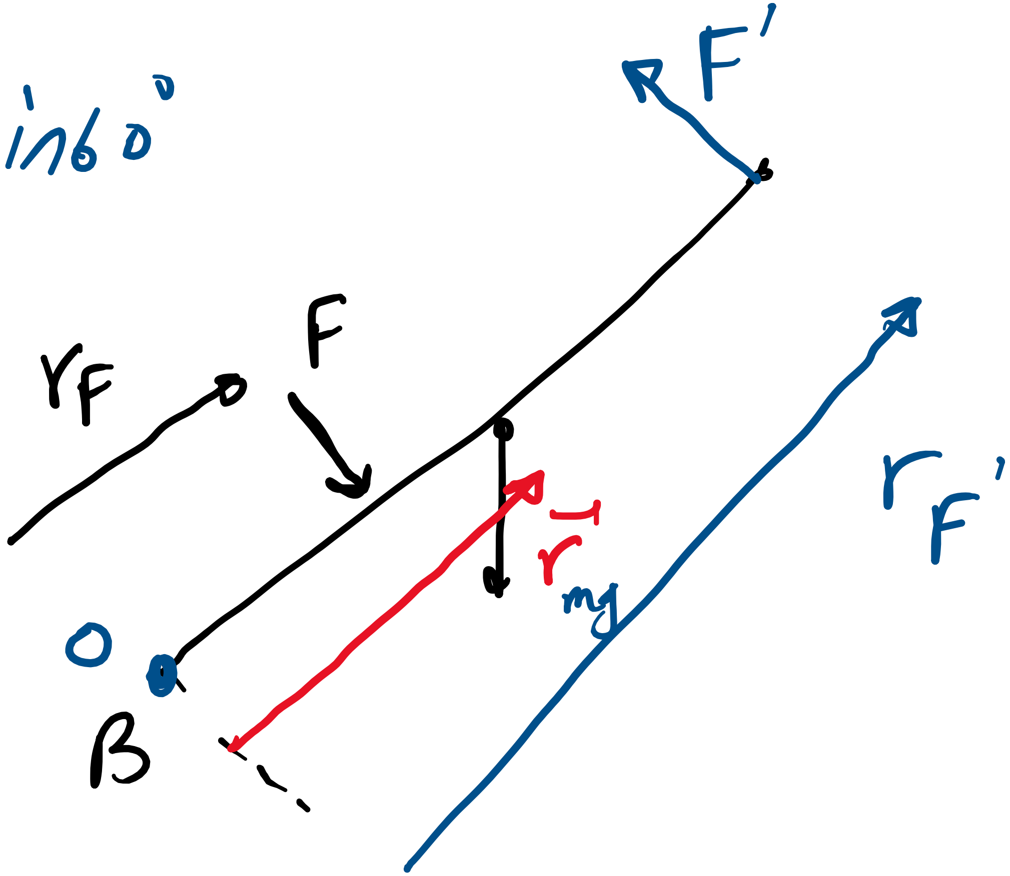
$$M_{F'/o} = M_F + \overset{mg\, r_{mg}}{M_{mg}}$$

$$2F' = \frac{2}{3} \times 19.62 + 2(1)\sin 60^\circ$$

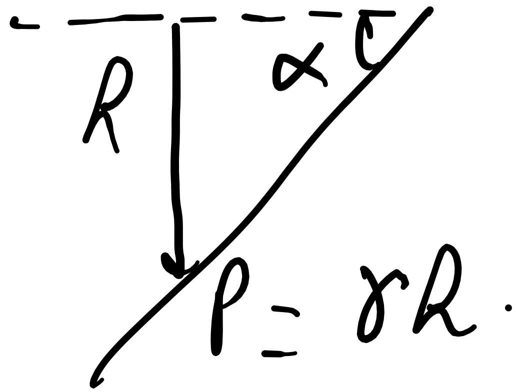
$$2F' = 13.08 + 1.7$$

$$2F' = 14.78$$

$$F' = 7.39 \text{ KN.}$$

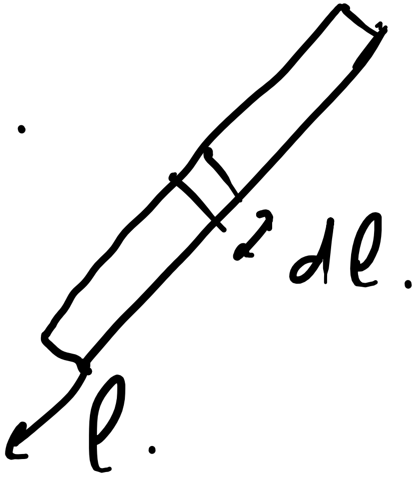


$$F_c = \gamma h_c A$$
$$= \iint p \, dA$$



$$p = \gamma \times h$$
$$= \gamma l \sin\alpha$$

$$dA = w \, dl$$



$$F_c = \iint p\,dA = \int_0^L \gamma\, l \sin\alpha \cdot w\,dl.$$

$$= \sin\alpha \cdot \gamma \; w \; \frac{l^2}{2} = \sin\alpha \times \gamma \times w \; \frac{L^2}{2}.$$

$$= \sin\alpha \cdot \gamma \; \frac{L}{2} \; A.$$

$$= \gamma \; \underbrace{l_c}_{\frac{L}{2} \cdot \sin\alpha} \; A.$$