Stress-Strain Behavior

6.3 A specimen of aluminum having a rectangular cross section 10 mm × 12.7 mm (0.4 in. × 0.5 in.) is pulled in tension with 35,500 N (8000 lb_f) force, producing only elastic deformation. Calculate the resulting strain.

Solution

This problem calls for us to calculate the elastic strain that results for an aluminum specimen stressed in tension. The cross-sectional area is just (10 mm) × (12.7 mm) = 127 mm^2 (= 1.27×10^{-4} m^2 = 0.20 $in.^2$); also, the elastic modulus for Al is given in Table 6.1 as 69 GPa (or 69×10^9 N/m^2). Combining Equations 6.1 and 6.5 and solving for the strain yields

$$\varepsilon = \frac{\sigma}{E} = \frac{F}{A_0 E} = \frac{35,500 \text{ N}}{\left(1.27 \times 10^{-4} \text{ m}^2\right)\left(69 \times 10^9 \text{ N/m}^2\right)} = 4.1 \times 10^{-3}$$

6.4 A cylindrical specimen of a titanium alloy having an elastic modulus of 107 GPa (15.5×10^6 psi) and an original diameter of 3.8 mm (0.15 in.) will experience only elastic deformation when a tensile load of 2000 N (450 lb_f) is applied. Compute the maximum length of the specimen before deformation if the maximum allowable elongation is 0.42 mm (0.0165 in.).

Solution

We are asked to compute the maximum length of a cylindrical titanium alloy specimen (before deformation) that is deformed elastically in tension. For a cylindrical specimen

$$A_0 = \pi\left(\frac{d_0}{2}\right)^2$$

where d_0 is the original diameter. Combining Equations 6.1, 6.2, and 6.5 and solving for l_0 leads to

$$l_0 = \frac{\Delta l}{\varepsilon} = \frac{\Delta l}{\dfrac{\sigma}{E}} = \frac{\Delta l\, E}{\dfrac{F}{A_0}} = \frac{\Delta l\, E\pi\left(\dfrac{d_0}{2}\right)^2}{F} = \frac{\Delta l\, E\pi\, d_0^2}{4F}$$

$$= \frac{(0.42 \times 10^{-3}\ \text{m})(107 \times 10^9\ \text{N/m}^2)(\pi)(3.8 \times 10^{-3}\ \text{m})^2}{(4)(2000\ \text{N})}$$

$$= 0.255\ \text{m} = 255\ \text{mm}\ (10.0\ \text{in.})$$

6.7 For a bronze alloy, the stress at which plastic deformation begins is 275 MPa (40,000 psi), and the modulus of elasticity is 115 GPa (16.7×10^6 psi).

(a) What is the maximum load that may be applied to a specimen with a cross-sectional area of 325 mm^2 (0.5 $in.^2$) without plastic deformation?

(b) If the original specimen length is 115 mm (4.5 in.), what is the maximum length to which it may be stretched without causing plastic deformation?

Solution

(a) This portion of the problem calls for a determination of the maximum load that can be applied without plastic deformation (F_y). Taking the yield strength to be 275 MPa, and employment of Equation 6.1 leads to

$$F_y = \sigma_y A_0 = (275 \times 10^6 \text{ N/m}^2)(325 \times 10^{-6} \text{ m}^2)$$

$$= 89{,}375 \text{ N} \quad (20{,}000 \text{ lb}_f)$$

(b) The maximum length to which the sample may be deformed without plastic deformation is determined from Equations 6.2 and 6.5 as

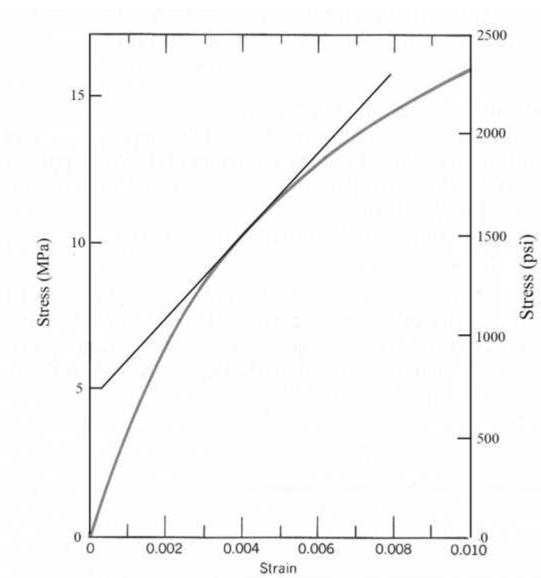
$$l_i = l_0\left(1 + \frac{\sigma}{E}\right)$$

$$= (115 \text{ mm})\left[1 + \frac{275 \text{ MPa}}{115 \times 10^3 \text{ MPa}}\right] = 115.28 \text{ mm} \quad (4.51 \text{ in.})$$

6.11 Figure 6.22 shows, for a gray cast iron, the tensile engineering stress–strain curve in the elastic region. Determine (a) the tangent modulus at 10.3 MPa (1500 psi), and (b) the secant modulus taken to 6.9 MPa (1000 psi).

Solution

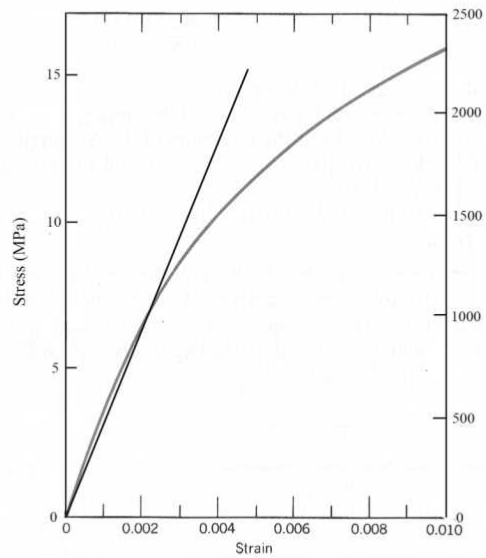
(a) This portion of the problem asks that the tangent modulus be determined for the gray cast iron, the stress-strain behavior of which is shown in Figure 6.22. In the figure below is shown a tangent draw on the curve at a stress of 10.3 MPa (1500 psi).



The slope of this line (i.e., $\Delta\sigma/\Delta\varepsilon$), the tangent modulus, is computed as follows:

$$\frac{\Delta\sigma}{\Delta\varepsilon} = \frac{15\text{ MPa} - 5\text{ MPa}}{0.0074 - 0.0003} = 1410\text{ MPa} = 1.41\text{ GPa } (2.04 \times 10^5\text{ psi})$$

(b) The secant modulus taken from the origin is calculated by taking the slope of a secant drawn from the origin through the stress-strain curve at 6.9 MPa (1,000 psi). This secant is drawn on the curve shown below:



The slope of this line (i.e., $\Delta\sigma/\Delta\varepsilon$), the secant modulus, is computed as follows:

$$\frac{\Delta\sigma}{\Delta\varepsilon} = \frac{15\text{ MPa} - 0\text{ MPa}}{0.0047 - 0} = 3190\text{ MPa} = 3.19\text{ GPa} \quad (4.63 \times 10^5\text{ psi})$$

Elastic Properties of Materials

6.15 A cylindrical specimen of aluminum having a diameter of 19 mm (0.75 in.) and length of 200 mm (8.0 in.) is deformed elastically in tension with a force of 48,800 N (11,000 lb_f). Using the data contained in Table 6.1, determine the following:

(a) The amount by which this specimen will elongate in the direction of the applied stress.

(b) The change in diameter of the specimen. Will the diameter increase or decrease?

Solution

(a) We are asked, in this portion of the problem, to determine the elongation of a cylindrical specimen of aluminum. Combining Equations 6.1, 6.2, and 6.5, leads to

$$\sigma = E\varepsilon$$

$$\frac{F}{\pi\left(\frac{d_0^2}{4}\right)} = E\,\frac{\Delta l}{l_0}$$

Or, solving for Δl (and realizing that E = 69 GPa, Table 6.1), yields

$$\Delta l = \frac{4F\,l_0}{\pi\,d_0^2 E}$$

$$= \frac{(4)(48{,}800\ \mathrm{N})\left(200 \times 10^{-3}\,\mathrm{m}\right)}{(\pi)\left(19 \times 10^{-3}\,\mathrm{m}\right)^2\left(69 \times 10^9\,\mathrm{N/m^2}\right)} = 5 \times 10^{-4}\ \mathrm{m} = 0.50\ \mathrm{mm}\ (0.02\ \mathrm{in.})$$

(b) We are now called upon to determine the change in diameter, Δd. Using Equation 6.8

$$\nu = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\Delta d / d_0}{\Delta l / l_0}$$

From Table 6.1, for aluminum, $\nu = 0.33$. Now, solving the above expression for Δd yields

$$\Delta d = -\frac{\nu\,\Delta l\,d_0}{l_0} = -\frac{(0.33)(0.50\ \mathrm{mm})(19\ \mathrm{mm})}{200\ \mathrm{mm}}$$

$$= -1.6 \times 10^{-2}\ \mathrm{mm}\ (-6.2 \times 10^{-4}\ \mathrm{in.})$$

The diameter will decrease.

6.17 A cylindrical specimen of some alloy 8 mm (0.31 in.) in diameter is stressed elastically in tension. A force of 15,700 N (3530 lb_f) produces a reduction in specimen diameter of 5×10^{-3} mm (2×10^{-4} in.). Compute Poisson's ratio for this material if its modulus of elasticity is 140 GPa (20.3×10^6 psi).

Solution

This problem asks that we compute Poisson's ratio for the metal alloy. From Equations 6.5 and 6.1

$$\varepsilon_z = \frac{\sigma}{E} = \frac{F}{A_0 E} = \frac{F}{\pi\left(\frac{d_0}{2}\right)^2 E} = \frac{4F}{\pi d_0^2 E}$$

Since the transverse strain ε_x is just

$$\varepsilon_x = \frac{\Delta d}{d_0}$$

and Poisson's ratio is defined by Equation 6.8, then

$$\nu = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\Delta d / d_0}{\left(\frac{4F}{\pi d_0^2 E}\right)} = -\frac{d_0 \Delta d \, \pi E}{4F}$$

$$= -\frac{(8 \times 10^{-3} \text{ m})(-5 \times 10^{-6} \text{ m})(\pi)(140 \times 10^9 \text{ N/m}^2)}{(4)(15,700 \text{ N})} = 0.280$$

6.18 A cylindrical specimen of a hypothetical metal alloy is stressed in compression. If its original and final diameters are 20.000 and 20.025 mm, respectively, and its final length is 74.96 mm, compute its original length if the deformation is totally elastic. The elastic and shear moduli for this alloy are 105 GPa and 39.7 GPa, respectively.

Solution

This problem asks that we compute the original length of a cylindrical specimen that is stressed in compression. It is first convenient to compute the lateral strain ε_x as

$$\varepsilon_x = \frac{\Delta d}{d_0} = \frac{20.025 \text{ mm} - 20.000 \text{ mm}}{20.000 \text{ mm}} = 1.25 \times 10^{-3}$$

In order to determine the longitudinal strain ε_z we need Poisson's ratio, which may be computed using Equation [illegible] solving for ν yields

$$\nu = \frac{E}{2G} - 1 = \frac{105 \times 10^3 \text{ MPa}}{(2)\left(39.7 \times 10^3 \text{ MPa}\right)} - 1 = 0.322$$

Now ε_z may be computed from Equation 6.8 as

$$\varepsilon_z = -\frac{\varepsilon_x}{\nu} = -\frac{1.25 \times 10^{-3}}{0.322} = -3.88 \times 10^{-3}$$

Now solving for l_0 using Equation 6.2

$$l_0 = \frac{l_i}{1 + \varepsilon_z}$$

$$= \frac{74.96 \text{ mm}}{1 - 3.88 \times 10^{-3}} = 75.25 \text{ mm}$$

6.19 Consider a cylindrical specimen of some hypothetical metal alloy that has a diameter of 8.0 mm (0.31 in.). A tensile force of 1000 N (225 lb_f) produces an elastic reduction in diameter of 2.8×10^{-4} mm (1.10×10^{-5} in.). Compute the modulus of elasticity for this alloy, given that Poisson's ratio is 0.30.

Solution

This problem asks that we calculate the modulus of elasticity of a metal that is stressed in tension. Combining Equations 6.5 and 6.1 leads to

$$E = \frac{\sigma}{\varepsilon_z} = \frac{F}{A_0 \varepsilon_z} = \frac{F}{\varepsilon_z \pi \left(\frac{d_0}{2}\right)^2} = \frac{4F}{\varepsilon_z \pi d_0^2}$$

From the definition of Poisson's ratio, (Equation 6.8) and realizing that for the transverse strain, $\varepsilon_x = \frac{\Delta d}{d_0}$

$$\varepsilon_z = -\frac{\varepsilon_x}{\nu} = -\frac{\Delta d}{d_0 \nu}$$

Therefore, substitution of this expression for ε_z into the above equation yields

$$E = \frac{4F}{\varepsilon_z \pi d_0^2} = \frac{4F\nu}{\pi d_0 \Delta d}$$

$$= \frac{(4)(1000\text{ N})(0.30)}{\pi (8 \times 10^{-3}\text{ m})(2.8 \times 10^{-7}\text{ m})} = 1.705 \times 10^{11}\text{ Pa} = 170.5\text{ GPa } (24.7 \times 10^{6}\text{ psi})$$

6.20 A brass alloy is known to have a yield strength of 275 MPa (40,000 psi), a tensile strength of 380 MPa (55,000 psi), and an elastic modulus of 103 GPa (15.0×10^6 psi). A cylindrical specimen of this alloy 12.7 mm (0.50 in.) in diameter and 250 mm (10.0 in.) long is stressed in tension and found to elongate 7.6 mm (0.30 in.). On the basis of the information given, is it possible to compute the magnitude of the load that is necessary to produce this change in length? If so, calculate the load. If not, explain why.

Solution

We are asked to ascertain whether or not it is possible to compute, for brass, the magnitude of the load necessary to produce an elongation of 7.6 mm (0.30 in.). It is first necessary to compute the strain at yielding from the yield strength and the elastic modulus, and then the strain experienced by the test specimen. Then, if

$$\varepsilon(\text{test}) < \varepsilon(\text{yield})$$

deformation is elastic, and the load may be computed using Equations 6.1 and 6.5. However, if

$$\varepsilon(\text{test}) > \varepsilon(\text{yield})$$

computation of the load is not possible inasmuch as deformation is plastic and we have neither a stress-strain plot nor a mathematical expression relating plastic stress and strain. We compute these two strain values as

$$\varepsilon(\text{test}) = \frac{\Delta l}{l_0} = \frac{7.6 \text{ mm}}{250 \text{ mm}} = 0.03$$

and

$$\varepsilon(\text{yield}) = \frac{\sigma_y}{E} = \frac{275 \text{ MPa}}{103 \times 10^3 \text{ MPa}} = 0.0027$$

Therefore, computation of the load is not possible since $\varepsilon(\text{test}) > \varepsilon(\text{yield})$.

6.21 A cylindrical metal specimen 12.7 mm (0.5 in.) in diameter and 250 mm (10 in.) long is to be VXEMHFWHG WR D WHQVLOH WHVW RI 0 3D SVL DW WKLV VWUHVV OHYHO WKH UHVXOWLQJ GHIRUPDWLRQ ZLOO EH WRWDOO elastic.

(a) If the elongation must be less than 0.080 mm (3.2×10^{-3} in.), which of the metals in Table 6.1 are suitable candidates? Why?

(b) If, in addition, the maximum permissible diameter decrease is 1.2×10^{-3} mm (4.7×10^{-5} in.) when the tensile stress of 28 MPa is applied, which of the metals that satisfy the criterion in part (a) are suitable candidates? Why?

Solution

(a) This part of the problem asks that we ascertain which of the metals in Table 6.1 experience an elongation of less than 0.080 mm when subjected to a tensile stress of 28 MPa. The maximum strain that may be sustained, (using Equation 6.2) is just

$$\varepsilon = \frac{\Delta l}{l_0} = \frac{0.080 \text{ mm}}{250 \text{ mm}} = 3.2 \times 10^{-4}$$

Since the stress level is given (50 MPa), using Equation 6.5 it is possible to compute the minimum modulus of elasticity which is required to yield this minimum strain. Hence

$$E = \frac{\sigma}{\varepsilon} = \frac{28 \text{ MPa}}{3.2 \times 10^{-4}} = 87.5 \text{ GPa}$$

Which means that those metals with moduli of elasticity greater than this value are acceptable candidates—namely, brass, Cu, Ni, steel, Ti and W.

(b) This portion of the problem further stipulates that the maximum permissible diameter decrease is 1.2×10^{-3} mm when the tensile stress of 28 MPa is applied. This translates into a maximum lateral strain ε_x(max) as

$$\varepsilon_x(\text{max}) = \frac{\Delta d}{d_0} = \frac{-1.2 \times 10^{-3} \text{ mm}}{12.7 \text{ mm}} = -9.45 \times 10^{-5}$$

But, since the specimen contracts in this lateral direction, and we are concerned that this strain be less than 9.45×10^{-5}, then the criterion for this part of the problem may be stipulated as $-\frac{\Delta d}{d_0} < 9.45 \times 10^{-5}$.

Now, Poisson's ratio is defined by Equation 6.8 as

$$\nu = -\frac{\varepsilon_x}{\varepsilon_z}$$

For each of the metal alloys let us consider a possible lateral strain, $\varepsilon_x = \frac{\Delta d}{d_0}$. Furthermore, since the deformation is elastic, then, from Equation 6.5, the longitudinal strain, ε_z is equal to

$$\varepsilon_z = \frac{\sigma}{E}$$

Substituting these expressions for ε_x and ε_z into the definition of Poisson's ratio we have

$$\nu = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\frac{\Delta d}{d_0}}{\frac{\sigma}{E}}$$

which leads to the following:

$$-\frac{\Delta d}{d_0} = \frac{\nu\sigma}{E}$$

Using values for ν and E found in Table 6.1 for the six metal alloys that satisfy the criterion for part (a), and for $\sigma = 28$ MPa, we are able to compute a $-\frac{\Delta d}{d_0}$ for each alloy as follows:

$$-\frac{\Delta d}{d_0}(\text{brass}) = \frac{(0.34)(28 \times 10^6 \text{ N/m}^2)}{97 \times 10^9 \text{ N/m}^2} = 9.81 \times 10^{-5}$$

$$-\frac{\Delta d}{d_0}(\text{copper}) = \frac{(0.34)(28 \times 10^6 \text{ N/m}^2)}{110 \times 10^9 \text{ N/m}^2} = 8.65 \times 10^{-5}$$

$$-\frac{\Delta d}{d_0}(\text{titanium}) = \frac{(0.34)(28 \times 10^6 \text{ N/m}^2)}{107 \times 10^9 \text{ N/m}^2} = 8.90 \times 10^{-5}$$

$$-\frac{\Delta d}{d_0}(\text{nickel}) = \frac{(0.31)(28 \times 10^6 \text{ N/m}^2)}{207 \times 10^9 \text{ N/m}^2} = 4.19 \times 10^{-5}$$

$$-\frac{\Delta d}{d_0}(\text{steel}) = \frac{(0.30)(28 \times 10^6 \text{ N/m}^2)}{207 \times 10^9 \text{ N/m}^2} = 4.06 \times 10^{-5}$$

$$-\frac{\Delta d}{d_0}(\text{tungsten}) = \frac{(0.28)(28 \times 10^6 \text{ N/m}^2)}{407 \times 10^9 \text{ N/m}^2} = 1.93 \times 10^{-5}$$

Thus, of the above six alloys, only brass will have a negative transverse strain that is greater than 9.45×10^{-5}. This means that the following alloys satisfy the criteria for both parts (a) and (b) of the problem: copper, titanium, nickel, steel, and tungsten.