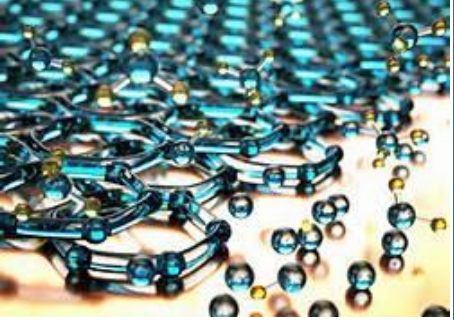


Materials Science

Lecture 12

Lebanese University - Faculty of Engineering – Branch 3

Fall 2022

Dr. Ali HARKOUS

Contents

Lecture 12:

⦿ Chap 5: Mechanical Behavior

5.2. Concepts of stress and strain

Tension Tests: Engineering stress

- The **output** of such a tensile test is **recorded** (usually on a computer) as **load** or force **versus elongation**.

- These **load–deformation** characteristics **depend** on the **specimen size**. **For example**, it requires twice the load to produce the same elongation if the cross-sectional area of the specimen is doubled.

- To **minimize** these **geometrical factors**, load and elongation are **normalized** to the respective parameters of **engineering stress** and **engineering strain**. Engineering stress s is defined by the relationship

$$\boxed{\sigma = \frac{F}{A_0}}$$

in which F is the instantaneous load applied perpendicular to the specimen cross section, in newtons (N) or pounds force (lb_f), and A_0 is the original cross-sectional area before any load is applied (m^2 or in^2). The units of engineering stress are megapascals, MPa (SI) and pounds force per square inch, psi (U.S.).

5.2. Concepts of stress and strain

Tension Tests: Engineering strain

- If an **axial load** is applied to the bar, it will change the bar's length L_0 to a length L_i.

- We will define the ***average normal strain ε*** (epsilon) of the bar as the change in its length **δ (delta) =** $L_i - L_0$ divided by its original length L_0, that is

$$\boxed{\epsilon = \frac{l_i - l_0}{l_0} = \frac{\Delta l}{l_0}}$$

in which l_0 is the original length before any load is applied and l_i is the instantaneous length. Sometimes the quantity l_i-l_0 is denoted as Δl and is the deformation elongation or change in length at some instant, as referenced to the original length. Engineering strain (subsequently called just strain) is unitless, but meters per meter or inches per inch is often used; the value of strain is obviously independent of the unit system. Sometimes strain is also expressed as a percentage, in which the strain value is multiplied by 100.

5.2. Concepts of stress and strain

Tension Tests: Engineering strain

- ***ε*** (or ***ε*** $_{avg}$) is a change in length per unit length, and it is **positive** when the initial line **elongates**, and **negative** when the line **contracts**.

- Normal strain is a ***dimensionless quantity***, since it is a ratio of two lengths.

- However, it is **sometimes** stated in terms of a ratio of length units. If the **SI** system is used, where the basic unit for length is the meter (m), then since ***ε*** is generally very small, for most engineering applications, measurements of strain will be in **micrometers per meter (μm/m)**, where 1 μm = 10^{-6} m.

- In the **Foot-Pound-Second system**, strain is often stated in units of **inches per inch (in./in.)**.

- For **experimental work**, strain is sometimes expressed as a **percent**. **For example**, a normal strain of $480(10^{-6})$ can be reported as **$480(10^{-6})$ in./in.**, **480 μm/m**, or **0.048%**.

5.2. Concepts of stress and strain

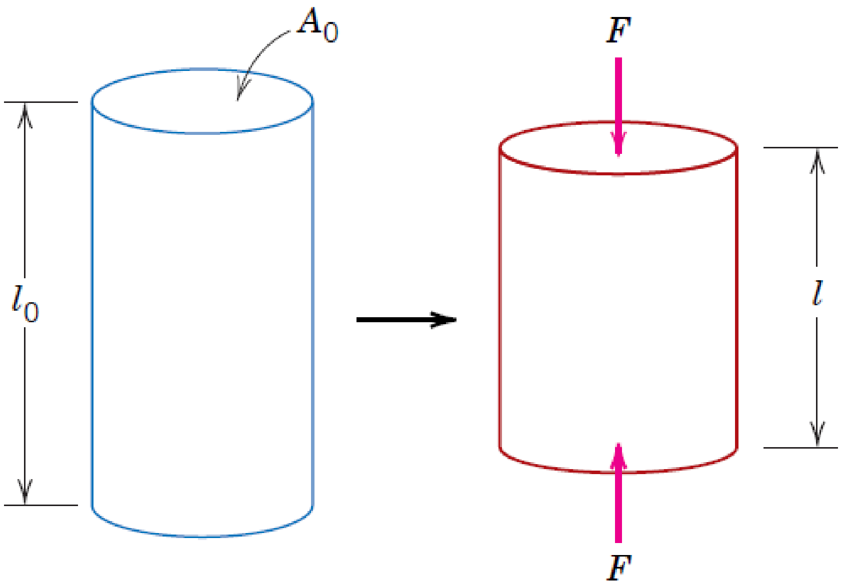
Compression Tests

- Compression stress–strain tests may be conducted **if in-service forces** are of **this type**.

- A compression test is conducted in a **manner similar** to the **tensile test**, except that the force is **compressive** and the **specimen contracts** along the direction of the stress.

- **Equations** of **tensile** stress and strain are utilized to compute compressive stress and strain, respectively.

- By convention, a **compressive force** is taken to be **negative**, which yields a **negative stress**.



Schematic illustration of how a compressive load produces contraction and a negative linear strain.

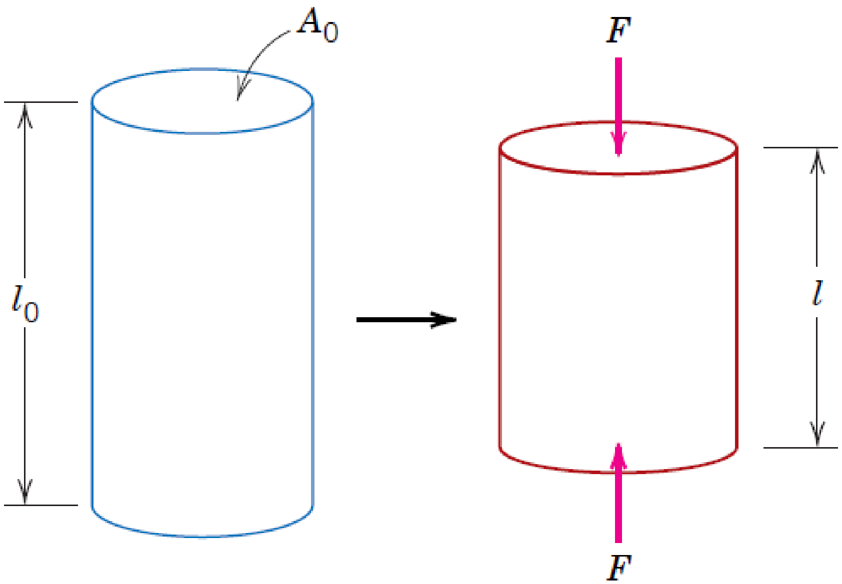
5.2. Concepts of stress and strain

Compression Tests

- Furthermore, because **l_0** is **greater** than **l_i**, **compressive strains** are necessarily also **negative**.

- **Tensile** tests are **more common** because they are **easier** to perform; also, for most materials used in structural applications, **very little additional information from compressive** tests.

- **Compressive** tests are used when a material's **behavior** under **large** and **permanent** (i.e., **plastic**) **strains** is **desired**, as in **manufacturing applications**, **or** when the material is **brittle in tension**.



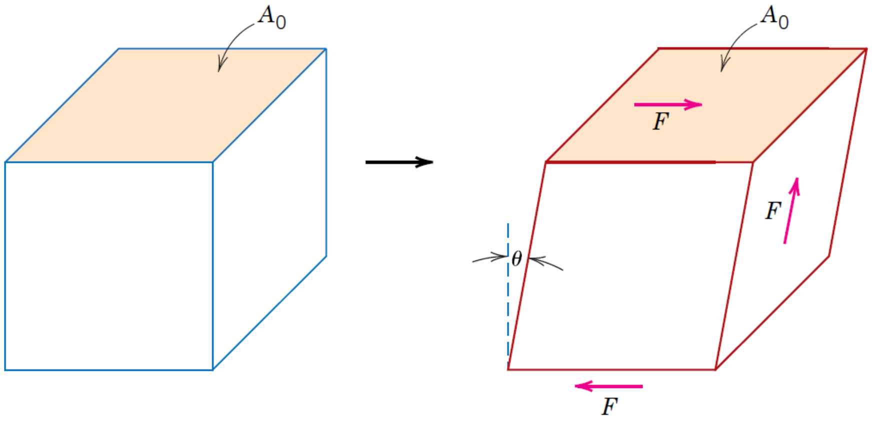
Schematic illustration of how a compressive load produces contraction and a negative linear strain.

5.2. Concepts of stress and strain

Shear and Torsional Tests

- For tests performed using a **pure shear force** as shown in the figure, the shear stress **τ** is computed according to

$$\boxed{\tau = \frac{F}{A_0}}$$



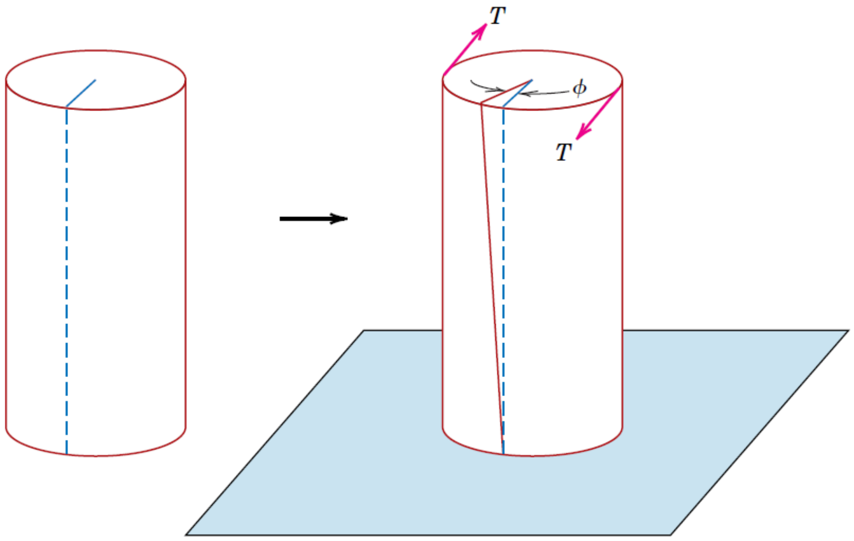
Schematic representation of shear strain γ, where γ=tan θ.

where F is the load or force imposed parallel to the upper and lower faces, each of which has an area of A_0. The shear strain γ is defined as the tangent of the strain angle θ, as indicated in the figure. The units for shear stress and strain are the same as for their tensile counterparts.

5.2. Concepts of stress and strain

Shear and Torsional Tests

- **Torsion** is a variation of **pure shear in which** a **structural member** is **twisted** in the manner of the figure.
- **Torsional forces** produce a **rotational motion** about the **longitudinal axis** of **one end** of the member **relative** to the **other** end.
- **Examples** of torsion are found for **machine axles** and drive **shafts** as well as for **twist drills**.
- Torsional tests are normally **performed** on **cylindrical solid shafts** or **tubes**.
- A shear **stress τ** is a function of the applied **torque T**, whereas **shear strain γ** is related to the **angle** of **twist**, **φ** in the figure.



Schematic representation of torsional deformation (i.e., angle of twist φ) produced by an applied torque T.

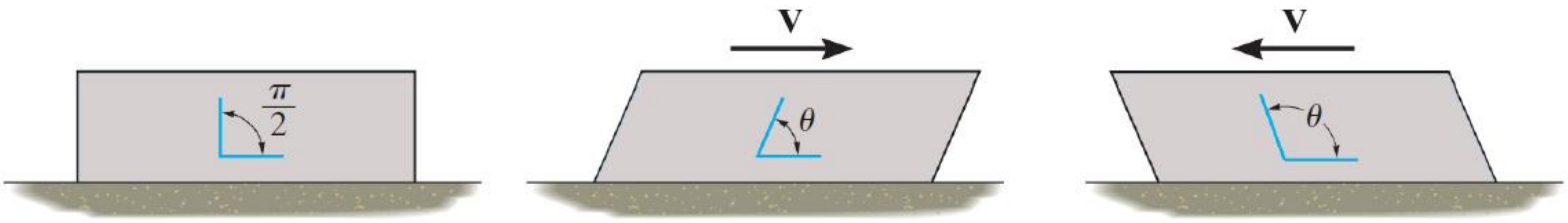
5.2. Concepts of stress and strain

Shear and Torsional Tests: Shear Strain

- **Deformations** not only cause line segments to elongate or contract, but they also cause them to **change direction**.

- **If** we select **two line segments** that are **originally perpendicular** to one another, then the **change in angle** that occurs between them is referred to as ***shear strain***.

- This **angle** is denoted by γ (gamma) and is always measured in radians (**rad**), which are **dimensionless**.



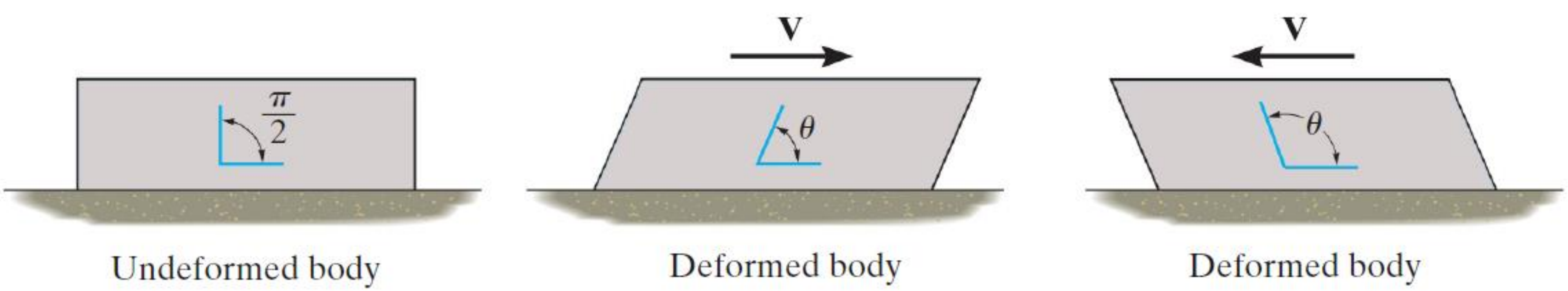
Undeformed body　　Deformed body　　Deformed body

5.2. Concepts of stress and strain

Shear and Torsional Tests: Shear Strain

- **For example**, consider the **two perpendicular** line **segments** at a point in the block shown in the figure. If an applied loading causes the block to deform so that the angle between the line segments becomes ***θ***, then the **shear strain** at the point **becomes**:

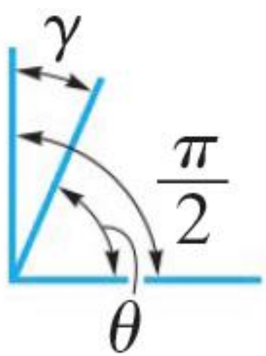
$$\boxed{\gamma = \frac{\pi}{2} - \theta}$$



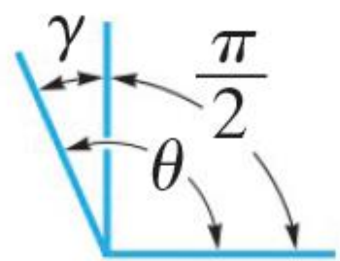
5.2. Concepts of stress and strain

Shear and Torsional Tests: Shear Strain

- Notice that if ***θ*** is **smaller than π/2** (figure), then the shear strain is ***positive***, whereas if ***θ*** is **larger than π/2**, then the shear strain is ***negative***.



Positive shear strain γ



Negative shear strain γ

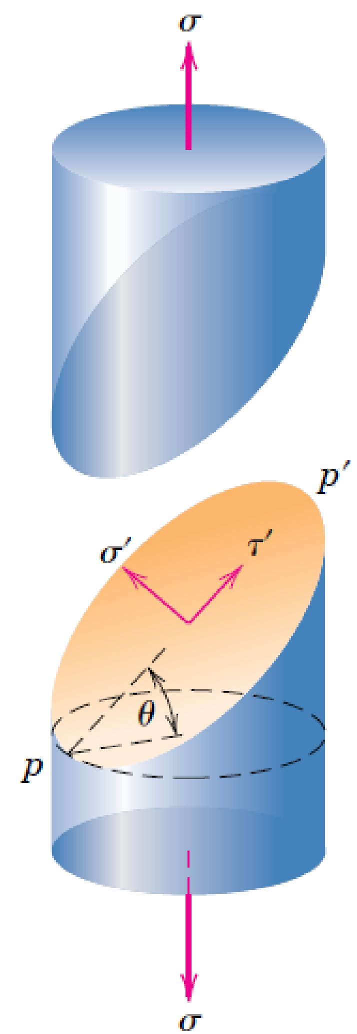
5.2. Concepts of stress and strain

Geometric Considerations of the Stress State

- **Stresses** that are computed from the tensile, compressive, shear, and torsional force states act either **parallel** or **perpendicular** to **planar faces** of the bodies represented in the previous illustrations.

- Note that **the stress state** is a function of the **orientations** of the **planes** upon which the stresses are taken to act. **For example**, consider the **cylindrical tensile specimen** of the **figure** that is subjected to a **tensile** stress **σ** applied parallel to its axis.

- Furthermore, consider also the **plane p-p'** that is **oriented** at **some arbitrary** angle **θ** relative to the plane of the specimen end-face.



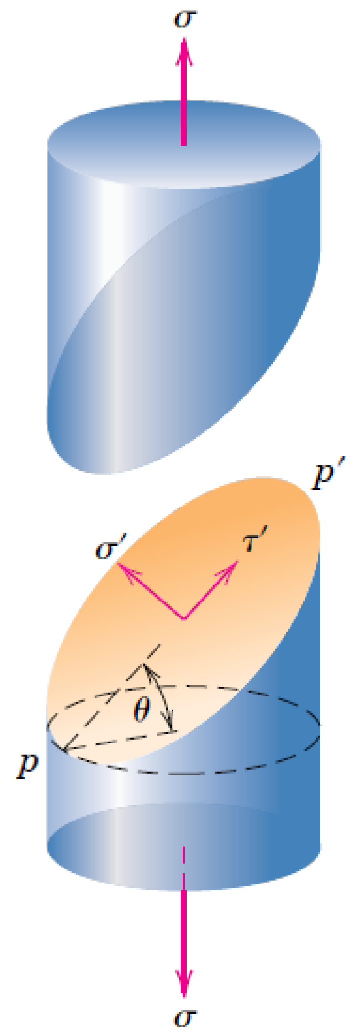
Schematic representation showing normal (σ') and shear (τ') stresses that act on a plane oriented at an angle θ relative to the plane taken perpendicular to the direction along which a pure tensile stress (σ) is applied.

5.2. Concepts of stress and strain

Geometric Considerations of the Stress State

- Upon this plane **p-p'**, the applied stress is **no longer a pure tensile one**.

- Rather, a **more complex stress state** is present that consists of a **tensile** (or normal) **stress σ'** that acts **normal** to **the p-p'** plane and, in addition, a **shear stress τ'** that acts **parallel** to this plane; both of these stresses are represented in the figure.



Schematic representation showing normal (σ') and shear (τ') stresses that act on a plane oriented at an angle θ relative to the plane taken perpendicular to the direction along which a pure tensile stress (σ) is applied.

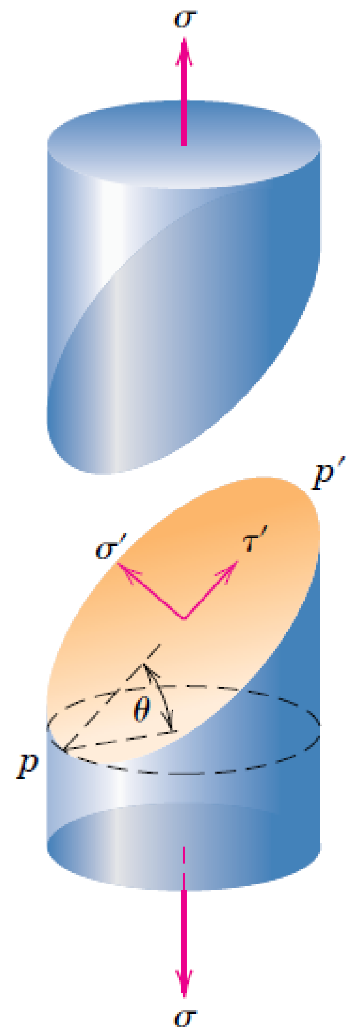
Geometric Considerations of the Stress State

- Using mechanics-of-materials principles, it is possible to develop equations for **σ' and τ' in terms of σ and θ**, as follows:

$$\boxed{\sigma' = \sigma\cos^2\theta = \sigma\left(\frac{1+\cos 2\theta}{2}\right)}$$

$$\boxed{\tau' = \sigma\sin\theta\cos\theta = \sigma\left(\frac{\sin 2\theta}{2}\right)}$$

- These same mechanics principles allow the **transformation** of stress components from one **coordinate system** to **another** coordinate system with a **different orientation**.



Schematic representation showing normal (σ') and shear (τ') stresses that act on a plane oriented at an angle θ relative to the plane taken perpendicular to the direction along which a pure tensile stress (σ) is applied.

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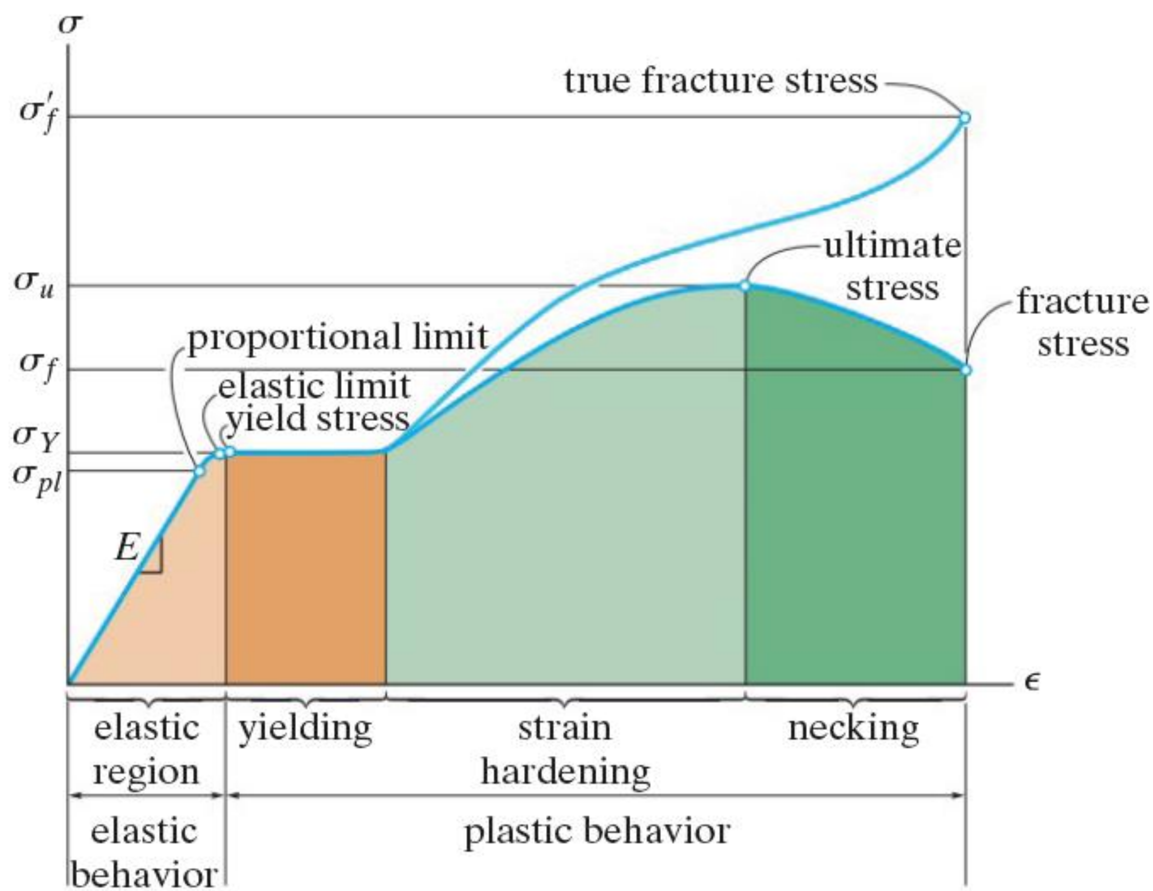
5.3. Stress–Strain behavior

- Once the **stress and strain** data from the test are **known**, then the results can be **plotted** to produce a curve called the ***stress–strain diagram***.

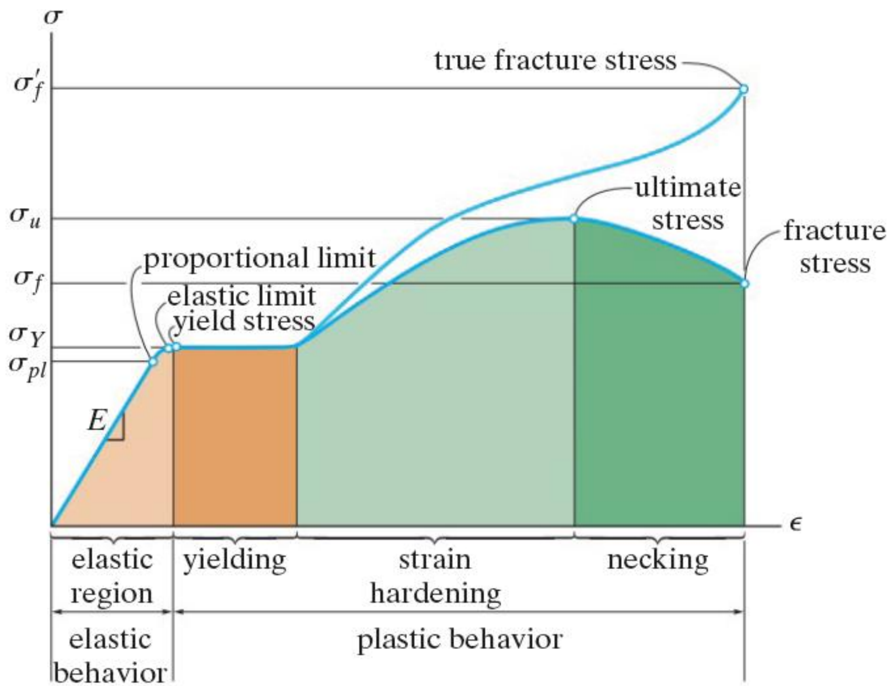
- This **diagram** is very useful since it applies to a specimen of the material made of ***any size***.



5.3. Stress–Strain behavior

Conventional Stress–Strain Diagram

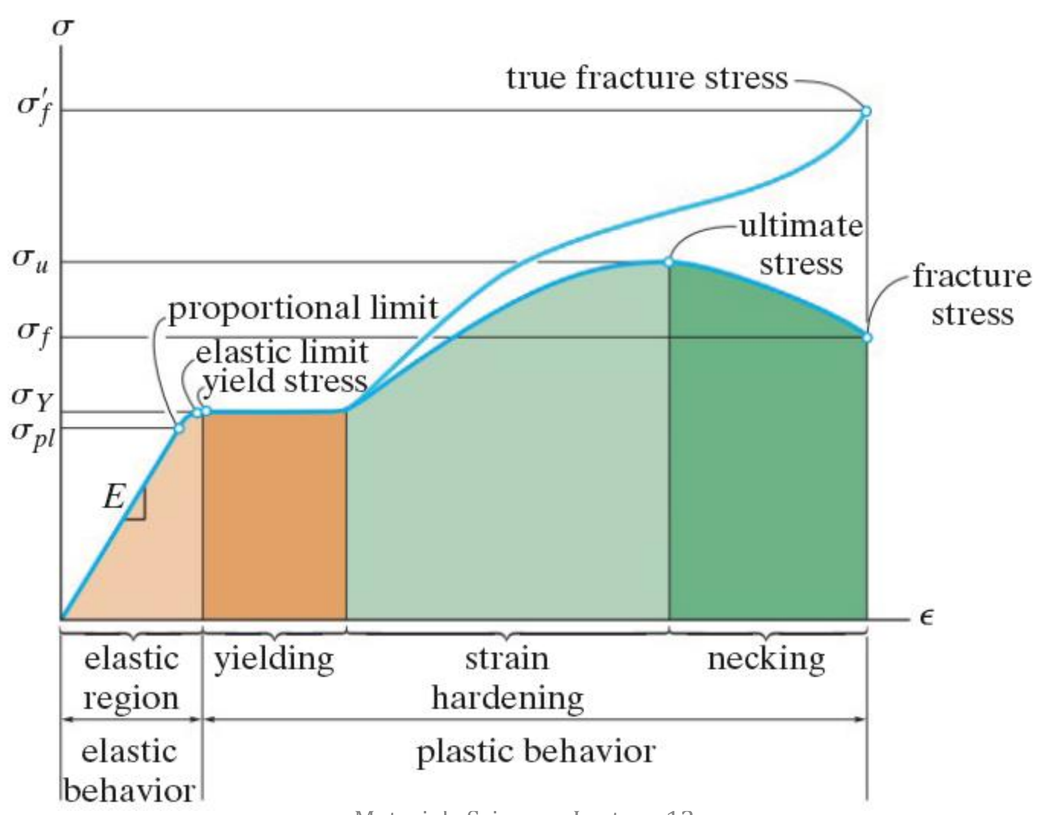
- **A typical example** of this curve is shown in the figure. Realize, however, that **two stress–strain diagrams** for a particular **material** will be quite **similar** but will **never be exactly the same**.
- This is because the results actually **depend** upon such variables as the material's **composition**, microscopic **imperfections**, the way the specimen is **manufactured**, the **rate of loading**, and the **temperature** during the time of the test.



5.3. Stress–Strain behavior

Conventional Stress–Strain Diagram

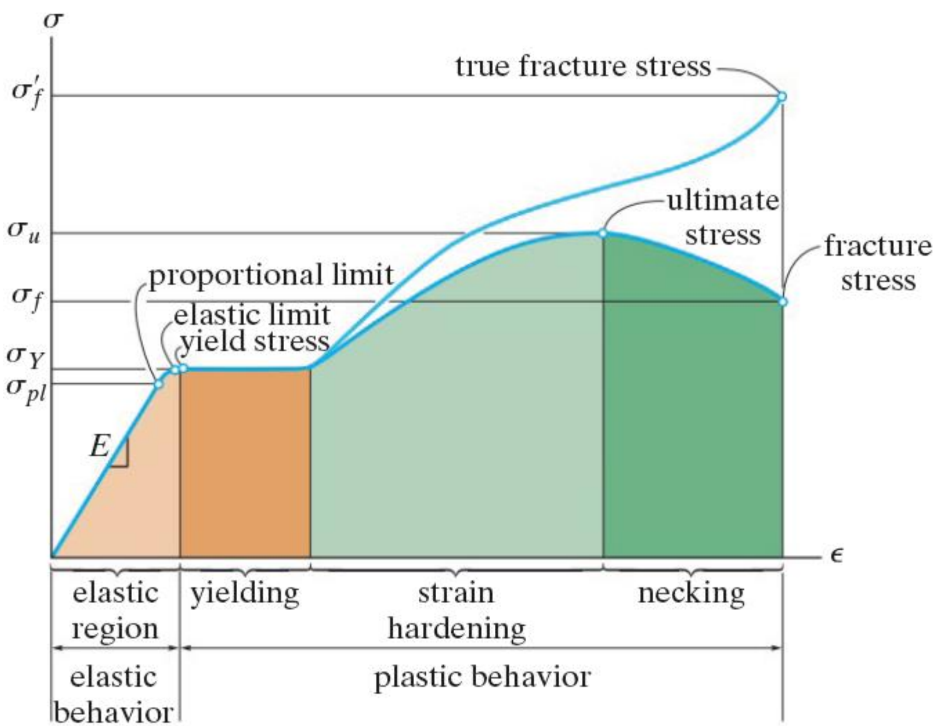
- From the curve, we can identify **four different regions** in which the material behaves in a unique way, **depending** on the **amount** of **strain** induced in the material.

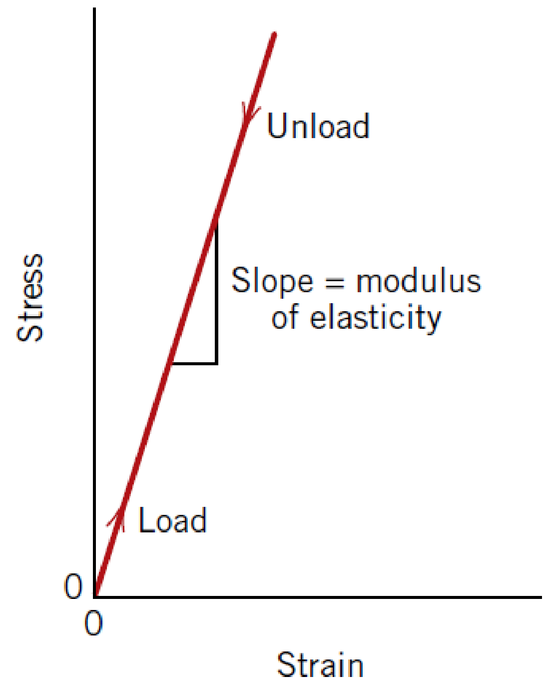


5.3. Stress–Strain behavior

Elastic Behavior: *Hooke's law*

- The initial region of the curve, indicated in **light orange**, is referred to as the **elastic region**.
- Here the curve is a ***straight line*** up to the point where the stress reaches the ***proportional limit***, $\boldsymbol{\sigma_{pl}}$.



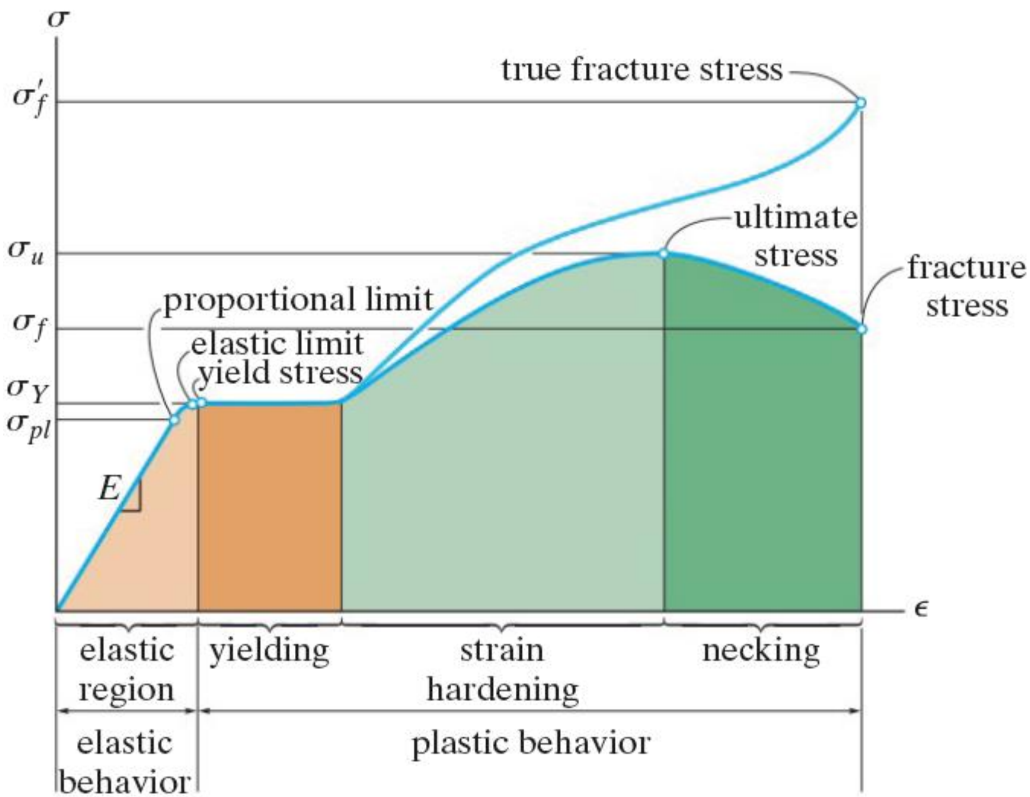


Schematic stress–strain diagram showing linear elastic deformation for loading and unloading cycles.

5.3. Stress–Strain behavior

Elastic Behavior: *Hooke's law*

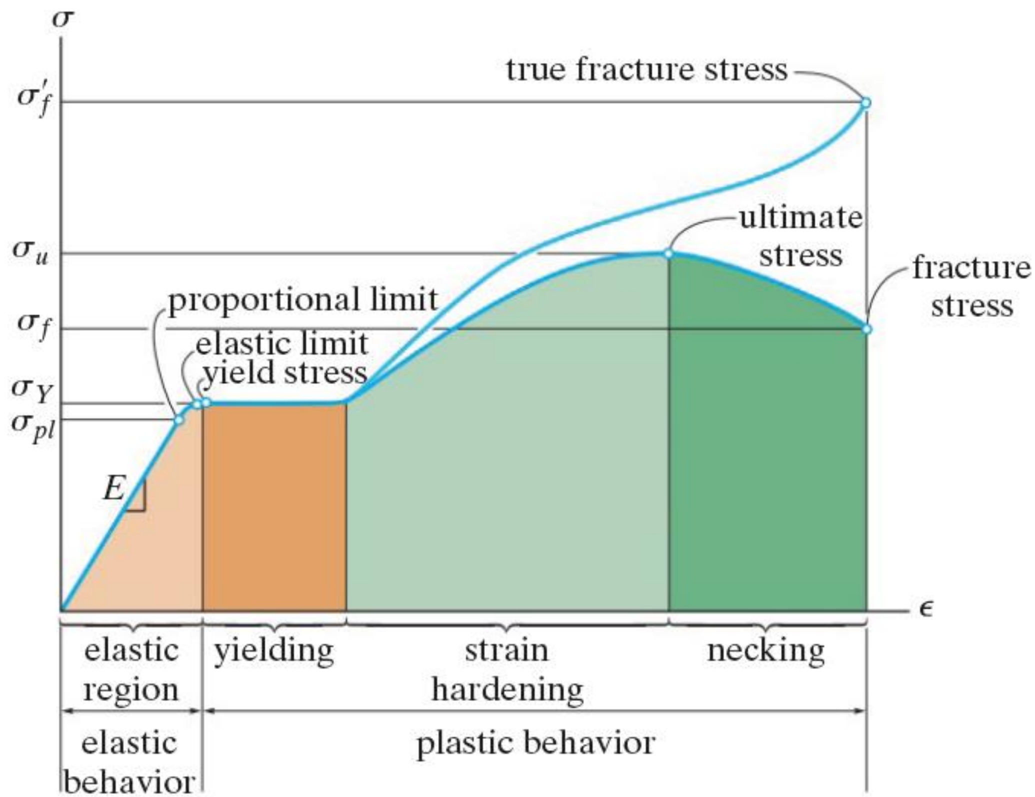
- When **the stress slightly exceeds** this value, the curve **bends until** the stress reaches an **elastic limit**. For most materials, **these points are very close**, and therefore it becomes rather difficult to distinguish their exact values.



5.3. Stress–Strain behavior

Elastic Behavior: *Hooke's law*

- What makes the elastic region unique, however, is that after **reaching σ_Y**, if the load **is removed**, the specimen will **recover** its **original shape**. In other words, no damage will be done to the material.



5.3. Stress–Strain behavior

Elastic Behavior: ***Hooke's law***

- Because the curve is a straight line up to σ_{pl}, any **increase in stress** will cause a **proportional increase** in **strain**. This fact was discovered in **1676** by Robert **Hooke**, **using springs**, and is known as ***Hooke's law***. It is expressed mathematically as:

$$\boxed{\sigma = E\epsilon}$$

- Here E represents the constant of proportionality, which is called the ***modulus of elasticity or Young's modulus***, named after **Thomas Young**, who published an account of it in **1807**.

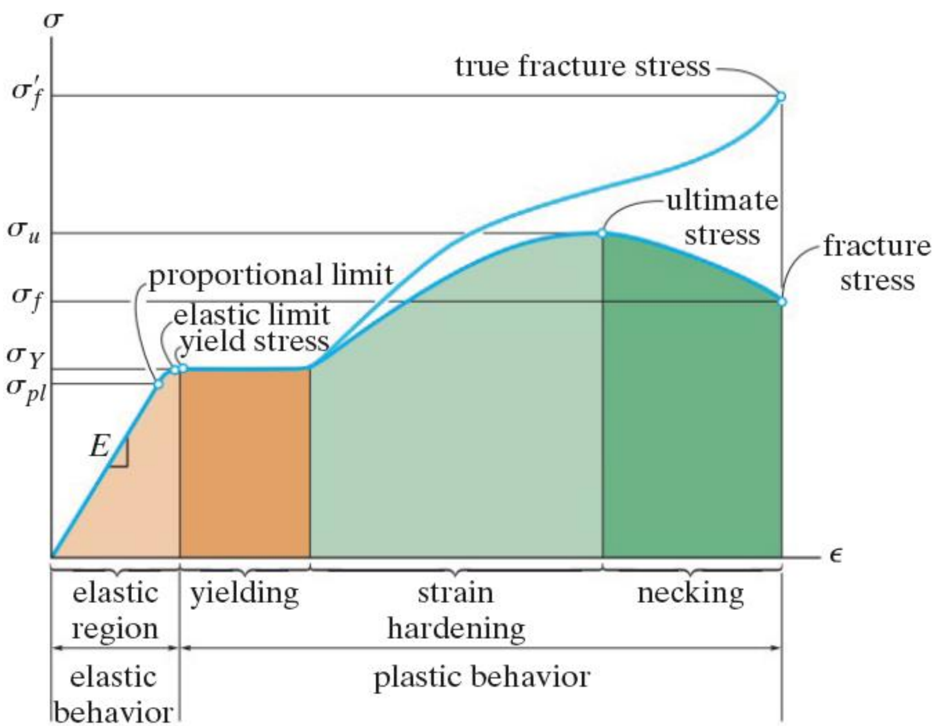
- This modulus may be thought of as **stiffness**, or a **material's resistance** to **elastic deformation**.

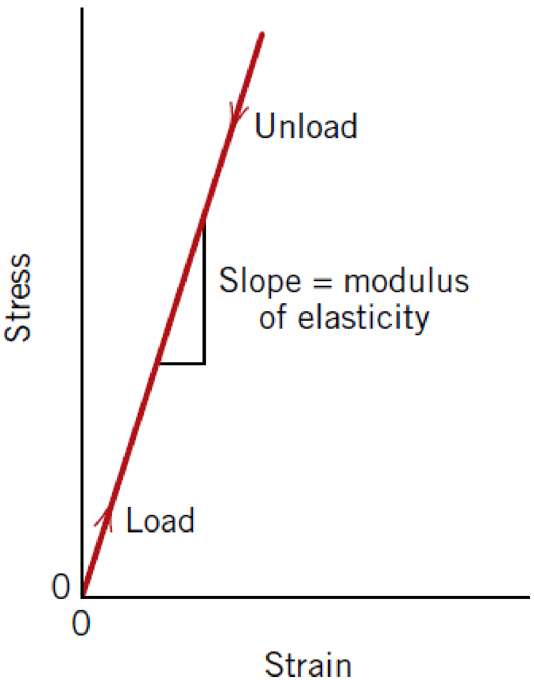
- The **greater** the **modulus**, the **stiffer** the **material**, or the **smaller** the **elastic** strain that results from the application of a given stress. The **modulus** is an **important design parameter** for computing **elastic deflections**.

5.3. Stress–Strain behavior

Elastic Behavior: *Hooke's law*

- As noted in the figure, the **modulus of elasticity** represents the ***slope*** of the straight line portion of the curve.
- Since **strain** is **dimensionless**, from Hooke's Law, **E** will have the **same units as stress,** such as **psi, ksi, or pascals**.





Schematic stress–strain diagram showing linear elastic deformation for loading and unloading cycles.

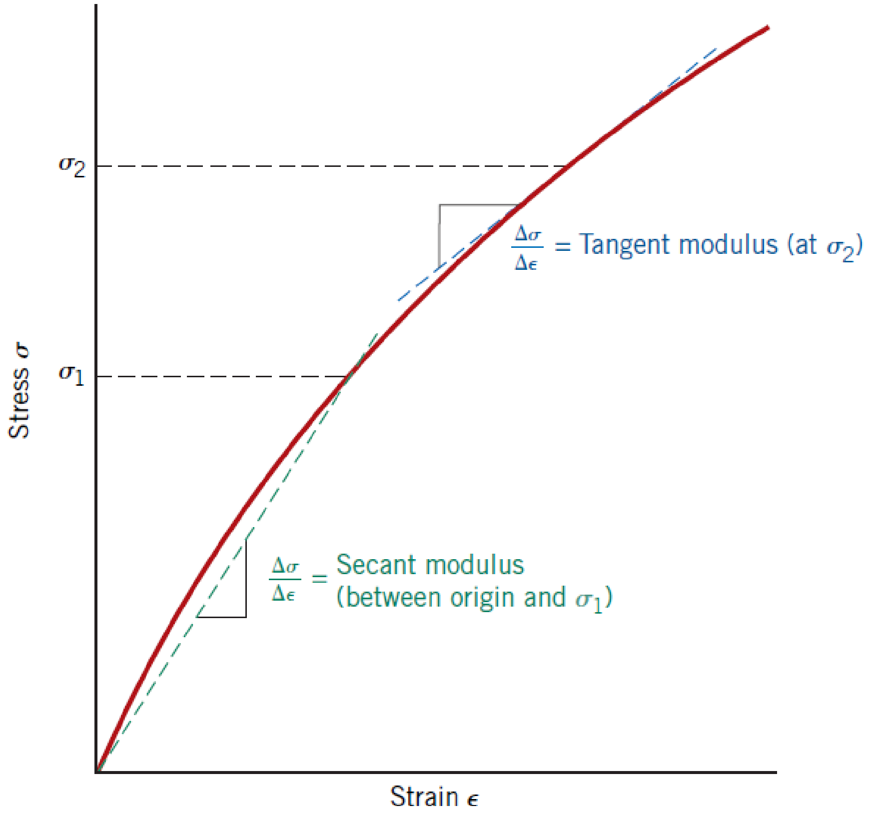
5.3. Stress–Strain behavior

- There are **some materials** (i.e., **gray cast iron, concrete, and many polymers**) for which this **elastic portion** of the stress–strain curve is **not linear**.

- Hence, it is **not possible** to **determine** a **modulus** of **elasticity** as described **previously**.

- For this **nonlinear behavior**, either the **tangent** or **secant** modulus is normally used.

- The **tangent** modulus is taken as the **slope** of the **stress–strain** curve at some specified level of stress, whereas the **secant modulus** represents the **slope** of a **secant** drawn **from the origin** to **some given point** of the σ-ε curve.

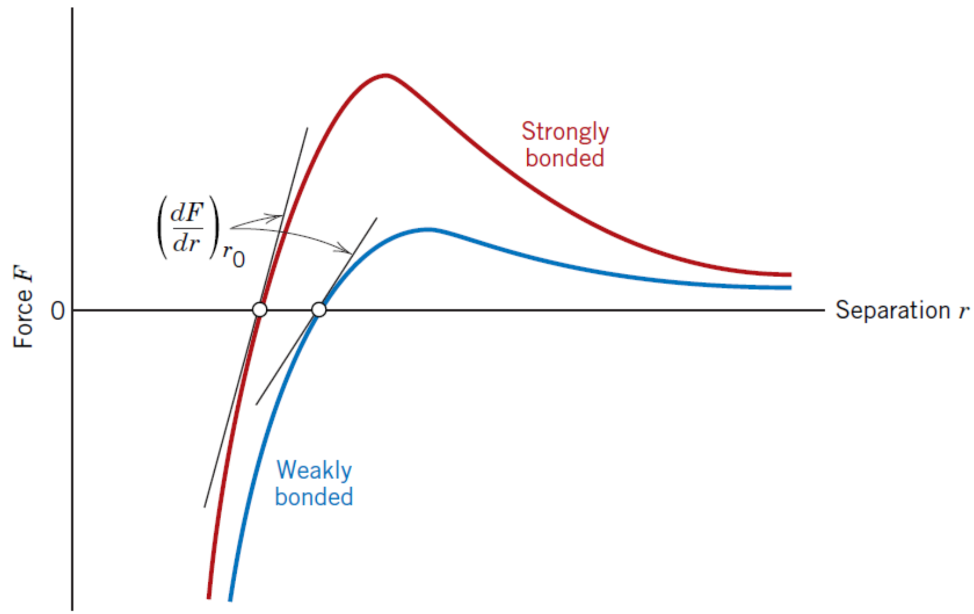


Schematic stress–strain diagram showing nonlinear elastic behavior and how secant and tangent moduli are determined.

5.3. Stress–Strain behavior

- On an **atomic scale**, **macroscopic elastic strain** is manifested as **small changes** in the **interatomic spacing** and the **stretching** of **interatomic bonds**.
- As a consequence, the **magnitude** of the **modulus of elasticity** is a measure of the **resistance to separation** of **adjacent atoms**, that is, the **interatomic bonding forces**.
- Furthermore, this **modulus** is **proportional** to the **slope** of the **interatomic force–separation curve** at the **equilibrium spacing**:

$$\boxed{E \propto \left(\frac{dF}{dr}\right)_{r_0}}$$



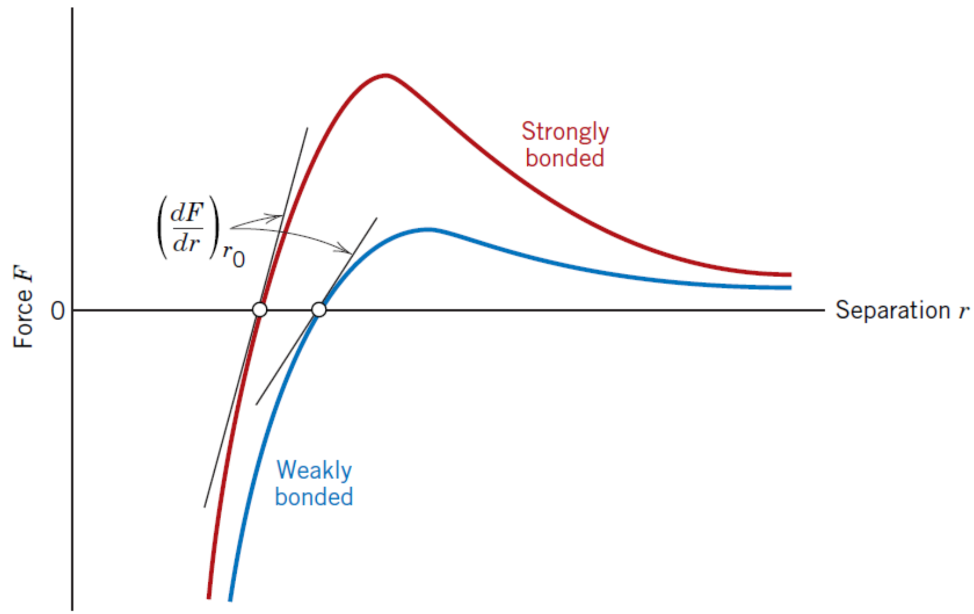
Force versus interatomic separation for weakly and strongly bonded atoms. The magnitude of the modulus of elasticity is proportional to the slope of each curve at the equilibrium interatomic separation r_0.

5.3. Stress–Strain behavior

- The figure shows the **force–separation** curves for materials having both **strong** and **weak interatomic bonds**; the **slope at r_0** is indicated for each.

- **Values** of the modulus of elasticity for **ceramic** materials are about the **same** as for **metals**; for **polymers** they are **lower**.

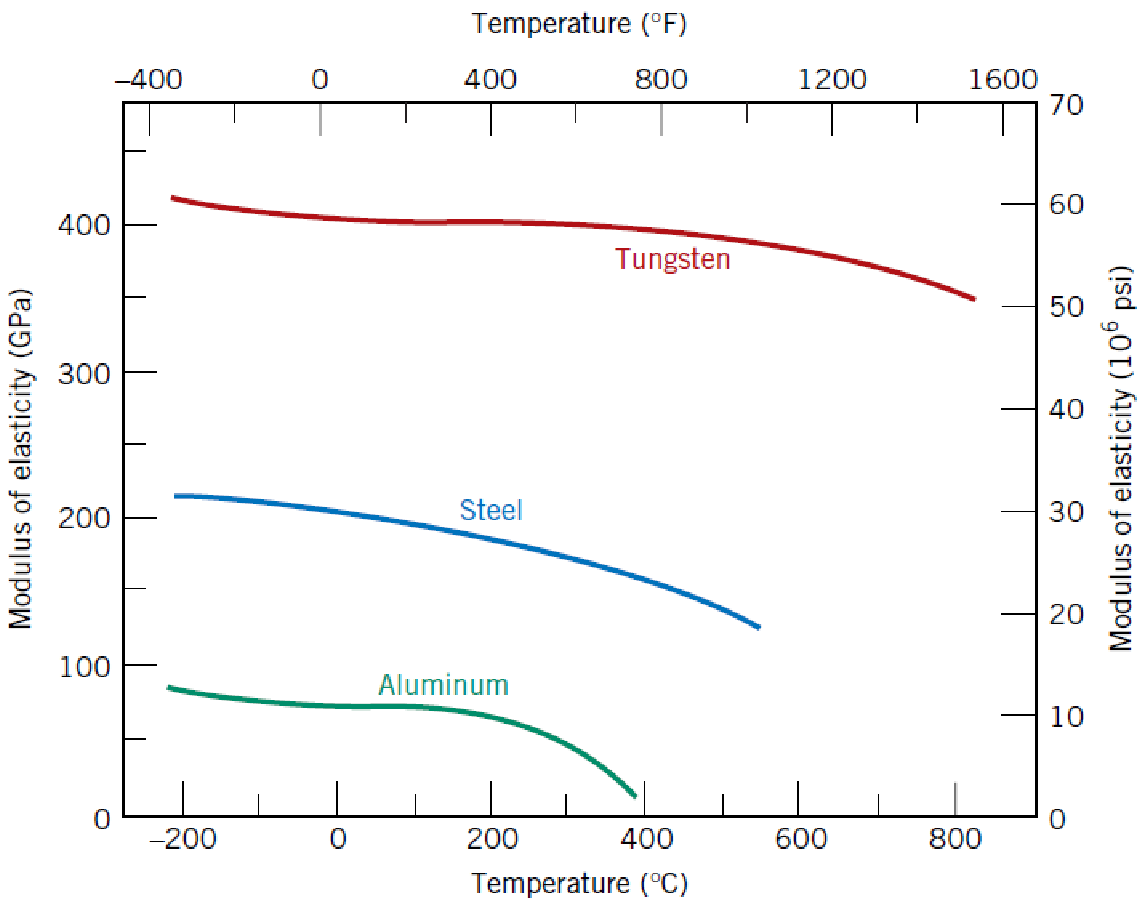
- These **differences** are a **direct consequence** of the **different types** of **atomic bonding** in the three materials types.



Force versus interatomic separation for weakly and strongly bonded atoms. The magnitude of the modulus of elasticity is proportional to the slope of each curve at the equilibrium interatomic separation r_0.

5.3. Stress–Strain behavior

- Furthermore, with **increasing temperature**, the **modulus of elasticity decreases**, as is shown for several metals.



Plot of modulus of elasticity versus temperature for tungsten, steel, and aluminum.

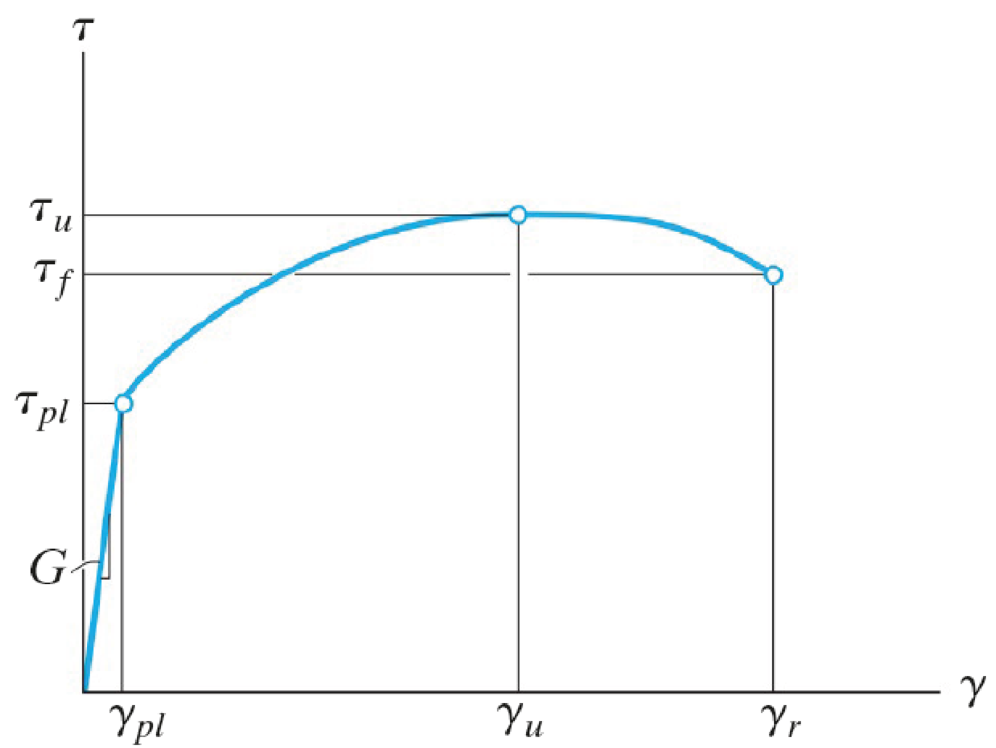
5.3. Stress–Strain behavior

Elastic deformation in Shear test

- As would be expected, the imposition of compressive, shear, or torsional stresses also evokes elastic behavior.

- The **stress–strain characteristics** at **low stress** levels are **virtually** the **same** for both **tensile** and **compressive** situations, to include the magnitude of the modulus of elasticity.

- **Like the tension test**, this material when subjected to shear will **exhibit linear elastic** behavior and it will have a defined **proportional limit τ_{pl}**.



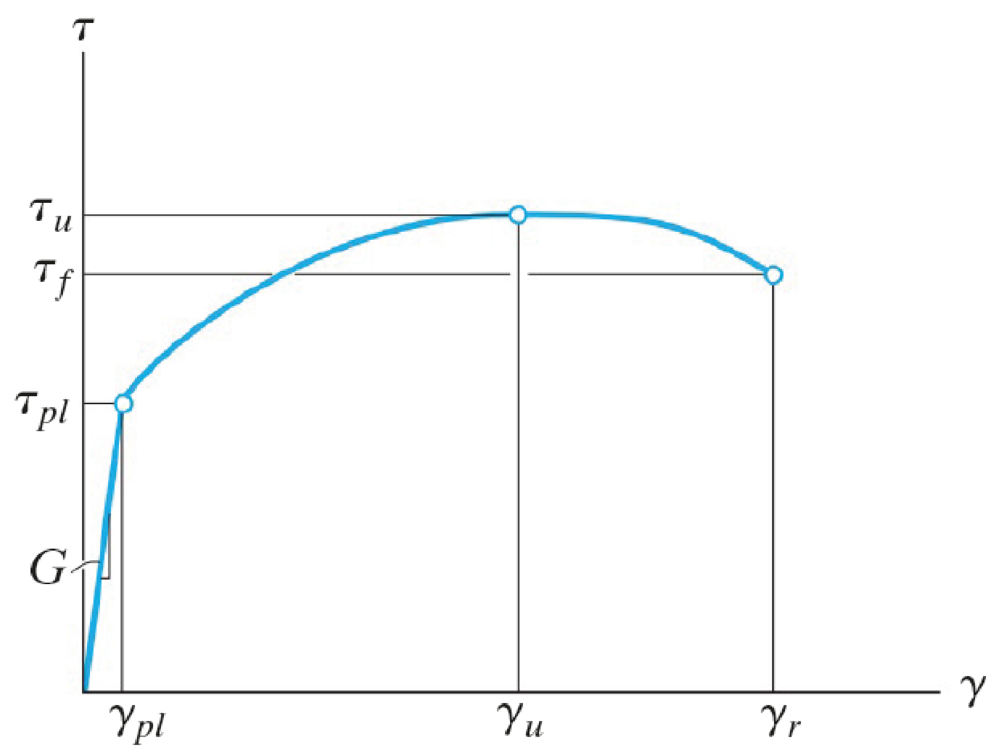
5.3. Stress–Strain behavior

Elastic deformation in Shear test

- Also, **strain hardening** will occur until an **ultimate shear stress τ_u** is reached. And finally, the material will begin to lose its shear strength until it reaches a point where it fractures, τ_f.
- Shear stress and strain are proportional to each other and so ***Hooke's law*** for shear can be written as:

$$\boxed{\tau = G\gamma}$$

- Here G is called the ***shear modulus of elasticity*** or the ***modulus of rigidity***.

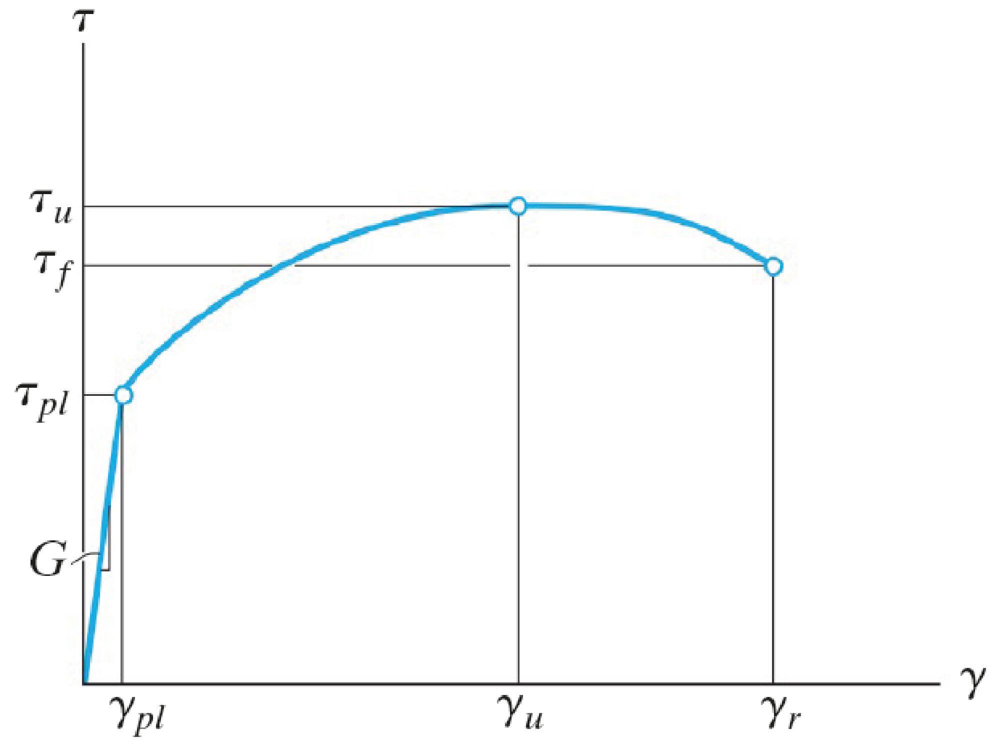


5.3. Stress–Strain behavior

Elastic deformation in Shear test

- Its value represents the ***slope*** of the line on the ***τ–γ diagram***, that is, ***$G=\tau_{pl}/\gamma_{pl}$.***

- Units of measurement for *G* will be the ***same*** as those for $\boldsymbol{\tau}$ (Pa or psi), since $\boldsymbol{\gamma}$ is measured in radians, a dimensionless quantity.



5.3. Stress–Strain behavior

- **Modulus of elasticity** and **shear modulus** values for **several metals** at **room temperature** are presented in the following table.

	Modulus of Elasticity		Shear Modulus		
Metal Alloy	*GPa*	*10^6 psi*	*GPa*	*10^6 psi*	*Poisson's Ratio*
Aluminum	69	10	25	3.6	0.33
Brass	97	14	37	5.4	0.34
Copper	110	16	46	6.7	0.34
Magnesium	45	6.5	17	2.5	0.29
Nickel	207	30	76	11.0	0.31
Steel	207	30	83	12.0	0.30
Titanium	107	15.5	45	6.5	0.34
Tungsten	407	59	160	23.2	0.28

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5.4. Anelasticity

- To this point, it has been **assumed that elastic deformation is time independent**—that is, that an applied stress produces an **instantaneous** elastic strain that remains constant over the period of time the **stress** is **maintained**.

- It has also been assumed that upon **release** of the **load**, the strain is **totally recovered**—that is, that the strain **immediately** returns to **zero**.

- In most engineering materials, however, there will also **exist a time-dependent** elastic strain component—that is, **elastic deformation will continue** after the stress application, and upon load **release**, some finite time is required for complete recovery.

- This **time-dependent elastic behavior** is known as **anelasticity**, and it is due to time-dependent **microscopic** and **atomistic processes** that are attendant to the deformation.

- **For metals**, the anelastic component is **normally small** and is often **neglected**.

- However, for some **polymeric materials**, its magnitude is **significant**; in this case it is termed **viscoelastic behavior**.

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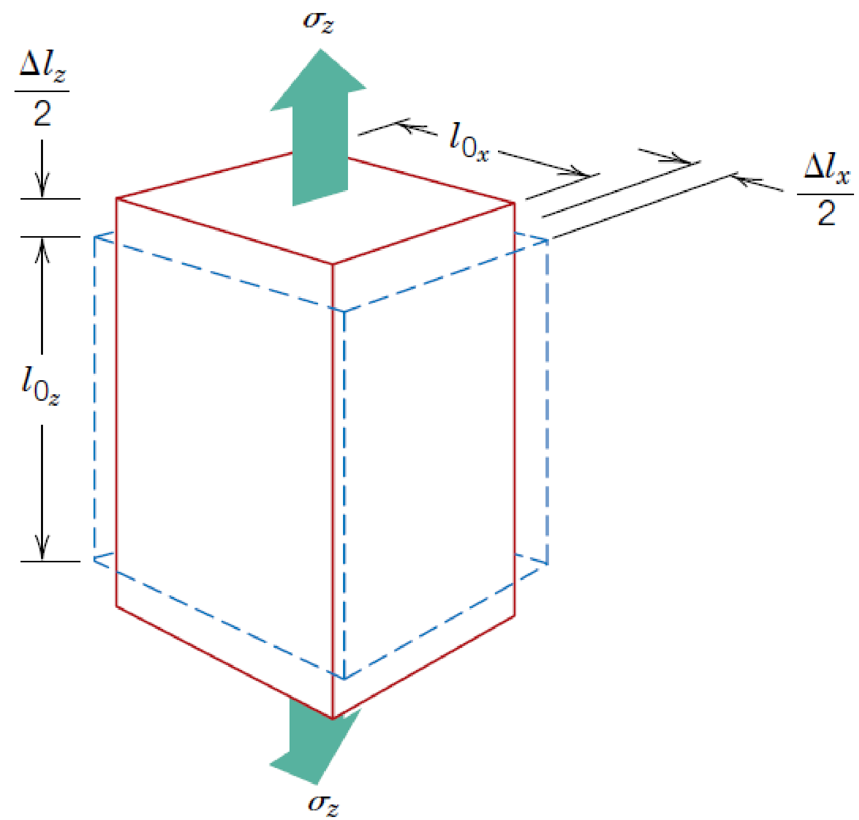
5.5. Elastic properties of materials

- When a **tensile** stress is imposed on a metal specimen, an **elastic elongation** and accompanying strain ε_z result in the direction of the applied stress (arbitrarily taken to be the z direction).

- As a result of this elongation, there will be **constrictions** in the **lateral** (**x and y**) directions **perpendicular** to the **applied stress**; from these contractions, the **compressive strains** ε_x and ε_y may be determined.

- **If** the applied **stress** is **uniaxial** (only in the z direction) and the material is **isotropic**, then $\varepsilon_x = \varepsilon_y$.



Axial (z) elongation (positive strain) and lateral (x and y) contractions (negative strains) in response to an imposed tensile stress. Solid lines represent dimensions after stress application; dashed lines, before.

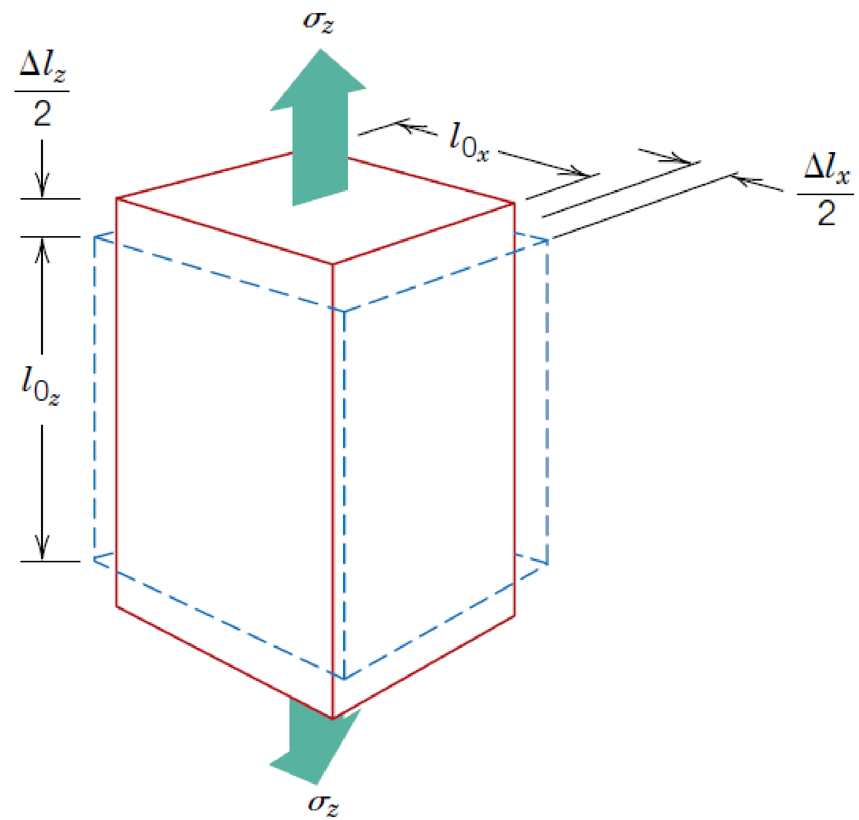
5.5. Elastic properties of materials

- A parameter termed **Poisson's ratio ν** is defined as the ratio of the **lateral** and **axial** strains:

$$\boxed{\nu = -\frac{\epsilon_x}{\epsilon_z} = -\frac{\epsilon_y}{\epsilon_z}}$$

- For virtually all structural materials, ε_x and ε_z will be of **opposite sign**; therefore, the **negative sign** is included in the preceding expression to ensure that **ν is positive.**

- For **many metals** and **other alloys**, values of **Poisson's ratio** range **between 0.25 and 0.35**.

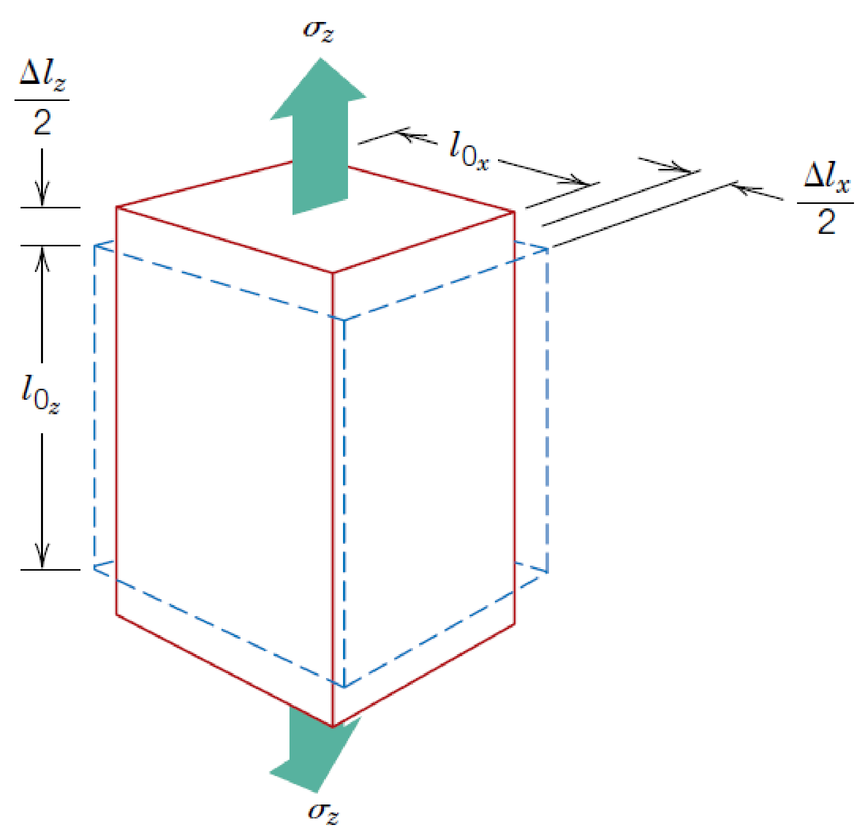


Axial (z) elongation (positive strain) and lateral (x and y) contractions (negative strains) in response to an imposed tensile stress. Solid lines represent dimensions after stress application; dashed lines, before.

5.5. Elastic properties of materials

- Keep in mind that these strains are caused only by the **single axial or longitudinal force *P***; i.e., **no force** acts in **a lateral direction** in order to strain the material in this direction.

- Poisson's ratio is a **dimensionless** quantity.

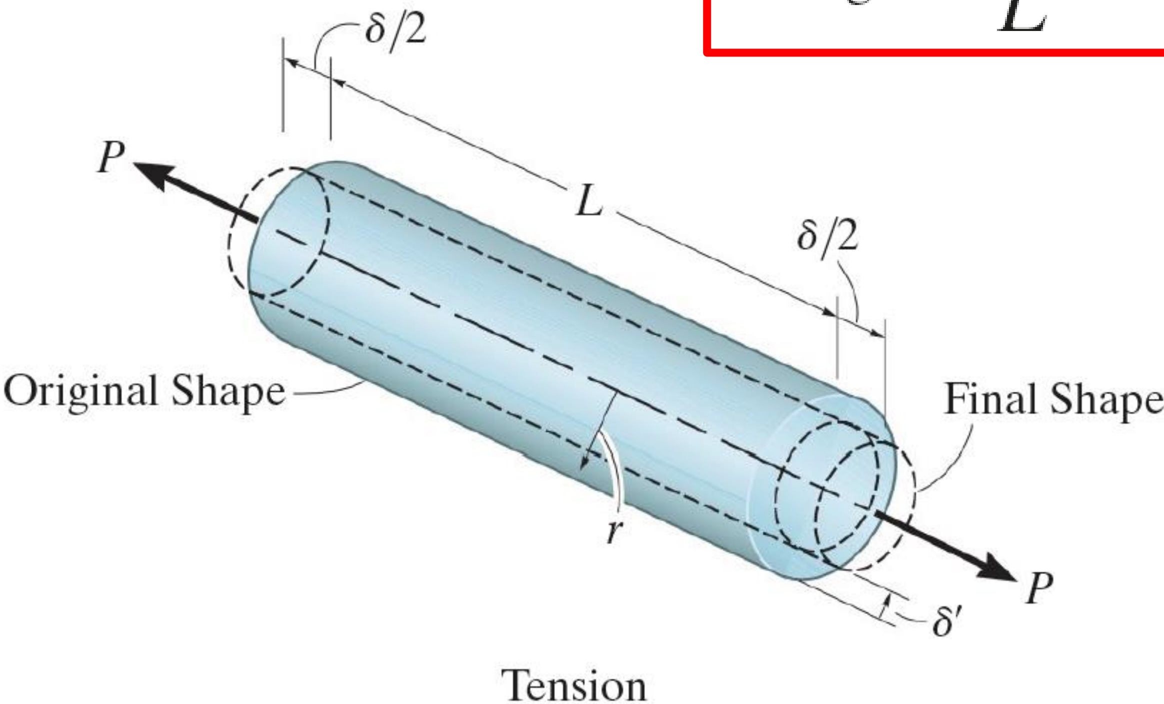


Axial (z) elongation (positive strain) and lateral (x and y) contractions (negative strains) in response to an imposed tensile stress. Solid lines represent dimensions after stress application; dashed lines, before.

5.5. Elastic properties of materials

- **Cylindrical example:** consider the bar in the figure that has an original radius ***r*** and length ***L***, and is subjected to the tensile ***force P.*** This force ***elongates*** the bar by an ***amount δ***, and its ***radius contracts*** by an ***amount δ'***.

$$\boxed{\epsilon_{\text{long}} = \frac{\delta}{L} \quad \text{and} \quad \epsilon_{\text{lat}} = \frac{\delta'}{r}}$$



$$\boxed{\nu = -\frac{\epsilon_{\text{lat}}}{\epsilon_{\text{long}}}}$$

5.5. Elastic properties of materials

- For **isotropic** materials, **shear** and **elastic moduli** are related to each other and to **Poisson's ratio** according to

$$\boxed{E = 2G(1+\nu)}$$

- Therefore, **if E and G are known**, the value of ν can then be **determined** from this equation rather than through experimental measurement.

- In **most metals**, **G** is about **0.4E**; thus, if the value of one modulus is known, the other may be approximated.

5.5. Elastic properties of materials

Anisotropic Materials

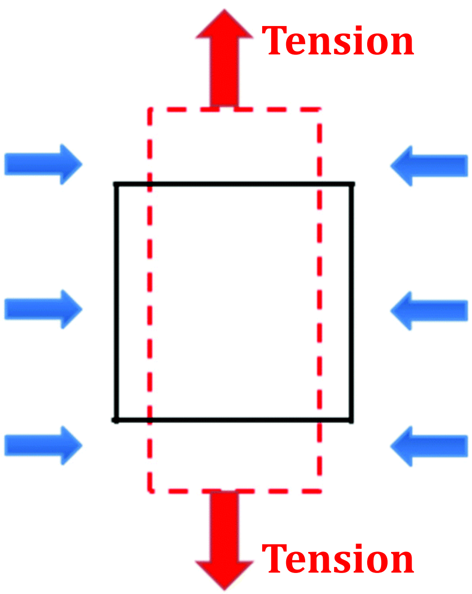
- Many materials are **elastically anisotropic**; that is, the **elastic behavior** (i.e., the magnitude of E) **varies** with **crystallographic direction**.

- Because the **grain orientation** is random in most **polycrystalline** materials, these **may be considered to be isotropic**; **inorganic ceramic glasses are also isotropic.**

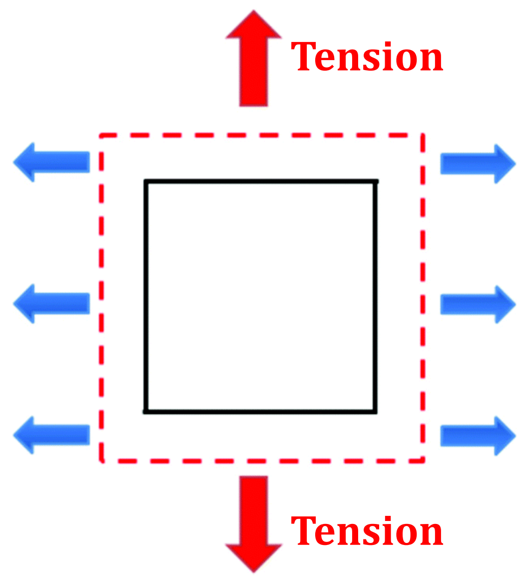
5.5. Elastic properties of materials

Auxetic Materials (a smart material)

- Some materials (e.g., specially **prepared polymer foams**) when pulled in **tension** actually **expand** in the **transverse direction**.
- In these materials, both ε_x and ε_z are **positive**, and thus **Poisson's ratio is negative**.
- Materials that exhibit this effect are termed **auxetics**.



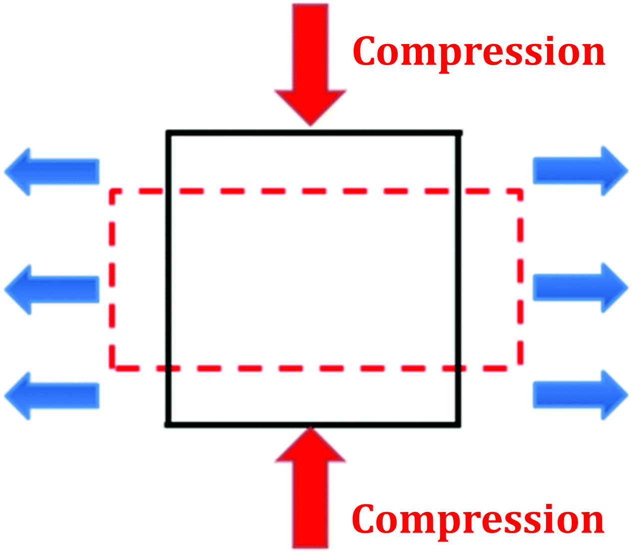
Conventional material (positive Poisson's ratio)



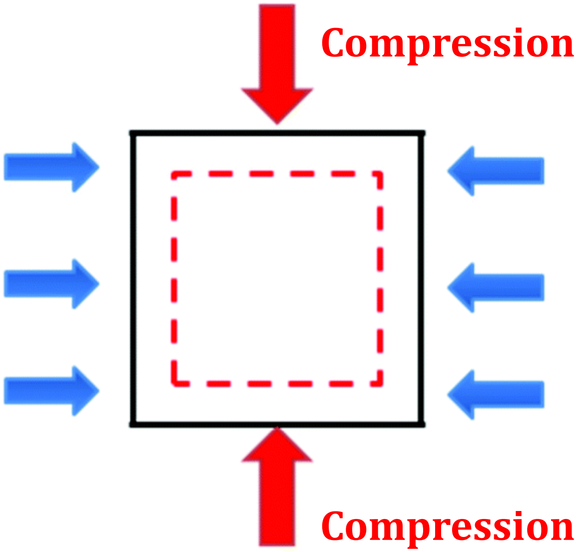
Auxetic material (negative Poisson's ratio)

5.5. Elastic properties of materials

Auxetic Materials (a smart material)



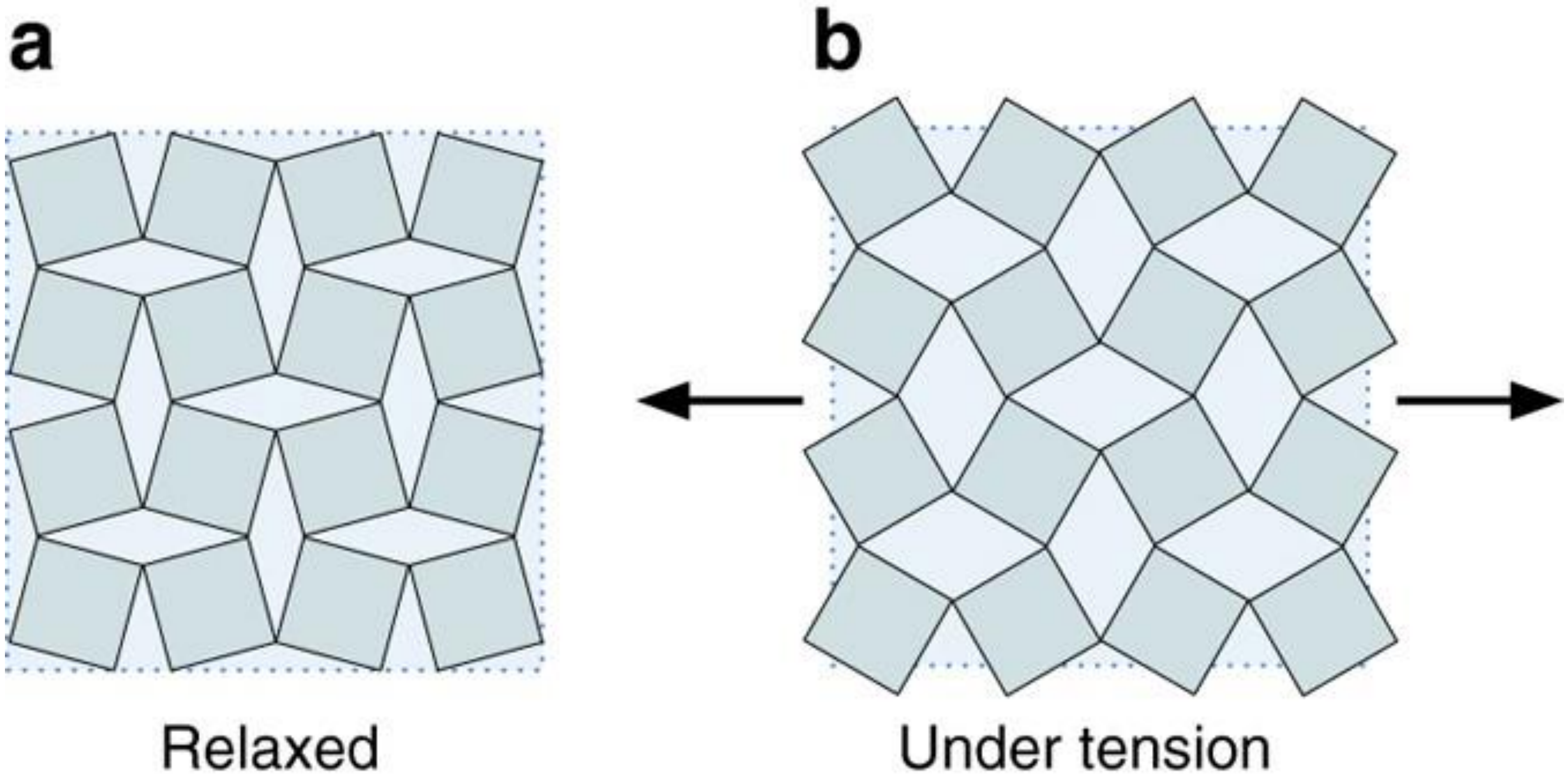
Conventional material
(positive Poisson's ratio)



Auxetic material
(negative Poisson's ratio)

5.5. Elastic properties of materials

Auxetic Materials (a smart material)



5.5. Elastic properties of materials

Auxetic Materials (a smart material)

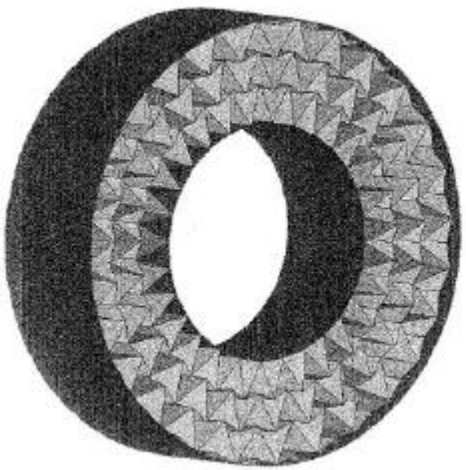
An application of lightweight auxetic composite panels to enhance the ballistic and impact resistance capabilities of armored vehicles.

Pain relief Auxetic structures are already used in Nike's Flyknit running shoe

Their ability to get thicker when stretched makes auxetics fascinating from a scientific and theoretical point of view. The shoe expands when a runner hits their foot on the ground, reducing uncomfortable pressure points in the process.

5.5. Elastic properties of materials

Auxetic Materials (a smart material)



Auxetics are used to make lightweight wheels and runflat tires

An auxetic seat belt, however, would get wider this would spread the loads over a much larger area, potentially reducing any injuries experienced

Blast protection curtains, crash helmet, projectile-resistant or bullet proof vest