Chapter 5

Control Charts for Attributes

5-1. Introduction

- Data that can be classified into one of several categories or classifications is known as attribute data.
- Classifications such as conforming and nonconforming are commonly used in quality control.
- Another example of attributes data is the count of defects.

5-2. Control Charts for Fraction Nonconforming

- Fraction nonconforming_ is the ratio of the number of nonconforming items in a population to the total number of items in that population.
- Control charts for fraction nonconforming are based on the binomial distribution.

5-2. Control Charts for Fraction Nonconforming

Recall: A quality characteristic follows a **binomial** distribution if:

1. All trials are independent.
2. Each outcome is either a "success" or "failure".
3. The probability of success on any trial is given as p. The probability of a failure is

   1-p.
4. The probability of a success is constant.

5-2. Control Charts for Fraction Nonconforming

- The binomial distribution with parameters $n \neq 0$ and $0 < p < 1$, is given by

$$p(x) = \binom{n}{x} p^{x}(1-p)^{n-x}$$

- The mean and variance of the binomial distribution are

$$\boldsymbol{\mu} = np \qquad \boldsymbol{\sigma}^{2} = np(1-p)$$

5-2. Control Charts for Fraction Nonconforming

Development of the Fraction Nonconforming Control Chart

Assume

- n = number of units of product selected at random.
- D = number of nonconforming units from the sample
- p= probability of selecting a nonconforming unit from the sample.
- Then:

$$P(D = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

5-2. Control Charts for Fraction Nonconforming

Development of the Fraction Nonconforming Control Chart

- The sample fraction nonconforming is given as

$$\hat{p} = \frac{D}{n}$$

where $\hat{p}$ is a random variable with mean and variance

$$\mu = p \qquad \sigma^2 = \frac{p(1-p)}{n}$$

5-2. Control Charts for Fraction Nonconforming

Standard Given

- If a standard value of p is given, then the control limits for the fraction nonconforming are

$$UCL = p + 3\sqrt{\frac{p(1-p)}{n}}$$

$$CL = p$$

$$LCL = p - 3\sqrt{\frac{p(1-p)}{n}}$$

5-2. Control Charts for Fraction Nonconforming

No Standard Given

- If no standard value of p is given, then the control limits for the fraction nonconforming are

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$CL = \bar{p}$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

where

$$\bar{p} = \frac{\sum_{i=1}^{m} D_i}{mn} = \frac{\sum_{i=1}^{m} \hat{p}_i}{m}$$

5-2. Control Charts for Fraction Nonconforming

Trial Control Limits

- Control limits that are based on a preliminary set of data can often be referred to as trial control limits.
- The quality characteristic is plotted against the trial limits, if any points plot out of control, assignable causes should be investigated and points removed.
- With removal of the points, the limits are then recalculated.

5-2. Control Charts for Fraction Nonconforming

Example

- A process that produces bearing housings is investigated. Ten samples of size 100 are selected.

Sample #	1	2	3	4	5	6	7	8	9	10
# Nonconf.	5	2	3	8	4	1	2	6	3	4

- Is this process operating in statistical control?

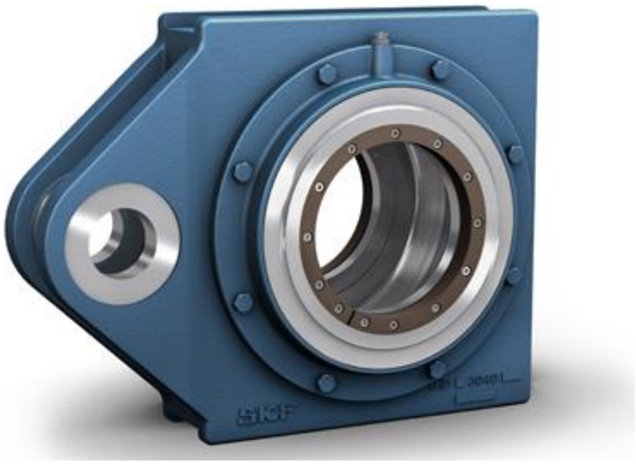
5-2. Control Charts for Fraction Nonconforming

Example

n = 100, m = 10

Sample #	1	2	3	4	5	6	7	8	9	10
# Nonconf.	5	2	3	8	4	1	2	6	3	4
Fraction Nonconf.	0.05	0.02	0.03	0.08	0.04	0.01	0.02	0.06	0.03	0.04

$$\overline{p} = \frac{\sum_{i=1}^{m} \hat{p}_i}{m} = 0.038$$



5-2. Control Charts for Fraction Nonconforming

Example

Control Limits are:

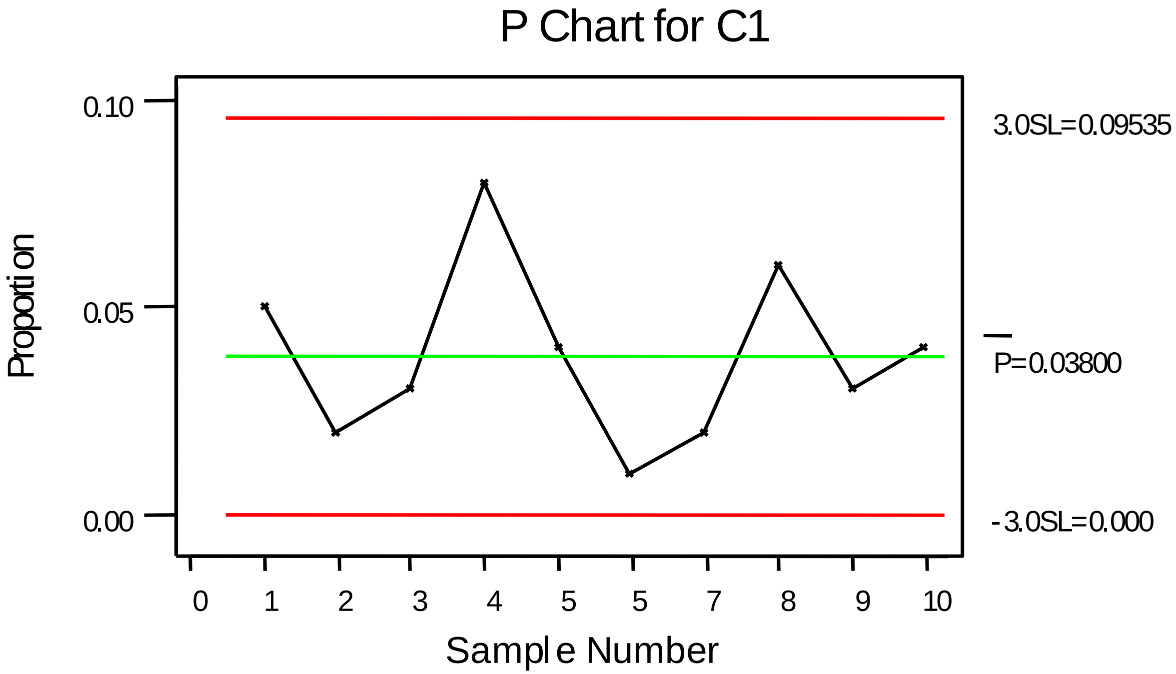
$$UCL = 0.038 + 3\sqrt{\frac{0.038(1-0.038)}{100}} = 0.095$$

$$CL = 0.038$$

$$LCL = 0.038 - 3\sqrt{\frac{0.038(1-0.038)}{100}} = -0.02 \rightarrow 0$$

5-2. Control Charts for Fraction Nonconforming

Example



5-2. Control Charts for Fraction Nonconforming

<u>Design of the Fraction Nonconforming Control Chart</u>

- The sample size can be determined so that a shift of some specified amount, δ can be *detected* with a stated level of probability (50% chance of detection). If δ is the magnitude of a process shift, then *n* must satisfy:

$$\delta = L\sqrt{\frac{p(1-p)}{n}}$$

Therefore,

$$n = \left(\frac{L}{\delta}\right)^2 p(1-p)$$

5-2. Control Charts for Fraction Nonconforming

Positive Lower Control Limit

- The sample size *n*, can be chosen so that the lower control limit would be nonzero:

$$LCL = p - L\sqrt{\frac{p(1-p)}{n}} > 0$$

and

$$n > \frac{(1-p)}{p}L^2$$

5-2. Control Charts for Fraction Nonconforming

Interpretation of Points on the Control Chart for Fraction Nonconforming

- Care must be exercised in interpreting points that plot *below* the lower control limit.
  - They are frequently caused by errors in the inspection process or improperly calibrated test and inspection equipment.
  - They often do not indicate a real improvement in process quality.

5-2. Control Charts for Fraction Nonconforming

The *np* control chart

- The actual number of nonconforming can also be charted. Let n = sample size, p = proportion of nonconforming. The control limits are:

$$\mathrm{UCL} = np + 3\sqrt{np(1-p)}$$

$$\mathrm{CL} = np$$

$$\mathrm{LCL} = np - 3\sqrt{np(1-p)}$$

(if a standard, p, is not given, use $\bar{p}$)

5-2.2 Variable Sample Size

- In some applications of the control chart for the fraction nonconforming, the sample is a 100% inspection of the process output over some period of time.
- Since different numbers of units could be produced in each period, the control chart would then have a variable sample size.

5-2.2 Variable Sample Size

Control Limits Based on an Average Sample Size

- Control charts based on the average sample size results in an approximate set of control limits.
- The average sample size is given by

$$\bar{n} = \frac{\sum_{i=1}^{m} n_i}{m}$$

- The upper and lower control limits are

$$\bar{p} \pm 3\sqrt{\frac{\bar{p}(1-\bar{p})}{\bar{n}}}$$

5-3. Control Charts for Nonconformities (Defects)

- There are many instances where an item will contain nonconformities but the item itself is not classified as nonconforming.
- It is often important to construct control charts for the total number of nonconformities or the average number of nonconformities for a given “area of opportunity”. The inspection unit must be the same for each unit.

5-3. Control Charts for Nonconformities (Defects)

<u>Poisson Distribution</u>

- The number of nonconformities in a given area can be modeled by the Poisson distribution. Let **c** be the parameter for a Poisson distribution, then the mean and variance of the Poisson distribution are equal to the value **c**.
- The probability of obtaining **x** nonconformities on a single inspection unit, when the average number of nonconformities is some constant, **c**, is found using:

$$p(x) = \frac{e^{-c} c^{x}}{x!}$$

5-3.1 Procedures with Constant Sample Size

c-chart

- Standard Given:

$$UCL = c + 3\sqrt{c}$$
$$CL = c$$
$$LCL = c - 3\sqrt{c}$$

- No Standard Given:

$$UCL = \bar{c} + 3\sqrt{\bar{c}}$$
$$CL = \bar{c}$$
$$LCL = \bar{c} - 3\sqrt{\bar{c}}$$

5-3.1 Procedures with Constant Sample Size

Choice of Sample Size: The *u* Chart

- If we find *c* total nonconformities in a sample of *n* inspection units, then the average number of nonconformities per inspection unit is $u = c/n$.
- The control limits for the average number of nonconformities is

$$\mathrm{UCL} = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}}$$

$$\mathrm{CL} = \bar{u}$$

$$\mathrm{LCL} = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}}$$

5-3.2 Procedures with Variable Sample Size

<u>Three Approaches for Control Charts with Variable Sample Size</u>

1. Variable Width Control Limits
2. Control Limits Based on Average Sample Size
3. Standardized Control Chart

5-3.2 Procedures with Variable Sample Size

<u>Control Limits Based on an Average Sample Size</u>

- Control charts based on the average sample size results in an approximate set of control limits.
- The average sample size is given by

$$\bar{n} = \frac{\sum_{i=1}^{m} n_i}{m}$$

- The upper and lower control limits are

$$\bar{u} \pm 3\sqrt{\frac{\bar{u}}{\bar{n}}}$$

5-3.3 Demerit Systems

- When **several less severe** or minor defects can occur, we may need some system for classifying nonconformities or defects according to severity; or to weigh various types of defects in some reasonable manner.

5-3.3 Demerit Systems

Demerit Schemes

1. Class A Defects - very serious
2. Class B Defects - serious
3. Class C Defects - Moderately serious
4. Class D Defects - Minor

- Let c_{iA}, c_{iB}, c_{iC}, and c_{iD} represent the number of units in each of the four classes.

5-3.3 Demerit Systems

Demerit Schemes

- The following weights are fairly popular in practice:
  - Class A-100, Class B - 50, Class C – 10, Class D - 1

$$d_i = 100c_{iA} + 50c_{iB} + 10c_{iC} + c_{iD}$$

d_i - the number of demerits in an inspection unit

5-3.3 Demerit Systems

Control Chart Development

- Number of demerits per unit:

$$u_i = \frac{D}{n}$$

where n = number of inspection units

$$D = \sum_{i=1}^{n} d_i$$

5-3.3 Demerit Systems

Control Chart Development

$$\mathrm{UCL} = \bar{u} + 3\hat{\sigma}_u$$

$$\mathrm{CL} = \bar{u}$$

$$\mathrm{LCL} = \bar{u} - 3\hat{\sigma}_u$$

where $$\bar{u} = 100\bar{u}_A + 50\bar{u}_B + 10\bar{u}_C + \bar{u}_D$$

and

$$\hat{\sigma}_u = \left[\frac{(100)^2\bar{u}_A + (50)^2\bar{u}_B + (10)^2\bar{u}_C + \bar{u}_D}{n}\right]^{1/2}$$