

* Chapter - 1:

$$PV = nRT = m \left(\frac{R}{M} \right) T$$

(N/mole) (kg)

Specific heat (heat needed to change temp. by 1°).

$$\delta Q = c dt \Rightarrow Q = m C \Delta T = N C \Delta T$$

(J/kgK) (J/Kmole)

isochoric = constant Volume ; isobaric = constant pressure

$$\delta W = P_c dU \rightarrow \begin{cases} \text{Compression} \Rightarrow W > 0 \\ \text{Expansion} \Rightarrow W < 0 \end{cases}$$

at reversible since there's an infinitesimal change $\Rightarrow W = - \int P dU$

$$\delta Q + \delta W = dU \Rightarrow Q + W = \Delta U$$

$$\Delta U = m C_v \Delta T ; \Delta H = m C_p \Delta T$$

$$Q + \int P dV = \Delta H$$

$$R = C_p - C_v$$

$$\text{Isothermal: } W = -NRT \ln\left(\frac{U_2}{U_1}\right) = NRT \ln\left(\frac{P_2}{P_1}\right) = PV \ln\left(\frac{P_2}{P_1}\right)$$

$$\text{Adiabatic (no heat exchange): } \begin{cases} P U^\gamma = \text{cte} \\ T U^{\gamma-1} = \text{cte} \\ \frac{T}{P^{\frac{\gamma}{\gamma-1}}} = \text{cte} \\ W = \frac{P_2 U_2 - P_1 U_1}{\gamma - 1} \end{cases} ; \gamma = \frac{C_p}{C_v} ; \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\text{Isolated} \Rightarrow Q = 0$$

* Formulas of ΔS : $\delta S \geq \frac{\delta Q}{T}$

$$\Delta S = C_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{U_2}{U_1}\right) = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$$

Isolated system: $dS \geq 0$
 (rev $dS = \frac{dQ}{T}$)
 (irrev. $dS > \frac{dQ}{T}$)
 in adiabatic: $\Delta S = \frac{Q}{T} = 0$

• Polytropic transformation: $PV^n = cte$.

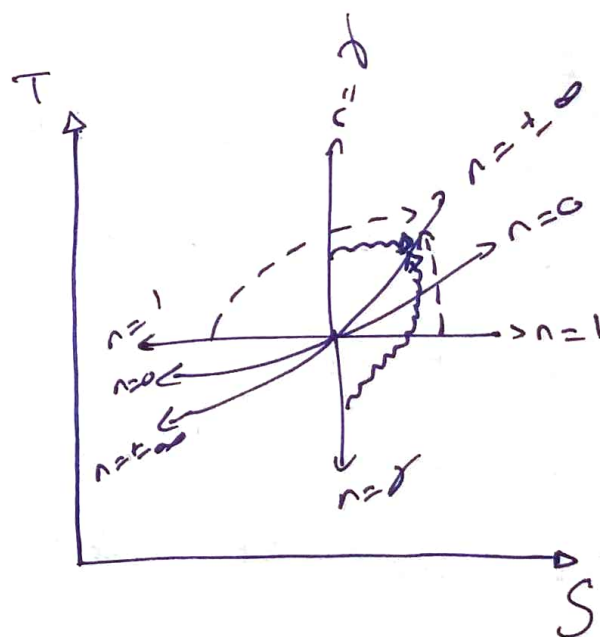
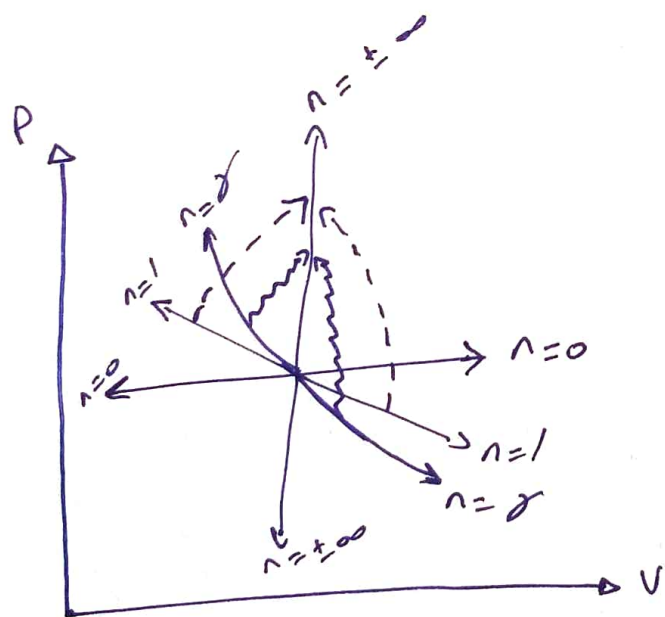
$$n = \frac{C - C_p}{C - C_v} \quad ; \quad C = \frac{n - \gamma}{n - 1} C_v = \frac{n - \gamma}{\gamma - 1} C_p.$$

$$W = \frac{P_2 V_2 - P_1 V_1}{n - 1}$$

$$\begin{cases} Q = \frac{n - \gamma}{n - 1} \Delta U \\ Q = \frac{n - \gamma}{\gamma - 1} W. \end{cases}$$

• Conclusion:

$n = 1 \Rightarrow PV = cte \Rightarrow \text{isothermal} \Rightarrow C \rightarrow \pm \infty$ ($T = cte$)
 $n = 0 \Rightarrow P = cte \Rightarrow \text{isobaric} \Rightarrow C = C_p$
 $n = \gamma \Rightarrow PV^\gamma = cte \Rightarrow \text{adiabatic} \Rightarrow C = 0$
 $n \rightarrow \pm \infty \Rightarrow V = cte \Rightarrow \text{isochoric} \Rightarrow C = C_v.$



• Compression for $n > \gamma$ or ~~detente~~ for $n < \gamma \Rightarrow S \uparrow \Rightarrow Q > 0$.

• Compression for $n > 1$ or detente for $n < 1 \Rightarrow T \uparrow \Rightarrow \Delta U > 0$

• Note:

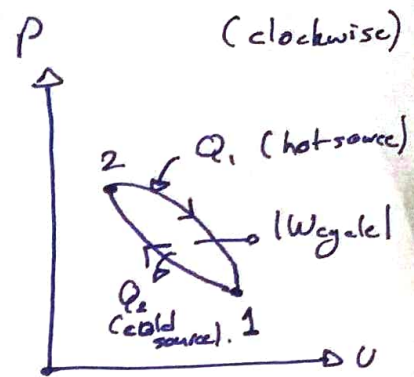
• Any Area on P-V diagram \Rightarrow Work.
 • " " " " T-S " \Rightarrow heat

* Cycles:

• Thermal Cycle: • Work of détente > Work of Compression

- Q_1 : heat gained from hot source.
- Q_2 : heat given from cold source
- $|W_{\text{cycle}}| = Q_1 - |Q_2|$.

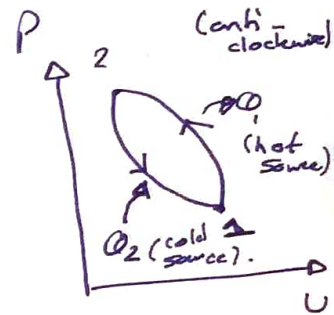
$$\Rightarrow \text{Cycle's efficiency} = \eta_{\text{th}} = \frac{|W_{\text{cycle}}|}{Q_1}$$



• Refrigeration Cycle: • expansion work < Compression work.

- Q_1 : heat transferred to hot source.
- Q_2 : heat taken from cold source.
- $W_{\text{cycle}} = Q_1 - Q_2$

$$\Rightarrow \text{Performance Coefficient: } e = \frac{Q_2}{W_{\text{cycle}}}$$



* Calculation of Work & ΔS in irreversible transformations:

(non-quasistatic \Rightarrow irreversible).

$$W = -P_e(V_f - V_o) = -P_f(V_f - V_o). \quad (W_{\text{rev}} < W_{\text{irreversible}})$$

• Thermostat: Remains Temp. constant (system not isolated).

• To calculate ΔS , we imagine that it's reversible b/w initial & final state.

$$Q + W = \Delta U = 0 \Rightarrow Q = -W$$

$$\Delta S = \frac{Q}{T_o} = \frac{-W}{T_o} = \frac{-NR T_o \ln(\frac{P_2}{P_1})}{T_o} = -NR \ln(\frac{P_2}{P_1}) < 0 \quad (\text{In non-isolated } \Delta S \text{ can be negative}).$$

* note:

$$\boxed{\dots \times 0.736 = \dots \text{ Kw}}$$

$$\Delta S_{12} = mC \ln\left(\frac{T_2}{T_1}\right)$$

$$L_{\text{atm}} = 101.31 \text{ J}$$

$$1 \text{ bar} = 100 \text{ J}$$

$$1 \text{ bar} = 10^5 \text{ Pa}$$

$$4.184 \text{ J} = 1 \text{ cal}$$

$$0.2 \text{ L} = 1 \text{ dm}^3$$

$$1 \text{ atm} = 101325 \text{ Pa}$$

$$= 1.01325$$

$$1 \text{ J} = \text{Pa} \cdot \text{m}^3$$

* Chapter-2:

* In a mixture each gas occupies the entire volume of the mixture and temp. of the mixture

* Total pressure of mixture = Σ Partial pressures of constituents. = ΣP_i .

* If we make $P_i = P_T \rightarrow$ we get Volume = V_i = Partial Volume, of this constituent.

* mass composition: $g_i = \frac{m_i}{m_{\text{total}}}$; $\Sigma g_i = 1$.
Kg

* Volume Composition: $U_i = \frac{N_i}{N}$; $\Sigma U_i = 1$.
K mole.

$$P_i = P \cdot u_i ; V_i = V \cdot u_i ; g_i = V_i \times \frac{M_i}{M}$$

Molar mass of Component
Total Molar mass.

$$P_i = \frac{P}{M_i}$$

* apparent molar mass = $M = \frac{m}{N} = \frac{1}{\Sigma \frac{g_i}{M_i}} = \frac{m}{\Sigma N_i} = \Sigma U_i M_i$.

$$g_i = V_i \times \frac{r_i}{r_i}$$

$$r = \Sigma g_i \cdot r_i$$

$$= \frac{1}{\Sigma \frac{U_i}{r_i}}$$

$$C = \Sigma U_i C_i \text{ (KJ / (K mole))}$$

$$C = \Sigma g_i C_i \text{ (KJ / Kg)}$$

* ideal mixture: $u = \Sigma N_i C_{v,i} T$; $H = \Sigma N_i C_{p,i} T$.

$$\begin{cases} \Delta U_{\text{mixture}} = \Sigma N_i C_{v,i} \Delta T ; \\ \Delta H_{\text{mixture}} = \Sigma N_i C_{p,i} \Delta T ; \\ \Delta S_{\text{mixture}} = \Sigma N_i C_{p,i} \ln\left(\frac{T_2}{T_1}\right). \end{cases}$$

* Types of ideal gas mixing:

a) Constant Volume: No heat exchange

$$U = \Sigma U_i$$

mixture after mixing components before mixing

$$\Delta U_{\text{sys}} = 0$$

$$T = \frac{\Sigma N_i \cdot C_{v,i} \cdot T_i}{\Sigma N_i C_{v,i}}$$

$$T = \Sigma U_i T_i$$

if atoms of same atomicity (since C_v becomes the same).

*Remark: when $N T = \sum N_i T_i$ (Same atomicity)

$$\Rightarrow P V = \sum P_i V_i$$



Pressure and Volume before mixing (not partial p & V).

b) Constant Pressure: No heat exchange

(we use masses on pistons to maintain $P = \text{cte}$).

$$\Rightarrow \Delta H_{\text{sys}} = 0 \quad ; \quad H = \sum H_i$$

$$\Rightarrow T = \frac{\sum N_i C_{p,i} T_i}{\sum N_i C_{p,i}}$$

if Same atomicity (Same C_p) $\Rightarrow T = \sum V_i T_i$

. In this case, part of energy in (PV) form transfers into internal energy (U)

c) Complete transformation of energy content of gas into internal energy:

(we don't put mass of a final mixture piston, only on initial piston).

. $P = \text{constant}$

after $U = U_0 + \sum H_i$ (gas before mixture)

$$\Rightarrow T = T_0 \cdot V_0 + \gamma \sum V_i T_i$$

(note: final temp higher than any temp of entering gases).

*Note:

$$P = \rho \cdot g \cdot h$$

$\frac{N}{m^2} \quad \frac{10 \text{ m/s}^2}$

$$1 \text{ kg/s}^2 = \text{N/m}$$

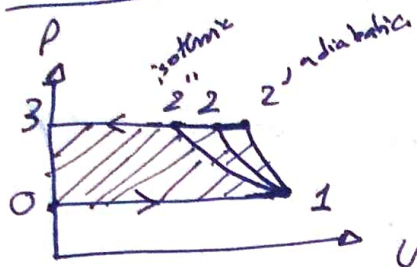
$$1 \text{ N.m} = \text{J}$$

*Remark:

when we calculate ΔS for a component in mixture we use partial pressure or partial volume.

* Chapter - 3: les compresseur.

I) Thermal Work diagram:



0-1: suction & filling of cylinder ($P = P_1$)

1-2: gas compression (2 values are lim.)

2-3: discharge of compressed gas ($P = P_2$)

$$W = \int_1^2 V dP \quad \text{(Compression Work of Cycle)} \\ \text{(theoretical comp. work)}$$

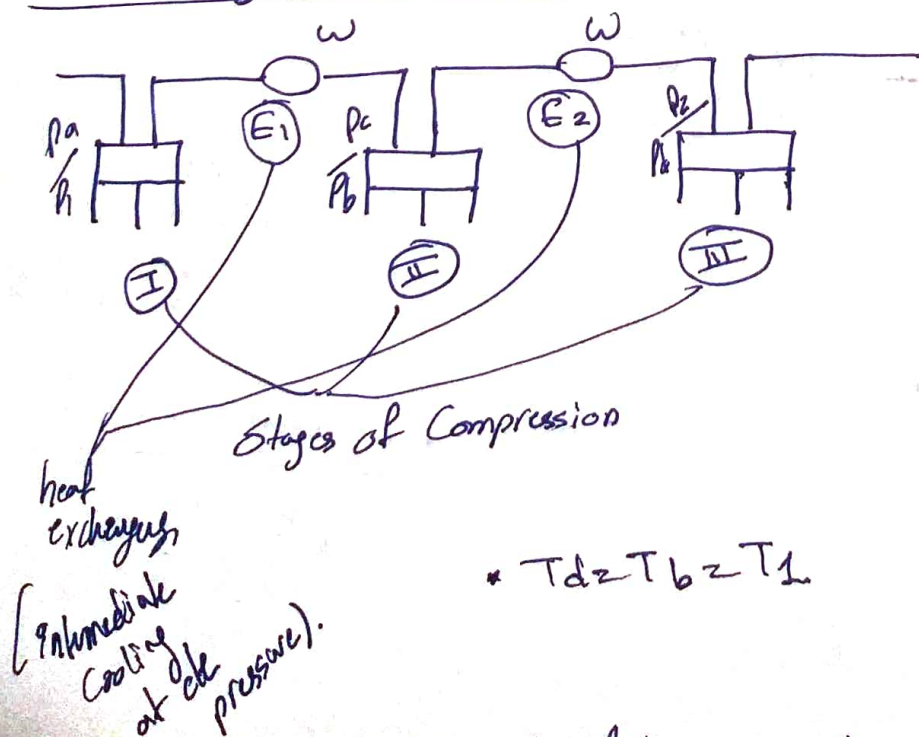
This work is called work with transfer.

$$W = \begin{cases} \text{Isothermal: } W_{iso} = W_{12} \\ \text{adiabatic: } W_{ad} = \gamma W_{12} \\ \text{Polytropic: } W_{pol.} = n W_{12} \end{cases}$$

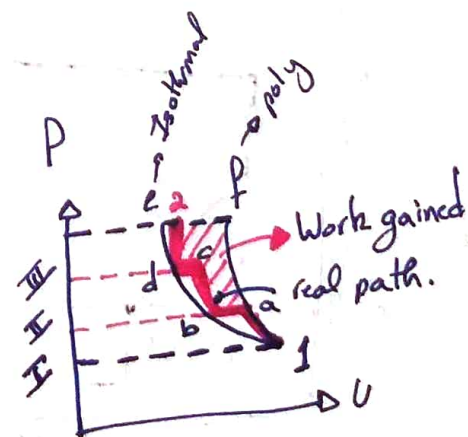
• General Formula of Work:

$$W = \frac{n}{n-1} (P_2 V_2 - P_1 V_1) = \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] \text{ KJ/cycle.} \\ \left. \begin{aligned} &= \frac{n}{n-1} R (T_2 - T_1) \text{ KJ/Kmol.} \\ &= \frac{n}{n-1} r (T_2 - T_1) \text{ KJ/Kg} \end{aligned} \right\} \begin{matrix} n n r T_1 \text{ or } m r T_1 \end{matrix}$$

* Multi-Stage Compression:



$$T_d \geq T_b \geq T_1$$



1 a, b, c, d 2 : Poly. comp. on stages I, II, III.

a, b, c, d Isochoric cooling

* note: We shouldn't use many stages to prevent pressure loss while cooling

- Compression on more than 1 stage (same ratio $\frac{P_2}{P_1}$) gives us:
 - gain in Work of cycle.
 - a lower temp. of compressed gas.

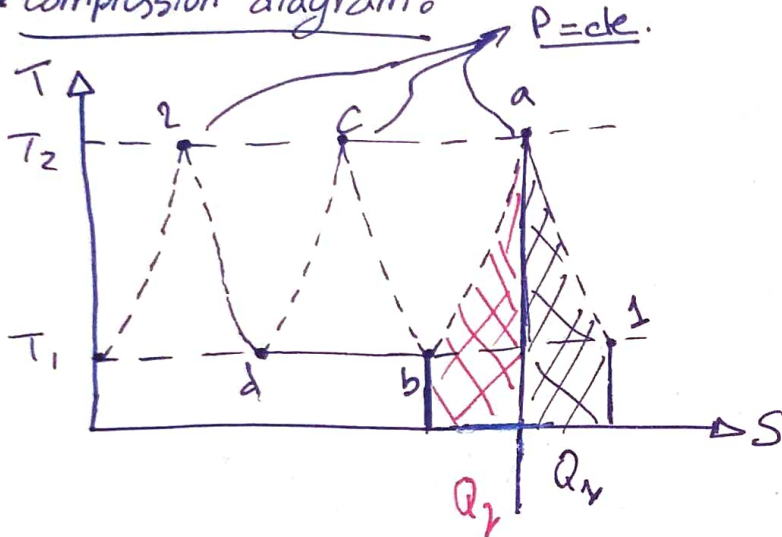
$$x = \frac{P_a}{P_1} = \frac{P_c}{P_a} = \frac{P_2}{P_c} \quad (\text{same ratio of pressure}) \quad (\text{to find heat efficiency}).$$

m: number of stages.

$$T_a = T_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{1}{m}} \right]^{\frac{n-1}{n}}$$

$$\left(\frac{P_2}{P_1} \right)^{\frac{1}{m}} = x$$

* Compression diagram:



* Q_1 = heat lost during polytropic compression.

$$Q_1 = C \Delta T$$

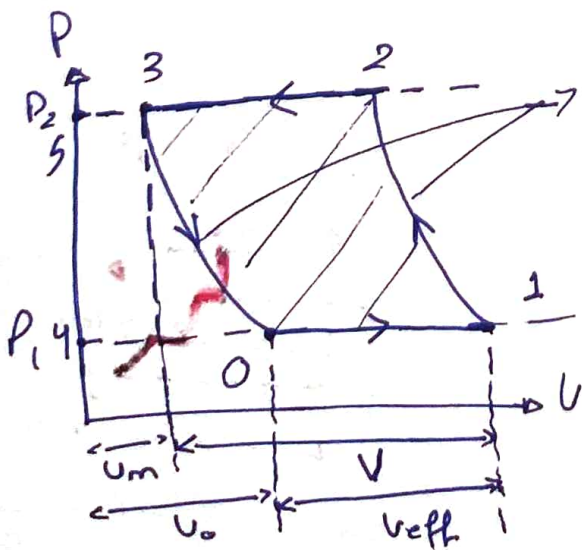
$$= C_u \cdot \frac{n-1}{n} \Delta T \quad \text{KJ/kg}$$

* Q_2 = heat lost during cooling of the 1st stage

$$Q_2 = C_p \Delta T$$

Note: if we have 10% decrease in pressure in exchanger we use $x = \left(\frac{P_2}{P_1 \times 0.9} \right)^{\frac{1}{m}}$ and complete exchanger if exchgs

II) Compression Cycle with dead volume (Volume mort):



$PV^n = \text{cte}$
(not necessary same n).

3-0: detante of residual gas.
(gas left as V_m expands to V_2).

• In aspiration:

$$\begin{cases} P_i < P_a \\ T_i > T_a \end{cases}$$

* n_1 : compression

* n_2 : detank.

* Percentage of Volumetric: $m = \frac{V_m}{V} \times 100\%$ (usually btw 2-15 %).

* V_{eff} : effective volume actually filled btw opening and closing of suction valve.

$V_{eff} = V_m + V - V_o$

(Volume generated by piston when moving btw extremities)

* rendement Volumétrique (Volumetric efficiency): $\eta_v = \frac{V_{eff}}{V} = 1 - m \left[\left(\frac{P_2}{P_1} \right)^{\frac{1}{\gamma_2}} - 1 \right]$

* Work = $W = W_{comp} - W_{ext atm}$

$$= \frac{n_1}{n_1 - 1} \cdot P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n_1 - 1}{n_1}} - 1 \right] - \frac{n_2}{n_2 - 1} P_2 V_o \left[\left(\frac{P_2}{P_1} \right)^{\frac{n_2 - 1}{n_2}} - 1 \right]$$

* filling (remplissage) efficiency: $\eta_r = \eta_v \times \frac{P_1}{P_a} \times \frac{T_a}{T_1}$; P_a, T_a parameters of air atmosphere.

* Compressor Output (productivité): $V_{comp} = V \times \eta_r \times N$ or $V \times \eta_v \times N$

(m^3/s) ($m^3/cycle$) (tr/s) (cycle/s)

* Power: $P = W \times N$ or $P = W \times m$

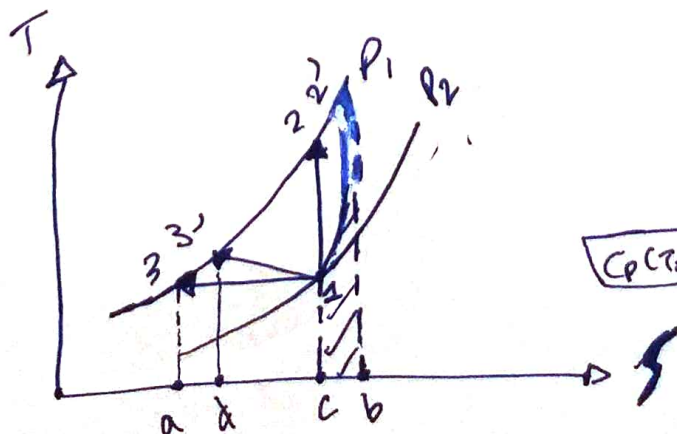
Kw ($KJ/cycle$) or RPM (convert to s) Kw (KJ/kg) Kg/s

II) Real evolution of gas in a compressor (with friction):

a) non-refrigeration machines

1-2: adiabatic

1-2': polytropic.



$$W_{ad} = \frac{\gamma}{\gamma - 1} \cdot r (T_2 - T_1)$$

$$= \Delta H_{12}$$

$$= \text{area } 23ac$$

$$C_p (T_2 - T_1)$$

$$W_{poly} = \frac{n}{n - 1} \cdot r (T_2 - T_1)$$

$$= \text{area } 12'3ac$$

$$= \Delta H_{12'} - Q_{12'}$$

$$W_{irr} = W_{pol.} + W_f = \Delta H_{12'} = \text{area } 2'3ab = C_p(T_{2'} - T_1)$$

\downarrow work really absorbed
 \downarrow friction work

$$\eta_{poly} = \frac{W_{poly.}}{W_{irr}} \quad \eta_{ad.} = \frac{W_{ad}}{W_{irr}}$$

b) Refrigeration machine:

1-3: isothermal (ideal)

1-3': polytropic (real).

$$W_{isothermal} = rT \ln\left(-\frac{P_2}{A}\right)$$

$= \text{area } 13ac$
 $= -Q_{13}$

$$W_{poly} = \frac{n}{n-1} \cdot r(T_{3'} - T_1)$$

$= \text{area } 13'3adc$
 (calculated before)

$$W_{irr} = W_{poly} + W_f$$

$$\eta_{poly} = \frac{W_{poly.}}{W_{irr.}} \quad \eta_{iso} = \frac{W_{isothermal}}{W_{irr}} \quad \eta_{ad} = \frac{W_{ad.}}{W_{irr.}}$$

$$W_{ad} = \frac{W_{ad}}{W_{irr}} = \frac{\Delta H_{12}}{\Delta H_{12'}} = \frac{m C_p \Delta T}{m R \ln \Delta T} = \frac{T_2 - T_1}{T_{2'} - T_1}$$