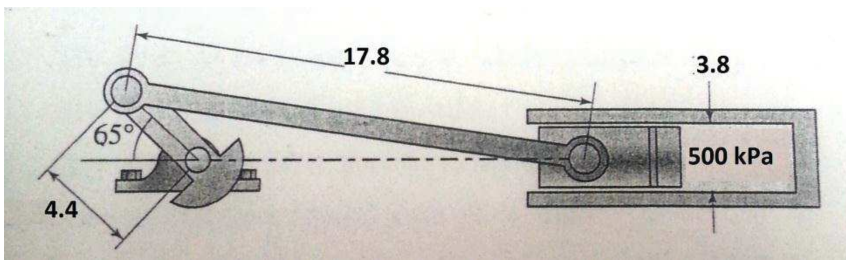
Problem 1

The compressor mechanism shown in the following figure is driven clockwise by a DC electric motor at a constant rate of 800 rpm. In the position shown, the pressure is 500 kPa.

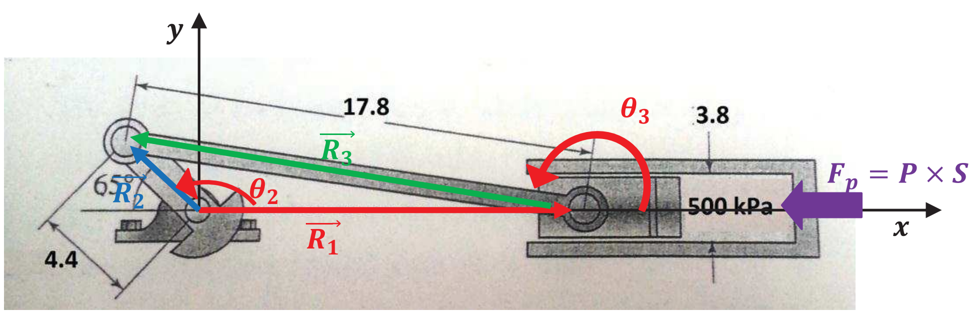
a) Find the position of the piston.
b) Find the velocity and acceleration of the piston.
c) Using Energy Method, determine the torque required from the motor to operate the compressor.

The piston has a mass $m_p = 0.45$ kg. Neglect the mass of other links. All dimensions are in cm.



Solution

a) Vector loop of OAB : $\overrightarrow{R_2} - \overrightarrow{R_3} - \overrightarrow{R_1} = 0$



For the slider-crank mechanism: firstly, determine the input angle and link length

Angle:

- $\theta_2 = 180° - 65° = 115°$ $(input\ angle)$

Link length

- a = 4.4cm, b = 17.8 cm, c = 0.

Then, calculate the output angle

- $\theta_{31} = \sin^{-1}\left(\frac{a\sin\theta_2 - c}{b}\right) = 12.946° \ (rejected)$
- $\theta_{32} = \sin^{-1}\left(-\frac{a\sin\theta_2 - c}{b}\right) + \pi = 167.05° \ (\boldsymbol{Accepted, open\ mechanism})$

The position of the piston w.r.t. O

$$d = a\ cos\theta_2 - b\cos\theta_3 = \mathbf{15.488\ cm}$$

b) Velocity analysis

Input velocity, $w_2 = \frac{-800\times 2\pi}{60} = -83.776\ rad/s$

- $w_3 = \frac{a}{b}\frac{cos\theta_2}{cos\theta_3}w_2 = \boldsymbol{-8.98\ rad/s}$
- $\dot{d} = -aw_2sin\theta_2 + bw_3sin\theta_3 = \mathbf{298.267\ cm/s}$

Acceleration analysis

Input acceleration, $\alpha_2 = 0$ (constant speed)

- $\alpha_3 = \frac{a\,\alpha_2\cos\theta_2 - a\,w_2^2\ \sin\theta_2 + b\,w_3^2\ \sin\theta_3}{b\ \cos\theta_3} = \boldsymbol{1594.8\ rad/s^2}$
- $\ddot{d} = -a\alpha_2\sin\theta_2 - aw_2^2\ \cos\theta_2 + b\,\alpha_3\ \sin\theta_3 + bw_3^2\ \cos\theta_3 = \boldsymbol{18102\ cm/s^2}$

c) Energy Method

$$\sum_{k=2}^{n}\overrightarrow{F_k}\,.\,\overrightarrow{V_k} + \sum_{k=2}^{n}\overrightarrow{T_k}\,.\,\overrightarrow{w_k} = \sum_{k=2}^{n}m_k\,.\,\overrightarrow{a_k}\,.\,\overrightarrow{V_k} + \sum_{k=2}^{n}I_k\,.\,\overrightarrow{\alpha_k}\,.\,\overrightarrow{w_k}\quad (\boldsymbol{eq.1})$$

The force due to pressure is :

$$F_p = (Pressure)\times(Surface) = P_i\ \times\pi\frac{d^2}{4} = 500\times 10^3\times\pi\frac{0.038^2}{4} = \mathbf{566.7}\ \boldsymbol{N}$$

and $\overrightarrow{F_p} = -566.7\,,\overrightarrow{V_p} = \dot{d} = 2.9867,\ a_p = 181.02\ m/s^2$

Then, eq.1 => $-566.77\times 2.9867 + T_{12}\times w_2 = m_p\times a_p\times v_p$

- $Power\ required:$

$$T_{12} \times w_2 = 0.45 \times 181.02 \times 2.98267 + 566.77 \times 2.9867 = \mathbf{1935.74\ Watt}$$

- $Torque\ required:$

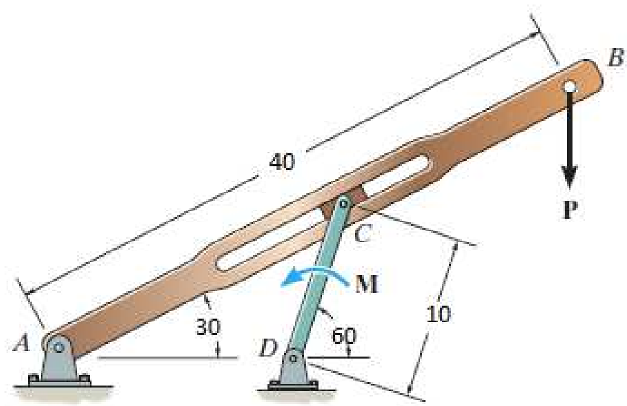
$$T_{12} = \frac{1935.74}{83.776} = \mathbf{23.1\ N.m}$$

Problem 2

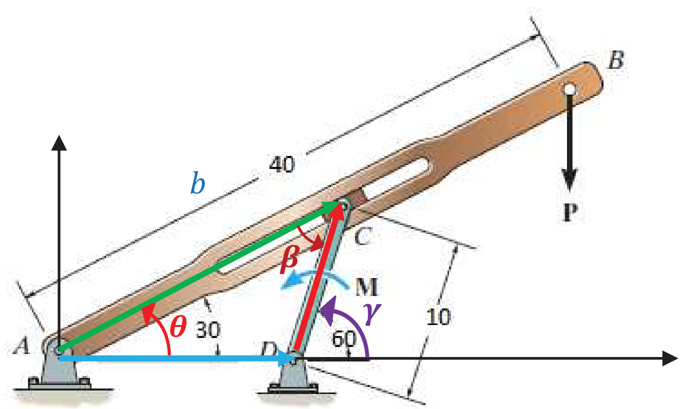
For the quick-return mechanism shown below, the rod DC rotates with a constant angular velocity of **1 rad/s CCW**.

a) For the position shown find the angular velocity of member AB and the velocity of sliding of block C within the member AB.
b) If a force P = 1 kN is applied vertically at B, determine the couple moment M that must be applied to member DC. Also find the force transmitted through the slider joint at C

Neglect the masses of all members. Dimensions are in cm.



Solution



Firstly, determine the input angle, link length and angular velocity :

Angle

- $\theta = 30°$
- $\beta = 180° - 30° - 120° = 30°$
- $\gamma = 60°$

Length

- ADC is an isosceles triangle => $AD = DC = 10\ cm$
- Calculation of AC :

$$\frac{\sin 120°}{AC} = \frac{\sin 30°}{10}$$

$$=> AC = \frac{\sin 120°}{\sin 30°} \times 10 = 17.32\ cm.$$

Angular velocity

$$\dot{\gamma} = 1\ rad/s$$

Vector loop equation of ACD :

$$\overrightarrow{AC} - \overrightarrow{DC} - \overrightarrow{AD} = 0$$

$$AC\ e^{j\theta} - DC\ e^{j\gamma} - AD\ e^{j0} = 0$$

Time derivative of Eq. 1

$$\dot{AC}\, e^{j\theta} + AC\,\dot{\theta}\, j\, e^{j\theta} \; - DC\,\dot{\gamma} j\, e^{j\gamma} = 0$$

We need to determine $\dot{AC}$ *and* $\dot{\theta}$.

$$\dot{AC}\,(\cos\theta + j\sin\theta) + AC\,\dot{\theta}\, j(\cos\theta + j\sin\theta) \; - DC\,\dot{\gamma}\, j(\cos\gamma + j\sin\gamma) = 0$$

$$\dot{AC}\,(\cos\theta + j\sin\theta) + AC\,\dot{\theta}\,(-\sin\theta + j\cos\theta) \; - DC\,\dot{\gamma}\,(-\sin\gamma + j\cos\gamma) = 0$$

- Real =0

$$\dot{AC}\;\cos\theta - AC\,\dot{\theta}\;\sin\theta + DC\,\dot{\gamma}\;\sin\gamma = 0$$

$$\boxed{\dot{AC}\;\cos 30° - 17.32\times\dot{\theta}\times\;\sin 30° + 10\times 1\times\sin 60° = 0\;(Eq.1)}$$

- Imag =0

$$\dot{AC}\;\sin\theta + AC\,\dot{\theta}\;\cos\theta\; - DC\,\dot{\gamma}\;\cos\gamma = 0$$

$$\boxed{\dot{AC}\;\sin 30° + 17.32\,\dot{\theta}\;\cos 30°\; - 10\times 1\times\;\cos 60° = 0\;(Eq.2)}$$

Solve the two equations 1 and 2 with the two unknowns, you get

- Velocity of sliding of block C: $\boldsymbol{\dot{AC} = -0.05\;m/s}$

- Angular velocity of member AB: $\boldsymbol{\dot{\theta} = 0.5\;rad/s}$

b) Energy method (No masses)

$$\sum \overrightarrow{F_k}\,.\overrightarrow{V_k} + \sum \overrightarrow{T_k}\,.\overrightarrow{w_k} = 0$$

$$=> \boldsymbol{\vec{P}\,.\overrightarrow{V_B} + M\,.\dot{\gamma} = 0}$$

with

- The velocity of B : $\boldsymbol{\overrightarrow{V_B} = AB.\dot{\theta}\,(-\sin 30° + j\;\cos 30°)}$

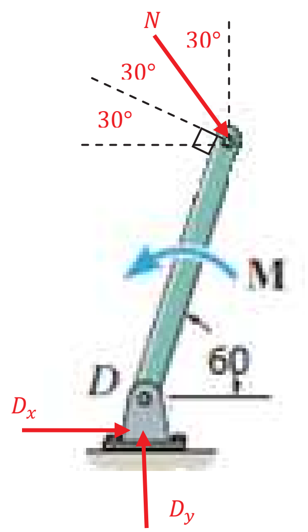
$$\boldsymbol{\overrightarrow{V_B} = 0.4\times 0.5\times(-\sin 30° + j\;\cos 30°)}$$

- The force at P : $\boldsymbol{\vec{P} = -1000\,j}$

and $\vec{P}.\overrightarrow{V_B} = -1000 \times (0.4 \times 0.5 \times \cos 30°) = -173.2$

then Eq. 4 => -173.2 + M x 1 = 0 => **M = 173.2 N.m**

F.B.D. of DC



N is perpendiculare to the axes of slip along the axis of transmission

$$\sum M_{/D} = 0 \quad (\alpha = 0, constant\ angular\ velocity)$$

$$M - N \times 0.1 \times \cos 30° = 0$$

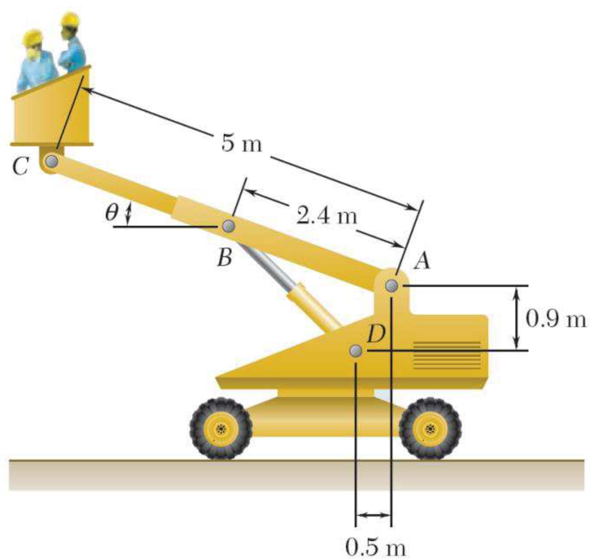
$$N = \frac{173.2}{0.1 \times \cos 30°} = \mathbf{2000N}$$

Problem 3

The telescoping arm *ABC* is used to provide an elevated platform for construction workers. The workers and the platform together have a mass of **200 kg** and have a combined center of gravity located directly above *C*. The hydraulic cylinder (DB) is designed to lift the platform so that the arm (ABC) rotates, about the pin A, with a constant angular speed of **0.1 rad/s CW**. For the position shown when $\boldsymbol{\theta = 20°}$**;**

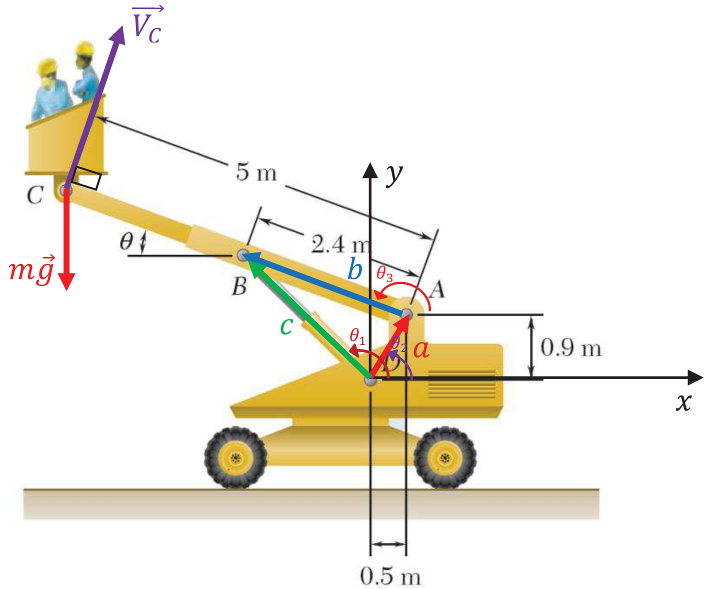
- **a) Draw clearly the vector loop (DAB).**
- **b) Find the velocity of extension of the hydraulic cylinder (DB) and its angular velocity.**
- **c) Find the velocity of center of gravity C.**
- **d) Using Energy Method, determine the force F required by the hydraulic cylinder (DB) to lift the platform.**

Neglect the masses of all members.



Solution

a) **Vector loop of DAB**



b) Firstly, determine the input angle and link length

Angle and angular velocity:

- $\theta_2 = \tan^{-1}\left(\frac{0.9}{0.5}\right) = 60.95°$
- $\theta_3 = 180 - \theta = 180 - 20 = 160°$
- $w_3 = -0.1\, rad/s$

Length

- $a = AD = \sqrt{0.5^2 + 0.9^2} = 1.0296\, m$
- b = AB = 2.4 m

$DA = a\, e^{j\theta_2},$	$AB = b\, e^{j\theta_3},$	$DB = c\, e^{j\theta_1}$

Vector loop of DAB

$$\overrightarrow{DA} + \overrightarrow{AB} - \overrightarrow{DB} = 0$$

$$a\, e^{j\theta_2} + b\, e^{j\theta_3} - c\, e^{j\theta_1} = 0\ (Eq\ 1)$$

$$a\,(\cos\theta_2 + j\sin\theta_2) + b\,(\cos\theta_3 + j\sin\theta_3) - c\,(\cos\theta_1 + j\sin\theta_1) = 0$$

- Real =0 :

$$a\ \cos\theta_2 + b\ \cos\theta_3 - c\cos\theta_1 = 0$$

$$1.0296\ \cos 60.95° + 2.4\ \cos 160° - c\cos\theta_1 = 0$$

$$\boxed{c\cos\theta_1 = -1.755\ (Eq\ 2)}$$

- Imag =0 :

$$a\ \sin\theta_2 + b\ \sin\theta_3 - c\sin\theta_1 = 0$$

$$1.0296\ \sin 60.95° + 2.4\ \sin 160° - c\sin\theta_1 = 0$$

$$\boxed{c\sin\theta_1 = 1.7209\ (Eq\ 3)}$$

then, solve the two equations 2 and 3 with the two unknowns, you get

$$\frac{c\sin\theta_1}{c\cos\theta_1} = \tan\theta_1 = \frac{1.7209}{-1.755} => \boldsymbol{\theta_1 = 135.56°}$$

and using equation 3, you find (c) => $c\sin 135.56° = 1.7209$ => $\boldsymbol{c = 2.458\ m}$

<u>Derivative of Eq 1:</u>

$$b\ w_3 j\ e^{j\theta_3} - \dot{c}e^{j\theta_1} - c\ w_1 j\ e^{j\theta_1} = 0$$

$$b\ w_3 j\ (\cos\theta_3 + j\sin\theta_3) - \dot{c}(\cos\theta_1 + j\sin\theta_1) - c\ w_1 j\ (\cos\theta_1 + j\sin\theta_1) = 0$$

$$b\ w_3\ (-\sin\theta_3 + j\cos\theta_3) - \dot{c}(\cos\theta_1 + j\sin\theta_1) - c\ w_1(-\sin\theta_1 + j\cos\theta_1) = 0$$

We need to determine w_1 *and* $\dot{c}$.

- Real =0 :

$$-b\ w_3\sin\theta_3 - \dot{c}\cos\theta_1 + c\ w_1\sin\theta_1 = 0$$

$$-2.4\ (-0.1)\sin 160 - \dot{c}\cos 135.56° + 2.458\ w_1\sin 135.56° = 0$$

$$\boxed{1.7209\ w_1 + 0.7139\ \dot{c} = -0.082\ (Eq. 4)}$$

- Imag =0 :

$$2.4\ (-0.1)\ \cos 160 - \dot{c}\ \sin 135.56° - 2.458\ w_1\cos 135.56° = 0$$

$$\boxed{1.755\ w_1 - 0.7\ \dot{c} = -0.2255\ (\text{Eq. } 5)}$$

Solve the two equations 4 and 5 with the two unknowns, you get

$$w_1 = \mathbf{-0.088\ rad/s}$$

$$\dot{c} = \mathbf{0.0993\ m/s}$$

c) **Velocity of point C**

$$\overrightarrow{R_C} = AC\ e^{j\theta_3}$$

$$\overrightarrow{V_C} = AC\ w_3\ j\ e^{j\theta_3}$$

$$\overrightarrow{V_C} = 5\ (-0.1)\ j\ (\cos\theta_3 + j\sin\theta_3)$$

$$\overrightarrow{V_C} = -0.5 \times (-\sin 160° + j\cos 160°) = 0.171 + j\ 0.469$$

Then

- Amplitude : $\left|\overrightarrow{V_C}\right| = \mathbf{0.499\ m/s}$
- Angle : $\theta = \mathbf{69.97°}$

d) Energy Method

$$\sum_{k=2}^{n} \overrightarrow{F_k} \cdot \overrightarrow{V_k} + \sum_{k=2}^{n} \overrightarrow{T_k} \cdot \overrightarrow{w_k} = \sum_{k=2}^{n} m_k \cdot \overrightarrow{a_k} \cdot \overrightarrow{V_k} + \sum_{k=2}^{n} I_k \cdot \overrightarrow{\alpha_k} \cdot \overrightarrow{w_k} \quad (eq.1)$$

- No external torque => $\sum_{k=2}^{n} \overrightarrow{T_k} \cdot \overrightarrow{w_k} = 0$
- Masses are neglected =>$\sum_{k=2}^{n} m_k \cdot \overrightarrow{a_k} \cdot \overrightarrow{V_k} + \sum_{k=2}^{n} I_k \cdot \overrightarrow{\alpha_k} \cdot \overrightarrow{w_k} = 0$

Then, eq.1 $\sum_{k=2}^{n} \overrightarrow{F_k} \cdot \overrightarrow{V_k} = 0$

$$m\vec{g} \cdot \overrightarrow{V_c} + \vec{F} \cdot \overrightarrow{V_{BC}} = 0$$

$$(-mg.j) \cdot \overrightarrow{V_c} + F \cdot \dot{c} = 0$$

$$(-200 \times 9.81j).(0.171 + j\ 0.469) + F \times 0.0993 = 0$$

$$-200 \times 9.81 \times 0.469 + F \times 0.0993 = 0$$

$$\mathbf{F = 9266.65\ N}$$