Chapter 3

Combining Factors and Spreadsheet Functions

LEARNING OUTCOMES

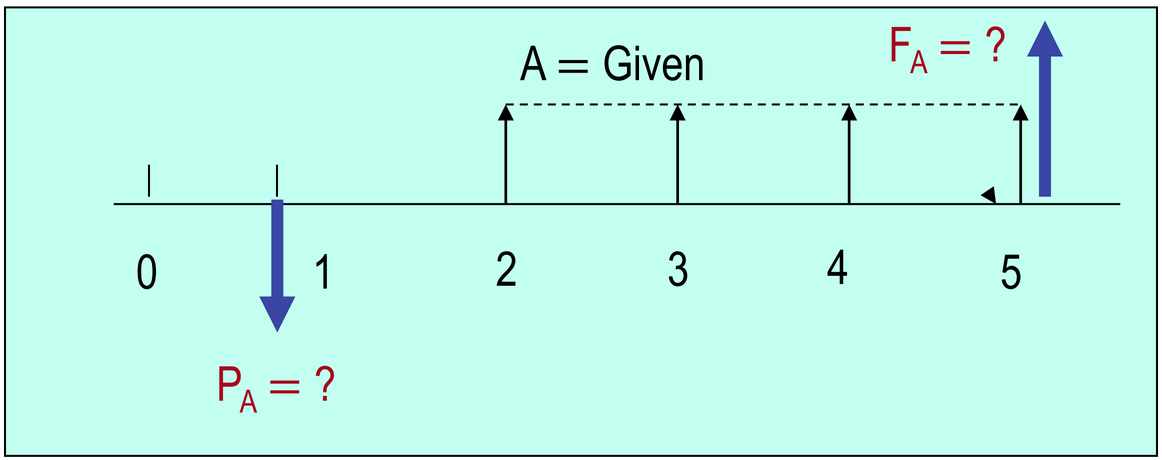
1. **Shifted uniform series**
2. **Shifted series and single cash flows**
3. **Shifted gradients**

Shifted Uniform Series

A shifted uniform series starts at a time *other than period 1*

The cash flow diagram below is an example of a shifted series

Series starts in period 2, not period 1



Shifted series usually require the use of *multiple factors*

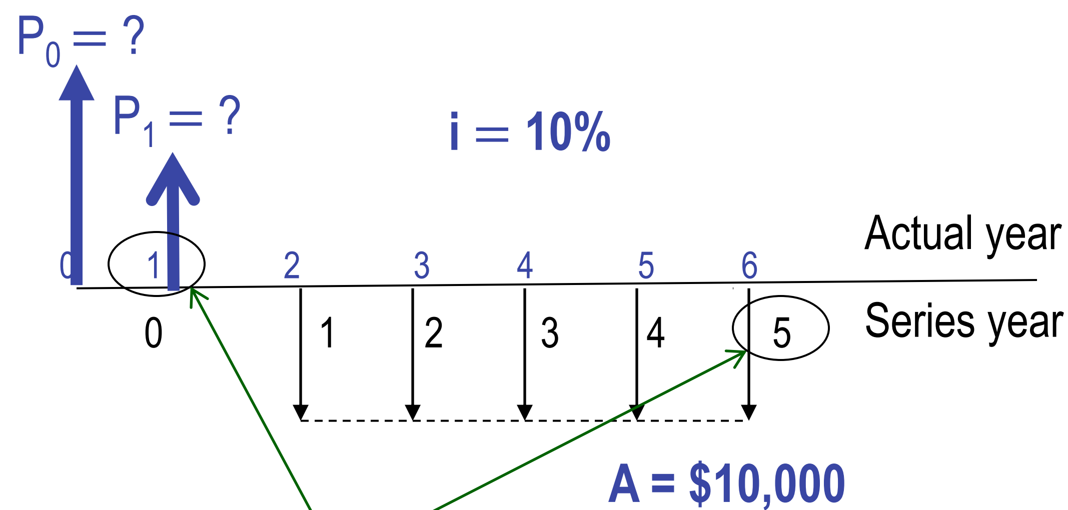
Remember: **When using P/A or A/P factor, P_A is always *one year ahead* of first A**

When using F/A or A/F factor, F_A is in *same year as last A*

Example Using P/A Factor: Shifted Uniform Series

The present worth of the cash flow shown below at $i = 10\%$ is:

(a) \$25,304 (b) \$29,562 (c) \$34,462 (d) \$37,908



Solution: (1) Use P/A factor with $n = 5$ (for 5 arrows) to get P_1 **in year 1**

(2) Use P/F factor with $n = 1$ to move P_1 back for P_0 **in year 0**

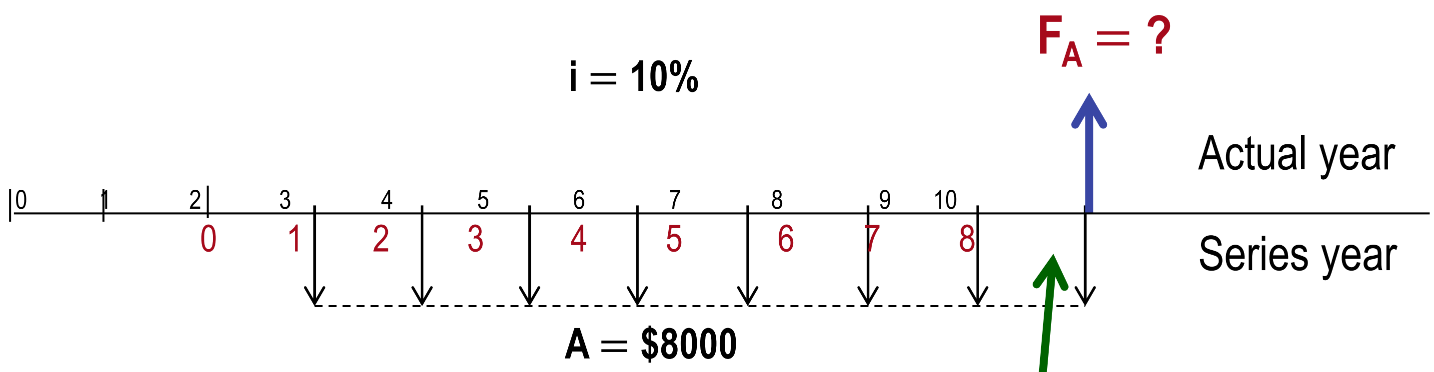
$$P_0 = P_1(P/F,10\%,1) = A(P/A,10\%,5)(P/F,10\%,1) = 10{,}000(3.7908)(0.9091) = \$34{,}462$$

Answer is (c)

Example Using F/A Factor: Shifted Uniform Series

How much money would be available in year 10 if \$8000 is deposited each year in years 3 through 10 at an interest rate of 10% per year?

Cash flow diagram is:



Solution: Re-number diagram to determine n = 8 (number of arrows)

$$F_A = 8000(F/A,10\%,8)$$
$$= 8000(11.4359)$$
$$= \mathbf{\$91,487}$$

Shifted Series and Random Single Amounts

For cash flows that include *uniform series* and *randomly placed single amounts*:

Uniform series procedures **are applied to the** ***series amounts***

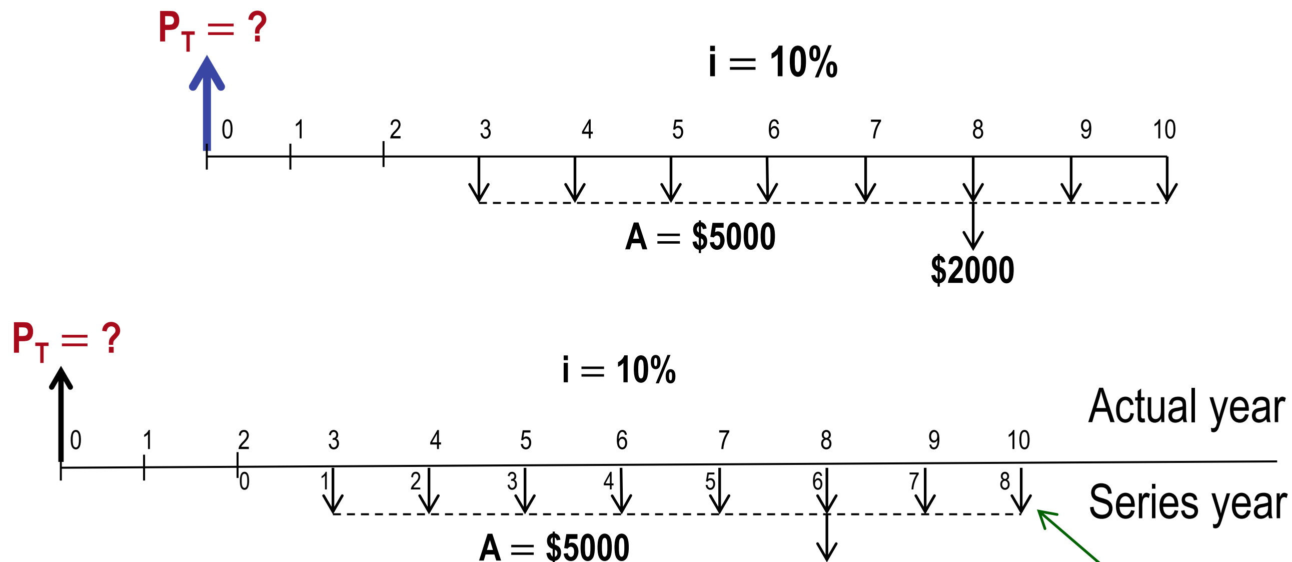
Single amount formulas **are applied to the** ***one-time cash flows***

The resulting values are then ***combined*** **per the problem statement**

The following slides illustrate the procedure

Example: Series and Random Single Amounts (1)

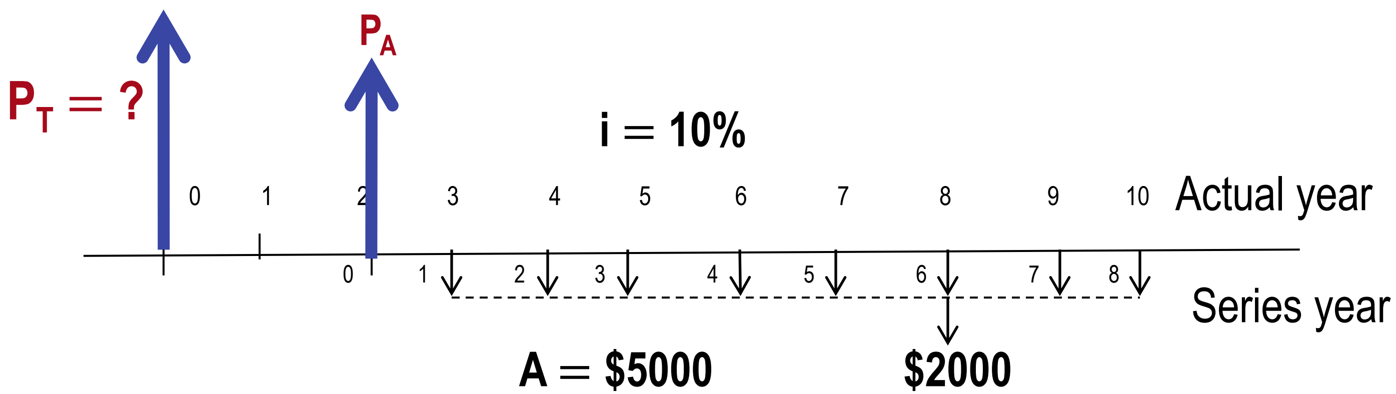
Find the present worth in year 0 for the cash flows shown using an interest rate of 10% per year.



Solution:

First, re-number cash flow diagram to get n for uniform series: $n = 8$

Example: Series and Random Single Amounts (2)



Use P/A to get P_A in year 2: $P_A = 5000(P/A,10\%,8) = 5000(5.3349) = \$26,675$

Move P_A back to year 0 using P/F: $P_0 = 26,675(P/F,10\%,2) = 26,675(0.8264) = \$22,044$

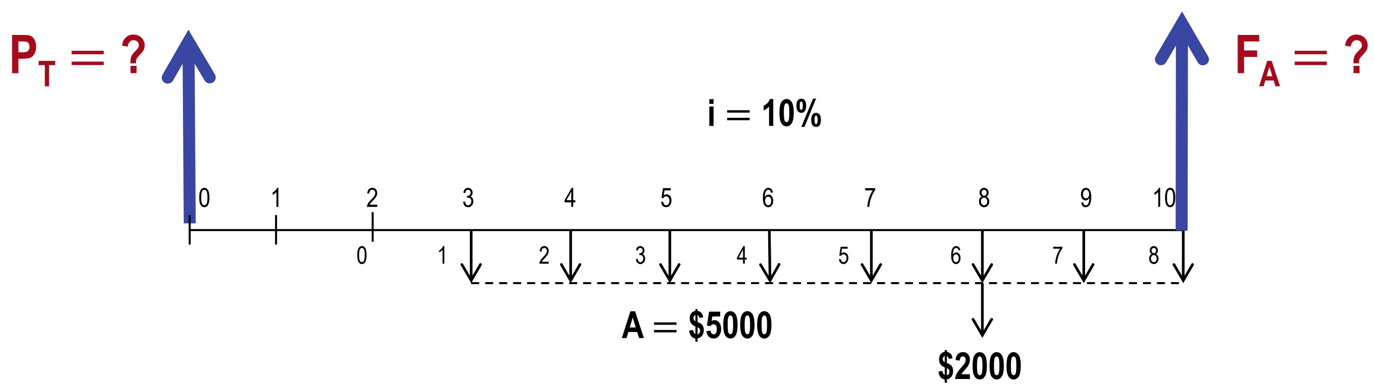
Move $2000 single amount back to year 0: $P_{2000} = 2000(P/F,10\%,8) = 2000(0.4665) = \$933$

Now, add P_0 and P_{2000} to get P_T: $P_T = 22,044 + 933 = \$22,977$

Example Worked a Different Way

(Using F/A instead of P/A for uniform series)

The same re-numbered diagram from the previous slide is used



Solution: Use F/A to get F_A in actual year 10: $F_A = 5000(F/A,10\%,8) = 5000(11.4359) = \$57,180$

Move F_A back to year 0 using P/F: $P_0 = 57,180(P/F,10\%,10) = 57,180(0.3855) = \$22,043$

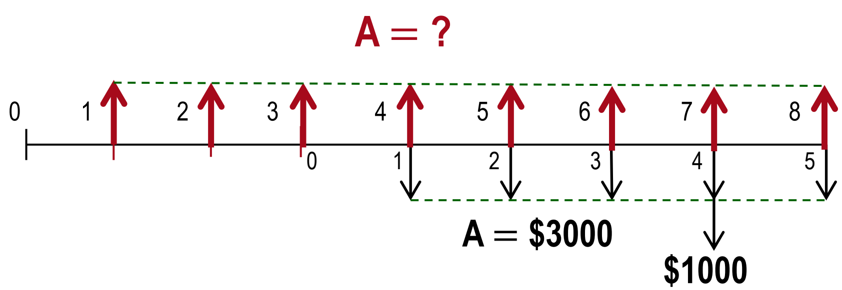
Move $2000 single amount back to year 0: $P_{2000} = 2000(P/F,10\%,8) = 2000(0.4665) = \$933$

Now, add two P values to get P_T: $P_T = 22,043 + 933 = \$22,976$ **Same as before**

As shown, there are usually multiple ways to work equivalency problems

Example: Series and Random Amounts

Convert the cash flows shown below (black arrows) into an equivalent annual worth A in years 1 through 8 (red arrows) at i = 10% per year.



i = 10%

Approaches: **1. Convert all cash flows into P in year 0** and use A/P with n = 8

2. Find F in year 8 and use A/F with n = 8

Solution:

Solve for F: $F = 3000(F/A,10\%,5) + 1000(F/P,10\%,1)$
$= 3000(6.1051) + 1000(1.1000)$
$= \$19{,}415$

Find A: $A = 19{,}415(A/F,10\%,8)$
$= 19{,}415(0.08744)$
$= \$1698$

Shifted Arithmetic Gradients

Shifted gradient begins at a time other than between periods 1 and 2

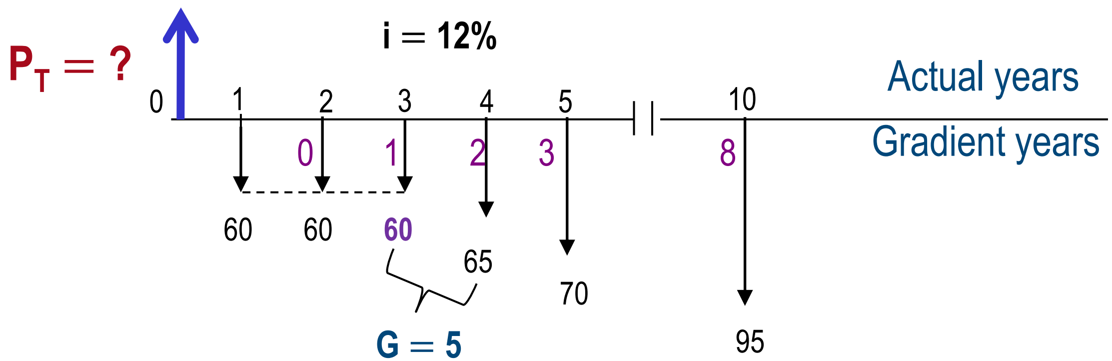
Present worth P_G is located 2 periods before gradient starts

Must use multiple factors to find P_T in actual year 0

To find equivalent A series, find P_T at actual time 0 and apply (A/P,i,n)

Example: Shifted Arithmetic Gradient

John Deere expects the cost of a tractor part to increase by \$5 per year beginning 4 years from now. If the cost in years 1-3 is \$60, determine the *present worth in year 0* of the cost through year 10 at an interest rate of 12% per year.



Solution: **First find P_2 for G = \$5 *and* base amount (\$60) in actual year 2**

$$P_2 = 60(P/A,12\%,8) + 5(P/G,12\%,8) = \$370.41$$

Next, move P_2 back to year 0 $P_0 = P_2(P/F,12\%,2) = \$295.29$

Next, find P_A for the \$60 amounts of years 1 and 2 $P_A = 60(P/A,12\%,2) = \$101.41$

Finally, add P_0 and P_A to get P_T in year 0 $\mathbf{P_T = P_0 + P_A = \$396.70}$

Shifted Geometric Gradients

Shifted gradient begins at a time other than between periods 1 and 2

Equation yields P_g for *all* cash flows (base amount A_1 is included)

Equation ($i \neq g$):

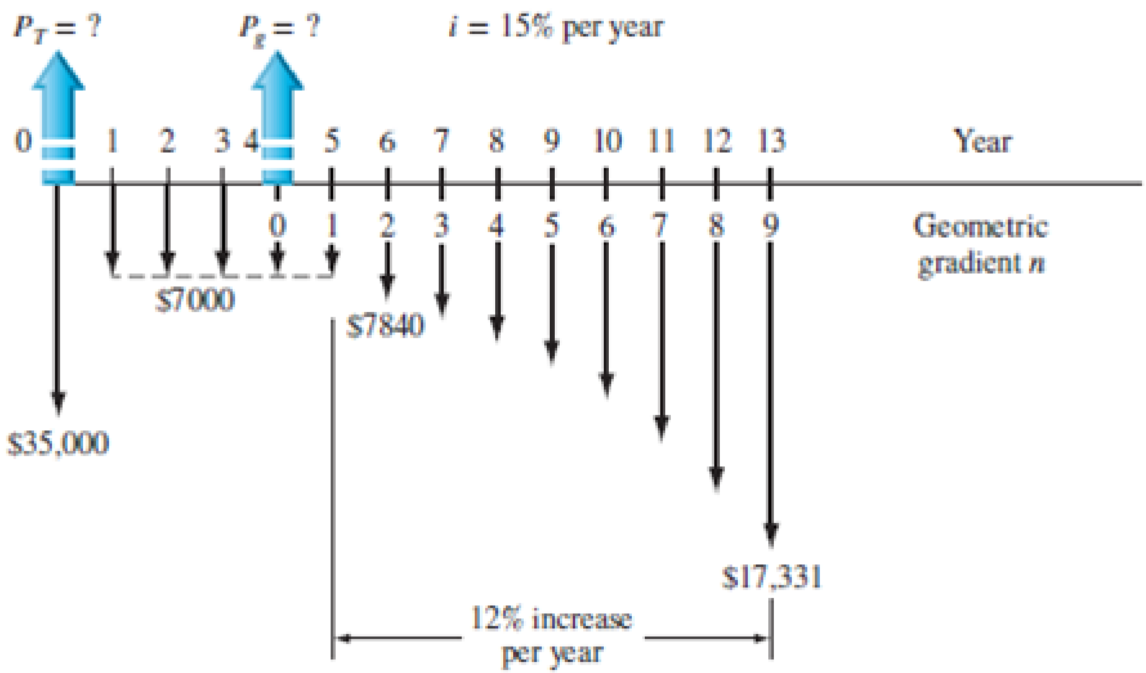
$$P_g = A_1\{1 - [(1+g)/(1+i)]^n/(i-g)\}$$

For negative gradient, change signs on both g values

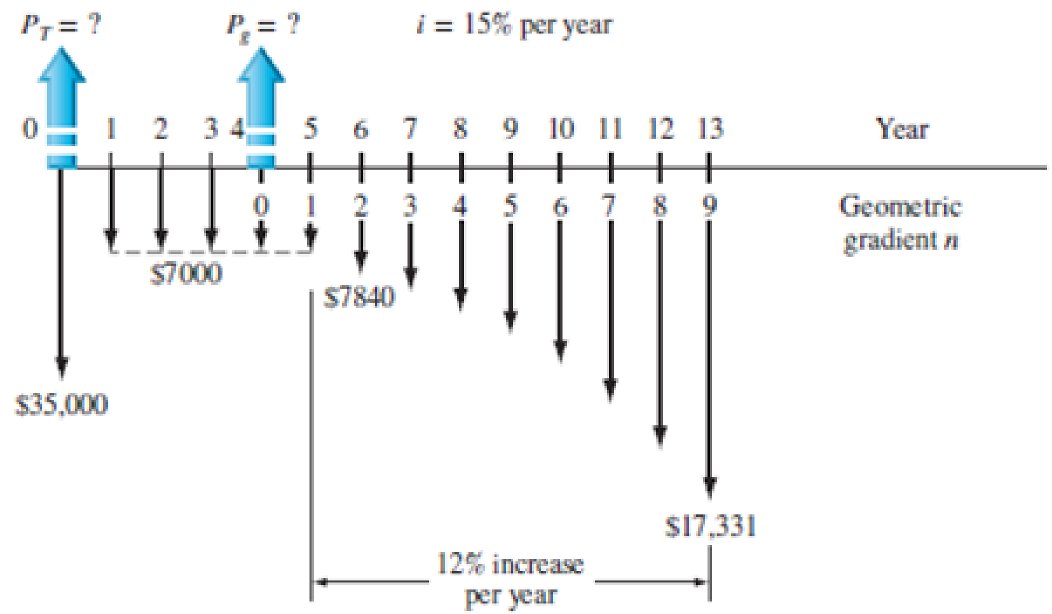
There are no tables for geometric gradient factors

Example: Shifted Geometric Gradient (1)

Weirton Steel signed a 5-year contract to purchase water treatment chemicals from a local distributor for \$7000 per year. When the contract ends, the cost of the chemicals is expected to increase by 12% per year for the next 8 years. If an initial investment in storage tanks is \$35,000, determine the equivalent present worth in year 0 of all of the cash flows at i = 15% per year.



Example: Shifted Geometric Gradient (2)



Gradient starts between actual years 5 and 6; these are gradient years 1 and 2.
P_g is located in gradient year 0, which is actual year 4

$$\mathbf{P_g = 7000\{1 - [(1 + 0.12)/(1 + 0.15)]^9/(0.15 - 0.12)\} = \$49,401}$$

Move P_g and other cash flows to year 0 to calculate P_T

$$\mathbf{P_T = 35,000 + 7000(P/A, 15\%, 4) + 49,401(P/F, 15\%, 4) = \$83,232}$$

Negative Shifted Gradients

For negative arithmetic gradients, change sign on G term from + to –

General equation for determining P: **P = present worth of base amount – P_G**

Changed from + to –

For negative geometric gradients, change signs on both g values

Changed from + to –

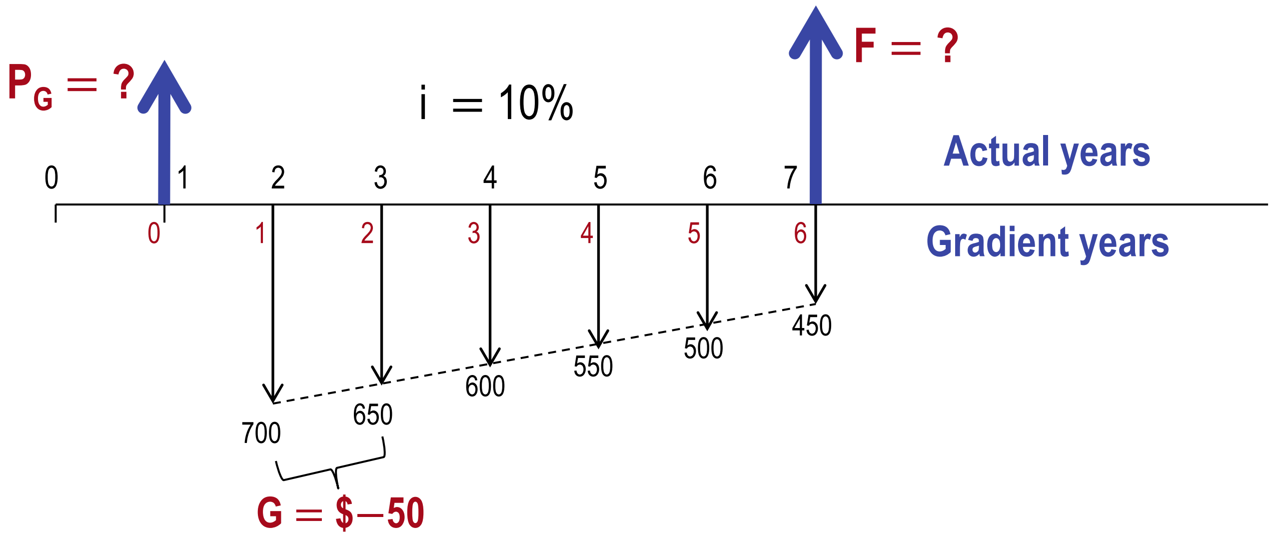
$$\mathbf{P_g = A1\{1 - [(1 - g)/(1 + i)]^n/(i + g)\}}$$

Changed from – to +

All other procedures are the same as for positive gradients

Example: Negative Shifted Arithmetic Gradient

For the cash flows shown, find the future worth in year 7 at i = 10% per year



Solution: Gradient G first occurs between actual years 2 and 3; these are gradient years 1 and 2

P_G is located in gradient year 0 (actual year 1); base amount of $700 is in gradient years 1-6

$$\mathbf{P_G = 700(P/A, 10\%, 6) - 50(P/G, 10\%, 6) = 700(4.3553) - 50(9.6842) = \$2565}$$

$$\mathbf{F = PG(F/P, 10\%, 6) = 2565(1.7716) = \$4544}$$

Summary of Important Points

P for shifted uniform series is *one period ahead* of first A; n is equal to number of A values

F for shifted uniform series is in *same period* as last A; n is equal to number of A values

For gradients, *first change* equal to G or g occurs between gradient years 1 and 2

For negative arithmetic gradients, change sign on G from + to −

For negative geometric gradients, change sign on g from + to −