Chapter-2: Design of Machinery / Kinematics fundamentals

Full joints
1 DOF

half joints
2 DOF

- order of joint = number of links − 1
- Kinematic chain: linkage assembled to provide output motion in response to a supplied input motion.
- Mechanism: Kinematic chain + at least one link connected to reference (ground).
- Machine: Mechanisms arranged to transmit forces & do work.

Link Classification:

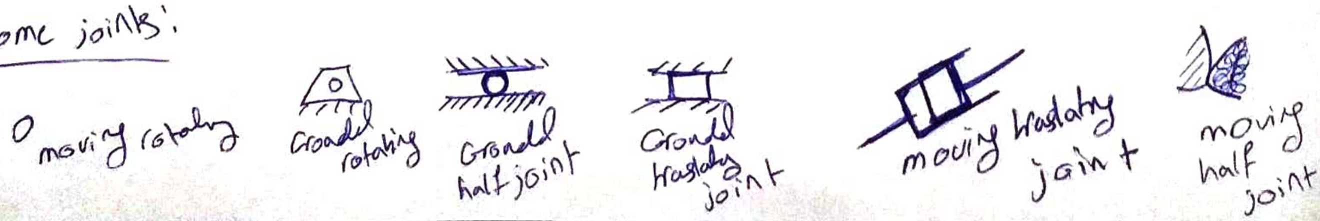
Crank: makes complete revolution and pivoted to ground.

Rocker: Oscillates and pivoted to ground.

Coupler: Complex motion and not connected to ground

Ground: fixed link or reference

Some joints:



* Mobility = DOF = number of inputs needed to create predictable output.
  = number of independent coordinates needed to define its position.

* Gruebler's equation: $M = 3(\overset{\text{Links including ground}}{L} - 1) - 2J$

M: Mobility

J: 1 for full joint, 1/2 for half joint

• **note:**

mechanism: DOF $= 1$

Structure: DOF $= 0$

Preloaded Structure: DOF $= -1$

Remark: • Redundant Links and their joints aren't calculated (Links with repeated motion - that doesn't affect the motion).

• if System and motion is symm, we take only the half (to get a reasonable result).

• sometimes () is calculated as full bcz rotation & translation are always synchronized and combined.

* if all joints are full ⇒ odd number of DOF requires even number of Links and vice-versa. (proof using gruebler's eqn).

$$\begin{cases} J = \dfrac{(2B + 3T + 4Q + 5P + 6H + \cdots)}{2} \\[2ex] J = \dfrac{3}{2}(L-1) - \dfrac{M}{2} \end{cases}$$

$L = B + T + Q + P + H + \cdots$

$$\Rightarrow \boxed{L - 3 - M = T + 2Q + 3P + 4H}$$ we use this eqn.

• Simplest 1DOF linkage is fourbar Linkage.

* **Linkage transformation:**

1. Revolute joint replaced by prismatic joints (slider – translating) with at least 2 revolute joints remaining in loop
   → no change in DOF.

2. Full joint replaced by half joint → increase DOF by 1

3. Removal of a Link → reduce DOF by 1

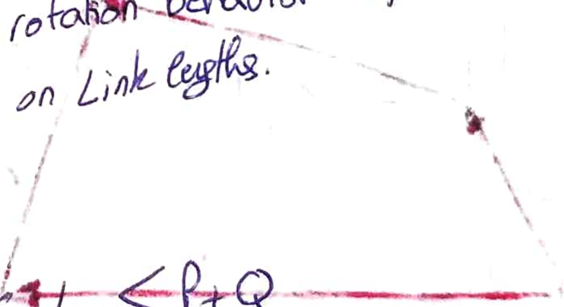
4. 2+3 → no change in DOF.

5. shrinking a link (to a lower order – Combining joints) → no change in DOF.

* Intermittent motion: output remains stationary for a period while input continues moving.

* Inversion: grounding a different Link in the chain.

* Grashof condition: relation that predicts the rotation behavior of a fourbar linkage's inversion based on Link lengths.



S = L. of Shortest Link.
L = L. " longest Link.
P = L. " remaining Link.
Q = " " " "

if $S+L \leq P+Q$

⇒ Linkage is Grashof.

means that at least one link will be capable of making a full revolution w.r.t. ground plane.

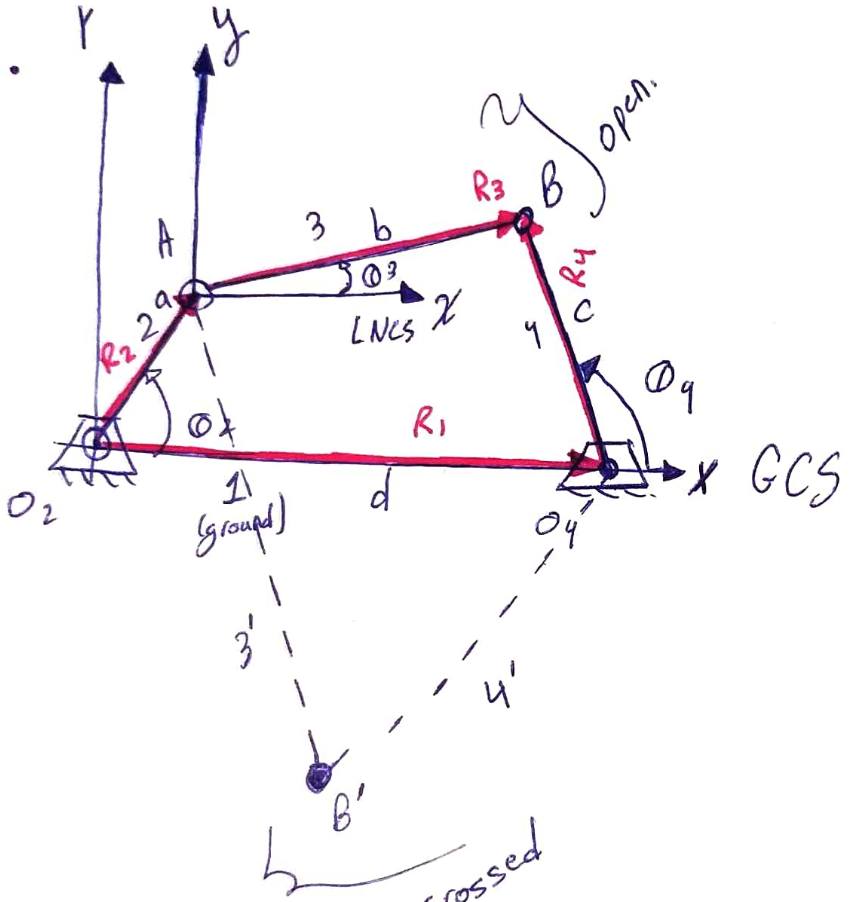
* Chapter 4: Position analysis.

- Global reference (GCS): attached to earth or frame reference.
- Local coordinate system: attached to a link at some point of interest.

- **Position vector:** $R_A = \sqrt{R_x^2 + R_y^2}$ $\quad \theta = \arctan\left(\frac{R_Y}{R_X}\right)$; $R = R_A \cdot e^{j\theta}$ $\quad$ ($R\cos\theta$, $R\sin\theta$)

note: we can transform the coordinates from one system to another by knowing the angle between 2 references.

- $R_{BA} = R_B - R_A$ (final − initial)
  displacement.



$\begin{cases} \theta_1 = 0 \\ \theta_2 \text{ input} \\ \theta_3, \theta_4 \text{ from formula sheet} \end{cases}$

we have θ_{31} and θ_{32} and similar for θ_4, so we choose the correct according to crossed or open.

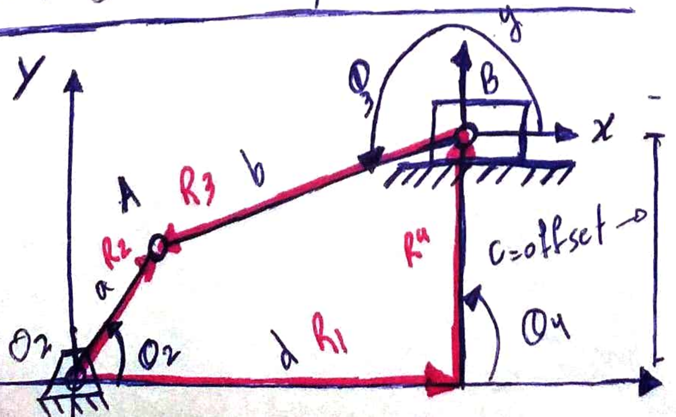
* Position vector loop for a fourbar linkage: $\boxed{R_2 + R_3 - R_4 - R_1 = 0}$

(similar loop can be written for any linkage).

* fourbar slider-crank position solution:

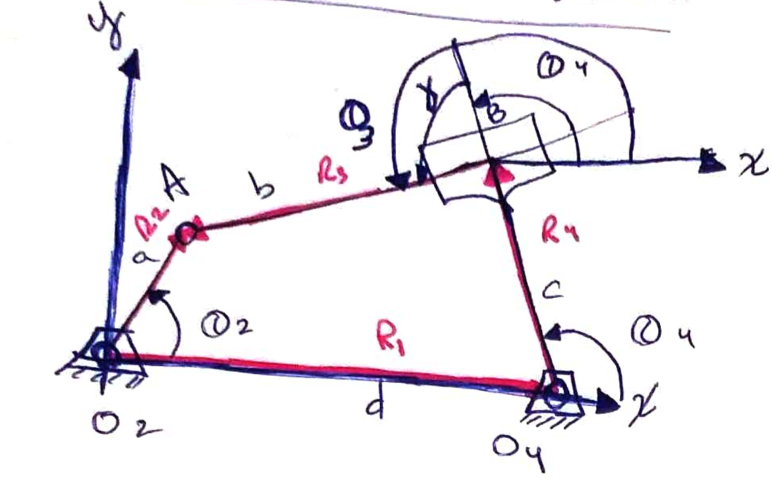
$\begin{cases} \theta_1 = 0° \\ \theta_4 = 90° \end{cases}$ $\begin{cases} \theta_2 \text{ input} \\ \theta_3, d \text{ formula sheet} \end{cases}$

$\boxed{R_2 - R_3 - R_4 - R_1 = 0}$



x (Parallel to slider axis toward the slider starting from O_2)

* Inverted Slider crank position Solution:



$$\boxed{R_2 - R_3 - R_4 - R_1 = 0}$$

$\begin{Bmatrix} \theta_1 = 0 \\ \theta_2 : \text{input} \end{Bmatrix}$; $\begin{Bmatrix} \theta_4 : \text{formula} \\ b \quad \text{sheet} \end{Bmatrix}$

$$\boxed{\theta_3 = \theta_4 \pm \gamma}$$

note: • Watt's sixbar = 2 fourbar in series.
• Stephenson's sixbar = 2 fourbar in parallel.

Remark: We can find position of any pt. by relating it to angles calculated and system found.

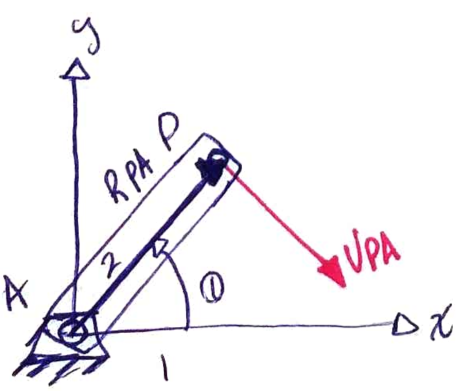
* Transmition angles: angle btw coupler link and output link

$$\theta_{trans} = |\theta_3 - \theta_4| \quad \text{(Should be always acute and 90)}$$

* Chapter 6 - Velocity analysis:

• linear velocity: $V = \frac{d\vec{R}}{dt}$ m/s.

• angular velocity: $\omega = \frac{d\theta}{dt}$ rad/s. (ccw → +ve ; cw → −ve)

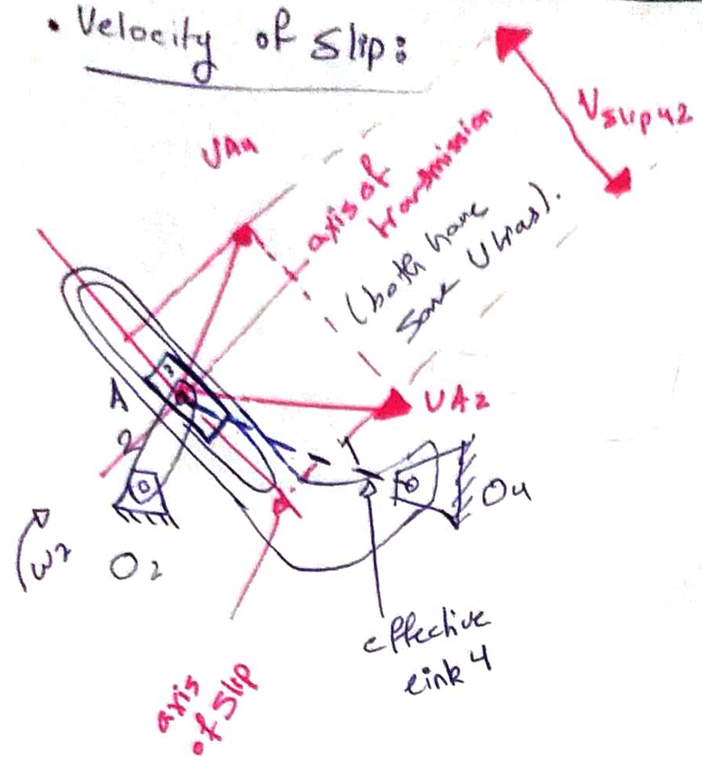
• 

. $R_{PA} = P e^{j\theta}$

. $V_{PA} = \frac{dR_{PA}}{dt} = P\omega j e^{j\theta}$ (since multiplied by j → ⊥ to R_{PA})

• $\boxed{V_P = V_A + V_{PA}}$

- **Velocity of slip:**

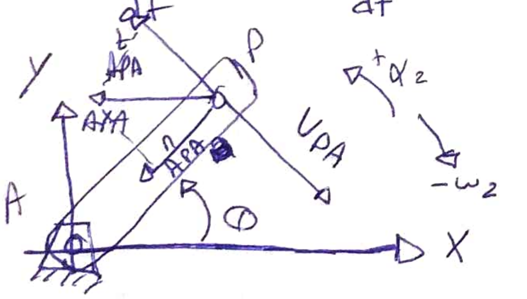


- Axis of slip: tangent to slider motion and along it. (all sliding occurs on it).
- transmission axis $\perp$ axis of slip and pass through slider joint (only line which we can transmit motion on ; except friction).

- angle of U after calculation should be verified in which quadrant.
- In slider crank: $\vec{U}_B = \dot{d}$ (slider velocity, velocity of slip).
- Remark: we can find velocity of any pt. by relating it to pts. with known velocities ($A, B, --$).

✱ Chapter 7: Acceleration Analysis:

$A = \frac{dU}{dt}$; $\alpha = \frac{d\omega}{dt}$

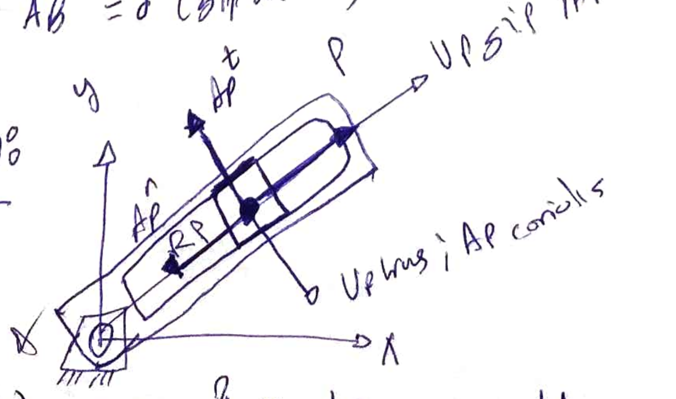


- $A_{PA} = \underbrace{P\alpha\, j e^{j\theta}}_{A^t_{PA}} + \underbrace{- P\omega^2 e^{j\theta}}_{A^n_{PA}}$

- $A_P = A_A + A_{PA}$.

- In slider crank: $A_B = \ddot{d}$ (slip acc —).

- Slider in rotation:



$A_P = \underset{(je^{j\theta})}{A_P^t} + \underset{(e^{j\theta})}{A_P^n} + \underset{(je^{j\theta}\ \text{with}\ \dot{P})}{A_{P\,cor.}} + \underset{(\ddot{P}\,-)}{A_{P\,slip}}$

- Jerk $= d(\text{acceleration})/dt$ } $\varphi = \frac{d\alpha}{dt}$; $J = \frac{dA}{dt}$.