This support is written to help students follow up during the class sessions and for their individual studies.
It is limited to a general introduction and simple examples, therefore, it will not replace textbook, discussed material and solved problems during class sessions.

Design of machinery
Chapter 11 Dynamic force analysis

Dr. Jaafar Hallal

Chapter 11 Dynamic force analysis

- 11.1 Newtonian solution method

This method gives the most information about internal forces.

$$\sum \mathbf{F} = m\mathbf{a} \qquad \sum \mathbf{T} = I_G\alpha$$

- Since it's a 2D problem:

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum T = I_G\alpha$$

- These equations should be applied on each link

Chapter 11 Dynamic force analysis

- 11.2 Single link in pure rotation

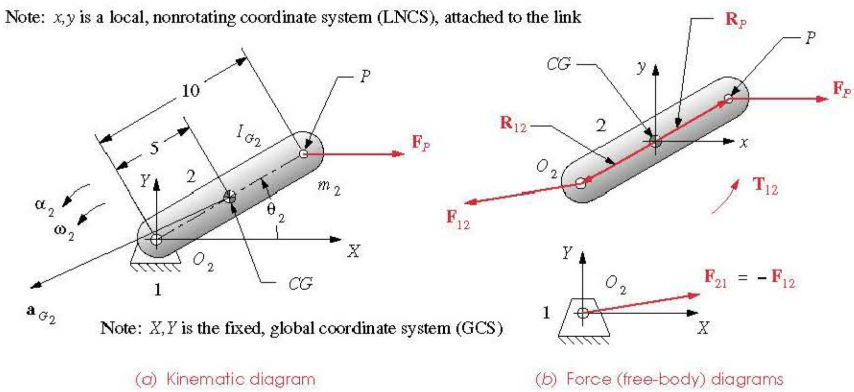
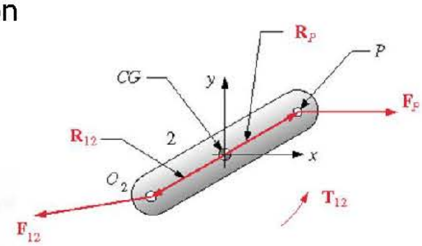


FIGURE 11-1

Dynamic force analysis of a single link in pure rotation

Chapter 11 Dynamic force analysis

- 11.2 Single link in pure rotation



$$\sum \mathbf{F} = \mathbf{F}_P + \mathbf{F}_{12} = m_2 \mathbf{a}_G$$

$$\sum \mathbf{T} = \mathbf{T}_{12} + (\mathbf{R}_{12} \times \mathbf{F}_{12}) + (\mathbf{R}_P \times \mathbf{F}_P) = I_G \alpha$$

$$F_{P_x} + F_{12_x} = m_2 a_{G_x}$$

$$F_{P_y} + F_{12_y} = m_2 a_{G_y}$$

$$T_{12} + \left(R_{12_x} F_{12_y} - R_{12_y} F_{12_x}\right) + \left(R_{P_x} F_{P_y} - R_{P_y} F_{P_x}\right) = I_G \alpha$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -R_{12_y} & R_{12_x} & 1 \end{bmatrix} \times \begin{bmatrix} F_{12_x} \\ F_{12_y} \\ T_{12} \end{bmatrix} = \begin{bmatrix} m_2 a_{G_x} - F_{P_x} \\ m_2 a_{G_y} - F_{P_y} \\ I_G \alpha - \left(R_{P_x} F_{P_y} - R_{P_y} F_{P_x}\right) \end{bmatrix}$$

Chapter 11 Dynamic force analysis

- 11.3 Force analysis of a threebar crank-slide linkage

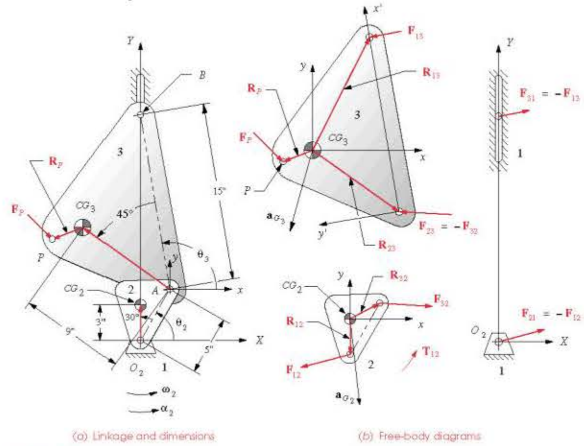
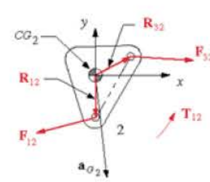


FIGURE 11-2
Dynamic force analysis of a slider-crank linkage

Chapter 11 Dynamic force analysis

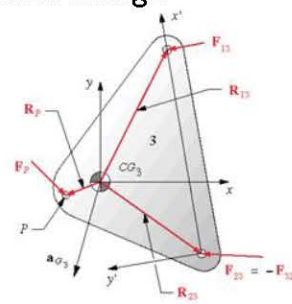
- 11.3 Force analysis of a threebar crank-slide linkage
- Link 2

$$\begin{cases} F_{12_x} + F_{32_x} = m_2 a_{G_{2_x}} \\ F_{12_y} + F_{32_y} = m_2 a_{G_{2_y}} \\ T_{12} + \left(R_{12_x} F_{12_y} - R_{12_y} F_{12_x}\right) + \left(R_{32_x} F_{32_y} - R_{32_y} F_{32_x}\right) = I_{G_2} \alpha_2 \end{cases}$$



Chapter 11 Dynamic force analysis

- 11.3 Force analysis of a threebar crank-slide linkage
- Link 3



In order to reduce the number of unknowns

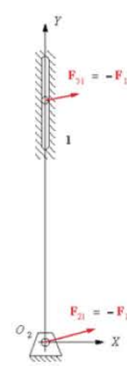
$$\mathbf{F}_{23} = -\mathbf{F}_{32}$$

$$\begin{cases} F_{13_x} - F_{32_x} + F_{P_x} = m_3 a_{G3_x} \\ F_{13_y} - F_{32_y} + F_{P_y} = m_3 a_{G3_y} \\ \left(R_{13_x} F_{13_y} - R_{13_y} F_{13_x}\right) - \left(R_{23_x} F_{32_y} - R_{23_y} F_{32_x}\right) + \left(R_{P_x} F_{P_y} - R_{P_y} F_{P_x}\right) = I_{G_3} \alpha_3 \end{cases}$$

Chapter 11 Dynamic force analysis

- 11.3 Force analysis of a threebar crank-slide linkage
- Now we have 6 equations and 7 unknowns, we need one more equation:

$$F_{13_y} = \mu F_{13_x}$$



- Substitute the equation of friction in the equations of motion:

$$F_{12_x} + F_{32_x} = m_2 a_{G_{2_x}}$$
$$F_{12_y} + F_{32_y} = m_2 a_{G_{2_y}}$$
$$T_{12} + R_{12_x} F_{12_y} - R_{12_y} F_{12_x} + R_{32_x} F_{32_y} - R_{32_y} F_{32_x} = I_{G_2} \alpha_2$$

$$F_{13_x} - F_{32_x} = m_3 a_{G_{3_x}} - F_{P_x}$$
$$\pm\mu F_{13_x} - F_{32_y} = m_3 a_{G_{3_y}} - F_{P_y}$$
$$\left(\pm\mu R_{13_x} - R_{13_y}\right) F_{13_x} - R_{23_x} F_{32_y} + R_{23_y} F_{32_x} = I_{G_3} \alpha_3 - R_{P_x} F_{P_y} + R_{P_y} F_{P_x}$$

Chapter 11 Dynamic force analysis

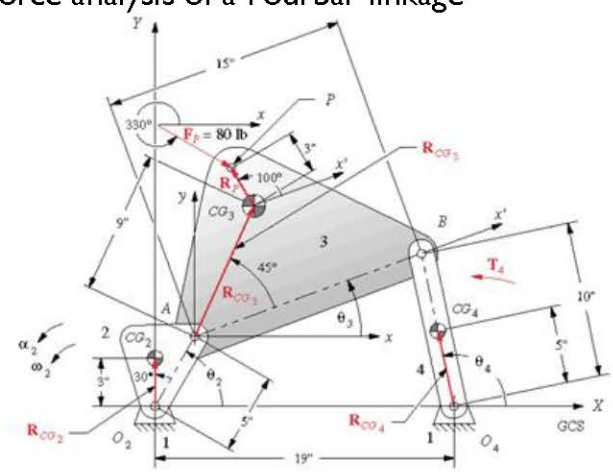
- 11.3 Force analysis of a threebar crank-slide linkage

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ -R_{12_y} & R_{12_x} & -R_{32_y} & R_{32_x} & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & -0 \\ 0 & 0 & 0 & -1 & \mu & 0 \\ 0 & 0 & R_{23_y} & -R_{23_x} & \left(\mu R_{13_x} - R_{13_y}\right) & 0 \end{bmatrix} \times \begin{bmatrix} F_{12_x} \\ F_{12_y} \\ F_{32_x} \\ F_{32_y} \\ F_{13_x} \\ T_{12} \end{bmatrix} = \begin{bmatrix} m_2 a_{G_{2x}} \\ m_2 a_{G_{2y}} \\ I_{G_2}\alpha_2 \\ m_3 a_{G_{3x}} - F_{P_x} \\ m_3 a_{G_{3y}} - F_{P_y} \\ I_{G_3}\alpha_3 - R_{P_x}F_{P_y} + R_{P_y}F_{P_x} \end{bmatrix}$$

- Solve this system for the unknowns.

Chapter 11 Dynamic force analysis

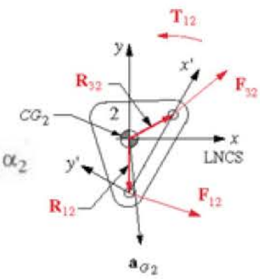
- 11.4 Force analysis of a Fourbar linkage



Chapter 11 Dynamic force analysis

- 11.4 Force analysis of a Fourbar linkage
- Link 2

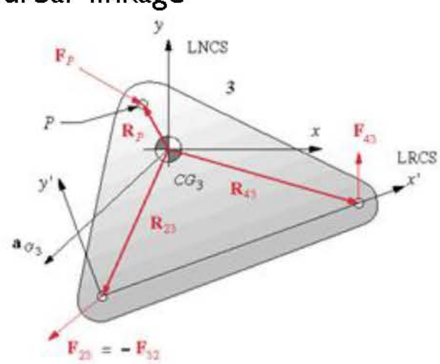
$$\begin{cases} F_{12_x} + F_{32_x} = m_2 a_{G_{2_x}} \\ F_{12_y} + F_{32_y} = m_2 a_{G_{2_y}} \\ T_{12} + \left(R_{12_x} F_{12_y} - R_{12_y} F_{12_x}\right) + \left(R_{32_x} F_{32_y} - R_{32_y} F_{32_x}\right) = I_{G_2} \alpha_2 \end{cases}$$



Chapter 11 Dynamic force analysis

- 11.4 Force analysis of a Fourbar linkage
- Link 3

Substitute $-\mathbf{F}_{32}$ for $\mathbf{F}_{23}$,

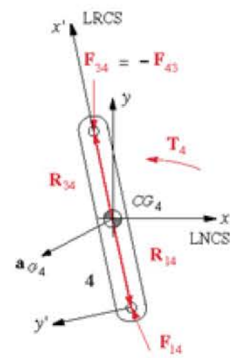


$$\begin{cases} F_{43_x} - F_{32_x} + F_{P_x} = m_3 a_{G_{3_x}} \\ F_{43_y} - F_{32_y} + F_{P_y} = m_3 a_{G_{3_y}} \\ \left(R_{43_x} F_{43_y} - R_{43_y} F_{43_x}\right) - \left(R_{23_x} F_{32_y} - R_{23_y} F_{32_x}\right) + \left(R_{P_x} F_{P_y} - R_{P_y} F_{P_x}\right) = I_{G_3} \alpha_3 \end{cases}$$

Chapter 11 Dynamic force analysis

- 11.4 Force analysis of a Fourbar linkage
- Link 4

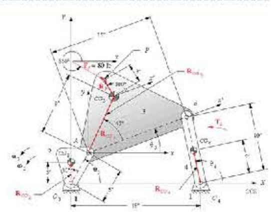
Substitute $-\mathbf{F}_{43}$ for $\mathbf{F}_{34}$



$$\begin{cases} F_{14_x} - F_{43_x} = m_4 a_{G4_x} \\ F_{14_y} - F_{43_y} = m_4 a_{G4_y} \\ \left(R_{14_x} F_{14_y} - R_{14_y} F_{14_x}\right) - \left(R_{34_x} F_{43_y} - R_{34_y} F_{43_x}\right) + T_4 = I_{G_4} \alpha_4 \end{cases}$$

Chapter 11 Dynamic force analysis

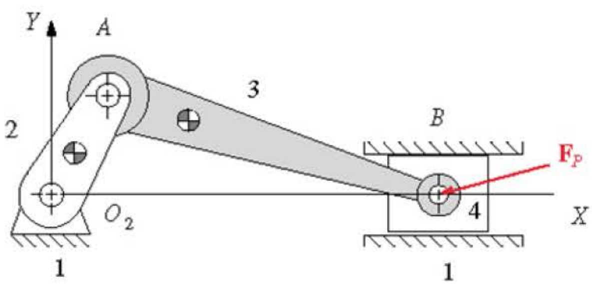
- 11.4 Force analysis of a Fourbar linkage



$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -R_{12_y} & R_{12_x} & -R_{32_y} & R_{32_x} & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & R_{23_y} & -R_{23_x} & -R_{43_y} & R_{43_x} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & R_{34_y} & -R_{34_x} & -R_{14_y} & R_{14_x} & 0 \end{bmatrix} \times \begin{bmatrix} F_{12_x} \\ F_{12_y} \\ F_{32_x} \\ F_{32_y} \\ F_{43_x} \\ F_{43_y} \\ F_{14_x} \\ F_{14_y} \\ T_{12} \end{bmatrix} = \begin{bmatrix} m_2 a_{G_{2x}} \\ m_2 a_{G_{2y}} \\ I_{G_2}\alpha_2 \\ m_3 a_{G_{3x}} - F_{P_x} \\ m_3 a_{G_{3y}} - F_{P_y} \\ I_{G_3}\alpha_3 - R_{P_x}F_{P_y} + R_{P_y}F_{P_x} \\ m_4 a_{G_{4x}} \\ m_4 a_{G_{4y}} \\ I_{G_4}\alpha_4 - T_4 \end{bmatrix}$$

Chapter 11 Dynamic force analysis

- 11.5 Force analysis of a Fourbar slider-crank linkage



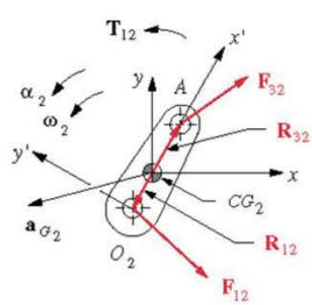
Chapter 11 Dynamic force analysis

- 11.5 Force analysis of a Fourbar slider-crank linkage
- Link 2

$$F_{12_x} + F_{32_x} = m_2 a_{G_{2x}}$$

$$F_{12_y} + F_{32_y} = m_2 a_{G_{2y}}$$

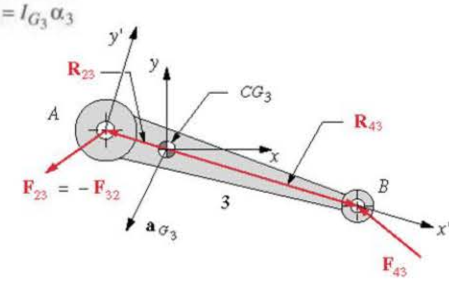
$$T_{12} + \left(R_{12_x} F_{12_y} - R_{12_y} F_{12_x}\right) + \left(R_{32_x} F_{32_y} - R_{32_y} F_{32_x}\right) = I_{G_2} \alpha_2$$



Chapter 11 Dynamic force analysis

- 11.5 Force analysis of a Fourbar slider-crank linkage
- Link 3

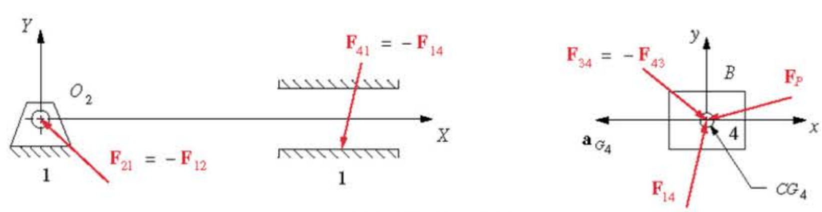
$$F_{43_x} - F_{32_x} = m_3 a_{G_{3_x}}$$
$$F_{43_y} - F_{32_y} = m_3 a_{G_{3_y}}$$
$$\left(R_{43_x} F_{43_y} - R_{43_y} F_{43_x}\right) - \left(R_{23_x} F_{32_y} - R_{23_y} F_{32_x}\right) = I_{G_3} \alpha_3$$



Chapter 11 Dynamic force analysis

- 11.5 Force analysis of a Fourbar slider-crank linkage
- Link 4 (Fp external force)

$$F_{14_x} - F_{43_x} + F_{P_x} = m_4 a_{G_{4_x}}$$
$$F_{14_y} - F_{43_y} + F_{P_y} = m_4 a_{G_{4_y}}$$
$$\left(R_{14_x} F_{14_y} - R_{14_y} F_{14_x}\right) - \left(R_{34_x} F_{43_y} - R_{34_y} F_{43_x}\right) + \left(R_{P_x} F_{P_y} - R_{P_y} F_{P_x}\right) = I_{G_4} \alpha_4$$



(b) Free-body diagrams

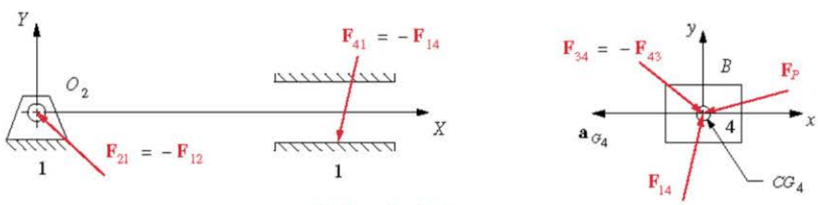
Chapter 11 Dynamic force analysis

- 11.5 Force analysis of a Fourbar slider-crank linkage
- Link 4

But $\alpha_4 = 0, \qquad a_{G_{4_y}} = 0$

and $F_{14_x} = \pm\mu F_{14_y}$

$$\Rightarrow \begin{cases} \pm\mu F_{14_y} - F_{43_x} + F_{P_x} = m_4 a_{G_{4_x}} \\ F_{14_y} - F_{43_y} + F_{P_y} = 0 \end{cases}$$

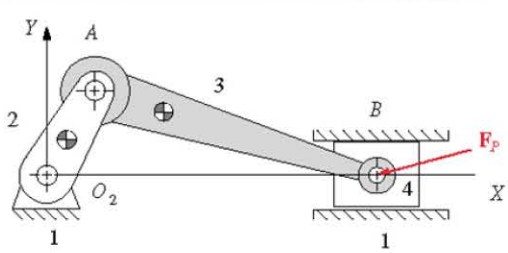


(b) Free-body diagrams

Chapter 11 Dynamic force analysis

- 11.5 Force analysis of a Fourbar slider-crank linkage

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ -R_{12_y} & R_{12_x} & -R_{32_y} & R_{32_x} & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & R_{23_y} & -R_{23_x} & -R_{43_y} & R_{43_x} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & \pm\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} F_{12_x} \\ F_{12_y} \\ F_{32_x} \\ F_{32_y} \\ F_{43_x} \\ F_{43_y} \\ F_{14_y} \\ T_{12} \end{bmatrix} = \begin{bmatrix} m_2 a_{G_{2_x}} \\ m_2 a_{G_{2_y}} \\ I_{G_2}\alpha_2 \\ m_3 a_{G_{3_x}} \\ m_3 a_{G_{3_y}} \\ I_{G_3}\alpha_3 \\ m_4 a_{G_{4_x}} - F_{P_x} \\ -F_{P_y} \end{bmatrix}$$



Chapter 11 Dynamic force analysis

- 11.6 Force analysis of an inverted slider-crank linkage

As you can see in this figure, the equations of motion of link 2 and link 3 are identical to the non-inverted slider crank

In order to guarantee that F34 and F43 are always perpendicular to the axis of slip:

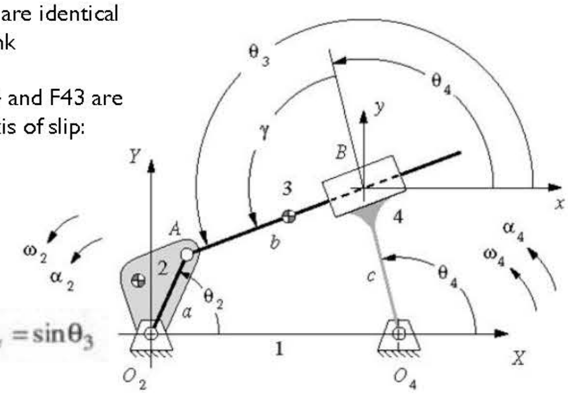
$$\hat{\mathbf{u}} \cdot \mathbf{F_{43}} = 0$$

➔

$$u_x F_{43_x} + u_y F_{43_y} = 0$$

Where:

$$u_x = \cos\theta_3, \qquad u_y = \sin\theta_3$$



Chapter 11 Dynamic force analysis

- 11.6 Force analysis of an inverted slider-crank linkage

Link 3: (Note that alpha3=alpha4)

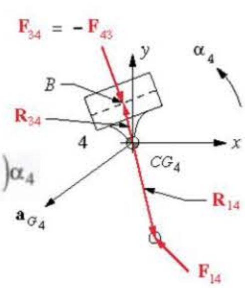
$$(\mathbf{R}_{43} \times \mathbf{F}_{43}) - (\mathbf{R}_{23} \times \mathbf{F}_{32}) = I_{G_3}\alpha_3 = I_{G_3}\alpha_4$$

+ Link 4:

$$(\mathbf{R}_{14} \times \mathbf{F}_{14}) - (\mathbf{R}_{34} \times \mathbf{F}_{43}) = I_{G_4}\alpha_4$$

=

$$(\mathbf{R}_{43} \times \mathbf{F}_{43}) - (\mathbf{R}_{23} \times \mathbf{F}_{32}) + (\mathbf{R}_{14} \times \mathbf{F}_{14}) - (\mathbf{R}_{34} \times \mathbf{F}_{43}) = (I_{G_3} + I_{G_4})\alpha_4$$



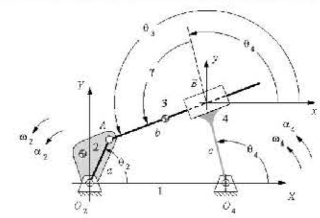
Expanding and collecting terms:

$$\left(R_{43_x} - R_{34_x}\right)F_{43_y} + \left(R_{34_y} - R_{43_y}\right)F_{43_x} - R_{23_x}F_{32_y} + R_{23_y}F_{32_x} + R_{14_x}F_{14_y} - R_{14_y}F_{14_x} = \left(I_{G_3} + I_{G_4}\right)\alpha_4$$

Chapter 11 Dynamic force analysis

- 11.6 Force analysis of an inverted slider-crank linkage

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -R_{12_y} & R_{12_x} & -R_{32_y} & R_{32_x} & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & R_{23_y} & -R_{23_x} & \left(R_{34_y}-R_{43_y}\right) & \left(R_{43_x}-R_{34_x}\right) & -R_{14_y} & R_{14_x} & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & u_x & u_y & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} F_{12_x} \\ F_{12_y} \\ F_{32_x} \\ F_{32_y} \\ F_{43_x} \\ F_{43_y} \\ F_{14_x} \\ F_{14_y} \\ T_{12} \end{bmatrix} = \begin{bmatrix} m_2 a_{G_{2_x}} \\ m_2 a_{G_{2_y}} \\ I_{G_2}\alpha_2 \\ m_3 a_{G_{3_x}} \\ m_3 a_{G_{3_y}} \\ \left(I_{G_3}+I_{G_4}\right)\alpha_4 \\ m_4 a_{G_{4_x}} \\ m_4 a_{G_{4_y}} \\ 0 \end{bmatrix}$$



Chapter 11 Dynamic force analysis

- 11.7 Force analysis: Linkages with more than four bars

$$\mathbf{F}_{ij} + \mathbf{F}_{jk} + \sum \mathbf{F}_{ext_j} = m_j \mathbf{a}_{G_j}$$

$$\left(\mathbf{R}_{ij} \times \mathbf{F}_{ij}\right) + \left(\mathbf{R}_{jk} \times \mathbf{F}_{jk}\right) + \sum \mathbf{T}_j + \left(\mathbf{R}_{ext_j} \times \sum \mathbf{F}_{ext_j}\right) = I_{G_j} \alpha_j$$

where:

$j = 2, 3, \ldots, n;\quad i = j - 1;\quad k = j + 1,\ j \neq n;\quad$ if $j = n, k = 1$

and $\quad \mathbf{F}_{ji} = -\mathbf{F}_{ij};\quad \mathbf{F}_{kj} = -\mathbf{F}_{jk}$

Chapter 11 Dynamic force analysis

11.8 Shaking forces and shaking torque

Interest to know the net effect of the dynamic forces as felt on the ground plane (Vibrations ...)

The **shaking forces** are the sum of all the force acting on the ground plane, in a Fourbar linkage:

$$\mathbf{F}_{21} = -\mathbf{F}_{12}$$

$$\mathbf{F}_{41} = -\mathbf{F}_{14}$$

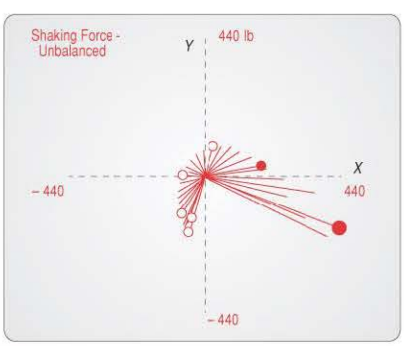
$$\mathbf{F}_s = \mathbf{F}_{21} + \mathbf{F}_{41}$$

The shaking torque is the reaction torque felt by the ground plane Ts:

$$\mathbf{T}_s = \mathbf{T}_{21} = -\mathbf{T}_{12}$$

Chapter 11 Dynamic force analysis

11.8 Shaking forces and shaking torque



Link No.	Length n	Mass Units	Inertia Units	CG Posit	at Deg	Ext. Force lb	at Deg
1	5.5						
2	2.0	.002	.004	1.0	0		
3	6.0	.030	.060	2.5	30	12	270
4	3.0	.010	.020	1.5	0	60	–45

Coupler pt. = 3 in @ 45°
Open/Crossed = open

Ext. Force 3 acts at 5 in @ 30° vs. CG of Link 3
Ext. Force 4 acts at 5 in @ 90° vs. CG of Link 4
Ext. Torque 3 = – 20 lb-in
Ext. Torque 4 = 25 lb-in

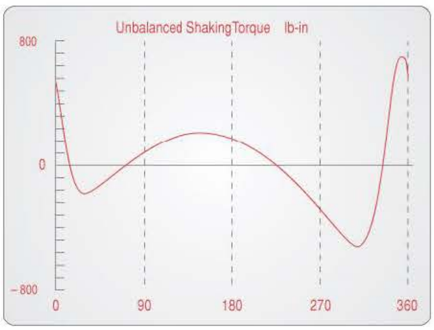
Start Alpha2 = 0 rad/sec^2
Start Omega2 = 50 rad/sec
Start Theta2 = 0°
Final Theta2 = 360°
Delta Theta2 = 10°

FIGURE 11-6

Linkage data and polar plot of shaking force for an unbalanced crank-rocker fourbar linkage from program FOURBAR

Chapter 11 Dynamic force analysis

- 11.8 Shaking forces and shaking torque



Link No.	Length in	Mass Units	Inertia Units	CG Posit	at Deg	Ext. Force lb	at Deg
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Ext. Torque 3 = – 20 lb-in
Ext. Torque 4 = 25 lb-in

Start Alpha2 = 0 rad/sec^2
Start Omega2 = 50 rad/sec
Start Theta2 = 0°
Final Theta2 = 360°
Delta Theta2 = 2°

FIGURE 11-7

Linkage data and shaking torque curve for an unbalanced crank-rocker fourbar linkage from program FOURBAR

Chapter 11 Dynamic force analysis

- 11.10 Linkage force analysis by energy methods

$$\sum_{k=2}^{n} \mathbf{F}_k \cdot \mathbf{v}_k + \sum_{k=2}^{n} \mathbf{T}_k \cdot \omega_k = \sum_{k=2}^{n} m_k \mathbf{a}_k \cdot \mathbf{v}_k + \sum_{k=2}^{n} I_k \alpha_k \cdot \omega_k$$

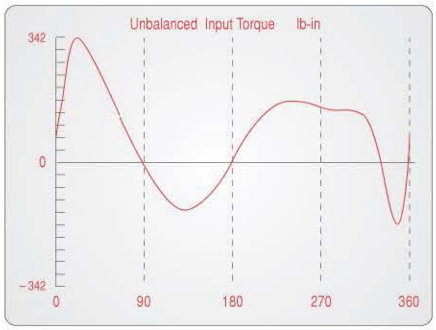
$$\begin{aligned}\left(\mathbf{F}_{P_3} \cdot \mathbf{v}_{P_3} + \mathbf{F}_{P_4} \cdot \mathbf{v}_{P_4}\right) + \left(\mathbf{T}_{12} \cdot \omega_2 + \mathbf{T}_3 \cdot \omega_3 + \mathbf{T}_4 \cdot \omega_4\right) = \\ \left(m_2 \mathbf{a}_{G_2} \cdot \mathbf{v}_{G_2} + m_3 \mathbf{a}_{G_3} \cdot \mathbf{v}_{G_3} + m_4 \mathbf{a}_{G_4} \cdot \mathbf{v}_{G_4}\right) \\ + \left(I_{G_2} \alpha_2 \cdot \omega_2 + I_{G_3} \alpha_3 \cdot \omega_3 + I_{G_4} \alpha_4 \cdot \omega_4\right)\end{aligned}$$

$$\begin{aligned}\left(F_{P_{3_x}} V_{P_{3_x}} + F_{P_{3_y}} V_{P_{3_y}}\right) + \left(F_{P_{4_x}} V_{P_{4_x}} + F_{P_{4_y}} V_{P_{4_y}}\right) + \left(T_{12}\omega_2 + T_3\omega_3 + T_4\omega_4\right) = \\ m_2\left(a_{G_{2_x}} V_{G_{2_x}} + a_{G_{2_y}} V_{G_{2_y}}\right) + m_3\left(a_{G_{3_x}} V_{G_{3_x}} + a_{G_{3_y}} V_{G_{3_y}}\right) \quad (1 \\ + m_4\left(a_{G_{4_x}} V_{G_{4_x}} + a_{G_{4_y}} V_{G_{4_y}}\right) + \left(I_{G_2}\alpha_2\omega_2 + I_{G_3}\alpha_3\omega_3 + I_{G_4}\alpha_4\omega_4\right)\end{aligned}$$

Chapter 11 Dynamic force analysis

▸11.11 Controlling input torque-Flywheels

Torque variation



Link No.	Length in	Mass Units	Inertia Units	CG Posit	at Deg	Ext. Force lb	at Deg
1	5.5						
2	2.0	.002	.004	1.0	0		
3	6.0	.030	.060	2.5	30	12	270
4	3.0	.010	.020	1.5	0	60	–45

Coupler pt. = 3 in @ 45°
Open/Crossed = open

Ext. Force 3 acts at 5 in @ 30° vs. CG of Link 3
Ext. Force 4 acts at 5 in @ 90° vs. CG of Link 4
Ext. Torque 3 = – 20 lb-in
Ext. Torque 4 = 25 lb-in
Start Alpha2 = 0 rad/sec^2
Start Omega2 = 50 rad/sec
Start Theta2 = 0°
Final Theta2 = 360°
Delta Theta2 = 2°

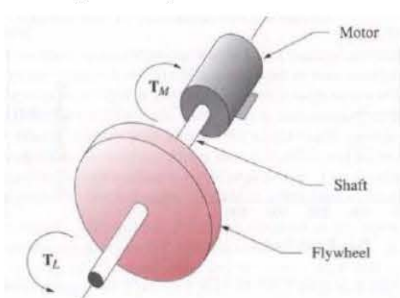
FIGURE 11-8

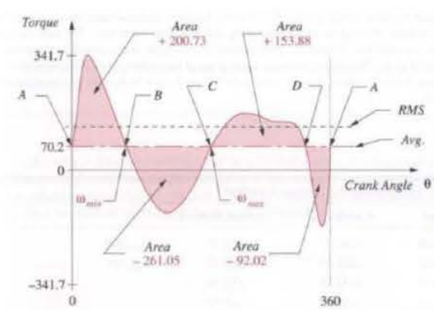
Linkage data and input torque curve for an unbalanced crank-rocker fourbar linkage from program FOURBAR

Chapter 11 Dynamic force analysis

▸11.11 Controlling input torque-Flywheels

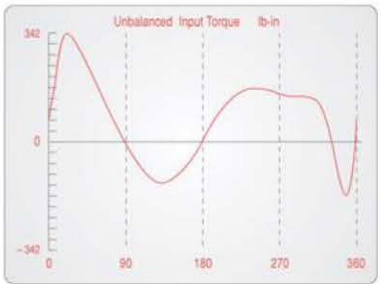
Sizing the flywheel



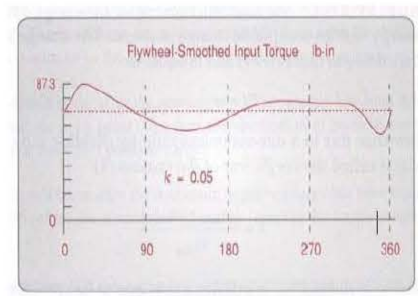


Chapter 11 Dynamic force analysis

11.11 Controlling input torque-Flywheels



Before

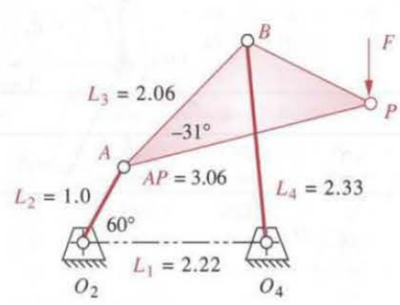


After

Chapter 11 Dynamic force analysis

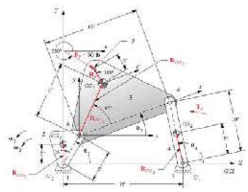
Problem 11.9

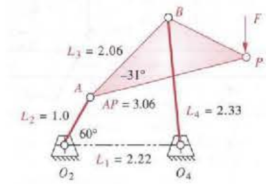
The steel crank and rocker have uniform cross sections 50 mm wide by 25 mm thick. The aluminum coupler is 25 mm thick. In the instantaneous position shown, the crank has w=10 rad/sec, and alpha=5/rad/sec2. There is a vertical force at P of F=100N. Find all pin forces and the torque needed to drive the crank at this instant.



Chapter 11 Dynamic force analysis

- Problem 11.9





$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -R_{12_y} & R_{12_x} & -R_{32_y} & R_{32_x} & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & R_{23_y} & -R_{23_x} & -R_{43_y} & R_{43_x} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & R_{34_y} & -R_{34_x} & -R_{14_y} & R_{14_x} & 0 \end{bmatrix} \times \begin{bmatrix} F_{12_x} \\ F_{12_y} \\ F_{32_x} \\ F_{32_y} \\ F_{43_x} \\ F_{43_y} \\ F_{14_x} \\ F_{14_y} \\ T_{12} \end{bmatrix} = \begin{bmatrix} m_2 a_{G_{2_x}} \\ m_2 a_{G_{2_y}} \\ I_{G_2}\alpha_2 \\ m_3 a_{G_{3_x}} - F_{P_x} \\ m_3 a_{G_{3_y}} - F_{P_y} \\ I_{G_3}\alpha_3 - R_{P_x}F_{P_y} + R_{P_y}F_{P_x} \\ m_4 a_{G_{4_x}} \\ m_4 a_{G_{4_y}} \\ I_{G_4}\alpha_4 - T_4 \end{bmatrix}$$

Chapter 11 Dynamic force analysis

- Problem 11.9

Given: Link lengths:

Link 2 (O_2 to A) $a := 1.00 \cdot m$ Link 3 (A to B) $b := 2.06 \cdot m$

Link 4 (B to O_4) $c := 2.33 \cdot m$ Link 1 (O_2 to O_4) $d := 2.22 \cdot m$

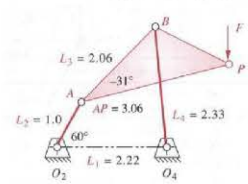
Coupler point: $R_{pa} := 3.06 \cdot m$ $\delta_3 := -31 \cdot deg$ $F := 100 \cdot N$ $T_4 := 0 \cdot N \cdot m$

Crank angle and motion: $\theta_2 := 60 \cdot deg$ $\omega_2 := 10 \cdot rad \cdot sec^{-1}$ $\alpha_2 := 5 \cdot rad \cdot sec^{-2}$

Link cross-section dims:

$w_2 := 50 \cdot mm$ $t_2 := 25 \cdot mm$ $t_3 := 25 \cdot mm$ $w_4 := 50 \cdot mm$ $t_4 := 25 \cdot mm$

Material specific weight: steel $\gamma_s := 0.3 \cdot lbf \cdot in^{-3}$ aluminum $\gamma_a := 0.1 \cdot lbf \cdot in^{-3}$



$F_{Px} := 0 \cdot N$ $F_{Py} := -F$

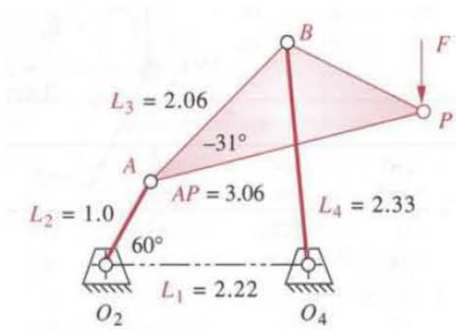
Chapter 11 Dynamic force analysis

- Problem 11.9

Position, velocity and acceleration analysis:

$\theta_3 := 44.732 \cdot deg$ $\quad \omega_3 := -3.669 \cdot rad \cdot sec^{-1}$ $\quad \alpha_3 := 55.752 \cdot rad \cdot sec^{-2}$

$\theta_4 := 96.322 \cdot deg$ $\quad \omega_4 := 1.442 \cdot rad \cdot sec^{-1}$ $\quad \alpha_4 := 67.103 \cdot rad \cdot sec^{-2}$



Chapter 11 Dynamic force analysis

- Problem 11.9

Determine the distance to the CG in the LRCS on each of the three moving links.

Links 2 and 4: $R_{CG2} := 0.5 \cdot a$ $\quad R_{CG2} = 0.500\,m$ $\quad R_{CG4} := 0.5 \cdot c$ $\quad R_{CG4} = 1.165\,m$

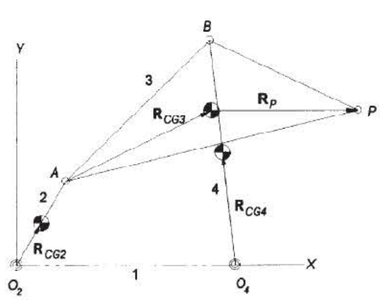
Link 3: $$R_{CG3x'} := \frac{R_{pa} \cdot \cos(\delta_3) + b}{3} \qquad R_{CG3x'} = 1.561\,m$$

$$R_{CG3y'} := \frac{R_{pa} \cdot \sin(\delta_3)}{3} \qquad R_{CG3y'} = -0.525\,m$$

$$R_{CG3} := \sqrt{R_{CG3x'}^{\,2} + R_{CG3y'}^{\,2}} \qquad R_{CG3} = 1.647\,m$$

At an angle with respect to the local x' axis of

$$\delta_{33} := atan2(R_{CG3x'}, R_{CG3y'}) \qquad \delta_{33} = -18.600\,deg$$



Chapter 11 Dynamic force analysis

- Problem 11.9

Determine the mass and moment of inertia of each link.

$$m_2 := w_2 \cdot t_2 \cdot a \cdot \frac{\gamma_s}{g} \qquad m_3 := \frac{1}{2} \cdot b \cdot \left| R_{pa} \cdot \sin(\delta_3) \right| \cdot t_3 \cdot \frac{\gamma_a}{g} \qquad m_4 := w_4 \cdot t_4 \cdot c \cdot \frac{\gamma_s}{g}$$

$$m_2 = 10.380\ kg \qquad m_3 = 112.332\ kg \qquad m_4 = 24.185\ kg$$

$$I_{G2} := \frac{m_2}{12} \cdot \left(w_2^{\,2} + a^2\right) \qquad I_{G2} = 0.867\ kg \cdot m^2$$

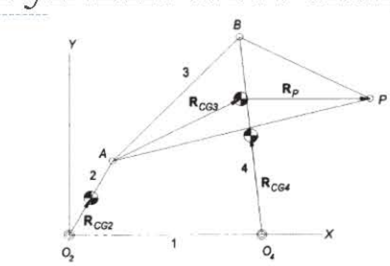
$$I_{G3} := \frac{m_3}{6} \cdot \left[b^2 + \left(R_{pa} \cdot \sin(\delta_3)\right)^2\right] \qquad I_{G3} = 125.951\ kg \cdot m^2$$

$$I_{G4} := \frac{m_4}{12} \cdot \left(w_4^{\,2} + c^2\right) \qquad I_{G4} = 10.947\ kg \cdot m^2$$

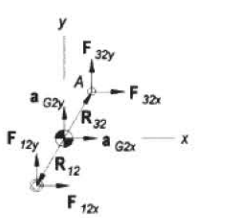
Chapter 11 Dynamic force analysis

- Problem 11.9

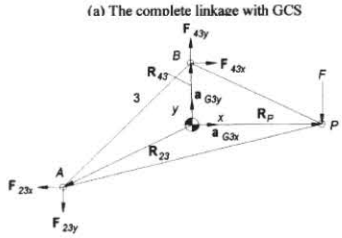
Set up all the LCS



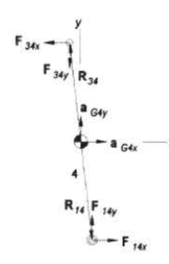
(a) The complete linkage with GCS



(b) FBD of Link 2



(c) FBD of Link 3



(d) FBD of Link 4

Chapter 11 Dynamic force analysis

- Problem 11.9

Position vectors

$R_{12x} := R_{CG2} \cdot cos(\theta_2 + 180 \cdot deg)$ $R_{12x} = -0.250\,m$

$R_{12y} := R_{CG2} \cdot sin(\theta_2 + 180 \cdot deg)$ $R_{12y} = -0.433\,m$

$R_{32x} := R_{CG2} \cdot cos(\theta_2)$ $R_{32x} = 0.250\,m$

$R_{32y} := R_{CG2} \cdot sin(\theta_2)$ $R_{32y} = 0.433\,m$

$R_{23x} := R_{CG3} \cdot cos(\delta_{33} + \theta_3 + 180 \cdot deg)$ $R_{23x} = -1.479\,m$

$R_{23y} := R_{CG3} \cdot sin(\delta_{33} + \theta_3 + 180 \cdot deg)$ $R_{23y} = -0.725\,m$

$R_{43x} := b \cdot cos(\theta_3) - R_{CG3} \cdot cos(\theta_3 + \delta_{33})$ $R_{43x} = -0.015\,m$

$R_{43y} := -(R_{CG3} \cdot sin(\theta_3 + \delta_{33}) - b \cdot sin(\theta_3))$ $R_{43y} = 0.724\,m$

$R_{34x} := R_{CG4} \cdot cos(\theta_4)$ $R_{34x} = -0.128\,m$

$R_{34y} := R_{CG4} \cdot sin(\theta_4)$ $R_{34y} = 1.158\,m$

$R_{14x} := R_{CG4} \cdot cos(\theta_4 + 180 \cdot deg)$ $R_{14x} = 0.128\,m$

$R_{14y} := R_{CG4} \cdot sin(\theta_4 + 180 \cdot deg)$ $R_{14y} = -1.158\,m$

$R_{Px} := R_{pa} \cdot cos(\theta_3 + \delta_3) - |R_{23x}|$ $R_{Px} = 1.494\,m$

$R_{Py} := R_{pa} \cdot sin(\theta_3 + \delta_3) - |R_{23y}|$ $R_{Py} = 9.865 \times 10^{-4}\,m$

Chapter 11 Dynamic force analysis

- Problem 11.9

Acceleration of CG

$\mathbf{a_{G2}} := R_{CG2} \cdot \alpha_2 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2)) - a \cdot \omega_2^2 \cdot (cos(\theta_2) + j \cdot sin(\theta_2))$

$a_{G2x} := Re(\mathbf{a_{G2}})$ $a_{G2x} = -52.165 \frac{m}{sec^2}$

$a_{G2y} := Im(\mathbf{a_{G2}})$ $a_{G2y} = -85.353 \frac{m}{sec^2}$

$\mathbf{a_A} := a \cdot \alpha_2 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2)) - a \cdot \omega_2^2 \cdot (cos(\theta_2) + j \cdot sin(\theta_2))$

$\mathbf{a_{CG3A}} := R_{CG3} \cdot \alpha_3 \cdot (-sin(\theta_3 + \delta_{33}) + j \cdot cos(\theta_3 + \delta_{33})) \ldots$
$+ -R_{CG3} \cdot \omega_3^2 \cdot (cos(\theta_3 + \delta_{33}) + j \cdot sin(\theta_3 + \delta_{33}))$

$\mathbf{a_{G3}} := \mathbf{a_A} + \mathbf{a_{CG3A}}$ $a_{G3x} := Re(\mathbf{a_{G3}})$ $a_{G3x} = -114.678 \frac{m}{sec^2}$

$a_{G3y} := Im(\mathbf{a_{G3}})$ $a_{G3y} = -11.429 \frac{m}{sec^2}$

$\mathbf{a_{G4}} := R_{CG4} \cdot \alpha_4 \cdot (-sin(\theta_4) + j \cdot cos(\theta_4)) - c \cdot \omega_4^2 \cdot (cos(\theta_4) + j \cdot sin(\theta_4))$

$a_{G4x} := Re(\mathbf{a_{G4}})$ $a_{G4x} = -77.166 \frac{m}{sec^2}$

$a_{G4y} := Im(\mathbf{a_{G4}})$ $a_{G4y} = -13.424 \frac{m}{sec^2}$

Chapter 11 Dynamic force analysis

Problem 11.9

Solve :

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -R_{12_y} & R_{12_x} & -R_{32_y} & R_{32_x} & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & R_{23_y} & -R_{23_x} & -R_{43_y} & R_{43_x} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & R_{34_y} & -R_{34_x} & -R_{14_y} & R_{14_x} & 0 \end{bmatrix} \times \begin{bmatrix} F_{12_x} \\ F_{12_y} \\ F_{32_x} \\ F_{32_y} \\ F_{43_x} \\ F_{43_y} \\ F_{14_x} \\ F_{14_y} \\ T_{12} \end{bmatrix} = \begin{bmatrix} m_2 a_{G_{2x}} \\ m_2 a_{G_{2y}} \\ I_{G_2}\alpha_2 \\ m_3 a_{G_{3x}} - F_{P_x} \\ m_3 a_{G_{3y}} - F_{P_y} \\ I_{G_3}\alpha_3 - R_{P_x}F_{P_y} \mp R_{P_y}F_{P_x} \\ m_4 a_{G_{4x}} \\ m_4 a_{G_{4y}} \\ I_{G_4}\alpha_4 - T_4 \end{bmatrix}$$

$F_{12x} = -13559\ N$ $\quad F_{12y} = -12294\ N$

$F_{32x} = 13018\ N$ $\quad F_{32y} = 11408\ N$

$F_{43x} = 136\ N$ $\quad F_{43y} = 10224\ N$

$F_{14x} = -1731\ N$ $\quad F_{14y} = 9899\ N$

$T_{12} = 5587\ N\cdot m$

Chapter 11 Dynamic force analysis

Problem

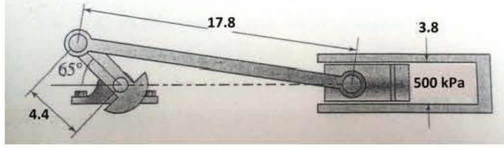
The compressor mechanism shown in the following figure is driven clockwise by a DC electric motor at a constant rate of 800 rpm. In the position shown, the pressure is 500 kPa;

a) Find the position of the piston.

b) Find the velocity and acceleration of the piston.

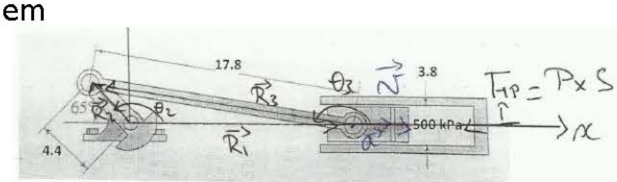
c) Using Energy Method, determine the power and torque required from the motor to operate the compressor.

The piston has a mass mp = 0.45 kg. Neglect the mass of other links. All dimensions are in cm.



Chapter 11 Dynamic force analysis

- Problem



Vector loop:

$$\vec{R_2} - \vec{R_3} - \vec{R_1} = 0$$

a) Position Analysis:

$a = 4.4$ cm, $b = 17.8$ cm, $c = 0$, $d = ??$

$\theta_2 = 180° - 65° = 115°$ ✓

$\theta_{31} = \sin^{-1}\left(\frac{a \sin\theta_2 - c}{b}\right) = 12.946°$ ✓ ✗

$\theta_{32} = \sin^{-1}\left(\frac{a \sin\theta_2 - c}{b}\right) + \pi = 167.05°$ ✓ ✓

$d = a\cos\theta_2 - b\cos\theta_3 = 15.488$ cm ✓

Chapter 11 Dynamic force analysis

- Problem

b) b.1 Velocity Analysis

$\omega_2 = -\frac{(800)(2\pi)}{60} = 83.776$ rad/s

$\omega_3 = \frac{a}{b}\frac{\cos\theta_2}{\cos\theta_3} \cdot \omega_2$

$\omega_3 = -9.98$ rad/s ✓

$\dot{d} = -a\omega_2 \sin\theta_2 + b\omega_3 \sin\theta_3$

$\dot{d} = 298.267$ cm/s ✓

b.2 Acceleration Analysis:

Constant speed $\Rightarrow \alpha_2 = 0$ ✓

$\alpha_3 = \frac{a\alpha_2 \cos\theta_2 - a\omega_2^2 \sin\theta_2 + b\omega_3^2 \sin\theta_3}{b\cos\theta_3}$

$\alpha_3 = 1594.8$ rad/s² ✓

$\ddot{d} = -a\alpha_2 \sin\theta_2 - a\omega_2^2 \cos\theta_2 + b\alpha_3 \sin\theta_3 + b\omega_3^2 \cos\theta_3$

$\ddot{d} = 18102$ cm/s² ✓

Chapter 11 Dynamic force analysis

- Problem

c) Emergy Method,

$$\sum_{K=2}^{m} \vec{F}_K \vec{V}_K + \sum_{K=2}^{m} \vec{T}_K \vec{\omega}_K = \sum_{K=2}^{m} m_K \vec{a}_K \vec{V}_K + \sum_{K=2}^{m} I_K \vec{\alpha}_K \vec{\omega}_K \quad \text{Eq.11}$$

0

the force due to pressure is $P_i = \frac{F_P}{\pi \frac{d^2}{4}}$

$\Rightarrow F_P = (\text{Pressure})(\text{Surface})$

$\Rightarrow F_P = 566.77\ N$

Eq.11 $-(566.77)(2.9867) + T_{12}\omega_2 = m_P \underset{\sim}{a_P}^{\ddot{d}} \underset{\sim}{v_P}^{\dot{d}}$

$\Rightarrow$ Power required $= T_{12}\omega_2 = 0.45(181.02)(2.98267) + (566.77)(2.9867)$

Power $= 1935.74$ Watt

$\Rightarrow$ Torque required $T_{12} = \frac{1935.74}{83.776} = 23.1\ N.m$