IV- Turbulent flow- smooth pipes: $2000 < R_e < 10^5$

In turbulent flow $\quad \tau = \tau_0 \left(1 - \frac{y}{R}\right) = \rho l^2 \left(\frac{dv}{dy}\right)^2$

$$\Rightarrow \frac{dv}{dy} = \sqrt{\frac{\tau_0}{\rho}} \frac{1}{Ky}$$

K = constant.

$$\Rightarrow \boxed{\frac{v - v_c}{v_*} = \frac{1}{K} \ln \frac{y}{R}} \quad (1)$$

$$\frac{v}{v_*} = 5.75 \log \frac{v_* y}{\nu} + 5.5 \qquad (2)$$

$$\frac{v_c}{V} = 1 + 4.07 \sqrt{\frac{\lambda}{8}} \qquad (3)$$

IV Turbulent flow _ Rough pipes $R_e > 10^5$

*) $$\frac{v}{v_*} = 5.75 \log \frac{y}{\varepsilon} + 8.5$$

$\frac{\varepsilon}{d}$ = relative roughness ε = grain diameter

*) $$\frac{V}{v_*} = 5.75 \log \frac{R}{\varepsilon} + 4.75$$ (Rough pipes)

*) $$\frac{1}{\sqrt{\lambda}} = 2 \log \frac{d}{\varepsilon} + 1.14 .$$ (Rough pipes).

Ex4:

water at 20°c flows in 75 mm diameter smooth pipeline. According to a wall shear stress $\tau_0 = 3.68\ N/m^2$ and $q_v = 5.7 \times 10^{-3}\ m^3/s$.

$= 3.68\ Pa$.

- Find v_*, V, δ, v_c, and velocity at 25 mm from the centerline.

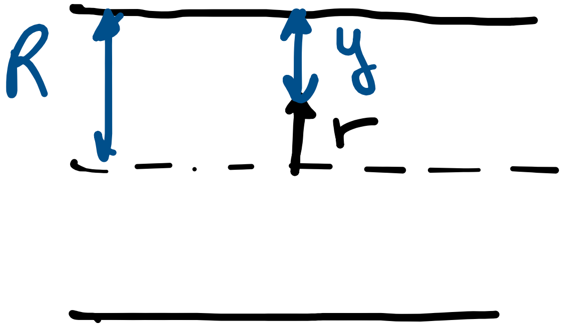
$$v_* = \sqrt{\frac{\tau_0}{\rho}} = V\sqrt{\frac{\lambda}{8}}$$

*) $$v_* = \sqrt{\frac{3.68}{10^3}} = 0.061 \text{ m/s.}$$

*) $$q_v = A \cdot V \Rightarrow \underset{\downarrow}{V} = \frac{q_v}{A} = \frac{5.7\times10^{-3}}{\frac{\pi}{4}(0.075)^2} = 1.29 \text{ m/s.}$$

V = mean velocity.

v = velocity at position y.



$r = R - y$.

*)

$$V_* = V\sqrt{\frac{\lambda}{8}} \Rightarrow \lambda = 8\left(\frac{V_*}{V}\right)^2 = 8\left(\frac{0.061}{1.29}\right)^2 = 0.018$$

$$Re = \frac{\rho d V}{\mu} = \frac{10^3 \times 0.075 \times 1.29}{10^{-3}} = 96750 < 10^5$$

smooth turbulent flow.

$$\frac{V_c}{V} = 1 + 4.07\sqrt{\frac{\lambda}{8}} = 1 + 4.07\sqrt{\frac{0.018}{8}} =$$

$$\Rightarrow V_c = 1.29\left(1 + 4.07\sqrt{\frac{0.018}{8}}\right) = 1.53 \text{ m/s}.$$

$V_c = 1.53\ m/s.$ $\qquad \nu = \frac{\mu}{\rho} = \frac{10^{-3}}{10^3} = 10^{-6}\ m^2/s$

$v = ??$ at $r = 25\,mm = 0.025\ m.$

$$y = R - r = \left(\frac{0.075}{2}\right) - 0.025 = 0.0125\ m.$$

Kin. viscosity.

$$\frac{v}{v_*} = 5.75 \log \frac{v_* y}{\nu} + 5.5 \quad ; \quad \nu = \frac{\mu}{\rho} = \frac{10^{-3}}{10^3} = 10^{-6}$$

$$\frac{v}{0.061} = 5.75 \log\left(\frac{0.061 \times 0.0125}{10^{-6}}\right) + 5.5 = 22.1$$

$$\Rightarrow v = 0.061(22.1) = 1.35\ m/s.$$

b) find the head loss in 1000 m of this pipe line.

$\rightarrow v_{mean}$.

$$h_L = \lambda \frac{L}{d} \frac{v^2}{2g} = 0.018 \times \frac{10^3}{0.075} \frac{(1.29)^2}{2(9.8)} \approx 20 \text{ m}$$

Ex5: water flows in 300 mm pipeline,

The mean velocity in 300 mm pipeline is 3 m/s

the relative roughness of pipe is $\frac{\varepsilon}{d} = 0.002$ and

$\nu = 9 \times 10^{-7}\ m^2/s$.

a- Find λ

b- v_c

c- v at 50 mm from the wall $v = ??$ $y = 50$ mm

d- $h_l = ??$ $L = 300$ m.

$$R_e = \frac{\rho d V}{\mu} = \frac{d V}{\left(\frac{\mu}{\rho}\right)} = \frac{d V}{\nu}$$

ν: Kinematic viscosity (m^2/s)

μ: dynamic viscosity (Pa.s).

$$R_e = \frac{dV}{\nu} = \frac{0.3(3)}{9\times10^{-7}} = \frac{9\times10^{-1}}{9\times10^{-7}} = 10^{6} > 10^{5}$$

$\Rightarrow$ rough turbulent flow.

a) $\frac{1}{\sqrt{\lambda}} = 2 \log \frac{d}{\varepsilon} + 1.14 = 2 \log\left(\frac{\phi}{0.002}\right) + 1.14 = 6.54$

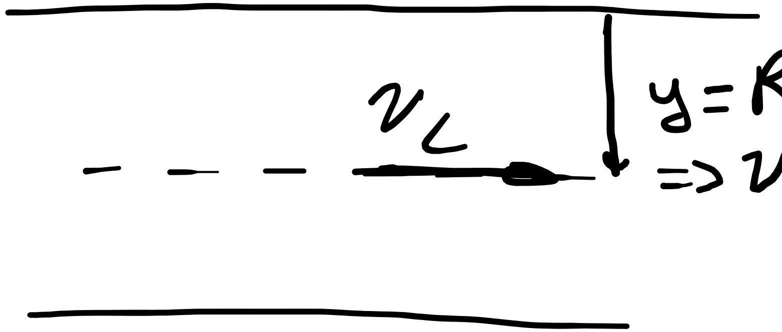
$\frac{1}{\sqrt{\lambda}} = 6.45 \Rightarrow \lambda = \frac{1}{6.54^2} = 0.0234.$

b) $v_* = ??$

c) $v_L = ??$

$v_* = V \sqrt{\frac{\lambda}{8}} = 3 \sqrt{\frac{0.0234}{8}} = 0.162 \, m/s$

$v_c = v$ when $y = R$.



$\Rightarrow v = v_c$.

*) $$\frac{v_c}{v_*} = 5.75 \log \frac{R}{\varepsilon} + 8.5$$

$$\frac{\varepsilon}{d} = \frac{\varepsilon}{2R} \Rightarrow \frac{\varepsilon}{R} = 2\frac{\varepsilon}{d}.$$

$$\frac{v_c}{0.162} = 5.75 \log \frac{1}{2}\left(\frac{1}{0.002}\right) + 8.5$$

$$\frac{R}{\varepsilon} = \frac{d}{2\varepsilon}.$$

$v_c = 3.61$ m/s.

d) $v = ??$ $y = 50 \text{ mm} = 0.05 \text{ m}.$

$\frac{\varepsilon}{d} = 0.002 \Rightarrow \varepsilon = 0.3 \times 0.002$

*) $\frac{v}{v_*} = 5.75 \log \frac{y}{\varepsilon} + 8.5 = 5.75 \log \frac{0.05}{0.3 \times 0.002} + 8.5$

$\frac{v}{0.162} = 19.5 \Rightarrow v = 3.16 \text{ m/s}.$

e) $h_L = f \frac{L}{d} \frac{v^2}{2g} = 0.0234 \frac{(300)}{0.3} \frac{3^2}{2(9.8)} = 10.7 \text{m}.$