Design of Machinery

Assignment

Chapters 4, 6 and 7

Position, velocity and acceleration analysis

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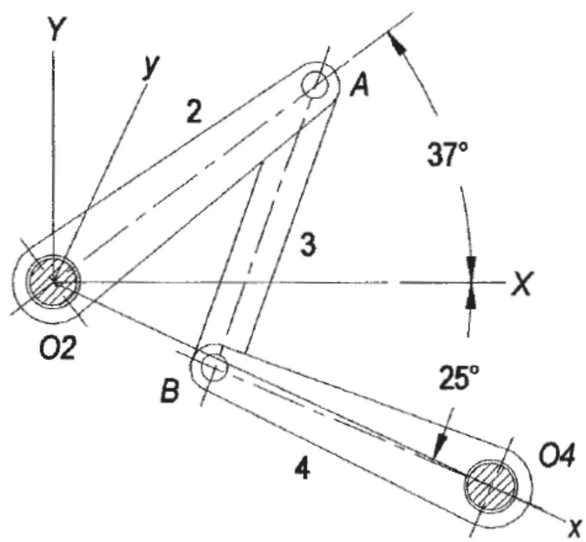
1. **Fourbar pin-jointed position analysis (4, 6.27, 7.21)**

The angle between the X and x axes is 25-deg. Find the angular displacement of link 4 when link 2 rotates clockwise from the position shown (+37 deg) to horizontal (0 deg).

Find ω_4, V_A and V_B in the local coordinate system if ω_2=15 rad/s CW.

Find alpha2, A_A and A_B in the global coordinate system if $\alpha 2$= 25 rad/s^2 CCW.

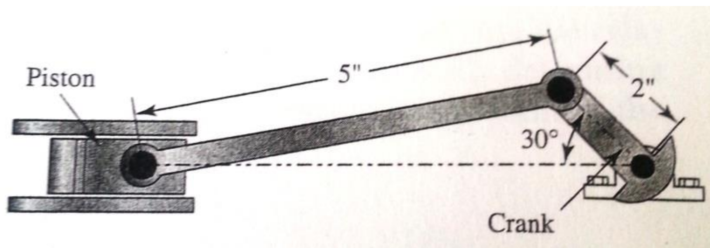
Take L2 = 116 mm., L3 = 108 mm., L4 = 110 mm and L1= 174 mm.



2. **Fourbar slider crank**

For the compressor shown in the following figure, find the position of the piston with respect to O_2. Also find the linear velocity of the piston as the crank rotates clockwise at constant rate of 120 rad/s.

Put clearly the vector loop and the global coordinate system. All dimensions are in inches.

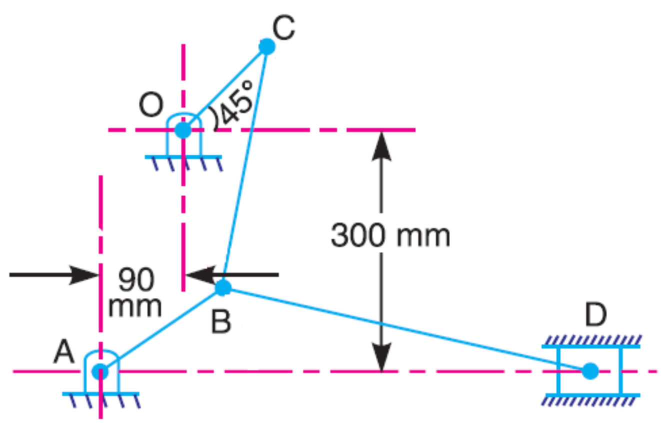


3. Sixbar

The device shown in figure below can be analyzed as a pin-jointed fourbar mechanism in series with a slider-crank mechanism. At the instant shown, the crank OC makes an angle 45^0 with the horizontal axis;

- Draw the vector loop of the two mechanisms separately
- Determine the position of the slider D with respect to A.

OC = 150 mm, AB = 200 mm, BC = 300 mm, BD = 450 mm.

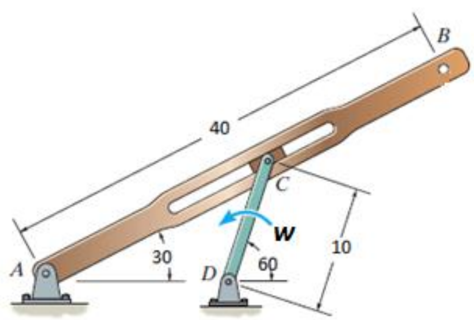


4. **Quick return mechanism**

For the quick-return mechanism shown below, the rod *DC* rotates with a constant angular velocity of 1 rad/s CCW. For the position shown find:

a) the position of the sliding block *C* with respect to A
b) the angular velocity of member *AB* and the velocity of sliding of block *C* within the member *AB*
c) the velocity of the node B.

Dimensions are in inches.

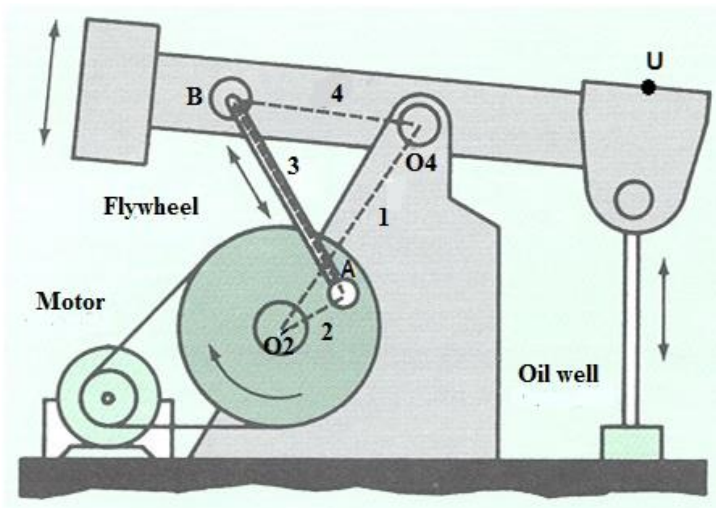


5. Fourbar pin-jointed mechanism

The device in the figure below is an oil well pump. Link number is shown on the figure.

L1=4m; L2=1m; L3=3.5m and L4=3m

In the local coordinate systems, take $\theta_2 = 315°$, $\omega 2 = 6$ rad/s, $\alpha 2 = -1$ rad/s, $\delta_4 = -60°$, u = 4m.

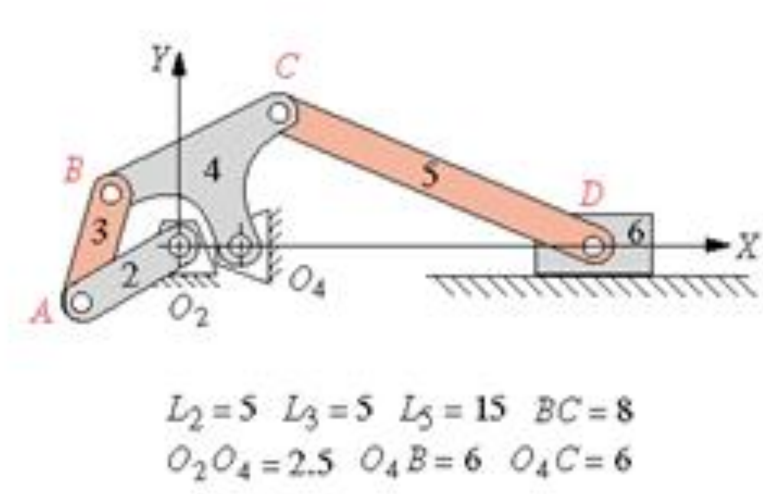


a) Draw the kinematic diagram of the mechanism and the vector loop.
b) Find the acceleration of point A, B and U

6. Sixbar mechanism

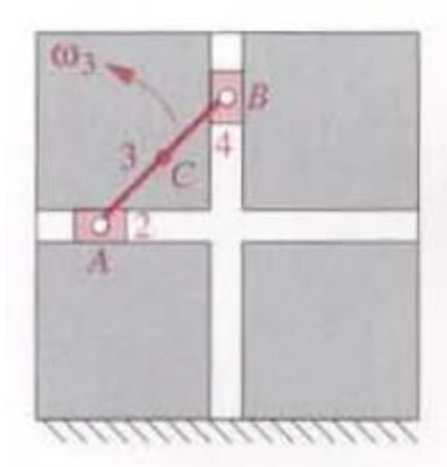
For the following six-bar drag-Link mechanism, the crank O_2A makes an angle $\theta_2 = 225^o$.

a) Determine the Grashof conditions and Barker classification for the first Fourbar (O2-A-B-O4)
b) Find the position of the Slider 6. (Dimensions are in Inch)



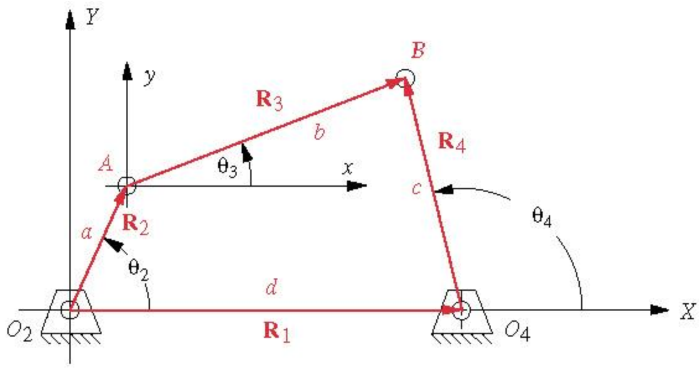
7. Elliptical trammel (4-58)

The elliptical trammel in the figure below must be driven by rotating link 3 in a full circle. Derive analytical expressions for the positions, velocity and accelerations of points A, B and a point C on link 3 midway between A and B as a function of θ_3 and the length AB of link 3. Use a vector loop equation.



Formula sheet

Position analysis: Pin-jointed Fourbar linkage



$$\boldsymbol{\theta_{4_{1,2}} = 2\,\mathrm{arctan}\left(\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}\right)}$$

- $A = cos\theta_2 - K_1 - K_2 cos\theta_2 + K_3$
- $B = -2sin\theta_2$
- $C = K_1 - (K_2 + 1)cos\theta_2 + K_3$
- $K_1 = \frac{d}{a}$
- $K_2 = \frac{d}{c}$
- $K_3 = \frac{a^2 - b^2 + c^2 + d^2}{2ac}$

$$\boldsymbol{\theta_{3_{1,2}} = 2\,arctan\left(\frac{-E \pm \sqrt{E^2 - 4DF}}{2D}\right)}$$

- $D = cos\theta_2 - K_1 - K_4 cos\theta_2 + K_5$
- $E = -2sin\theta_2$
- $F = K_1 + (K_4 - 1)cos\theta_2 + K_5$
- $K_4 = \frac{d}{b}$
- $K_5 = \frac{c^2 - d^2 - a^2 - b^2}{2ab}$

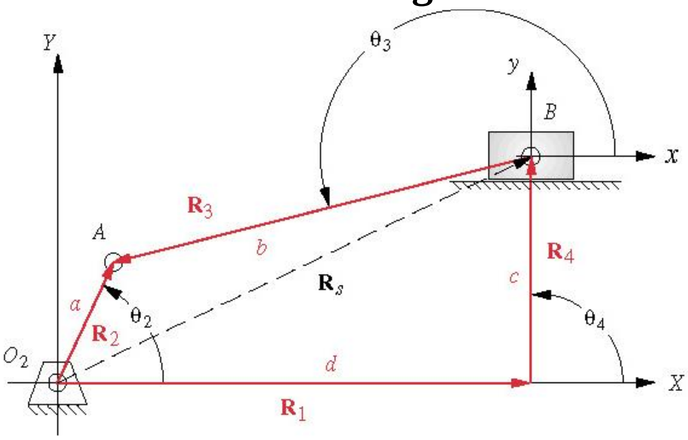
If θ_4 is calculated before you can use the equations below to solve for θ_3 :

1. $\boldsymbol{bcos\theta_3 = -acos\theta_2 + ccos\theta_4 + d}$

or

2. $\boldsymbol{bsin\theta_3 = -asin\theta_2 + csin\theta_4}$

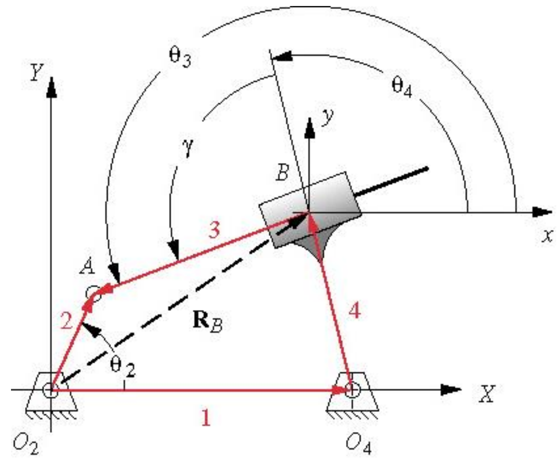
Position analysis: Slider-crank linkage



$$\theta_{3_1} = arcsin\left(\frac{asin\theta_2 - c}{b}\right) \quad or \quad \theta_{3_2} = arcsin\left(-\frac{asin\theta_2 - c}{b}\right) + \pi$$

$$d = acos\theta_2 - bcos\theta_3$$

Position analysis: Inverted Slider-crank linkage



$$\boldsymbol{\theta_{4\,1,2} = 2arctan\left(\frac{-T \pm \sqrt{T^2 - 4SU}}{2S}\right)}$$

$$\boldsymbol{\theta_3 = \theta_4 \pm \gamma} \qquad \boldsymbol{b = \frac{asin\theta_2 - csin\theta_4}{sin\theta_3}}$$

$$P = asin\theta_2 sin\,\gamma + (acos\theta_2 - d)cos\gamma$$

$$Q = -asin\theta_2 cos\,\gamma + (acos\theta_2 - d)sin\gamma$$

$$R = -csin\,\gamma; \quad S = R - Q; \qquad T = 2P; \qquad U = Q + R$$

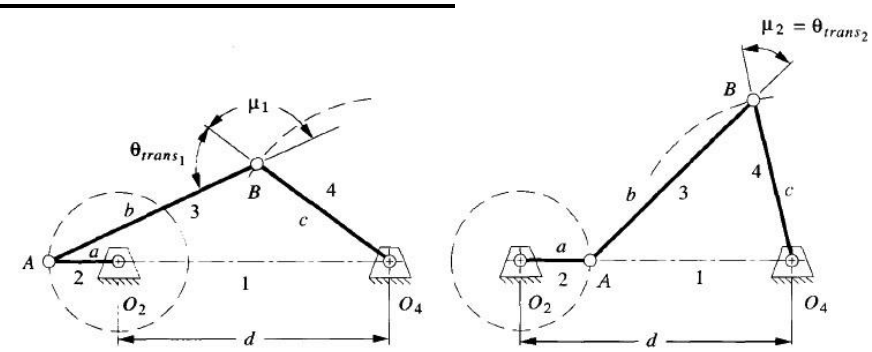
Transmission Angles for a pin-jointed fourbar linkage

$$\theta_{trans} = |\theta_3 - \theta_4|$$

$$\text{if} \qquad \theta_{trans} > \frac{\pi}{2} \qquad \text{then} \qquad \theta_{trans} = \pi - \theta_{trans}$$

Extreme Values of the Transmission Angle

1. **Grashof crank-rocker-rocker**



(a) Extended (b) Overlapped

$$\mu_1 = \arccos\left[\frac{b^2 + c^2 - (d+a)^2}{2bc}\right] \qquad \mu_2 = \arccos\left[\frac{b^2 + c^2 - (d-a)^2}{2bc}\right]$$

2. **Grashof rocker-crank-rocker**

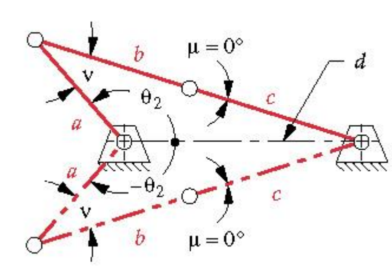
   The transmission angle can vary from 0 to 90 degrees

3. **Non-Grashof triple-rocker**

   The minimum transmission angle is 0 degree

Toggle Positions of the Non-Grashof triple-rocker

$$\theta_{2_{toggle}} = \arccos\left(\frac{a^2 + d^2 - b^2 - c^2}{2ad} \pm \frac{bc}{ad}\right); \qquad 0 \le \theta_{2_{toggle}} \le \pi$$



Design of Machinery

(a) Toggle positions for links b and c

TABLE 2-4 **Barker's Complete Classification of Planar Fourbar Mechanisms**

Adapted from ref. (10). s = shortest link, l = longest link, Gxxx = Grashof, RRRx = non-Grashof, Sxx = Special case

Type	$s + l$ vs. $p + q$	Inversion	Class	Barker's Designation	Code	Also Known As
1	<	$L_1 = s$ = ground	I-1	Grashof crank-crank-crank	GCCC	double-crank
2	<	$L_2 = s$ = input	I-2	Grashof crank-rocker-rocker	GCRR	crank-rocker
3	<	$L_3 = s$ = coupler	I-3	Grashof rocker-crank-rocker	GRCR	double-rocker
4	<	$L_4 = s$ = output	I-4	Grashof rocker-rocker-crank	GRRC	rocker-crank
5	>	$L_1 = l$ = ground	II-1	Class 1 rocker-rocker-rocker	RRR1	triple-rocker
6	>	$L_2 = l$ = input	II-2	Class 2 rocker-rocker-rocker	RRR2	triple-rocker
7	>	$L_3 = l$ = coupler	II-3	Class 3 rocker-rocker-rocker	RRR3	triple-rocker
8	>	$L_4 = l$ = output	II-4	Class 4 rocker-rocker-rocker	RRR4	triple-rocker
9	=	$L_1 = s$ = ground	III-1	change point crank-crank-crank	SCCC	SC* double -crank
10	=	$L_2 = s$ = input	III-2	change point crank-rocker-rocker	SCRR	SC crank-rocker
11	=	$L_3 = s$ = coupler	III-3	change point rocker-crank-rocker	SRCR	SC double-rocker
12	=	$L_4 = s$ = output	III-4	change point rocker-rocker-crank	SRRC	SC rocker-crank
13	=	two equal pairs	III-5	double change point	S2X	parallelogram or deltoid
14	=	$L_1 = L_2 = L_3 = L_4$	III-6	triple change point	S3X	square

* SC = special case.