6.4 ANELASTICITY

To this point, it has been assumed that elastic deformation is time independent—that is, that an applied stress produces an instantaneous elastic strain that remains constant over the period of time the stress is maintained. It has also been assumed that upon release of the load, the strain is totally recovered—that is, that the strain immediately returns to zero. In most engineering materials, however, there will also exist a time-dependent elastic strain component—that is, elastic deformation will continue after the stress application, and upon load release, some finite time is required for complete recovery. This time-dependent elastic behavior is known as **anelasticity,** and it is due to time-dependent microscopic and atomistic processes that are attendant to the deformation. For metals, the anelastic component is normally small and is often neglected. However, for some polymeric materials, its magnitude is significant; in this case it is termed *viscoelastic behavior,* which is the discussion topic of Section 15.4.

anelasticity

EXAMPLE PROBLEM 6.1

Elongation (Elastic) Computation

A piece of copper originally 305 mm (12 in.) long is pulled in tension with a stress of 276 MPa (40,000 psi). If the deformation is entirely elastic, what will be the resultant elongation?

Solution

Because the deformation is elastic, strain is dependent on stress according to Equation 6.5. Furthermore, the elongation Δl is related to the original length l_0 through Equation 6.2. Combining these two expressions and solving for Δl yields

$$\sigma = \epsilon E = \left(\frac{\Delta l}{l_0}\right)E$$

$$\Delta l = \frac{\sigma l_0}{E}$$

The values of σ and l_0 are given as 276 MPa and 305 mm, respectively, and the magnitude of E for copper from Table 6.1 is 110 GPa (16×10^6 psi). Elongation is obtained by substitution into the preceding expression as

$$\Delta l = \frac{(276 \text{ MPa})(305 \text{ mm})}{110 \times 10^3 \text{ MPa}} = 0.77 \text{ mm } (0.03 \text{ in.})$$

6.5 ELASTIC PROPERTIES OF MATERIALS

When a tensile stress is imposed on a metal specimen, an elastic elongation and accompanying strain ϵ_z result in the direction of the applied stress (arbitrarily taken to be the z direction), as indicated in Figure 6.9. As a result of this elongation, there will be constrictions in the lateral (x and y) directions perpendicular to the applied stress; from these contractions, the compressive strains ϵ_x and ϵ_y may be determined. If the applied stress is uniaxial (only in the z direction) and the material is isotropic, then $\epsilon_x = \epsilon_y$. A parameter termed **Poisson's ratio** ν is defined as the ratio of the lateral and axial strains, or

Poisson's ratio

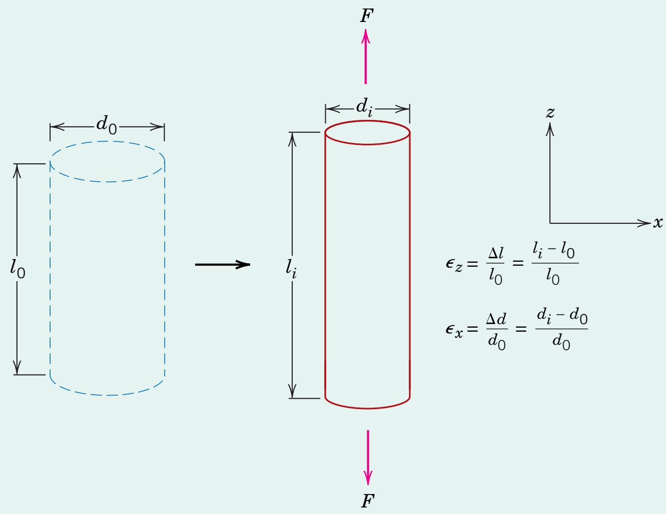
Definition of Poisson's ratio in terms of lateral and axial strains

$$\nu = -\frac{\epsilon_x}{\epsilon_z} = -\frac{\epsilon_y}{\epsilon_z} \tag{6.8}$$

EXAMPLE PROBLEM 6.2

Computation of Load to Produce Specified Diameter Change

A tensile stress is to be applied along the long axis of a cylindrical brass rod that has a diameter of 10 mm (0.4 in.). Determine the magnitude of the load required to produce a 2.5×10^{-3}-mm (10^{-4}-in.) change in diameter if the deformation is entirely elastic.



Solution

This deformation situation is represented in the accompanying drawing.

When the force F is applied, the specimen will elongate in the z direction and at the same time experience a reduction in diameter, Δd, of 2.5×10^{-3} mm in the x direction. For the strain in the x direction,

$$\epsilon_x = \frac{\Delta d}{d_0} = \frac{-2.5 \times 10^{-3}\text{ mm}}{10\text{ mm}} = -2.5 \times 10^{-4}$$

which is negative because the diameter is reduced.

It next becomes necessary to calculate the strain in the z direction using Equation 6.8. The value for Poisson's ratio for brass is 0.34 (Table 6.1), and thus

$$\epsilon_z = -\frac{\epsilon_x}{\nu} = -\frac{(-2.5 \times 10^{-4})}{0.34} = 7.35 \times 10^{-4}$$

The applied stress may now be computed using Equation 6.5 and the modulus of elasticity, given in Table 6.1 as 97 GPa (14×10^6 psi), as

$$\sigma = \epsilon_z E = (7.35 \times 10^{-4})(97 \times 10^3\text{ MPa}) = 71.3\text{ MPa}$$

Finally, from Equation 6.1, the applied force may be determined as

$$F = \sigma A_0 = \sigma\left(\frac{d_0}{2}\right)^2 \pi$$

$$= (71.3 \times 10^6\text{ N/m}^2)\left(\frac{10 \times 10^{-3}\text{ m}}{2}\right)^2 \pi = 5600\text{ N } (1293\text{ lb}_f)$$

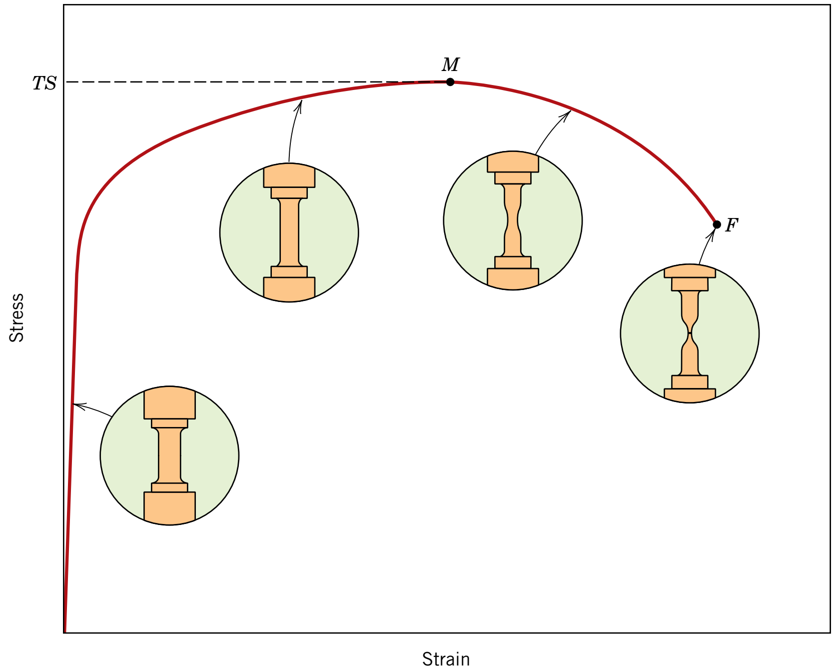


Figure 6.11 Typical engineering stress–strain behavior to fracture, point *F*. The tensile strength *TS* is indicated at point *M*. The circular insets represent the geometry of the deformed specimen at various points along the curve.

and fracture ultimately occurs at the neck.[11] The fracture strength corresponds to the stress at fracture.

Tensile strengths vary from 50 MPa (7000 psi) for an aluminum to as high as 3000 MPa (450,000 psi) for the high-strength steels. Typically, when the strength of a metal is cited for design purposes, the yield strength is used because by the time a stress corresponding to the tensile strength has been applied, often a structure has experienced so much plastic deformation that it is useless. Furthermore, fracture strengths are not normally specified for engineering design purposes.

EXAMPLE PROBLEM 6.3

Mechanical Property Determinations from Stress–Strain Plot

From the tensile stress–strain behavior for the brass specimen shown in Figure 6.12, determine the following:

(a) The modulus of elasticity
(b) The yield strength at a strain offset of 0.002
(c) The maximum load that can be sustained by a cylindrical specimen having an original diameter of 12.8 mm (0.505 in.)
(d) The change in length of a specimen originally 250 mm (10 in.) long that is subjected to a tensile stress of 345 MPa (50,000 psi)

Solution

(a) The modulus of elasticity is the slope of the elastic or initial linear portion of the stress–strain curve. The strain axis has been expanded in the inset of Figure 6.12 to facilitate

[11]The apparent decrease in engineering stress with continued deformation past the maximum point of Figure 6.11 is due to the necking phenomenon. As explained in Section 6.7, the true stress (within the neck) actually increases.

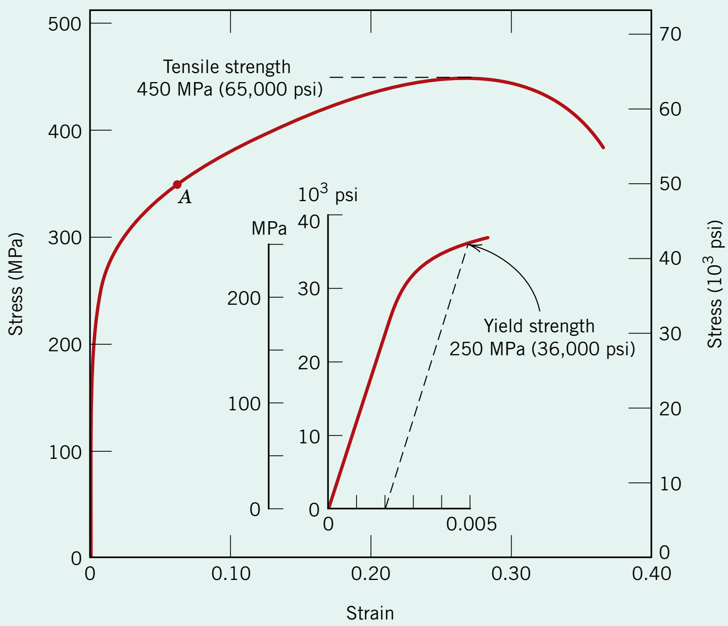


Figure 6.12 The stress–strain behavior for the brass specimen discussed in Example Problem 6.3.

this computation. The slope of this linear region is the rise over the run, or the change in stress divided by the corresponding change in strain; in mathematical terms,

$$E = \text{slope} = \frac{\Delta\sigma}{\Delta\epsilon} = \frac{\sigma_2 - \sigma_1}{\epsilon_2 - \epsilon_1} \tag{6.10}$$

Inasmuch as the line segment passes through the origin, it is convenient to take both σ_1 and ϵ_1 as zero. If σ_2 is arbitrarily taken as 150 MPa, then ϵ_2 will have a value of 0.0016. Therefore,

$$E = \frac{(150 - 0)\text{ MPa}}{0.0016 - 0} = 93.8\text{ GPa } (13.6 \times 10^6\text{ psi})$$

which is very close to the value of 97 GPa (14×10^6 psi) given for brass in Table 6.1.

(b) The 0.002 strain offset line is constructed as shown in the inset; its inter-section with the stress–strain curve is at approximately 250 MPa (36,000 psi), which is the yield strength of the brass.

(c) The maximum load that can be sustained by the specimen is calculated by using Equation 6.1, in which σ is taken to be the tensile strength, from Figure 6.12, 450 MPa (65,000 psi). Solving for F, the maximum load, yields

$$F = \sigma A_0 = \sigma\left(\frac{d_0}{2}\right)^2 \pi$$

$$= (450 \times 10^6\text{ N/m}^2)\left(\frac{12.8 \times 10^{-3}\text{ m}}{2}\right)^2 \pi = 57{,}900\text{ N } (13{,}000\text{ lb}_\text{f})$$

(d) To compute the change in length, Δl, in Equation 6.2, it is first necessary to determine the strain that is produced by a stress of 345 MPa. This is accomplished by locating the stress point on the stress–strain curve, point A, and reading the corresponding strain from the strain axis, which is approximately 0.06. Inasmuch as $l_0 = 250$ mm, we have

$$\Delta l = \epsilon l_0 = (0.06)(250\text{ mm}) = 15\text{ mm } (0.6\text{ in.})$$

Table 6.4
The n and K Values (Equation 6.19) for Several Alloys

Material	*n*	*K*	
		MPa	*psi*
Low-carbon steel (annealed)	0.21	600	87,000
4340 steel alloy (tempered @ 315°C)	0.12	2650	385,000
304 stainless steel (annealed)	0.44	1400	205,000
Copper (annealed)	0.44	530	76,500
Naval brass (annealed)	0.21	585	85,000
2024 aluminum alloy (heat-treated—T3)	0.17	780	113,000
AZ-31B magnesium alloy (annealed)	0.16	450	66,000

EXAMPLE PROBLEM 6.4

Ductility and True-Stress-at-Fracture Computations

A cylindrical specimen of steel having an original diameter of 12.8 mm (0.505 in.) is tensile-tested to fracture and found to have an engineering fracture strength σ_f of 460 MPa (67,000 psi). If its cross-sectional diameter at fracture is 10.7 mm (0.422 in.), determine

(a) The ductility in terms of percentage reduction in area
(b) The true stress at fracture

Solution

(a) Ductility is computed using Equation 6.12, as

$$\%\,\mathrm{RA} = \frac{\left(\dfrac{12.8\text{ mm}}{2}\right)^2 \pi - \left(\dfrac{10.7\text{ mm}}{2}\right)^2 \pi}{\left(\dfrac{12.8\text{ mm}}{2}\right)^2 \pi} \times 100$$

$$= \frac{128.7\text{ mm}^2 - 89.9\text{ mm}^2}{128.7\text{ mm}^2} \times 100 = 30\%$$

(b) True stress is defined by Equation 6.15, where, in this case, the area is taken as the fracture area A_f. However, the load at fracture must first be computed from the fracture strength as

$$F = \sigma_f A_0 = (460 \times 10^6\text{ N/m}^2)(128.7\text{ mm}^2)\left(\frac{1\text{ m}^2}{10^6\text{ mm}^2}\right) = 59{,}200\text{ N}$$

Thus, the true stress is calculated as

$$\sigma_T = \frac{F}{A_f} = \frac{59{,}200\text{ N}}{(89.9\text{ mm}^2)\left(\dfrac{1\text{ m}^2}{10^6\text{ mm}^2}\right)}$$

$$= 6.6 \times 10^8\text{ N/m}^2 = 660\text{ MPa (95,700 psi)}$$

EXAMPLE PROBLEM 6.5

Calculation of Strain-Hardening Exponent

Compute the strain-hardening exponent n in Equation 6.19 for an alloy in which a true stress of 415 MPa (60,000 psi) produces a true strain of 0.10; assume a value of 1035 MPa (150,000 psi) for K.

Solution

This requires some algebraic manipulation of Equation 6.19 so that n becomes the dependent parameter. This is accomplished by taking logarithms and rearranging. Solving for n yields

$$n = \frac{\log \sigma_T - \log K}{\log \epsilon_T}$$

$$= \frac{\log(415\text{ MPa}) - \log(1035\text{ MPa})}{\log(0.1)} = 0.40$$

6.8 ELASTIC RECOVERY AFTER PLASTIC DEFORMATION

Upon release of the load during the course of a stress–strain test, some fraction of the total deformation is recovered as elastic strain. This behavior is demonstrated in Figure 6.17, a schematic engineering stress–strain plot. During the unloading cycle, the curve traces a near straight-line path from the point of unloading (point D), and its slope is virtually identical to the modulus of elasticity, or parallel to the initial elastic portion of the curve. The magnitude of this elastic strain, which is regained during unloading, corresponds to the strain recovery, as shown in Figure 6.17. If the load is reapplied, the curve will traverse essentially the same linear portion in the direction opposite to unloading; yielding will again occur at the unloading stress level where the unloading began. There will also be an elastic strain recovery associated with fracture.

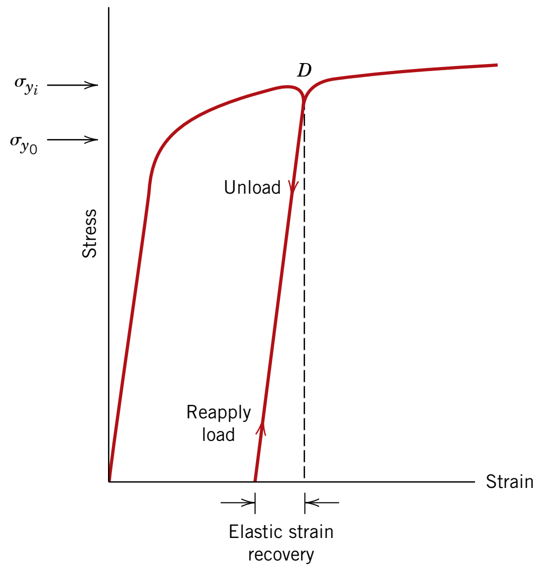


Figure 6.17 Schematic tensile stress–strain diagram showing the phenomena of elastic strain recovery and strain hardening. The initial yield strength is designated as σ_{y_0}; σ_{y_i} is the yield strength after releasing the load at point D and then upon reloading.

where n is the number of observations or measurements and x_i is the value of a discrete measurement.

Furthermore, the standard deviation s is determined using the following expression:

Computation of standard deviation

$$s = \left[\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n - 1} \right]^{1/2} \tag{6.22}$$

where x_i, $\bar{x}$, and n were defined earlier. A large value of the standard deviation corresponds to a high degree of scatter.

EXAMPLE PROBLEM 6.6

Average and Standard Deviation Computations

The following tensile strengths were measured for four specimens of the same steel alloy:

Sample Number	*Tensile Strength (MPa)*
1	520
2	512
3	515
4	522

(a) Compute the average tensile strength.
(b) Determine the standard deviation.

Solution

(a) The average tensile strength ($\overline{TS}$) is computed using Equation 6.21 with $n = 4$:

$$\overline{TS} = \frac{\sum_{i=1}^{4} (TS)_i}{4}$$

$$= \frac{520 + 512 + 515 + 522}{4}$$

$$= 517 \text{ MPa}$$

(b) For the standard deviation, using Equation 6.22, we obtain

$$s = \left[\frac{\sum_{i=1}^{4} \{(TS)_i - \overline{TS}\}^2}{4 - 1} \right]^{1/2}$$

$$= \left[\frac{(520 - 517)^2 + (512 - 517)^2 + (515 - 517)^2 + (522 - 517)^2}{4 - 1} \right]^{1/2}$$

$$= 4.6 \text{ MPa}$$

Figure 6.20 presents the tensile strength by specimen number for this example problem and also how the data may be represented in graphical form. The tensile strength data point (Figure 6.20*b*) corresponds to the average value $\overline{TS}$, and scatter is depicted by error bars (short horizontal lines) situated above and below the data point symbol and connected to this symbol by vertical lines. The upper error bar is positioned at a value of the average value plus the standard deviation ($\overline{TS} + s$), and the lower error bar corresponds to the average minus the standard deviation ($\overline{TS} - s$).

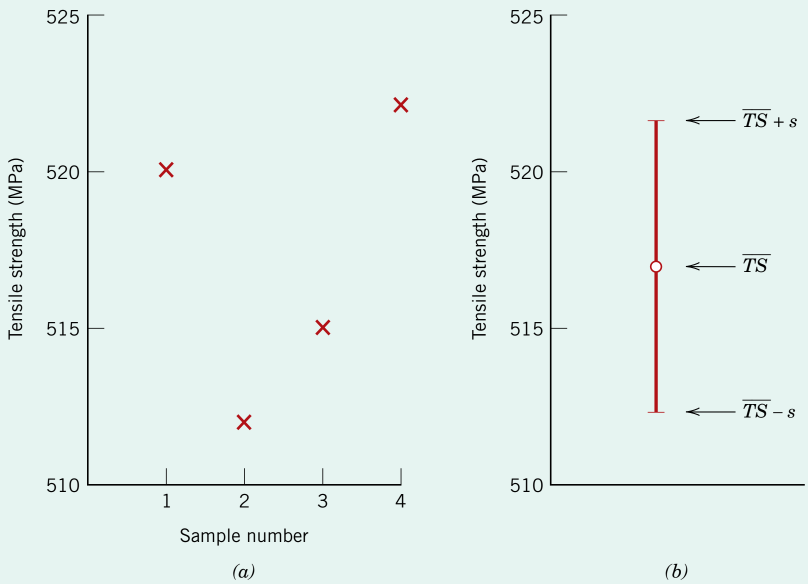


Figure 6.20 (*a*) Tensile strength data associated with Example Problem 6.6. (*b*) The manner in which these data could be plotted. The data point corresponds to the average value of the tensile strength ($\overline{TS}$); error bars that indicate the degree of scatter correspond to the average value plus and minus the standard deviation ($\overline{TS} \pm s$).

6.12 DESIGN/SAFETY FACTORS

There will always be uncertainties in characterizing the magnitude of applied loads and their associated stress levels for in-service applications; typically, load calculations are only approximate. Furthermore, as noted in Section 6.11, virtually all engineering materials exhibit a variability in their measured mechanical properties, have imperfections that were introduced during manufacture, and, in some instances, will have sustained damage during service. Consequently, design approaches must be employed to protect against unanticipated failure. During the 20th century, the protocol was to reduce the applied stress by a *design safety factor*. Although this is still an acceptable procedure for some structural applications, it does not provide adequate safety for critical applications such as those found in aircraft and bridge structural components. The current approach for these critical structural applications is to utilize materials that have adequate toughnesses and also offer redundancy in the structural design (i.e., excess or duplicate structures), provided there are regular inspections to detect the presence of flaws and, when necessary, safely remove or repair components. (These topics are discussed in Chapter 8, *Failure*—specifically Section 8.5.)

design stress

For less critical static situations and when tough materials are used, a **design stress,** σ_d, is taken as the calculated stress level σ_c (on the basis of the estimated maximum load) multiplied by a *design factor, N′*; that is,

$$\sigma_d = N'\sigma_c \tag{6.23}$$