EXAMPLE PROBLEM 6.5

Calculation of Strain-Hardening Exponent

Compute the strain-hardening exponent n in Equation 6.19 for an alloy in which a true stress of 415 MPa (60,000 psi) produces a true strain of 0.10; assume a value of 1035 MPa (150,000 psi) for K.

Solution

This requires some algebraic manipulation of Equation 6.19 so that n becomes the dependent parameter. This is accomplished by taking logarithms and rearranging. Solving for n yields

$$n = \frac{\log \sigma_T - \log K}{\log \epsilon_T}$$

$$= \frac{\log(415 \text{ MPa}) - \log(1035 \text{ MPa})}{\log(0.1)} = 0.40$$

6.8 ELASTIC RECOVERY AFTER PLASTIC DEFORMATION

Upon release of the load during the course of a stress–strain test, some fraction of the total deformation is recovered as elastic strain. This behavior is demonstrated in Figure 6.17, a schematic engineering stress–strain plot. During the unloading cycle, the curve traces a near straight-line path from the point of unloading (point D), and its slope is virtually identical to the modulus of elasticity, or parallel to the initial elastic portion of the curve. The magnitude of this elastic strain, which is regained during unloading, corresponds to the strain recovery, as shown in Figure 6.17. If the load is reapplied, the curve will traverse essentially the same linear portion in the direction opposite to unloading; yielding will again occur at the unloading stress level where the unloading began. There will also be an elastic strain recovery associated with fracture.

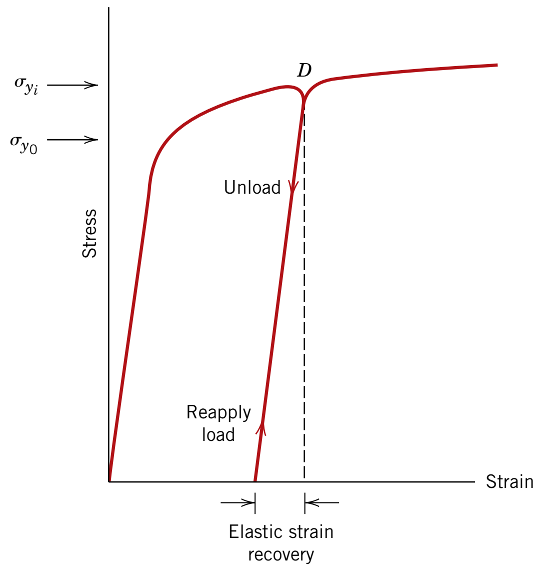

Figure 6.17 Schematic tensile stress–strain diagram showing the phenomena of elastic strain recovery and strain hardening. The initial yield strength is designated as σ_{y_0}; σ_{y_i} is the yield strength after releasing the load at point D and then upon reloading.

6.9 COMPRESSIVE, SHEAR, AND TORSIONAL DEFORMATIONS

Of course, metals may experience plastic deformation under the influence of applied compressive, shear, and torsional loads. The resulting stress–strain behavior into the plastic region is similar to the tensile counterpart (Figure 6.10*a*: yielding and the associated curvature). However, for compression, there is no maximum because necking does not occur; furthermore, the mode of fracture is different from that for tension.

Concept Check 6.3 Make a schematic plot showing the tensile engineering stress–strain behavior for a typical metal alloy to the point of fracture. Now superimpose on this plot a schematic compressive engineering stress–strain curve for the same alloy. Explain any differences between the two curves.

[*The answer may be found at www.wiley.com/college/callister (Student Companion Site).*]

6.10 HARDNESS

hardness

Another mechanical property that may be important to consider is **hardness,** which is a measure of a material's resistance to localized plastic deformation (e.g., a small dent or a scratch). Early hardness tests were based on natural minerals with a scale constructed solely on the ability of one material to scratch another that was softer. A qualitative and somewhat arbitrary hardness indexing scheme was devised, termed the *Mohs scale*, which ranged from 1 on the soft end for talc to 10 for diamond. Quantitative hardness techniques have been developed over the years in which a small indenter is forced into the surface of a material to be tested under controlled conditions of load and rate of application. The depth or size of the resulting indentation is measured and related to a hardness number; the softer the material, the larger and deeper the indentation, and the lower the hardness index number. Measured hardnesses are only relative (rather than absolute), and care should be exercised when comparing values determined by different techniques.

Hardness tests are performed more frequently than any other mechanical test for several reasons:

1. They are simple and inexpensive—typically, no special specimen need be prepared, and the testing apparatus is relatively inexpensive.
2. The test is nondestructive—the specimen is neither fractured nor excessively deformed; a small indentation is the only deformation.
3. Other mechanical properties often may be estimated from hardness data, such as tensile strength (see Figure 6.19).

Rockwell Hardness Tests[13]

The Rockwell tests constitute the most common method used to measure hardness because they are so simple to perform and require no special skills. Several different scales may be used from possible combinations of various indenters and different loads a process that permits the testing of virtually all metal alloys (as well as some polymers).

[13]ASTM Standard E18, "Standard Test Methods for Rockwell Hardness of Metallic Materials."

Indenters include spherical and hardened steel balls having diameters of $\frac{1}{16}$, $\frac{1}{8}$, $\frac{1}{4}$, and $\frac{1}{2}$ in. (1.588, 3.175, 6.350, and 12.70 mm, respectively), as well as a conical diamond (Brale) indenter, which is used for the hardest materials.

With this system, a hardness number is determined by the difference in depth of penetration resulting from the application of an initial minor load followed by a larger major load; utilization of a minor load enhances test accuracy. On the basis of the magnitude of both major and minor loads, there are two types of tests: Rockwell and superficial Rockwell. For the Rockwell test, the minor load is 10 kg, whereas major loads are 60, 100, and 150 kg. Each scale is represented by a letter of the alphabet; several are listed with the corresponding indenter and load in Tables 6.5 and 6.6a. For superficial tests, 3 kg is the minor load; 15, 30, and 45 kg are the possible major load values. These scales are identified by a 15, 30, or 45 (according to load), followed by N, T, W, X, or Y, depending on the indenter. Superficial tests are frequently performed on thin specimens. Table 6.6b presents several superficial scales.

When specifying Rockwell and superficial hardnesses, both hardness number and scale symbol must be indicated. The scale is designated by the symbol HR followed by the appropriate scale identification.[14] For example, 80 HRB represents a Rockwell hardness of 80 on the B scale, and 60 HR30W indicates a superficial hardness of 60 on the 30W scale.

For each scale, hardnesses may range up to 130; however, as hardness values rise above 100 or drop below 20 on any scale, they become inaccurate; and because the scales have some overlap, in such a situation it is best to utilize the next-harder or next-softer scale.

Inaccuracies also result if the test specimen is too thin, if an indentation is made too near a specimen edge, or if two indentations are made too close to one another. Specimen thickness should be at least 10 times the indentation depth, whereas allowance should be made for at least three indentation diameters between the center of one indentation and the specimen edge, or to the center of a second indentation. Furthermore, testing of specimens stacked one on top of another is not recommended. Also, accuracy is dependent on the indentation being made into a smooth flat surface.

The modern apparatus for making Rockwell hardness measurements is automated and very simple to use; hardness is read directly, and each measurement requires only a few seconds. This apparatus also permits a variation in the time of load application. This variable must also be considered in interpreting hardness data.

Brinell Hardness Tests[15]

In Brinell tests, as in Rockwell measurements, a hard, spherical indenter is forced into the surface of the metal to be tested. The diameter of the hardened steel (or tungsten carbide) indenter is 10.00 mm (0.394 in.). Standard loads range between 500 and 3000 kg in 500-kg increments; during a test, the load is maintained constant for a specified time (between 10 and 30 s). Harder materials require greater applied loads. The Brinell hardness number, HB, is a function of both the magnitude of the load and the diameter of the resulting indentation (see Table 6.5).[16] This diameter is measured with a special low-power microscope using a scale that is etched on the eyepiece. The measured diameter is then converted to the appropriate HB number using a chart; only one scale is employed with this technique.

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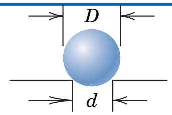
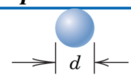
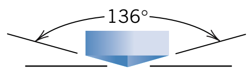
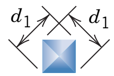
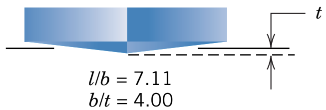
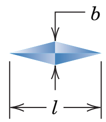
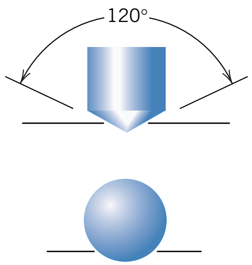
[14] Rockwell scales are also frequently designated by an R with the appropriate scale letter as a subscript, for example, R_C denotes the Rockwell C scale.

[15] ASTM Standard E10, "Standard Test Method for Brinell Hardness of Metallic Materials."

[16] The Brinell hardness number is also represented by BHN.

Table 6.5 Hardness-Testing Techniques

Test	*Indenter*	*Shape of Indentation* — *Side View*	*Shape of Indentation* — *Top View*	*Load*	*Formula for Hardness Number*[a]
Brinell	10-mm sphere of steel or tungsten carbide			P	$\mathrm{HB} = \dfrac{2P}{\pi D[D - \sqrt{D^2 - d^2}]}$
Vickers microhardness	Diamond pyramid			P	$\mathrm{HV} = 1.854P/d_1^2$
Knoop microhardness	Diamond pyramid			P	$\mathrm{HK} = 14.2P/l^2$
Rockwell and superficial Rockwell	Diamond cone: $\frac{1}{16}$-, $\frac{1}{8}$-, $\frac{1}{4}$-, $\frac{1}{2}$- in. diameter steel spheres			60 kg 100 kg 150 kg } Rockwell 15 kg 30 kg 45 kg } Superficial Rockwell	

[a]For the hardness formulas given, P (the applied load) is in kg, and D, d, d_1, and l are all in millimeters.

Source: Adapted from H. W. Hayden, W. G. Moffatt, and J. Wulff, *The Structure and Properties of Materials,* Vol. III, *Mechanical Behavior*. Copyright © 1965 by John Wiley & Sons, New York.

Table 6.6a Rockwell Hardness Scales

Scale Symbol	*Indenter*	*Major Load (kg)*
A	Diamond	60
B	$\frac{1}{16}$-in. ball	100
C	Diamond	150
D	Diamond	100
E	$\frac{1}{8}$-in. ball	100
F	$\frac{1}{16}$-in. ball	60
G	$\frac{1}{16}$-in. ball	150
H	$\frac{1}{8}$-in. ball	60
K	$\frac{1}{8}$-in. ball	150

Table 6.6b Superficial Rockwell Hardness Scales

Scale Symbol	*Indenter*	*Major Load (kg)*
15N	Diamond	15
30N	Diamond	30
45N	Diamond	45
15T	$\frac{1}{16}$-in. ball	15
30T	$\frac{1}{16}$-in. ball	30
45T	$\frac{1}{16}$-in. ball	45
15W	$\frac{1}{8}$-in. ball	15
30W	$\frac{1}{8}$-in. ball	30
45W	$\frac{1}{8}$-in. ball	45

Semiautomatic techniques for measuring Brinell hardness are available. These employ optical scanning systems consisting of a digital camera mounted on a flexible probe, which allows positioning of the camera over the indentation. Data from the camera are transferred to a computer that analyzes the indentation, determines its size, and then calculates the Brinell hardness number. For this technique, surface finish requirements are normally more stringent than those for manual measurements.

Maximum specimen thickness and indentation position (relative to specimen edges) as well as minimum indentation spacing requirements are the same as for Rockwell tests. In addition, a well-defined indentation is required; this necessitates a smooth, flat surface in which the indentation is made.

Knoop and Vickers Microindentation Hardness Tests[17]

Two other hardness-testing techniques are the Knoop (pronounced *nūp*) and Vickers tests (sometimes also called *diamond pyramid*). For each test, a very small diamond indenter having pyramidal geometry is forced into the surface of the specimen. Applied loads are much smaller than for the Rockwell and Brinell tests, ranging between 1 and 1000 g. The resulting impression is observed under a microscope and measured; this measurement is then converted into a hardness number (Table 6.5). Careful specimen surface preparation (grinding and polishing) may be necessary to ensure a well-defined indentation that may be measured accurately. The Knoop and Vickers hardness numbers are designated by HK and HV, respectively,[18] and hardness scales for both techniques are approximately equivalent. The Knoop and Vickers techniques are referred to as *microindentation-testing methods* on the basis of indenter size. Both are well suited for measuring the hardness of small, selected specimen regions; furthermore, the Knoop technique is used for testing brittle materials such as ceramics (Section 12.11).

The modern microindentation hardness-testing equipment has been automated by coupling the indenter apparatus to an image analyzer that incorporates a computer and software package. The software controls important system functions, including indent location, indent spacing, computation of hardness values, and plotting of data.

[17]ASTM Standard E92, "Standard Test Method for Vickers Hardness of Metallic Materials," and ASTM Standard E384, "Standard Test Method for Microindentation Hardness of Materials."

[18]Sometimes KHN and VHN are used to denote Knoop and Vickers hardness numbers, respectively.

Other hardness-testing techniques are frequently employed but will not be discussed here; these include ultrasonic microhardness, dynamic (Scleroscope), durometer (for plastic and elastomeric materials), and scratch hardness tests. These are described in references provided at the end of the chapter.

Hardness Conversion

The facility to convert the hardness measured on one scale to that of another is most desirable. However, because hardness is not a well-defined material property, and because of the experimental dissimilarities among the various techniques, a comprehensive conversion scheme has not been devised. Hardness conversion data have been determined experimentally and found to be dependent on material type and characteristics. The most reliable conversion data exist for steels, some of which are presented in Figure 6.18 for Knoop, Brinell, and two Rockwell scales; the Mohs scale is also included. Detailed conversion tables for various other metals and alloys are contained in ASTM Standard

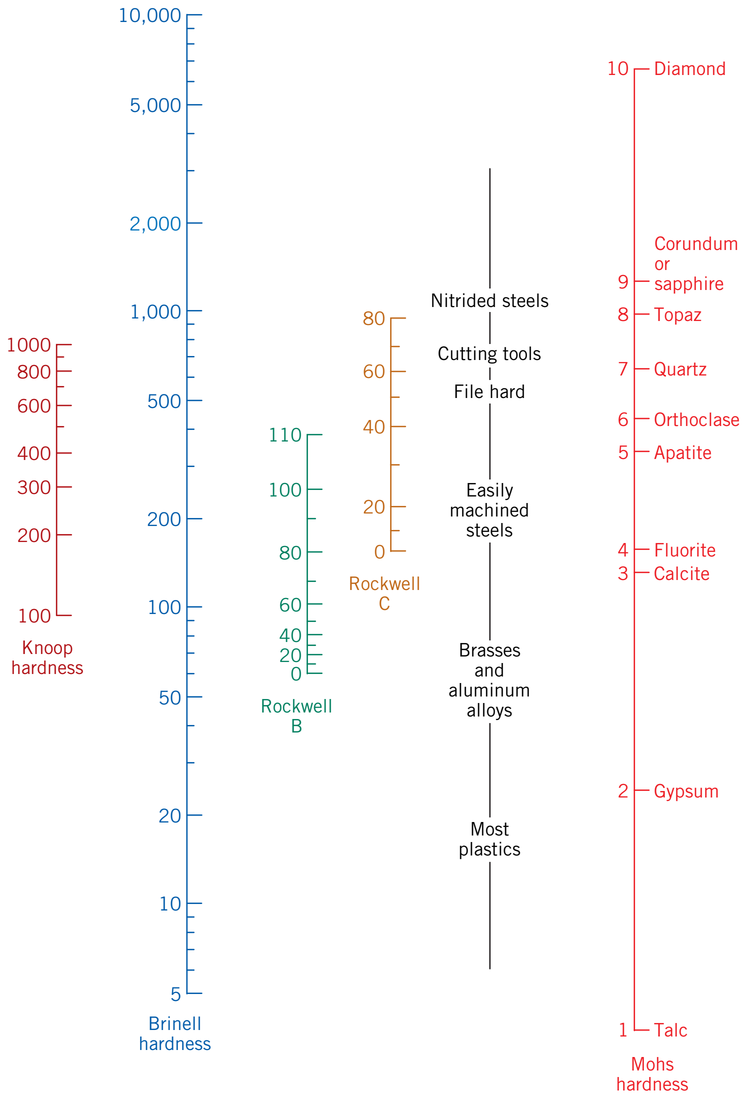


Figure 6.18 Comparison of several hardness scales. (Adapted from G. F. Kinney, *Engineering Properties and Applications of Plastics,* p. 202. Copyright © 1957 by John Wiley & Sons, New York. Reprinted by permission of John Wiley & Sons, Inc.)

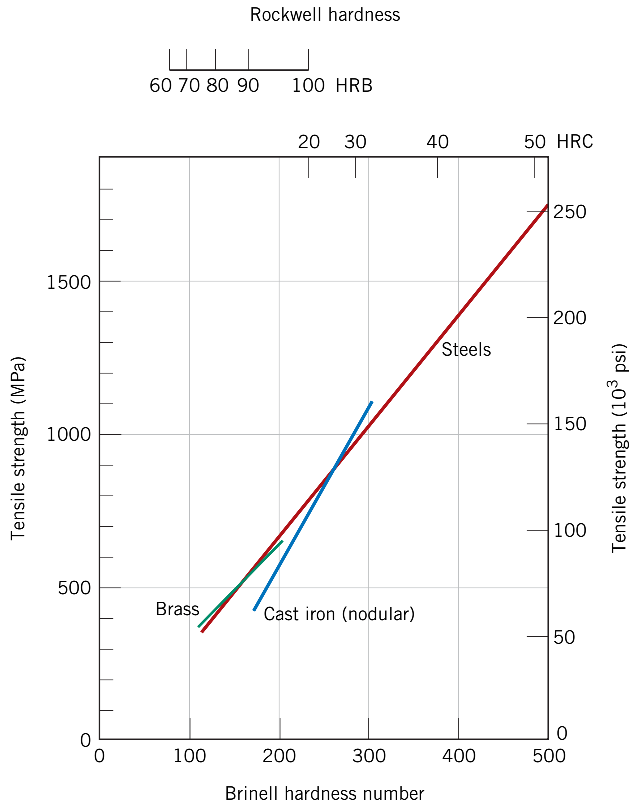


Figure 6.19 Relationships between hardness and tensile strength for steel, brass, and cast iron. [Adapted from *Metals Handbook: Properties and Selection: Irons and Steels,* Vol. 1, 9th edition, B. Bardes (Editor), 1978; and *Metals Handbook: Properties and Selection: Nonferrous Alloys and Pure Metals,* Vol. 2, 9th edition, H. Baker (Managing Editor), 1979. Reproduced by permission of ASM International, Materials Park, OH.]

E140, "Standard Hardness Conversion Tables for Metals." In light of the preceding discussion, care should be exercised in extrapolation of conversion data from one alloy system to another.

Correlation between Hardness and Tensile Strength

Both tensile strength and hardness are indicators of a metal's resistance to plastic deformation. Consequently, they are roughly proportional, as shown in Figure 6.19, for tensile strength as a function of the HB for cast iron, steel, and brass. The same proportionality relationship does not hold for all metals, as Figure 6.19 indicates. As a rule of thumb, for most steels, the HB and the tensile strength are related according to

For steel alloys, conversion of Brinell hardness to tensile strength

$$TS\,(\text{MPa}) = 3.45 \times \text{HB} \tag{6.20a}$$

$$TS\,(\text{psi}) = 500 \times \text{HB} \tag{6.20b}$$

Concept Check 6.4 Of those metals listed in Table 6.3, which is the hardest? Why?

[*The answer may be found at www.wiley.com/college/callister (Student Companion Site).*]

This concludes our discussion on the tensile properties of metals. By way of summary, Table 6.7 lists these properties, their symbols, and their characteristics (qualitatively).

Table 6.7 Summary of Mechanical Properties for Metals

Property	*Symbol*	*Measure of*
Modulus of elasticity	E	Stiffness—resistance to elastic deformation
Yield strength	σ_y	Resistance to plastic deformation
Tensile strength	TS	Maximum load-bearing capacity
Ductility	%EL, %RA	Degree of plastic deformation at fracture
Modulus of resilience	U_r	Energy absorption—elastic deformation
Toughness (static)	—	Energy absorption—plastic deformation
Hardness	e.g., HB, HRC	Resistance to localized surface deformation

Property Variability and Design/Safety Factors

6.11 VARIABILITY OF MATERIAL PROPERTIES

At this point, it is worthwhile to discuss an issue that sometimes proves troublesome to many engineering students—namely, that measured material properties are not exact quantities. That is, even if we have a most precise measuring apparatus and a highly controlled test procedure, there will always be some scatter or variability in the data that are collected from specimens of the same material. For example, consider a number of identical tensile samples that are prepared from a single bar of some metal alloy, which samples are subsequently stress–strain tested in the same apparatus. We would most likely observe that each resulting stress–strain plot is slightly different from the others. This would lead to a variety of modulus of elasticity, yield strength, and tensile strength values. A number of factors lead to uncertainties in measured data, including the test method, variations in specimen fabrication procedures, operator bias, and apparatus calibration. Furthermore, there might be inhomogeneities within the same lot of material and/or slight compositional and other differences from lot to lot. Of course, appropriate measures should be taken to minimize the possibility of measurement error and mitigate those factors that lead to data variability.

It should also be mentioned that scatter exists for other measured material properties, such as density, electrical conductivity, and coefficient of thermal expansion.

It is important for the design engineer to realize that scatter and variability of materials properties are inevitable and must be dealt with appropriately. On occasion, data must be subjected to statistical treatments and probabilities determined. For example, instead of asking, "What is the fracture strength of this alloy?" the engineer should become accustomed to asking, "What is the probability of failure of this alloy under these given circumstances?"

It is often desirable to specify a typical value and degree of dispersion (or scatter) for some measured property; this is commonly accomplished by taking the average and the standard deviation, respectively.

Computation of Average and Standard Deviation Values

An *average value* is obtained by dividing the sum of all measured values by the number of measurements taken. In mathematical terms, the average $\overline{x}$ of some parameter x is

Computation of average value

$$\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n} \tag{6.21}$$

where n is the number of observations or measurements and x_i is the value of a discrete measurement.

Furthermore, the standard deviation s is determined using the following expression:

Computation of standard deviation

$$s = \left[\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}\right]^{1/2} \tag{6.22}$$

where x_i, $\bar{x}$, and n were defined earlier. A large value of the standard deviation corresponds to a high degree of scatter.

EXAMPLE PROBLEM 6.6

Average and Standard Deviation Computations

The following tensile strengths were measured for four specimens of the same steel alloy:

Sample Number	*Tensile Strength (MPa)*
1	520
2	512
3	515
4	522

(a) Compute the average tensile strength.
(b) Determine the standard deviation.

Solution

(a) The average tensile strength ($\overline{TS}$) is computed using Equation 6.21 with $n = 4$:

$$\overline{TS} = \frac{\sum_{i=1}^{4}(TS)_i}{4}$$

$$= \frac{520 + 512 + 515 + 522}{4}$$

$$= 517 \text{ MPa}$$

(b) For the standard deviation, using Equation 6.22, we obtain

$$s = \left[\frac{\sum_{i=1}^{4}\{(TS)_i - \overline{TS}\}^2}{4-1}\right]^{1/2}$$

$$= \left[\frac{(520-517)^2 + (512-517)^2 + (515-517)^2 + (522-517)^2}{4-1}\right]^{1/2}$$

$$= 4.6 \text{ MPa}$$

Figure 6.20 presents the tensile strength by specimen number for this example problem and also how the data may be represented in graphical form. The tensile strength data point (Figure 6.20*b*) corresponds to the average value $\overline{TS}$, and scatter is depicted by error bars (short horizontal lines) situated above and below the data point symbol and connected to this symbol by vertical lines. The upper error bar is positioned at a value of the average value plus the standard deviation ($\overline{TS} + s$), and the lower error bar corresponds to the average minus the standard deviation ($\overline{TS} - s$).

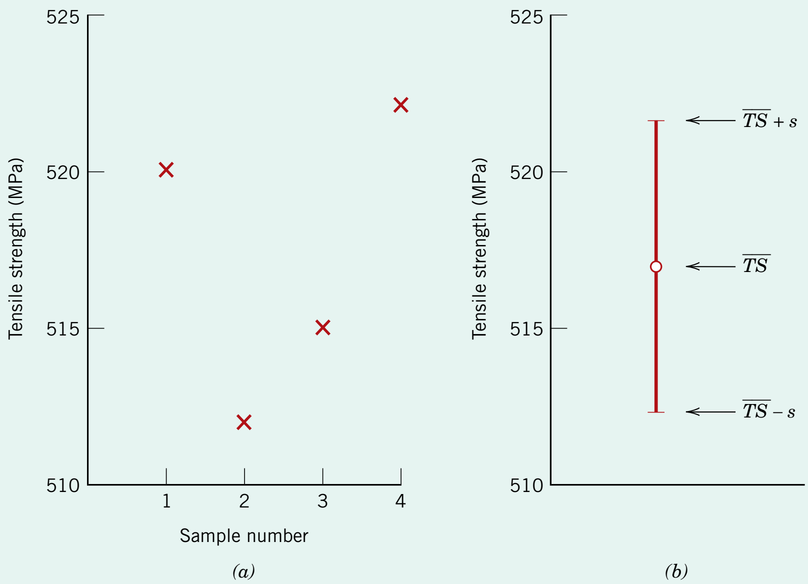


Figure 6.20 (a) Tensile strength data associated with Example Problem 6.6. (b) The manner in which these data could be plotted. The data point corresponds to the average value of the tensile strength ($\overline{TS}$); error bars that indicate the degree of scatter correspond to the average value plus and minus the standard deviation ($\overline{TS} \pm s$).

6.12 DESIGN/SAFETY FACTORS

There will always be uncertainties in characterizing the magnitude of applied loads and their associated stress levels for in-service applications; typically, load calculations are only approximate. Furthermore, as noted in Section 6.11, virtually all engineering materials exhibit a variability in their measured mechanical properties, have imperfections that were introduced during manufacture, and, in some instances, will have sustained damage during service. Consequently, design approaches must be employed to protect against unanticipated failure. During the 20th century, the protocol was to reduce the applied stress by a *design safety factor*. Although this is still an acceptable procedure for some structural applications, it does not provide adequate safety for critical applications such as those found in aircraft and bridge structural components. The current approach for these critical structural applications is to utilize materials that have adequate toughnesses and also offer redundancy in the structural design (i.e., excess or duplicate structures), provided there are regular inspections to detect the presence of flaws and, when necessary, safely remove or repair components. (These topics are discussed in Chapter 8, *Failure*—specifically Section 8.5.)

design stress

For less critical static situations and when tough materials are used, a **design stress,** σ_d, is taken as the calculated stress level σ_c (on the basis of the estimated maximum load) multiplied by a *design factor, N′*; that is,

$$\sigma_d = N'\sigma_c \tag{6.23}$$

where N' is greater than unity. Thus, the material to be used for the particular application is chosen so as to have a yield strength at least as high as this value of σ_d.

safe stress

Alternatively, a **safe stress** or *working stress,* σ_w, is used instead of design stress. This safe stress is based on the yield strength of the material and is defined as the yield strength divided by a *factor of safety,* N, or

Computation of safe (or working) stress

$$\sigma_w = \frac{\sigma_y}{N} \tag{6.24}$$

Utilization of design stress (Equation 6.23) is usually preferred because it is based on the anticipated maximum applied stress instead of the yield strength of the material; normally, there is a greater uncertainty in estimating this stress level than in the specification of the yield strength. However, in the discussion of this text, we are concerned with factors that influence the yield strengths of metal alloys and not in the determination of applied stresses; therefore, the succeeding discussion deals with working stresses and factors of safety.

The choice of an appropriate value of N is necessary. If N is too large, then component overdesign will result; that is, either too much material or an alloy having a higher-than-necessary strength will be used. Values normally range between 1.2 and 4.0. Selection of N will depend on a number of factors, including economics, previous experience, the accuracy with which mechanical forces and material properties may be determined, and, most important, the consequences of failure in terms of loss of life and/or property damage. Because large N values lead to increased material cost and weight, structural designers are moving toward using tougher materials with redundant (and inspectable) designs, where economically feasible.

DESIGN EXAMPLE 6.1

Specification of Support-Post Diameter

A tensile-testing apparatus is to be constructed that must withstand a maximum load of 220,000 N (50,000 lb_f). The design calls for two cylindrical support posts, each of which is to support half of the maximum load. Furthermore, plain-carbon (1045) steel ground and polished shafting rounds are to be used; the minimum yield and tensile strengths of this alloy are 310 MPa (45,000 psi) and 565 MPa (82,000 psi), respectively. Specify a suitable diameter for these support posts.

Solution

The first step in this design process is to decide on a factor of safety, N, which then allows determination of a working stress according to Equation 6.24. In addition, to ensure that the apparatus will be safe to operate, we also want to minimize any elastic deflection of the rods during testing; therefore, a relatively conservative factor of safety is to be used, say $N = 5$. Thus, the working stress σ_w is just

$$\sigma_w = \frac{\sigma_y}{N}$$
$$= \frac{310\text{ MPa}}{5} = 62\text{ MPa (9000 psi)}$$

From the definition of stress, Equation 6.1,

$$A_0 = \left(\frac{d}{2}\right)^2 \pi = \frac{F}{\sigma_w}$$