This support is written to help students follow up during the class sessions and for their individual studies.
It is limited to a general introduction and simple examples, therefore, it will not replace textbook, discussed material and solved problems during class sessions.

Design of machinery
Chapter 4 Position analysis

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Chapter 4 Position analysis

- 4.0 Introduction
- A principle goal of kinematics analysis is to determine the accelerations of all moving parts in the assembly. Using Newton's law we can determine forces.

- In order to calculate the **accelerations**, we must first find the **positions** of all the links or elements in the mechanism for each increment of input motion, and then differentiate the position equations versus time to find **velocities**, and then differentiate again to obtain the expressions for acceleration

Chapter 4 Position analysis

- 4.1 Coordinate systems

Global or absolute coordinate system: (GCS, XY)

Attached to the earth or another ground plane such as frame of reference.

Local coordinate systems typically attached to a link at some point of interest (pin joint, center of Gravity ...)

Local non rotating coordinate system: (LNCS, xy)

Local rotating coordinate systems: (LRCS, x'y')

Chapter 4 Position analysis

- 4.2 Position and displacement

Position vector: The position of a point in the plane can be defined by the use of a position vector.

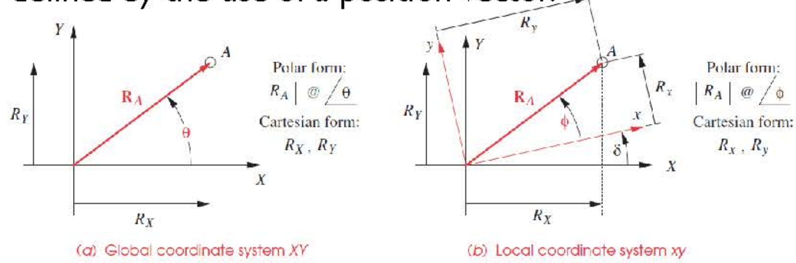


FIGURE 4-1

A position vector in the plane - expressed in both global and local coordinates

$$\boxed{R_A = \sqrt{R_X^2 + R_X^2}}$$

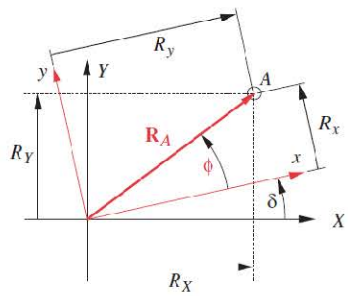
$$\boxed{\theta = \arctan\left(\frac{R_Y}{R_X}\right)}$$

Chapter 4 Position analysis

- 4.2 Position and displacement

Coordinate transformation:

It is often necessary to transform the coordinates of a point defined in one system to coordinates in another.



$$R_X = R_x \cos\delta - R_y \sin\delta$$
$$R_Y = R_x \sin\delta + R_y \cos\delta$$

Chapter 4 Position analysis

- 4.2 Position and displacement

Displacement of a point is the change in its position and can be defined as the straight line distance between the initial and final position of a point which has moved in the reference frame.

$$\mathbf{R}_{BA} = \mathbf{R}_B - \mathbf{R}_A$$

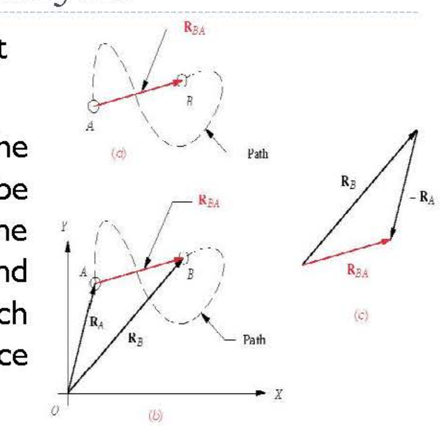


FIGURE 4-2
Position difference and relative position

Case 1: One body in 2 successive position ➔ **position difference**
Case2: Two bodies simultaneously in separate positions ➔ **relative position**

Chapter 4 Position analysis

- 4.3 Translation, rotation and complex motion

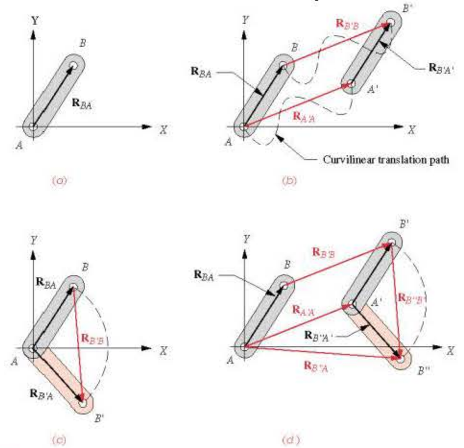


FIGURE 4-3
Translation, rotation, and complex motion

Chapter 4 Position analysis

- 4.4 Graphical position analysis of linkages

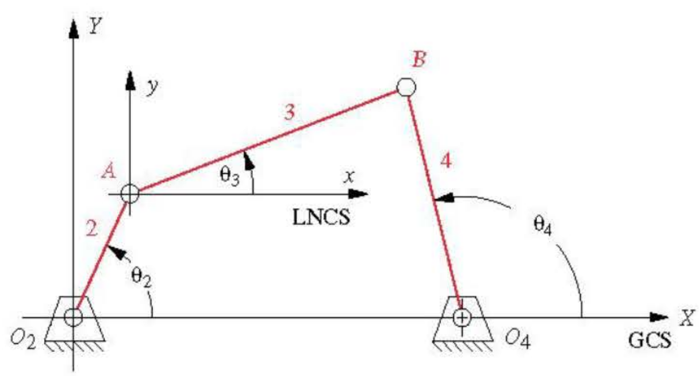


FIGURE 4-4
Measurement of angles in the fourbar linkage

Chapter 4 Position analysis

- 4.4 Graphical position analysis of linkages

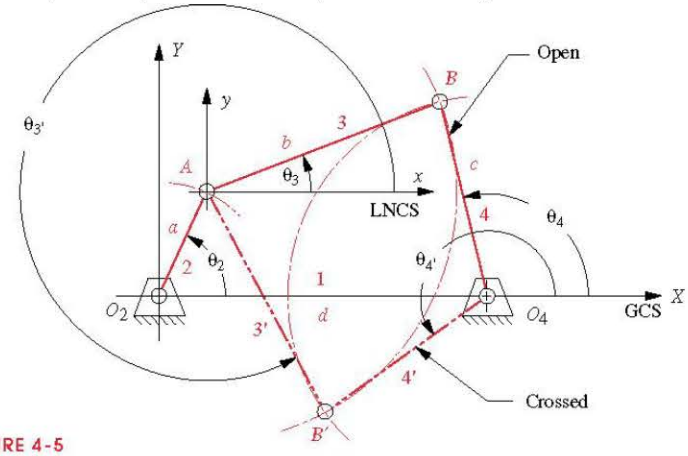


FIGURE 4-5
Graphical position solution to the open and crossed configurations of the fourbar linkage

Chapter 4 Position analysis

- 4.5 Algebraic position analysis of linkages

$$A_x = a\cos\theta_2$$

$$A_y = a\sin\theta_2$$

Point B situated on the intersection of 2 circles :
1) Center A radius b
2) Center O4 radius c

$$b^2 = (B_x - A_x)^2 + (B_y - A_y)^2$$

$$c^2 = (B_x - d)^2 + B_y^2$$

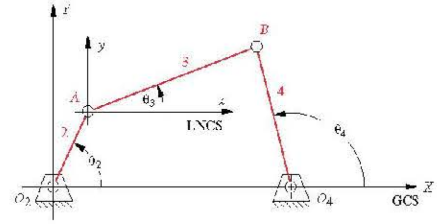


FIGURE 4-4
Measurement of angles in the fourbar linkage

Chapter 4 Position analysis

- 4.5 Algebraic position analysis of linkages

$$B_x = \frac{a^2 - b^2 + c^2 - d^2}{2(A_x - d)} - \frac{2A_y B_y}{2(A_x - d)} = S - \frac{2A_y B_y}{2(A_x - d)}$$

$$B_y^2 + \left(S - \frac{A_y B_y}{A_x - d} - d\right)^2 - c^2 = 0$$

$$B_y = \frac{-Q \pm \sqrt{Q^2 - 4PR}}{2P}$$

$$P = \frac{A_y^2}{(A_x - d)^2} + 1 \qquad Q = \frac{2A_y(d - S)}{A_x - d}$$

$$R = (d - S)^2 - c^2 \qquad S = \frac{a^2 - b^2 + c^2 - d^2}{2(A_x - d)}$$

$$\boxed{\theta_3 = \tan^{-1}\left(\frac{B_y - A_y}{B_x - A_x}\right) \qquad \theta_4 = \tan^{-1}\left(\frac{B_y}{B_x - d}\right)}$$

Chapter 4 Position analysis

- 4.5 Algebraic position analysis of linkages

Vector loop representation of linkages:

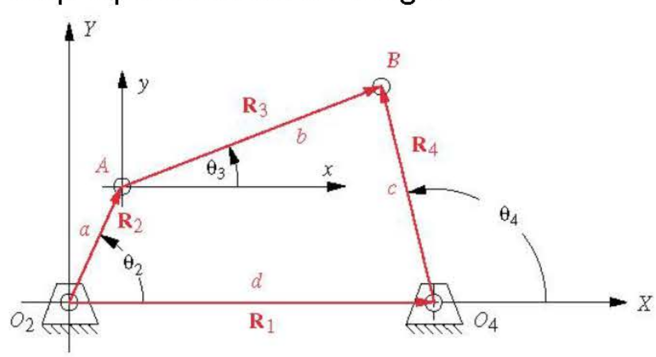


FIGURE 4-6
Position vector loop for a fourbar linkage

Chapter 4 Position analysis

- 4.5 Algebraic position analysis of linkages

Complex numbers as vectors

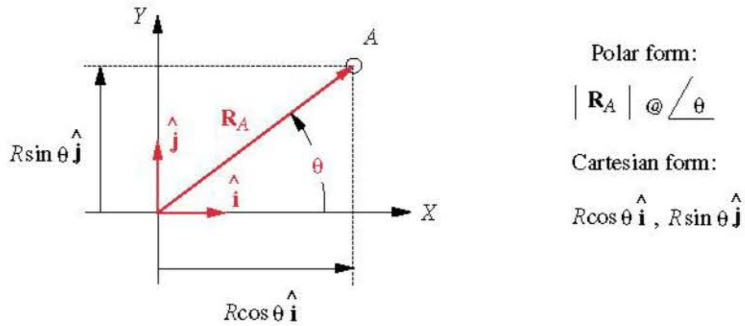


FIGURE 4-7

Unit vector notation for position vectors

Chapter 4 Position analysis

- 4.5 Algebraic position analysis of linkages

Complex numbers as vectors

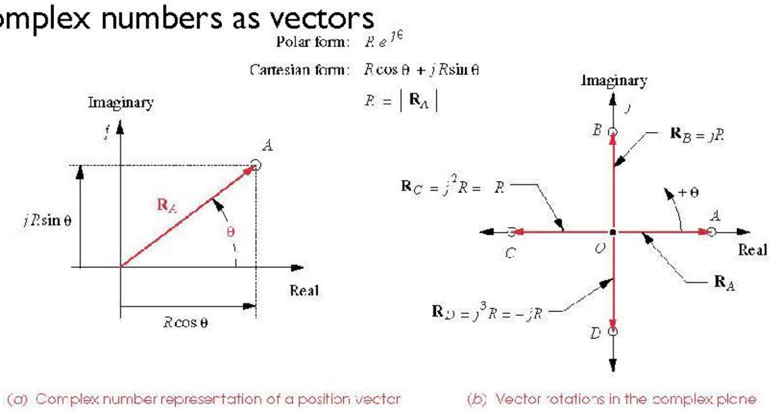


FIGURE 4-8

Complex number representation of vectors in the plane

Chapter 4 Position analysis

- 4.5 Algebraic position analysis of linkages

Notation used:

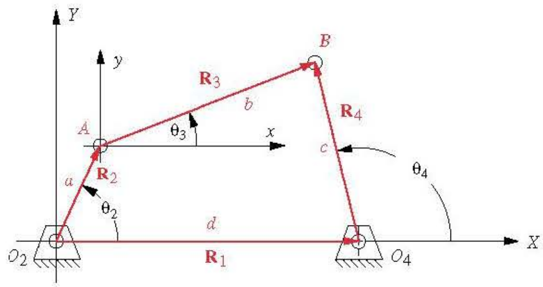


FIGURE 4-6

Position vec-or loop for a fourbar linkage

The angle of a vector is always measured at its root.

Chapter 4 Position analysis

- 4.5 Algebraic position analysis of linkages

The vector loop equation for a Fourbar pin jointed linkage

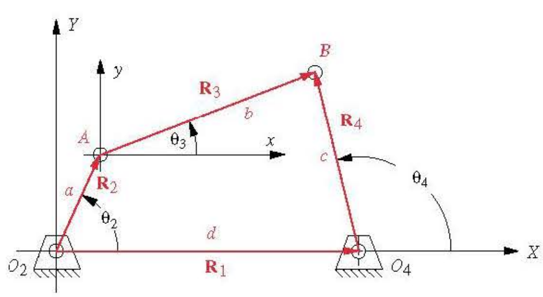


FIGURE 4-6

Position vec-or loop for a fourbar linkage

$$\mathbf{R}_2 + \mathbf{R}_3 - \mathbf{R}_4 - \mathbf{R}_1 = 0$$

Chapter 4 Position analysis

- 4.5 Algebraic position analysis of linkages

The vector loop equation for a Fourbar pin jointed linkage

$$\mathbf{R}_2 + \mathbf{R}_3 - \mathbf{R}_4 - \mathbf{R}_1 = 0$$

$$\mathbf{R}_A + \mathbf{R}_{BA} - \mathbf{R}_{BO_4} - \mathbf{R}_{O_4} = 0$$

$$a e^{j\theta_2} + b e^{j\theta_3} - c e^{j\theta_4} - d e^{j\theta_1} = 0$$

$$a(\cos\theta_2 + j\sin\theta_2) + b(\cos\theta_3 + j\sin\theta_3) - c(\cos\theta_4 + j\sin\theta_4) - d(\cos\theta_1 + j\sin\theta_1) = 0$$

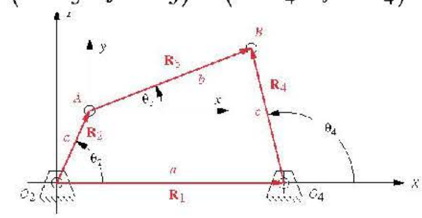


FIGURE 4-6

Position vector loop for a fourbar linkage

Chapter 4 Position analysis

- 4.5 Algebraic position analysis of linkages

The vector loop equation for a Fourbar pin jointed linkage

real part (x component):

$$a\cos\theta_2 + b\cos\theta_3 - c\cos\theta_4 - d\cos\theta_1 = 0$$

but: $\theta_1 = 0$, so:

$$\boxed{a\cos\theta_2 + b\cos\theta_3 - c\cos\theta_4 - d = 0}$$

2 equations with 2 unknowns

imaginary part (y component):

$$ja\sin\theta_2 + jb\sin\theta_3 - jc\sin\theta_4 - jd\sin\theta_1 = 0$$

but: $\theta_1 = 0$, and the j's divide out, so:

$$\boxed{a\sin\theta_2 + b\sin\theta_3 - c\sin\theta_4 = 0}$$

Chapter 4 Position analysis

- 4.5 Algebraic position analysis of linkages

The vector loop equation for a Fourbar pin jointed linkage

Real $a\cos\theta_2 + b\cos\theta_3 - c\cos\theta_4 - d = 0$

Imaginary $a\sin\theta_2 + b\sin\theta_3 - c\sin\theta_4 = 0$

- Solution for θ_4
- Isolate θ_3

$$\begin{cases} b\cos\theta_3 = -a\cos\theta_2 + c\cos\theta_4 + d \\ b\sin\theta_3 = -a\sin\theta_2 + c\sin\theta_4 \end{cases}$$

- Square both sides of equations and add them

$$b^2(\underbrace{\cos^2\theta_3 + \sin^2\theta_3}_{=1}) = (-a\sin\theta_2 + c\sin\theta_4)^2 + (-a\cos\theta_2 + c\cos\theta_4 + d)^2$$

$$\Rightarrow b^2 = a^2 + c^2 + d^2 - 2ad\cos\theta_2 + 2cd\cos\theta_4 - 2ac(\sin\theta_2\sin\theta_4 + \cos\theta_2\cos\theta_4)$$

Chapter 4 Position analysis

- 4.5 Algebraic position analysis of linkages

The vector loop equation for a Fourbar pin jointed linkage

- To simplify, constants are define in terms of the constant link length

$$\boxed{K_1 = \frac{d}{a} \qquad K_2 = \frac{d}{c} \qquad K_3 = \frac{a^2 - b^2 + c^2 + d^2}{2ac}}$$

and $\Rightarrow K_1\cos\theta_4 - K_2\cos\theta_2 + K_3 = \cos\theta_2\cos\theta_4 + \sin\theta_2\sin\theta_4$

- Substituting the identity $\cos(\theta_2 - \theta_4) = \cos\theta_2\cos\theta_4 + \sin\theta_2\sin\theta_4$
- Freudenstein's equation $K_1\cos\theta_4 - K_2\cos\theta_2 + K_3 = \cos(\theta_2 - \theta_4)$
- Using the half angle identities

$$\sin\theta_4 = \frac{2\tan\left(\frac{\theta_4}{2}\right)}{1 - \tan^2\left(\frac{\theta_4}{2}\right)} \qquad \cos\theta_4 = \frac{1 - \tan^2\left(\frac{\theta_4}{2}\right)}{1 + \tan^2\left(\frac{\theta_4}{2}\right)}$$

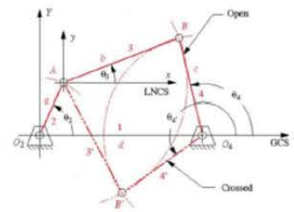
Chapter 4 Position analysis

4.5 Algebraic position analysis of linkages

- Simplified form

$$\Rightarrow A\tan^2\left(\frac{\theta_4}{2}\right)+B\tan\left(\frac{\theta_4}{2}\right)+C=0$$

$$\boxed{\begin{aligned}A&=\cos\theta_2-K_1-K_2\cos\theta_2+K_3\\ B&=-2\sin\theta_2\\ C&=K_1-(K_2+1)\cos\theta_2+K_3\end{aligned}}$$



- The equation is quadratic and the solution is

$$\tan\theta_4=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

$$\boxed{\Rightarrow \theta_{4_{1,2}}=2\arctan\left(\frac{-B\pm\sqrt{B^2-4AC}}{2A}\right)}$$

Note :
- If $B^2-4AC\ <0$ => complex conjugate solution) => the link lengths chosen are not capable of connection
- The solution will usually be real and unequal => there are two values of θ_4
  - :for crossed configuration of link
  - for open configuration of link

Chapter 4 Position analysis

4.5 Algebraic position analysis of linkages

➢Solution for θ_3

First case

- *If θ_4 is calculated the angle θ_3 can be determined using one of the following equation*

$$\begin{cases} b\cos\theta_3=-a\cos\theta_2+c\cos\theta_4+d\\ b\sin\theta_3=-a\sin\theta_2+c\sin\theta_4\end{cases}$$

Chapter 4 Position analysis

- 4.5 Algebraic position analysis of linkages
  - Solution for $\theta_{3:}$

  second case *(if θ_4 is not calculated)*

  - Isolate θ_4

$$c\cos\theta_4 = a\cos\theta_2 + b\cos\theta_3 - d$$
$$c\sin\theta_4 = a\sin\theta_2 + b\sin\theta_3$$

  - Square and add

$$K_1\cos\theta_3 - K_4\cos\theta_2 + K_5 = \cos\theta_2\cos\theta_3 + \sin\theta_2\sin\theta_3$$

$$\boxed{K_4 = \frac{d}{b} \qquad K_5 = \frac{c^2 - d^2 - a^2 - b^2}{2ab}}$$

$$\Rightarrow D\tan^2\left(\frac{\theta_3}{2}\right) + E\tan\left(\frac{\theta_3}{2}\right) + F = 0$$

$$\boxed{\begin{cases} D = \cos\theta_2 - K_1 + K_4\cos\theta_2 + K_5 \\ E = -2\sin\theta_2 \\ F = K_1 + (K_4 - 1)\cos\theta_2 + K_5 \end{cases}}$$

Chapter 4 Position analysis

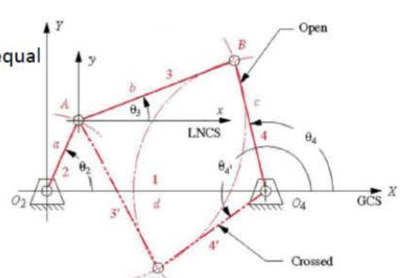
- 4.5 Algebraic position analysis of linkages
- The solution :

$$\theta_{3_{1,2}} = 2\arctan\left(\frac{-E \pm \sqrt{E^2 - 4DF}}{2D}\right)$$

- If the solution is complex conjugate, the link lengths chosen are not capable of connection
- The solution will usually be real and unequal

2 possible solutions :

  - For open configuration
  - For Crossed configuration



Chapter 4 Position analysis

- Problem 4-7

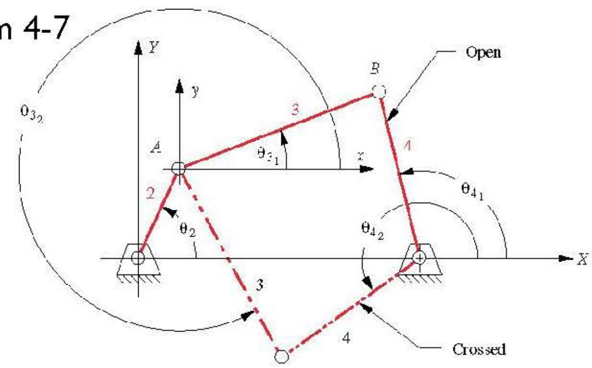


FIGURE P4-1‡

Problems 4-6 to 4-7. General configuration and terminology for the fourbar linkage

Note that link 1 (O2O4) should be always along x axis. If not you should create a local coordinate system where link 1 is along x' axis.

Chapter 4 Position analysis

- Problem 4-7

TABLE P4-1 Data for Problems 4-6, 4-7 and 4-13 to 4-15

Row	Link 1	Link 2	Link 3	Link 4	θ_2
a	6	2	7	9	30
b	7	9	3	8	85
c	3	10	6	8	45
d	8	5	7	6	25
e	8	5	8	6	75
f	5	8	8	9	15
g	6	8	8	9	25
h	20	10	10	10	50
i	4	5	2	5	80
j	20	10	5	10	33
k	4	6	10	7	88
l	9	7	10	7	60
m	9	7	11	8	50
n	9	7	11	6	120

Chapter 4 Position analysis

- Problem 4-7

Solution

$K_1 = \frac{d}{a}$ =3

$K_2 = \frac{d}{c}$ =0.6667

$K_3 = \frac{a^2 - b^2 + c^2 + d^2}{2ac}$ =2

$A = \cos\theta_2 - K_1 - K_2\cos\theta_2 + K_3$ =-0.7113

$B = -2\sin\theta_2$ =-1

$C = K_1 - (K_2 + 1)\cos\theta_2 + K_3$ =3.5566

$$\theta_{4_{1,2}} = 2\arctan\left(\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}\right)$$

$$b\sin\theta_3 = -a\sin\theta_2 + c\sin\theta_4$$

$\theta_{31} := 88.84 \cdot deg$

$\theta_{41} := 117.29 \cdot deg$

$\theta_{32} := 360 \cdot deg - 115.21 \cdot deg$ $\quad \theta_{32} = 244.790\ deg$

$\theta_{42} := 360 \cdot deg - 143.66 \cdot deg$ $\quad \theta_{42} = 216.340\ deg$

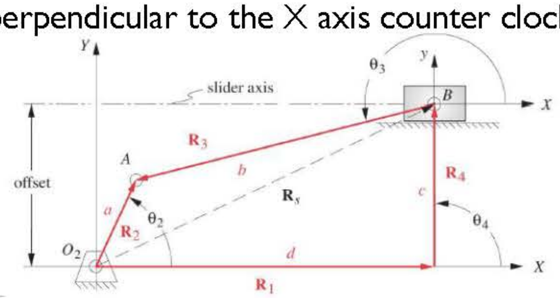
Chapter 4 Position analysis

- 4.6 The Fourbar slider-crank position solution

The slider crank is a mechanism that transform the rotation of a crank into translation of slider and vice versa.

X axis should be taken parallel to the slider axis (toward the slider)

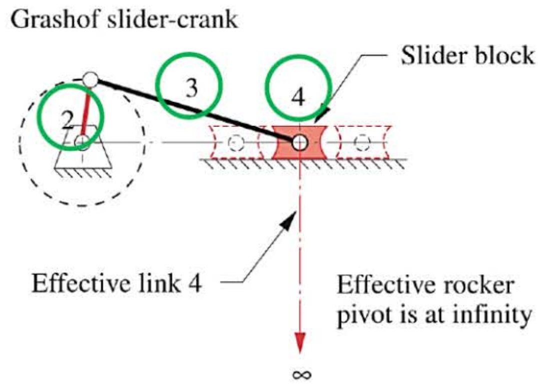
Y axis is perpendicular to the X axis counter clockwise.



Chapter 4 Position analysis

- 4.6 The Fourbar slider-crank position solution

Non offset slider crank linkage: Slider axis extended pass through the crank pivot ➔ offset = zero.



Chapter 4 Position analysis

- 4.6 The Fourbar slider-crank position solution

$$\mathbf{R}_2 - \mathbf{R}_3 - \mathbf{R}_4 - \mathbf{R}_1 = 0$$

$$a e^{j\theta_2} - b e^{j\theta_3} - c e^{j\theta_4} - d e^{j\theta_1} = 0$$

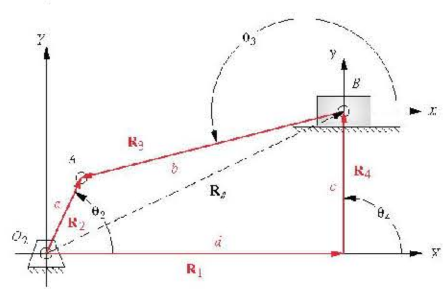


FIGURE 4-9

Position vector loop for a fourbar slider-crank linkage

$$a(\cos\theta_2 + j\sin\theta_2) - b(\cos\theta_3 + j\sin\theta_3) \\ - c(\cos\theta_4 + j\sin\theta_4) - d(\cos\theta_1 + j\sin\theta_1) = 0$$

Chapter 4 Position analysis

- 4.6 The Fourbar slider-crank position solution

Note that we have always:

$\theta_1 = 0^\circ$ and $\theta_4 = 90^\circ$

Therefore R_1 is position of the slider and R_4 the offset.

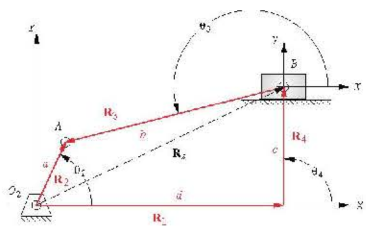


FIGURE 4-9
Position vector loop for a fourbar slider-crank linkage

Chapter 4 Position analysis

- 4.6 The Fourbar slider-crank position solution

real part (x component):

$$a\cos\theta_2 - b\cos\theta_3 - c\cos\theta_4 - d\cos\theta_1 = 0$$

but : $\theta_1 = 0$, so :

$$\boxed{a\cos\theta_2 - b\cos\theta_3 - c\cos\theta_4 - d = 0}$$

imaginary part (y component):

$$ja\sin\theta_2 - jb\sin\theta_3 - jc\sin\theta_4 - jd\sin\theta_1 = 0$$

but : $\theta_1 = 0$, and the j's divide out, so :

$$\boxed{a\sin\theta_2 - b\sin\theta_3 - c\sin\theta_4 = 0}$$

$$\boxed{\begin{gathered}\theta_{3_1} = \arcsin\left(\frac{a\sin\theta_2 - c}{b}\right) \\ d = a\cos\theta_2 - b\cos\theta_3\end{gathered}}$$

$$\boxed{\theta_{3_2} = \arcsin\left(-\frac{a\sin\theta_2 - c}{b}\right) + \pi}$$

Chapter 4 Position analysis

- Problem 4.10

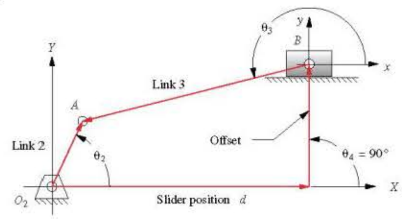


FIGURE P4-2

Problems 4-9 to 4-10 Open configuration and terminology for a fourbar slider-crank linkage

TABLE P4-2 Data for Problems 4-9 to 4-10

Row	Link 2	Link 3	Offset	θ_2
a	1.4	4	1	45
b	2	6	-3	60
c	3	8	2	-30
d	3.5	10	1	120
e	5	20	-5	225
f	3	13	0	100
g	7	25	10	330

Chapter 4 Position analysis

- Problem 4.10

Crossed:

$$\theta_{32} := asin\left(\frac{a \cdot sin(\theta_2) - c}{b}\right) \qquad \theta_{32} = -0.144\,deg$$

$$d_2 := a \cdot cos(\theta_2) - b \cdot cos(\theta_{32}) \qquad d_2 = -3.010\,in$$

Open:

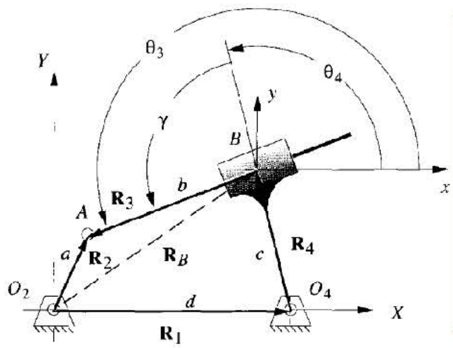
$$\theta_{31} := asin\left(-\frac{a \cdot sin(\theta_2) - c}{b}\right) + \pi \qquad \theta_{31} = 180.144\,deg$$

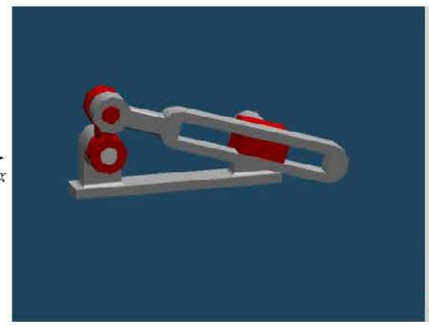
$$d_1 := a \cdot cos(\theta_2) - b \cdot cos(\theta_{31}) \qquad d_1 = 4.990\,in$$

Chapter 4 Position analysis

- 4.7 An inverted slider crank position solution

$$\theta_3 = \theta_4 \pm \gamma$$





Chapter 4 Position analysis

- 4.7 An inverted slider crank position solution

Vector loop →

$$\theta_3 = \theta_4 \pm \gamma$$

$$a\cos\theta_2 - b\cos\theta_3 - c\cos\theta_4 - d = 0$$

$$a\sin\theta_2 - b\sin\theta_3 - c\sin\theta_4 = 0$$

Solution:

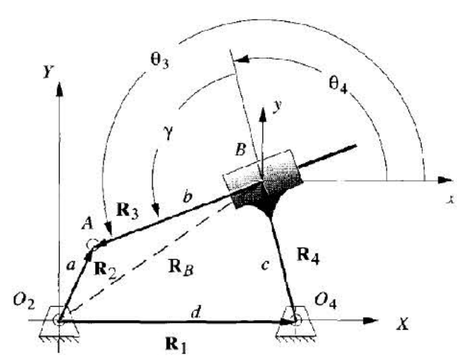
$$P = a\sin\theta_2 \sin\gamma + (a\cos\theta_2 - d)\cos\gamma$$

$$Q = -a\sin\theta_2 \cos\gamma + (a\cos\theta_2 - d)\sin\gamma$$

$$R = -c\sin\gamma$$

$$S = R - Q; \qquad T = 2P; \qquad U = Q + R$$

$$\boxed{\theta_{4_{1,2}} = 2\arctan\left(\frac{-T \pm \sqrt{T^2 - 4SU}}{2S}\right)}$$



Chapter 4 Position analysis

- Problem 4.12

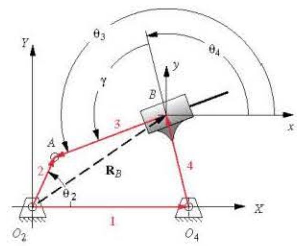


FIGURE P4-3
Problems 4-11 to 4-12 Terminology for inversion #3 of the fourbar slider-crank linkage

TABLE P4-3 Data for Problems 4-11 to 4-12

Row	Link 1	Link 2	Link 4	γ	θ_2
a	6	2	4	90	30
b	7	9	3	75	85
c	3	10	6	45	45
d	8	5	3	60	25
e	8	4	2	30	75
f	5	8	8	90	150

Chapter 4 Position analysis

- Problem 4.12

$P := a\cdot sin(\theta_2)\cdot sin(\gamma) + (a\cdot cos(\theta_2) - d)\cdot cos(\gamma)$ $\quad P = 1.000\ in$

$Q := -a\cdot sin(\theta_2)\cdot cos(\gamma) + (a\cdot cos(\theta_2) - d)\cdot sin(\gamma)$ $\quad Q = -4.268\ in$

$R := -c\cdot sin(\gamma)$ $\quad R = -4.000\ in$ $\quad T := 2\cdot P$ $\quad T = 2.000\ in$

$S := R - Q$ $\quad S = 0.268\ in$ $\quad U := Q + R$ $\quad U = -8.268\ in$

OPEN $\quad \theta_{41} := 2\cdot atan2\left(2\cdot S, -T + \sqrt{T^2 - 4\cdot S\cdot U}\right)$ $\quad \theta_{41} = 142.667\ deg$

CROSSED $\quad \theta_{42} := 2\cdot atan2\left(2\cdot S, -T - \sqrt{T^2 - 4\cdot S\cdot U}\right)$ $\quad \theta_{42} = -169.041\ deg$

OPEN $\quad b_1 := \dfrac{a\cdot sin(\theta_2) - c\cdot sin(\theta_{41})}{sin(\theta_{41} + \gamma)}$ $\quad b_1 = 1.793\ in$

CROSSED $\quad b_2 := \left|\dfrac{a\cdot sin(\theta_2) - c\cdot sin(\theta_{42})}{sin(\theta_{42} + \gamma)}\right|$ $\quad b_2 = 1.793\ in$

Chapter 4 Position analysis

- Problem 4.12

OPEN $\mathbf{R_{B1}} := a\cdot(\cos(\theta_2) + j\cdot\sin(\theta_2)) - b_1\cdot(\cos(\theta_{31}) + j\cdot\sin(\theta_{31}))$

$R_{B1} := |\mathbf{R_{B1}}|$ $\quad$ $R_{B1} = 3.719\ in$

$\theta_{B1} := arg(\mathbf{R_{B1}})$ $\quad$ $\theta_{B1} = 40.707\ deg$

CROSSED $\mathbf{R_{B2}} := a\cdot(\cos(\theta_2) + j\cdot\sin(\theta_2)) - b_2\cdot(\cos(\theta_{32}) + j\cdot\sin(\theta_{32}))$

$R_{B2} := |\mathbf{R_{B2}}|$ $\quad$ $R_{B2} = 2.208\ in$

$\theta_{B2} := arg(\mathbf{R_{B2}})$ $\quad$ $\theta_{B2} = -20.145\ deg$

Chapter 4 Position analysis

- 4.8 Linkages of more than four bars

The geared Fivebar linkage

gear ratio λ
phase angle ϕ

$$\boxed{\theta_5 = \lambda\theta_2 + \phi}$$

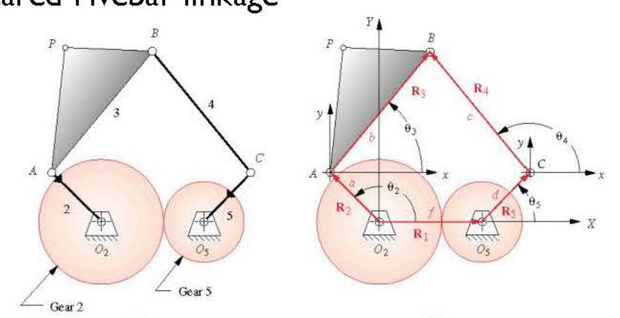


FIGURE 4-11
The geared fivebar linkage and its vector loop

$$\mathbf{R}_2 + \mathbf{R}_3 - \mathbf{R}_4 - \mathbf{R}_5 - \mathbf{R}_1 = 0$$

$$a e^{j\theta_2} + b e^{j\theta_3} - c e^{j\theta_4} - d e^{j\theta_5} - f e^{j\theta_1} = 0$$

Chapter 4 Position analysis

- 4.8 Linkages of more than four bars

The geared Fivebar linkage

$$A = 2c\left[d\cos(\lambda\theta_2+\phi) - a\cos\theta_2 + f\right]$$
$$B = 2c\left[d\sin(\lambda\theta_2+\phi) - a\sin\theta_2\right]$$
$$C = a^2 - b^2 + c^2 + d^2 + f^2 - 2af\cos\theta_2 - 2d(a\cos\theta_2 - f)\cos(\lambda\theta_2+\phi) - 2ad\sin\theta_2\sin(\lambda\theta_2+\phi)$$
$$D = C - A; \qquad E = 2B; \qquad F = A + C$$

$$\boxed{\theta_{4_{1,2}} = 2\arctan\left(\frac{-E \pm \sqrt{E^2 - 4DF}}{2D}\right)}$$

$$G = 2b\left[a\cos\theta_2 - d\cos(\lambda\theta_2+\phi) - f\right]$$
$$H = 2b\left[a\sin\theta_2 - d\sin(\lambda\theta_2+\phi)\right]$$
$$K = a^2 + b^2 - c^2 + d^2 + f^2 - 2af\cos\theta_2 - 2d(a\cos\theta_2 - f)\cos(\lambda\theta_2+\phi) - 2ad\sin\theta_2\sin(\lambda\theta_2+\phi)$$
$$L = K - G; \qquad M = 2H; \; N = G + K$$

$$\boxed{\theta_{3_{1,2}} = 2\arctan\left(\frac{-M \pm \sqrt{M^2 - 4LN}}{2L}\right)}$$

Chapter 4 Position analysis

- Problem 4.17

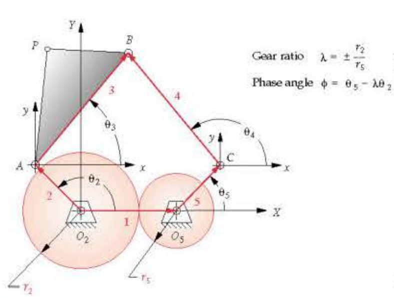


FIGURE P4-4
Problems 4-16 to 4-17 Open configuration and terminology for the geared fivebar linkage

TABLE P4-4 Data for Problems 4-16 to 4-17

Row	Link 1	Link 2	Link 3	Link 4	Link 5	λ	ϕ	θ_2
a	6	1	7	9	4	2	30	60
b	6	5	7	8	4	−2.5	60	30
c	3	5	7	8	4	−0.5	0	45
d	4	5	7	8	4	−1	120	75
e	5	9	11	8	8	3.2	−50	−39
f	10	2	7	5	3	1.5	30	120
g	15	7	9	11	4	2.5	−90	75
h	12	8	7	9	4	−2.5	60	55
i	9	7	8	9	4	−4	120	100

Chapter 4 Position analysis

- Problem 4.17

$A := 2 \cdot c \cdot \left(d \cdot cos(\lambda \cdot \theta_2 + \phi) - a \cdot cos(\theta_2) + f\right)$ $\quad A = 36.6462\ in^2$

$B := 2 \cdot c \cdot \left(d \cdot sin(\lambda \cdot \theta_2 + \phi) - a \cdot sin(\theta_2)\right)$ $\quad B = 20.412\ in^2$

$C := \left(a^2 - b^2 + c^2 + d^2 + f^2\right) - 2 \cdot a \cdot f \cdot cos(\theta_2) \ldots$
$+ -\left[2 \cdot d \cdot \left(a \cdot cos(\theta_2) - f\right) \cdot cos(\lambda \cdot \theta_2 + \phi)\right] \ldots$
$+ -2 \cdot a \cdot d \cdot sin(\theta_2) \cdot sin(\lambda \cdot \theta_2 + \phi)$ $\quad C = 37.4308\ in^2$

$D := C - A$ $\quad D = 0.78461\ in^2$

$E := 2 \cdot B$ $\quad E = 40.823\ in^2$

$F := A + C$ $\quad F = 74.077\ in^2$

$G := 2 \cdot b \cdot \left[-\left(d \cdot cos(\lambda \cdot \theta_2 + \phi)\right) + a \cdot cos(\theta_2) - f\right]$ $\quad G = -28.503\ in^2$

$H := 2 \cdot b \cdot \left[-\left(d \cdot sin(\lambda \cdot \theta_2 + \phi)\right) + a \cdot sin(\theta_2)\right]$ $\quad H = -15.876\ in^2$

$K := \left(a^2 + b^2 - c^2 + d^2 + f^2\right) - 2 \cdot a \cdot f \cdot cos(\theta_2) \ldots$
$+ -\left[2 \cdot d \cdot \left(a \cdot cos(\theta_2) - f\right) \cdot cos(\lambda \cdot \theta_2 + \phi)\right] \ldots$
$+ -2 \cdot a \cdot d \cdot sin(\theta_2) \cdot sin(\lambda \cdot \theta_2 + \phi)$ $\quad K = -26.569\ in^2$

$L := K - G$ $\quad L = 1.933\ in^2$

$M := 2 \cdot H$ $\quad M = -31.751\ in^2$

$N := G + K$ $\quad N = -55.072\ in^2$

Chapter 4 Position analysis

- Problem 4.17

OPEN $\quad \theta_{31} := 2 \cdot \left(atan2\left(2 \cdot L, -M + \sqrt{M^2 - 4 \cdot L \cdot N}\right)\right)$ $\quad \theta_{31} = 173.642\ deg$

$\theta_{41} := 2 \cdot \left(atan2\left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F}\right)\right)$ $\quad \theta_{41} = -177.715\ deg$

CROSSED $\quad \theta_{32} := 2 \cdot \left(atan2\left(2 \cdot L, -M - \sqrt{M^2 - 4 \cdot L \cdot N}\right)\right)$ $\quad \theta_{32} = -115.407\ deg$

$\theta_{42} := 2 \cdot \left(atan2\left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F}\right)\right)$ $\quad \theta_{42} = -124.050\ deg$

Chapter 4 Position analysis

- 4.8 Linkages of more than four bars

Watt's sixbar is essentially two fourbar linkages in series.

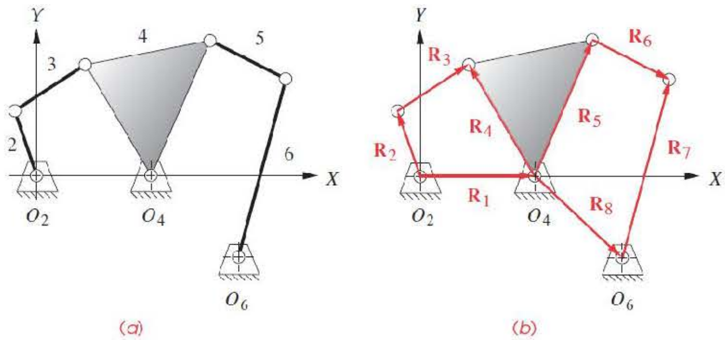


FIGURE 4-12

Watt's sixbar linkage and vector loop

Chapter 4 Position analysis

- 4.8 Linkages of more than four bars

Stephenson's sixbar is more complicated to analyze.

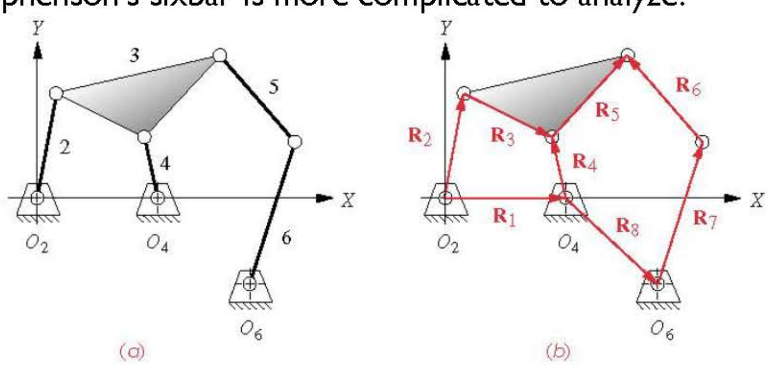


FIGURE 4-13

Stephenson's sixbar linkage and vector loop

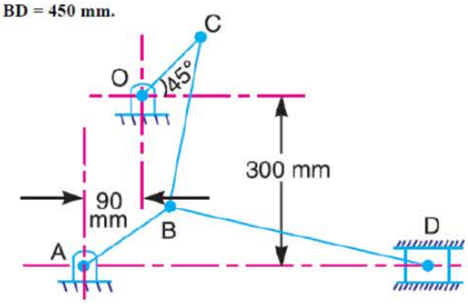
Chapter 4 Position analysis

- Problem

The device shown in figure below can be analyzed as a pin-jointed fourbar mechanism in series with a slider-crank mechanism. At the instant shown, the crank OC makes an angle 45 with the horizontal axis;

1) Draw the vector loop of the two mechanisms separately
2) Determine the position of the slider D with respect to A.

OC = 150 mm, AB = 200 mm, BC = 300 mm, BD = 450 mm.



Chapter 4 Position analysis

- 4.9 Position of any point on a linkage

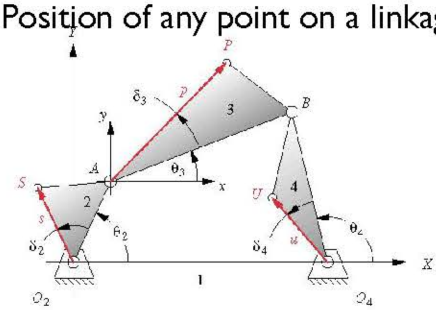


FIGURE 4-14
Positions of points on the links

$$\mathbf{R}_{SO_2} = \mathbf{R}_S = se^{j(\theta_2+\delta_2)} = s\left[\cos(\theta_2+\delta_2)+j\sin(\theta_2-\delta_2)\right]$$

$$\mathbf{R}_{UO_4} = ue^{j(\theta_4+\delta_4)} = u\left[\cos(\theta_4+\delta_4)+j\sin(\theta_4+\delta_4)\right]$$

$$\mathbf{R}_P = \mathbf{R}_A + \mathbf{R}_{PA} \qquad \mathbf{R}_{PA} = pe^{j(\theta_3+\delta_3)} = p\left[\cos(\theta_3+\delta_3)+j\sin(\theta_3+\delta_3)\right]$$

Chapter 4 Position analysis

- 4.10 Transmission angles

The **transmission angle** μ is defined as *the angle between the output link and the coupler*. It is usually taken as the absolute value of the acute angle of the pair of angles at the intersection of the two links and varies continuously from some minimum to some maximum value as the linkage goes through its range of motion. It is a measure of the quality of force transmission at the joint.

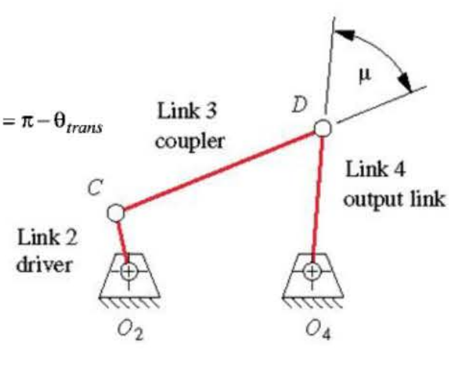
Chapter 4 Position analysis

- 4.10 Transmission angles

We take the absolute value of the difference and force it to be an acute angle.

$$\theta_{trans} = |\theta_3 - \theta_4|$$

$$\text{if} \quad \theta_{trans} > \frac{\pi}{2} \quad \text{then} \quad \theta_{trans} = \pi - \theta_{trans}$$



(a) Linkage transmission angle μ

Chapter 4 Position analysis

- 4.10 Transmission angles

Extreme values of the transmission angle

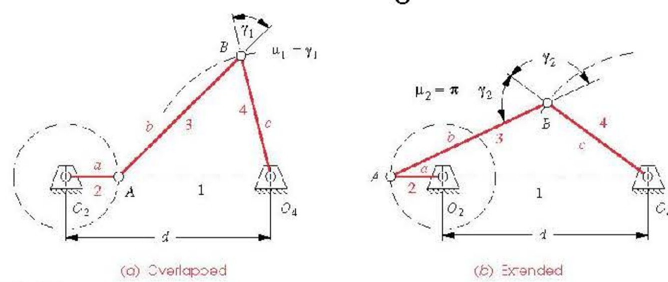


FIGURE 4-15

The minimum transmission angle in the Grashof crank-rocker fourbar linkage occurs in one of two positions

Min transmission angle in a Grashof crank rocker is the smaller of :

$$\boxed{\mu_1 = \arccos\left[\frac{b^2 + c^2 - (d+a)^2}{2bc}\right]}$$

$$\boxed{\mu_2 = \arccos\left[\frac{b^2 + c^2 - (d-a)^2}{2bc}\right]}$$

Chapter 4 Position analysis

- 4.10 Transmission angles

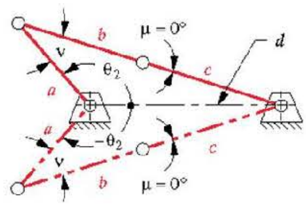
For a Grashof double rocker linkage the transmission angle vary from 0 to 90 degrees because the coupler can make a full revolution with respect to other links.

Chapter 4 Position analysis

- 4.11 Toggle positions

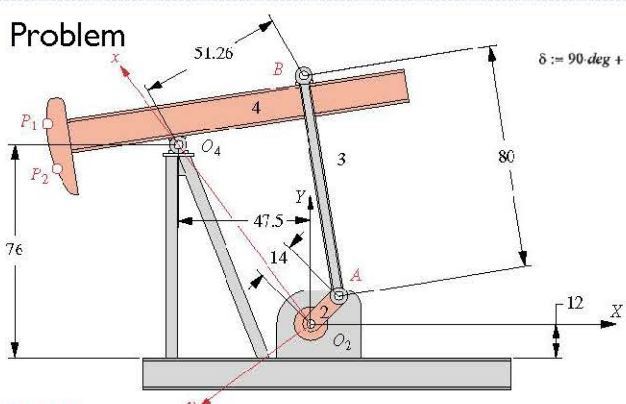
The input link angles which correspond to the toggle positions (stationary configurations) of the **non-Grashof triple-rocker** can be calculated by the following method, using trigonometry.

$$\theta_{2_{toggle}} = \cos^{-1}\left(\frac{a^2+d^2-b^2-c^2}{2ad} \pm \frac{bc}{ad}\right) \quad 0 \le \theta_{2_{toggle}} \le \pi$$



Chapter 4 Position analysis

- Problem



$\delta := 90 \cdot deg + atan\left(\frac{47.5}{64}\right) \qquad \delta = 126.582\,deg$

FIGURE P4-16

Problems 4-52 to 4-54 An oil field pump - dimensions in inches

Input (O_2A)	$a := 14$	Coupler (AB)	$b := 80$
Rocker (O_4B)	$c := 51.26$	Ground link	$d := 79.70$

Chapter 4 Position analysis

- Problem

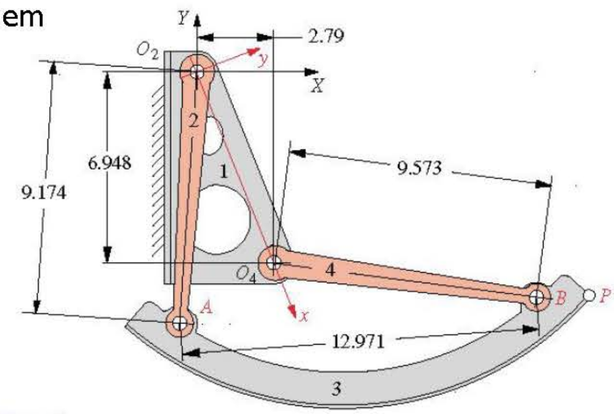


FIGURE P4-17

Problems 4-55 to 4-57 An aircraft overhead bin mechanism - dimensions in inches

Chapter 4 Position analysis

- Problem

For the quick-return mechanism shown below, the rod DC rotates with a constant angular velocity of 1 rad/s CCW. For the position shown find:
a) the position of the sliding block C with respect to A
b) the angular velocity of member AB and the velocity of sliding of block C within the member AB
c) the velocity of the node B.