Stress-Strain Behavior

6.3 A specimen of aluminum having a rectangular cross section 10 mm × 12.7 mm (0.4 in. × 0.5 in.) is pulled in tension with 35,500 N (8000 lb_f) force, producing only elastic deformation. Calculate the resulting strain.

Solution

This problem calls for us to calculate the elastic strain that results for an aluminum specimen stressed in tension. The cross-sectional area is just (10 mm) × (12.7 mm) = 127 mm^2 (= 1.27×10^{-4} m^2 = 0.20 in.^{2}); also, the elastic modulus for Al is given in Table 6.1 as 69 GPa (or 69×10^9 N/m^2). Combining Equations 6.1 and 6.5 and solving for the strain yields

$$\varepsilon = \frac{\sigma}{E} = \frac{F}{A_0 E} = \frac{35,500 \text{ N}}{(1.27 \times 10^{-4} \text{ m}^2)(69 \times 10^9 \text{ N/m}^2)} = 4.1 \times 10^{-3}$$

6.4 A cylindrical specimen of a titanium alloy having an elastic modulus of 107 GPa (15.5×10^6 psi) and an original diameter of 3.8 mm (0.15 in.) will experience only elastic deformation when a tensile load of 2000 N (450 lb_f) is applied. Compute the maximum length of the specimen before deformation if the maximum allowable elongation is 0.42 mm (0.0165 in.).

Solution

We are asked to compute the maximum length of a cylindrical titanium alloy specimen (before deformation) that is deformed elastically in tension. For a cylindrical specimen

$$A_0 = \pi \left(\frac{d_0}{2}\right)^2$$

where d_0 is the original diameter. Combining Equations 6.1, 6.2, and 6.5 and solving for l_0 leads to

$$l_0 = \frac{\Delta l}{\varepsilon} = \frac{\Delta l}{\dfrac{\sigma}{E}} = \frac{\Delta l\, E}{\dfrac{F}{A_0}} = \frac{\Delta l\, E \pi \left(\dfrac{d_0}{2}\right)^2}{F} = \frac{\Delta l\, E \pi\, d_0^2}{4F}$$

$$= \frac{(0.42 \times 10^{-3}\ \mathrm{m})(107 \times 10^9\ \mathrm{N/m^2})\,(\pi)(3.8 \times 10^{-3}\ \mathrm{m})^2}{(4)(2000\ \ \mathrm{N})}$$

$$= 0.255\ \mathrm{m} = 255\ \mathrm{mm}\ (10.0\ \mathrm{in.})$$

6.7 For a bronze alloy, the stress at which plastic deformation begins is 275 MPa (40,000 psi), and the modulus of elasticity is 115 GPa (16.7×10^6 psi).

(a) What is the maximum load that may be applied to a specimen with a cross-sectional area of 325 mm^2 (0.5 $in.^2$) without plastic deformation?

(b) If the original specimen length is 115 mm (4.5 in.), what is the maximum length to which it may be stretched without causing plastic deformation?

Solution

(a) This portion of the problem calls for a determination of the maximum load that can be applied without plastic deformation (F_y). Taking the yield strength to be 275 MPa, and employment of Equation 6.1 leads to

$$F_y = \sigma_y A_0 = (275 \times 10^6 \text{ N/m}^2)(325 \times 10^{-6} \text{ m}^2)$$

$$= 89{,}375 \text{ N} \quad (20{,}000 \text{ lb}_f)$$

(b) The maximum length to which the sample may be deformed without plastic deformation is determined from Equations 6.2 and 6.5 as

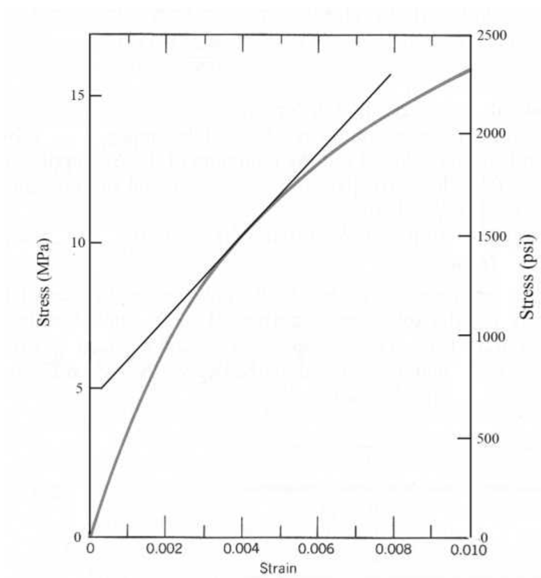
$$l_i = l_0\left(1 + \frac{\sigma}{E}\right)$$

$$= (115 \text{ mm})\left[1 + \frac{275 \text{ MPa}}{115 \times 10^3 \text{ MPa}}\right] = 115.28 \text{ mm} \quad (4.51 \text{ in.})$$

6.11 Figure 6.22 shows, for a gray cast iron, the tensile engineering stress–strain curve in the elastic region. Determine (a) the tangent modulus at 10.3 MPa (1500 psi), and (b) the secant modulus taken to 6.9 MPa (1000 psi).

Solution

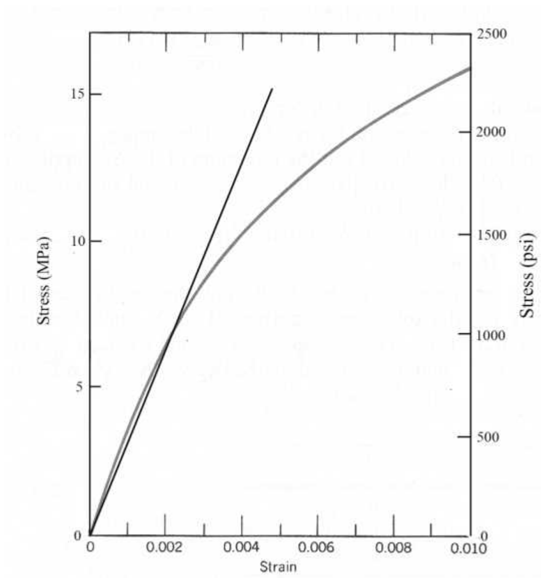
(a) This portion of the problem asks that the tangent modulus be determined for the gray cast iron, the stress-strain behavior of which is shown in Figure 6.22. In the figure below is shown a tangent draw on the curve at a stress of 10.3 MPa (1500 psi).



The slope of this line (i.e., $\Delta\sigma/\Delta\varepsilon$), the tangent modulus, is computed as follows:

$$\frac{\Delta\sigma}{\Delta\varepsilon} = \frac{15 \text{ MPa} - 5 \text{ MPa}}{0.0074 - 0.0003} = 1410 \text{ MPa} = 1.41 \text{ GPa} \ (2.04 \times 10^5 \text{ psi})$$

(b) The secant modulus taken from the origin is calculated by taking the slope of a secant drawn from the origin through the stress-strain curve at 6.9 MPa (1,000 psi). This secant is drawn on the curve shown below:



The slope of this line (i.e., $\Delta\sigma/\Delta\varepsilon$), the secant modulus, is computed as follows:

$$\frac{\Delta\sigma}{\Delta\varepsilon} = \frac{15\text{ MPa} - 0\text{ MPa}}{0.0047 - 0} = 3190\text{ MPa} = 3.19\text{ GPa} \quad (4.63 \times 10^{5}\text{ psi})$$

Elastic Properties of Materials

6.15 A cylindrical specimen of aluminum having a diameter of 19 mm (0.75 in.) and length of 200 mm (8.0 in.) is deformed elastically in tension with a force of 48,800 N (11,000 lb_f). Using the data contained in Table 6.1, determine the following:

(a) The amount by which this specimen will elongate in the direction of the applied stress.

(b) The change in diameter of the specimen. Will the diameter increase or decrease?

Solution

(a) We are asked, in this portion of the problem, to determine the elongation of a cylindrical specimen of aluminum. Combining Equations 6.1, 6.2, and 6.5, leads to

$$\sigma = E\varepsilon$$

$$\frac{F}{\pi\left(\dfrac{d_0^2}{4}\right)} = E\,\frac{\Delta l}{l_0}$$

Or, solving for Δl (and realizing that E = 69 GPa, Table 6.1), yields

$$\Delta l = \frac{4F\,l_0}{\pi\, d_0^2 E}$$

$$= \frac{(4)(48{,}800\ \mathrm{N})\left(200 \times 10^{-3}\,\mathrm{m}\right)}{(\pi)\left(19 \times 10^{-3}\,\mathrm{m}\right)^2\left(69 \times 10^{9}\,\mathrm{N/m^2}\right)} = 5 \times 10^{-4}\ \mathrm{m} = 0.50\ \mathrm{mm}\ \ (0.02\ \mathrm{in.})$$

(b) We are now called upon to determine the change in diameter, Δd. Using Equation 6.8

$$\nu = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\Delta d / d_0}{\Delta l / l_0}$$

From Table 6.1, for aluminum, $\nu = 0.33$. Now, solving the above expression for Δd yields

$$\Delta d = -\frac{\nu\,\Delta l\, d_0}{l_0} = -\frac{(0.33)(0.50\ \mathrm{mm})(19\ \mathrm{mm})}{200\ \mathrm{mm}}$$

$$= -1.6 \times 10^{-2}\ \mathrm{mm}\ \ (-6.2 \times 10^{-4}\ \mathrm{in.})$$

The diameter will decrease.

6.17 A cylindrical specimen of some alloy 8 mm (0.31 in.) in diameter is stressed elastically in tension. A force of 15,700 N (3530 lb_f) produces a reduction in specimen diameter of 5×10^{-3} mm (2×10^{-4} in.). Compute Poisson's ratio for this material if its modulus of elasticity is 140 GPa (20.3×10^6 psi).

Solution

This problem asks that we compute Poisson's ratio for the metal alloy. From Equations 6.5 and 6.1

$$\varepsilon_z = \frac{\sigma}{E} = \frac{F}{A_0 E} = \frac{F}{\pi \left(\frac{d_0}{2}\right)^2 E} = \frac{4F}{\pi d_0^2 E}$$

Since the transverse strain ε_x is just

$$\varepsilon_x = \frac{\Delta d}{d_0}$$

and Poisson's ratio is defined by Equation 6.8, then

$$\nu = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\Delta d / d_0}{\left(\frac{4F}{\pi d_0^2 E}\right)} = -\frac{d_0 \Delta d \,\pi E}{4F}$$

$$= -\frac{(8 \times 10^{-3}\ \text{m})(-5 \times 10^{-6}\ \text{m})\,(\pi)(140 \times 10^9\ \text{N/m}^2)}{(4)(15{,}700\ \ \text{N})} = 0.280$$

6.18 A cylindrical specimen of a hypothetical metal alloy is stressed in compression. If its original and final diameters are 20.000 and 20.025 mm, respectively, and its final length is 74.96 mm, compute its original length if the deformation is totally elastic. The elastic and shear moduli for this alloy are 105 GPa and 39.7 GPa, respectively.

Solution

This problem asks that we compute the original length of a cylindrical specimen that is stressed in compression. It is first convenient to compute the lateral strain ε_x as

$$\varepsilon_x = \frac{\Delta d}{d_0} = \frac{20.025\text{ mm} - 20.000\text{ mm}}{20.000\text{ mm}} = 1.25 \times 10^{-3}$$

In order to determine the longitudinal strain ε_z we need Poisson's ratio, which may be computed using Equation 6.9; solving for ν yields

$$\nu = \frac{E}{2G} - 1 = \frac{105 \times 10^3\text{ MPa}}{(2)\left(39.7 \times 10^3\text{ MPa}\right)} - 1 = 0.322$$

Now ε_z may be computed from Equation 6.8 as

$$\varepsilon_z = -\frac{\varepsilon_x}{\nu} = -\frac{1.25 \times 10^{-3}}{0.322} = -3.88 \times 10^{-3}$$

Now solving for l_0 using Equation 6.2

$$l_0 = \frac{l_i}{1 + \varepsilon_z}$$

$$= \frac{74.96\text{ mm}}{1 - 3.88 \times 10^{-3}} = 75.25\text{ mm}$$

6.19 Consider a cylindrical specimen of some hypothetical metal alloy that has a diameter of 8.0 mm (0.31 in.). A tensile force of 1000 N (225 lb_f) produces an elastic reduction in diameter of 2.8×10^{-4} mm (1.10×10^{-5} in.). Compute the modulus of elasticity for this alloy, given that Poisson's ratio is 0.30.

Solution

This problem asks that we calculate the modulus of elasticity of a metal that is stressed in tension. Combining Equations 6.5 and 6.1 leads to

$$E = \frac{\sigma}{\varepsilon_z} = \frac{F}{A_0 \varepsilon_z} = \frac{F}{\varepsilon_z \pi \left(\frac{d_0}{2}\right)^2} = \frac{4F}{\varepsilon_z \pi d_0^2}$$

From the definition of Poisson's ratio, (Equation 6.8) and realizing that for the transverse strain, $\varepsilon_x = \frac{\Delta d}{d_0}$

$$\varepsilon_z = -\frac{\varepsilon_x}{\nu} = -\frac{\Delta d}{d_0 \nu}$$

Therefore, substitution of this expression for ε_z into the above equation yields

$$E = \frac{4F}{\varepsilon_z \pi d_0^2} = \frac{4F\nu}{\pi d_0 \Delta d}$$

$$= \frac{(4)(1000\text{ N})(0.30)}{\pi \left(8 \times 10^{-3}\text{ m}\right)\left(2.8 \times 10^{-7}\text{ m}\right)} = 1.705 \times 10^{11}\text{ Pa} = 170.5\text{ GPa } (24.7 \times 10^{6}\text{ psi})$$

6.20 A brass alloy is known to have a yield strength of 275 MPa (40,000 psi), a tensile strength of 380 MPa (55,000 psi), and an elastic modulus of 103 GPa (15.0×10^6 psi). A cylindrical specimen of this alloy 12.7 mm (0.50 in.) in diameter and 250 mm (10.0 in.) long is stressed in tension and found to elongate 7.6 mm (0.30 in.). On the basis of the information given, is it possible to compute the magnitude of the load that is necessary to produce this change in length? If so, calculate the load. If not, explain why.

Solution

We are asked to ascertain whether or not it is possible to compute, for brass, the magnitude of the load necessary to produce an elongation of 7.6 mm (0.30 in.). It is first necessary to compute the strain at yielding from the yield strength and the elastic modulus, and then the strain experienced by the test specimen. Then, if

$$\varepsilon(\text{test}) < \varepsilon(\text{yield})$$

deformation is elastic, and the load may be computed using Equations 6.1 and 6.5. However, if

$$\varepsilon(\text{test}) > \varepsilon(\text{yield})$$

computation of the load is not possible inasmuch as deformation is plastic and we have neither a stress-strain plot nor a mathematical expression relating plastic stress and strain. We compute these two strain values as

$$\varepsilon(\text{test}) = \frac{\Delta l}{l_0} = \frac{7.6 \text{ mm}}{250 \text{ mm}} = 0.03$$

and

$$\varepsilon(\text{yield}) = \frac{\sigma_y}{E} = \frac{275 \text{ MPa}}{103 \times 10^3 \text{ MPa}} = 0.0027$$

Therefore, computation of the load is not possible since $\varepsilon(\text{test}) > \varepsilon(\text{yield})$.

6.21 A cylindrical metal specimen 12.7 mm (0.5 in.) in diameter and 250 mm (10 in.) long is to be VXEMHFWHG WR D WHQVLOH VWUHVV RI 0 3D SVL DW WKLV VWUHVV OHYHO WKH UHVXOWLQJ GHIRUPDWLRQ ZLOO EH WRWDOO elastic.

(a) If the elongation must be less than 0.080 mm (3.2×10^{-3} in.), which of the metals in Table 6.1 are suitable candidates? Why?

(b) If, in addition, the maximum permissible diameter decrease is 1.2×10^{-3} mm (4.7×10^{-5} in.) when the tensile stress of 28 MPa is applied, which of the metals that satisfy the criterion in part (a) are suitable candidates? Why?

Solution

(a) This part of the problem asks that we ascertain which of the metals in Table 6.1 experience an elongation of less than 0.080 mm when subjected to a tensile stress of 28 MPa. The maximum strain that may be sustained, (using Equation 6.2) is just

$$\varepsilon = \frac{\Delta l}{l_0} = \frac{0.080 \text{ mm}}{250 \text{ mm}} = 3.2 \times 10^{-4}$$

Since the stress level is given (50 MPa), using Equation 6.5 it is possible to compute the minimum modulus of elasticity which is required to yield this minimum strain. Hence

$$E = \frac{\sigma}{\varepsilon} = \frac{28 \text{ MPa}}{3.2 \times 10^{-4}} = 87.5 \text{ GPa}$$

Which means that those metals with moduli of elasticity greater than this value are acceptable candidates—namely, brass, Cu, Ni, steel, Ti and W.

(b) This portion of the problem further stipulates that the maximum permissible diameter decrease is 1.2×10^{-3} mm when the tensile stress of 28 MPa is applied. This translates into a maximum lateral strain $\varepsilon_x(\text{max})$ as

$$\varepsilon_x(\text{max}) = \frac{\Delta d}{d_0} = \frac{-1.2 \times 10^{-3} \text{ mm}}{12.7 \text{ mm}} = -9.45 \times 10^{-5}$$

But, since the specimen contracts in this lateral direction, and we are concerned that this strain be less than 9.45×10^{-5}, then the criterion for this part of the problem may be stipulated as $-\frac{\Delta d}{d_0} < 9.45 \times 10^{-5}$.

Now, Poisson's ratio is defined by Equation 6.8 as

$$\nu = -\frac{\varepsilon_x}{\varepsilon_z}$$

For each of the metal alloys let us consider a possible lateral strain, $\varepsilon_x = \frac{\Delta d}{d_0}$. Furthermore, since the deformation is elastic, then, from Equation 6.5, the longitudinal strain, ε_z is equal to

$$\varepsilon_z = \frac{\sigma}{E}$$

Substituting these expressions for ε_x and ε_z into the definition of Poisson's ratio we have

$$\nu = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\frac{\Delta d}{d_0}}{\frac{\sigma}{E}}$$

which leads to the following:

$$-\frac{\Delta d}{d_0} = \frac{\nu \sigma}{E}$$

Using values for ν and E found in Table 6.1 for the six metal alloys that satisfy the criterion for part (a), and for $\sigma = 28$ MPa, we are able to compute a $-\frac{\Delta d}{d_0}$ for each alloy as follows:

$$-\frac{\Delta d}{d_0}(\text{brass}) = \frac{(0.34)(28 \times 10^6\ \text{N/m}^2)}{97 \times 10^9\ \text{N/m}^2} = 9.81 \times 10^{-5}$$

$$-\frac{\Delta d}{d_0}(\text{copper}) = \frac{(0.34)(28 \times 10^6\ \text{N/m}^2)}{110 \times 10^9\ \text{N/m}^2} = 8.65 \times 10^{-5}$$

$$-\frac{\Delta d}{d_0}(\text{titanium}) = \frac{(0.34)(28 \times 10^6\ \text{N/m}^2)}{107 \times 10^9\ \text{N/m}^2} = 8.90 \times 10^{-5}$$

$$-\frac{\Delta d}{d_0}(\text{nickel}) = \frac{(0.31)(28 \times 10^6\ \text{N/m}^2)}{207 \times 10^9\ \text{N/m}^2} = 4.19 \times 10^{-5}$$

$$-\frac{\Delta d}{d_0}(\text{steel}) = \frac{(0.30)(28 \times 10^6\ \text{N/m}^2)}{207 \times 10^9\ \text{N/m}^2} = 4.06 \times 10^{-5}$$

$$-\frac{\Delta d}{d_0}(\text{tungsten}) = \frac{(0.28)(28 \times 10^6\ \text{N/m}^2)}{407 \times 10^9\ \text{N/m}^2} = 1.93 \times 10^{-5}$$

Thus, of the above six alloys, only brass will have a negative transverse strain that is greater than 9.45×10^{-5}. This means that the following alloys satisfy the criteria for both parts (a) and (b) of the problem: copper, titanium, nickel, steel, and tungsten.

6.22 Consider the brass alloy for which the stress-strain behavior is shown in Figure 6.12. A cylindrical specimen of this material 6 mm (0.24 in.) in diameter and 50 mm (2 in.) long is pulled in tension with a force of 5000 N (1125 lb_f). If it is known that this alloy has a Poisson's ratio of 0.30, compute: (a) the specimen elongation, and (b) the reduction in specimen diameter.

Solution

(a) This portion of the problem asks that we compute the elongation of the brass specimen. The first calculation necessary is that of the applied stress using Equation 6.1, as

$$\sigma = \frac{F}{A_0} = \frac{F}{\pi\left(\frac{d_0}{2}\right)^2} = \frac{5000 \text{ N}}{\pi\left(\frac{6 \times 10^{-3}\text{ m}}{2}\right)^2} = 177 \times 10^6 \text{ N/m}^2 = 177 \text{ MPa} \quad (25,000 \text{ psi})$$

From the stress-strain plot in Figure 6.12, this stress corresponds to a strain of about 2.0×10^{-3}. From the definition of strain, Equation 6.2

$$\Delta l = \varepsilon l_0 = (2.0 \times 10^{-3})(50 \text{ mm}) = 0.10 \text{ mm} \quad (4 \times 10^{-3} \text{ in.})$$

(b) In order to determine the reduction in diameter Δd, it is necessary to use Equation 6.8 and the definition of lateral strain (i.e., $\varepsilon_x = \Delta d/d_0$) as follows

$$\Delta d = d_0\varepsilon_x = -d_0 \nu \varepsilon_z = -(6 \text{ mm})(0.30)\ (2.0 \times 10^{-3})$$

$$= -3.6 \times 10^{-3} \text{ mm} \ (-1.4 \times 10^{-4} \text{ in.})$$

6.23 A cylindrical rod 100 mm long and having a diameter of 10.0 mm is to be deformed using a tensile load of 27,500 N. It must not experience either plastic deformation or a diameter reduction of more than 7.5×10^{-3} mm. Of the materials listed as follows, which are possible candidates? Justify your choice(s).

Material	**Modulus of Elasticity (GPa)**	**Yield Strength (MPa)**	**Poisson's Ratio**
Aluminum alloy	70	200	0.33
Brass alloy	101	300	0.34
Steel alloy	207	400	0.30
Titanium alloy	107	650	0.34

Solution

This problem asks that we assess the four alloys relative to the two criteria presented. The first criterion is that the material not experience plastic deformation when the tensile load of 27,500 N is applied; this means that the stress corresponding to this load not exceed the yield strength of the material. Upon computing the stress

$$\sigma = \frac{F}{A_0} = \frac{F}{\pi\left(\frac{d_0}{2}\right)^2} = \frac{27,500 \text{ N}}{\pi\left(\frac{10 \times 10^{-3}\text{ m}}{2}\right)^2} = 350 \times 10^6 \text{ N/m}^2 = 350 \text{ MPa}$$

Of the alloys listed, the Ti and steel alloys have yield strengths greater than 350 MPa.

Relative to the second criterion (i.e., that Δd be less than 7.5×10^{-3} mm), it is necessary to calculate the change in diameter Δd for these three alloys. From Equation 6.8

$$\nu = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\frac{\Delta d}{d_0}}{\frac{\sigma}{E}} = -\frac{E\,\Delta d}{\sigma\, d_0}$$

Now, solving for Δd from this expression,

$$\Delta d = -\frac{\nu\,\sigma\, d_0}{E}$$

For the steel alloy

$$\Delta d = -\frac{(0.30)(350\ \mathrm{MPa})(10\ \mathrm{mm})}{207 \times 10^3\ \mathrm{MPa}} = -5.1 \times 10^{-3}\ \mathrm{mm}$$

Therefore, the steel is a candidate.

For the Ti alloy

$$\Delta d = -\frac{(0.34)(350\ \mathrm{MPa})(10\ \mathrm{mm})}{107 \times 10^3\ \mathrm{MPa}} = -11.1 \times 10^{-3}\ \mathrm{mm}$$

Hence, the titanium alloy is not a candidate.

6.27 A load of 85,000 N (19,100 lb_f) is applied to a cylindrical specimen of a steel alloy (displaying the stress–strain behavior shown in Figure 6.21) that has a cross-sectional diameter of 15 mm (0.59 in.).

(a) Will the specimen experience elastic and/or plastic deformation? Why?

(b) If the original specimen length is 250 mm (10 in.), how much will it increase in length when this load is applied?

Solution

This problem asks us to determine the deformation characteristics of a steel specimen, the stress-strain behavior for which is shown in Figure 6.21.

(a) In order to ascertain whether the deformation is elastic or plastic, we must first compute the stress, then locate it on the stress-strain curve, and, finally, note whether this point is on the elastic or plastic region. Thus, from Equation 6.1

$$\sigma = \frac{F}{A_0} = \frac{85{,}000 \text{ N}}{\pi \left(\dfrac{15 \times 10^{-3}\text{ m}}{2}\right)^2} = 481 \times 10^6 \text{ N/m}^2 = 481 \text{ MPa} \quad (69{,}900 \text{ psi})$$

The 481 MPa point is beyond the linear portion of the curve, and, therefore, the deformation will be both elastic and plastic.

(b) This portion of the problem asks us to compute the increase in specimen length. From the stress-strain curve, the strain at 481 MPa is approximately 0.0135. Thus, from Equation 6.2

$$\Delta l = \varepsilon l_0 = (0.0135)(250 \text{ mm}) = 3.4 \text{ mm} \quad (0.135 \text{ in.})$$

6.35 A cylindrical metal specimen having an original diameter of 12.8 mm (0.505 in.) and gauge length of 50.80 mm (2.000 in.) is pulled in tension until fracture occurs. The diameter at the point of fracture is 6.60 mm (0.260 in.), and the fractured gauge length is 72.14 mm (2.840 in.). Calculate the ductility in terms of percent reduction in area and percent elongation.

Solution

This problem calls for the computation of ductility in both percent reduction in area and percent elongation. Percent reduction in area is computed using Equation 6.12 as

$$\%RA = \frac{\pi\left(\frac{d_0}{2}\right)^2 - \pi\left(\frac{d_f}{2}\right)^2}{\pi\left(\frac{d_0}{2}\right)^2} \times 100$$

in which d_0 and d_f are, respectively, the original and fracture cross-sectional areas. Thus,

$$\%RA = \frac{\pi\left(\frac{12.8\text{ mm}}{2}\right)^2 - \pi\left(\frac{6.60\text{ mm}}{2}\right)^2}{\pi\left(\frac{12.8\text{ mm}}{2}\right)^2} \times 100 = 73.4\%$$

While, for percent elongation, we use Equation 6.11 as

$$\%EL = \left(\frac{l_f - l_0}{l_0}\right) \times 100$$

$$= \frac{72.14\text{ mm} - 50.80\text{ mm}}{50.80\text{ mm}} \times 100 = 42\%$$

6.36 Calculate the moduli of resilience for the materials having the stress–strain behaviors shown in Figures 6.12 and 6.21.

Solution

This problem asks us to calculate the moduli of resilience for the materials having the stress-strain behaviors shown in Figures 6.12 and 6.21. According to Equation 6.14, the modulus of resilience U_r is a function of the yield strength and the modulus of elasticity as

$$U_r = \frac{\sigma_y^2}{2E}$$

The values for σ_y and E for the brass in Figure 6.12 are determined in Example Problem 6.3 as 250 MPa (36,000 psi) and 93.8 GPa (13.6×10^6 psi), respectively. Thus

$$U_r = \frac{(250\text{ MPa})^2}{(2)(93.8 \times 10^3\text{ MPa})} = 3.32 \times 10^5\text{ J/m}^3 \quad (48.2\text{ in.-lb}_f/\text{in.}^3)$$

Values of the corresponding parameters for the steel alloy (Figure 6.21) are determined in Problem 6.25 as 400 MPa (58,000 psi) and 200 GPa (29×10^6 psi), respectively, and therefore

$$U_r = \frac{(400\text{ MPa})^2}{(2)(200 \times 10^3\text{ MPa})} = 4.0 \times 10^5\text{ J/m}^3 \quad (58\text{ in.-lb}_f/\text{in.}^3)$$

EXAMPLE PROBLEM 6.5

Calculation of Strain-Hardening Exponent

Compute the strain-hardening exponent n in Equation 6.19 for an alloy in which a true stress of 415 MPa (60,000 psi) produces a true strain of 0.10; assume a value of 1035 MPa (150,000 psi) for K.

Solution

This requires some algebraic manipulation of Equation 6.19 so that n becomes the dependent parameter. This is accomplished by taking logarithms and rearranging. Solving for n yields

$$n = \frac{\log \sigma_T - \log K}{\log \epsilon_T}$$

$$= \frac{\log(415 \text{ MPa}) - \log(1035 \text{ MPa})}{\log(0.1)} = 0.40$$

6.8 ELASTIC RECOVERY AFTER PLASTIC DEFORMATION

Upon release of the load during the course of a stress–strain test, some fraction of the total deformation is recovered as elastic strain. This behavior is demonstrated in Figure 6.17, a schematic engineering stress–strain plot. During the unloading cycle, the curve traces a near straight-line path from the point of unloading (point D), and its slope is virtually identical to the modulus of elasticity, or parallel to the initial elastic portion of the curve. The magnitude of this elastic strain, which is regained during unloading, corresponds to the strain recovery, as shown in Figure 6.17. If the load is reapplied, the curve will traverse essentially the same linear portion in the direction opposite to unloading; yielding will again occur at the unloading stress level where the unloading began. There will also be an elastic strain recovery associated with fracture.

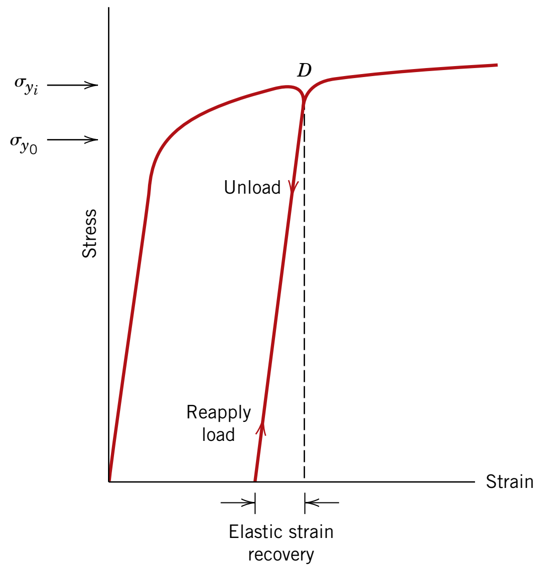


Figure 6.17 Schematic tensile stress–strain diagram showing the phenomena of elastic strain recovery and strain hardening. The initial yield strength is designated as σ_{y_0}; σ_{y_i} is the yield strength after releasing the load at point D and then upon reloading.

6.43 For some metal alloy, a true stress of 415 MPa (60,175 psi) produces a plastic true strain of 0.475. How much will a specimen of this material elongate when a true stress of 325 MPa (46,125 psi) is applied if the original length is 300 mm (11.8 in.)? Assume a value of 0.25 for the strain-hardening exponent n.

Solution

Solution of this problem requires that we utilize Equation 6.19. It is first necessary to solve for K from the given true stress and strain. Rearrangement of this equation yields

$$K = \frac{\sigma_T}{(\varepsilon_T)^n} = \frac{415 \text{ MPa}}{(0.475)^{0.25}} = 500 \text{ MPa} \quad (72,500 \text{ psi})$$

Next we must solve for the true strain produced when a true stress of 325 MPa is applied, also using Equation 6.19. Thus

$$\varepsilon_T = \left(\frac{\sigma_T}{K}\right)^{1/n} = \left(\frac{325 \text{ MPa}}{500 \text{ MPa}}\right)^{1/0.25} = 0.179 = \ln\left(\frac{l_i}{l_0}\right)$$

Now, solving for l_i gives

$$l_i = l_0 e^{0.179} = (300 \text{ mm})\, e^{0.179} = 358.8 \text{ mm} \quad (14.11 \text{ in.})$$

And finally, the elongation Δl is just

$$\Delta l = l_i - l_0 = 358.8 \text{ mm} - 300 \text{ mm} = 58.8 \text{ mm} \quad (2.31 \text{ in.})$$

6.45 For a brass alloy, the following engineering stresses produce the corresponding plastic engineering strains, prior to necking:

Engineering Stress (MPa)	Engineering Strain
235	0.194
250	0.296

On the basis of this information, compute the engineering stress necessary to produce an engineering strain of 0.25.

Solution

For this problem we first need to convert engineering stresses and strains to true stresses and strains so that the constants K and n in Equation 6.19 may be determined. Since $\sigma_T = \sigma(1 + \varepsilon)$ then

$$\sigma_{T1} = (235 \text{ MPa})(1 + 0.194) = 280 \text{ MPa}$$

$$\sigma_{T2} = (250 \text{ MPa})(1 + 0.296) = 324 \text{ MPa}$$

Similarly for strains, since $\varepsilon_T = \ln(1 + \varepsilon)$ then

$$\varepsilon_{T1} = \ln\,(1 + 0.194) = 0.177$$

$$\varepsilon_{T2} = \ln\,(1 + 0.296) = 0.259$$

Taking logarithms of Equation 6.19, we get

$$\log \sigma_T = \log K + n \log \varepsilon_T$$

which allows us to set up two simultaneous equations for the above pairs of true stresses and true strains, with K and n as unknowns. Thus

$$\log\,(280) = \log K + n \log\,(0.177)$$

$$\log\,(324) = \log K + n \log\,(0.259)$$

Solving for these two expressions yields K = 543 MPa and n = 0.383.

Now, converting $\varepsilon = 0.25$ to true strain

$$\varepsilon_T = \ln(1 + 0.25) = 0.223$$

The corresponding σ_T to give this value of ε_T (using Equation 6.19) is just

$$\sigma_T = K\varepsilon_T^n = (543 \text{ MPa})(0.223)^{0.383} = 306 \text{ MPa}$$

Now converting this value of σ_T to an engineering stress using Equation 6.18a gives

$$\sigma = \frac{\sigma_T}{1 + \varepsilon} = \frac{306 \text{ MPa}}{1 + 0.25} = 245 \text{ MPa}$$

6.50 A steel alloy specimen having a rectangular cross section of dimensions 12.7 mm × 6.4 mm (0.5 in. × 0.25 in.) has the stress–strain behavior shown in Figure 6.21. If this specimen is subjected to a tensile force of 38,000 N (8540 lb_f) then

(a) Determine the elastic and plastic strain values.

(b) If its original length is 460 mm (18.0 in.), what will be its final length after the load in part (a) is applied and then released?

<u>Solution</u>

(a) We are asked to determine both the elastic and plastic strain values when a tensile force of 38,000 N (8540 lb_f) is applied to the steel specimen and then released. First it becomes necessary to determine the applied WWHVVXVLQJ (TXDWLRQ ¶ WKXV

$$\sigma = \frac{F}{A_0} = \frac{F}{b_0 d_0}$$

where b_0 and d_0 are cross-sectional width and depth (12.7 mm and 6.4 mm, respectively). Thus

$$\sigma = \frac{38{,}000 \text{ N}}{\left(12.7 \times 10^{-3} \text{ m}\right)\left(6.4 \times 10^{-3} \text{ m}\right)} = 468 \times 10^{6} \text{ N/m}^2 = 468 \text{ MPa} \ (68,300 \text{ psi})$$

From Figure 6.21, this point is in the plastic region so the specimen will be both elastic and plastic strains. The total strain at this point, ε_t, is about 0.010. We are able to estimate the amount of permanent strain recovery ε_e from Hooke's law, Equation 6.5 as

$$\varepsilon_e = \frac{\sigma}{E}$$

And, since E = 207 GPa for steel (Table 6.1)

$$\varepsilon_e = \frac{468 \text{ MPa}}{207 \times 10^3 \text{ MPa}} = 0.00226$$

The value of the plastic strain, ε_p is just the difference between the total and elastic strains; that is

$$\varepsilon_p = \varepsilon_t - \varepsilon_e = 0.010 - 0.00226 = 0.00774$$

(b) If the initial length is 460 mm (18.0 in.) then the final specimen length l_i may be determined from a rearranged form of Equation 6.2 using the plastic strain value as

$$l_i = l_0(1 + \varepsilon_p) = (460 \text{ mm})(1 + 0.00774) = 463.6 \text{ mm } (18.14 \text{ in.})$$

Hardness

6.51 (a) A 10-mm-diameter Brinell hardness indenter produced an indentation 1.62 mm in diameter in a steel alloy when a load of 500 kg was used. Compute the HB of this material.

(b) What will be the diameter of an indentation to yield a hardness of 450 HB when a 500 kg load is used?

Solution

(a) We are asked to compute the Brinell hardness for the given indentation. It is necessary to use the equation in Table 6.5 for HB, where P = 500 kg, d = 1.62 mm, and D = 10 mm. Thus, the Brinell hardness is computed as

$$\mathrm{HB} = \frac{2P}{\pi D\left[D - \sqrt{D^2 - d^2}\right]}$$

$$= \frac{(2)(500\ \mathrm{kg})}{(\pi)(10\ \mathrm{mm})\left[10\ \mathrm{mm} - \sqrt{(10\ \mathrm{mm})^2 - (1.62\ \mathrm{mm})^2}\right]} = 241$$

(b) This part of the problem calls for us to determine the indentation diameter d which will yield a 450 HB when P = 500 kg. Solving for d from the equation in Table 6.5 gives

$$d = \sqrt{D^2 - \left[D - \frac{2P}{(\mathrm{HB})\pi D}\right]^2}$$

$$= \sqrt{(10\,\mathrm{mm})^2 - \left[10\ \mathrm{mm} - \frac{(2)(500\ \mathrm{kg})}{(450)(\pi)(10\ \mathrm{mm})}\right]^2} = 1.19\ \mathrm{mm}$$

6.55 Below are tabulated a number of Rockwell B hardness values that were measured on a single steel specimen. Compute average and standard deviation hardness values.

83.3	80.7	86.4
88.3	84.7	85.2
82.8	87.8	86.9
86.2	83.5	84.4
87.2	85.5	86.3

Solution

The average of the given hardness values is calculated using Equation 6.21 as

$$\overline{\mathrm{HRB}} = \frac{\sum_{i=1}^{15} \mathrm{HRB}_i}{15}$$

$$= \frac{83.3 + 88.3 + 82.8 \ . \ . \ . \ . + 86.3}{15} = 85.3$$

And we compute the standard deviation using Equation 6.22 as follows:

$$s = \sqrt{\frac{\sum_{i=1}^{15} \left(\mathrm{HRB}_i - \overline{\mathrm{HRB}}\right)^2}{15 - 1}}$$

$$= \left[\frac{(83.3 - 85.3)^2 + (88.3 - 85.3)^2 + . \ . \ . \ . + (86.3 - 85.3)^2}{14}\right]^{1/2}$$

$$= \sqrt{\frac{60.31}{14}} = 2.08$$