This support is written to help students follow up during the class sessions and for their individual studies.
It is limited to a general introduction and simple examples, therefore, it will not replace textbook, discussed material and solved problems during class sessions.

Design of machinery
Chapter 6 Velocity analysis

Dr. Jaafar Hallal

Chapter 6 Velocity analysis

- 6.0 Introduction

The objective is to determine velocities of all links and points of interest in a mechanism.

How?: Derive the analytical solution.

Chapter 6 Velocity analysis

- 6.1 Definition of velocity

The velocity is defined as the time rate change of the position.

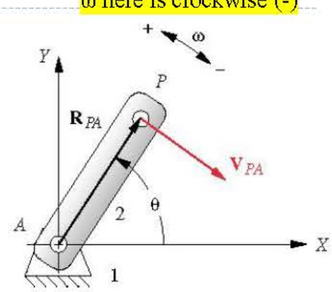
The velocity can be linear or angular.

Linear velocity $: V = \frac{d\vec{R}}{dt}$ m/s

Angular velocity : $\omega = \frac{d\theta}{dt}$ rad/s

Chapter 6 Velocity analysis

ω here is clockwise (-)



- 6.1 Definition of velocity

Link in pure rotation

Position vector: $\mathbf{R}_{PA} = pe^{j\theta}$

Time derivative :

$$\mathbf{V}_{PA} = \frac{d\mathbf{R}_{PA}}{dt} = p\,je^{j\theta}\frac{d\theta}{dt} = p\omega\,je^{j\theta}$$

Multiplying by j rotates the vector by 90°.

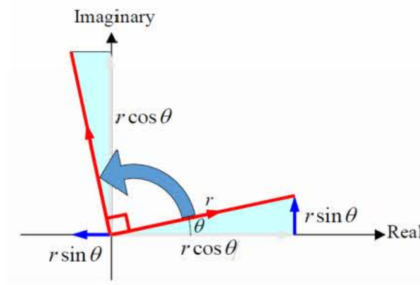
Note that AP=p is constant here

$$\mathbf{V}_{PA} = p\,\omega\,j(\cos\theta + j\sin\theta) = p\omega(-\sin\theta + j\cos\theta)$$

Chapter 6 Velocity analysis

- 6.1 Definition of velocity

Polar form of vector



Vector $\vec{r}$ can be written as:

$\vec{r} = re^{j\theta} = r(cos\theta + jsin\theta)$

Multiplying by j gives:

$jre^{j\theta} = r(-sin\theta + jcos\theta)$

➔ Multiplying by *j* rotates a vector by 90˚

Chapter 6 Velocity analysis

- 6.1 Definition of velocity

Case 1: *Two points in the same body* => **velocity difference**

Case 2: *Two points in different bodies* => **relative velocity**

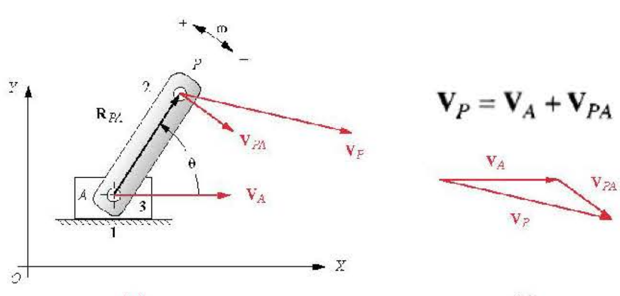


FIGURE 6-2

Velocity difference

Chapter 6 Velocity analysis

- 6.1 Definition of velocity

CASE 1: *Two points in the same body* => **velocity difference**

CASE 2: *Two points in different bodies* => **relative velocity**

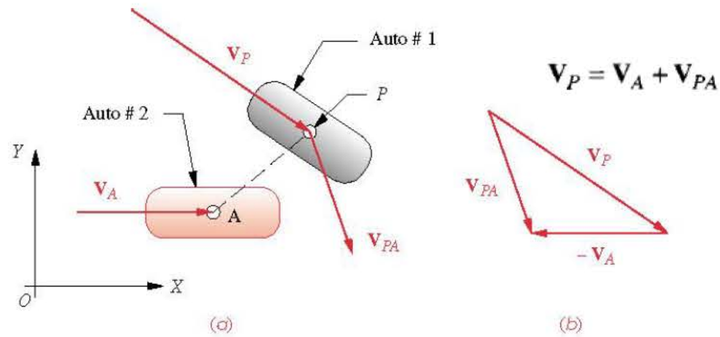
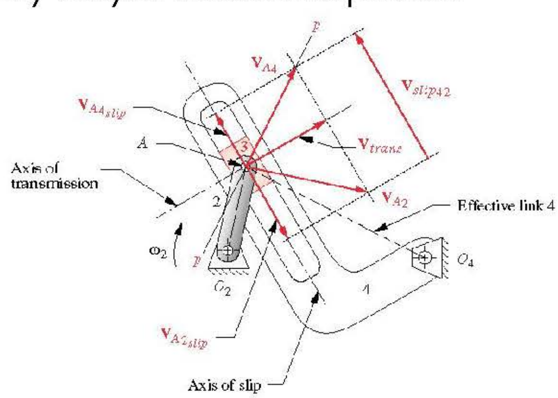


FIGURE 6-3

Relative velocity

Chapter 6 Velocity analysis

- 6.6 Velocity of slip
- Sliding joint between two links neither one is the ground, the velocity analysis is more complicated.



Chapter 6 Velocity analysis

- 6.6 Velocity of slip

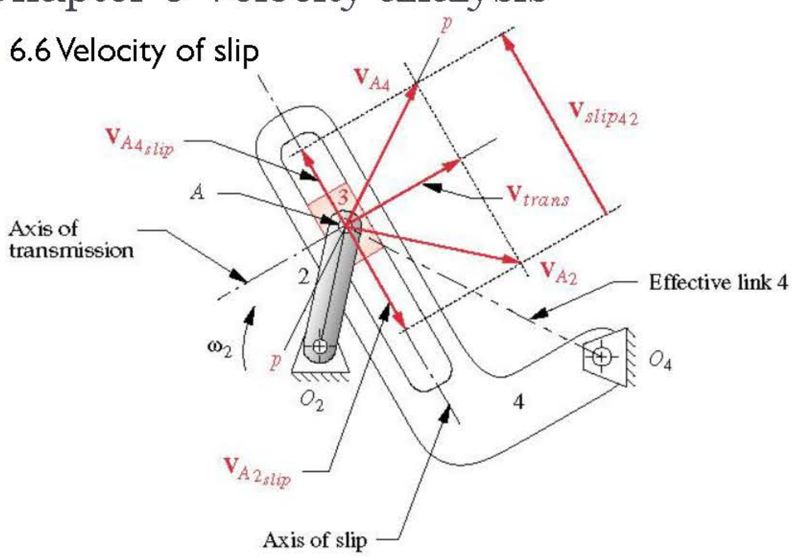
Axis of slip is tangent to the slider motion and is along which all sliding occurs.

Axis of transmission is perpendicular to the axis of slip and pass through the slider joint.

The axis of transmission is the only line along which we can transmit motion or force across the slider joint, except for friction.

Chapter 6 Velocity analysis

- 6.6 Velocity of slip



Chapter 6 Velocity analysis

- 6.6 Velocity of slip

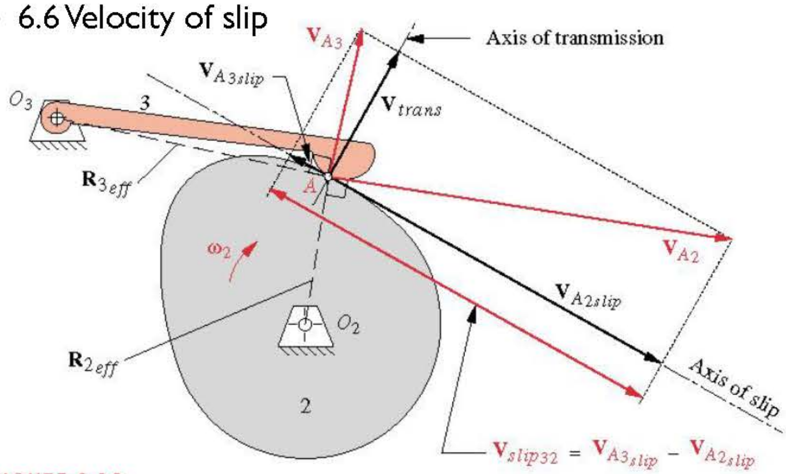


FIGURE 6-19

Graphical velocity analysis of a cam and follower

Chapter 6 Velocity analysis

- 6.7 Analytical solution for velocity analysis

The Fourbar pin-jointed linkage

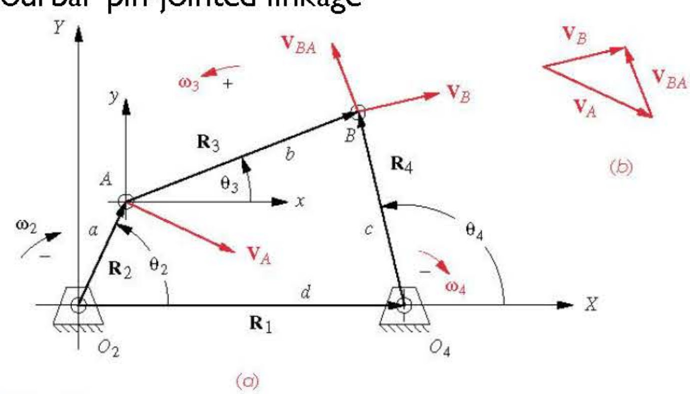


FIGURE 6-20

Position vector loop for a fourbar linkage showing velocity vectors for a negative (cw) ω_2

Chapter 6 Velocity analysis

- 6.7 Analytical solution for velocity analysis

The Fourbar pin-jointed linkage

The vector loop equation

$$\mathbf{R}_2 + \mathbf{R}_3 - \mathbf{R}_4 - \mathbf{R}_1 = 0$$

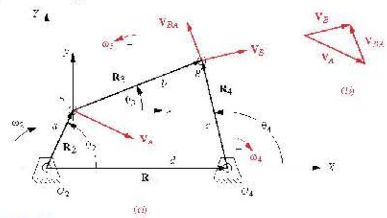


FIGURE 6-20

Position vector loop for a fourbar linkage showing velocity vectors for a negative (cw) ω_2

Vector equation:

$$a e^{j\theta_2} + b e^{j\theta_3} - c e^{j\theta_4} - d e^{j\theta_1} = 0$$

Differentiate:

$$ja e^{j\theta_2} \frac{d\theta_2}{dt} + jb e^{j\theta_3} \frac{d\theta_3}{dt} - jc e^{j\theta_4} \frac{d\theta_4}{dt} = 0$$

But $\omega = \frac{d\theta}{dt}$; then:

$$ja\omega_2 e^{j\theta_2} + jb\omega_3 e^{j\theta_3} - jc\omega_4 e^{j\theta_4} = 0$$

Chapter 6 Velocity analysis

- 6.7 Analytical solution for velocity analysis

The Fourbar pin-jointed linkage

$$ja\omega_2 e^{j\theta_2} + jb\omega_3 e^{j\theta_3} - jc\omega_4 e^{j\theta_4} = 0$$

Substitute Euler equation:

$$ja\omega_2(cos\theta_2 + jsin\theta_2) + jb\omega_3(cos\theta_3 + jsin\theta_3) - jc\omega_4(cos\theta_4 + jsin\theta_4) = 0$$

Multiply by j, and substitute $j^2 = -1$

➔

$$a\omega_2(jcos\theta_2 - sin\theta_2) + b\omega_3(jcos\theta_3 - sin\theta_3) - c\omega_4(jcos\theta_4 - sin\theta_4) = 0$$

Chapter 6 Velocity analysis

- 6.7 Analytical solution for velocity analysis

The Fourbar pin-jointed linkage

$$\omega_2(j\cos\theta_2 - \sin\theta_2) + b\omega_3(j\cos\theta_3 - \sin\theta_3) - c\omega_4(j\cos\theta_4 - \sin\theta_4) = 0$$

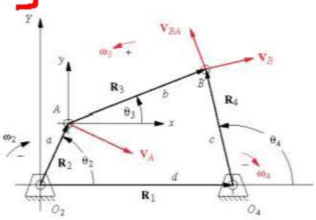
real part (x component):

$$-a\omega_2 \sin\theta_2 - b\omega_3 \sin\theta_3 + c\omega_4 \sin\theta_4 = 0$$

imaginary part (y component):

$$a\omega_2 \cos\theta_2 + b\omega_3 \cos\theta_3 - c\omega_4 \cos\theta_4 = 0$$

$$\boxed{\omega_3 = \frac{a\omega_2}{b}\frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)} \qquad \omega_4 = \frac{a\omega_2}{c}\frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)}}$$



Chapter 6 Velocity analysis

- 6.7 Analytical solution for velocity analysis

The Fourbar pin-jointed linkage

Linear velocities:

$$ja\omega_2 e^{j\theta_2} + jb\omega_3 e^{j\theta_3} - jc\omega_4 e^{j\theta_4} = 0$$

$\overrightarrow{V_A}$ $\qquad$ $\overrightarrow{V_{B/A}}$ $\qquad$ $\overrightarrow{V_B}$

$$\boxed{\begin{aligned} \mathbf{V}_A &= ja\omega_2(\cos\theta_2 + j\sin\theta_2) = a\omega_2(-\sin\theta_2 + j\cos\theta_2) \\ \mathbf{V}_{BA} &= jb\omega_3(\cos\theta_3 + j\sin\theta_3) = b\omega_3(-\sin\theta_3 + j\cos\theta_3) \\ \mathbf{V}_B &= jc\omega_4(\cos\theta_4 + j\sin\theta_4) = c\omega_4(-\sin\theta_4 + j\cos\theta_4) \end{aligned}}$$

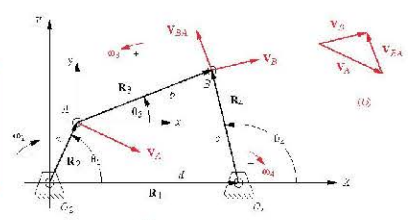


FIGURE 6-20

Position vector loop for a fourbar linkage showing velocity vectors for a negative (cw) ω_2

Chapter 6 Velocity analysis

- 6.7 Analytical solution for velocity analysis

Example: Given a fourbar linkage with the link lengths L_1=d= 100 mm, L_2 = a= 40 mm, L_3=b= 120 mm, L_4=c= 80mm. For θ_2= 400and ω_2=25 rad/s find the values of ω_3 and ω_4, V_A, V_{BA} and V_B for the open circuit linkage.

Use the angles found for the same linkage and position in Example 4-1.

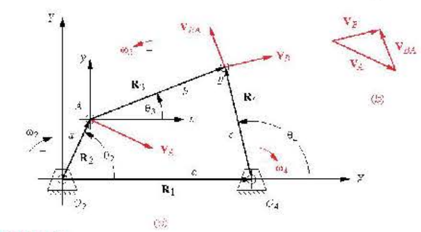


FIGURE 6-20

Position vector loop for a fourbar linkage showing velocity vectors for a negative (cw) ω_2

Chapter 6 Velocity analysis

- 6.7 Analytical solution for velocity analysis

Solution:

$$\omega_3 = \frac{a\omega_2}{b}\frac{sin(\theta_4 - \theta_2)}{sin(\theta_3 - \theta_4)} = \frac{40(25)}{180}\frac{sin(57.325^o - 40^o)}{sin(20.298^o - 57.325^o)} = -4.121 rad/sec$$

$$\omega_4 = \frac{a\omega_2}{c}\frac{sin(\theta_2 - \theta_3)}{sin(\theta_4 - \theta_3)} = \frac{40(25)}{180}\frac{sin(40^o - 20.298^o)}{sin(57.325^o - 20.298^o)} = 6.998 rad/sec$$

$$V_A = a\omega_2(j\cos\theta_2 - sin\theta_2) = 40(25)(-sin40^o + jcos40^o) = -642.79 + j766.04$$

$$V_{BA} = b\omega_2(j\cos\theta_3 - sin\theta_3) = 120(-4.121)(-sin20.298^o + jcos20.298^o) = 171.55 + j463.08$$

$$V_B = c\omega_4(j\cos\theta_4 - sin\theta_4) = 80(6.998)(-sin57.325^o + jcos57.325^o) = -471.242 + j302.243$$

$$Magnitude = \sqrt{real^2 + imag^2}$$

$$Angle = arctan\left(\frac{imag}{real}\right)$$

Angle should be verified in which quadrant

Chapter 6 Velocity analysis

- Problem 6.30

Crank angle: $\theta_2 := 57 \cdot deg$ Global XY system

Input crank angular velocity $\omega_2 := 20 \cdot rad \cdot sec^{-1}$

Coordinate rotation angle $\alpha := -36 \cdot deg$ Global XY system to local xy system

$K_1 := \frac{d}{a}$ $\quad K_2 := \frac{d}{c}$

$K_1 = 4.0500$ $\quad K_2 = 1.3279$

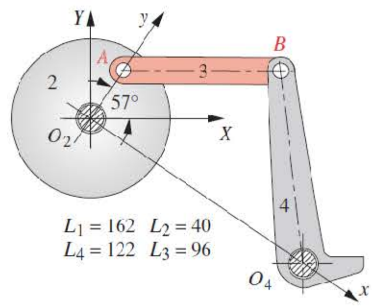
$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)}$ $\quad K_3 = 3.4336$ $\quad \theta_2 := \theta_{21} - \alpha$

$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$

$B := -2 \cdot \sin(\theta_2)$

$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$

$A = -0.5992$ $\quad B = -1.9973$ $\quad C = 7.6054$



Chapter 6 Velocity analysis

- Problem 6.30

$\theta_4 = 132.386\, deg$

$\theta_3 = 31.504\, deg$

$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)}$ $\quad \omega_3 = -5.385 \frac{rad}{sec}$

$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)}$ $\quad \omega_4 = 5.868 \frac{rad}{sec}$

$V_A := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$

$V_A = -798.904 - 41.869j \frac{mm}{sec}$ $\quad |V_A| = 800.000 \frac{mm}{sec}$ $\quad arg(V_A) = -177.000\, deg$

$V_B := c \cdot \omega_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4))$

$V_B = -528.774 - 482.608j \frac{mm}{sec}$ $\quad |V_B| = 715.900 \frac{mm}{sec}$ $\quad arg(V_B) = -137.614\, deg$

Chapter 6 Velocity analysis

- 6.7 Analytical solution for velocity analysis

The Fourbar slider crank

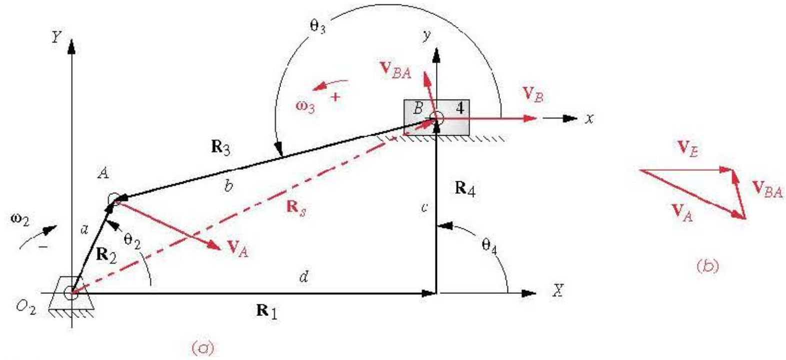
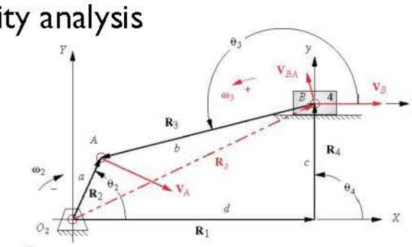


FIGURE 6-21
Position vector loop for a fourbar slider-crank linkage showing velocity vectors for a negative (cw) ω_2

Chapter 6 Velocity analysis

- 6.7 Analytical solution for velocity analysis

The Fourbar slider crank



The vector loop:

$$\mathbf{R}_2 - \mathbf{R}_3 - \mathbf{R}_4 - \mathbf{R}_1 = 0$$

The vector equation :

$$ae^{j\theta_2} - be^{j\theta_3} - ce^{j\theta_4} - de^{j\theta_1} = 0$$

Differentiate <u>(Note that d is not constant here)</u>

$$ja\omega_2 e^{j\theta_2} - jb\omega_3 e^{j\theta_3} - \dot{d} = 0$$

Substitute Euler equation:

$$ja\omega_2(cos\theta_2 + jsin\theta_2) - jb\omega_3(cos\theta_3 + jsin\theta_3) - \dot{d} = 0$$

Chapter 6 Velocity analysis

- 6.7 Analytical solution for velocity analysis

The Fourbar slider crank

$$ja\omega_2(cos\theta_2 + jsin\theta_2) - jb\omega_3(cos\theta_3 + jsin\theta_3) - \dot{d} = 0$$

Simplify:

$$a\omega_2(jcos\theta_2 - sin\theta_2) - b\omega_3(jcos\theta_3 - sin\theta_3) - \dot{d} = 0$$

real part (x component):

$$-a\omega_2 \sin\theta_2 + b\omega_3 \sin\theta_3 - \dot{d} = 0$$

imaginary part (y component):

$$a\omega_2 \cos\theta_2 - b\omega_3 \cos\theta_3 = 0$$

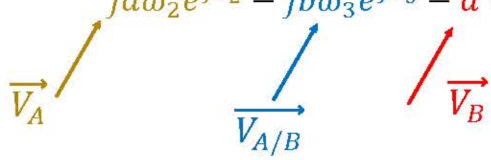
$$\boxed{\begin{gathered}\omega_3 = \frac{a}{b}\frac{\cos\theta_2}{\cos\theta_3}\omega_2 \\ \dot{d} = -a\omega_2 \sin\theta_2 + b\omega_3 \sin\theta_3\end{gathered}}$$

Chapter 6 Velocity analysis

- 6.7 Analytical solution for velocity analysis

The Fourbar slider crank: Linear velocity

$$ja\omega_2 e^{j\theta_2} - jb\omega_3 e^{j\theta_3} - \dot{d} = 0$$



$V_B = \dot{d}$ Slider velocity (The velocity of slip)

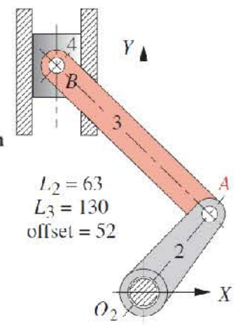
$$\boxed{\begin{aligned}\mathbf{V}_A &= a\omega_2(-\sin\theta_2 + j\cos\theta_2) \\ \mathbf{V}_{AB} &= b\omega_3(-\sin\theta_3 + j\cos\theta_3) \\ \mathbf{V}_{BA} &= -\mathbf{V}_{AB}\end{aligned}}$$

Chapter 6 Velocity analysis

- Problem 6.34

Link 2	$a := 63 \cdot mm$
Link 3	$b := 130 \cdot mm$
Offset	$c := -52 \cdot mm$
Crank angle:	$\theta_2 := 141 \cdot deg$
Input crank angular velocity	$\omega_2 := -25 \cdot rad \cdot sec^{-1}$

Local *xy* coordinate system



$$\theta_3 := asin\left(\frac{a \cdot sin(\theta_2) - c}{b}\right) \qquad \theta_3 = 44.828\,deg$$

$$d := a \cdot cos(\theta_2) - b \cdot cos(\theta_3) \qquad d = -141.160\,mm$$

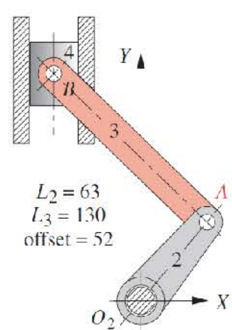
$$V_A := a \cdot \omega_2 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2))$$

$$V_A = 991.180 + 1224.005i\,\frac{mm}{sec} \qquad |V_A| = 1575.000\,\frac{mm}{sec} \qquad arg(V_A) = 51.000\,deg$$

In the global coordinate system, $\theta_{VA} := arg(V_A) - 90 \cdot deg$ $\qquad \theta_{VA} = -39.000\,deg$

Chapter 6 Velocity analysis

- Problem 6.34



$$V_B := -a \cdot \omega_2 \cdot sin(\theta_2) + b \cdot \omega_3 \cdot sin(\theta_3) \qquad V_B = 2207.849\,\frac{mm}{sec}$$

$$\mathbf{V_B} := V_B$$

In the global coordinate system, $\theta_{VB} := arg(\mathbf{V_B}) - 90 \cdot deg$ $\qquad \theta_{VB} = -90.000\,deg$

Chapter 6 Velocity analysis

- 6.7 Analytical solution for velocity analysis

The Fourbar inverted slider crank

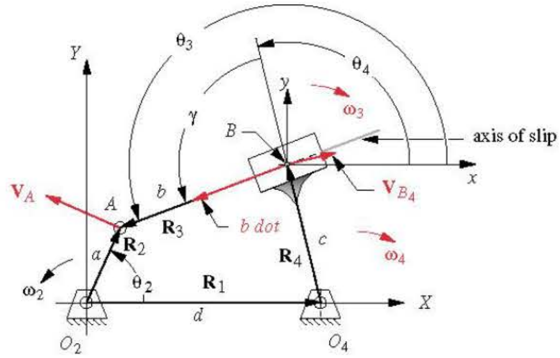


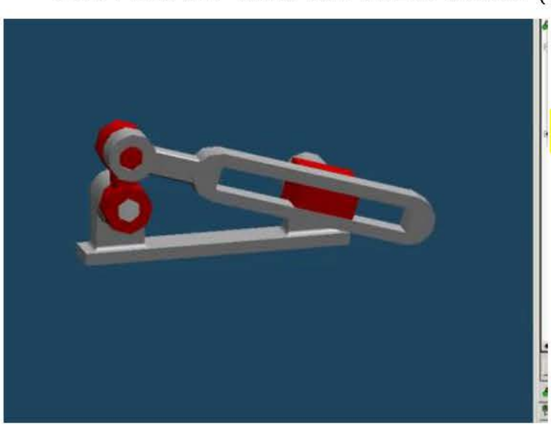
FIGURE 6-22

Velocity analysis of inversion #3 of the slider-crank fourbar linkage

Chapter 6 Velocity analysis

- 6.7 Analytical solution for velocity analysis

The Fourbar inverted slider crank (Example)



In this fourbar inverted slider crank example:

$c=0$ and $\gamma=90^{\circ}$

Chapter 6 Velocity analysis

- 6.7 Analytical solution for velocity analysis

The Fourbar inverted slider crank

The same procedure as before:

$$ja\omega_2 e^{j\theta_2} - jb\omega_3 e^{j\theta_3} - \dot{b}e^{j\theta_3} - jc\omega_4 e^{j\theta_4} = 0$$

Note that b is not constant here

$\theta_3 = \theta_4 \pm \gamma$ therefore $\omega_3 = \omega_4$

Substitute Euler equation:

$$ja\omega_2(\cos\theta_2 + j\sin\theta_2) - jb\omega_3(\cos\theta_3 + j\sin\theta_3)$$
$$-\dot{b}(\cos\theta_3 + j\sin\theta_3) - jc\omega_4(\cos\theta_4 + j\sin\theta_4) = 0$$

Chapter 6 Velocity analysis

- 6.7 Analytical solution for velocity analysis

real part (x component):

$$-a\omega_2\sin\theta_2 + b\omega_4\sin\theta_3 - \dot{b}\cos\theta_3 + c\omega_4\sin\theta_4 = 0$$

imaginary part (y component):

$$a\omega_2\cos\theta_2 - b\omega_4\cos\theta_3 - \dot{b}\sin\theta_3 - c\omega_4\cos\theta_4 = 0$$

The velocity of slip at point B

$$\boxed{\dot{b} = \frac{-a\omega_2\sin\theta_2 + \omega_4(b\sin\theta_3 + c\sin\theta_4)}{\cos\theta_3}}$$

$$\boxed{\omega_4 = \frac{a\omega_2\cos(\theta_2 - \theta_3)}{b + c\cos(\theta_4 - \theta_3)}}$$

$$-\gamma = \theta_4 - \theta_3$$

Chapter 6 Velocity analysis

- 6.7 Analytical solution for velocity analysis

The Fourbar inverted slider crank: Linear velocity

$$\underbrace{ja\omega_2 e^{j\theta_2}}_{\overrightarrow{V_A}} \underbrace{- jb\omega_3 e^{j\theta_3} - \dot{b}e^{j\theta_3}}_{\overrightarrow{V_{A/B}}} \underbrace{- jc\omega_4 e^{j\theta_4}}_{\overrightarrow{V_B}} = 0$$

The velocity of point B on link 4.

$$\boxed{\mathbf{V}_{B_4} = jc\omega_4 e^{j\theta_4} = c\omega_4\left(-\sin\theta_4 + j\cos\theta_4\right)}$$

The velocity of transmission is the component of V_{B4} normal to the axis of slip.

Chapter 6 Velocity analysis

- 6.7 Analytical solution for velocity analysis

The Fourbar inverted slider crank: Example

Consider a typical inverted slider-crank linkage as shown in the previous figures. Given:

Link lengths: Link 1 = 6 in, Link 2 = 2 in, link 4 = 4 in. Positions: $\gamma = 90^\circ$, $\theta_2 = 70^\circ$, $\theta_4 = 115.717^\circ$, $\theta_3 = 205.717^\circ$ and link 3 = b = 3.9739 in. Link 2 velocity $\omega_2 = 10$ rad/s

Calculate the angular velocity of links 4 and 3 and the velocity of slip at point B.

Chapter 6 Velocity analysis

- 6.7 Analytical solution for velocity analysis

The Fourbar inverted slider crank: Example

$$\omega_4 = \frac{a\omega_2 \cos(\theta_2 - \theta_3)}{b + c\cos(\theta_4 - \theta_3)}$$ = -3,6031 rad/sec

$$\omega_3 = \omega_4$$

$$\dot{b} = \frac{-a\omega_2 \sin\theta_2 + \omega_4(b\sin\theta_3 + c\sin\theta_4)}{\cos\theta_3}$$ = 28,376 in/s

$\dot{b}$ is the velocity of slip at point B

Chapter 6 Velocity analysis

- 6.8 Velocity analysis of the geared Fivebar linkage

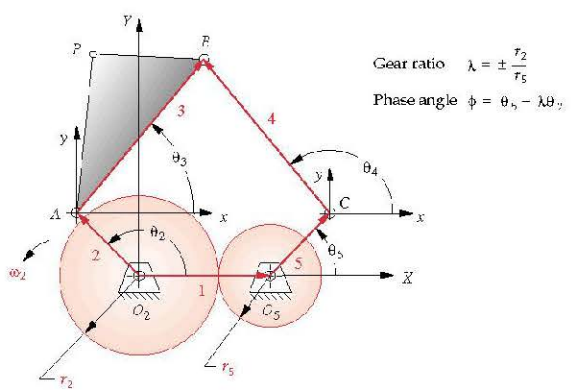


FIGURE P6-4

Configuration and terminology for problems 6-10 to 6-11

Chapter 6 Velocity analysis

- 6.8 Velocity analysis of the geared Fivebar linkage

$$ae^{j\theta_2} + be^{j\theta_3} - ce^{j\theta_4} - de^{j\theta_5} - fe^{j\theta_1} = 0$$

$$a\omega_2 je^{j\theta_2} + b\omega_3 je^{j\theta_3} - c\omega_4 je^{j\theta_4} - d\omega_5 je^{j\theta_5} = 0$$

$$a\omega_2 j(\cos\theta_2 + j\sin\theta_2) + b\omega_3 j(\cos\theta_3 + j\sin\theta_3) \\ -c\omega_4 j(\cos\theta_4 + j\sin\theta_4) - d\omega_5 j(\cos\theta_5 + j\sin\theta_5) = 0$$

$$\theta_5 = \lambda\theta_2 + \phi$$

$$\omega_5 = \lambda\omega_2$$

$$\text{real}: \quad -a\omega_2 \sin\theta_2 - b\omega_3 \sin\theta_3 + c\omega_4 \sin\theta_4 + d\omega_5 \sin\theta_5 = 0$$

$$\text{imaginary}: \quad a\omega_2 \cos\theta_2 + b\omega_3 \cos\theta_3 - c\omega_4 \cos\theta_4 - d\omega_5 \cos\theta_5 = 0$$

Chapter 6 Velocity analysis

- 6.8 Velocity analysis of the geared Fivebar linkage

$$\left.\begin{aligned}&\text{real}: \quad -a\omega_2 \sin\theta_2 - b\omega_3 \sin\theta_3 + c\omega_4 \sin\theta_4 + d\omega_5 \sin\theta_5 = 0\\ &\text{imaginary}: \quad a\omega_2 \cos\theta_2 + b\omega_3 \cos\theta_3 - c\omega_4 \cos\theta_4 - d\omega_5 \cos\theta_5 = 0\end{aligned}\right\}$$

$$\boxed{\omega_3 = -\frac{2\sin\theta_4\left[a\omega_2\sin(\theta_2 - \theta_4) + d\omega_5\sin(\theta_4 - \theta_5)\right]}{b\left[\cos(\theta_3 - 2\theta_4) - \cos\theta_3\right]}}$$

$$\boxed{\omega_4 = \frac{a\omega_2\sin\theta_2 + b\omega_3\sin\theta_3 - d\omega_5\sin\theta_5}{c\sin\theta_4}}$$

$$\boxed{\begin{aligned}\mathbf{V}_A &= a\omega_2(-\sin\theta_2 + j\cos\theta_2)\\ \mathbf{V}_{BA} &= b\omega_3(-\sin\theta_3 + j\cos\theta_3)\\ \mathbf{V}_C &= d\omega_5(-\sin\theta_5 + j\cos\theta_5)\\ \mathbf{V}_B &= \mathbf{V}_A + \mathbf{V}_{BA}\end{aligned}}$$

Chapter 6 Velocity analysis

- 6.9 Velocity of any point on a linkage

Once the angular velocities of all the links are found it is easy to define and calculate the velocity of *any point on any link* for any input position of the linkage.

We will develop algebraic expressions for the velocities of representative points (S,P,U) on the links (or any other points).

Chapter 6 Velocity analysis

- 6.9 Velocity of any point on a linkage

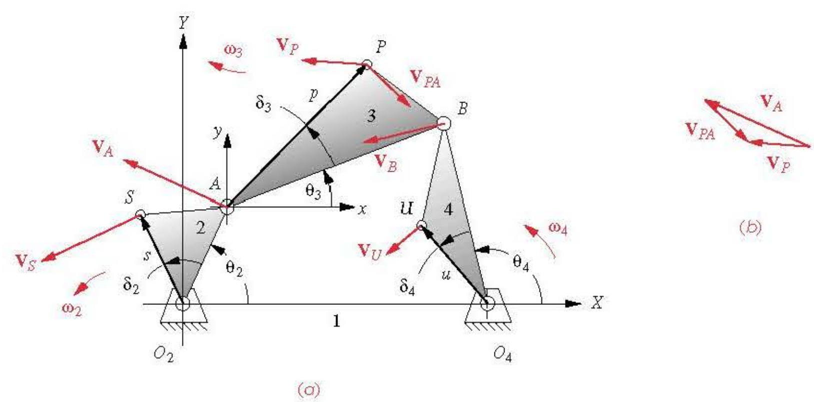
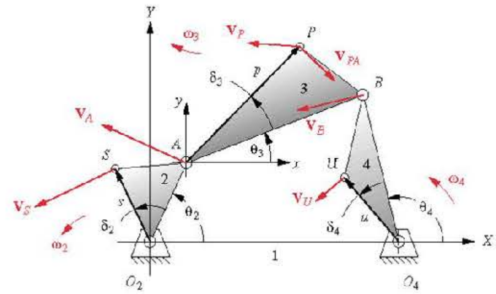


FIGURE 6-23
Finding the velocities of points on the links

Chapter 6 Velocity analysis

- 6.9 Velocity of any point on a linkage

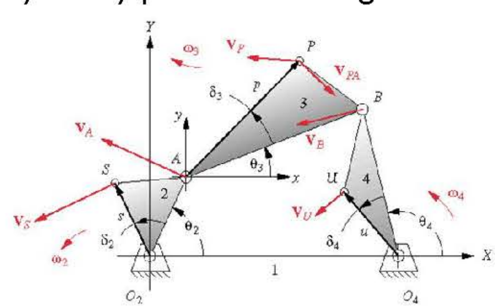


$$\mathbf{R}_{SO_2} = \mathbf{R}_S = se^{j(\theta_2+\delta_2)} = s\left[\cos(\theta_2+\delta_2)+j\sin(\theta_2+\delta_2)\right]$$

$$\boxed{\mathbf{V}_S = jse^{j(\theta_2+\delta_2)}\omega_2 = s\omega_2\left[-\sin(\theta_2+\delta_2)+j\cos(\theta_2+\delta_2)\right]}$$

Chapter 6 Velocity analysis

- 6.9 Velocity of any point on a linkage



$$\mathbf{R}_{UO_4} = ue^{j(\theta_4+\delta_4)} = u\left[\cos(\theta_4+\delta_4)+j\sin(\theta_4+\delta_4)\right]$$

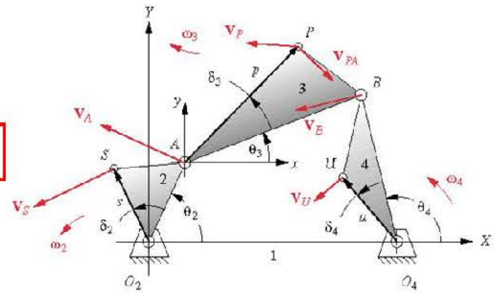
$$\boxed{\mathbf{V}_U = jue^{j(\theta_4+\delta_4)}\omega_4 = u\omega_4\left[-\sin(\theta_4+\delta_4)+j\cos(\theta_4+\delta_4)\right]}$$

Chapter 6 Velocity analysis

- 6.9 Velocity of any point on a linkage

$$\mathbf{R}_P = \mathbf{R}_A + \mathbf{R}_{PA}$$

$$\boxed{\mathbf{V}_P = \mathbf{V}_A + \mathbf{V}_{PA}}$$



$$\mathbf{R}_{PA} = pe^{j(\theta_3+\delta_3)} = p\left[\cos(\theta_3+\delta_3) + j\sin(\theta_3+\delta_3)\right]$$

$$\boxed{\mathbf{V}_{PA} = jpe^{j(\theta_3+\delta_3)}\omega_3 = p\omega_3\left[-\sin(\theta_3+\delta_3) + j\cos(\theta_3+\delta_3)\right]}$$

Chapter 6 Velocity analysis

- Problem 6.84

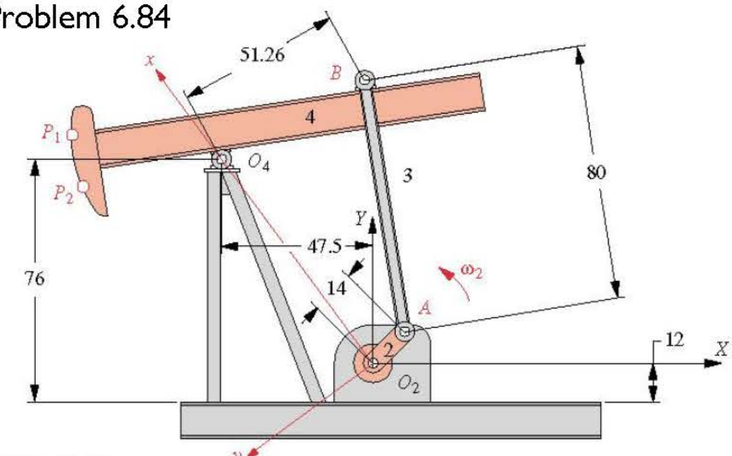
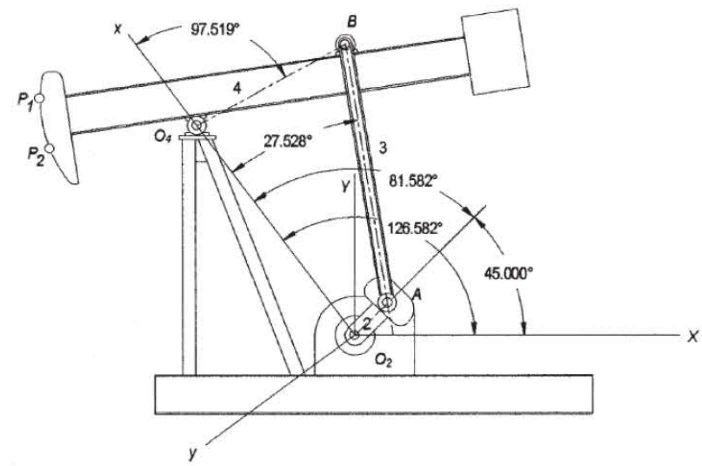


FIGURE P6-30

Problems 6-83 to 6-85 An oil field pump - dimensions in inches

Chapter 6 Velocity analysis

- Problem 6.84



Chapter 6 Velocity analysis

- Problem 6.84

Link 2 (O_2 to A)	$a := 14.00 \cdot in$	Link 3 (A to B)	$b := 80.00 \cdot in$
Link 4 (O_4 to B)	$c := 51.26 \cdot in$		
Link 1 X-offset	$d_X := 47.5 \cdot in$	Link 1 Y-offset	$d_Y := 76.00 \cdot in - 12.00 \cdot in$
Coupler point x-offset	$p_x := 114.68 \cdot in$	Coupler point y-offset	$p_y := 33.19 \cdot in$

Crank angle: $\theta_{2XY} := 45 \cdot deg$ XY coord system

Coordinate transformation angle: $\delta := 126.582 \cdot deg$

Input crank angular velocity $\omega_2 := 10 \cdot rad \cdot sec^{-1}$ CCW

$\theta_2 := \theta_{2XY} - \delta$ $\theta_2 = -81.582\, deg$

Distance O_2O_4: $d := \sqrt{d_X^2 + d_Y^2}$ $d = 79.701\, in$

Chapter 6 Velocity analysis

- Problem 6.84

Position analysis

$K_1 := \frac{d}{a}$ $\quad K_2 := \frac{d}{c}$ $\quad A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$

$B := -2 \cdot \sin(\theta_2)$

$K_1 = 5.6929$ $\quad K_2 = 1.5548$ $\quad C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$

$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)}$ $\quad K_3 = 1.9340$ $\quad A = -3.8401 \quad B = 1.9785 \quad C = 7.2529$

$\theta_4 = 262.482\ deg \quad \theta_3 = 332.475\ deg$

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)}$$

$$\omega_3 = -0.511 \frac{rad}{sec}$$

$$\omega_4 = 2.353 \frac{rad}{sec}$$

$\mathbf{V_A} := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$

$\mathbf{V_A} = 138.492 + 20.495j \frac{in}{sec}$ $\quad |\mathbf{V_A}| = 140.000 \frac{in}{sec}$

$\theta_{VAXY} := \arg(\mathbf{V_A}) + \delta$ $\quad \theta_{VAXY} = 135.000\ deg$

$\mathbf{V_B} := c \cdot \omega_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4))$

$\mathbf{V_B} = 119.588 - 15.781j \frac{in}{sec}$ $\quad |\mathbf{V_B}| = 120.624 \frac{in}{sec}$

$\theta_{VBXY} := \arg(\mathbf{V_B}) + \delta$ $\quad \theta_{VBXY} = 119.064\ deg$

Chapter 6 Velocity analysis

- Problem 6.84

Calculate the distance from O_4 to P_1 and the angle BO_4P_1.

Distance from O_4 to P_1: $\quad e := \sqrt{(p_x - d)^2 + (p_y)^2}$ $\quad e = 48.219\ in$

$$\delta_4 := 180 \cdot deg - \operatorname{atan}\left(\frac{p_y}{p_x - d}\right) \qquad \delta_4 = 136.503\ deg$$

Determine the velocity of point P_1 using equations 6.35.

$$\mathbf{V_{P1}} := e \cdot \omega_4 \cdot (-\sin(\theta_4 + \delta) + j \cdot \cos(\theta_4 + \delta))$$

$\mathbf{V_{P1}} = -55.122 + 99.181j \frac{in}{sec}$ $\quad |\mathbf{V_{P1}}| = 113.469 \frac{in}{sec}$

$\theta_{VPXY} := \arg(\mathbf{V_{P1}}) + \delta$ $\quad \theta_{VPXY} = 245.646\ deg$

Chapter 6 Velocity analysis

- Problem

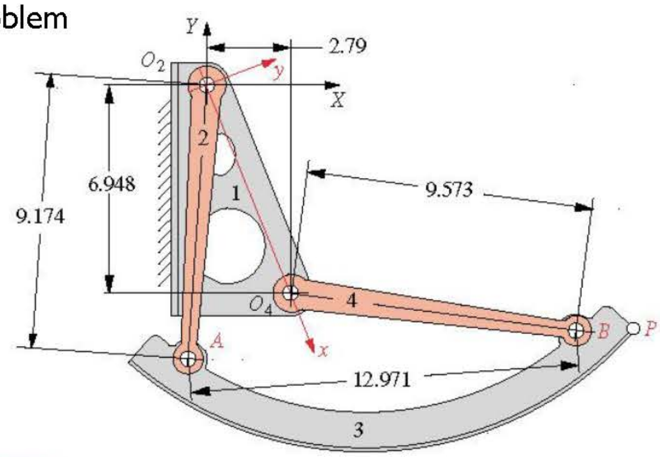


FIGURE P6-31

Problems 6-86 and 6-87 An aircraft overhead bin mechanism - dimensions in inches

Chapter 6 Velocity analysis

- Problem