6.4 ANELASTICITY

anelasticity

To this point, it has been assumed that elastic deformation is time independent—that is, that an applied stress produces an instantaneous elastic strain that remains constant over the period of time the stress is maintained. It has also been assumed that upon release of the load, the strain is totally recovered—that is, that the strain immediately returns to zero. In most engineering materials, however, there will also exist a time-dependent elastic strain component—that is, elastic deformation will continue after the stress application, and upon load release, some finite time is required for complete recovery. This time-dependent elastic behavior is known as **anelasticity,** and it is due to time-dependent microscopic and atomistic processes that are attendant to the deformation. For metals, the anelastic component is normally small and is often neglected. However, for some polymeric materials, its magnitude is significant; in this case it is termed *viscoelastic behavior,* which is the discussion topic of Section 15.4.

EXAMPLE PROBLEM 6.1

Elongation (Elastic) Computation

A piece of copper originally 305 mm (12 in.) long is pulled in tension with a stress of 276 MPa (40,000 psi). If the deformation is entirely elastic, what will be the resultant elongation?

Solution

Because the deformation is elastic, strain is dependent on stress according to Equation 6.5. Furthermore, the elongation Δl is related to the original length l_0 through Equation 6.2. Combining these two expressions and solving for Δl yields

$$\sigma = \epsilon E = \left(\frac{\Delta l}{l_0}\right)E$$

$$\Delta l = \frac{\sigma l_0}{E}$$

The values of σ and l_0 are given as 276 MPa and 305 mm, respectively, and the magnitude of E for copper from Table 6.1 is 110 GPa (16×10^6 psi). Elongation is obtained by substitution into the preceding expression as

$$\Delta l = \frac{(276 \text{ MPa})(305 \text{ mm})}{110 \times 10^3 \text{ MPa}} = 0.77 \text{ mm } (0.03 \text{ in.})$$

6.5 ELASTIC PROPERTIES OF MATERIALS

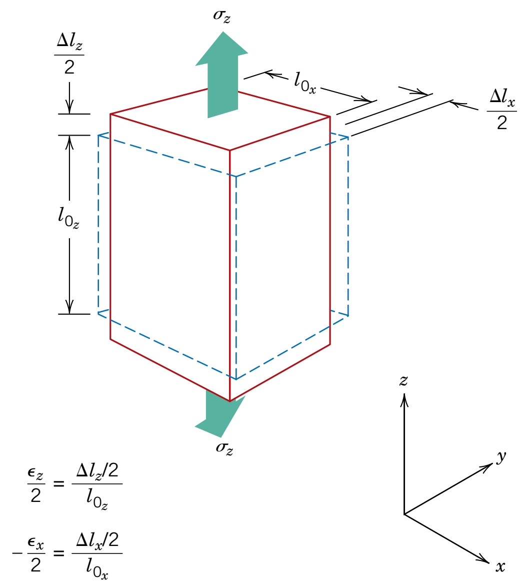
Poisson's ratio

When a tensile stress is imposed on a metal specimen, an elastic elongation and accompanying strain ϵ_z result in the direction of the applied stress (arbitrarily taken to be the z direction), as indicated in Figure 6.9. As a result of this elongation, there will be constrictions in the lateral (x and y) directions perpendicular to the applied stress; from these contractions, the compressive strains ϵ_x and ϵ_y may be determined. If the applied stress is uniaxial (only in the z direction) and the material is isotropic, then $\epsilon_x = \epsilon_y$. A parameter termed **Poisson's ratio** ν is defined as the ratio of the lateral and axial strains, or

Definition of Poisson's ratio in terms of lateral and axial strains

$$\nu = -\frac{\epsilon_x}{\epsilon_z} = -\frac{\epsilon_y}{\epsilon_z} \tag{6.8}$$

Figure 6.9 Axial (z) elongation (positive strain) and lateral (x and y) contractions (negative strains) in response to an imposed tensile stress. Solid lines represent dimensions after stress application; dashed lines, before.



For virtually all structural materials, ϵ_x and ϵ_z will be of opposite sign; therefore, the negative sign is included in the preceding expression to ensure that ν is positive.[7] Theoretically, Poisson's ratio for isotropic materials should be $\frac{1}{4}$; furthermore, the maximum value for ν (or the value for which there is no net volume change) is 0.50. For many metals and other alloys, values of Poisson's ratio range between 0.25 and 0.35. Table 6.1 shows ν values for several common metallic materials.

For isotropic materials, shear and elastic moduli are related to each other and to Poisson's ratio according to

Relationship among elastic parameters—modulus of elasticity, shear modulus, and Poisson's ratio

$$E = 2G(1 + \nu) \tag{6.9}$$

In most metals, G is about $0.4E$; thus, if the value of one modulus is known, the other may be approximated.

Many materials are elastically anisotropic; that is, the elastic behavior (i.e., the magnitude of E) varies with crystallographic direction (see Table 3.4). For these materials, the elastic properties are completely characterized only by the specification of several elastic constants, their number depending on characteristics of the crystal structure. Even for isotropic materials, for complete characterization of the elastic properties, at least two constants must be given. Because the grain orientation is random in most polycrystalline materials, these may be considered to be isotropic; inorganic ceramic glasses are also isotropic. The remaining discussion of mechanical behavior assumes isotropy and polycrystallinity because this is the character of most engineering materials.

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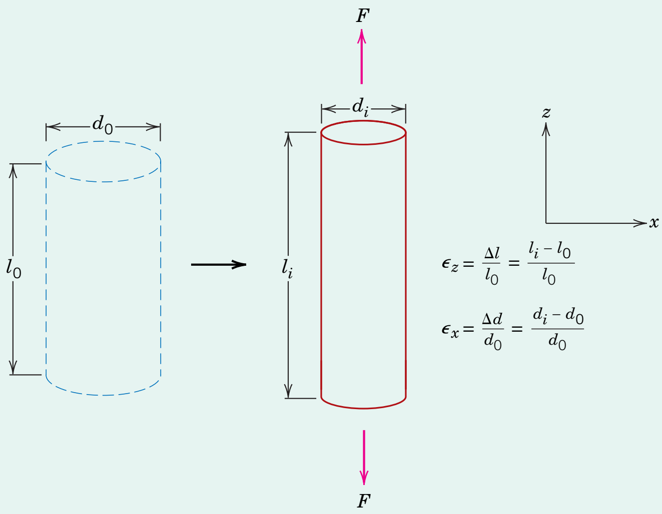
[7] Some materials (e.g., specially prepared polymer foams) when pulled in tension actually expand in the transverse direction. In these materials, both ϵ_x and ϵ_z of Equation 6.8 are positive, and thus Poisson's ratio is negative. Materials that exhibit this effect are termed *auxetics*.

EXAMPLE PROBLEM 6.2

Computation of Load to Produce Specified Diameter Change

A tensile stress is to be applied along the long axis of a cylindrical brass rod that has a diameter of 10 mm (0.4 in.). Determine the magnitude of the load required to produce a 2.5×10^{-3}-mm (10^{-4}-in.) change in diameter if the deformation is entirely elastic.



Solution

This deformation situation is represented in the accompanying drawing.

When the force F is applied, the specimen will elongate in the z direction and at the same time experience a reduction in diameter, Δd, of 2.5×10^{-3} mm in the x direction. For the strain in the x direction,

$$\epsilon_x = \frac{\Delta d}{d_0} = \frac{-2.5 \times 10^{-3}\ \text{mm}}{10\ \text{mm}} = -2.5 \times 10^{-4}$$

which is negative because the diameter is reduced.

It next becomes necessary to calculate the strain in the z direction using Equation 6.8. The value for Poisson's ratio for brass is 0.34 (Table 6.1), and thus

$$\epsilon_z = -\frac{\epsilon_x}{\nu} = -\frac{(-2.5 \times 10^{-4})}{0.34} = 7.35 \times 10^{-4}$$

The applied stress may now be computed using Equation 6.5 and the modulus of elasticity, given in Table 6.1 as 97 GPa (14×10^6 psi), as

$$\sigma = \epsilon_z E = (7.35 \times 10^{-4})(97 \times 10^3\ \text{MPa}) = 71.3\ \text{MPa}$$

Finally, from Equation 6.1, the applied force may be determined as

$$F = \sigma A_0 = \sigma \left(\frac{d_0}{2}\right)^2 \pi$$

$$= (71.3 \times 10^6\ \text{N/m}^2)\left(\frac{10 \times 10^{-3}\ \text{m}}{2}\right)^2 \pi = 5600\ \text{N}\ (1293\ \text{lb}_\text{f})$$

Plastic Deformation

plastic deformation

For most metallic materials, elastic deformation persists only to strains of about 0.005. As the material is deformed beyond this point, the stress is no longer proportional to strain (Hooke's law, Equation 6.5, ceases to be valid), and permanent, nonrecoverable, or **plastic deformation** occurs. Figure 6.10*a* plots schematically the tensile stress–strain behavior into the plastic region for a typical metal. The transition from elastic to plastic is a gradual one for most metals; some curvature results at the onset of plastic deformation, which increases more rapidly with rising stress.

From an atomic perspective, plastic deformation corresponds to the breaking of bonds with original atom neighbors and then the re-forming of bonds with new neighbors as large numbers of atoms or molecules move relative to one another; upon removal of the stress, they do not return to their original positions. The mechanism of this deformation is different for crystalline and amorphous materials. For crystalline solids, deformation is accomplished by means of a process called *slip,* which involves the motion of dislocations as discussed in Section 7.2. Plastic deformation in noncrystalline solids (as well as liquids) occurs by a viscous flow mechanism, which is outlined in Section 12.10.

6.6 TENSILE PROPERTIES

Yielding and Yield Strength

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Metal Alloys

yielding

Most structures are designed to ensure that only elastic deformation will result when a stress is applied. A structure or component that has plastically deformed—or experienced a permanent change in shape—may not be capable of functioning as intended. It is therefore desirable to know the stress level at which plastic deformation begins, or where the phenomenon of **yielding** occurs. For metals that experience this gradual

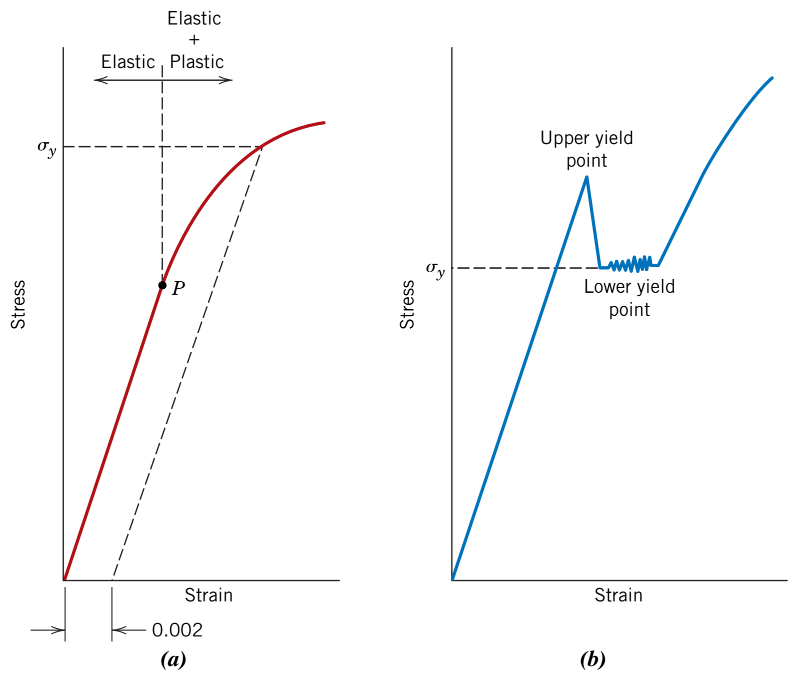


Figure 6.10 (*a*) Typical stress–strain behavior for a metal showing elastic and plastic deformations, the proportional limit *P*, and the yield strength σ_y, as determined using the 0.002 strain offset method. (*b*) Representative stress–strain behavior found for some steels demonstrating the yield point phenomenon.

elastic–plastic transition, the point of yielding may be determined as the initial departure from linearity of the stress–strain curve; this is sometimes called the

proportional limit

proportional limit, as indicated by point P in Figure 6.10*a*, and represents the onset of plastic deformation on a microscopic level. The position of this point P is difficult to measure precisely. As a consequence, a convention has been established by which a straight line is constructed parallel to the elastic portion of the stress–strain curve at some specified strain offset, usually 0.002. The stress corresponding to the intersection of this line and the stress–strain curve as it bends over in the plastic region is defined as the **yield strength** σ_y.[8] This is demonstrated in Figure 6.10*a*. The units of yield strength are MPa or psi.[9]

yield strength

For materials having a nonlinear elastic region (Figure 6.6), use of the strain offset method is not possible, and the usual practice is to define the yield strength as the stress required to produce some amount of strain (e.g., $\epsilon = 0.005$).

Some steels and other materials exhibit the tensile stress–strain behavior shown in Figure 6.10*b*. The elastic–plastic transition is very well defined and occurs abruptly in what is termed a *yield point phenomenon*. At the upper yield point, plastic deformation is initiated with an apparent decrease in engineering stress. Continued deformation fluctuates slightly about some constant stress value, termed the *lower yield point*; stress subsequently rises with increasing strain. For metals that display this effect, the yield strength is taken as the average stress that is associated with the lower yield point because it is well defined and relatively insensitive to the testing procedure.[10] Thus, it is not necessary to employ the strain offset method for these materials.

The magnitude of the yield strength for a metal is a measure of its resistance to plastic deformation. Yield strengths may range from 35 MPa (5000 psi) for a low-strength aluminum to greater than 1400 MPa (200,000 psi) for high-strength steels.

Concept Check 6.1 Cite the primary differences between elastic, anelastic, and plastic deformation behaviors.

[*The answer may be found at www.wiley.com/college/callister (Student Companion Site).*]

Tensile Strength

After yielding, the stress necessary to continue plastic deformation in metals increases to a maximum, point M in Figure 6.11, and then decreases to the eventual fracture, point F. The **tensile strength** TS (MPa or psi) is the stress at the maximum on the engineering stress–strain curve (Figure 6.11). This corresponds to the maximum stress that can be sustained by a structure in tension; if this stress is applied and maintained, fracture will result. All deformation to this point is uniform throughout the narrow region of the tensile specimen. However, at this maximum stress, a small constriction or neck begins to form at some point, and all subsequent deformation is confined at this neck, as indicated by the schematic specimen insets in Figure 6.11. This phenomenon is termed *necking,*

tensile strength

[8] *Strength* is used in lieu of *stress* because strength is a property of the metal, whereas stress is related to the magnitude of the applied load.

[9] For customary U.S. units, the unit of kilopounds per square inch (ksi) is sometimes used for the sake of convenience, where 1 ksi = 1000 psi.

[10] Note that to observe the yield point phenomenon, a "stiff" tensile-testing apparatus must be used; by "stiff," it is meant that there is very little elastic deformation of the machine during loading.

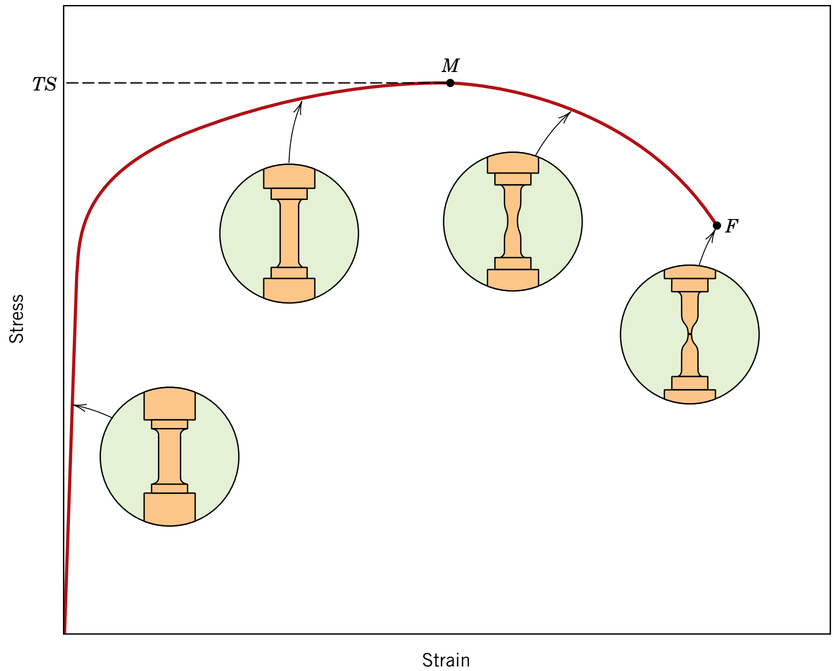


Figure 6.11 Typical engineering stress–strain behavior to fracture, point *F*. The tensile strength *TS* is indicated at point *M*. The circular insets represent the geometry of the deformed specimen at various points along the curve.

and fracture ultimately occurs at the neck.[11] The fracture strength corresponds to the stress at fracture.

Tensile strengths vary from 50 MPa (7000 psi) for an aluminum to as high as 3000 MPa (450,000 psi) for the high-strength steels. Typically, when the strength of a metal is cited for design purposes, the yield strength is used because by the time a stress corresponding to the tensile strength has been applied, often a structure has experienced so much plastic deformation that it is useless. Furthermore, fracture strengths are not normally specified for engineering design purposes.

EXAMPLE PROBLEM 6.3

Mechanical Property Determinations from Stress–Strain Plot

From the tensile stress–strain behavior for the brass specimen shown in Figure 6.12, determine the following:

(a) The modulus of elasticity
(b) The yield strength at a strain offset of 0.002
(c) The maximum load that can be sustained by a cylindrical specimen having an original diameter of 12.8 mm (0.505 in.)
(d) The change in length of a specimen originally 250 mm (10 in.) long that is subjected to a tensile stress of 345 MPa (50,000 psi)

Solution

(a) The modulus of elasticity is the slope of the elastic or initial linear portion of the stress–strain curve. The strain axis has been expanded in the inset of Figure 6.12 to facilitate

[11]The apparent decrease in engineering stress with continued deformation past the maximum point of Figure 6.11 is due to the necking phenomenon. As explained in Section 6.7, the true stress (within the neck) actually increases.

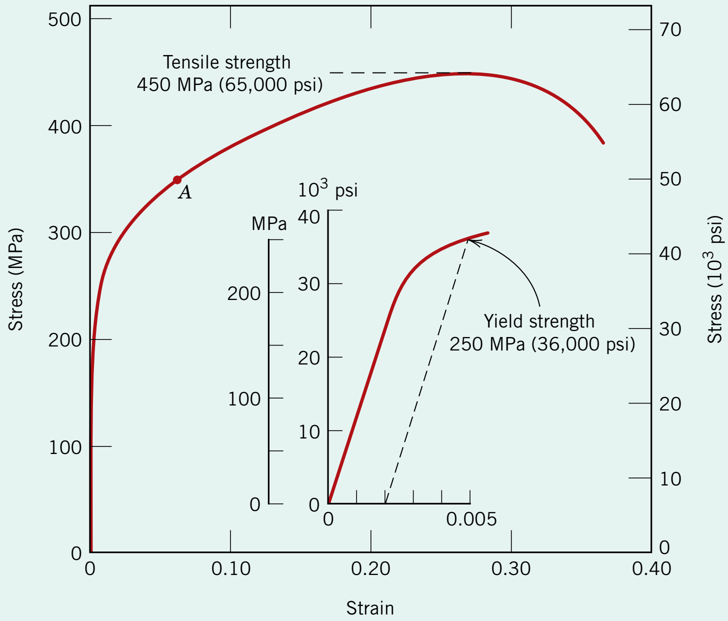


Figure 6.12 The stress–strain behavior for the brass specimen discussed in Example Problem 6.3.

this computation. The slope of this linear region is the rise over the run, or the change in stress divided by the corresponding change in strain; in mathematical terms,

$$E = \text{slope} = \frac{\Delta\sigma}{\Delta\epsilon} = \frac{\sigma_2 - \sigma_1}{\epsilon_2 - \epsilon_1} \tag{6.10}$$

Inasmuch as the line segment passes through the origin, it is convenient to take both σ_1 and ϵ_1 as zero. If σ_2 is arbitrarily taken as 150 MPa, then ϵ_2 will have a value of 0.0016. Therefore,

$$E = \frac{(150 - 0)\text{ MPa}}{0.0016 - 0} = 93.8\text{ GPa }(13.6 \times 10^6\text{ psi})$$

which is very close to the value of 97 GPa (14×10^6 psi) given for brass in Table 6.1.

(b) The 0.002 strain offset line is constructed as shown in the inset; its inter-section with the stress–strain curve is at approximately 250 MPa (36,000 psi), which is the yield strength of the brass.

(c) The maximum load that can be sustained by the specimen is calculated by using Equation 6.1, in which σ is taken to be the tensile strength, from Figure 6.12, 450 MPa (65,000 psi). Solving for F, the maximum load, yields

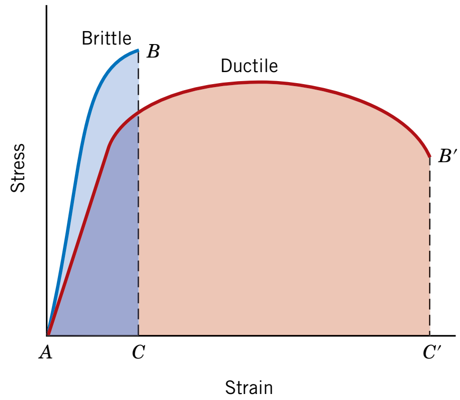
$$F = \sigma A_0 = \sigma\left(\frac{d_0}{2}\right)^2 \pi$$

$$= (450 \times 10^6\text{ N/m}^2)\left(\frac{12.8 \times 10^{-3}\text{ m}}{2}\right)^2 \pi = 57{,}900\text{ N }(13{,}000\text{ lb}_\text{f})$$

(d) To compute the change in length, Δl, in Equation 6.2, it is first necessary to determine the strain that is produced by a stress of 345 MPa. This is accomplished by locating the stress point on the stress–strain curve, point A, and reading the corresponding strain from the strain axis, which is approximately 0.06. Inasmuch as $l_0 = 250$ mm, we have

$$\Delta l = \epsilon l_0 = (0.06)(250\text{ mm}) = 15\text{ mm }(0.6\text{ in.})$$

Figure 6.13 Schematic representations of tensile stress–strain behavior for brittle and ductile metals loaded to fracture.



Ductility

ductility

Ductility is another important mechanical property. It is a measure of the degree of plastic deformation that has been sustained at fracture. A metal that experiences very little or no plastic deformation upon fracture is termed *brittle*. The tensile stress–strain behaviors for both ductile and brittle metals are schematically illustrated in Figure 6.13.

Ductility may be expressed quantitatively as either *percent elongation* or *percent reduction in area*. Percent elongation (%EL) is the percentage of plastic strain at fracture, or

Ductility, as percent elongation

$$\%\text{EL} = \left(\frac{l_f - l_0}{l_0}\right) \times 100 \tag{6.11}$$

where l_f is the fracture length[12] and l_0 is the original gauge length as given earlier. Inasmuch as a significant proportion of the plastic deformation at fracture is confined to the neck region, the magnitude of %EL will depend on specimen gauge length. The shorter l_0, the greater the fraction of total elongation from the neck and, consequently, the higher the value of %EL. Therefore, l_0 should be specified when percent elongation values are cited; it is commonly 50 mm (2 in.).

Percent reduction in area (%RA) is defined as

Ductility, as percent reduction in area

$$\%\text{RA} = \left(\frac{A_0 - A_f}{A_0}\right) \times 100 \tag{6.12}$$

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Tutorial Video: Tensile Test

How do I determine ductility in percent elongation and percent reduction in area?

where A_0 is the original cross-sectional area and A_f is the cross-sectional area at the point of fracture.[12] Values of percent reduction in area are independent of both l_0 and A_0. Furthermore, for a given material, the magnitudes of %EL and %RA will, in general, be different. Most metals possess at least a moderate degree of ductility at room temperature; however, some become brittle as the temperature is lowered (Section 8.6).

Knowledge of the ductility of materials is important for at least two reasons. First, it indicates to a designer the degree to which a structure will deform plastically before

[12]Both l_f and A_f are measured subsequent to fracture and after the two broken ends have been repositioned back together.

Table 6.2
Typical Mechanical Properties of Several Metals and Alloys in an Annealed State

Metal Alloy	*Yield Strength, MPa (ksi)*	*Tensile Strength, MPa (ksi)*	*Ductility, %EL [in 50 mm (2 in.)]*
Aluminum	35 (5)	90 (13)	40
Copper	69 (10)	200 (29)	45
Brass (70Cu–30Zn)	75 (11)	300 (44)	68
Iron	130 (19)	262 (38)	45
Nickel	138 (20)	480 (70)	40
Steel (1020)	180 (26)	380 (55)	25
Titanium	450 (65)	520 (75)	25
Molybdenum	565 (82)	655 (95)	35

fracture. Second, it specifies the degree of allowable deformation during fabrication operations. We sometimes refer to relatively ductile materials as being "forgiving," in the sense that they may experience local deformation without fracture, should there be an error in the magnitude of the design stress calculation.

Brittle materials are *approximately* considered to be those having a fracture strain of less than about 5%.

Thus, several important mechanical properties of metals may be determined from tensile stress–strain tests. Table 6.2 presents some typical room-temperature values of yield strength, tensile strength, and ductility for several common metals. These properties are sensitive to any prior deformation, the presence of impurities, and/or any heat treatment to which the metal has been subjected. The modulus of elasticity is one mechanical parameter that is insensitive to these treatments. As with modulus of elasticity, the magnitudes of both yield and tensile strengths decline with increasing temperature; just the reverse holds for ductility—it usually increases with temperature. Figure 6.14 shows how the stress–strain behavior of iron varies with temperature.

Resilience

resilience

Resilience is the capacity of a material to absorb energy when it is deformed elastically and then, upon unloading, to have this energy recovered. The associated property is the *modulus of resilience,* U_r, which is the strain energy per unit volume required to stress a material from an unloaded state up to the point of yielding.

Computationally, the modulus of resilience for a specimen subjected to a uniaxial tension test is just the area under the engineering stress–strain curve taken to yielding (Figure 6.15), or

Definition of modulus of resilience

$$U_r = \int_0^{\epsilon_y} \sigma \, d\epsilon \tag{6.13a}$$

Assuming a linear elastic region, we have

Modulus of resilience for linear elastic behavior

$$U_r = \frac{1}{2}\sigma_y \epsilon_y \tag{6.13b}$$

in which ϵ_y is the strain at yielding.

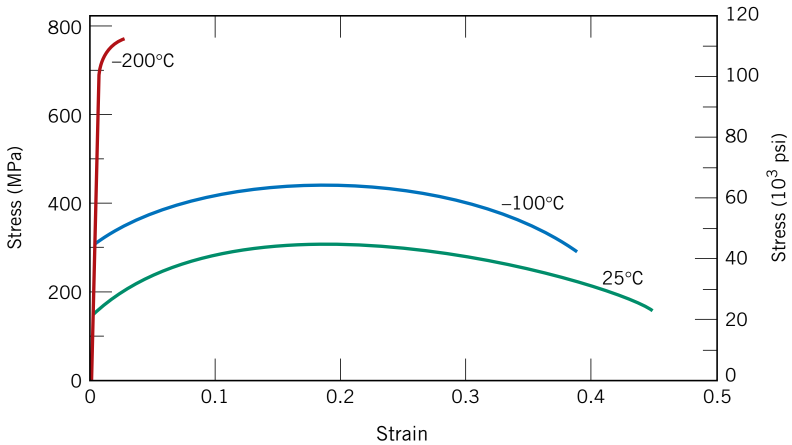

Figure 6.14 Engineering stress–strain behavior for iron at three temperatures.

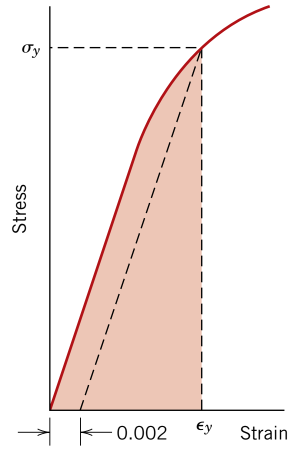

Figure 6.15 Schematic representation showing how modulus of resilience (corresponding to the shaded area) is determined from the tensile stress–strain behavior of a material.

The units of resilience are the product of the units from each of the two axes of the stress–strain plot. For SI units, this is joules per cubic meter (J/m^3, equivalent to Pa), whereas with customary U.S. units it is inch-pounds force per cubic inch ($in.\text{-}lb_f/in.^3$, equivalent to psi). Both joules and inch-pounds force are units of energy, and thus this area under the stress–strain curve represents energy absorption per unit volume (in cubic meters or cubic inches) of material.

Incorporation of Equation 6.5 into Equation 6.13b yields

Modulus of resilience for linear elastic behavior, and incorporating Hooke's law

$$U_r = \frac{1}{2}\sigma_y \epsilon_y = \frac{1}{2}\sigma_y\left(\frac{\sigma_y}{E}\right) = \frac{\sigma_y^2}{2E} \tag{6.14}$$

Thus, resilient materials are those having high yield strengths and low moduli of elasticity; such alloys are used in spring applications.

Toughness

toughness

Toughness is a mechanical term that may be used in several contexts. For one, toughness (or more specifically, *fracture toughness*) is a property that is indicative of a material's resistance to fracture when a crack (or other stress-concentrating defect) is present (as discussed in Section 8.5). Because it is nearly impossible (as well as costly) to manufacture materials with zero defects (or to prevent damage during service), fracture toughness is a major consideration for all structural materials.

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Tutorial Video: Tensile Test

What is toughness and how do I determine its value?

Another way of defining toughness is as the ability of a material to absorb energy and plastically deform before fracturing. For dynamic (high strain rate) loading conditions and when a notch (or point of stress concentration) is present, *notch toughness* is assessed by using an impact test, as discussed in Section 8.6.

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Tutorial Video: Mechanical Property Calculations from Tensile Test Measurements

For the static (low strain rate) situation, a measure of toughness in metals (derived from plastic deformation) may be ascertained from the results of a tensile stress–strain test. It is the area under the σ–ϵ curve up to the point of fracture. The units are the same as for resilience (i.e., energy per unit volume of material). For a metal to be tough, it must display both strength and ductility. This is demonstrated in Figure 6.13, in which the stress–strain curves are plotted for both metal types. Hence, even though the brittle metal has higher yield and tensile strengths, it has a lower toughness than the ductile one, as can be seen by comparing the areas ABC and $AB'C'$ in Figure 6.13.

Concept Check 6.2 Of those metals listed in Table 6.3,

(a) Which will experience the greatest percentage reduction in area? Why?

(b) Which is the strongest? Why?

(c) Which is the stiffest? Why?

[*The answer may be found at www.wiley.com/college/callister (Student Companion Site).*]

Table 6.3 **Tensile Stress–Strain Data for Several Hypothetical Metals to Be Used with Concept Checks 6.2 and 6.4**

Material	*Yield Strength (MPa)*	*Tensile Strength (MPa)*	*Strain at Fracture*	*Fracture Strength (MPa)*	*Elastic Modulus (GPa)*
A	310	340	0.23	265	210
B	100	120	0.40	105	150
C	415	550	0.15	500	310
D	700	850	0.14	720	210
E	Fractures before yielding			650	350

6.7 TRUE STRESS AND STRAIN

From Figure 6.11, the decline in the stress necessary to continue deformation past the maximum—point M—seems to indicate that the metal is becoming weaker. This is not at all the case; as a matter of fact, it is increasing in strength. However, the cross-sectional area is decreasing rapidly within the neck region, where deformation is occurring. This results in a reduction in the load-bearing capacity of the specimen. The stress, as computed from Equation 6.1, is on the basis of the original cross-sectional area before any deformation and does not take into account this reduction in area at the neck.

true stress

Sometimes it is more meaningful to use a true stress–true strain scheme. **True stress** σ_T is defined as the load F divided by the instantaneous cross-sectional area A_i over which deformation is occurring (i.e., the neck, past the tensile point), or

Definition of true stress

$$\sigma_T = \frac{F}{A_i} \tag{6.15}$$

true strain

Furthermore, it is occasionally more convenient to represent strain as **true strain** ϵ_T, defined by

Definition of true strain

$$\epsilon_T = \ln \frac{l_i}{l_0} \tag{6.16}$$

If no volume change occurs during deformation—that is, if

$$A_i l_i = A_0 l_0 \tag{6.17}$$

—then true and engineering stress and strain are related according to

Conversion of engineering stress to true stress

$$\sigma_T = \sigma(1 + \epsilon) \tag{6.18a}$$

Conversion of engineering strain to true strain

$$\epsilon_T = \ln(1 + \epsilon) \tag{6.18b}$$

Equations 6.18a and 6.18b are valid only to the onset of necking; beyond this point, true stress and strain should be computed from actual load, cross-sectional area, and gauge length measurements.

A schematic comparison of engineering and true stress–strain behaviors is made in Figure 6.16. It is worth noting that the true stress necessary to sustain increasing strain continues to rise past the tensile point M'.

Coincident with the formation of a neck is the introduction of a complex stress state within the neck region (i.e., the existence of other stress components in addition to the axial stress). As a consequence, the correct stress (*axial*) within the neck is slightly lower than the stress computed from the applied load and neck cross-sectional area. This leads to the "corrected" curve in Figure 6.16.

For some metals and alloys the region of the true stress–strain curve from the onset of plastic deformation to the point at which necking begins may be approximated by

True stress–true strain relationship in the plastic region of deformation (to the point of necking)

$$\sigma_T = K\epsilon_T^n \tag{6.19}$$

In this expression, K and n are constants; these values vary from alloy to alloy and also depend on the condition of the material (whether it has been plastically deformed, heat-treated, etc.). The parameter n is often termed the *strain-hardening exponent* and has a value less than unity. Values of n and K for several alloys are given in Table 6.4.

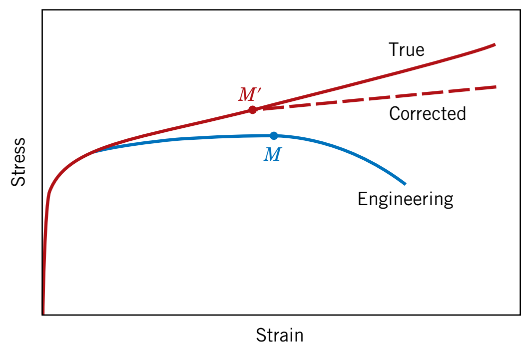

Figure 6.16 A comparison of typical tensile engineering stress–strain and true stress–strain behaviors. Necking begins at point M on the engineering curve, which corresponds to M' on the true curve. The "corrected" true stress–strain curve takes into account the complex stress state within the neck region.

Table 6.4
The n and K Values (Equation 6.19) for Several Alloys

		K	
Material	***n***	***MPa***	***psi***
Low-carbon steel (annealed)	0.21	600	87,000
4340 steel alloy (tempered @ 315°C)	0.12	2650	385,000
304 stainless steel (annealed)	0.44	1400	205,000
Copper (annealed)	0.44	530	76,500
Naval brass (annealed)	0.21	585	85,000
2024 aluminum alloy (heat-treated—T3)	0.17	780	113,000
AZ-31B magnesium alloy (annealed)	0.16	450	66,000

EXAMPLE PROBLEM 6.4

Ductility and True-Stress-at-Fracture Computations

A cylindrical specimen of steel having an original diameter of 12.8 mm (0.505 in.) is tensile-tested to fracture and found to have an engineering fracture strength σ_f of 460 MPa (67,000 psi). If its cross-sectional diameter at fracture is 10.7 mm (0.422 in.), determine

(a) The ductility in terms of percentage reduction in area
(b) The true stress at fracture

Solution

(a) Ductility is computed using Equation 6.12, as

$$\%\,\mathrm{RA} = \frac{\left(\dfrac{12.8\ \mathrm{mm}}{2}\right)^2 \pi - \left(\dfrac{10.7\ \mathrm{mm}}{2}\right)^2 \pi}{\left(\dfrac{12.8\ \mathrm{mm}}{2}\right)^2 \pi} \times 100$$

$$= \frac{128.7\ \mathrm{mm}^2 - 89.9\ \mathrm{mm}^2}{128.7\ \mathrm{mm}^2} \times 100 = 30\%$$

(b) True stress is defined by Equation 6.15, where, in this case, the area is taken as the fracture area A_f. However, the load at fracture must first be computed from the fracture strength as

$$F = \sigma_f A_0 = (460 \times 10^6\ \mathrm{N/m^2})(128.7\ \mathrm{mm}^2)\left(\frac{1\ \mathrm{m}^2}{10^6\ \mathrm{mm}^2}\right) = 59{,}200\ \mathrm{N}$$

Thus, the true stress is calculated as

$$\sigma_T = \frac{F}{A_f} = \frac{59{,}200\ \mathrm{N}}{(89.9\ \mathrm{mm}^2)\left(\dfrac{1\ \mathrm{m}^2}{10^6\ \mathrm{mm}^2}\right)}$$

$$= 6.6 \times 10^8\ \mathrm{N/m^2} = 660\ \mathrm{MPa}\ (95{,}700\ \mathrm{psi})$$