Ex 1

a) From the appendix table $(n = +5)$: $A_2 = 0.577$, $D_3 = 0$, $D_4 = 2.114$ and $d_2 = 2.326$

$\bar{\bar{x}} = 34.0041$ and $\bar{R} = 4.708$

$\bar{x}$ Chart

$UCL = \bar{\bar{x}} + A_2\bar{R} = 36.7206$

$Center\ Line = \bar{\bar{x}} = 34.0041$

$LCL = \bar{\bar{x}} - A_2\bar{R} = 31.2875$

R Chart

$UCL = D_4\bar{R} = 9.952$

$Center\ Line = \bar{R} = 4.708$

$LCL = D_3\bar{R} = 0$

The process is not in statistical in control; $\bar{x}$ is beyond the upper control limit for both sample 12 and sample 15. Assuming an assignable cause is found for these points out-of-control points, the two points can be excluded from the control limit calculations. The new process parameter estimates are:

From the appendix table $(n = +5)$: $A_2 = 0.577$, $D_3 = 0$, $D_4 = 2.114$ and $d_2 = 2.326$

$\bar{\bar{x}} = 33.6545$ and $\bar{R} = 4.499 \approx 4.5$

$\bar{x}$ Chart

$UCL_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 36.2509$

$Center\ Line = \bar{\bar{x}} = 33.6545$

$LCL_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 31.058$

R Chart

$UCL_{\mathrm{R}} = D_4\bar{R} = 9.5128$

$Center\ Line = \bar{R} = 4.5$

$LCL_{\mathrm{R}} = D_3\bar{R} = 0$

b) The process is now in control with $\sigma = \frac{\bar{R}}{d_2} = 1.93$ from the appendix

$$P(x < +20) + P(x > +40) = P\left(z < \frac{20 - 33.65}{1.93}\right) + P\left(z > \frac{40 - 33.65}{1.93}\right)$$

$$P(z < -7.07) + P(z > +3.29)$$

$$F(-7.07) + F(-3.29) = 2 - F(+7.07) - F(+3.29) = 0.00050$$

$$C_p = \frac{USL - LSL}{6\sigma} = \frac{20}{6 \times 1.93} = 1.727$$

Ex 2

a) From the appendix table ($n = +4$) : $A_2 = 0.729$, $D_3 = 0$, $D_4 = 2.282$ and $d_2 = 2.059$

$\bar{\bar{x}} = 10.325$ and $\bar{R} = 6.25$

$\bar{x}$ Chart

$UCL = \bar{\bar{x}} + A_2\bar{R} = 14.881$

$Center\ Line = \bar{\bar{x}} = 10.325$

$LCL = \bar{\bar{x}} - A_2\bar{R} = 5.768$

R Chart

$UCL = D_4\bar{R} = 14.262$

$Center\ Line = \bar{R} = 6.25$

$LCL = D_3\bar{R} = 0$

The process is in statistical in control. $\sigma = \frac{\bar{R}}{d_2} = 3.035$

$$C_p = \frac{USL - LSL}{6\sigma} = \frac{50 - (-50)}{6 \times 3.035} = 5.49$$

The process is capable

$$P\left(z < \frac{-50 - 10.325}{3.035}\right) + P\left(z > \frac{50 - 10.325}{3.035}\right)$$

$$P(z < -19.87) + P(z > +13.07) = 0$$

Ex3

a) From the appendix table ($n = +5$) : $A_2 = 0.577$, $D_3 = 0$, $D_4 = 2.114$ and $d_2 = 2.326$

$\bar{\bar{x}} = 10.9$ and $\bar{R} = 63.5$

$\bar{x}$ Chart

$UCL = \bar{\bar{x}} + A_2\bar{R} = 47.53$

$Center\ Line = \bar{\bar{x}} = 10.9$

$LCL = \bar{\bar{x}} - A_2\bar{R} = -25.73$

R Chart

$UCL = D_4\bar{R} = 134.239$

$Center\ Line = \bar{R} = 63.5$

$LCL = D_3\bar{R} = 0$

The process is in statistical in control. $\sigma = \frac{\bar{R}}{d_2} = 27.3$

$$C_p = \frac{USL - LSL}{6\sigma} = \frac{200}{6 \times 27.3} = 1.22$$

So the process is capable

Ex7

$n = 6$ items/sample; $\sum_{i=1}^{50} \bar{x}_i = 2000$; $\sum_{i=1}^{50} R_i = 200$; $m = 50$ samples

(a)

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{50} \bar{x}_i}{m} = \frac{2000}{50} = 40; \quad \bar{R} = \frac{\sum_{i=1}^{50} R_i}{m} = \frac{200}{50} = 4$$

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} = 40 + 0.483(4) = 41.932$$

$$\text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R} = 40 - 0.483(4) = 38.068$$

$$\text{UCL}_R = D_4\bar{R} = 2.004(4) = 8.016$$

$$\text{LCL}_R = D_3\bar{R} = 0(4) = 0$$

b) **Natural tolerance limits**

$$\bar{\bar{x}} \pm 3\sigma = 40 \pm 3\left(\frac{4}{2.534}\right) = [35.264, 44.736]$$

$$C_p = \frac{5-(-5)}{6\times1.5.79} = 1.056 \text{ so the process is not capable}$$

c) **Scarp**

$$P\left(z < \frac{36-40}{1.579}\right) = P(z < -2.533) = 0.0057$$

d) **Rework**

$$P\left(z > \frac{46-40}{1.579}\right)$$

$$P(z > 3.799) = 0.00007$$

e) **Center the process at 41 not 40 and reduce scrap and rework costs. Second reduce the variability by:**
The natural process tolerance limits are, closer, say to $\sigma = 1.253$