III USING IT TO DESIGN

We began utterly wrong in England, and we have gone on wrongly, and the consequence is that it is only the exceptional person who learns to draw very well. Now in my experiments I have reversed that process, and I find that not only does every person when he is taught rationally, and intelligently in the same way that he is taught Latin, and Greek, and mathematics, learn to draw well, but also to paint well, and to design well. But it is on a wholly different principle from that on which he is taught here in England. . . . We have developed an intellectual method of teaching drawing, more industrial, more practical, more artistic, and infinitely more successful. . . . I propose to show how art education can be as sensibly treated as Latin and mathematics. I have a great wealth of illustration here, which, if you allow me, I shall be very happy to submit to you. Probably it would interest you if I showed you some of the actual work of the children in the public schools of Boston and Massachusetts.

—Walter Smith

Design Is Calculating

Metaphors are good heuristics. In fact, there's this metaphor for metaphors and heuristics. And there's also the one I started with

(1) design is calculating

as something to try and to prove. An alternative to it is an equivalence when I use the mathematics of part II. It's this

(2) drawing is calculating

where drawing is both seeing and doing. Whenever I put pencil to paper, I'm calculating with shapes or symbols. But there's nothing to code in a drawing, so I don't have to use symbols in place of shapes to calculate. Shapes are fine by themselves without underlying descriptions or representations. In the introduction, I said that I wanted to do this book in a rigorous way with shapes and no words. Now I can, and in a sense, this part is a first try—at least there are a lot of drawings that are all just as rigorous and formal as symbols and code. Shapes in rules show how to change shapes in an open-ended process for drawing and design. This is to see, but it's also included logically in statements 1 and 2. They combine in the formula

(3) design is drawing

and I like to think that the corollary

(4) design is calculating when you don't know what you're going to see and do next

follows, as well. This completes the transition from the mostly verbal discussion in the introduction through the unfolding visual argument in parts I and II to the mostly visual presentation now. It's going from calculating by counting to calculating by seeing with the shifting (subjective) viewpoints this implies.

Statement 4 pretty much sums up how I want to approach design with shapes and rules. I've been trying to show that calculating with shapes and rules is inclusive enough to deal with anything that might come up in design when it's done visually. That's the reason embedding and transformations are needed to apply rules. First, the embedding relation—what you see is there if you can trace it out, no matter what has gone on before. Second, the transformations—what you see is like given examples of what to look for, maybe things that were noticed in the past and used. And together—embedding and transformations interact as rules are tried to calculate with shapes.

The details aren't too far from drawing when you pay attention to what you're seeing and doing. The trick is to slow this process down a little bit to describe what's going on in a mechanical way. It's easy to say and, more important, easy to see. The beginning isn't much—start with any shape

C

Intuitively, there are lines on paper, but C can also include points, planes, solids, labels, weights, etc. Shapes can have any dimension you please, both in terms of basic elements and how they're combined. In drawings, lines are one dimensional and located on planes that are two. It all depends on the algebra in which you calculate. Then a rule

$A \rightarrow B$

that shows two shapes A and B applies to C whenever

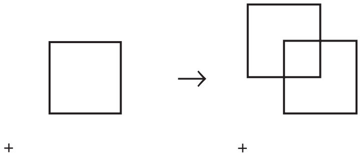
$t(A) \leq C$

that is to say, there's a transformation t such that the shape $t(A)$ is part of (embedded in) C. The rule $A \rightarrow B$ is an example of what I want to see and do, and $t(A)$ is some part of C that catches my eye because it looks like A. The result of this is another shape

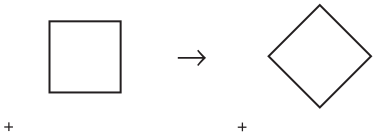
$(C - t(A)) + t(B)$

The shape $t(A)$ is taken away from C, and the shape $t(B)$ that looks like B is the new part that's added back. This is the same drawing with pencil and paper, although replacing parts with new ones isn't set out explicitly. The beauty of the process—with rules, or pencil and paper—is that what I do now doesn't restrict what I see next. Shapes fuse and divide freely, whether or not they're drawn according to rules.

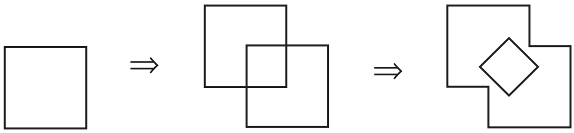
For example, I can add a square to a square with the rule



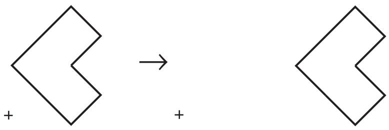
and then rotate a third square



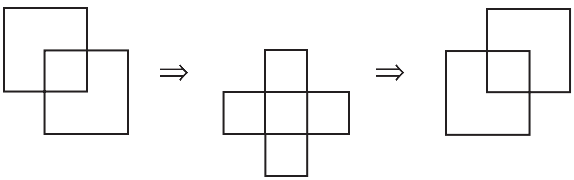
that's neither one of the two squares I put together originally



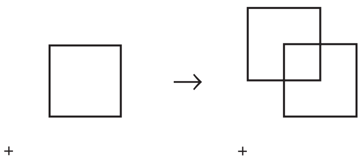
Or I can translate a chevron



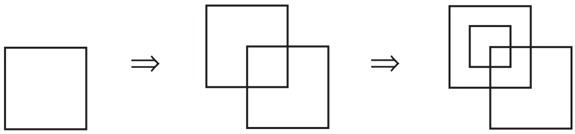
to make a cross, and move another chevron in the opposite way to rotate squares



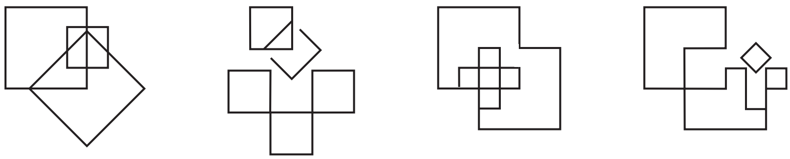
But my rule for adding squares



does more when I apply it in another way



Then what I'm doing can repeat with plenty of surprises—here are four of them



It's a lot easier to follow this with your eyes than to be consistent about it in words. Certainly, I haven't succeeded, fooling around with the squares I add and the squares I see and moving chevrons that pop up in unexpected ways. It's all a little crazy when I can change what I see and do this freely. It's hard to know what to say that isn't misleading or wrong. And really, that's the whole story. I can use rules to change shapes without being consistent about what I see. This doesn't sound like calculating, but it is. And it's what you need to design, so that you can change your mind about what you see and do as you go on. What it shows is that statement 4 isn't that far-fetched—

design is calculating when you don't know what you're going to see and do next

What a shape is depends on what rules are used, and when and how. This can vary for different rules, and, in actual fact, it changes every time any rule is tried. Yet there are some other things to consider that open up calculating even more.

As an example of what I want to see and do, the rule $A \rightarrow B$ may be too narrow. This was the problem in part II for squares and quadrilaterals. But there was a straightforward solution I could try. Instead of drawing rules or pointing to specific shapes, I could use schemas. This made calculating easier in some nice ways without compromising the embedding relation.

The new setup is pretty much the same as the one I have for rules, with the addition of variables, assignments, and predicates. There's a shape

C

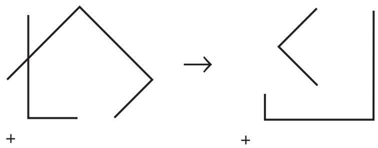
and now a schema

$x \rightarrow y$

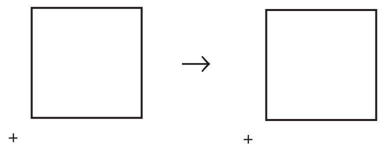
where x and y are variables that take shapes as values. These are given in an assignment g that may be restricted in some way by a predicate. The assignment defines the rule

$g(x) \rightarrow g(y)$

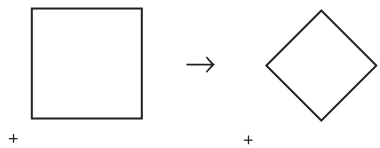
according to the predicate. This can go from any rule—then the values for x and y are simply shapes



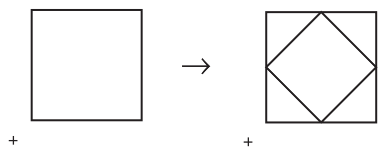
—to rules with certain prescribed properties. Maybe $y = x$ to define identities for polygons



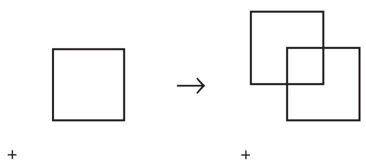
Or perhaps $y = t(x)$ for a transformation t of x



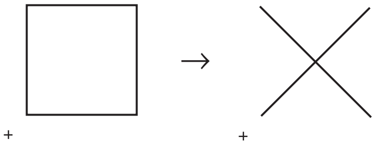
or $y = x + t(x)$



so that once a rule is applied to a part like x it can be applied again to a part like $t(x)$. This kind of recursion is neat, but it's not all that can happen in shapes, as the rule



demonstrates with the surprise square in its right side. And single rules are also possible—then x and y are given shapes, say, this square and its diagonals



that I draw or point to in the usual way to show what they are. But no matter what rule is defined, it applies exactly as my two formulas describe. Still, it may be worthwhile to say this for schemas directly. The schema $x \to y$ applies to the shape C whenever an assignment g and a transformation t define a shape $t(g(x))$ that's part of C

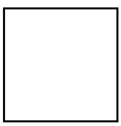
$$t(g(x)) \leq C$$

The result of this is another shape

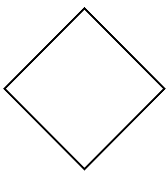
$$(C - t(g(x))) + t(g(y))$$

produced by replacing $t(g(x))$ with the shape $t(g(y))$. In most of the examples that follow, I'll be using schemas. I'll usually define them with one or two of the rules they determine under specific assignments with a short verbal description of the predicate involved, just as I've been doing. This avoids the need for a lot of technical details and lets me present everything visually. We'll just agree that predicates can be filled in as necessary. And I promise not to do anything where the details are mysterious.

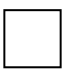
A useful way to explain my formulas for schemas is to notice that the composition $t \cdot g$ in the expressions $t(g(x))$ and $t(g(y))$ generalizes the transformations. What it means for shapes to look alike can range very broadly and be decided in all sorts of different ways. I tend to show examples that depend only on Euclidean transformations—for example, shapes are alike if they're copies of the square



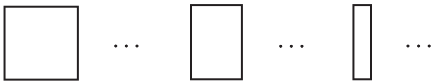
either because they're congruent



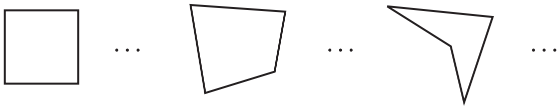
or because they're geometrically similar



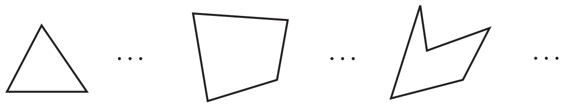
but really anything will do. All rectangles may be alike



or quadrilaterals



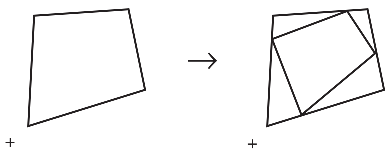
or polygons



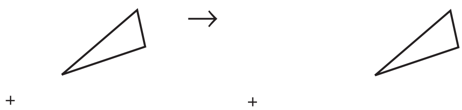
In fact, I can make the predicate that defines the assignment g as wild as I please. Maybe x in the schema $x \to y$ is a blob of some special kind

or any simple closed curve—whatever makes sense when you look is OK as long as there's a general transformation $t \cdot g$ for it.

It's also worth repeating that the predicates I use to define rules needn't be preserved as I calculate. I can put two quadrilaterals together



not unlike adding polygons and their transformations in the schema $x \to x + t(x)$, and then move triangles



according to the schema $x \to t(x)$. Predicates are simply a convenient way of describing rules—a way that doesn't carry over to calculating. Then descriptions don't count. There's no reason for me to be consistent about the things I see. Consistency is at root conservative—insisting that the language I've used in the past is kept as I see and do more makes calculating much too rigid. Certainly, it's logical—words make sense, so that things stay the way you say they are, and behave properly in the way you expect them to—but it isn't necessary. Why should words trump shapes when their parts can change freely in an open-ended, creative process? There's plenty of ambiguity and the paradigm (gestalt) switches that go along with it—what you see is what you get, any time you try another rule. And you can go on from whatever it is you see without ever thinking about it. There's always more to do. Ambiguity is to use. There's no reason to hesitate—

No one ever says ''Stop!'' Nothing blocks my way. Whatever I see is OK. Neither descriptions nor representations of shapes are needed to calculate—they don't intrude when rules are used to see and do.

So far, so good, but what about design?—that's what I want to use rules and schemas for. The problem here is to convince you that design is calculating. This is harder than it looks—not because it's difficult to produce designs by calculating, but because whatever I do may not be what you call design. Design varies so widely and unpredictably from person to person and from time to time that any calculating I do isn't likely to cover very much of it. Calculating to make designs will just have to do, with the promise that if you want to do it in another way, then shapes and rules are up to the task. And this doesn't mean starting over from scratch every time you have another idea or want to try something new, but rather adding rules or relaxing the ones you've already got to allow for more, so that designing, like seeing, is an open-ended process. This seems OK, but maybe I should look for a way to prove my claim that design is calculating. Well, this probably won't work—it's more a question of confirming a thesis in ongoing practice, and, in fact, there's a strong precedent for this in the argument that algorithms are Turing machines. I like to think that there's plenty of solid evidence in my examples. I'll trace these in a kind of personal history that shows some of the things I've tried to design by calculating. Most of what I say is pretty impressionistic, although not because I have to be so. The details are given elsewhere, so they needn't be a distraction here. It's more important now to emphasize key ideas that run through what I try and how they're expressed in rules and schemas. There's a lot to see that shows what rules can do, and most of it is just like drawing—but surely, this is already evident from statement 3

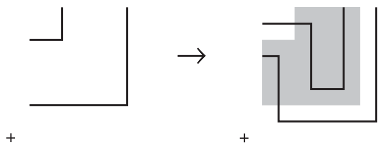
design is drawing

and what follows in statement 4 for shapes of all kinds.

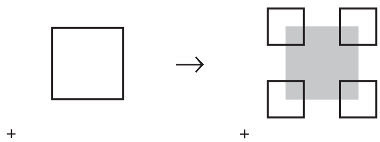
Tell Me What Schema to Use

I started out with shapes and rules to generate paintings that were mostly about seeing—designs like this one

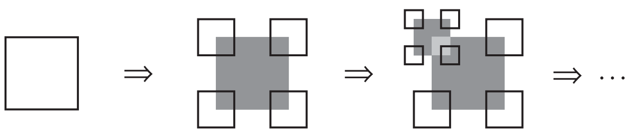
It's defined in an algebra $U_{12} + W_{22}$ for shapes made up of lines, and planes with weights—here, weights are colors that change from dark to light as they add up to produce an ambiguous kind of layering. My original rules were a little different from the rules I use now. They separated shapes and colors in order to generate designs, but in concert they worked pretty much as the rule



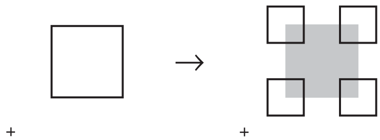
The idea was to connect squares in continuous paths that followed one another and intertwined. This elaborated the rule



that's much easier to describe in words without confusing what's going on. The rule colors square regions in a recursive process



The lines in the left side of the rule



fix the boundary of the square plane in the right side, and they're replaced with lines for additional squares at the four corners of the plane. This is expressed inclusively to allow for other shapes besides squares in the schema

$$b(x) \to x + t_0(b(x)) + \cdots + t_n(b(x))$$

It's a little too complicated for my taste, so breaking it down into a pair of easier schemas is helpful. First, I have

$$b(x) \to x$$

to produce areas or regions (maximal planes) from the sum of their boundaries, and then next comes

$$b(x) \to t_0(b(x)) + \cdots + t_n(b(x))$$

to replace a shape with a number of geometrically similar copies. But I've already shown this in summary form

$$x \to \sum t(x)$$

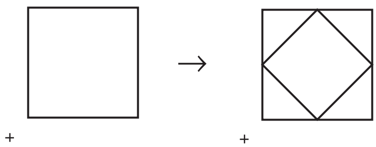
for fractals. In fact, the design

was inspired by David Hilbert's famous curve. More generally, though, I was applying two easy schemas from part II—schemas that did a lot of other things, as well, and that I still use today. This is something to keep in mind. The same schemas can be used to produce widely different results in different contexts, although the calculating doesn't change. This provides a positive reply to students when they try to calculate with shapes and rules for the first time—"What schema should I use?" It's a good question that experience finally answers. But, in the meantime, schemas and the rules they define can be shared and used in your own way—it's copying without repeating the past. There's something to learn in the classroom that works in the studio when students ask, "What should I draw?" Creative design can be taught like language and mathematics in school, with examples, rules, and practice and the opportunity to experiment freely. Rules make this possible because they apply to shapes in terms of embedding and transformations. But I'll come back to teaching a little later on, after I've presented more evidence to back up my claim that calculating with shapes and rules is creative.

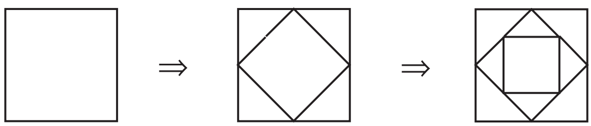
I've been talking about schemas and rules for my designs. I did a lot of them, but one of the key ideas that worked over and over again in surprising ways was to add shapes and their transformations. This is minimally expressed in the schema

$x \to x + t(x)$

that defines the rule



used for the shapes in the series



And, in fact, this provides for the six designs in figure 1 that keep to what I did with the schema $b(x) \to x + t_0(b(x)) + \cdots + t_n(b(x))$ to explore alternative ways of coloring regions. Whatever I've said about counting in the past, permuting colors is sometimes informative—as long as you know there's always more to see in another way. And as I went on, it seemed obvious that with all I could see and do with shapes and rules—my two examples merely begin to scratch the surface—designing and calculating weren't too far apart. But not everyone agreed.

The problem was evident—I was calculating to make designs in my own way. I could do anything I wanted and call it design. How about producing things that were designed by others with sensitivity and skill? What about some real examples that had the stamp of prior approval? Certainly, this wasn't possible, especially for things that weren't as regular and hard-edged as my designs. But, as luck would have it, I didn't have to search very far to go on. There were plenty of things to design with shapes and rules wherever I happened to look. It was surprisingly easy, and there were others to help who saw this, too. Design was calculating. But in fact I knew that, and I already had a way to go.

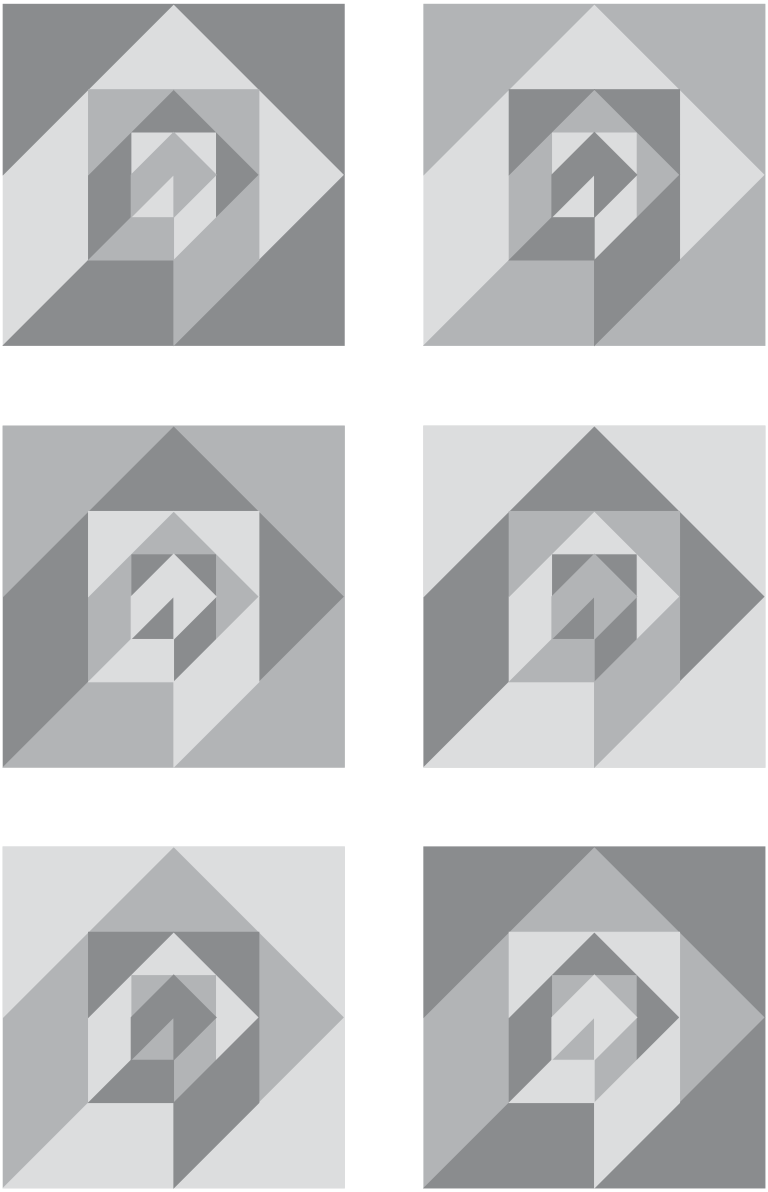
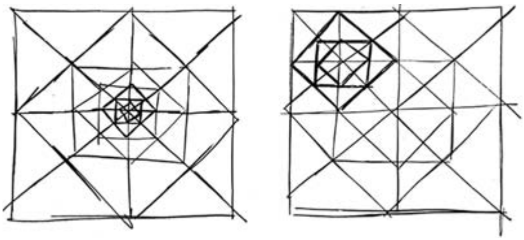


Figure 1

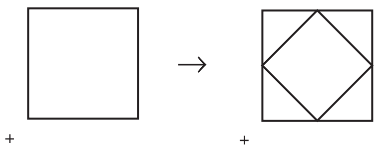
What the Thinking Eye Sees

After I tried schemas for my own designs and saw how well they worked to define rules, I tested them out on examples from Paul Klee's *The Thinking Eye*. I've already talked about the opening lessons in Klee's *Pedagogical Sketchbook* and the way they suggest a kind of visual calculating with shapes and rules. But there are more examples in *The Thinking Eye* of far greater variety. It's a perfect laboratory for schemas with exciting experiments ready to try.

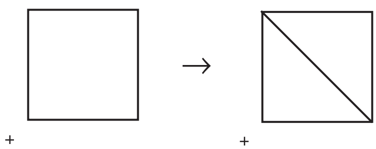
There are plenty of designs like these two



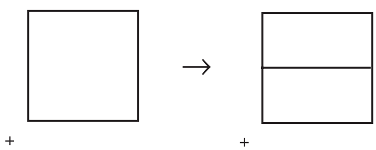
that I can get directly with rules. The rule



that's defined for squares in the schema $x \rightarrow x + t(x)$ is already familiar. And two additional rules introduce the idea of dividing shapes or the regions they bound—here, for the diagonals of squares



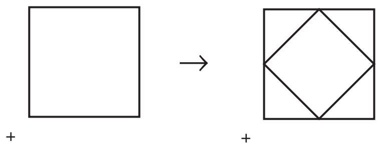
and for horizontal or vertical cuts



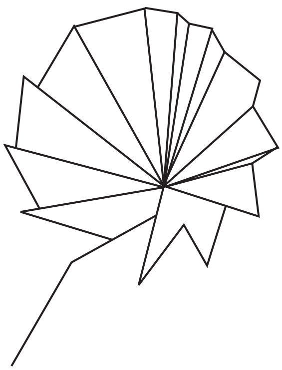
This implies division rules in a new schema

$x \rightarrow \mathrm{div}(x)$

—divide x—that extends my kind of calculating. In fact, $x \to x + t(x)$ and $x \to \text{div}(x)$ overlap in some nice ways. The rule



answers to both—its right side is the sum of two squares, or it's divided into four triangles and also a square. Together, the two schemas do a lot of neat things, and in the next section, I'll show this in a series of interrelated examples. But first I need a more typical example from *The Thinking Eye* where schemas are conspicuously required. There's something useful on almost every page—something surprising to see—but one design is especially right for how it looks and for what it shows. This is the "palm-leaf umbrella"

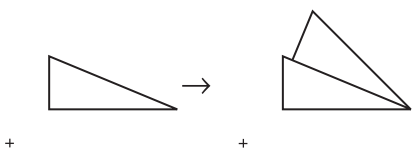


and it's within the immediate reach of the schemas I have right now, in particular, the pair of schemas above: $x \to x + t(x)$ and $x \to \text{div}(x)$. Only a couple of small changes are needed for the job. In many ways, schemas are like shapes—they're easy to change.

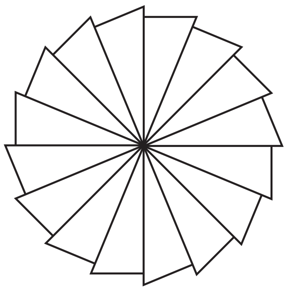
The palm-leaf umbrella is a playful design, yet Klee presents it under a serious rubric—

Irregularity means greater freedom without transgressing the law.

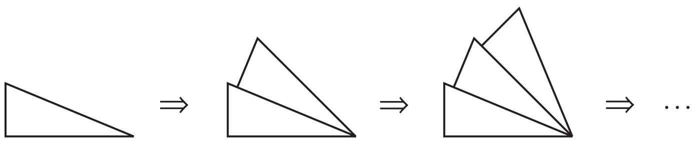
But is this the kind of freedom calculating allows? Certainly, calculating is a rigorous process that means following the letter of the law. The rule



is defined in the schema $x \to x + t(x)$, and it evidently produces a pinwheel that's cyclically symmetric



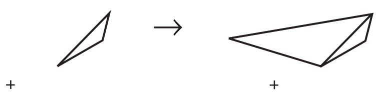
when calculating goes like this



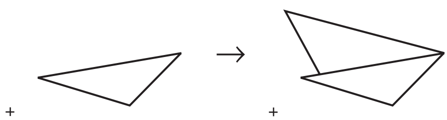
Not a bad design, easy to get, and a little something like Klee's palm-leaf umbrella. Nonetheless, everything seems far too regular in the way you'd expect when recursively applying a single rule that adds two congruent right triangles hypotenuse to side. But this is readily fixed in the new schema

$x \to x + x'$

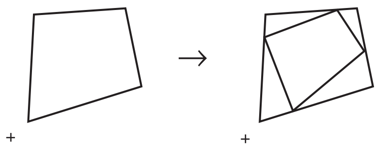
where x and x' are both triangles, and not always similar ones x and $t(x)$. (The schema is another version of the schema $x \to A + B$ for spatial relations. I showed how to use it for arbitrary shapes A and B, along with its inverse $A + B \to x$, in several examples in part II.) The rule



is defined in my new schema, and so is the alternative rule



In fact, I can show how all of the triangles in the palm-leaf umbrella are related—no surprise. This sort of parametric variation in rules is already allowed in my schema for quadrilaterals that includes the rule

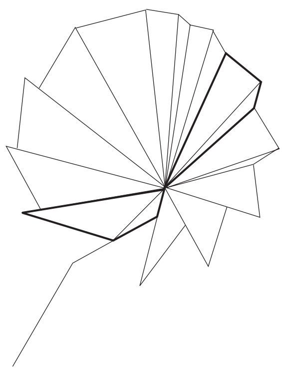


Perhaps the schema $x \to x + t(x)$ would be more productive if it were expressed in some other way that's a little more relaxed, maybe as

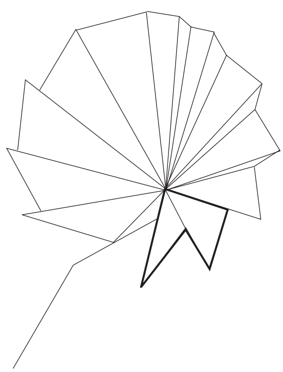
$$x \to x + t \cdot g(x)$$

in terms of a general transformation defined in a composition $t \cdot g$. There's probably some abstract algebra here that's worth pursuing. I'd have to be more explicit about what I wanted, but really that's an exercise to try. Right now, it makes more sense to go on calculating with its concrete results. The various rules in the schema $x \to x + x'$ give me the series of shapes in figure 2.

Still, the final shape I show in figure 2 isn't Klee's palm-leaf umbrella—there are also quadrilaterals



and a pentagon



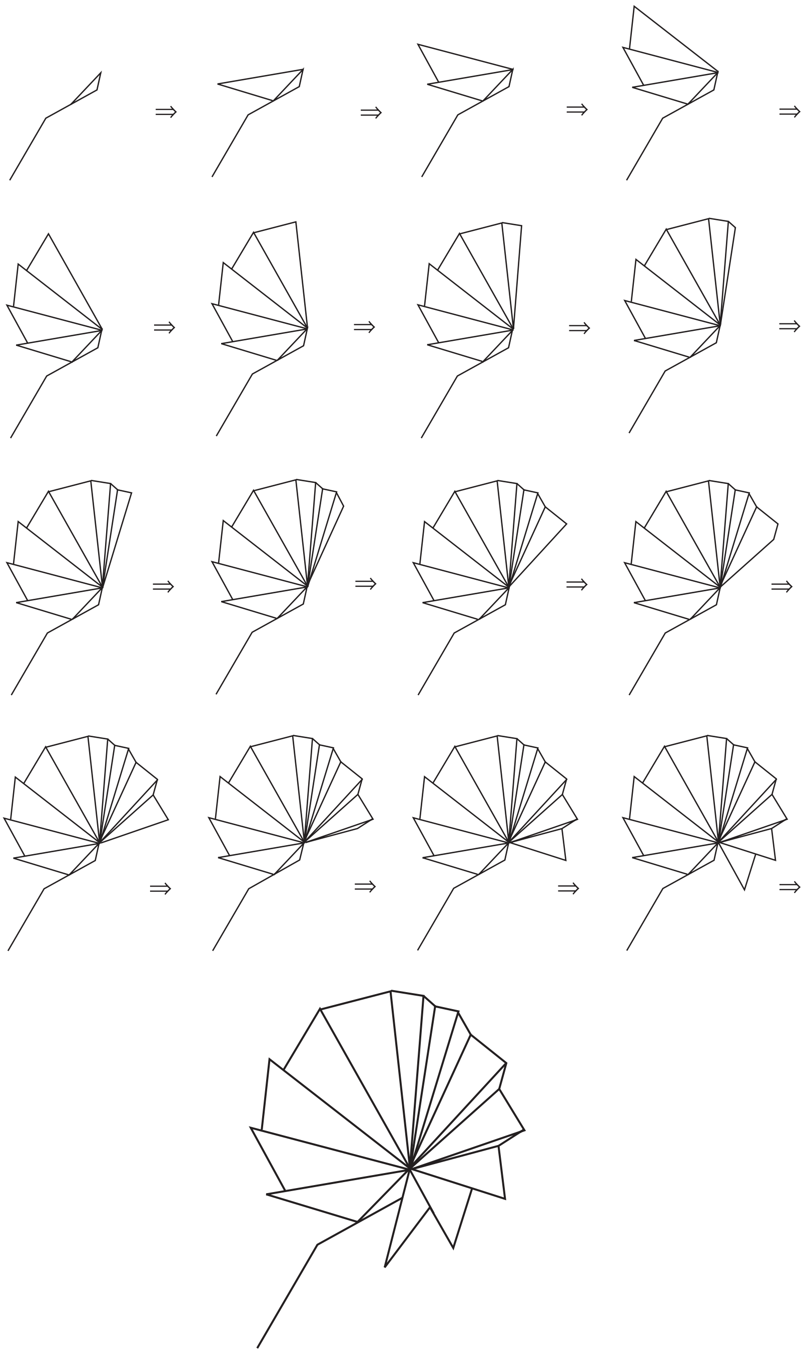
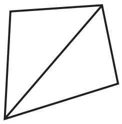
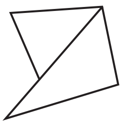


Figure 2

I seem to be stuck. But no, this isn't the end of it—I can do much more with my schemas to get exactly the results I want. Look at it another way and see how it works. The diagonal of a quadrilateral forms twin triangles



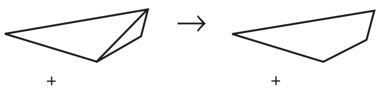
and it's the same for a pentagon with a concavity, at least in the special case where a diagonal and an edge are collinear



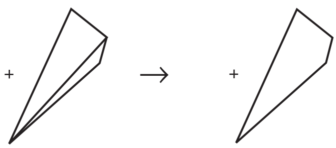
Shapes are always ambiguous, and the ambiguity is something to use. It seems that I already know how to go on. If I apply the inverse of my division schema $x \rightarrow \mathrm{div}(x)$, that is to say, the schema

$\mathrm{div}(x) \rightarrow x$

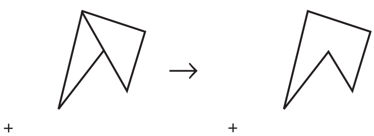
I can erase diagonals in polygons in just the right way. For quadrilaterals in the palm-leaf umbrella, I need the rule



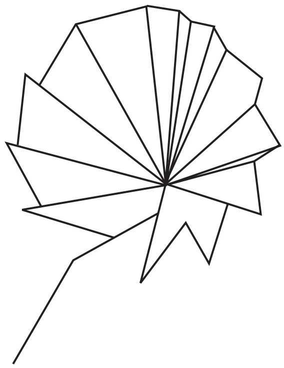
and this one



and for the pentagon, the rule



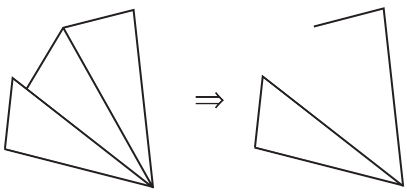
is fine. Inverses often come in handy—the schema $x \rightarrow b(x)$ determines the boundary of a shape with lines, planes, or solids, while its inverse $b(x) \rightarrow x$ works to color regions in my designs. And then there's the palm-leaf umbrella



But be careful. Inverses may not do what you expect. For example, the inverse

$x + x' \rightarrow x$

of the schema $x \rightarrow x + x'$ doesn't work for quadrilaterals and pentagons. When I use the rules it defines, I'm left with an ugly gap



Yet beauty is in the eye of the beholder—perhaps this outcome is good for something else. And I can always go on from anything I have with shapes and rules.

Working through Klee's drawing in this way isn't meant to recapitulate what he did—no one knows for sure, and it's no good relying on what he says. Rather, I'm trying to show that schemas are up to Klee's kind of designs. The rules I need are included in a handful of schemas that are easy to use. And the rules themselves are easy to apply to shapes in terms of embedding and transformations. Embedding ensures there's no letter of the law—I can calculate with triangles and see quadrilaterals and pentagons—and general transformations allow for as much irregularity as I want—parametric variation of any kind is possible. Rules and freedom go hand in hand—the one implies the other.

I'm staking the plausibility of the metaphor "design is calculating" as a heuristic and equivalence squarely on my success with schemas, especially as I'm able to use them more and more to calculate with shapes and to make designs in different styles from different places and times. This is another kind of freedom, and it provides a very practical kind of proof that calculating works. There's seeing and doing, and that's all it takes when rules apply to shapes. I guess "design is calculating" really is more of a thesis than a theorem. The evidence adds up, but not to logical certainty, just to empirical belief.

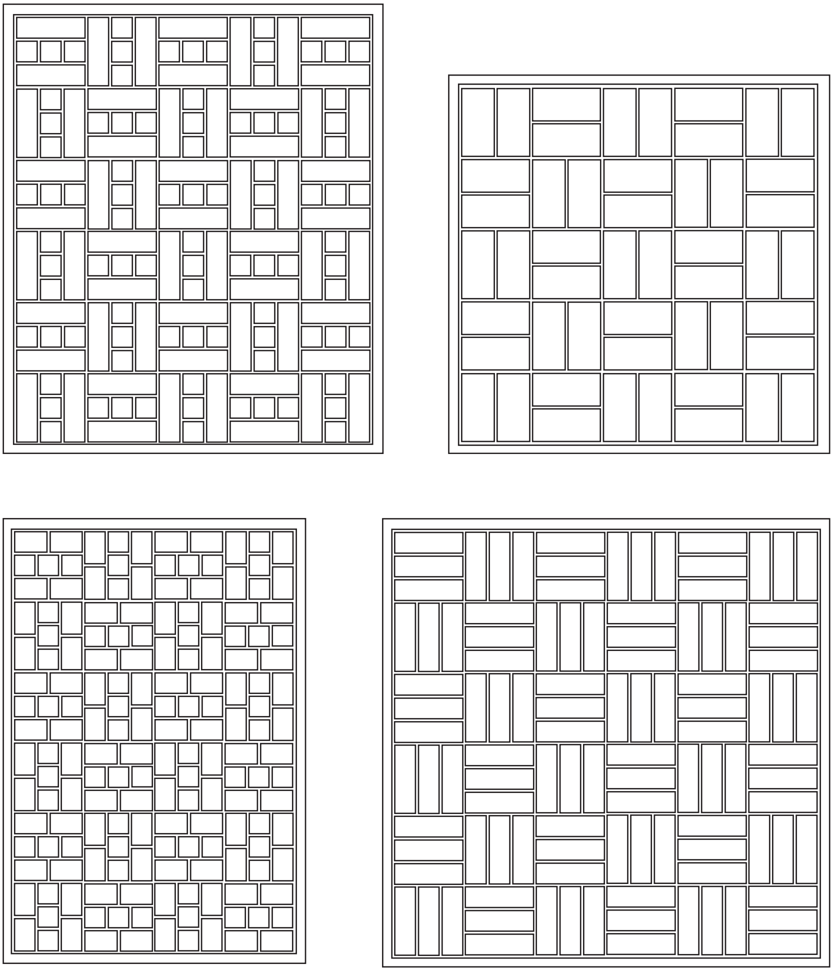
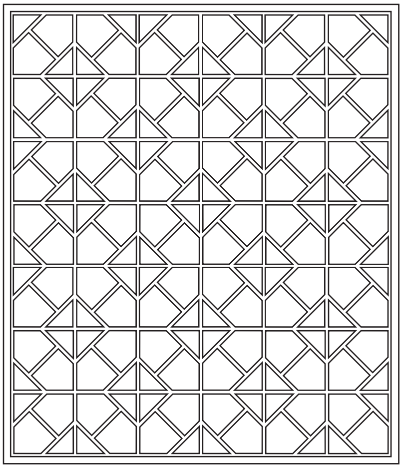


Figure 3

Chinese Lattice Designs—Seeing What You Do

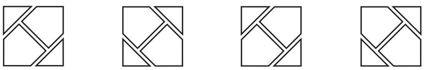
There are myriad ways schemas can be used to produce designs. I especially like the Chinese lattice designs that fill window frames. Most of these form regular patterns of the kind shown in figure 3. They're taken from Daniel Sheets Dye's *A Grammar of Chinese Lattice*. It's a wonderful and extensive catalogue that's hard to put down. It's a joy to see. And in fact, like *The Thinking Eye*, it holds the material for endless experiments with your eyes.

The four lattices I've shown are checkerboard designs like this one

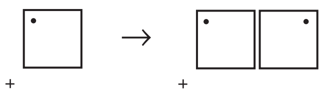


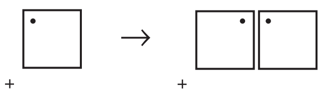
in which a given motif

is inscribed in squares in a rectangular array. The "H" alternates this way and that, left to right from side to side

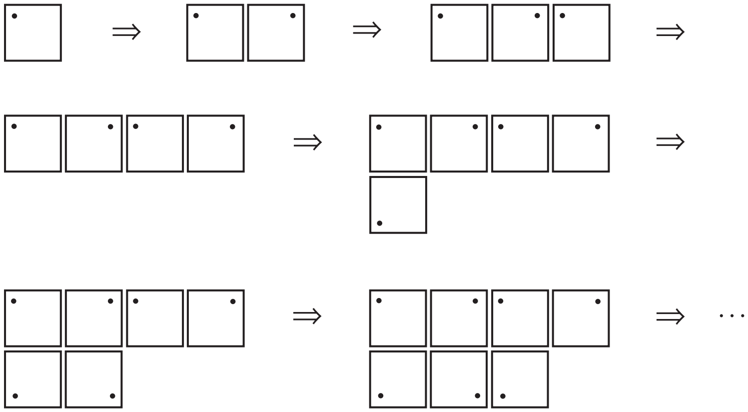


to correspond to contrasting colors or distinct labels (symbols). This makes it easy to use my schemas—first, $x \rightarrow x + t(x)$ to produce the checkerboard pattern, followed by $x \rightarrow \mathrm{div}(x)$ to inscribe the H and divide squares. There are two rules for the checkerboard

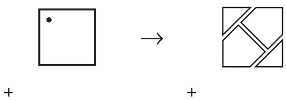
(1) 

(2) 

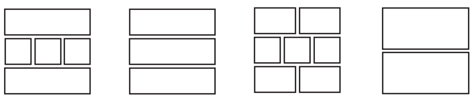
where x is a square and an asymmetrically placed point to orient the H with respect to left and right diagonals. And going back and forth from rule 1 to rule 2, I get the series of shapes



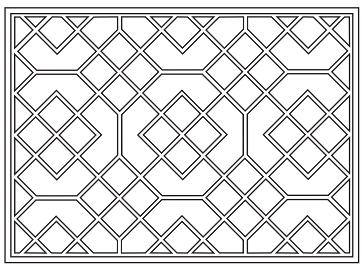
Then there's a single rule for the H to divide squares



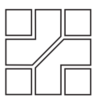
Of course, the H I've given isn't the only motif I can use. Squares are divided in many different ways in Chinese lattices to make checkerboard patterns. In the designs in figure 3, the motifs vary in the following way



to define new regions, and in this lattice



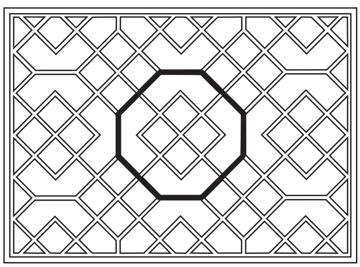
there's a surprising checkerboard pattern, too, when squares in a diagonal grid are divided into regions so



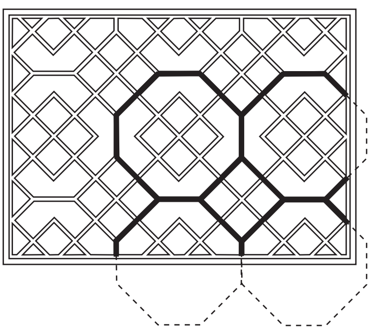
This raises an important issue. I began with a corpus of shapes—designs in a given style—and specified rules in my schemas to define the corpus and to produce other designs of the same sort, and nothing else. There's even an easy kind of stylistic change where I add rules to divide squares with new motifs. In many ways, this is all very convincing. I can recognize what's in the style—do my rules generate it?—and I can produce novel instances of the style that haven't been seen before. I can go from known to new, and I can say exactly what's happening in terms of the rules I try. And that's the problem. When I look at the lattice just above—or the one with the H motif, for that matter—I don't automatically see the checkerboard or how its squares are divided. In other rule-based systems in which shapes are represented in terms of symbols (vocabulary and syntax), this is just something to get used to—learning the "right" way to see according to given rules. There's no ambiguity. Nonetheless, I have a creative way to handle any discrepancies between the rules I apply to produce designs and what designs look like. This is what I set out to do at the beginning of this book with embedding and transformations, and the effort pays off handsomely when I calculate with shapes and rules. That's why the mathematics was worth doing and getting right in the first place. Now calculating seems almost too easy, and it may be something of an anticlimax. I simply augment my rules with identities defined in the schema

$x \rightarrow x$

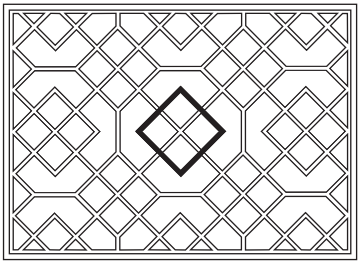
to pick out any other parts I see—according to Dye, "octagons"



"octagon-squares"



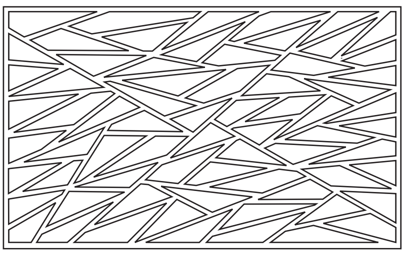
and "supplemental squares"

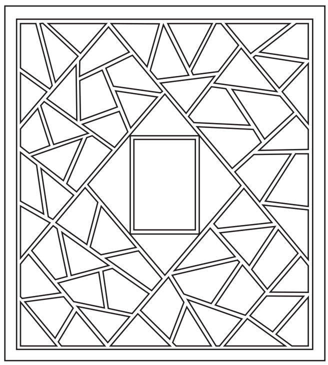


But, really, I can try any identity or rule—I'm free to see and do as I please.

A strictly generative account of style may not be enough. Saying what you see as you make what you want matters, too. Ambiguity isn't noise, it's something to use. And this is how shapes and rules work. I can always add the identities I need to see what's important and describe it independent of the rules I apply to produce designs. Embedding and transformations make this possible once shapes fuse, so that generative and descriptive aspects of style are on equal footing and can change with ongoing experience. Understanding a style is more than connoisseurship and forgery. There are other things to see and to say, as well, that go beyond recognizing instances and copying them in a particular way, or branching out to make new ones. And I needn't miss any of this when I calculate with shapes and rules.

There's always more to see and do. Not all of the lattice designs in Dye's *Grammar* are regular patterns. In particular, there are marvelous "ice-rays"

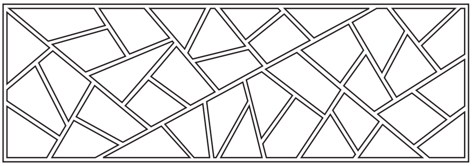




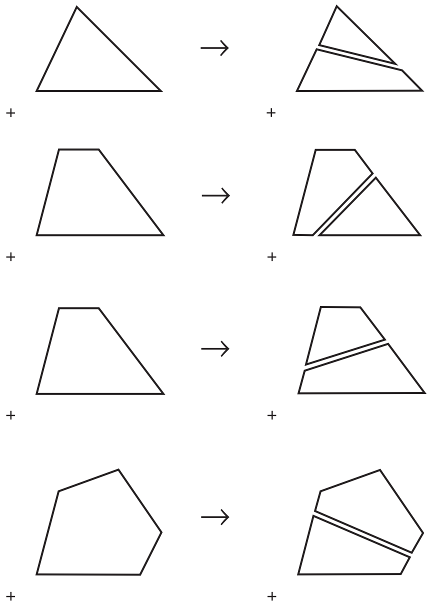
To appreciate [these] designs ... one needs to see ice forming on quiet water on a cold night. Straight lines meet longer lines, making unique and beautiful patterns. The Chinese term this *ice-line*, or lines formed by cracking ice; I have described it as the result of a molecular strain in shrinking or breaking, but more recent observations and photographs seem to prove that it is a conventionalization of ice-formation which has become traditional.

And by now, it shouldn't be a surprise that ice-rays are made by calculating.

Let's try the ice-ray lattice



It's one of my favorites—in this case because what I see looks hard to do, but isn't. I need four kinds of rules like these



perhaps from the schema $x \rightarrow \mathrm{div}(x)$ to divide triangles, quadrilaterals in alternative ways, and pentagons. Then it's one more case of parametric variation using general transformations $t \cdot g$. Dye summarizes the process nicely—

In the case of the ice-ray pattern [the artisan] divides the whole area into large and equal light spots, and then subdivides until he reaches the size desired; he seldom uses dividers in this work.

Or I might think of my rules in another way in terms of a fractal-like schema

$x \rightarrow x' + x''$

where one polygon x is replaced by the sum of two, x' and x''. Whatever I do, it's the same—visually, there's the series of shapes in figure 4. Maybe the series is excessive and even a little indulgent—I like to draw—but it seems to me that it's worth seeing once for ice-rays that rules work throughout; and after all, I'm only showing half of

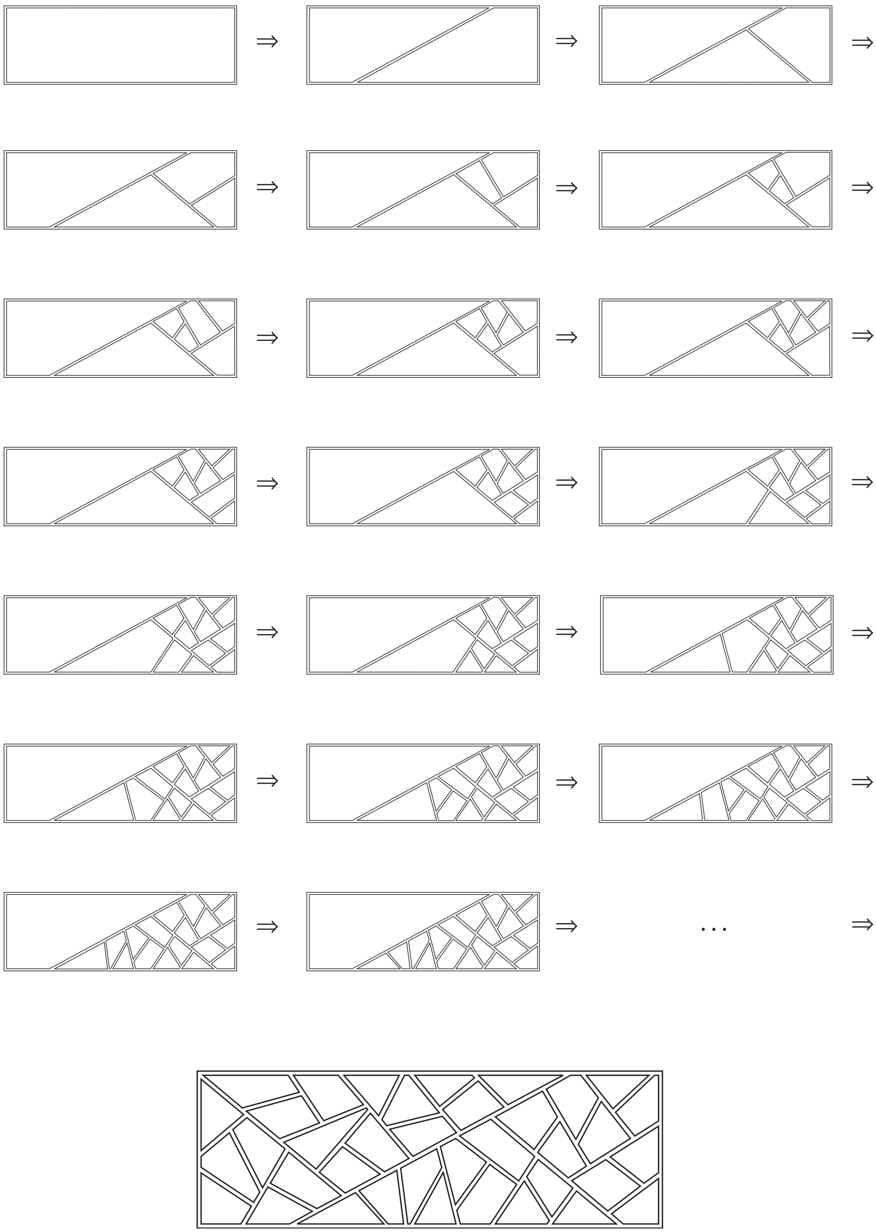


Figure 4

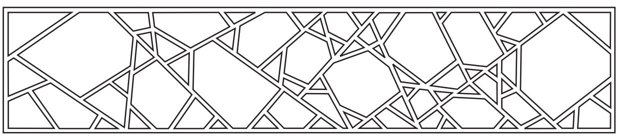
the process. In fact, the process isn't hard to describe. When I first thought about making ice-rays, I had Dye and the schema $x \rightarrow \mathrm{div}(x)$ in mind. This is what I said—

One can imagine a Chinese artisan, summoned to a building site, bringing with him tools and implements and a collection of finely finished sticks. Shown a rectangular window frame, he is asked to create an ice-ray lattice. He begins his design by selecting a stick of the appropriate length and carefully attaching it between two edges of the existing rectangular frame, thus forming two quadrilateral regions. He continues his work by subdividing one of these areas into a triangle and a pentagon. He further divides the triangle into a triangle and a quadrilateral; he divides the pentagon into a quadrilateral and a pentagon. Each subdivision is made in the same way: attach an appropriately sized stick between two edges of a previously constructed triangle or quadrilateral or pentagon, so that it does not cross previously inserted pieces. Each stage of the construction is stable; each stage follows the same rules.

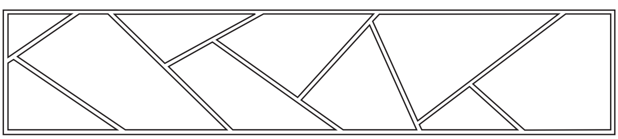
Ice-rays make it easy to be a ''rationalist''—to believe that available technologies, properties of materials, and methods of construction determine designs. And, to some extent, these constraints do matter as they're expressed in schemas and rules. Rules allow for freedom and constraints—going back and forth from one to the other is always possible. In a way, though, it's ironic. Many times, the truly hard problems are to fix constraints that aren't immediately spatial, for example, so that rules apply in the right logical or temporal sequence. But the seemingly effortless switch from constraints to freedom via identities and other rules is more impressive. That's where embedding and transformations really make a difference.

I've tacitly assumed all along that rules are nondeterministic, that is to say, they can be applied in various sequences under alternative transformations. This is the source of creativity, in Chomsky's sense, for vocabulary and syntax when different things are produced combining the same constituents, and there's a kind of ambiguity, too, when the same thing results in different ways. But shapes and rules give much more. Creativity and ambiguity have an Aristotelian origin, as well, that's evident when I use the schema $x \rightarrow x$ to define identities. Then I can divide shapes to see as I please.

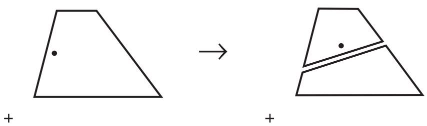
For lattice designs, there are unlimited opportunities for variety and novelty in a traditional style. Even so, rules may apply in more specific ways. For example, not counting the pair of leftmost divisions, the ice-ray



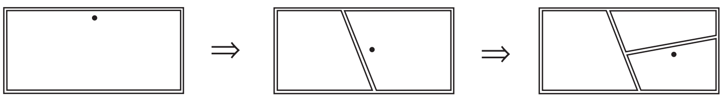
is produced from left to right



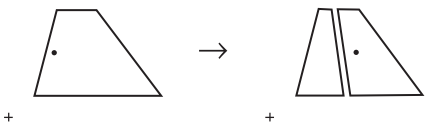
But this may be a fluke—lucky accidents and surprises where there's something unexpected to see are a wonderful part of art. Nonetheless, I can also use some familiar tricks from part II to make rules work the way I want, to guarantee reliable results. Perhaps it's important for the main divisions in ice-rays to be ''orthogonal'' as my example shows—then rules like this one



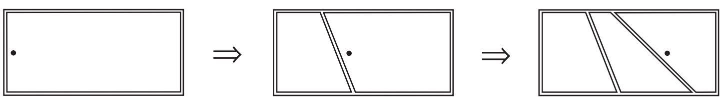
that incorporates a point will do the job



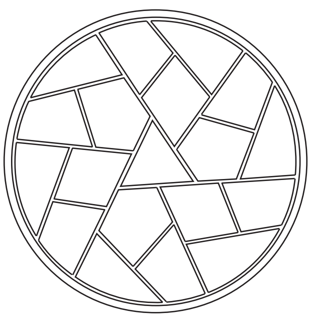
Or maybe ''parallel'' divisions that don't intersect are more desirable—then I can use a point in the rule



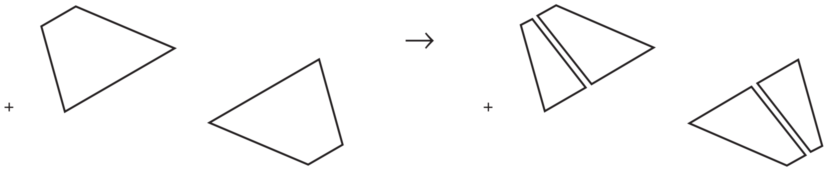
and in other rules like it to calculate so



And I can also give rules for lattice designs with other kinds of symmetry, for example, where divisions rotate in a nice way



It's easy to do this for any transformation—simply show how it works in both sides of a rule in the schema $x \rightarrow \text{div}(x)$. In particular, the rule



is good for a half-turn π. Then the compound schema

$$x + \pi(x) \rightarrow \mathrm{div}(x) + \pi(\mathrm{div}(x))$$

gives rules for any polygon x, even if it looks a little complicated. Still, there's no reason to worry. I can recast what the schema does in a two stage process in which the schema $x \rightarrow \mathrm{div}(x)$ is applied first to x and next to $\pi(x)$. This guarantees that $\mathrm{div}(x)$ and $\pi(\mathrm{div}(x))$ are both in the design as long as $\mathrm{div}(\pi(x)) = \pi(\mathrm{div}(x))$. Or equivalently, there's the schema

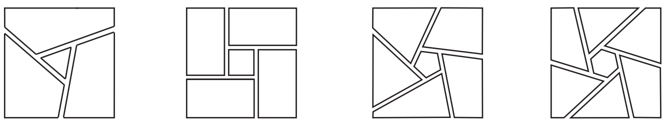
$$x + \pi(x) \rightarrow \mathrm{div}(x) + \mathrm{div}(\pi(x))$$

But in fact, $x \rightarrow \mathrm{div}(x)$ alone may really be enough—simply let

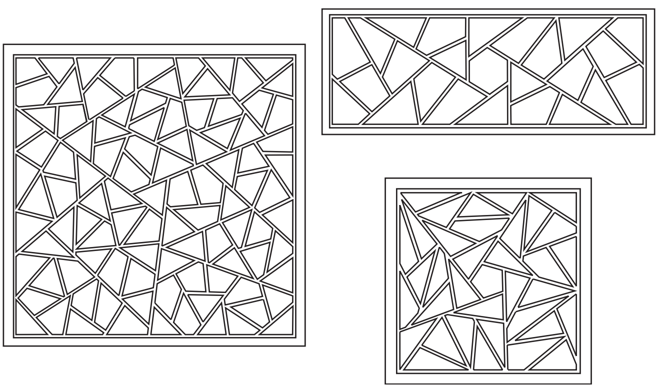
$$x + \pi(x) \rightarrow \mathrm{div}(x + \pi(x))$$

I can always divide more than polygons. Schemas and the rules they define work with remarkable ease.

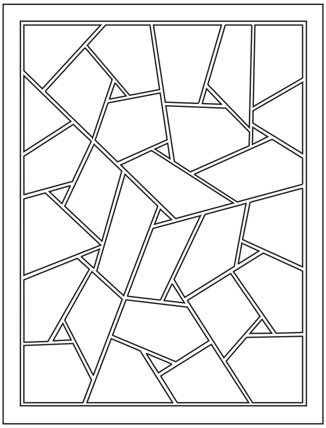
Of course, there are other ways to change ice-rays that are of equal interest. Just as checkerboard lattice designs contain different motifs, ice-rays can be divided in alternative ways. In particular, there are axial motifs with three to six arms



Usually, these devices are used once centrally to start

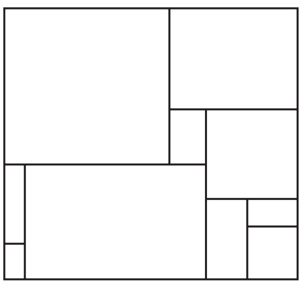


but occasionally they repeat in some delightful ways

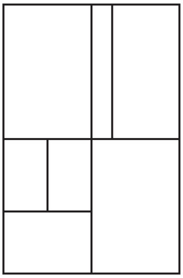


With shapes and rules, it's easy to add new motifs and to take advantage of what they do. It requires nothing more than a drawing to show what I want to see and do. That's how rules are defined. And that's all I need to calculate.

Earlier on, near the beginning of this part, I said that the same schemas worked in different contexts to produce different results. The schema $x \rightarrow \mathrm{div}(x)$ that I've been using to divide regions in ice-rays is a perfect example of this. I've already shown that the schema is good for other kinds of lattice designs and, as an inverse, for Klee's palm-leaf umbrella. But more conspicuously, it works in various algebras for painting and architecture. The painter Georges Vantongerloo did a large number of designs like this one



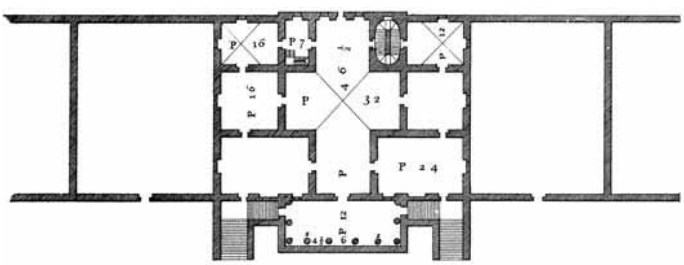
in which he starts with a pinwheel division and then makes orthogonal cuts. No one would ever call it an ice-ray, but it really is. And then the Portuguese architect Alvaro Siza relies on the same idea in designs for floor plans in his ongoing housing project at Malagueira—about twelve hundred units to date that repeat at least thirty-five plans. Here's one design that's produced with a biaxial cut followed by three more divisions



Ice-rays can be used widely in many different ways; but, of much greater importance, schemas can, too. This is certainly prima facie evidence that design is calculating, although showing that calculating pays off in studio teaching and creative practice is the real test. Professional disciplines are apt to have stubborn customs and recalcitrant standards—that keeps them exclusive. And art and design are hardly exceptions to this rule, so it helps to have more to go on before asking artists and designers to try something new that breaks with traditional methods of teaching and practice. Whatever they are, teaching and practice simply can't be calculating with shapes and rules. No—calculating is never ever creative.

They're Shapes before They're Plans

I'm going to do Andrea Palladio's famous villa plans, with the Villa Foscari as my main example



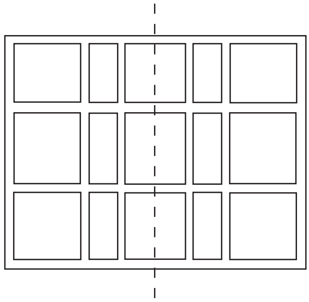
They form a style apart, vital and never diminishing in the capacity to entice others into emulation, but evasive of definition.

I'm interested in Palladio, just as I'm interested in Klee and Chinese window grilles, and my paintings, as well, to show that design is calculating. But of greater importance for buildings is that functional, social, and aesthetic relationships—things in the architect's Vitruvian canon of firmness, commodity, and delight—aren't beyond shapes and rules. Many find this surprising, even though these relationships are expressed in form. They're all about shapes, so calculating according to rules makes sense. However, I'm not going to do Palladio's system of proportions as part of what I show. Numerical relationships are easy in my schemas and rules, yet calculating with numbers doesn't buy anything new and may give the wrong impression. Rather, it's the look of Palladian villa plans—this is more than the usual stuff of calculating—that's my central concern. But first, notice a few additional things that this isn't about. Architects can be remarkably insecure when it comes to what they do, so it's worth trying to avoid common misconceptions from the start. Some ambiguity is lost in prefatory disclaimers, although there may be more to gain in other ways. What I'm doing isn't a commentary on contemporary architecture and how it's practiced, or an argument for classical principles of building or for the plan as a method of designing. Nor is my interest in Palladio historical. I don't know what he did to get his villa plans, although he's pretty good at saying how to design them in *The Four Books of Architecture*, and that's really as far as my scholarship goes. What Palladio draws, not what he builds, is what I'm interested in—plans are shapes before they're plans. That's why I can calculate with them. But the technical devices I use aren't necessarily the same as Palladio's—showing how to calculate always takes precedence, even if the results I get and the steps along the way are like his in many ways. My only stake in this is to show more about calculating with shapes and rules, and that calculating includes design, at least to the extent that I can make convincing plans without too much fuss. In fact, the schemas and rules I use are not unlike the ones I've used multiple times before. This buttresses my repertoire of schemas and recommends them again for teaching and practice—in particular, it extends what I can do in the classroom and how this works creatively in the studio. My way of calculating with shapes and rules is always open-ended. Knowing how to design something spatial—Palladian villa plans—may go farther than you expect.

It seems intuitive enough that there's more to villa plans than there is to Chinese lattice designs to make my case that design is calculating. Yet it's difficult to be sure about judgments like this when every shape of dimension one or more—even a single line

—has indefinitely many parts. There's a lot going on in shapes, all of them, that encourages wandering around in an aimless way. You never know how it's going to turn out. What there is in a line is always a surprise. And in fact I've been arguing from the start that being able to handle whatever comes up—to deal with the ambiguity—is a prerequisite if calculating is going to be of any use in design. It's calculating by seeing, yet once you can do it, you can also proceed with a plan in mind. Sometimes, it helps to calculate in stages—now, so that villa plans are produced in a straightforward manner that's easy to follow and explain. Four stages work perfectly for the schemas and rules I want to use, and they make good architectural sense—there are (1) walls, (2) rooms, (3) porticoes, and (4) windows and doors. What works for one thing, though, shouldn't be taken too far in a different direction. This is a sequence just for calculating, and it doesn't imply a hierarchy in plans. Hierarchies and such are always possible in other ways, for example, retrospectively in my topologies, but they aren't something I'm going to pursue seriously here. Defining villa plans is my only practical concern, and it's a must before I can try anything else.

This is the grid of walls



for the Villa Foscari, with an axis running through the middle column of rectangles. Bilateral symmetry is a characteristic feature of villa plans, and it's preserved in everything I do.

The rooms ought to be distributed on each side of the entry and hall: and it is to be observed, that those on the right correspond to those on the left, that so the fabrick may be the same in one place as in the other.

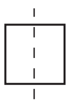
I can define the grid in terms of the schema

$x \to x + x'$

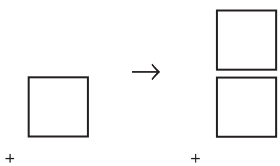
where x and x' are rectangles with dimensions that fit in Palladio's system of proportions. And the trick I used for symmetric ice-rays works perfectly again, to reflect the schema in this way

$x + R(x) \to x + x' + R(x + x')$

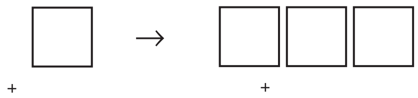
so that both sides of the grid are produced at the same time. Rectangles are added either vertically or horizontally. This is evident in two separate cases. First consider what happens when x is on axis, so that $x + R(x) = x$



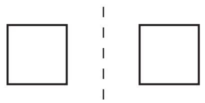
Then, either $x' + R(x') = x'$, as in the rule



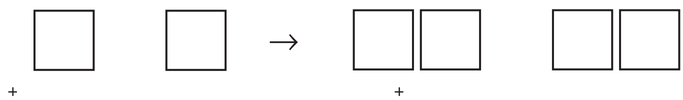
that builds a column of rectangles, or this isn't so, and I have a rule



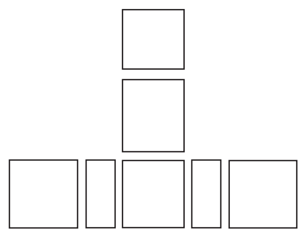
to start rectangles in a row. And second, it's possible for x to be off axis, so that $x + R(x)$ is



with the result that



to continue a row of rectangles. And all of this comes together to arrange rectangles in an inverted T



But I still need rules to fill in open areas, and to add a bounding rectangle. Two schemas do the job, and they make a good exercise—draw what you see and then what you want to specify the left and right sides of a rule of the kind you need. I'll say a little more about defining schemas in this way later for the Villa Barbaro, with a drawn example. In the meantime, there's something else I need to do of more immediate concern.

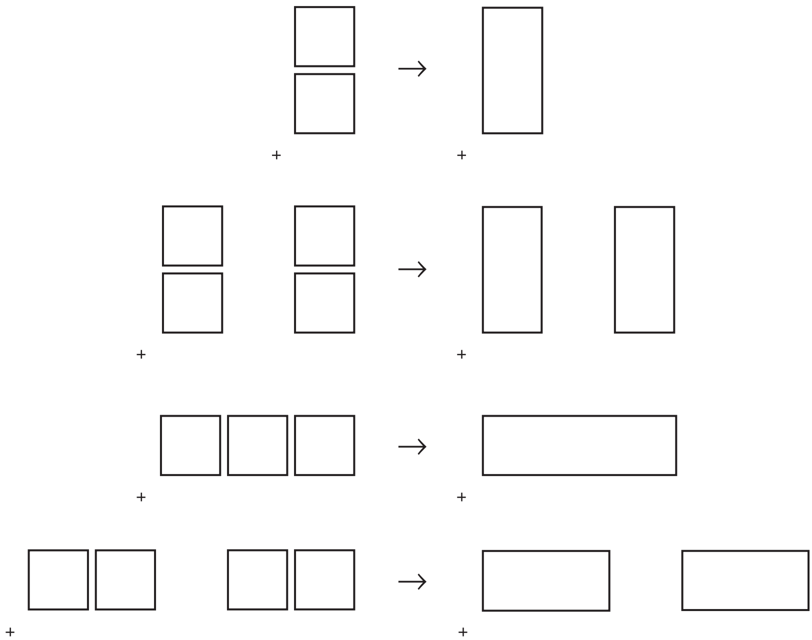
The problem is to combine rectangles to get the right room layout. To me, this is the heart of Palladian design—

> A unique feature of Palladio's sketches or the plates to his *Quattro Libri* is the uniformity of schema in plans.

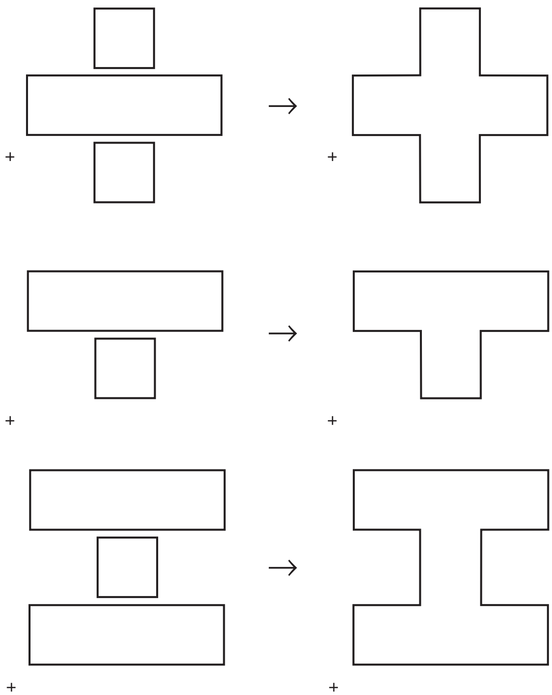
The schema $x \to \mathrm{div}(x)$ provides for the details that are needed to make this work. More precisely, I'll use the reflection of the schema's inverse—it's the opposite of what I did in ice-ray lattices—in this way

$$\mathrm{div}(x) + R(\mathrm{div}(x)) \to x + R(x)$$

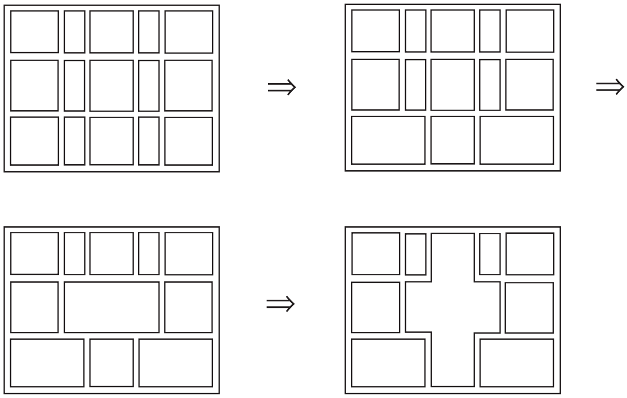
Rules of the following kind are defined in this schema to form larger rectangular spaces in a recursive process



and also central crosses, T’s, and I’s on the axis of the plan



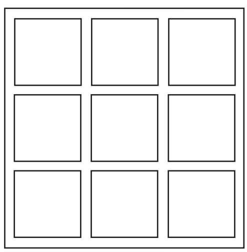
The rules are applied in this four-term series



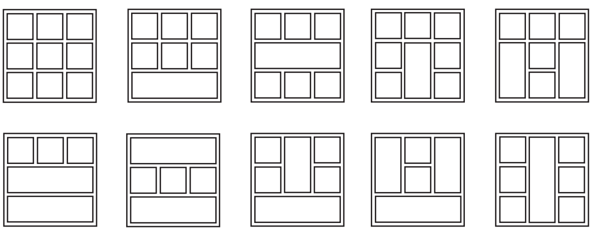
for the Villa Foscari. Of course, additional room layouts are readily defined as well. And it would be nice to know how many.

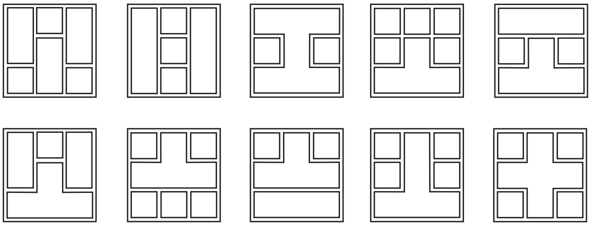
Among the twenty surviving villas and twenty-odd projects known from drawings and the *Quattro Libri*, there are few instances of a repeated plan, motive or composition in mass; Palladio would produce at most two or three versions of a particular scheme before reaching out in an entirely new direction. The common core within this variety is a particular conception of architectural harmony and composition.

Questions of proportion aside, this depends on the number of rectangles in the original grid of walls. For a three-by-three grid



there are twenty distinct possibilities



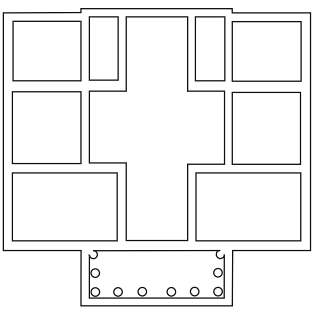


of which two—the fourth one with a central rectangle surrounded by seven squares, so that there’s an open side, and the last one with a cross—appear in *The Four Books of Architecture* in the Villa Angarano and the Villa Barbaro. Then for the Villa Foscari and a five-by-three grid, there are two hundred ten alternative room layouts. Palladio includes seven of these in *The Four Books*, and one twice—it’s remarkable how much more there is to do. Of course, this is the kind of configurational enumeration that I’ve been loath to recommend. Nonetheless, it’s not without precedent in design. In fact, there are pattern books and the like—ones of late vintage and of combinatorial interest include polyominos, rectangular dissections, and Lionel March’s remarkable taxonomy of all floor plans in terms of planar maps. It’s useful to know what’s possible, unless it interferes with seeing. Then the course is clear—stop counting!

Porticoes come next.

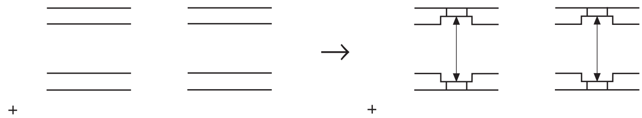
> Often the porch is the only antique reference in the design; all the rest of the detail is simple geometry, which is consistent with the concept of a hierarchy of elements.

Porticoes and like devices line up with interior walls, and when there’s more than one portico or whatever, they line up together, as well. There are various schemas for this that form a kind of architectural lexicon—I show how to define one entry below for the Villa Barbaro—but the logic of schemas is pretty straightforward. On one side of the plan, there’s almost always a portico—either in antis or prostyle, as in the Villa Foscari. And then on the opposite side of the plan, there’s a portico in antis, a wall inflection like this one

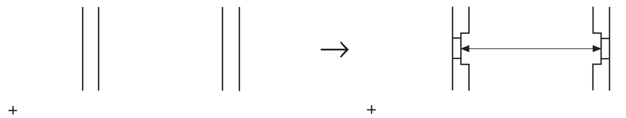


at the back of the Villa Foscari, or nothing at all.

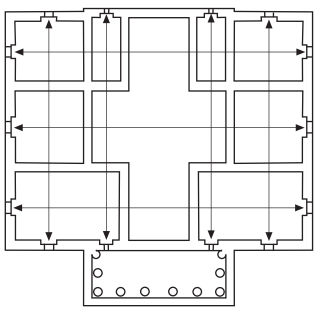
Everything is ready for windows and doors. On axis, there are main entrances—at least one and usually two—that are obligatory, and interior doors line up with them. By now, the schemas for this should be evident. But off axis, things are a little more challenging. There are also enfilades, that is to say, windows and doors are aligned as the main entrances and interior doors are on axis. Drawing makes this practicable in rules. Put a line between two external walls and insert windows—vertically to respect the symmetry of the plan in the obvious way



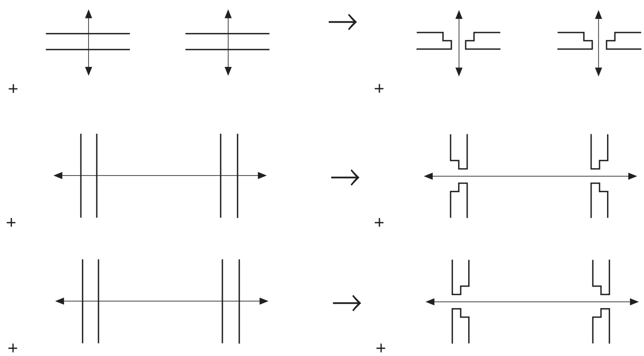
and horizontally, too



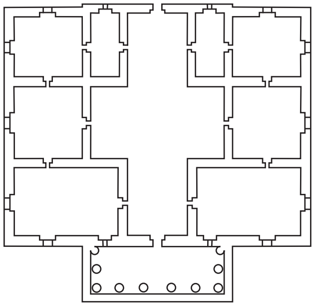
with this result



Then, where lines cut internal walls, put doors



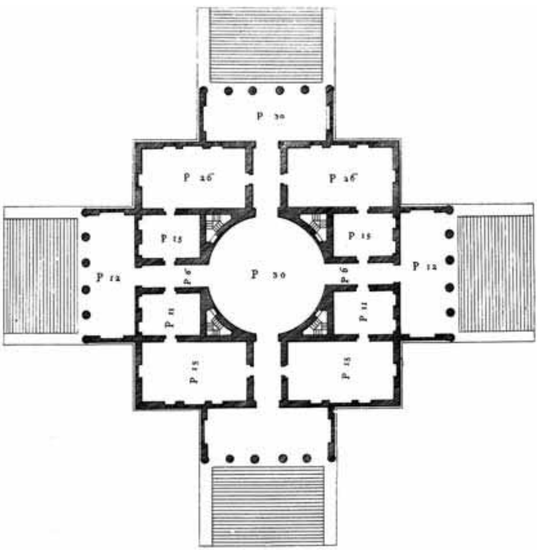
It's really kind of neat to see how easy this actually is, just seeing and doing wall by wall and door by door



so that enfilades are formed automatically.

That's it. There's the Villa Foscari, only what else is possible? That's what schemas are for, to define rules that do more. And the examples in figure 5 show a representative sample of results—some of them are from Palladio and others are forgeries, and the two kinds aren't easy to tell apart. This can be a lot of fun, and it seems to me that it's in the spirit of *The Four Books of Architecture*. I can't help thinking Palladio meant his treatise to be used, and not simply as a static record of his designs. Drawings without instructions would have been enough for that. With shapes and rules, I have drawings and, in fact, drawings as instructions. Once again, design is calculating.

But there's a lot more to Palladio than I've shown with my schemas. I can use them speculatively in various ways, perhaps to track down Palladio's kin in design or to explore a host of novelties that bear a family resemblance. Palladio's most famous design is the Villa Rotonda



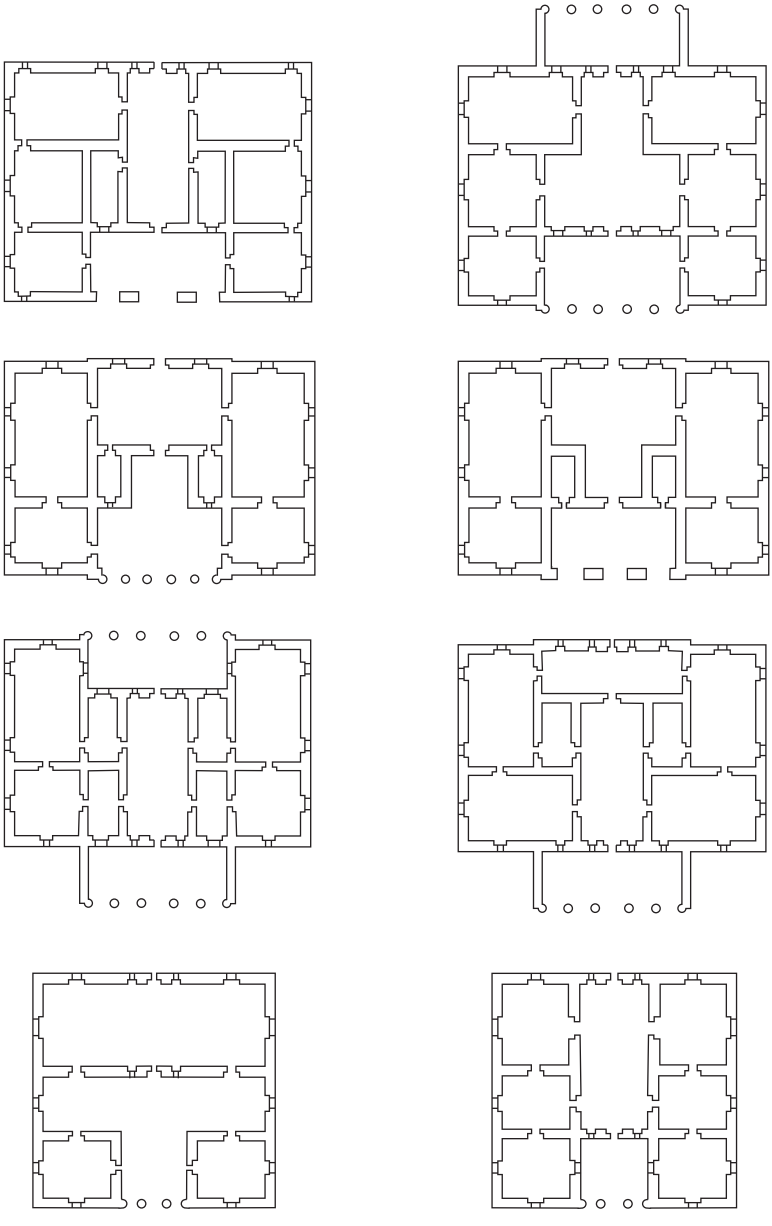
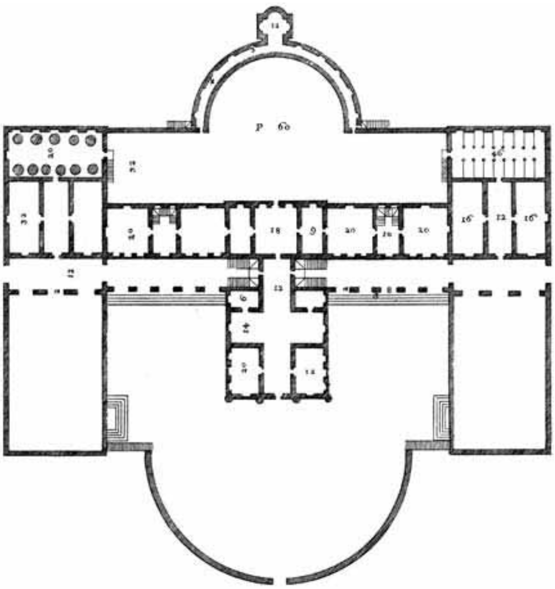


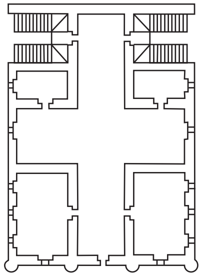
Figure 5

But this is probably too easy given the schemas I already have. It's enough to reflect them with respect to a second axis of symmetry located in a few distinct ways, and then to add the little more that's needed to handle centrally placed domes. The technique is already established in what I've been doing. So let's try another design that doesn't seem as straightforward at first blush and see how it goes. The Villa Barbaro at Maser—I mentioned it earlier when I set out my catalogue of three-by-three room layouts—is a good example. This is how Palladio draws it in *The Four Books of Architecture*

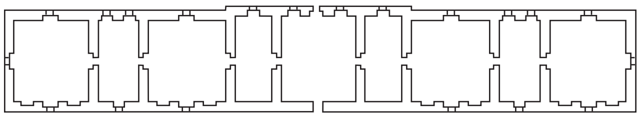


To understand how this design works tests schemas and rules in a new way. The villa was built for Daniele and Marcantonio Barbaro. It includes common public spaces in the main rectangular block and a row of private living spaces that the brothers shared equally, one of them to the left and the other to the right. And certainly, the design is different from the villa plans shown in figure 5. In particular, look at the stairs that divide the building close to its middle. What kind of entranceways are these, from the sides, symmetric but off axis? Moreover, the villa is complicated in other interesting ways. I like to describe it as separate buildings, one in front of the other, and there's good evidence for this in the actual villa, in photographs, and in other drawings (notably, the famous drawings of Bertotti Scamozzi). But is there anything to suggest two distinct buildings in Palladio's plan as it's actually drawn in his *Four Books*? Suppose this is exclusively an exercise with respect to the schemas I have for villa plans. The austerity and rigor smack of logic and reason, and this isn't at all misguided. Schemas and rules can be applied forensically to explain existing designs, just as they're used creatively to produce new ones. And my schemas give a definite answer—the Villa Barbaro is, in fact, the sum of two buildings.

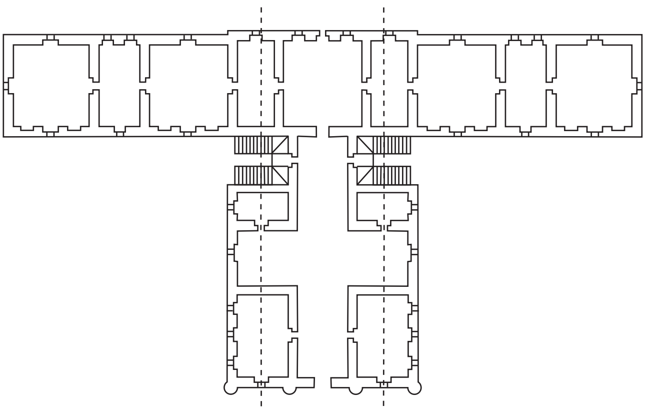
Let's see how this works in terms of specific schemas and rules. First there's the main rectangular block



and then, just behind it, a single row of nine spaces

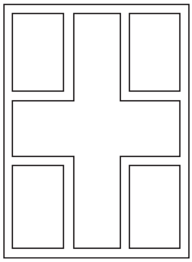


The central space in the row and the twin flanking spaces abut the main block, but the row and the block must be separate structures. This is the only way to ensure that there are enfilades—otherwise, windows and doors wouldn't make any sense. They would look like this

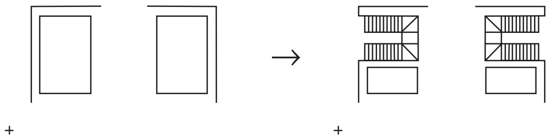


in a haphazard arrangement where not everything aligns as it should. My schemas don't allow for this, and Palladio simply wouldn't do it.

Now, it's easy to make plans. The back building is merely a symmetric row of rectangles—not a typical Palladian room layout, yet one nonetheless with a wall inflection and a horizontal enfilade—and the front building is in my catalogue of three-by-three room layouts, here as a plan with the correct dimensions



Still, there are the stairs—what should I do about them? They partially fill the two corner rectangles at the top, according to the rule



and thereby provide an alternative "portico." The rule is easy enough to define in the following way: draw what you want to change—that's the left side of the rule—and then draw what you want this to be—that's the right side. This seems almost trivial given the usual difficulties writing computer code, when I have to divide things into constituents, define them separately, and specify how they interact now and forever. But nothing has to be segmented or rationalized in this way to calculate. Drawing the rule works flawlessly, precisely because I don't have to say what anything is. Shapes don't have definite parts. I just trace what I see and do, with no divisions whatsoever, and go on—perhaps next to entrances that are put in like windows to define enfilades. And with a little jiggling, the rule is in a schema that applies generally. Shapes show how to change shapes as I calculate. This is calculating by drawing, first to define rules and then to use them. It's exactly what I want, and it's perfect for design.

But don't rules freeze Palladio in an academic style, so that his villas are set permanently outside of ongoing experience? This is the question I asked earlier for checkerboard lattice designs, and my answer remains exactly the same—

NO!

Now it's an emphatic "no" to academicism and the reductive nothing-but-ism it inevitably implies. And it's a rigorous "no" that I can justify on technical grounds, not a romantic response that simply feels good. The romantic "no" is an answer that's easy to give, easy to believe, easy to applaud, and hard to prove. It's right, but not at the expense of calculating. When you look at it another way, there's no reason for a split between romantic freedom and doing it by the numbers with shapes and rules. Calculating and freedom aren't incommensurable. Because rules are defined in terms of embedding and transformations, I can apply the same rules in different ways to change myriad parts of shapes, and I can alter rules and add them any way I please without ever having to stop what I'm doing and start over. There's no discrepancy between what I do and what I see next to go on—the one includes the other as shapes fuse and divide. It's calculating, through and through. The identities defined in the schema $x \rightarrow x$ may not do very much, but they let me see lattice designs in novel ways and show how calculating can be open-ended. In fact, the way identities work guarantees this kind of freedom and flexibility for all rules. The ease with which I can define schemas and use them to play around—for example, with the plan for the Villa Barbaro—is added proof. No wonder styles change and evolve freely. It's the chance to go on to something else whenever I look again that makes calculating with shapes and rules worthwhile. Calculating and experience are the same—there are rules for whatever I see and do. But these aren't rules for the academy—not with a grammarian's small "a" to enforce current standards, conventions, and norms, and not with Plato's capital "A" to grasp the eternal and unchangeable, either. There's no vocabulary in practice to distinguish good and bad usage when it comes to design. Shapes are filled with ambiguity—there's a lot to see and do wandering around. Calculating and experience are always new.

Seeing Won't Do—Design Needs Words

I'm afraid I may be giving a false impression. Design is drawing—true enough, it's calculating with shapes and rules. Yet most of the time words are involved, too, to say what designs are for and to connect them to other things. I've already done a lot of talking to describe rules as they apply to produce Chinese lattice designs and Palladian villa plans. Take my rules for villa plans—they're about villas and not just the shapes that go together to make them. When I combined the spaces in a grid of walls, I was describing rooms—central ones and the symmetric ones to their left and right. And when I defined an additional rule to alter the corner spaces in my room layout of the Villa Barbaro, I wasn't thinking about rectangles per se but about entrances, stairs, and porticoes. Villa plans are of functional, social, and aesthetic interest—they involve more than shapes—and different aspects of these concerns play into design at different times. What I say matters as much as drawing and calculating. Seeing won't do—design needs words. How can I manage this with rules, and how can I connect these rules and the rules I use to calculate with shapes in a single process? Filling in the details is important to make design seamless, so that a confluence of diverse and changing interests can be handled all at once. Designers and especially architects like to say that they're "generalists," and maybe it's really true. Design may not be about anything in particular but about a host of different things at different times. I'm going to show how this works using rules to calculate with shapes and using rules to calculate with words when both kinds of rules are used together.

I already said pretty much what's involved in part II, with topologies and combining algebras in sums and products. On the one hand, I showed how to define topologies for shapes retrospectively in terms of the rules I tried—with particular attention to erasing rules in the schema

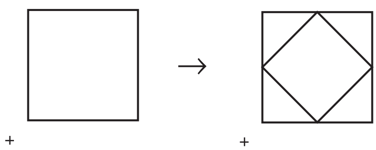
$x \rightarrow$

that help to count, and identities and other rules in the schema

$x \rightarrow t(x)$

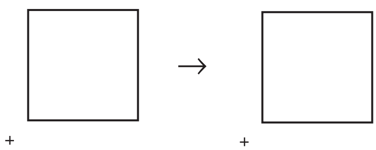
Topologies are simply descriptions that show how I'm calculating and how shapes and their parts change continuously as I go on. But I may want to name parts and give them meaning in alternative ways. So, on the other hand, I sketched a general approach in which different algebras are combined in sums and products to define compound shapes. It includes both what I did in algebras like $U_{12} + W_{22}$ with lines, planes, and colors (weights) to produce paintings and what I did originally when I handled shapes and colors separately. Now I'm going to give a couple of examples to illustrate some of the things I proposed, strictly with shapes and words—actually numbers, but these are words as well, and are largely equivalent in practice. Using words and using numbers both require looking at what's there. Words and numbers divide shapes, naming their parts, although numbers do it sequentially in order to count them up. The two work together to describe visual experience. They're there to give shapes meaning and they tend to keep it constant. The consistency seems right, yet it isn't necessary. Ordinarily, it's expected, and it lets us anticipate the future and plan for it without having to worry that everything might change without rhyme or reason. Perhaps this explains why creative activity, drawing and playing around with shapes when their parts alter erratically, is apt to seem ineffable. Yes, there are always topologies, although they aren't exactly everyday descriptions in words. But words and descriptions don't have to be used in design all the time, only if they're useful. And they are in my two examples. The second is the more elaborate. It uses schemas and rules for Palladian villa plans to explain how shapes and words (numbers) work and what this shows.

The main idea is easy enough—I'm going to associate description rules with the rules I apply to shapes, and use both kinds of rules to calculate in parallel. Every time I give a rule for shapes, for example, this one

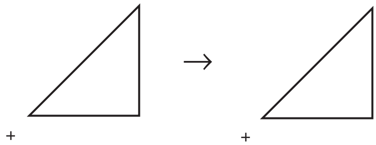


there are description rules for something else:

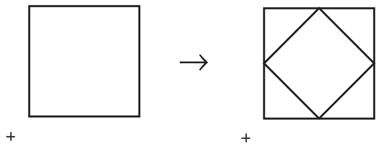
(1) If the new shape is not the previous one, then the total number of squares I've inscribed increases by one, where squares are defined in the identity



(2) Furthermore, if there are more squares, then there are four more triangles, where triangles are defined in the identity



These description rules apply in conjunction with the rule

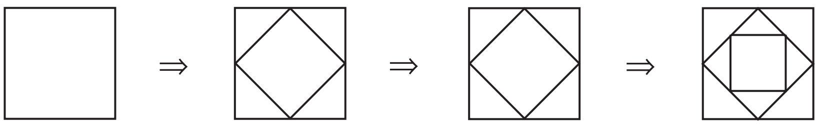


to say what happens as I continue to draw lines in a certain way—I get additional squares and triangles. Symbolically, I might summarize this with the rules

(1′) number of squares $\rightarrow$ number of squares $+ 1$

(2′) number of triangles $\rightarrow$ number of triangles $+ 4$

Then, for the series of shapes



there's the corresponding series of descriptions

from step	0	1	2	3
number of squares	1	2	2	3
number of triangles	0	4	4	8

I'm calculating in parallel to produce shapes and the descriptions that go with them. And notice that I can even distinguish descriptions that are the same, if I count the steps in which they're produced. To start, there's a zero for bookkeeping, and then, the description rule

step $\rightarrow$ step $+ 1$

There's a lot going on to tell me what I'm doing. The mechanics for all of it is straightforward enough, and extends neatly to many other things. For example, I can record the transformations I use to apply rules and define my topologies to show how rules pick out parts of shapes. Then there's more than mere mechanics that concludes with dry numbers. Well, the mechanics still isn't much—that's the way I like it when I calculate—but it shows that knowing how to count means first knowing how to see. This is where meaning starts and how it grows, and the process is far from dry. ''To [observe] well,'' notes J. S. Mill, ''is a rare talent.'' Shapes are filled with meaning as rules are tried. And it seems to me that this is the only kind of meaning that matters for shapes, whether in words, numbers, or topologies. Perhaps it's the only kind of meaning, period. It depends on what I see and do, and on what I make of it as I go on to see and do more. Rules apply to shapes and bring in words and numbers to make this possible. It's a process in which everything can interact and connect, and it's all calculating.

Let's try something that appears to be a little harder, although meaning doesn't have to be when it's a question of seeing. Suppose I'm looking at the rooms in a Palladian villa plan, and that I want to count them. Once again, there are numbers, and this involves distinguishing spaces and naming them in terms of what they're for. And, in fact, both naming and counting depend mainly on the rules I apply to pick out parts and change them as I calculate to produce the plan.

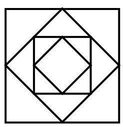
Let L and R be the number of polygons in the left side and the right side of a rule, and let k be defined as follows

$$k = L - R$$

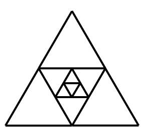
This seems to be OK, but there's really a lot more to it than I've said. Rules are made with shapes that aren't divided in advance, and not with distinct symbols that are ready to count. This isn't generative grammar or syntax and words, so I have to figure out a way to count polygons. But remember what I did to define squares and triangles with identities from the schema $x \rightarrow x$. In much the same way, I can use erasing rules from another schema

$$\text{poly} \rightarrow$$

to count what I want. These rules pick out polygons—there are rectangles, crosses, T's, and I's—so that I add plus one every time a rule is tried with no chance of double-counting. There's an example of this kind in part II for squares and triangles in the shape

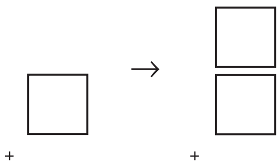


only it gives multiple answers between four and eight, and there's another example in part I for triangles in the shape



that works in the same way with answers from four to six. But now, the erasing rules in my schema apply to give definite results—polygons are numerically distinct. And now, these rules apply to shapes in rules. This is important. Shapes are shapes wherever they are—it doesn't matter whether they're in rules or not. Either way, they're exactly as they're given, without finer divisions. Divisions are determined as I calculate.

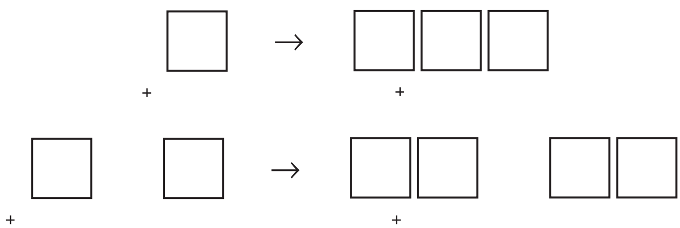
Once k is defined, it's easy to say what rules do. First, there's an $m \times n$ grid of walls that's described by the coordinate pair (m, n), where m is the number of rectangles in a row and n is the number of rectangles in a column. (This counts columns before rows. It's interesting how things can get switched or turned around in shapes and words alike.) These values are set in the inverted T used to produce the grid. If I try a rule like this



to add a rectangle vertically, my description rule is

$(m, n) \rightarrow (m, n - k)$

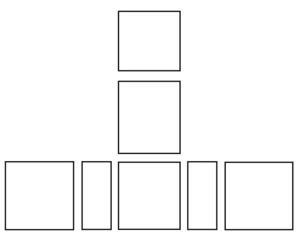
where $k = -1$. And in the corresponding way, if I apply a rule to add rectangles horizontally



then my description rule is

$(m, n) \rightarrow (m - k, n)$

and $k = -2$. So, in the Villa Foscari with the inverted T



I have descriptions that look like this

from step	0	1	2	3	4
m	1	1	1	3	5
n	1	2	3	3	3

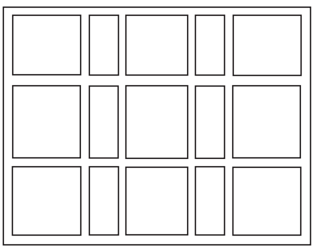
if the central column of rectangles is produced before the row that forms the base of the grid. And likewise, I can have the descriptions

from step	0	1	2	3	4
m	1	3	5	5	5
n	1	1	1	2	3

when I do all of the horizontal additions before the vertical ones. Both of these series begin and end in the same way, although the start is obligatory, and they have different terms in between. The variation isn't always gratuitous. It may be useful to guide or structure an assembly or manufacturing process—there are different numbers of rectangles at different times, so supplying and managing components, or resources generally, may be better in one way than another—or the variation may help to define and sequence the stages in a construction project. But for my purposes right now, both series are just the same because they end that way. This can be useful when rules apply to shapes nondeterministically. Then a successful conclusion may be the only thing that matters, not the different things that happen along the way. Still, there's a little more to do—an inverted T isn't a grid of walls. The T has to be completed—I need to fill in spaces and add a bounding rectangle. The description rules for this are given in the numerical identity

$(m, n) \rightarrow (m, n)$

The grid of walls for the Villa Foscari



is produced in nine more steps after the inverted T is in place. This gives the following descriptions as the grid is finished off

from step	4	5	...	13
m	5	5	...	5
n	3	3	...	3

with the same repetition of numbers—5 . . . 5 or 3 . . . 3—at the end of both series. Evidently, the order in which rectangles are added continues to be irrelevant, and there's no reason to keep a running count.

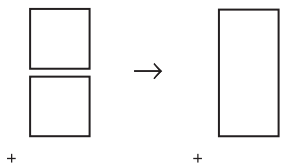
Given a grid of walls and with its description (m, n) in hand, I know there are N rooms, that is to say, N is given in the formula

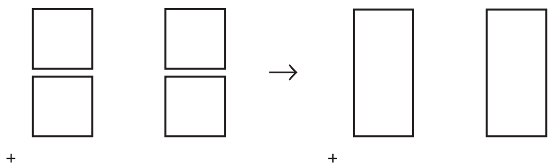
$$N = mn$$

This changes according to the description rule

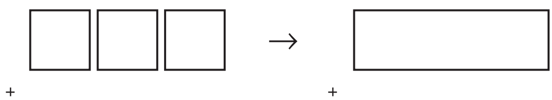
$$N \rightarrow N - k$$

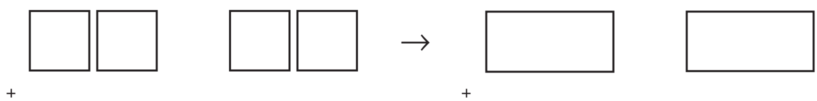
as I use rules to combine rectangles. In particular, for rules like these



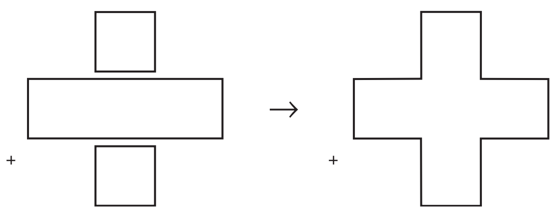


that put rectangles together vertically, $k = 1$ and $k = 2$. Then, for rules like these

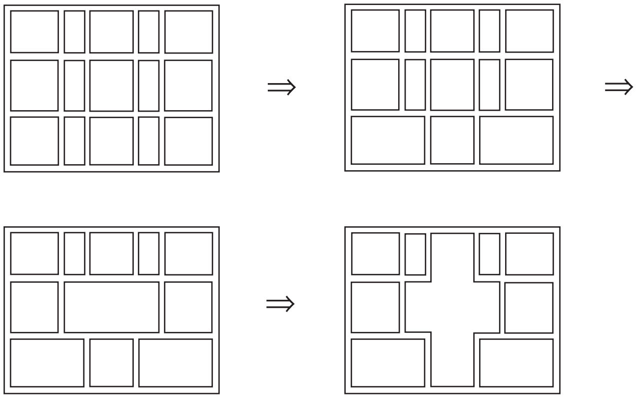




where spaces change horizontally, $k = 2$. And for the cross that combines three rectangles—two small ones and a big one—



$k = 2$. As rooms are formed in the Villa Foscari

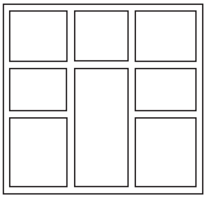


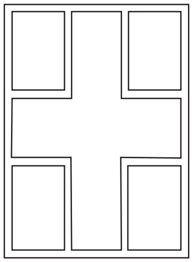
three descriptions change in this way

from step	13	14	15	16
N	15	13	11	9

My descriptions for grids of walls and room layouts fit together with almost classical precision to explain what my rules are doing as I apply them to shapes. In fact, there's a common parameter k in different description rules that's a kind of underlying module—the unity is undeniable. It's uncanny, but everything has got to be right once it's so clearly understood. That's why words and numbers work for a lot of design—to make it understood. Nonetheless, there are many aspects of design that seem exclusively visual—for example, mannerist details and more experimental devices are sometimes like this. Then, words and numbers—descriptions of all sorts, for that matter—may actually intrude. Words can bias the eye and obscure what there is to see and do. Luckily, there's plenty of opportunity to use rules and descriptions promiscuously in the kind of open-ended process I'm talking about. Even so, unbounded freedom may be burdensome when I have set responsibilities to meet and there are important goals to achieve. Goals and such are also descriptions, and they can be used to circumscribe the options I have as I calculate with shapes and rules—but this story is better told a little later on to add to my picture of design. Meanwhile, descriptions do something else in design that's worth a closer look. They help me organize what I'm doing in a variety of useful ways.

Descriptions determine relations to classify designs and to order them. In fact, I've already used descriptions to classify Palladian villa plans according to the values of m and n in the coordinate pair (m, n) that gives the number of rectangles in a row and a column in a grid of walls. In this sense, all of the entries in my catalogue of three-by-three room layouts are alike—they're the same "size." In particular, the plans for Palladio's Villa Angarano and Villa Barbaro





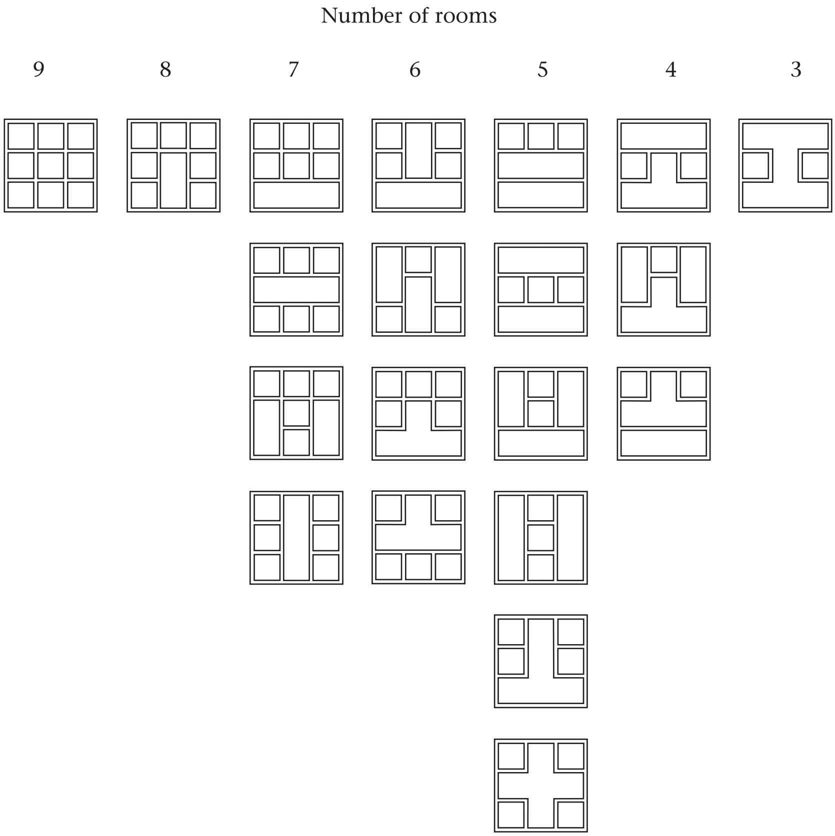
are equivalent. This is a little crude, but descriptions allow for finer distinctions, as well. I can also group $m \times n$ villa plans according to the number N of rooms they contain, where, once again

$$N = mn$$

in a grid of walls and varies according to the description rule

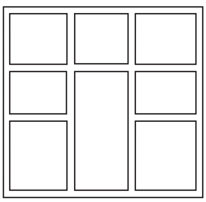
$$N \rightarrow N - k$$

Then my catalogue of three-by-three room layouts is partitioned in this way

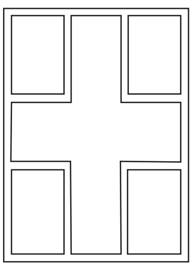


I guess it's about time that I call room layouts plans or villa plans. This may be overly optimistic—even reckless—but I've been leaning in that direction for a while, and words can change meaning as I calculate, just as shapes can when rules are tried. Now, two plans in my catalogue are equivalent when they have the same number of rooms, so the Villa Angarano and the Villa Barbaro are no longer the same. Nonetheless, there are five other plans that are still equivalent to the Villa Barbaro's, even if it's the only plan with a central cross. If I wish, I can augment my description rules to take care of this and classify central rooms, too. Then four plans are equivalent, with central rectangles, while one has a T and the other one has the cross. I can also go on to distinguish the plans with rectangular rooms in terms of description rules—it's a good exercise—but it may not be necessary. Descriptions give me a way of focusing on the kinds of things I want generally, with a free choice among equivalent designs. And this is how a lot of design seems to work in practice. It's a common occurrence that there are many designs that satisfy the same constraints and requirements. There's plenty of room to play around. And in fact, that's one reason why neat formulas like "form follows function" are always heuristics and only partially explain what happens in design. It seems that whatever I say about function is never enough to determine a unique form. And in fact, I may not want this. Otherwise, the myriad "undefined" things—at least there aren't any words for them—that I contribute as a designer are likely to be lost. Free choice or not, I'm calculating—it's just a lot more fun when things are open-ended. But descriptions work in another way to help with important decisions. Descriptions classify designs, and they order them—there's equivalence and also value. How does this work, and how is it part of the design process?

Once again, my catalogue of three-by-three villa plans is a pretty good place to start. I can count rooms and order plans up to equivalence according to the distinct numbers I get. The Villa Angarano



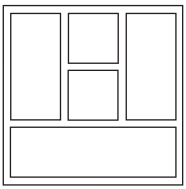
contains more rooms than the Villa Barbaro



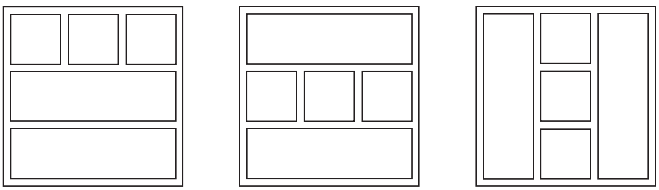
and in fact, more rooms than any other plan in the catalogue except the complete grid of walls. If I want to move beyond the grid and get something that's a little more expressive, and the number of rooms in a plan is the only additional thing I'm considering—the number should be high—then the Villa Angarano is the best I can do. But numbers may measure other properties, perhaps to optimize physical performance or social interactions. I might even try and apply George Birkhoff's aesthetic measure

$$M = \frac{O}{C}$$

to get something of delight from the plans in my catalogue, or use the corresponding measure of my own—E_Z—that I described along with Birkhoff's in the introduction. I wasn't keen on any of this then, and in practice I'm still not. Nonetheless, E_Z makes an instructive example without my having to rehearse any technical details. The results are enough. In terms of E_Z, the Villa Barbaro is better than the Villa Angarano, but it's only the second best among the plans with five rooms, after this one



And notice that the three remaining plans with only rectangular rooms

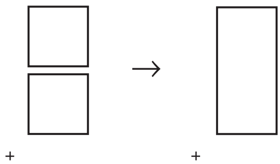


are the same. In fact in many ways, they look it—each with twin three-by-one or one-by-three spaces. But really, the main thing to get from all of this is that equivalence and value both depend on how descriptions are defined as I calculate with shapes. And there's plenty of room for descriptions to vary—they can be given in lots and lots of ways, and the ones to use are up for grabs. The perfect description now needn't be right the next time I look, and it's easy to forget as I go on. Things change whenever I try a rule. I'm always free to design some more.

I've kept shapes and descriptions together in the way I have so that when I apply rules to shapes, descriptions are automatically defined in parallel. But it's time to flip this around in an inverse way to show something else that's important in design. Descriptions can be defined independently to provide design specifications and requirements—even architectural programs—that set goals to guide and control the design process. For example, I can give a number N of rooms and then search through villa plans to find those with exactly N rooms. My description rules can be applied by themselves to define N, and this in turn tells me how I should try rules to produce plans. Suppose that $N = 8$ in a three-by-three villa plan. Then my description rule is

$N \rightarrow N - 1$

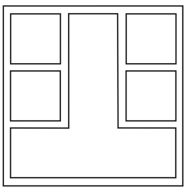
and I have to apply the rule



as I did for the Villa Angarano. But if $N = 5$, there are six options. In particular, if the number five is obtained in a three-step series—maybe this one

from step	8	9	10	11
N	9	7	6	5

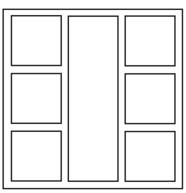
—then the plan



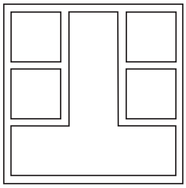
is the only one I can produce. Two other three-step series are also possible

from step	8	9	10	11
N	9	8	6	5
N	9	8	7	5

The first results in the same plan—remember that rules apply nondeterministically—while the second is simply a dead end. No plan can be produced according to the descriptions in the series—there's no way to go from step 10 with seven rooms to step 11 with five rooms, that is to say, from the plan



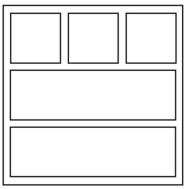
to the plan

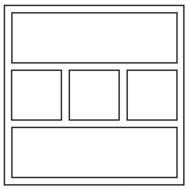


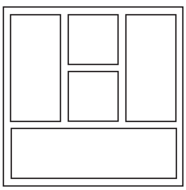
Descriptions combine too copiously in too many series. You can ask for something that's meaningful according to my description rules—at least it's "grammatical"—that you can't get calculating with villa plans. It seems to me that sometimes imagination works in this way, too, when words promise more than I can do with shapes. Only perhaps this is just my imagination. And even if it's not, it doesn't make up for anything words miss. What I can say never trumps what I can see and do. But let's go on. The single two-step series

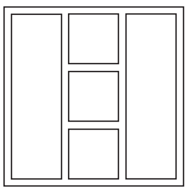
from step	8	9	10
N	9	7	5

already gives the Villa Barbaro, and also these four plans

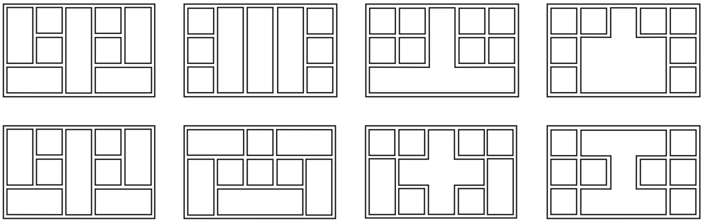








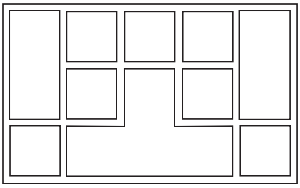
But notice that the plan for the Villa Barbaro is the only one that's defined uniquely—two rules are applied: first, one to produce a three-by-one rectangle, and then another to produce the cross. Each of the plans above, however, is defined in two ways—again rules apply nondeterministically. In three plans, large rectangular rooms come in pairs. Horizontally, the rectangles can be formed either one before the other, and vertically, from the top or the bottom. And in the remaining plan, the two symmetrical rectangles can come before or after the large rectangle at the base. Next, suppose $N = 9$ and the grid of walls is five by three to start—as in the Villa Foscari. Then, there are forty-three possibilities, of which these eight are a nice sample



There are plans with central rectangles, crosses, T's and I's. But of far more interest now, plans fall into two separate and distinct groups. There are fourteen where descriptions are produced in a four-step series, perhaps this one

from step	13	14	15	16	17
N	15	13	11	10	9

of which the plan



and the four in the top row are examples. And there are twenty-nine plans where descriptions are produced in the familiar three-step series

from step	13	14	15	16
N	15	13	11	9

for the Villa Foscari. The series also works for the four plans in the bottom row. Still, in practice things may not be as straightforward. Programs like plans can always be revised, and in fact the real goal may be to optimize both as they influence one another in various ways. This means changing programs and plans in parallel. Form and function, etc., interact, so that to some extent each implies and affects the other in a confluence of mixed interests and goals. This is beginning to feel like real design, and it continues to look like calculating.

The descriptions I've been using are fairly rudimentary, and they aren't anything to recommend in practice, although sometimes the descriptions that are used in practice aren't any more sophisticated. But it's not really descriptions that are at stake here, but rather description rules and how they're defined and used to describe what happens as I calculate. The idea is to combine shapes with words and numbers in a recursive process. Doing multiple things at once with shapes and words seems natural enough when you're in the middle of it—there's a knack to drawing and talking at the same time. But giving rules for this can be confusing at first, especially when the rules that describe what's going on interact in parallel and in other ways with the rules that apply to shapes. To get things off the ground, I've tried to avoid unnecessary details in a few very easy examples. Nonetheless, the key conclusions are pretty evident, and they're the same whatever descriptions are used. Design may be about more than shapes, but this isn't more than calculating allows. There are a host of different things involving functional, social, and aesthetic relationships that can be defined in myriad ways. Using models and mathematics is one way, as in the formulas $N = mn$ and $M = O/C$. I guess the formulas are kind of silly, but bringing in mathematics (topology, etc.) is a serious step. Description rules are made for it—they describe designs and can help to guide the design process. But none of this is what descriptions are actually about. The trick to descriptions is that they're never the final word. Things change as I go on calculating. It's worth saying over and over—there's more to see. This may coincide with what I've already said, but it's independent of the past. I may not remember what there was before, and in fact I may just as well be looking for the first time with no one to tell me what to see and do to go on. There's no reason to be coherent. Logic (rationality and science), morality, history, the authority of the ancients, and like imperatives are simply beside the point. Yes, consistency and words are useful more times than not, and they make a difference when I look again—but they don't have to. Otherwise, there's nothing really new to create. It's all been said before. Design goes away. Perhaps that's why the identities in the schema

$$x \rightarrow x$$

provide such good examples when it comes to seeing what rules do. Identities show everything new about shapes and calculating, and they can always be tried once more in another way.

Getting in the Right Frame of Mind

The picture of design I've been painting is all about schemas and rules, and about calculating in parallel with shapes and words, especially when shapes aren't divided into units in advance. Seeing is at the heart of it. I made a point of this near the end of part II, quoting William James, who was quoting John Stuart Mill on the importance of observation. Observing well—dividing things into useful and meaningful parts—takes a rare talent.

> It would be possible to point out what qualities of mind, and modes of mental culture, fit a person for being a good observer: that, however, is a question not of Logic, but of the Theory of Education, in the most enlarged sense of the term. There is not properly an Art of Observing. There may be rules for observing. But these, like rules for inventing, are properly instructions for the preparation of one's own mind; for putting it into the state in which it will be most fitted to observe, or most likely to invent.

Mill's rules for observing and inventing have to do with getting in the right frame of mind to see and do. From my point of view this is marshaling schemas and rules to use. In fact, the schemas and rules I've proposed up to now provide a pretty good course in design—at least they're something to teach to "fit a person for being a good observer"—but there's more on education to come. Right now, it's time to take stock of the various schemas I have to use in teaching and practice, and to organize them in a catalogue raisonné.

I can go through schemas $x \to y$ in a more or less systematic way in terms of the variables x and y in their left and right sides. In the easiest case, both x and y are empty

$\to$

This seems inane, but it nonetheless defines a rule that's indispensable when I'm applying rules in parallel to everything at once and want to leave what's independent of my immediate concerns alone. Next, I have the erasing schema

$x \to$

and, in particular, the restriction

poly $\to$

for polygons, maybe with additional qualifications that make them convex or regular or that number their sides. The rules defined in both of these schemas let me count in another way, and they do in fact change what's there. The inverse of $x \to$ gives the schema

$\to x$

that appears to produce something from nothing. This seems like magic, but $\to x$ like $\to$ proves its worth calculating with rules in parallel—I can add to one thing in terms of something else that's not part of it. There are a couple of nice examples of this in the product of the algebras U_{12} and $U_{12} + U_{22}$, and in the algebra $U_{12} + W_{22}$ in part II, and my paintings work in the same way. Furthermore, the rules defined in $\to x$ help me to get going. When I calculate, I always need someplace definite to start. Nonetheless, $\to x$ may be unnecessary for this. If I wish, I can use identities in the schema

$x \to x$

to start with something interesting I happen to see instead. And then, of course, there's the general schema

$x \to y$

Most of the time, the variables x and y in the schema $x \to y$ are related in some way. This is already evident for the identities in the schema $x \to x$. Moreover, I have the schema

$x \to t(x)$

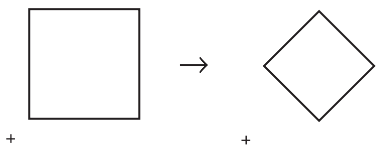
that includes the identities, and the schema

$x \to b(x)$

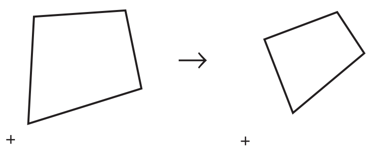
where there's a Euclidean transformation t or a boundary operator b that changes x. Or I might define another schema as an extension or counterpart of $x \to t(x)$, maybe

$x \to x'$

where x' is a parametric variation of x. Then I can relax the relationships between lines in the rule



for squares with a pair of arbitrary quadrilaterals



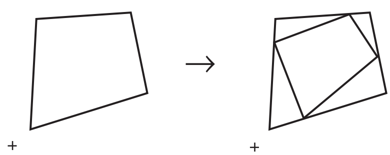
And for each of these four schemas, there are the sums formed when I add their left sides to their right sides. Ignoring the identities that stay the same, I get the trio of schemas

$x \to x + t(x)$

$x \to x + b(x)$

$x \to x + x'$

for rules that apply without erasing anything. Again, quadrilaterals are a good example



Inverses are very useful, too. The following ones are different from the schemas from which they're defined

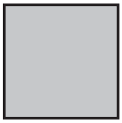
$b(x) \to x$

$x + t(x) \to x$

$x + b(x) \to x$

$x + x' \to x$

while the schema $x \to x$ and its inverse are equivalent, along with the twin schemas $x \to t(x)$ and $x \to x'$ and their inverses. But notice, too, that in practice $b(x) \to x$ works for $x + b(x) \to x$—here, for a square and its boundary



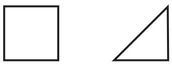
I've also tried a few general-purpose schemas of another sort. For example, there's

$x \to \mathrm{div}(x)$

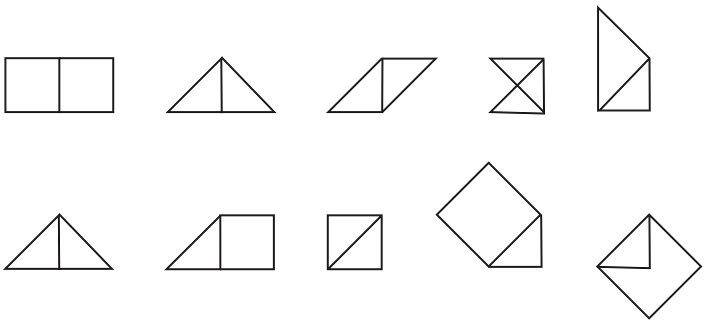
to divide shapes in various ways—typically, the shapes are polygons as in ice-ray window grilles, housing plans, and paintings, although polygons aren't required for the schema to work—and then there's my old standby from "Kindergarten Grammars"

$x \to A + B$

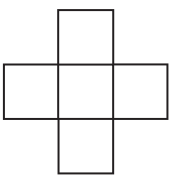
for spatial relations that are defined either from a given vocabulary of shapes, maybe a square and a triangle



combined side to side in the following ten ways, where squares and triangles vary in size and orientation



or from parts selected using identities in the schema $x \to x$, perhaps when x is a polygon and identities are applied to the shape

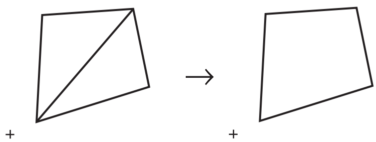


These were the exercises I tried for shapes and rules in "Two Exercises in Formal Composition." They set everything up for design synthesis and stylistic analysis. Spatial relations were enumerated in terms of a given vocabulary of shapes, but calculating with shapes wasn't restricted in this way. And once more, inverses are defined in the additional schemas

$\mathrm{div}(x) \to x$

$A + B \to x$

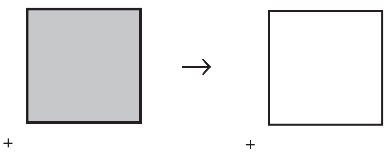
I used polygons in $\mathrm{div}(x) \to x$ for rules like this one



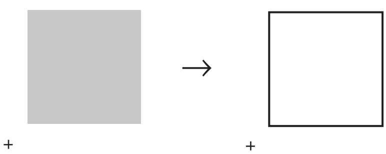
to complete Klee's palm-leaf umbrella. Of interest, too, I can recast the schema $x \to x + b(x)$ in terms of the schema $x \to A + B$, so that x in the left side of $x \to x + b(x)$ is a variable over A and B, and in the right side, x is A and $b(x)$ is B. This isn't exactly normal mathematics—it's more like Ludwig Wittgenstein's kind of calculating where "the figures on paper alter erratically"—but it does let me add the "usable" schema

$x + b(x) \to b(x)$

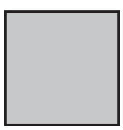
to my list of inverses for rules like this one



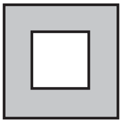
when $x \to b(x)$, that is to say



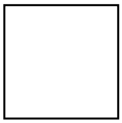
applies too broadly—perhaps to the shape



to produce



instead of



I can combine schemas in a variety of ways, too. For symmetrical arrangements and fractal designs, there's the easy schema

$$x \to \sum t(x)$$

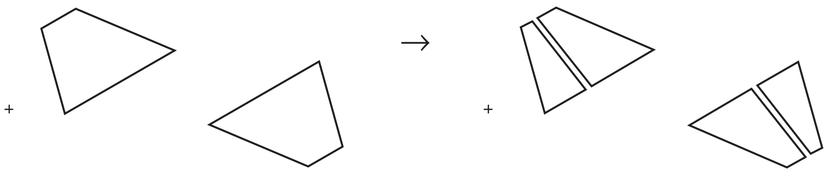
when the schema

$$x \to x + t(x)$$

is used repeatedly with the generators of a symmetry group, or with transformations that include a change of a scale. Then, for shapes and fractal-like boundaries as in my paintings at the beginning of this part, there's

$$b(x) \to x + \sum t(b(x))$$

And, in special cases, there are rules like this one



for symmetrical ice-rays that suggest schemas for grids

$$x + R(x) \to x + x' + R(x + x')$$

and schemas for villa plans (room layouts)

$$\mathrm{div}(x) + R(\mathrm{div}(x)) \to x + R(x)$$

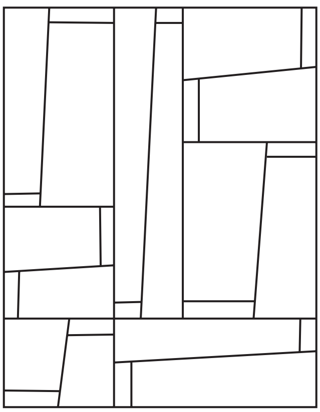
in which transformations are used—in the two schemas here, there's a reflection R—to ensure symmetry of a certain kind. In particular, villa plans are bilaterally symmetric with respect to an axis that runs through rooms.

All of this gives me a handle on style and stylistic change, where schemas, assignments, and transformations in given algebras play important if not exclusive parts. It's easy to imagine one or more designers with a core repertoire of schemas that they combine and then apply in their own fashion to define a style that can change—even erratically—as different assignments and transformations are tried. This begins a good story, although designers are merely incidental. The problem, put simply, is to find rules that explain known examples of some kind that appear to hang together, with the ability to go on to new things like them, either recognized later somewhere else or produced from scratch. What designers say about what they do can help to define the rules, but personal testimony isn't indispensable and may be misleading or wrong. It's the same trying to find a grammar for a natural language like English—the speaker is only one of many who have a stake in what's said and how this works. And in literary criticism, writers are related to their work in the same way. What they say isn't decisive—others have a say, too. The evidence I need varies in a community enterprise. There are no necessary facts. Anything can contribute to the definition of a style at any time, especially when it's something new to see. Even if there's a definition that's affirmed and widely accepted, it's never final and conclusive. Everything can change as I go on. And that's exactly what new schemas allow when they're used to define rules to calculate with shapes. Minimally, of course, I can always add the schema $x \rightarrow x$ for identities. But in fact, keeping schemas the same—adding to them only once in a while, as I did for the Villa Barbaro—may be enough. Then there are assignments and transformations to stir things up.

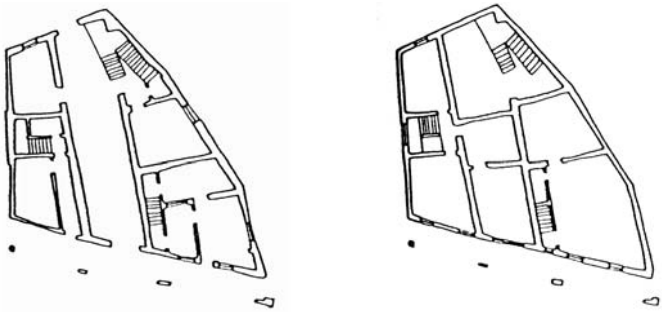
Earlier, when I was first talking about style, I showed how a single schema

$x \rightarrow \mathrm{div}(x)$

could be used in diverse ways for Chinese lattice designs—checkerboard patterns and traditional ice-rays—floor plans in a major housing project, and nonrepresentational paintings. And it's easy to go on. The schema also works for paintings in the style of Fritz Glarner—his own, of course, and a host of new designs, maybe this one



Then there are vernacular building plans in medieval Treviso—in two floors, for example



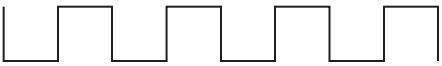
and many other things. There's plenty to see and do in various algebras as the schema is applied in terms of different assignments and transformations. Perhaps there's a formal method here—combining schemas from a common pool and trying a range of assignments and transformations to define styles and to change them. Terry Knight has shown with remarkable precision and scrupulous attention to detail—her taxonomy is uncanny—that the schema

$x \to A + B$

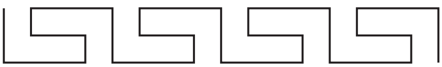
and its larger sums, first off

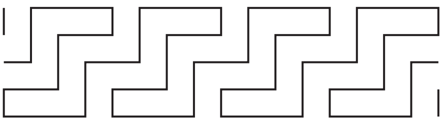
$x \to A + B + C$

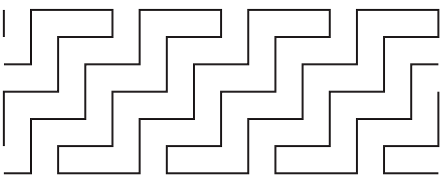
where x is a combination of A, B, and C, work to distinguish workshops and periods of ancient Greek art in the Geometric style. There are battlement meanders



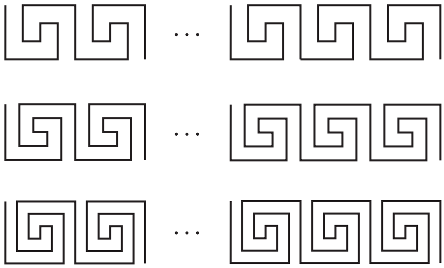
single (running) and multistage meanders



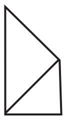




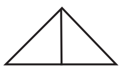
and spiraling meanders



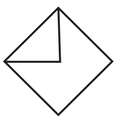
that vary in terms of the spatial relations given to define rules in her schemas. Assignments allow for shapes to be rearranged in spatial relations and for new shapes to be introduced. For example, I might start with the spatial relation



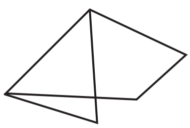
between two triangles, flip the large triangle to get the spatial relation



and then replace the large triangle with a square

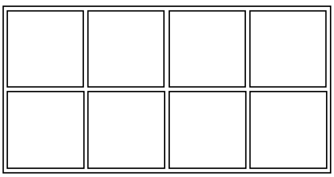


all in accordance with the enumeration I set out before. And I can continue this in other ways, perhaps ''dragging'' points

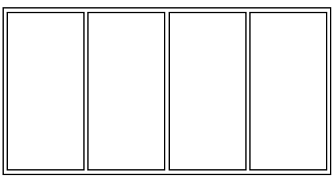


to turn the square into a quadrilateral with corresponding changes to the triangle. The repertoire of schemas from the series $x \to A + B$, $x \to A + B + C, \ldots$ is more or less constant, while rules are defined in alternative ways to produce different results. Knight proves the general paradigm in a special case. It's marvelous—styles and how they change depend on schemas, assignments, and transformations in given algebras. But undoubtedly I've missed a lot in my gloss of the essential facts. Knight would probably see it another way, as well, and there's no one better to believe than the author. One view is as good as another calculating with shapes, and no view lasts forever. This goes for styles themselves and ways of defining them. It strikes me that my description of style is simply a generalization of what's been tried, with more freedom and flexibility when it comes to schemas, using them, and changing them, and with greater emphasis on how assignments and transformations vary to tie things together. In addition, my approach to style corresponds to what I'm doing now with schemas and rules in different algebras of shapes, etc. It's a useful fit that's temporary if not evanescent, yet no less rigorous for that. It's being right without being eternal and unchangeable. It's just calculating with shapes.

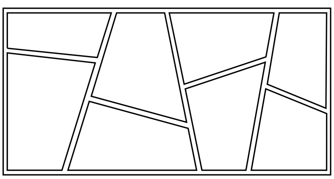
Choosing assignments and transformations is another way to get in the right frame of mind to design, although I really don't have anything more specific to recommend than what I've already shown by haphazardly dividing shapes, defining spatial relations, etc. Building up a repertoire of examples and motifs that can be used in assignments—perhaps this is what designers mean by defining a vocabulary—repays the effort. In fact, this is as much as I know how to do, in addition to augmenting my catalogue of schemas. Even so, it seems there are too many options to be systematic. In principle, anything will work. However, the idea is clear. Given an assignment g and a transformation t, the composition $t \cdot g$ determines when parts are alike and how they change as rules are tried. There's plenty of room for variation and a myriad of expressive possibilities. The proof is easy to see in a small way in ice-ray lattice designs. Starting with a rectangular frame, orthogonal compositions produce



and parallel ones



But then there's also this



I said I could augment my catalogue of schemas and rules, and indeed, it's never set once and for all. I can add to it as I please, and I have no reason to be parsimonious. Actually, I enjoy being messy and prodigal. There are many ways to frame schemas that are useful, even if the new ones I get are included in the ones I already have or are derived from them in some way. For example, six of the schemas I've given are ordered in terms of inclusion in the lattice

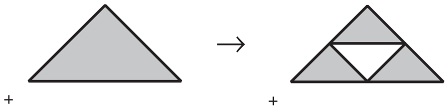
$$\begin{tikzcd}
& & x \to \sum x' \arrow[dll, dash] \arrow[d, dash] \arrow[drr, dash] & & \\
x \to x' \arrow[dr, dash] & & x \to \sum t(x) \arrow[dl, dash] \arrow[dr, dash] & & x \to x + x' \arrow[dl, dash] \\
& x \to t(x) \arrow[dr, dash] & & x \to x + t(x) \arrow[dl, dash] & \\
& & x \to x & &
\end{tikzcd}$$

with a new schema at the top—

$$x \to \sum x'$$

—that extends the second schema $x \to x' + x''$ I gave for ice-rays. And building from scratch, the schema $x \to A + B + C$ and larger sums are defined when the schema $x \to A + B$ is applied recursively. But why should I bother with relationships like this? Trying to decide whether one schema is included in another or whether to reduce everything to a few "primitive" schemas seldom repays the effort. In general, each is difficult to do, and either way there's an inevitable loss of immediacy—all of a sudden, things are harder than they look. It's better to define schemas, so that they capture what I want to see and do now—whatever that is. Schemas should be easy to use. Some organization may help to remember what I have in my catalogue, although it doesn't go much farther than that. There's little to gain—the descriptions schemas contain don't carry over to shapes as I calculate. Seeing more to get new designs is what's important, not elegant schemas in a formal catalogue. Nonetheless, my catalogue is a database, and I'm told that's vital for cognition and thought. But more rigor and formality seems unnecessary, and even what there is may be too much. Adding to my catalogue, so that there are always schemas to use, is all that counts.

Where do I find schemas to add? Well, I've already given some hints. I can combine the sides of the ones I've got in sums, or switch sides to define inverses. Or I can combine schemas in various ways—as above in $x \to \sum x'$, or by incorporating transformations, etc., for grids and villa plans. (Many of the technical devices in my *Pictorial and Formal Aspects of Shape and Shape Grammars* are useful in this regard.) Moreover, I can learn from experience, adding to my catalogue as I try new things and see what they do. Perhaps I have a list of identities that are useful, and expand it according to what I produce when I calculate. Then what I see next depends on what I've done before. And, in fact, I did this for fractals in part II to show how they were self-similar. I defined identities according to the shapes—lines and planes—I produced calculating with the rule



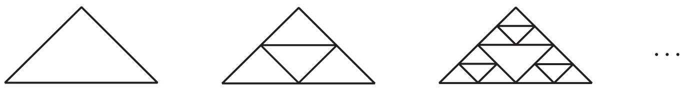
in the schema

$$x \to \sum t(x)$$

for a triangle and its boundary. Alternatively, I could use the schema

$$x + b(x) \to b(x)$$

where $x + b(x)$ is anything I produce with my rule, to play around with triangles made up of lines



But getting back to my catalogue, I can apply the schema $x \to A + B$ a couple of times and use identities to define rules in the schema $x \to A + B + C$. And I can go on in this way for larger sums. Shapes go together when I calculate, and they go together in rules, as well.

But there's still another way to find new schemas—I can borrow them from others. Whatever I see anyone else do will work in my catalogue—perhaps I use identities again to get new rules and schemas as I did for the Villa Barbaro. But whatever I do, the way it works isn't rote copying. No one confuses ice-rays and Glarner's paintings or imagines they're the same. There are different algebras, assignments, and transformations for the schemas I have. That's the beauty of it. Because of the way shapes are defined in terms of embedding, schemas and the rules they define are readily transferable and can be used in many creative ways. Being original isn't something you have to do all on your own. In fact in many ways, creativity implies community—both to make new things and to recognize their originality—and it's something that follows automatically when you calculate with shapes and rules.

I haven't paid very much attention to descriptions—despite this book, I really prefer seeing and doing more than saying what this is about. Nonetheless, it's easy to include schemas for descriptions in my catalogue, so that shapes and words interact usefully in design. I already have a number of schemas for counting and topologies of different kinds from part II, and surely there are schemas for many other things, as well. I can always find something else to say.

Latin and Greek, and Mathematics

I started this book with a question and an answer. I wanted to know how to draw lines on a blank sheet of paper. My question seemed easy enough, yet Miss H——'s response implied that I should look elsewhere—

If you don't know that, you'll never be an artist.

School wasn't the place for an answer. There wasn't anything to teach in the classroom. Explicit instruction didn't work—in fact, Miss H—— proved it in a neat reductio ad absurdum when she asked the kids in my class to figure out the possibilities in a drawing exercise. Drawing wasn't like the three R's—arithmetic and spelling were pointless without definite results. Everyone had a different way to draw—the results were your own and could vary in many ways. You had to look around to make sense of what was going on—to understand what you were doing and what others were trying. All of this was a surprise and something to get used to. Learning to draw meant learning to see and do—but seeing and doing what? It seemed that seeing was never really finished. Drawings were filled with ambiguity and could change freely at any time. There was more to see whenever you looked again and another way to go on. These were problems to solve. Yet everyone looked askance—nothing seemed to work. What good were problems that no one knew how to handle or wanted to try? Miss H—— was a conscientious teacher—why couldn't she help? There had to be something to learn that was definite and useful. You could start out with the basics in other subjects—so what about drawing? Education and training were important, but how? Did anyone know? What did the experts recommend?

Walter Smith pioneered art education in Boston public schools when Massachusetts made drawing a required subject. The law for this was passed in 1870 to gain a competitive edge for the state in world trade markets. And today, there's evident progress. Seeing doesn't matter anymore, but the goal hasn't changed. Instead of drawing, kids are required to take rigorous tests in reading, writing, and arithmetic to show they're worth hiring. There's a single standard for everyone that measures what's taught and what's learned. The idea is to "teach to the test" in each subject, so that everything is clear to teachers and students alike. There's no guesswork, only certainty and permanent results. All of this seems right, and it's easy to join in when everyone is saying exactly the same thing. No one ever disagrees. Nonetheless, it may be wrong, and saying why may not be what you want to hear. No one likes to talk about it in public—the reason is usually an embarrassment that's better to whisper and hide. What Søren Kierkegaard said about results also goes for tests—they're impossible without cheating.

While objective thought translates everything into results and helps all mankind to cheat, by copying these off and reciting them by rote, subjective thought puts everything in process and omits the results.

It's the same problem for everyone when what's taught and what's learned are translated into objective results that can be recited by rote. What are schools for and why bother to go if teachers cheat and kids follow their example? Without subjective thought, everything in process is another arrangement of given units—the only task is to combine them and count the results. It's the same both to teach and to learn. Schooling is senseless—there's little to say when there's nothing new to see and do. But what does Kierkegaard know about education today? He lived in nineteenth-century Denmark and was mostly ignored. Miss H—— didn't think drawing (subjective thought) belonged in the classroom and neither do the citizens of twenty-first-century Massachusetts. It's the law—good schools mean hard work and the chance to fail on objective tests. It's tough, it's fair, and it's a huge success. Everyone is asked the same questions and trained to give the same answers. There's no ambiguity. Everyone is prepared for the same future that's been decided in advance by someone else, and everyone is held accountable in the same way. It's simple enough: everyone is the same and sees the same things. Of course, 1870 was different—it was another time and another law. The decision was easy. Massachusetts hired Smith. He had a plan, he put it in place, and he made it work.

I find that not only does every person when he is taught rationally, and intelligently in the same way that he is taught Latin, and Greek, and mathematics, learn to draw well, but also to paint well, and to design well.

Then, as it does now, design mattered. It was drawing, and Smith wanted to teach drawing in the classroom along with language and mathematics. His teaching methods relied heavily on copying figures drawn on the blackboard, memorizing forms of objects and arrangements of them—was this vocabulary and syntax?—and repetition and practice. It was drill, drill, drill. Smith justified this in two ways: (1) copying was the only rational way to learn because drawing was essentially copying, and (2) it was the only practical way to teach to large classes that met just for short periods. This sounds pretty grim and horrible, and it goes hard against the emphasis on creative activity and open-ended experiment that's standard today in most design education. That's why there are studios instead of classrooms. And that's why students want them. But perhaps there's something to Smith's pedagogy. Copying needn't be as empty as it seems at first nor always produce rote results—combining predefined units in the way Kierkegaard scorns. Look again. Copying may hold something creative and original.

With shapes and rules, things change. That's been an important lesson throughout this book, and it's still the same. So it should come as no surprise that copying is in many ways at the root of calculating. When I apply a rule $A \rightarrow B$, I find a copy of A and replace it with a copy of B. An identity in the schema

$$x \rightarrow x$$

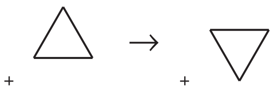
is a good example. I first erase the shape assigned to x and then draw it again exactly as it was. This is copying according to the formula

$$(x - x) + x$$

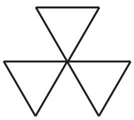
But there's more to rules than identities that keep shapes the same—even if this changes the way shapes look. Rules in the schema

$$x \rightarrow t(x)$$

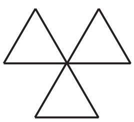
copy a shape or any part of one, erasing it and drawing it someplace else. The schema shows again why embedding—fusing shapes and then dividing to pick out parts—is so important. What you copy depends on reciprocal tests—what you can see with your eyes or trace out with your hands. And there's nothing rote about the results when there aren't any units to keep you from going on in your own way. Copying triangles with the rule



to turn the shape



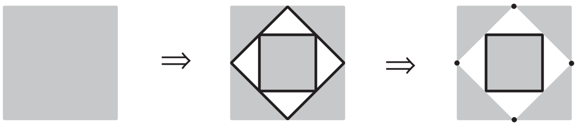
upside down



proves it beyond doubt. Copying three triangles gives two new ones to copy, and then this goes in reverse from two triangles to three. There's magic in the scribe's hand when there's always something new to see. (This may not be scholarship, but it does seem to be art.) Or perhaps boundaries are better as copies to delineate regions and mark endpoints. This is one way drawing works to copy what's there. Then the rules in the schema

$$x \rightarrow b(x)$$

are perfect. The schema shows even more how copying changes according to what you see and what you draw, as boundaries are decided in different ways—here in a collage of points, lines, and planes



that's made first by outlining four triangles that are ''embedded'' in a square plane and then by highlighting the vertices in one of the squares that these triangles contain. Going from one part to the next skips all over the place, but it's still copying. And in addition, there's the more elaborate schema

$x \to x + b(x)$

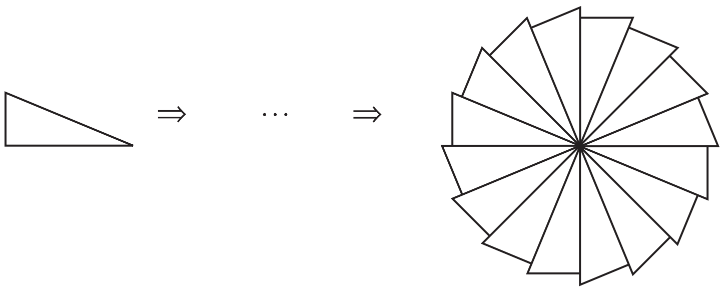
that's much the same, and its counterpart

$b(x) \to x + b(x)$

to connect points and fill in areas in coloring-book exercises. Rules in these schemas produce copies without erasing as copies alternate between parts and their boundaries. Copying without erasing is also the case for rules in the schema

$x \to x + t(x)$

Now, parts can be copied more than once in different ways. This already leads to a host of creative possibilities. At least there are symmetrical patterns of all sorts—here's one I considered earlier



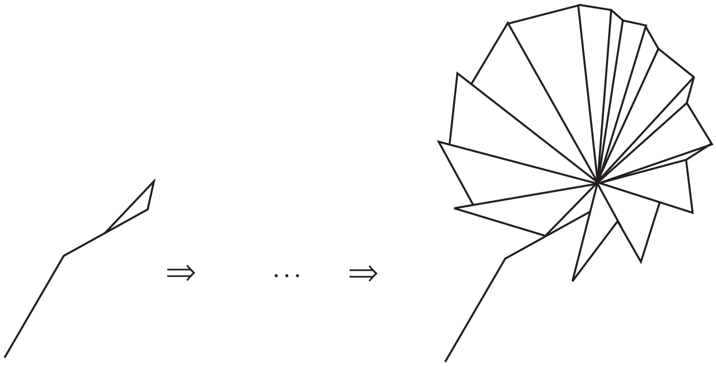
And, of course, there are fractals. But this kind of repetition isn't very exciting, when the same thing is copied over and over again in the same way. Klee wanted more freedom within the law—to see in different ways and to vary what he copied. With rules in the schema

$x \to x'$

and ones in the schema

$x \to x + x'$

copies may vary freely with parametric abandon. It's easy to be surprised—it's really almost effortless as long as you're ready and willing to calculate with shapes and rules



It's going on, seeing and doing in whatever way you like.

And there are other ways to copy shapes, too. In particular, I can combine algebras in products and copy across their components. This uses rules in the erasing schema

$$x \rightarrow$$

rules in its inverse

$$\rightarrow x$$

and identities $x \rightarrow x$ to calculate in parallel. For example, two new schemas can be defined—something more to put in my catalogue—that look like this

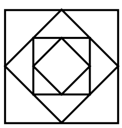
$$x, - \rightarrow -, x$$

$$x, - \rightarrow x, x$$

where the dash (–) indicates the empty shape. The first schema has a neat symmetry—it combines $x \rightarrow$ and $\rightarrow x$, so that the component or the part of it that's copied is erased. But in the second schema, $x \rightarrow x$ and $\rightarrow x$ are combined to preserve what's copied in its original place without changing anything. A slightly different example is in part II on pages 287–290. The schema

$$x, - \rightarrow -, y$$

is used to decompose the shape



into squares and triangles in five distinct ways. Simply put, these polygons in x are copied with the lines or the plane given in y. And this kind of copying is easy to elaborate in lots of ways—with added transformations, the boundary operator, parametric variation, etc. I can even use weights to get colors or other properties that interact as I draw, perhaps as in the example on pages 290–291, or in my design with Hilbert's curve and in the similar ones in figure 1. The conclusion is evident—once I can copy, I can calculate. (In fact this follows the letter of the law when triangles are copied in rules for Turing machines.) And once I can calculate, there's drawing and design, and the many delightful things that shapes and rules imply.

Smith really got it right—copying works, even if shapes, schemas, and rules may be a little more than he had in mind. It makes sense to go on from what others say and do. But perhaps there's more to this than blind luck and empty coincidence—there's also Smith's marvelous idea that drawing can be taught like language and mathematics in the classroom with worked examples and explicit instruction. And it's no surprise that this goes for what I've been showing for shapes, schemas, and rules. Does this mean I've changed my mind? I've spent a good portion of this book arguing that drawing isn't language. What can I possibly teach without vocabulary and syntax? Something is wrong. Well maybe not, vocabulary and syntax aren't at stake. Teaching drawing like language doesn't prove that

drawing is language

It's still a metaphor, although many accept it today as a heuristic or assume there's equivalence. And that's where the problem lies—not with teaching but with heuristics and equivalence. Certainly, all of the schemas I've described are things to teach in the classroom and to explain at the blackboard. There's a lot of copying to do to write schemas down and to try them out in exercises. And there's also showing why drawing isn't a language, how mathematics describes shapes and rules, and how shapes and words are connected when I calculate—it's Latin and Greek, and mathematics in another way. Many useful lessons are worth teaching in the classroom—there's everything in this book—before the vital transition into the studio to experiment freely with schemas. The opportunities for creativity and originality seem to be unlimited once you've learned to copy.

Of course, the idea of open-ended experiment in art and design is key in American art education after Smith—in particular, with Denman Ross at Harvard and Arthur Dow at Columbia University. Mine Ozkar traces the history of this—learning to draw wasn't rote copying, that is to say, blankly following instructions and blindly doing what you're told in the normal way you're trained to calculate. There was seeing, too.

> For a great while we have been teaching art through imitation—of nature and the "historic styles"—leaving structure to take care of itself . . . so much modern painting is but picture-writing; only story-telling, not art; and so much architecture and decoration only dead copies of conventional motives.

For Ross and Dow, "picture-writing"—what a felicitous compound given what I've been saying about drawing and why it isn't language with letters and words—wasn't near enough for art. And "dead copies" of familiar devices—correct spelling—didn't work in design. Never mind that words are easy to mix up—imitation and copying take care of structure automatically; the latter is included in the former. But it's the intention that counts. Ross and Dow were good guys—you get what they mean in spite of their words:

> Their pedagogies show how to set up temporary frameworks, instead of submitting to any predetermined structures. Frameworks develop based on variance in sense perception.

Sense perception lets me calculate my way—this depends on shapes and how rules are tried to pick out parts. And in keeping with this, work in the studio is trying what you've been taught in the classroom to go on to new things. It's also defining new schemas in terms of what you've done, and seeing what others are doing. It's learning from experience. It's another valuable way to add to your catalogue of schemas and to the rules you have to use in practice. It's becoming a creative designer. In the studio and in practice, designs (intellectual property) are free and only to share. This is copying to be original in the way schemas allow when they're used to define rules and calculate with shapes. Copying, community, and creativity run throughout design practice and education, and shapes and rules tie the three of them together. Both practice and teaching are seeing what to do in an open-ended process that involves us all. There's lasting community in the things we make—drawings, designs, etc.—not because we see them in the same way but because we don't have to. There's no common pool of assumptions or knowledge—tacit or not—that everyone has to accept in order to join in and take part. It's unnecessary to have shared anything ahead of time to go on. Otherwise, it would be enough to work alone—what's gained by working together is the other guy's point of view. Each of us is free to see and do in his or her own way without prior training or outside coercion. That's why shapes and rules and calculating with them are so crucially important—they let all of this happen. Design is never working alone, even if it seems that way. It isn't a solitary pursuit—there are just too many things to see.

When I calculate with shapes and rules, I get the easy corollary

design is copying

I really like this a lot because it's so practical—I know where I can look to see what to do—and it seems to reassure most people when they wonder where new designs come from. This is probably because it's an unremarkable answer, and copying is something everyone does—it works and it works well. The corollary allows for effective practice in design and shows how design can be taught without giving up any creativity. There's as much creativity in copying as there is in anything else. There are sources and precedents for just about everything. Almost nothing is aboriginal. But copying is PLAGIARISM—perhaps it is, but teaching and learning are more important. Plagiarism doesn't matter when there are no rote results. Then it isn't cheating. What you see and do belongs only to you—it's always your own work. There's something new and original every time you look, and who knows, the results may be spectacular. It's hard to imagine how to teach design without any plagiarism, much less how to design anything novel in practice. (Is it any surprise that design seems incongruous in great universities with their single-minded focus on academic integrity? The irony is that honor codes, student and faculty handbooks, and standards for research overlap from school to school, so that the ''rules'' to stop copying seem to be copied. Nonetheless, design studios are out. They rely on seeing what your classmates are doing and putting it to use as your own. This condones copying and encourages student work that looks the same—it's uncanny how often studio projects are alike and how no one ever seems to notice. But copying is no way to show what you can do in research—so much for reverse engineering and repeatable results in science—and in super-tough university classes where you can earn an F. Copying isn't allowed. It's a question of ownership—everyone has to do ''original'' work to guarantee that it's his or hers. That's how teaching and learning are measured—by the work you do on your own—and it's what it means to be creative. There's just no place for design!) As long as I don't assume there are units in a combinatorial process—that's been the big problem all along: how to calculate without symbols—it's easy to go on in new ways and to encourage others to do the same. Design needn't be a mystery. You can teach it and foster mastery instead. Design and teaching how to do it show meaningful results when there's copying. It's best practice, and it's always in process so that nothing is ever final. Results aren't set in advance and can't be repeated (recited) by rote. It may be that there are some useful alternatives to copying—anything can happen—but I wouldn't bank on it. I'll put my money on shapes and rules. Calculating—yes, copying—provides a competitive advantage that's hard to beat.

One of the things I've been trying to show throughout this book is that it's always possible to go on in new ways when you calculate with shapes and rules. It's about seeing for the first time again and again without ever having to start over. Going on, to be sure, is the way to design, but it's hardly the way to end a book. That calls for a conclusion. I started out in the introduction talking about seeing and doing in this way:

[Calculating with shapes and rules] is subjective and variable—the shape grammarist's voice is ineluctably personal.

And this seems to be the right ending, too. In many ways, it's just what this part of my book has shown. I've taken you through many examples of what I do when I use rules to calculate with shapes. Seeing and saying what I see are always personal. There are no rote results, whether I copy what I see or call this something else—descriptions don't count. My eyes have only their own way of knowing. That's a good reason to calculate, and it's why calculating works in design.

Background

The background to this part is pretty straightforward, especially if I stick to what I've said in the order I've said it. The quotation I start off with is from Walter Smith's testimony to the Royal Commission on Technical Instruction of 1883.[1] Smith sets the tone for what I go on to say. There are three points of interest. First, he equates drawing with painting and design. Second, he's sure that drawing can be taught rationally and intelligently in school like language and mathematics. And third, he turns to ''a great wealth of illustration'' to prove it. That design is drawing is where I begin, and that design can be taught in the classroom is where I end, but probably not with the kind of rationality and intelligence—copying—Smith has in mind. Within this locus, there's plenty of illustration. And, in fact, that's what really matters—what I can calculate with shapes and rules, and what you can see with your eyes.

Painting is how I started with shapes and rules, although this is recast here in terms of the algebras and rules I use now.[2] I first used schemas to produce Chinese lattice designs.[3] My discussion of these traditional window grilles—ice-rays included—hasn't changed much, except for the key addition of identities, etc., to handle descriptive aspects of style over and above what it takes to produce designs. This is easy with schemas as they're currently defined using twin variables.[4] The material on Palladio is from three early papers, but my discussion of the Villa Barbaro hasn't been seen before.[5] I tried description rules a little after Palladio, only not for his villa plans.[6] Their use in this way is new. The idea of calculating with rules in parallel, however, is already clear in *Pictorial and Formal Aspects of Shape and Shape Grammars*.[7] My original discussion of design synthesis and stylistic analysis is in ''Two Exercises in Formal Composition,'' with a lot more in ''Kindergarten Grammars'' on design synthesis, spatial relations, and the schema $x \to A + B$ and its inverse.[8]

Of course, I owe much to many others in all of my examples. Paul Klee's drawings—in particular, the ''palm-leaf umbrella''—are in *The Thinking Eye*, together with the quote on irregularity and the law.[9] Then there's Daniel Sheets Dye's *A Grammar of Chinese Lattice Designs*, which is valuable for both the drawings and the words it contains.[10] The plans for the Villa Foscari, the Villa Rotonda, and the Villa Barbaro are from Andrea Palladio's *The Four Books of Architecture*, and there's the quotation on page 343.[11] I've also relied on James Ackerman's book *Palladio* for useful commentary in four places—pages 341, 344, 346, and 347.[12] I don't discuss Palladio's system of building proportions, but Lionel March provides a pathbreaking analysis that the shape grammarist can't help but admire.[13] Terry Knight gives a comprehensive and critical account of different approaches to style and stylistic analysis that concludes with shapes and rules.[14] She goes on to investigate stylistic change in an original and effective way using a generalization of the schema $x \to A + B$ and its inverse. This includes her analysis of Greek meander designs, and a detailed discussion of the paintings of Georges Vantongerloo and Fritz Glarner—I borrow from her examples for all three. Alvaro Siza's ground plan at Malagueira is adapted from José Duarte's discursive survey of the project.[15] Also of interest, Andrew Li provides an elegant application of description rules to elaborate the building principles in the *Yingzao Fashi*.[16] Perhaps this isn't that surprising for a notoriously rigorous building manual. Nonetheless, to see the rules actually work is pretty impressive. And notice, too, that the relationship between rules and description rules mimics Gottlob Frege's principle of compositionality that links syntax and semantics—the meaning of a logical expression or a sentence in English is a function of the meanings of its parts and how they're combined. Of course, this fails for shapes because their parts aren't set in advance. There are options in retrospect—perhaps using topologies—but these probably wouldn't satisfy Frege and his current followers. They relish hard problems where things stay the same, yet sometimes problems are hard only because of the way they're set up to start. It's easier not to worry about this and to let things alter freely, as they do calculating with shapes or copying Wittgenstein's "figures on paper." An account of George Birkhoff's aesthetic measure and its information-theoretic counterpart E_Z is in *Algorithmic Aesthetics*.[17] For an incisive discussion of value in design, see March's classic essay "The Logic of Design and the Question of Value."[18] March goes on to consider C. S. Peirce's famous trio of inferential categories in design—these are deduction and induction, as expected, and then, in addition, abduction. Roughly speaking, going from shapes to their descriptions corresponds to deduction, going from descriptions to shapes to abduction, and figuring out schemas and rules in the first place to induction. March puts this together in a cyclic model of design that applies in some telling ways. Peirce is keen on clarity, albeit each kind of inference holds alternative choices with a combinatorial (syntactic) uncertainty or ambiguity of its own. The lines I've quoted form Søren Kierkegaard are copied from my introduction. Mine Ozkar chronicles the artistic adventures of Denman Ross and Arthur Dow, and the incidental interactions of the former with William James at Harvard and the noteworthy relationship of the latter with John Dewey at Columbia.[19] More important, though, shapes and rules are compatible with open-ended experiment in the design studio.

> All this prevails in the distinctive pedagogical standpoint shared by Ross and Dow. . . . Individuals are encouraged in their unique ways, which can only be represented in temporary and discardable conceptual structures.[20]

Better still, every rule—even an identity—implies a new (original) conceptual structure every time it's applied to a shape. The way topologies are redefined on the fly is a good example of how this works, but there are many other examples throughout this book. Ordinarily, design and calculating are worlds apart. Sometimes, a brief alliance is formed when calculating produces things designers want to use and can't make on their own. Then it's one way calculating and another way when design works its magic—there's a discrepancy between what the computer does and what the designer sees. This exposes shortcomings in both calculating and design, and the divide between them. Yet now, the "two cultures" fuse. My initial metaphor is equality—

design = calculating

—not because design is calculating in the usual way, but because calculating is more than it's supposed to be. Design is calculating with shapes and rules. It's seeing and doing, and all that follows as I go on.

Today, there are far too many examples of shape grammars in design—including architecture and engineering—to be cited item by item. Most of this material is in *Environment and Planning B: Planning and Design* from 1976. Nonetheless, there are two pioneers of the subject whose work has been widely influential—Ulrich Flemming and Terry Knight.[21]