Numerical Computation

Goals

- be able to answer the prompt: write a function that will find the square root of a number
- Understand multiple possible algorithms for finding a square root
- See python code that implements the algorithms
- Run python notebook with code demonstrating the algorithms

Understanding the computer's abilities

In order to write an algorithm, you must know how the computer "thinks"

- computer can only do small, simple steps
- a computer can only do one at a time
- computers are good at repeating
- computers must be provided some information
- computers cannot invent anything
- a computer can DERIVE information from prior information

Write a function that will find the square root of a number

Guess and Check

Figuring out the square root of a number (poorly)

Essential information: The **number** that should be square-rooted

- This will be a parameter to a function

Logical Steps:

- guess a random number as the solution
- confirm or deny by squaring it
- repeat until solution is found

Pseudo Code:

- loop while solution has not been found
  - create random guess
  - square random guess
  - if random guess squared IS the original number
    - return random guess!
  - Otherwise start process again

guess and check

Bad example!

```python
# Guess and Check

import random

def squareroot(number: int) -> int:
  """Guess integer roots and check."""

  while True:

    guess = random.randint(0, number)

    if guess**2 == number:
      return guess
```

- loop while solution has not been found
  - create random guess
  - square random guess
  - if random guess squared IS the original number
    - return random guess!
  - Otherwise start process again

guess and check

Write a function that will find the square root of a number

Organized Guessing

Organized guessing…?

Imagine: What is the square root of 625?

The solution should be somewhere between 1 and 625

[1,2,3,4,5,6,7,8,9,10,11,……,24,25,26,27,.....625]

Exhaustive enumeration

Figuring out the square root of a number

Essential information: The number that should be square-rooted

- This will be a parameter to a function

Logical Steps:

- check every number in a range, in order, exhaustively
- confirm or deny by squaring it
- repeat until solution is found

Pseudo Code:

- loop through the range
  - Consider item in the range
  - square the item
  - if item squared IS the original number
    - return it!
  - Otherwise move on to next item in the range

Exhaustive enumeration

```python
# Exhaustive Enumeration for perfect squares

def squareroot(number: int) -> int:
  """Exaustively check integer roots."""

  for possible_answer in range(number):

    if possible_answer**2 == number:
      return possible_answer

  return -1
```

- loop through the range
  - Consider item in the range
  - square the item
  - if item squared IS the original number
    - return it!
  - Otherwise move on to next item in the range

Exhaustive enumeration

What about non-perfect squares?

The solution will not be an integers…..so how should organized guessing work?

Exhaustive enumeration

What about non-perfect squares?

```python
# Exhaustive Enumeration for non-perfect squares

def squareroot_ee(number: int) -> float:
  """Exhaustively check all possible roots for numbers >= 1."""

  if number < 1:
    return -1

  epsilon = 0.01 # margin of error, could be a parameter
  step_size = epsilon**2
  possible_answer = 0

  while abs(number - possible_answer**2) > epsilon
    possible_answer += step_size

  return possible_answer
```

Exhaustive enumeration

```
and possible_answer**2
< number + epsilon:
```

```
while abs(number - possible_answer**2) > epsilon and possible_answer**2 < number + epsilon:
```

- epsilon is a margin of error
- as you saw in the first lab, floating point numbers are not perfect
- It will never be possible to find the exact answer, so a margin of error is needed!
- the and is needed to cut off an infinite loop caused by a step size that misses the margin of error

Write a function that will find the square root of a number

Efficient Guessing!!!

Figuring out the square root of a number

Essential information: The **number** that should be square-rooted

- This will be a parameter to a function

Logical Steps:

- check middle number in a range
- confirm or deny by squaring it
- cut the range in half intelligently
- repeat until solution is found

Pseudo Code:

- loop while solution has not been found
  - consider middle item
  - square item
  - if item squared IS the original number
    - return it!
  - Otherwise
    - if the item squared was too large
      - adjust the search range to be the lower half of the range
    - otherwise
      - adjust the search range to be the upper half of the range
  - repeat

Bisection Search (divide and check)

Bisection Search to find square root of 144

min = 0, max = 144, midpoint = 72

72*72 is too large!

min = 0, max = 72, midpoint = 36

36*36 is too large!

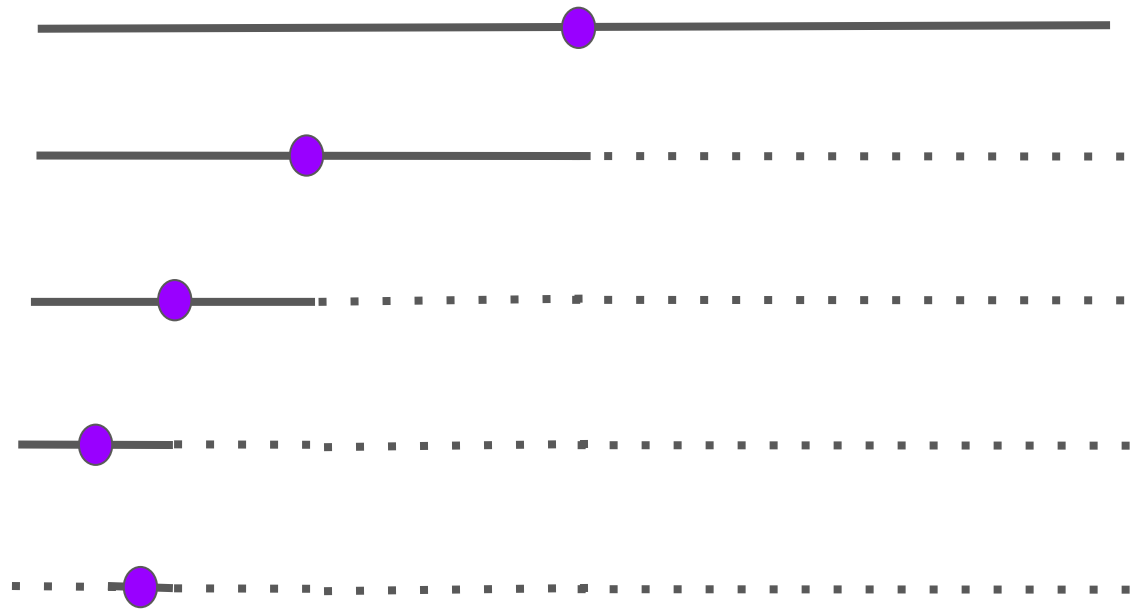
min = 0, max = 36, midpoint = 18

18*18 is too large!

min = 0, max = 18, midpoint = 9

9*9 is too small!

min = 9, max = 18, midpoint = 13.5 etc.



Search space is cut in half every time! Amazing, compared to exhaustive enumeration

Does that picture remind you of anything?

Like using a real Merriam Webster Dictionary!

```python
# Bisection Search for non-perfect squares

def squareroot_bs(number: int) -> float:
  """Perform bisection search to find root."""

  if number < 1:
    return -1

  epsilon = 0.01 # margin of error, could be a parameter
  lower_bound = 0
  upper_bound = number
  midpoint = (lower_bound+upper_bound)/2

  while abs(number - midpoint**2) > epsilon:

    if midpoint**2 > number:
      upper_bound = midpoint
    else:
      lower_bound = midpoint

    midpoint = (lower_bound+upper_bound)/2

  return midpoint
```

- loop while solution has not been found
  - consider middle item
  - square item
  - if item squared IS the original number
    - return it!
  - Otherwise
    - if the item squared was too large
      - adjust the search range to be the lower half of the range
    - otherwise
      - adjust the search range to be the upper half of the range
  - repeat

Bisection Search (divide and check)

General things to note

- simple algorithms are usually less efficient
- efficiency improves as the algorithms are refined and include sophisticated steps