Optimization

Guttag Chapter 14

Goals

Understand Knapsack Problem

- Values
- Weights
- Constraint
- Optimization Objective

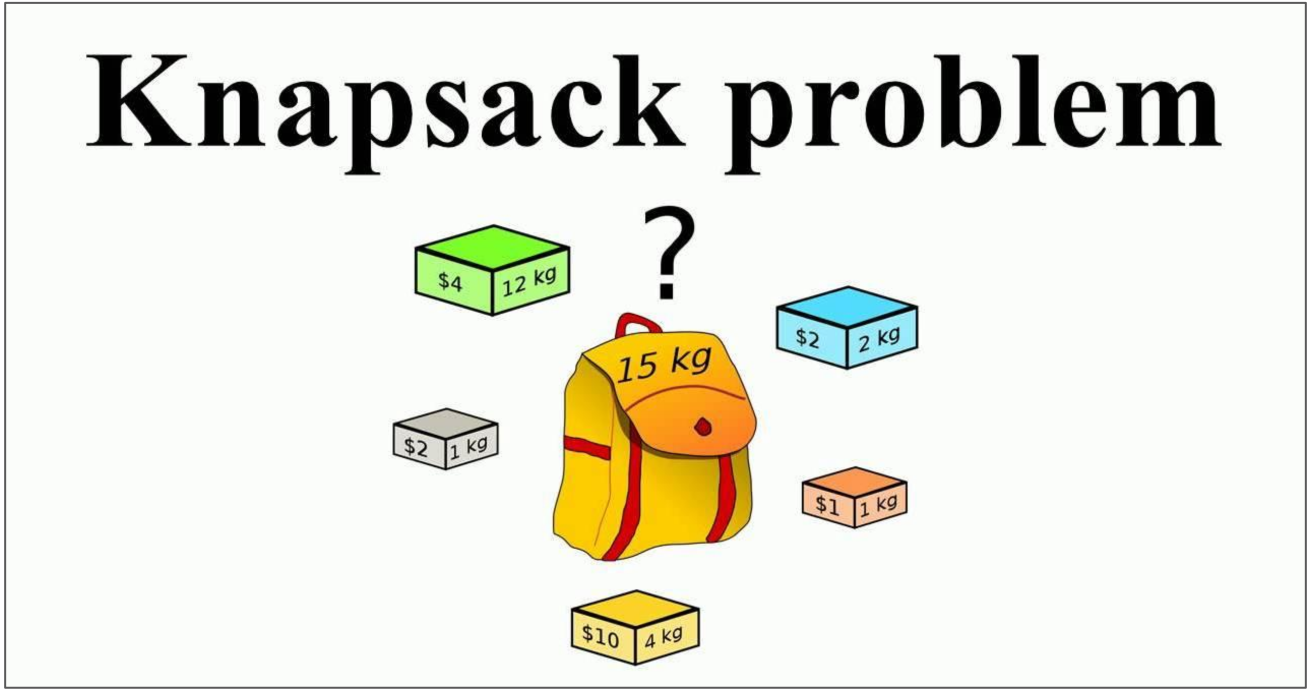
Powerset Solution

Greedy Algorithm Solution

Code Example

Understanding the Knapsack

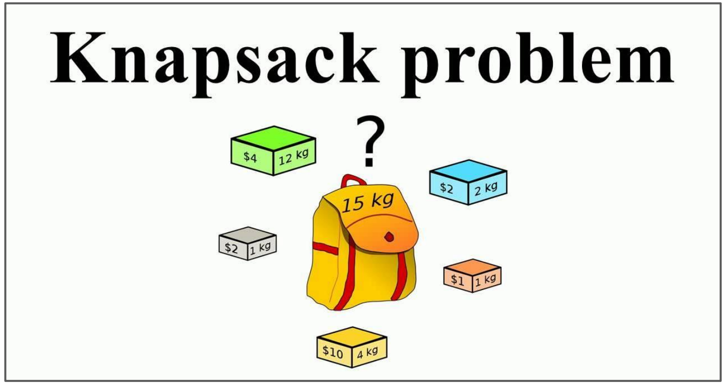
Knapsack Problem



Knapsack Problem

Definition:

- a knapsack has limited room for items inside.
- The packed knapsack can only contain a <u>subset</u> of items
- The problem is finding the **optimal** subset

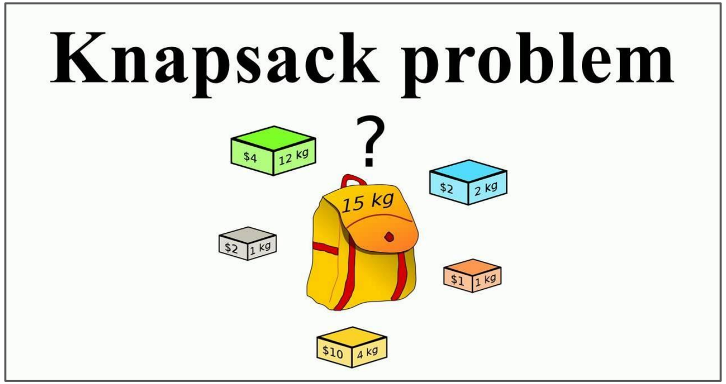


Knapsack Problem

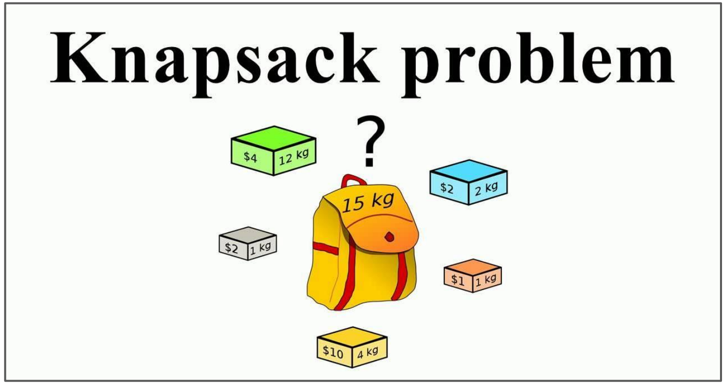
Example:

- the knapsack can only contain 15 kg
- which subset of items should fill the sack?

Lime: \$4, 12 kg
Gray: \$2, 1 kg
Blue: \$2, 2 kg
Orange: \$1, 1 kg
Yellow: \$10, 4 kg



Knapsack Problem



Example:

- the knapsack can only contain 15 kg
- which subset of items should fill the sack?

Lime: $4, 12 kg
Gray: $2, 1 kg
Blue: $2, 2 kg
Orange: $1, 1 kg
Yellow: $10, 4 kg

Each item has a **value** ($)

Each item has a **weight** (kg)

The knapsack is **constrained** to have a max weight

Optimization **Objective**

Definition:

- Choosing the ideal items
- Follow a goal

Example

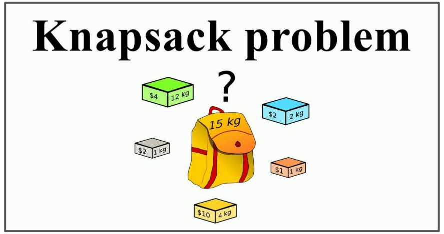
- choose items to maximize value
- choose items to minimize value (e.g. let's say calories)
- choose items to maximize weight
- choose items to maximize density (e.g. value/weight)

Powerset Solution

Powerset Solution

The objective is to maximize the value in a knapsack that can only hold 15 kg max

Lime: $4, 12 kg
Gray: $2, 1 kg
Blue: $2, 2 kg
Orange: $1, 1 kg
Yellow: $10, 4 kg



1. Generate the Powerset:

{}, {L}, {G}, {GL}, {B}, {BL}, {BG}, {BGL}, {O}, {OL}, {OG}, {OGL}, {OB}, {OBL}, {OBG}, {OBGL}, {Y}, {YL}, {YG}, {YGL}, {YB}, {YBL}, {YBG}, {YBGL}, {YO}, {YOL}, {YOG}, {YOGL}, {YOB}, {YOBL}, {YOBG}, {YOBGL}

2. Search through the powerset to find the subset that has largest $ value under 15 kg !

Critical Thinking

1. Generate the Powerset:

{}, {L}, {G}, {GL}, {B}, {BL}, {BG}, {BGL}, {O}, {OL}, {OG}, {OGL}, {OB}, {OBL}, {OBG}, {OBGL}, {Y}, {YL}, {YG}, {YGL}, {YB}, {YBL}, {YBG}, {YBGL}, {YO}, {YOL}, {YOG}, {YOGL}, {YOB}, {YOBL}, {YOBG}, {YOBGL}

2. Search through the powerset to find the subset that has largest $ value under 15 kg

What does the search entail? Be ready to answer out loud

Pseudo Code

```
# for each subset

# set up a value tracker, and weight tracker variable

# loop through everything in the subset one at a time

# increment the value and weight as you go

# if the max weight is exceeded, stop and move on to the Next SUBSET

# if no weight issue, only save the subset if it has the best value you've seen so far
```

Greedy Approximation

Greedy

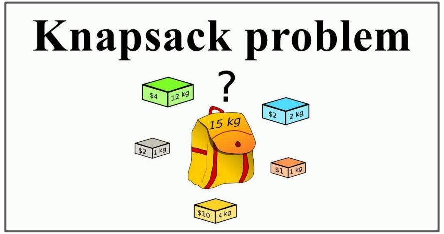
Definition

- take the best thing that fits within the **constraint**
- best is guided by the **objective**
- repeat the process until the **constraint** is hit

Greedy Solution

The objective is to maximize the value in a knapsack that can only hold 15 kg max

Lime: $4, 12 kg
Gray: $2, 1 kg
Blue: $2, 2 kg
Orange: $1, 1 kg
Yellow: $10, 4 kg



1. SORT your items by value
2. Take the best $ value first → Yellow
3. Check that the constraint is not violated (11 kg remain)
4. Take the best $ value again → Lime
5. Check that the constraint is not violated (-1 kg remain XXXX)
6. Discard Lime and continue greedy algorithm

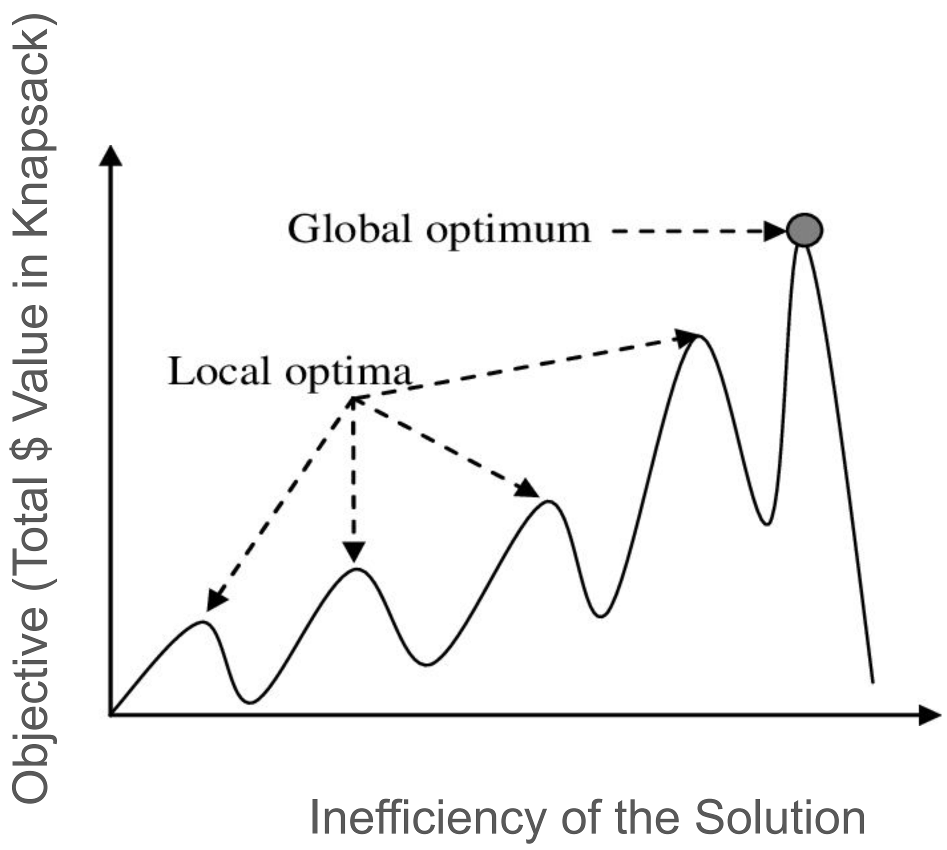
Greedy vs Powerset

The **Global Optimum** will be found with the **powerset**.

- But the search is extremely inefficient! $O(2^n)$

A **Local Optimum** will be found with the **Greedy Algorithm**

- Greedy is efficient O(n)



Code Example

For the Code Example

Think of 8 foods that you find delicious

- **Value** will be deliciousness value for you
- **Weight** will be the calories in the food
- **Objective** will be getting the most value while not exceeding some calorie **constraint**