CMPSC 102
Discrete Structures
Fall 2019

Practical 8: The Fibonacci Sequence at an Arm's Length

Refer to your notes, slides and sample Python code from this week and other weeks. In particular, follow the python code that we created in class or check on line for interesting pieces of code to help you in your programming.

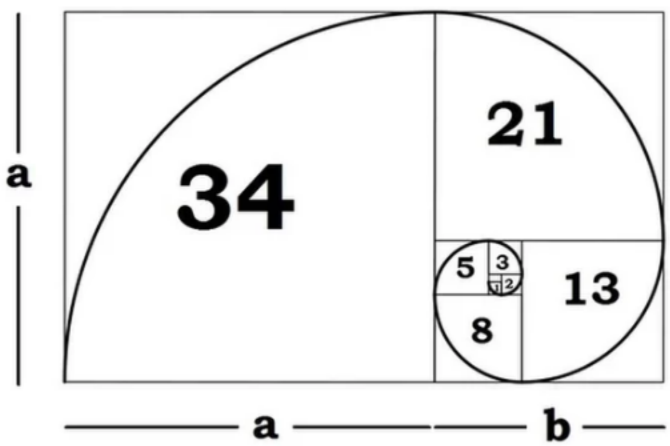

Figure 1: The *Golden Ratio* (ϕ) is an irrational number (1.61803398875) that can be found all over nature, including the Fibonacci sequence. For example, ϕ may be found in the quotient of tangent (i.e., side by side) Fibonacci sequence pairs, f_m and f_n by the following calculation $\left(\frac{f_{n-2}}{f_{n-1}}\right)$. This ratio becomes more accurate (i.e., a better approximation to the real value) as the sequence pairs in this fraction become increasingly large. Here, these values represent surface areas of rectangles.

GitHub Starter Link

https://classroom.github.com/a/dZBHQgCD

To use this link, please follow the steps below.

- Click on the link and accept the assignment.
- Once the importing task has completed, click on the created assignment link which will take you to your newly created GitHub repository for this lab.
- Clone this repository (bearing your name) and work on the practical locally.
- As you are working on your practical, you are to commit and push regularly. You can use the following commands to add a single file, you must be in the directory where the file is located (or add the path to the file in the command):

```
– git commit <nameOfFile> -m ''Your notes about commit here''
– git push
```

Alternatively, you can use the following commands to add multiple files from your repository:

```
– git add -A
– git commit -m ''Your notes about commit here''
– git push
```

Summary

In this practical, you be studying yet another fascinating way to determine the Fibonacci sequence using a special relationship between the numbers, shown in Figure 1. In this task, your approach will be to determine the *Golden Ratio* (ϕ), that will be used to grow out the terms of the Fibonacci sequence. To make this determination, you will use mere measurements from your arm, as shown in Figure 2. The value of the length of your shoulder to your fingers, over the value of the length of your elbow to your fingers, is (coincidentally) an approximation of the same ratio of tangent Fibonacci numbers, discussed below. *Note: if you do not wish to use your arm's measurements, then you are invited to use the general terms found in the sample output shown below, or you can determine values using other means.*

Fibonacci's Sequence and Phi (called, ϕ)

For example, the 18th, 19th and 20th values for the Fibonacci sequence are the following.

$$\{\texttt{2584, 4181, 6765}\}$$

Using these numbers, we apply division to get the following quotients.

$$\phi \sim \frac{6765}{4181} = 1.6180339631667064$$

and

$$\phi \sim \frac{4181}{2584} = 1.618034055727554$$

and (using really big values of the sequence)

$$\phi \sim \frac{9969216677189303386214405760200}{6161314747715278029583501626149} = 1.618033988749895$$

(the 150th and 149th values)

The larger the value of the Fibonacci sequence pairs, the better the approximation to the actual ϕ value you will find. Although the quotients taken from lower values of the sequence may not equal the actual value of ϕ (which is 1.61803398875, according to Google), we note that these quotients are still *very* close and should work well for the task of growing out the Fibonacci sequence after applying an initial value.

An Equation to Note

The equation to grow a sequence term (f_n) from a previous term (f_m) is the following.

For the sequence,

$$\{..., f_m, f_n, ...\},$$

where f_n and f_m are sequential terms of the Fibonacci sequence, and ϕ is the *Golden Ratio*, we note that the term f_n (i.e., the next term in the sequence) may be found by the following equation.

$$f_n = \phi * f_m.$$

If you find that your sequence appears to be growing with an error, you might want to try experimenting with the `math.ceil()` or `math.floor()` functions in your code to round your value up (or down) to the nearest integers to reduce the errors that naturally arise in your calculations. This line of code involves the `math` library in Python3 and would return `6` for `math.ceil(5.5)` and would return 5 for `math.floor(5.5)`. The following line may help to reduce error.

$$f_n = \texttt{math.ceil(}\phi * f_m\texttt{)}.$$

The File `src/fibArm_starter.py`, that you will modify, already imports the `math` library for this function to be available in your code.

The Steps to Complete

1. Locate the incomplete Python source file in `src/fibArm_starter_i.py` to modify. The code already produces the correct Fibonacci sequence that you can use for comparison to your own sequence values.

2. To make your own Fibonacci sequence values, you are to modify the file Python code to accept the user inputs of the arm (i.e., shoulder to fingers and elbow to fingers, shown in Figure 2) to calculate the ratio of $\phi = \frac{f_n}{f_m}$. Note: please refer to Figure 2 to determine which value is which. Then you will write a function (called, `fibByArm()`) to calculate the Golden Ratio (ϕ) and to calculate the next sequence value.

3. You will also add code to ask the user how many iterations of the sequence to produce. This could be another user input or could be simply hard-coded into the source file.

4. Please note that due to natural errors in determining the necessary arm measurements, your ϕ value is likely to be prone to error and your sequence values will be consequently slightly different to the actual Fibonacci sequence values. In this case, your sequence may be called a Lucas Sequence which grow similarly to Fibonacci sequences (read more about Lucas numbers at `https://en.wikipedia.org/wiki/Lucas_number`.)

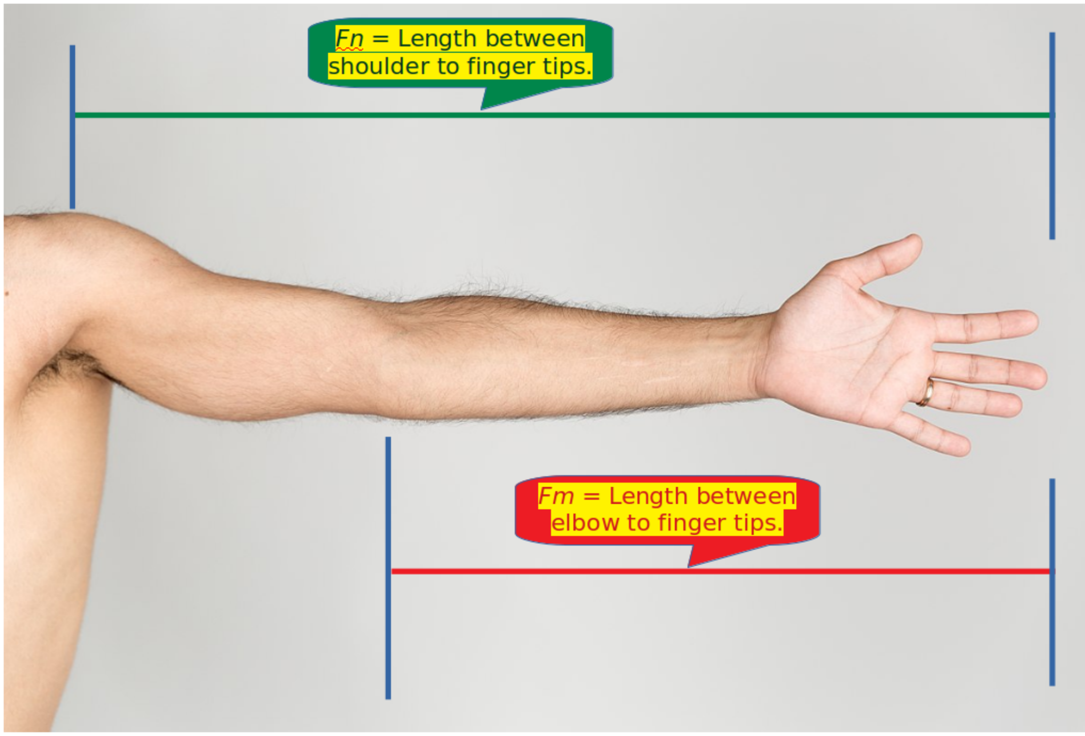

Figure 2: Your measurements to determine the *Golden Ratio* are extracted by measuring the distance between your longest finger and your shoulder (f_n) and the distance between your elbow to your longest finger (f_m). The *Golden Ratio* can be found here by finding the quotient of $\frac{f_n}{f_m}$. Yes, this is my arm. :-)

Output

The output from the source code will look like the following. Note, the sequence may start at 2, or the third term of the Fibonacci sequence.

```
The Real Fibonacci Sequence for 15 iterations:
  [1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610]

 Length of shoulder tip of longest finger : 28
 Length of elbow to tip of longest finger : 17.3
 From these inputs, the ratio is : 1.6184971098265895
   n Sequence term
   0 1
   1 1
   2 2
```

My value of ϕ above is already off by a small amount. You will note that even with fairly accurate measurements, approximation errors happen and prevent the your sequence from being exactly the same as the real Fibonacci sequence. Perhaps your measurements will be more accurate and your sequence will look better than the one above.

Deliverables

1. Your completed (and working) Python code (`src/fibArm_starter.py`)
2. Extra Credit: If you go to Friday's talk on Ethical Robotics and write up a short statement about what you learned and submit the file in writing/statement.md, then you can earn ten extra credit exam points.

```
- .... . / ..-. .. -... --- -. .- -.-. -.-. .. /
        ... . --.- ..- . -. -.-. . /
         .. ... /
            . ...- . .-. -.-- .-- .... . .-. . -.-.--
```