# CMPSC 390
# Crypto Fundamentals

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Credit: Authors of "Bitcoin and Cryptocurrency Technologies"

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# Cryptocurrency Components

- Identity: an account (node) in the system.
- Transactions: sending and receiving units of cryptocurrency.
- Distributed Ledger: a public record of transaction history (blockchain).
- Trustless Consensus: agreement on changes to the ledger.

# Cryptographic Hash Functions

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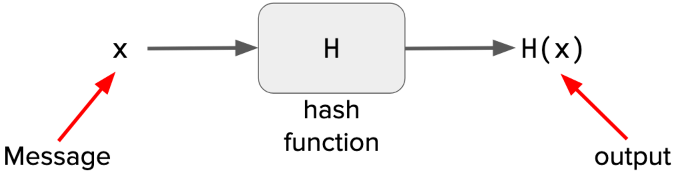
- Input of any size, fixed-size output (e.g., 256 bits).

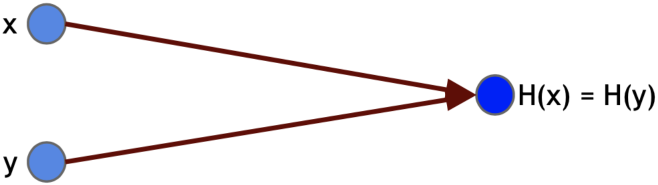


# Security Properties

**Hash property 1:**

**Collision resistance**

Nobody can find $x$ and $y$ such that $x! = y$ and $H(x) = H(y)$.




Nobody can find this!

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- There is 99.8% chance that two of them will collide.
- This works no matter what $H$ is ... but it takes too long to matter.
- No $H$ has been **proven** collision-free.

# Security Properties

**Hash property 2**:

**Hide information**

Given $H(x)$, it is infeasible to find $x$.

# Commitment API

```
(com, key) := commit(msg)
match := verify(com, key, msg)

To seal msg in envelope:
    (com, key) := commit(msg) -- then publish com
To open envelope:
    publish key, msg
    anyone can use verify() to check validity
```

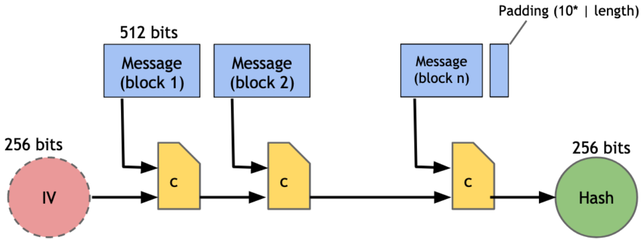
# Security Properties

**Hash property 3**:

**Computational efficiency**

Set of computation to get a hash should not take a long time.

# SHA-256 hash function




Theorem: If c is collision-free, then SHA-256 is collision-free.

- SHA-256: A cryptographic hash function designed by the NSA.
- Bitcoin uses $SHA - 256^2$ ("SHA-256 squared"), meaning that $H(x)$ actually means $SHA256(SHA256(x))$.

# Generating hash function in Python

```
import hashlib

hashlib.sha256(string.encode()).hexdigest()
```

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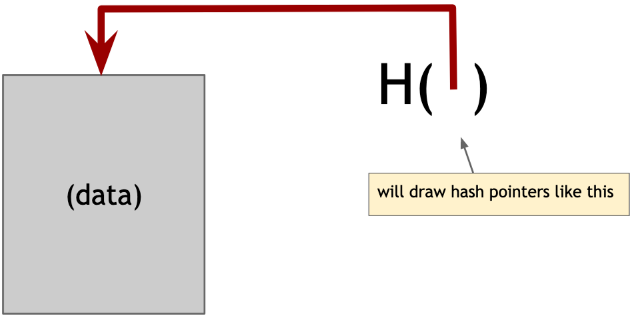
*With a hash pointer, we can*:

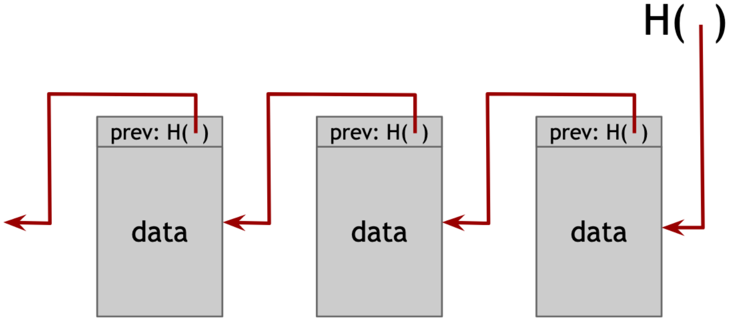
- ask to get the info back, and
- verify that it hasn't changed.

# Key Idea: build data structures with hash pointers



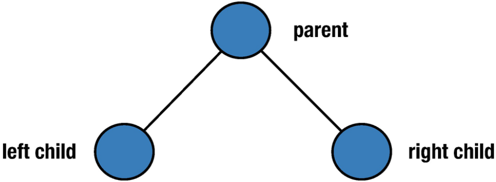
# Blockchain

Linked list with hash pointers = “block chain”



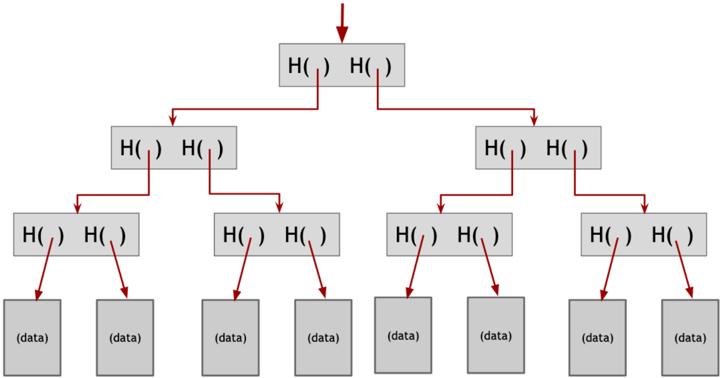
# Binary Tree

Binary tree: a tree data structure with each node having at most two children.



# Merkle Tree

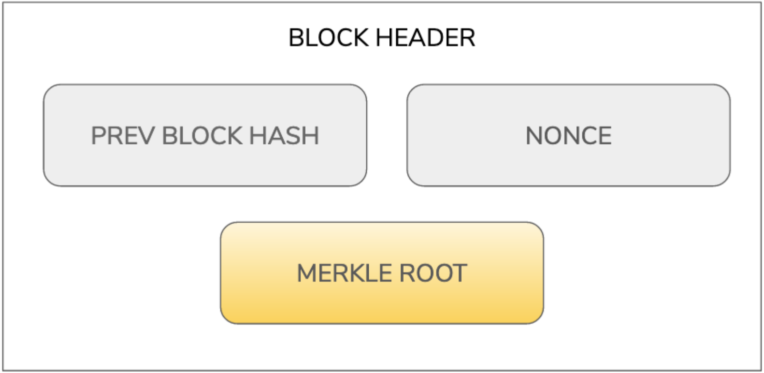
A binary tree with hash pointers



# Advantages of Merkle Trees

- Just need to remember the root hash.
- Can verify membership in $O(logn)$ time/space.

# Blockchain



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- Only you can sign, buy anyone can verify.
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**Public Key Cryptography:**

a cryptographic system that allows for secure dissemination of identity and authentication of valid messages.

# API for Digital Signatures

(sk, pk) := generateKeys(keysize)

sk: secret signing key

pk: public verification key

sig := sign(sk, message)

can be randomized algorithms

isValid := verify(pk, message, sig)

# ECDSA standard: Elliptic Curve Digital Signature Algorithm

- The algorithm used by Bitcoin to generate public keys and verify transactions.
- A variant of standard DSA but with elliptic curves.
- Good randomness is essential.

# ECDSA Example

Instructor's chicken scratches ...

# Public Key Generation

```
Input: public key
Output: corresponding private key

256 bit private key, takes O(sqrt(n)) operations to crack
15 * pow(2,40) hashes per second on the ENTIRE Bitcoin network

pow(2,128) / (15 * pow(2,40)) / 3600 / 24 / 365.25
= 0.6537992112229596e18

650 million billion years
```

# Public Key as Identity

Make new identity:

- Create a new, random key-pair $(sk, pk)$.
- You control the identity, because only you know the secret key, $sk$.

# Decentralized Identity Management

- Anyone can make a new identity at any time, as many as want.
- There is no central point of coordination.
- Identities are called addresses in Bitcoin.

# Privacy

- Addresses not directly connected to real-world identity.
- But observer can link together an address's activity over time, make inferences.
- Will discuss privacy in Bitcoin in more detail later.