Week 1: practice problems
ECON 100A

TA Allegra Saggese

Date 29 september 2025

Professor: Natalia Lazzati

Topic: math review

Problem 0

What is a Lagrangian function? What is the role of (*often* λ) the Lagrange multiplier?

Problem 1

Find the derivatives of the following functions:

1. $-7x^3$

2. $\frac{(x-1)}{(x+1)}$

3. $12x^{-2}$

4. $4x^5 - 3x^{\frac{1}{2}}$

Find the partial derivatives with respect to x, y of the following functions:

1. $4x^2y - 3xy^3 + 6x$

2. xy^2

Problem 2

1.
$$\begin{cases} x + y = 5 \\ 2x - y = 1 \end{cases}$$

2.
$$\begin{cases} x + 2y = 7 \\ 4x - y = 5 \end{cases}$$

3.
$$\begin{cases} x + y = 6 \\ \dfrac{x}{y} = 2 \end{cases}$$

Problem 3

1. Maximize $f(x) = -x^2 + 6x, \quad x \geq 0$

2. Maximize $f(x, y) = -x^2 - y^2 + 4x + 2y - 5, \quad x, y \geq 0$

Problem 4

Solve with Lagrange method:

$$\max_{x,y \geq 0} u(x, y) = xy \quad \text{s.t.} \quad 2x + y = 10.$$
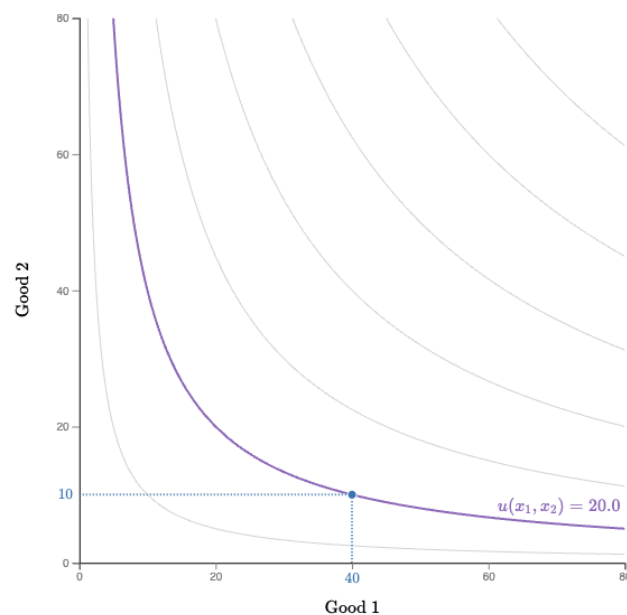
Week 2: practice problems
ECON 100A

TA Allegra Saggese
Date October 6, 2025

Professor: Natalia Lazzati
Topic: consumer problem

Problem 0

What is the relationship between a utility function and indifference curves?

Problem 1

If the price of good 1 doubles and the price of good 2 triples, does the budget line become flatter or steeper?

Problem 2

Suppose income is $m = 120$, prices $p_1 = 3$, $p_2 = 6$. Imagine there are only two goods in the economy.

1. Is the bundle $(25, 10)$ affordable?

2. Suppose a specific tax of \$3 is imposed on good 1. Write down the new budget equation.

3. After the tax, is the bundle $(25, 10)$ still affordable?

4. If the consumer buys $(12, 8)$ after the tax, how much tax revenue is collected?

Problem 3

Let $u(x_1, x_2) = x_1^2 x_2^5$.

1. Compute MU_1 and MU_2.

2. Derive the marginal rate of substitution $\text{MRS}_{1,2}$.

Problem 4

Let's compare two utility functions:

Consumer A: $u(x_1, x_2) = (x_1 + 2x_2)^2$

Consumer B: $v(x_1, x_2) = x_1 + 2x_2$

1. Find marginal utilities and $\text{MRS}_{1,2}$ for both.

2. Do they represent the same preferences? Explain briefly.

Problem 5

Optimal movie watching experience At the movies, you and your friends can choose to buy popcorn (P), soda (S), and candy (C). Everyone gets their own snacks, but one of your friends, Bacchus, has decided he will buy the same combo of concessions based on a fun index he has created. We will help Bacchus identify the differences between the bundles and to compare them. Define

$$A = P + S + C \quad \text{(total items)}, \qquad B = 3P + 2S + 4C \quad \text{(weighted "treat points")},$$

and let N be the number of visits. The *Fun Index* is

$$\text{FI} = \frac{A\,B}{N}.$$

Four friends had the following purchases last month:

Friend	N	P	S	C
Apollo	10	6	8	3
Vesta	12	8	6	5
Jupiter	9	5	10	2
Edesia	11	7	7	7

1. For each friend, compute A, B, and FI.

2. Rank the friends from highest to lowest by FI.

3. In the (A, B)-plane, plot each friend's (A, B) point and draw the *indifference curve* through it. For a fixed $\text{FI} = \bar{r}$ and given N, indifference curves satisfy

$$\frac{A\,B}{N} = \bar{r} \quad \Longleftrightarrow \quad B = \frac{N\bar{r}}{A}.$$

For each friend, use $N\bar{r} = A_0 B_0$ at their point, so the curve is $B = \frac{A_0 B_0}{A}$.

Week 3: practice problems
ECON 100A

TA Allegra Saggese
Date October 10, 2025

Professor: Natalia Lazzati
Topic: miterm 1 review

Problem 0

Suppose a consumer buys wood for a stove, where each piece of wood (1 unit of wood), provides k units of heat in the house. Assume that the consumer also buys insulation (1 unit of insulation at a time), which provides 1 unit of protection. If I is the number of insulation units and W is the quantity of wood put on the fire (consumed), then the utility function for protection and heat is

$$U(I, kW) = \min\{I,\ kW\}.$$

You know that for this consumer heat and protection are perfect complements. The per-unit prices of insulation and wood are p_I and p_W, respectively, and income is m.

(a) Set up the maximization problem and derive the demand functions for I^* and W^*.

(b) How does W^* change when k increases?

(c) Show how part (b) would change if heat and shelter were perfect substitutes.

Problem 1

A consumer buys two goods, x_1 and x_2. Initially, prices are $(p_1, p_2) = (15, 10)$ and the consumer chooses basket 1:

$$(x_1, x_2) = (10, 3).$$

Later, prices change to $(p_1, p_2) = (12, 12)$, and the consumer chooses basket 2:

$$(x_1, x_2) = (5, 10).$$

(a) Are these two choices consistent with utility maximization?

Problem 2

Can you explain why taking a monotonic transformation of a utility function does not change the marginal rate of substitution (MRS)?

Week 5: practice problems
ECON 100A

TA Allegra Saggese   **Professor:** Natalia Lazzati
Date October 29, 2025   **Topic:** WARP, revealed preferences

Problem 0

Suppose that you are given the following partial information about a consumer's purchases. He consumes only two goods.

$$\text{Year 1:} \quad p_1 = 100, \quad p_2 = 100, \quad x_1 = 100, \quad x_2 = 100$$
$$\text{Year 2:} \quad p_1 = 100, \quad p_2 = 80, \quad x_1 = 120, \quad x_2 = m$$

Over what range of m would you conclude:

1. that his behavior is inconsistent with the Weak Axiom of Revealed Preference (WARP)?

2. that the consumption bundle in Year 1 is revealed preferred to that in Year 2?

Problem 1

Suppose that you are given the following partial information about a consumer's purchasing decisions. The consumer gets only two goods.

$$\text{Year 1:} \quad p_1 = 10, \quad p_2 = 10, \quad y = 2000, \quad x_1 = 50, \quad x_2 = 150$$
$$\text{Year 2:} \quad p_1 = 15, \quad p_2 = 5, \quad y = 2000, \quad x_1 = M, \quad x_2 = N$$

Suppose the consumer exhausts all his income in the two years.

1. Describe all pairs (M, N) that make consumer's behavior consistent with WARP.

2. Let (M', N') be a pair of choices in Year 2 consistent with WARP. Can you state whether the consumer prefers bundle $(50, 150)$ or (M', N')?

Week 6: practice problems
ECON 100A

TA Allegra Saggese
Date November 3, 2025

Professor: Natalia Lazzati
Topic: Intertemporality

Problem 0

As the interest rate rises, does the intertemporal budget constraint become steeper or flatter?
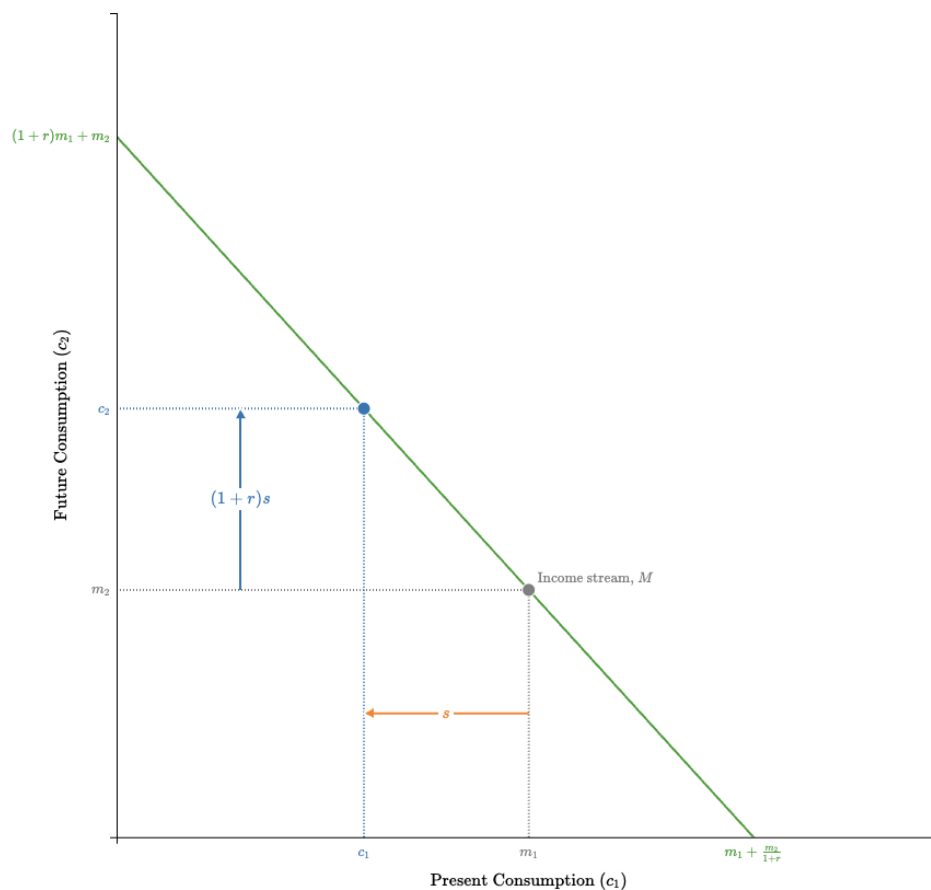

Figure 1: Intertemporal budget constraint (two periods)

Problem 1

A consumer, who is initially a lender, remains a lender even after a decline in interest rates. Is this consumer better off or worse off after the change in interest rates? If the consumer becomes a borrower after the change is he better off or worse off?

Problem 2

Preferences

$$U(c_1, c_2) = (c_1)^{1/2}(c_2)^{1/2}$$

Prices $p_1 = p_2 = 1$, interest rate $r = 0.20$ Incomes $m_1 = 18{,}000$, $m_2 = 24{,}000$

(a) Find the optimal consumption (c_1^*, c_2^*)

(b) Determine saving $s = m_1 - c_1^*$ and state whether the consumer is a saver or borrower

Problem 3

Consider an individual named Jordan who is deciding whether to enroll in a one-year graduate program in economics. Tuition and materials cost \$30,000. If Jordan does not attend, she could work and earn \$50,000 over the year. If she does attend, she expects that, starting next year, her annual salary will be \$70,000 instead of \$50,000. Assume no savings and that the market interest rate is $r = 0.05$.

(a) What is the **explicit cost** of pursuing the graduate program?

(b) What is the **opportunity cost** of enrolling? Please give an intuitive explanation.

(c) Write down a simple present-value comparison of the two choices: work now or study now.

Hint: Think about opportunity cost as "the full value of the best alternative given up" it is not only in terms of money, but also in time, effort, and utility.

Week 7: practice problems
ECON 100A

TA Allegra Saggese   **Professor:** Natalia Lazzati
Date November 6, 2025   **Topic:** midterm 2 review

*The material on the exam, as discussed in class on 5th November will include one problem from the previous midterm. This could be any **consumer problem**, including constrained optimization (utility maximization), taxes/subsidies, and other consumer choice issues. There will be one question on **intertemporal choice** and one question on **WARP or revealed preferences**.*

Problem 1a

Intertemporal choice

Molly has a utility function $U(c_1, c_2) = c_1 + 2c_2$, where c_1 and c_2 are her consumptions in periods 1 and 2, respectively. Suppose that in period 1 her income is $m_1 = 100$. In period 2 her income is $m_2 = 200$. In addition the interest rate is $r = 0.1$

1. Write down her budget constraint in terms of future values

2. Find her optimal consumption levels in periods 1 and 2, i.e., c_1^* and c_2^*

3. Suppose the government imposes an income tax of 20 dollars only on period 2. Find her optimal consumption levels in periods 1 and 2 after the tax, i.e., c_1^* and c_2^*

Problem 1b

Intertemporal choice

Anna's utility function is $U(c_1, c_2) = c_1 + c_2$, where c_1 is her consumption of bread in period 1 and c_2 is her consumption of bread in period 2. The price of bread is \$1 per loaf in periods 1 and 2. The interest rate is $r = 0.20$. Anna's income is $m_1 = 2000$ in period 1 and $m_2 = 1200$ in period 2.

1. Write Anna's budget constraint in terms of future value.

2. How much bread does Anna optimally consume in the first period and how much money does she save?

3. Write Anna's budget constraint in terms of future value if she faces an income tax of \$100 in period 1 and of \$200 in period 2.

Problem 2

Revealed preferences

Suppose that you are given the following partial information about a consumer's purchases. He consumes only two goods.

$$\text{Year 1:} \quad p_1 = 100, \quad p_2 = 100, \quad x_1 = 100, \quad x_2 = 100$$
$$\text{Year 2:} \quad p_1 = 100, \quad p_2 = 80, \quad x_1 = 120, \quad x_2 = m$$

Over what range of m would you conclude:

1. that his behavior is inconsistent with the Weak Axiom of Revealed Preference (WARP)?

2. that the consumption bundle in Year 1 is revealed preferred to that in Year 2?

Problem 3

Review of consumer theory

Suppose that Dan consumes only two goods, 1 and 2, with prices $p_1 = 5$ and $p_2 = 3$. He has an income level of $m = 150$. The utility function of Dan is

$$U(x_1, x_2) = x_1^{1/4} x_2^{3/4}$$

[(a)]Calculate the marginal utility of good 1 and the marginal rate of substitution How many units of goods 1 and 2 would Dan optimally consume Re-do part (b) assuming $U(x_1, x_2) = \frac{1}{4} \ln x_1 + \frac{3}{4} \ln x_2$

Week 8: practice problems
ECON 100A

TA Allegra Saggese

Date November 17, 2025

Professor: Natalia Lazzati

Topic: Firms: monopolies

Problem 1

0.1 What are monopolies?

Can you come up with an example of a monopoly in the current US economy? Why do you believe it is a monopoly? On what basis can we *assess or prove* the existence of a monopoly? *Discuss this question with your neighbor.*

0.2 What are natural monopolies?

Provide an example of a firm or service that is potentially a **natural monopoly,** such that consumers actually benefit from having one firm selling a good or providing service. Through what economic phenomena are natural monopolies not inefficient?

Problem 2

The monopoly faces a demand curve given by

$$D(p) = 100 - 4p.$$

Its cost function is

$$c(y) = 5y.$$

1. What is its optimal level of output and price?

2. What is the elasticity of demand at the optimal level of production (i.e., $\varepsilon(y^*)$)?

3. What are the profits of the firm at the optimal solution?

Problem 3

A monopolist serves two identifiable groups of consumers. The firm can prevent resale and therefore charge each group a different price. Consumers in group 1 and group 2 have inverse demand curves:

$$p_1(y_1) = 40 - y_1, \qquad p_2(y_2) = 30 - \tfrac{1}{2}y_2.$$

The monopolist has a single production process with total cost

$$C(y_1 + y_2) = 10(y_1 + y_2),$$

so marginal cost is constant at $MC = 10$.

1. Write down the profit maximization problem of the monopolist.

2. Solve for the optimal outputs y_1^* and y_2^*.

3. Compute the prices charged to each group, p_1^* and p_2^*.

4. Compute the price elasticity of demand for each group at the optimal outputs. Which group pays a higher price? Explain.

5. Compute the monopolist's total profit under price discrimination.

Week 9: practice problems
ECON 100A

TA Allegra Saggese
Date November 24, 2025

Professor: Natalia Lazzati
Topic: Markets - review

Problem 0: Explanation

Define the following three types of markets:

1. Perfectly competitive

2. Monopolies

3. Oligopoly

Problem 1: Definition

Define a competitive equilibrium (use both descriptive words and math, where applicable) in:

1. Short run

2. Long run

Problem 2: Perfect competition

Consider a market with the following demand and supply curves:

$$Q_d(p) = 30 - p, \qquad Q_s(p) = 2p.$$

1. Find the competitive equilibrium price and quantity.

2. Draw a supply-and-demand diagram and clearly label the equilibrium.

3. Using the equilibrium values, compute consumer surplus (CS). Recall that CS is the area of a triangle above the price and below the demand curve.

4. Compute producer surplus (PS). Recall that PS is the area of a triangle below the price and above the supply curve.

5. What is the total surplus in this market?

Problem 3: Monopoly

The monopoly faces a demand curve

$$D(p) = 100 - 4p$$

Its cost function is

$$c(y) = 5y$$

1. What is its optimal level of output and price?

2. What is the elasticity of demand at the optimal level of production (i.e., $\varepsilon(y^*)$)?

3. What are the profits of the firm at the optimal solution?

Problem 4: Duopoly

Firm 1 produces a product with cost

$$C_1(Q_1) = Q_1.$$

Firm 2 produces the same product with cost

$$C_2(Q_2) = Q_2.$$

Market demand is

$$P(Q_1 + Q_2) = 16 - (Q_1 + Q_2).$$

1. Set up the maximization problem; write down the profit expression to be maximized for each firm.

2. Solve for each firm's reaction curve.

3. Find the Cournot–Nash equilibrium.

4. Find the optimal price if the firms coordinate strategies (collude).

Week 10: practice problems
ECON 100A

TA Allegra Saggese

Professor: Natalia Lazzati

Date December 1, 2025

Topic: Final exam review

Logistics: During Week 10, the professor began Monday's lecture with a discussion of oligopoly, and then the case where oligopolists (or duopolists) will coordinate - i.e. collusion. She compared this with perfect competition and monopolist behavior. Then we reviewed problem set 6, looking at the conditional input demand.

The final exam will cover all topics from the class and will be a **2 hour exam**. Recall the professor told us there will be one problem on competitive markets, one on monopoly, one on oligopoly, and one from the earlier content in the course. That is four total questions.

The remainder of classes will cover:

- Reviewing problem set 7 on oligopoly for Wednesday lecture

- Reviewing practice final for Friday lecture

- Final exam next week - *bring blue book!*

Problem 0: Cornout competition (duopoly)

There are two firms in the market that compete in quantities (Cournot competition). The cost of firm 1 is given by
$$C_1(Q_1) = 4Q_1$$
Firm 2 produces the same product with the same total cost as Firm 1, i.e.,
$$C_2(Q_2) = 4Q_2.$$
Market demand is given by
$$P(Q_1, Q_2) = 20 - 2(Q_1 + Q_2)$$

1. Find the equilibrium level of production of each firm and the equilibrium market price.

2. Find the optimal price if these firms decide to coordinate strategies, i.e., if they collude.

Problem 1: Cornout competition (duopoly)

Two firms compete in the Cournot game. Market demand is
$$P(Q_1 + Q_2) = 100 - (Q_1 + Q_2),$$
and each firm has the same cost function
$$C_i(Q_i) = 20Q_i$$

1. Write the profit function for each firm.

2. Derive each firm's reaction function.

3. Solve for the Cournot-Nash equilibrium quantities Q_1^* and Q_2^*.

4. Compute the equilibrium market price.

5. Suppose the firms collude and behave as a monopolist. What total output would maximize joint profits?