Delta-Alpha Zeta Zero: a Geometric Transformation Linking Primes to Zeta Zeros

by Allen Proxmire

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Abstract.

This work is based on a simple geometric transformation of the Prime Number Sequence which creates right triangles from all consecutive pairs of primes (P_n, P_{n+1}). From the resulting alpha angles (α_n) of these so-called "Prime Triangles", the instantaneous change of the alpha angles ($\Delta\alpha$) can be analyzed as a time-series signal.

This Delta-Alpha ($\Delta\alpha$) Signal is shown to 1.) display a well-ordered and invariant resonance structure, 2.) have perfect PSD alignment with the derivative of the normalized prime gap, 3.) be inconsistent with the Random Matrix Theory (RMT) predictions, and 4.) scale directly to the non-trivial zeros of the Zeta Function.

This work overturns the notion that the prime gaps are locally random and best described by statistical models. It bridges the prime domain to the zeta domain by replacing the unwieldy Explicit Formula with a single, numerical constant.

The remaining challenge is to derive this scaling constant from the first principles and fundamental constants of number theory, turning this empirical observation into a proven theorem.

Part One. Building the Geometric Foundation

The key to understanding the prime number sequence is in studying consecutive pairs of primes (P_n, P_{n+1}). There has, in fact, been extensive, even exhaustive, analysis of the prime gaps (P_{n+1} - P_n). The prime gaps, specifically, the first difference of the gaps, are a "muted version" or simplified analog of the signal described below.

The consecutive primes are used to convert the linear analysis of the primes into a two-dimensional one. P_n is used as the x-coordinate and P_{n+1} as the y-coordinate on a Cartesian plane. Connecting (P_n, 0) to (0, P_{n+1}) creates a "Prime Triangle" from which the alpha angle (α_n) can be calculated:

$$\alpha = \arctan\ (P_{n+1} / P_n)$$

The change to alpha ($\Delta\alpha$), can then be analyzed as a time-series signal, the $\Delta\alpha$ Signal:

$$\Delta\alpha = \alpha_{n+1} - \alpha_n.$$

The $\Delta\alpha$ Signal, though analogous to the change to the prime gaps, is more subtle and carries more information. The extra information comes from the Fourier Transform (FFT) and Power Spectral Density (PSD) analysis performed on the $\Delta\alpha$ Signal . Three invariant spectral peaks at $f(A) \approx 0.35153$, $f(B) \approx 0.38895$, $f(C) \approx 0.47871$, dominate the initial results. These and other peaks are collectively termed the Prime Frequencies (f_k).

It is from this geometric transformation that we are able to relate the Prime Number Sequence to the Zeta Function's non-trivial zeros (t_n).

Part Two. Validating Structure and Refuting Randomness.

The Prime Frequencies survive all robustness tests across various sample sizes that were used to verify their statistical significance. A final, "smoking gun" test compares the PSD of the $\Delta\alpha$ Signal to that of the derivative (γ'_n) of the normalized prime gap:

$$\gamma_n = ((P_{n+1}) - P_n) / \ln(P_n).$$

The two spectra align perfectly. The $\Delta\alpha$ Signal equates to the theoretical derivative of the normalized prime gap, a precise analytic formula from foundational number theory.

The current standard model for understanding the spacing of the Riemann Zeta Zeros (t_n) is the RMT. It asserts that the spacings between Zeta Zeros are random, and that the primes behave like an unpredictable, statistical fluid.

The $\Delta\alpha$ Signal does not match the RMT's predicted nearest-neighbor spacing ratio of $< r > \approx 0.536$. Again proving that the $\Delta\alpha$ Signal is not random, but the product of a fixed, resonance mechanism.

Part Three. Connecting Delta-Alpha to the Zeta Zeros

Understanding that the Prime Frequencies (f_k) were indeed structured and wave-like, and were derived from the differences of primes (i.e. $\Delta\alpha$, the change to alpha), it was hypothesized that the Prime Frequencies (f_k) were a scaled representation of the Zeta Zeros' differences, or "beat frequencies", ($|t_n - t_m|$):

$$c \approx f(\mathrm{A}) / |t_3 - t_2|$$
$$c \approx 0.35153 / (25.0108575 - 21.0220396)$$
$$c \approx 0.35153 / 3.98881$$
$$\boldsymbol{c \approx 0.088128}$$

Using this hypothesized constant ($c \approx 0.088128$), and the Zeta Zeros' differences for $|t_8 - t_7|$, $|t_4 - t_3|$, produced the frequencies for $f(\mathrm{B}) \approx 0.38895$ and $f(\mathrm{C}) \approx 0.47871$.

Using this hypothesized constant ($c \approx 0.088128$), and the Zeta differences for $|t_6 - t_2|$, $|t_5 - t_2|$, and $|t_4 - t_2|$, and PSD analysis, found distinct, statistically significant peaks at exactly the predicted locations of 0.8287, 1.0498, 1.4597, respectively.

This empirically demonstrates that c is the scaling constant required to connect the Zeta function to the geometric structure of consecutive primes (the $\Delta\alpha$ Signal).

The remaining challenge is to derive c from first principles, complete a mathematical proof for this relationship, and elevate this from convincing empirical argument to a theorem for the mechanism behind prime number distribution.