Event Density: A Mathematical Formalization

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1. Introduction

Event Density (ED) is a framework for describing reality in terms of *becoming* rather than in terms of objects, states, or trajectories. The earlier papers in this research program develop the metaphysical, phenomenological, and applied aspects of this idea. This paper provides the mathematical core. Its purpose is to isolate the minimal formal ingredients required to treat ED as a mathematical structure, independent of any particular interpretation or application.

The central claim of the ED program is that becoming has a measurable structure. This structure does not presuppose time, geometry, causality, or probability. Instead, it begins with a domain of primitive events and assigns to finite configurations of these events a non-negative quantity called *event density*. This quantity expresses how much becoming is present in a configuration. The goal of this paper is to formalize this idea in a way that is both minimal and general.

The mathematical framework developed here is intentionally sparse. The event domain is treated as a bare set with no intrinsic structure. All relevant structure enters through the admissible configurations and the density function defined on them. This choice ensures that ED remains compatible with the broader program: additional structure can be introduced in later papers without contradicting the core definitions presented here.

The axioms introduced in this paper capture the essential behavior of event density: non-negativity, a null baseline, monotonicity with respect to inclusion, and a form of subadditivity that reflects the non-linear character of becoming. These axioms are weak enough to accommodate a wide range of ED models, yet strong enough to support meaningful mathematical analysis.

This paper also introduces morphisms between ED systems, allowing us to compare different realizations of event density and to identify when two systems represent the same underlying structure. This provides a categorical perspective that will be useful in later work.

Finally, the paper situates ED within the landscape of existing mathematical frameworks. Although ED resembles measure theory, point processes, and certain structures in statistical mechanics, it is not reducible to any of them. The comparison clarifies both the generality of ED and the specific commitments that distinguish it.
The remainder of the paper is organized as follows. Section 2 introduces the primitive data and basic definitions.

Section 3 presents the axioms for event density systems. Section 4 develops the notion of morphisms and equivalence. Section 5 relates ED to existing mathematical frameworks. Section 6 provides worked examples, including a connection to the Temporal Tension papers. Section 7 discusses open questions and future directions.

2. Primitive Data and Basic Definitions

This section introduces the minimal mathematical ingredients required for an Event Density (ED) system. The goal is to specify the underlying data without assuming any additional structure such as order, topology, geometry, or time. All structure relevant to ED enters through the admissible configurations and the density function defined on them.

2.1 Event Domain

An **event domain** is a bare set

$$E.$$

No assumptions are made about the internal structure of E. In particular, E is not required to carry a topology, metric, algebraic structure, or partial order. This choice ensures that ED remains compatible with a wide range of interpretations and applications. Any additional structure required for specific models can be introduced later without altering the core definitions.

Elements of E are called **events**. The framework does not interpret events beyond their role as primitive units of becoming.

2.2 Configurations

A **configuration** is a finite subset of the event domain. Let

$$P_{\text{fin}}(E)$$

denote the set of all finite subsets of E. An ED system specifies a collection

$$C \subseteq P_{\text{fin}}(E)$$

of **admissible configurations**.

The choice of C determines which finite combinations of events are meaningful for the density function. In the simplest case, $C = P_{\text{fin}}(E)$. More restrictive choices allow ED to encode structural constraints without modifying the event domain itself.

Configurations support the following basic operations:
- **Inclusion:** $C \subseteq D$
- **Union:** $C \cup D$
- **Disjointness:** $C \cap D = \varnothing$
- **Restriction:** For $F \subseteq E$, the restriction of C to F is $C \cap F$

These operations will be used in the axioms governing event density.

2.3 Event Density

An **event density** is a function

$$\rho: C \to [0,\infty)$$

assigning a non-negative real number to each admissible configuration. The value $\rho(C)$ represents the amount of becoming associated with the configuration C.

The framework does not impose an interpretation on ρ. It may represent intensity, weight, tension, or any other structural quantity depending on the application. The axioms introduced in Section 3 specify the minimal constraints required for ρ to behave coherently.

Several derived notions are useful:

Support:
- The support of ρ is the set
$$\text{supp}(\rho) = \{e \in E : \rho(\{e\}) > 0\}.$$

Increment:
- For $e \notin C$, the increment of adding e to C is
$$\Delta_e(C) = \rho(C \cup \{e\}) - \rho(C).$$

Local density:
- For $C \in C$, the local density at C is the collection of increments
$$\{\Delta_e(C) : e \notin C\}.$$

These derived quantities are not part of the primitive data but will be useful in analyzing specific ED systems and in comparing different models.

2.4 Why Density Lives on Configurations
Event Density is defined on **configurations** rather than on individual events.

This is not a technical convenience but a structural commitment of the ED framework.

A single event, taken in isolation, carries no internal structure.
It has no relations, no tension, no contrast, and no participation.
Becoming arises only when events are considered **together**, as a finite group.

Configurations are therefore the smallest units in which becoming can appear.

Defining density on configurations allows ED to capture interaction, constraint, and non‑linearity.
If density were defined on individual events, the theory would collapse into a pointwise assignment with no room for emergent structure.

By placing density on finite subsets, ED treats becoming as a **relational** phenomenon: something that happens *between* events rather than *within* them.

This architectural choice prepares the ground for the axioms in Section 3, especially monotonicity and subadditivity, which express how becoming accumulates and how it is shared across configurations.

3. Axioms for Event Density Systems
An **Event Density system** consists of a triple
$$(E, C, \rho)$$

where E is an event domain, C is a collection of admissible finite configurations, and $\rho: C \to [0,\infty)$ is an event density function.

This section introduces the axioms that govern the behavior of ρ.

The axioms are minimal: they ensure coherence without imposing additive or probabilistic structure.

3.1 Non‑negativity

For every admissible configuration $C \in C$,

$$\rho(C) \geq 0.$$

This axiom ensures that event density behaves as a non-negative quantity.

3.2 Null Configuration
The empty configuration carries no density:

$$\rho(\varnothing) = 0.$$

This establishes a baseline for all other configurations.

3.3 Monotonicity
If $C, D \in C$ and $C \subseteq D$, then

$$\rho(C) \leq \rho(D).$$

Adding events to a configuration cannot decrease its density.

This axiom reflects the idea that becoming accumulates.

3.4 Subadditivity
If $C, D \in C$ are disjoint, then

$$\rho(C \cup D) \leq \rho(C) + \rho(D).$$

Subadditivity captures the non-linear character of event density.

It allows interactions or constraints between disjoint configurations without requiring full additivity.

3.5 Locality and Compositionality
For any configuration $C \in C$ that can be expressed as a union of subconfigurations

$$C = C_1 \cup C_2 \cup \cdots \cup C_k,$$

the density of C is determined by the densities of its parts through a system-specific compositional rule:

$$\rho(C) = F(\rho(C_1), \rho(C_2), \ldots, \rho(C_k)).$$

The function F is not fixed by the axioms.

Different choices of F correspond to different ED models.

This axiom ensures that ED systems behave coherently under decomposition and recomposition of configurations.

3.6 Optional Structural Axioms
Depending on the intended application, an ED system may satisfy additional axioms.

These are not part of the core framework but are included here for completeness.

3.6.1 Symmetry
If a permutation of E preserves admissibility, then it preserves density:

$$\rho(\sigma(C)) = \rho(C).$$

3.6.2 Invariance under relabeling
If $f : E \to E$ is a bijection such that $f(C) \in C$ for all $C \in C$, then
$$\rho(f(C)) = \rho(C).$$

3.6.3 Structural constraints
Additional axioms may restrict C or ρ to encode temporal, causal, geometric, or number-theoretic structure.

3.6.4 Maximum speed of causal influence
The ED network admits a finite maximum propagation rate for causal updates; denote this rate by c. In regions of increased local participation, the effective propagation rate is reduced by a competition term k, yielding a local propagation speed of $c - k$. No admissible sequence of events may require propagation faster than c, since each event must be grounded in the updated state of its neighbors.

These constraints are model-dependent and are introduced in later papers.

4. Morphisms and Equivalence of ED Systems
This section introduces structure-preserving maps between Event Density systems.

Morphisms allow us to compare different systems, transfer density information across domains, and identify when two systems represent the same underlying structure.

The definitions are minimal and do not assume additional structure on the event domains.
Let
$$(E, C, \rho) \text{ and } (E', C', \rho')$$

be two ED systems.

4.1 Morphisms of ED Systems
A **morphism of ED systems** is a function
$$f : E \to E'$$

satisfying the following conditions:

1. Configuration preservation:
    For every $C \in C$, the image
$$f(C) = \{f(e) : e \in C\}$$

lies in C'.

2. Density preservation:
    For every $C \in C$,
$$\rho'(f(C)) = \rho(C).$$

A morphism preserves both the admissible configurations and their densities.

It provides a way to represent one ED system inside another without altering the structure of becoming.

4.2 Isomorphisms

A morphism $f : E \to E'$ is an **isomorphism** if it is bijective and its inverse f^{-1} is also a morphism.

In this case, the two ED systems are said to be **isomorphic**, written
$$(E, C, \rho) \cong (E', C', \rho').$$

Isomorphic systems represent the same ED structure, possibly with different labels for events.

4.3 Subsystems

A **subsystem** of an ED system (E, C, ρ) is a triple
$$(F, C_F, \rho_F)$$

where:
- $F \subseteq E$,
- $CF = \{C \in C : C \subseteq F\}$,
- ρ_F is the restriction of ρ to C_F.

Subsystems allow the analysis of ED structure on restricted domains without introducing new axioms.

4.4 Quotients

Let ~ be an equivalence relation on E.

A **quotient ED system** identifies events that are equivalent under ~.

The quotient domain is
$$E/\sim,$$

and configurations are images of admissible configurations under the quotient map.

Density is defined by
$$\rho \sim([C]) = \rho(C),$$

where $[C]$ is the image of C under the quotient.

Quotients allow the collapse of redundant or indistinguishable events while preserving density.

4.5 Category of ED Systems

ED systems and their morphisms form a category, denoted ED.

Objects are ED systems, and morphisms are structure‑preserving maps as defined above.

Composition is given by composition of functions, and identity morphisms are identity functions on event domains.

This categorical structure provides a uniform framework for comparing ED systems, constructing new ones, and analyzing their relationships.

5. Relations to Existing Mathematical Frameworks

Event Density (ED) resembles several established mathematical structures but is not reducible to any of them.

This section clarifies these relationships. The goal is not to derive ED from existing theories, but to identify points of contact and separation. This situates ED within the broader mathematical landscape and highlights the generality of the axioms introduced in Section 3.

5.1 Measure Theory

ED shares several features with measure theory:
- Both assign non-negative quantities to collections of elements.
- Both satisfy monotonicity.
- Both allow subadditivity.

However, ED differs from measure theory in three fundamental ways:

1. Domain of definition:
> A measure is defined on a σ-algebra of subsets, typically infinite and closed under countable unions.
> ED is defined only on finite configurations.

2. Additivity:
> Measures are additive on disjoint sets.
> ED requires only subadditivity; full additivity is not assumed.

3. Interpretation:
> Measures quantify size or probability.
> ED quantifies becoming, which is not a size, frequency, or likelihood.

Thus, ED generalizes some structural aspects of measure theory while remaining independent of its probabilistic or geometric interpretations.

5.2 Probability Theory

ED is not a probabilistic framework.

The density $\rho(C)$ does not represent uncertainty, likelihood, or frequency.

Nevertheless, there are superficial similarities:
- Both assign non-negative values to sets.
- Both may exhibit subadditivity.
- Both can be used to compare relative magnitudes.

The key distinction is conceptual: probability measures uncertainty about events, while ED measures the structural intensity of events themselves.

No normalization condition (such as total mass 1) is imposed on ED.

5.3 Point Processes and Intensity Functions
Point processes assign random configurations of points to a domain, often equipped with an intensity function describing expected density.

ED differs in several respects:
- ED is deterministic; no randomness is assumed.
- ED assigns density to actual configurations, not expected ones.
- ED does not require a background space with geometric or topological structure.

The analogy is useful: ED can be viewed as a deterministic analogue of an intensity function, but without stochastic assumptions or geometric commitments.

5.4 Order Theory and Causal Sets
If additional structure is added to the event domain—such as a partial order—ED can interact with order-theoretic frameworks.

For example, if events are partially ordered by causal precedence, configurations may be required to respect this order.

However:
- ED does not assume any order on E.
- ED does not require transitivity, acyclicity, or any causal structure.
- ED can be applied to domains where no causal interpretation is available.

Thus, ED is compatible with causal set theory but not dependent on it.

5.5 Statistical Mechanics and Partition Functions
Statistical mechanics assigns weights to microstates, often through energy functions or partition functions.

ED resembles this structure in that:
- Configurations receive non-negative weights.
- Subadditivity can reflect interactions between components.
- Density can be interpreted as a structural weight.

However:
- ED does not assume a Hamiltonian, temperature, or probabilistic ensemble.
- ED does not require normalization or thermodynamic limits.
- ED applies to arbitrary event domains, not physical microstates.

The analogy is structural rather than physical.

6. Worked Examples
This section presents several examples of Event Density (ED) systems.

The goal is not to model specific physical or metaphysical scenarios, but to illustrate how the axioms operate in concrete settings.

Each example specifies an event domain, a set of admissible configurations, and a density function satisfying the axioms of Section 3.

6.1 Finite Toy Model
Let
$$E = \{a, b, c\}$$

and let
$$C = P_{\text{fin}}(E).$$

Define ρ by:
$$\rho(C) = |C|.$$

This system satisfies all axioms:
- Non-negativity: $|C| \geq 0$.
- Null configuration: $|\varnothing| = 0$.
- Monotonicity: if $C \subseteq D$, then $|C| \leq |D|$.
- Subadditivity: for disjoint C, D, $|C \cup D| = |C| + |D|$.
- Locality: density is determined by cardinality, which is compositional.

This is the simplest ED system: density equals the number of events.

6.2 Weighted Toy Model
Let
$$E = \{e_1, e_2, e_3\} \text{ and } C = P_{\text{fin}}(E).$$

Assign weights
$$w(e_1) = 1, w(e_2) = 2, w(e_3) = 4.$$

Define
$$\rho(C) = \sum_{e \in C} w(e).$$

This system is additive but not uniform.

It satisfies all axioms and illustrates how ED can encode heterogeneous contributions from different events.

6.3 Subadditive Interaction Model
Let
$$E = \{x, y\} \text{ and } C = P_{\text{fin}}(E).$$

Define:
$$\rho(\{x\}) = 1, \qquad \rho(\{y\}) = 1, \qquad \rho(\{x,y\}) = 1.5.$$

This system is strictly subadditive:
$$\rho(\{x, y\}) < \rho(\{x\}) + \rho(\{y\}).$$

It satisfies all axioms and demonstrates how ED can encode interaction or constraint between events.

6.4 Number‑Theoretic Example (Factor Skyline Bridge)

Let
$$E = N_{\geq 2}, \text{ the set of integers greater than 1.}$$

Let configurations be finite intervals:
$$C = \{\{n, n+1, \ldots, m\} : 2 \leq n \leq m\}.$$

Let $h(k)$ denote the height of the Factor Skyline at integer k.

Define:
$$\rho(\{n, \ldots, m\}) = \sum_{k=n}^{m} h(k).$$

This system satisfies:
- Non‑negativity: skyline heights are non‑negative.
- Null configuration: the empty interval has density 0.
- Monotonicity: extending an interval adds non‑negative height.
- Subadditivity: disjoint intervals satisfy

$$\rho([n_1,m_1] \cup [n_2,m_2]) = \rho([n_1,m_1]) + \rho([n_2,m_2]).$$

This example shows how ED can encode structural information about the integers.

6.5 Temporal Example (Temporal Tension Bridge)

Let E be a set of micro‑events associated with a temporal process.

Let configurations be finite subsets of E that lie within a bounded temporal window.

Let $T : E \to R$ assign a timestamp to each event.

Define density by:
$$\rho(C) = \max_{e \in C} T(e) - \min_{e \in C} T(e).$$

This system satisfies:
- Non‑negativity: time differences are non‑negative.
- Null configuration: $\rho(\varnothing)=0$.
- Monotonicity: adding events can only increase the temporal span.
- Subadditivity: for disjoint windows, the span of the union is at most the sum of spans.

This example shows how ED can encode temporal tension without assuming a global time structure.

6.6 Deterministic Intensity Model

Let

$$E \text{ be any set and } C = P_{\text{fin}}(E).$$

Let

$$w : E \rightarrow [0,\infty) \text{ be a weight function.}$$

Define:

$$\rho(C) = \left(\sum_{e \in C} w(e)^p\right)^{1/p}$$

for some fixed $p \geq 1$.

This is a deterministic analogue of an L^p intensity.

It satisfies all axioms and illustrates how ED can generalize classical norms.

7. Discussion and Outlook

This paper has introduced the mathematical core of Event Density (ED). The framework is intentionally minimal: an event domain treated as a bare set, a collection of admissible finite configurations, and a density function satisfying a small set of axioms. These ingredients are sufficient to capture the structural behavior of becoming without assuming temporal, geometric, causal, or probabilistic structure. The resulting formalism is general enough to support a wide range of models while remaining precise enough to enable mathematical analysis.

The axioms presented here establish a baseline for ED systems. They guarantee coherence, monotonicity, and subadditivity, but they do not impose additivity, normalization, or continuity. This leaves room for diverse realizations of ED, including systems that resemble measures, systems that resemble intensities, and systems that exhibit interaction or constraint. The examples in Section 6 illustrate this range.

The notion of morphisms provides a way to compare ED systems and identify when two systems represent the same underlying structure. This categorical perspective will be useful in later work, particularly when analyzing how ED behaves under transformations, restrictions, or extensions of the event domain.
Several directions for further development remain open:

1. Extended configuration spaces.
The present framework restricts attention to finite configurations. Extending ED to infinite configurations would require additional axioms and may connect ED to measure theory or topology.

2. Additional structure on the event domain.
Specific applications may introduce order, geometry, or algebraic structure on E. Understanding how ED interacts with such structure is an important next step.

3. Model classification.
Identifying canonical families of ED systems, or characterizing systems satisfying additional constraints, may lead to a taxonomy of ED models.

4. Connections to physics and computation.

ED may provide a structural foundation for temporal asymmetry, causal emergence, or computational processes. These possibilities require further exploration.

5. Integration with the broader ED program.

The mathematical core developed here supports the metaphysical exposition of Paper 1, the number‑theoretic analysis of Paper 2, the generalizations of Paper 3, and the temporal analysis of Paper 4. Future work will refine these connections and develop additional applications.

Event Density is intended as a foundational framework. The formalism presented in this paper establishes the basic structure on which the rest of the ED program depends. The next steps involve developing specific models, analyzing their properties, and exploring their implications for mathematics, philosophy, and the sciences.

Appendix A — Plain‑Language Companion

This appendix provides an intuitive, non‑technical explanation of the core concepts introduced in the paper.

Each subsection mirrors the structure of the main text and offers:

- a plain‑language explanation
- a key intuition
- a short takeaway

The goal is to make the framework accessible to readers from philosophy, physics, and mathematics who may not be accustomed to abstract structural formalism.

A.1 Event Domain

Plain Language.

The event domain is simply a collection of events — the raw material of becoming.

No structure is assumed: no time, no space, no causality, no geometry.

It is just "the things that can happen."

Key Intuition.

Becoming begins with events before any structure is imposed.

Short Takeaway.

The event domain is a bare set of events with no built‑in structure.

A.2 Configurations

Plain Language.

A configuration is a small group of events considered together.

Event Density is defined on these groups, not on individual events.

Configurations are the first place where relational structure can appear.

Key Intuition.

Becoming is relational — it emerges when events are grouped, not in isolated events.

Short Takeaway.

Configurations are finite groups of events.

Event Density is defined on configurations, not single events.

The basic set operations on configurations are the "moves" allowed in the ED framework.

A.3 Event Density

Plain Language.

Event Density assigns a non‑negative number to each configuration.

This number represents how much becoming is present in that group of events.

Density is never defined on single events — only on configurations.

Key Intuition.

Becoming is something a configuration *does*, not something an event *has*.

Short Takeaway.

Density measures how much becoming a configuration embodies.

It is always non‑negative and always defined on groups.

A.4 Why Density Lives on Configurations

Plain Language.

A single event has no internal structure.

Becoming arises only when events participate together.

Configurations are the smallest units where interaction, tension, and emergence can appear.

Key Intuition.

Becoming is fundamentally relational.

Density lives on configurations because that is where structure arises.

Short Takeaway.

A single event cannot exhibit becoming.

Density must be defined on groups of events.

A.5 Axioms for Event Density (Section 3)

A.5.1 Monotonicity

Plain Language.

Adding events to a configuration cannot reduce its density.

Key Intuition.

Becoming never decreases when more events occur.

Short Takeaway.

Adding events cannot reduce density.

A.5.2 Subadditivity

Plain Language.

When two disjoint configurations are combined, the density of the whole is at most the sum of the parts.

This captures tension, interference, and shared structure.

Key Intuition.

Events can constrain each other.

Becoming is not simply additive.

Short Takeaway.

The whole is never more than the sum of the parts.

A.5.3 Locality and Compositionality

Plain Language.

The density of a whole configuration is determined by the densities of its parts, using a rule specific to the ED model.

Key Intuition.

ED is compositional: parts determine wholes through a model-specific rule.

Short Takeaway.

Density must be internally consistent when configurations are decomposed or recombined.

A.5.4 Optional Structural Axioms

Plain Language.

These axioms add symmetry, invariance, or domain-specific constraints when needed (temporal, causal, geometric, number-theoretic, etc.).

Key Intuition.

Optional axioms refine ED for specialized applications.

Short Takeaway.

Use optional axioms when the model requires symmetry or additional structure.

A.6 Morphisms and Equivalence (Section 4)

A.6.1 Morphisms
Plain Language.
A morphism is a map between ED systems that preserves which configurations are allowed and how much becoming they have.
Key Intuition.
Morphisms let ED systems "talk to each other" without distorting structure.
Short Takeaway.
A morphism preserves admissibility and density.

A.6.2 Isomorphisms
Plain Language.
Two ED systems are isomorphic if they are the same structure with different labels.
Key Intuition.
Isomorphism expresses structural sameness.
Short Takeaway.
Isomorphic ED systems differ only in naming.

A.6.3 Subsystems
Plain Language.
A subsystem is a smaller ED system carved out of a larger one.
Key Intuition.
Zooming in preserves structure.
Short Takeaway.
Subsystems inherit configurations and density from the parent system.

A.6.4 Quotients
Plain Language.
A quotient ED system merges events that are considered equivalent.
Key Intuition.
Quotients simplify ED systems by collapsing indistinguishable events.
Short Takeaway.
Quotients identify events while preserving density.

A.6.5 The Category of ED Systems
Plain Language.
ED systems and their morphisms form a category.
Key Intuition.
ED fits naturally into the language of category theory.
Short Takeaway.
ED is a full mathematical structure with maps, identities, and composition.

A.7 Relations to Existing Frameworks (Section 5)

Plain Language.

ED resembles measures, probabilities, intensities, causal sets, and partition functions — but it is none of them.
It is more primitive and more general.

Key Intuition.

ED captures becoming, not size, chance, or energy.

Short Takeaway.

ED is structurally similar to many frameworks but reducible to none.

A.8 Worked Examples (Section 6)

Plain Language.

Examples show how ED can model:
- simple counting
- weighted contributions
- subadditive tension
- number-theoretic structure
- temporal span
- intensity norms

Key Intuition.

The same axioms support simple, complex, temporal, arithmetic, and geometric models.

Short Takeaway.

ED is flexible — it can model arithmetic, time, tension, or simple counts.

A.9 Outlook (Section 7)

Plain Language.

ED is a minimal structural framework for becoming.
It is general enough to support many models and specific enough to be mathematically meaningful.

Key Intuition.

This paper lays the foundation.
Future work builds structure on top of it.

Short Takeaway.

ED is a foundational ontology with room for expansion into time, causality, number theory, and physics.