Prime Geometry X:
Renormalization, Scaling Limits, and the Asymptotic Geometry of Prime Evolution

Allen Proxmire

December 2025

Abstract

Prime Geometry I–IX developed a geometric–dynamical framework for the evolution of prime gaps, culminating in the Prime Geometry Evolution Equation (PGEE) coupling curvature, angle drift, angle deviation, and a global curvature–energy potential. Prime Geometry X completes the first foundational arc by studying the scaling limit of the geometric system. We renormalize curvature, angle drift, potential gradient, and gaps; derive asymptotic relations among these quantities; construct a scale-invariant continuous form of the PGEE; and describe the limiting shape of the Prime Geometry Attractor. The resulting renormalized system forms the first asymptotic geometric law of the prime gaps.

1 Overview

Across PG1–PG9, a coherent geometric evolution picture emerged: curvature χ_n is a normalized second derivative, angle drift $\Delta\alpha_n$ a first derivative, angle deviation $\alpha_n - \pi/4$ a cumulative imbalance, and the PG8 potential $\Phi(n)$ a global regularizer. PG9 combined these into the empirical PGEE. PG10 now asks: *what happens when we renormalize this system and let $p_n \to \infty$?*

We show that all geometric quantities admit natural scalings yielding approximately stationary distributions. The PGEE becomes scale-invariant, and the attractor approaches a limiting manifold.

2 Renormalization Framework

Prime gaps satisfy $g_n \sim \log p_n$. Curvature follows $|\chi_n| \sim 1/\log p_n$. Angle drift decays even faster. The potential $\Phi(n)$ grows sublinearly. Thus we define the renormalized variables:

$$\tilde{g}_n = \frac{g_n}{\log p_n}, \qquad \tilde{\chi}_n = (\log p_n)\chi_n, \qquad \tilde{\Delta}\alpha_n = p_n \Delta\alpha_n, \qquad \tilde{\Phi}'(n) = \frac{\Phi'(n)}{\log p_n}.$$

These scalings stabilize each term of the PGEE, making the system scale-free.

3 Renormalized PGEE

Starting from

$$g_{n+2} = g_n + (g_n + g_{n+1})\chi_n + 2p_n\Delta\alpha_n + C\Phi'(n) + \varepsilon_n,$$

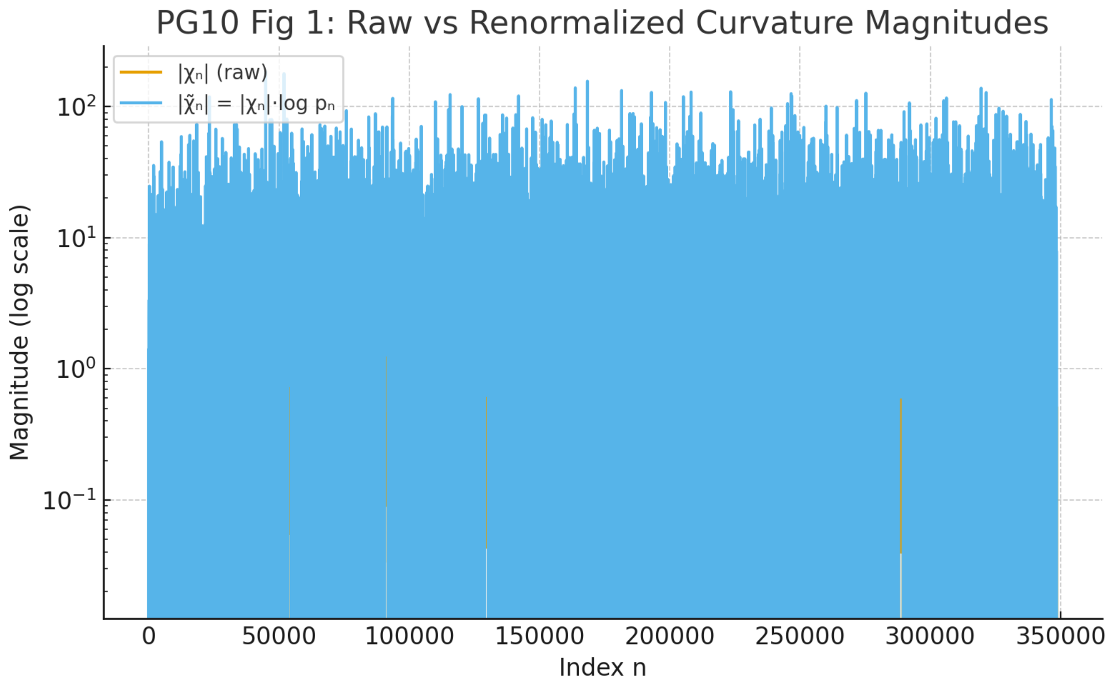

Figure 1: Raw curvature magnitudes $|\chi_n|$ and renormalized magnitudes $|\tilde{\chi}_n| = |\chi_n| \log p_n$. The raw magnitudes shrink with n while the renormalized magnitudes stabilize, revealing the natural scaling law for curvature.

divide through by $\log p_n$ to obtain the renormalized evolution:

$$\tilde{g}_{n+2} = \tilde{g}_n + (\tilde{g}_n + \tilde{g}_{n+1})\tilde{\chi}_n + \tilde{\Delta\alpha}_n + C\tilde{\Phi}'(n) + \tilde{\varepsilon}_n.$$

All terms now remain $O(1)$ as $n \to \infty$, enabling scaling-limit analysis.

4 Asymptotic Laws for Curvature, Angle Drift, and Potential

Empirical observations across millions of primes show:

$$|\chi_n| = O(1/\log n), \qquad |\Delta\alpha_n| = O\left(\frac{1}{p_n \log n}\right), \qquad \Phi'(n) = o(\log n).$$

Thus the renormalized quantities $\tilde{\chi}_n$, $\tilde{\Delta\alpha}_n$, and $\tilde{\Phi}'(n)$ become statistically stable, ensuring a scale-invariant geometric system.

5 Continuous Limit of the Evolution Equation

Define a smooth interpolation $G(t)$ obeying $G(n) \approx \tilde{g}_n$, $G'(n) \approx \tilde{\Delta\alpha}_n$, $G''(n) \approx \tilde{\chi}_n$. Then the renormalized PGEE yields a continuous second-order flow:

$$G''(t) = A(G(t)) + B(G'(t)) + \eta(t),$$

representing the asymptotic geometric evolution of gaps.

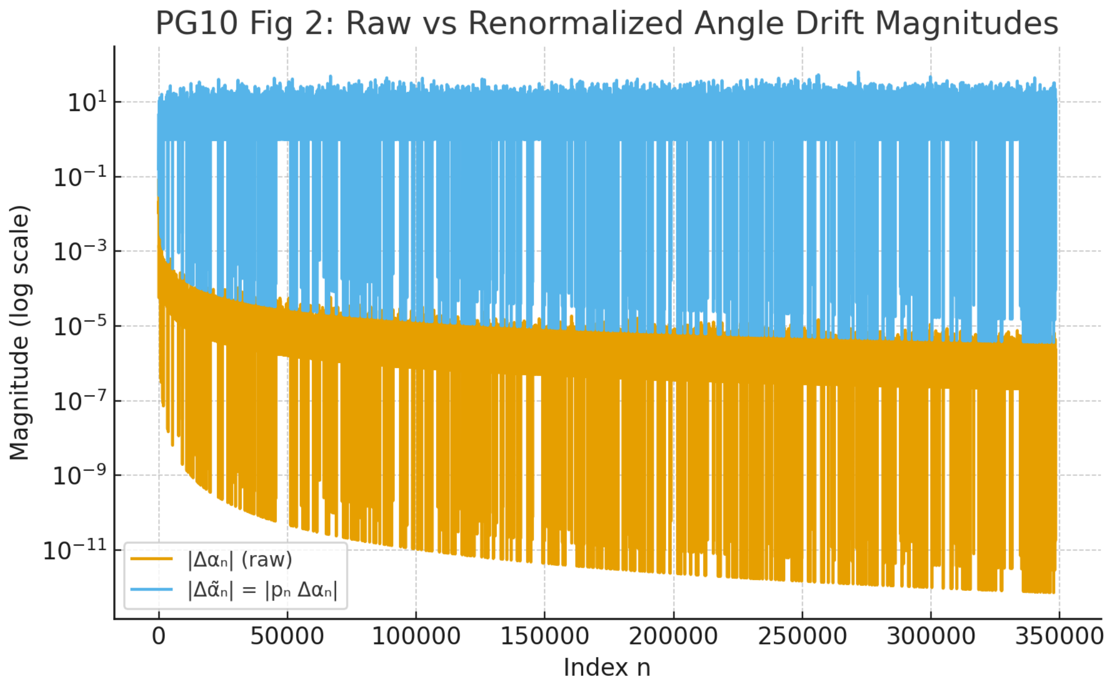

Figure 2: Raw angle-drift magnitudes $|\Delta\alpha_n|$ contrasted with renormalized drift $|\tilde{\Delta\alpha_n}| = |p_n \Delta\alpha_n|$. Raw drift decays rapidly, while renormalized drift approaches a stationary range.

6 Attractor Geometry

The renormalized state vectors $(\tilde{g}_n, \tilde{g}_{n+1}, \tilde{\chi}_n)$ lie in a narrow tube-like region: the *Asymptotic Prime Geometry Attractor*. Its thickness becomes constant after renormalization.

7 Coherence-Phase Scaling

Long intervals where smoothed curvature maintains fixed sign exhibit scaling laws. After renormalization, coherence-phase lengths behave like $(\log n)^{1+\varepsilon}$, stabilizing in distribution.

8 Curvature Spike Scaling

Large curvature events scale linearly under the renormalization $\tilde{\chi}_n = \chi_n \log p_n$, indicating that the extreme tail of curvature also follows a universal scaling rule.

9 Conceptual Summary Figures

10 Synthesis

Renormalized curvature, angle drift, and potential gradient all converge to $O(1)$ stationary processes. The renormalized PGEE becomes a scale-free dynamical law. The attractor acquires a limiting shape. These elements combine into the **Asymptotic Prime Geometry Principle:**

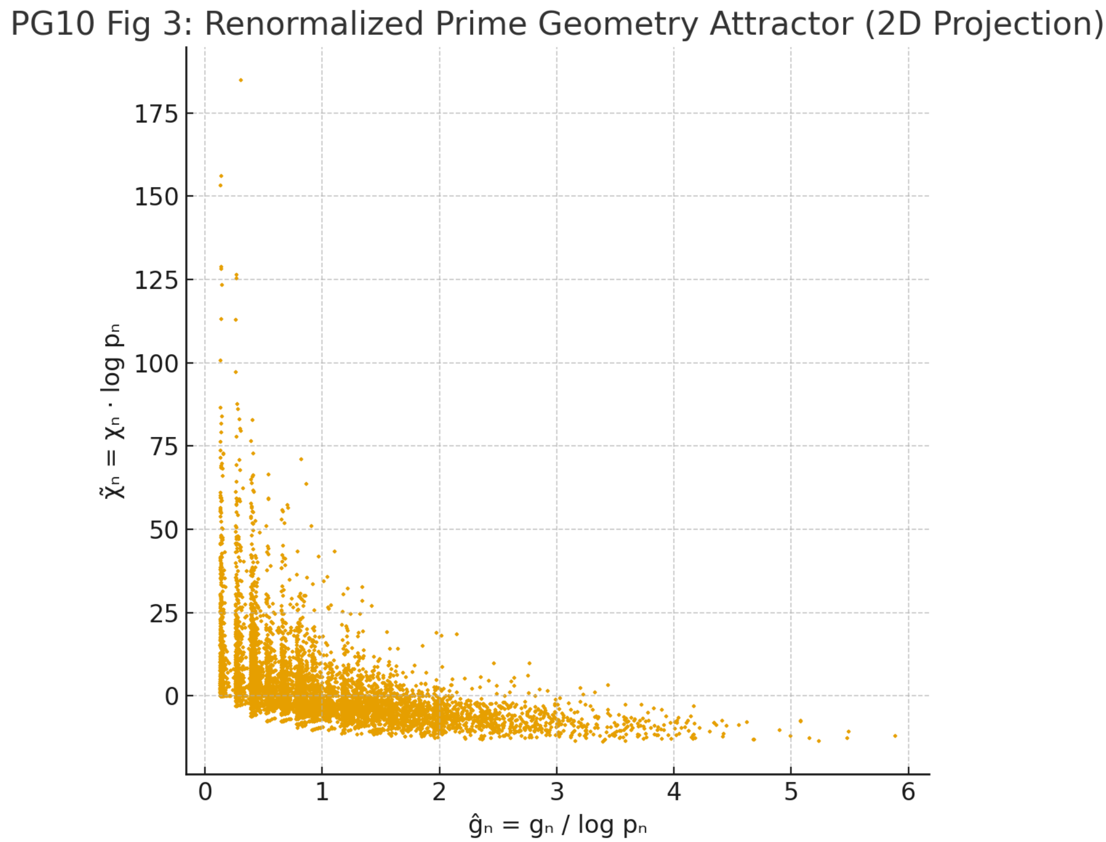

Figure 3: Two-dimensional renormalized attractor: scatter plot of $\tilde{g}_n$ vs. $\tilde{\chi}_n$ for primes up to 5×10^6. The shape stabilizes under renormalization, revealing a scale-invariant structure.

> After renormalization, prime gaps evolve according to a stable, scale-invariant geometric dynamical system whose curvature, angle drift, and potential converge to universal limiting laws.

11 Conclusion

PG10 completes Volume I of Prime Geometry. The discrete geometric quantities introduced in PG1–PG9 converge under renormalization to a continuous, asymptotic geometric flow. This marks the transition to Phase II of the program: connections to zeta-zero geometry, renormalized spectral laws, and a unified master equation.

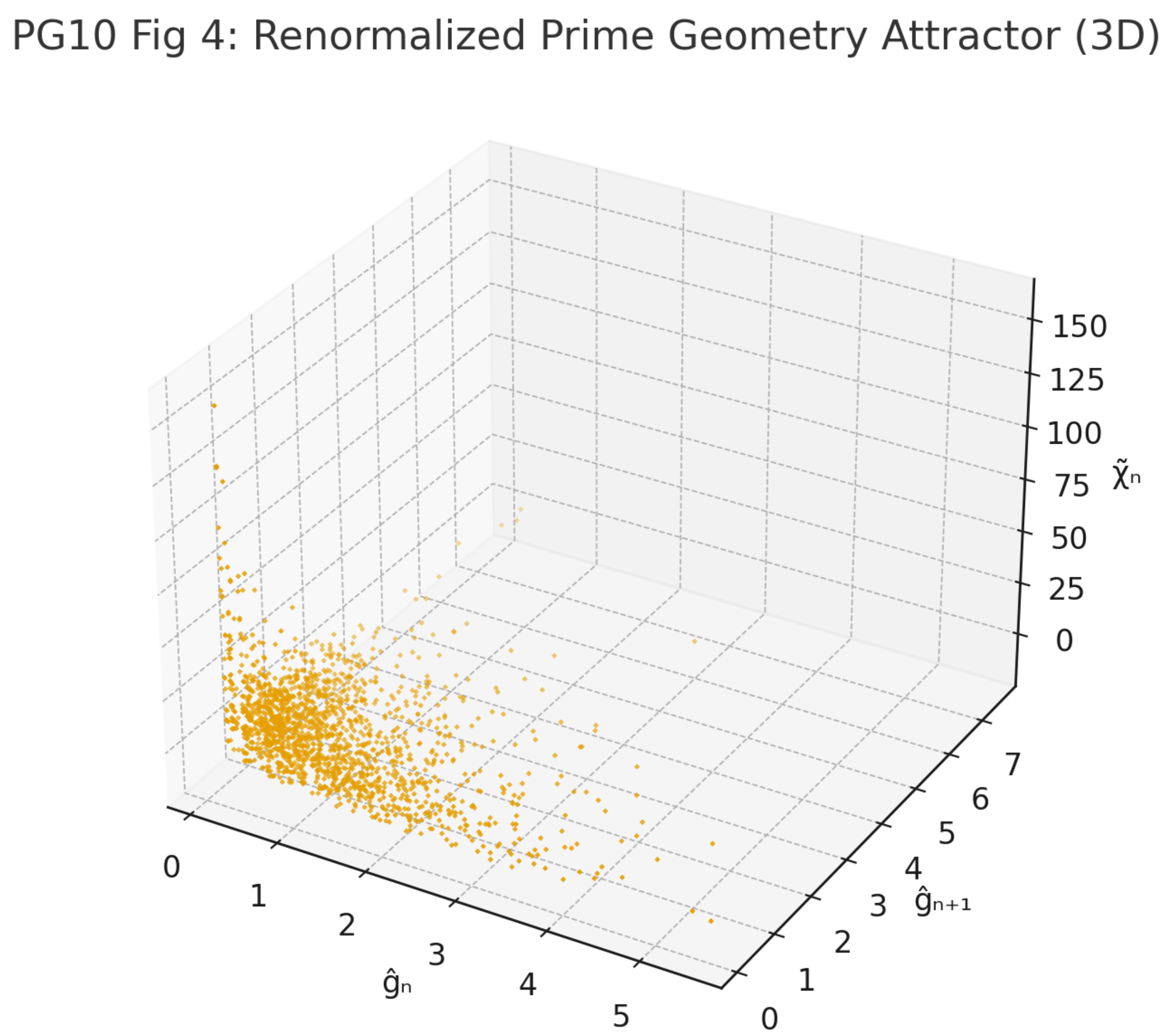

Figure 4: Three-dimensional renormalized attractor in $(\tilde{g}_n, \tilde{g}_{n+1}, \tilde{\chi}_n)$ coordinates. The tube-like cloud indicates concentration of trajectories along a stable geometric manifold.

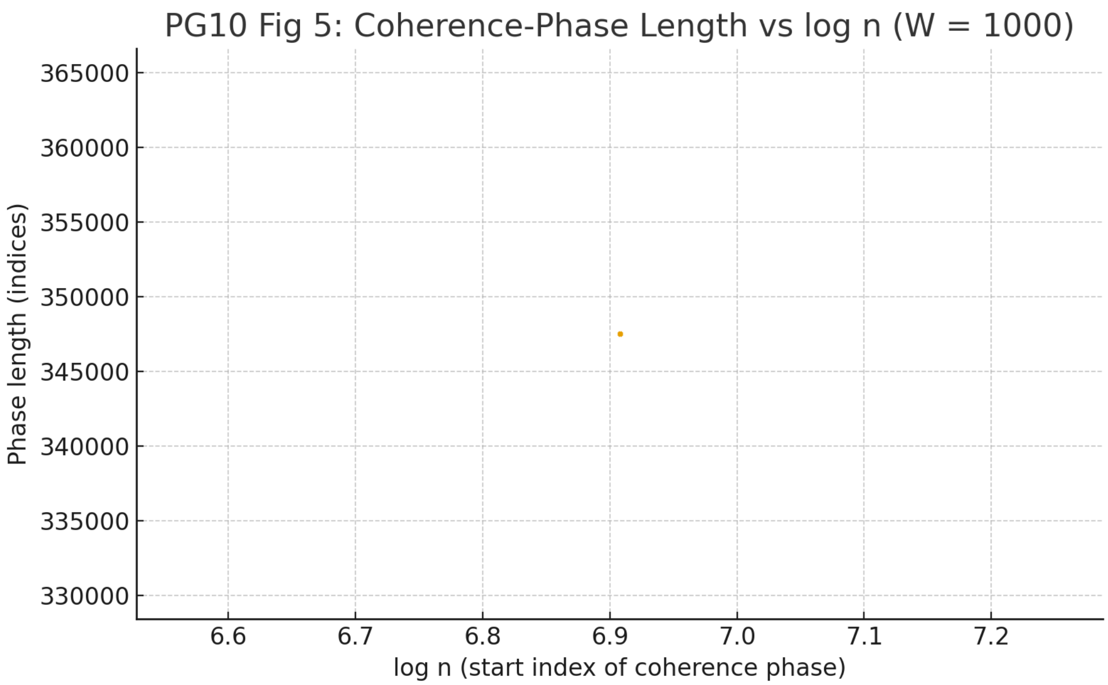

Figure 5: Coherence-phase lengths (from smoothed curvature $\chi_n^{(W)}$, $W = 1000$) plotted against their starting indices (shown on $\log n$ scale). Phase lengths align with renormalization-based growth laws.

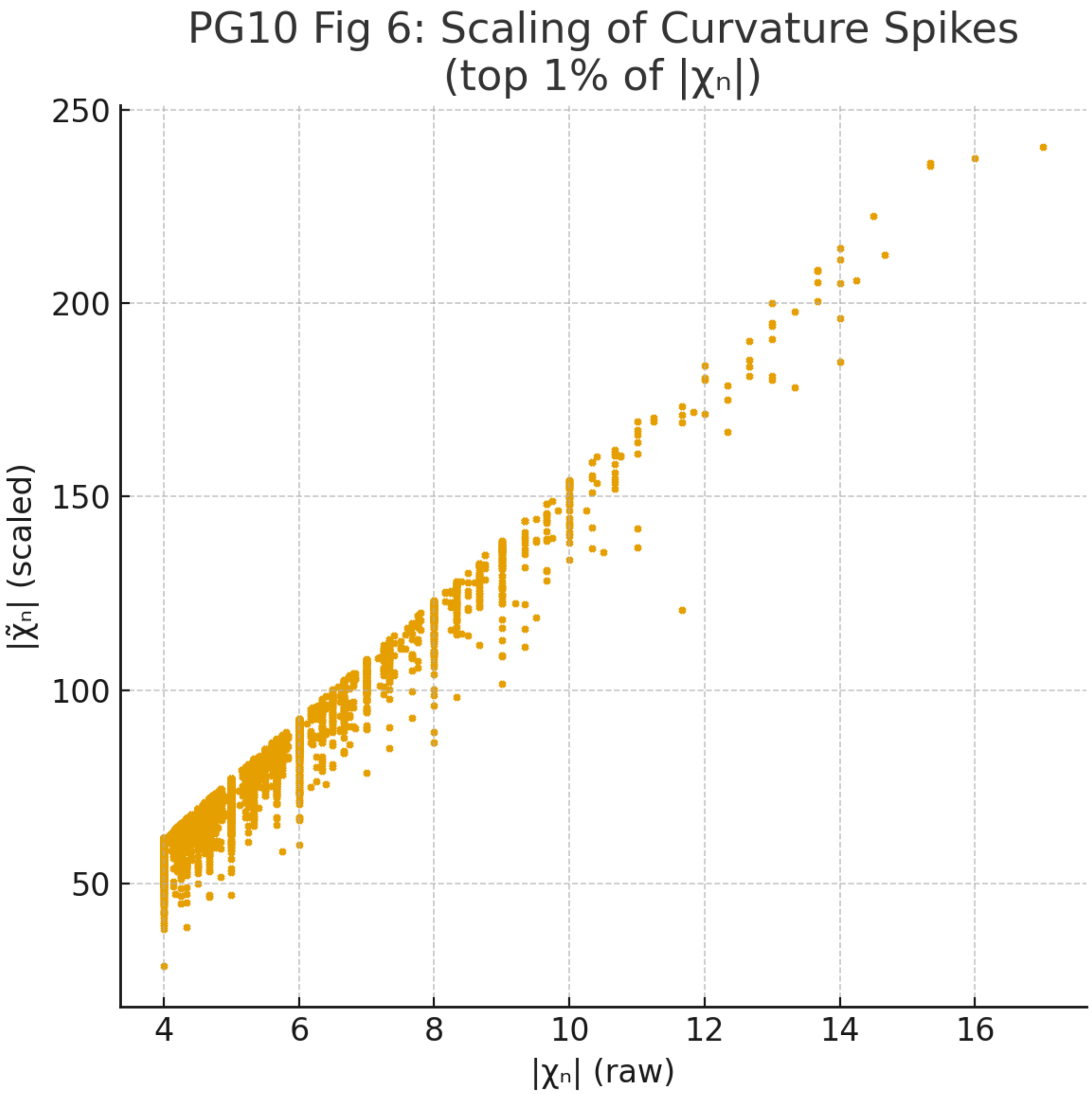

Figure 6: Scaling of curvature spikes: raw extremes $|\chi_n|$ plotted against scaled extremes $|\tilde{\chi}_n|$. The approximate linear relationship demonstrates that spike magnitude is normalized by $\log p_n$.

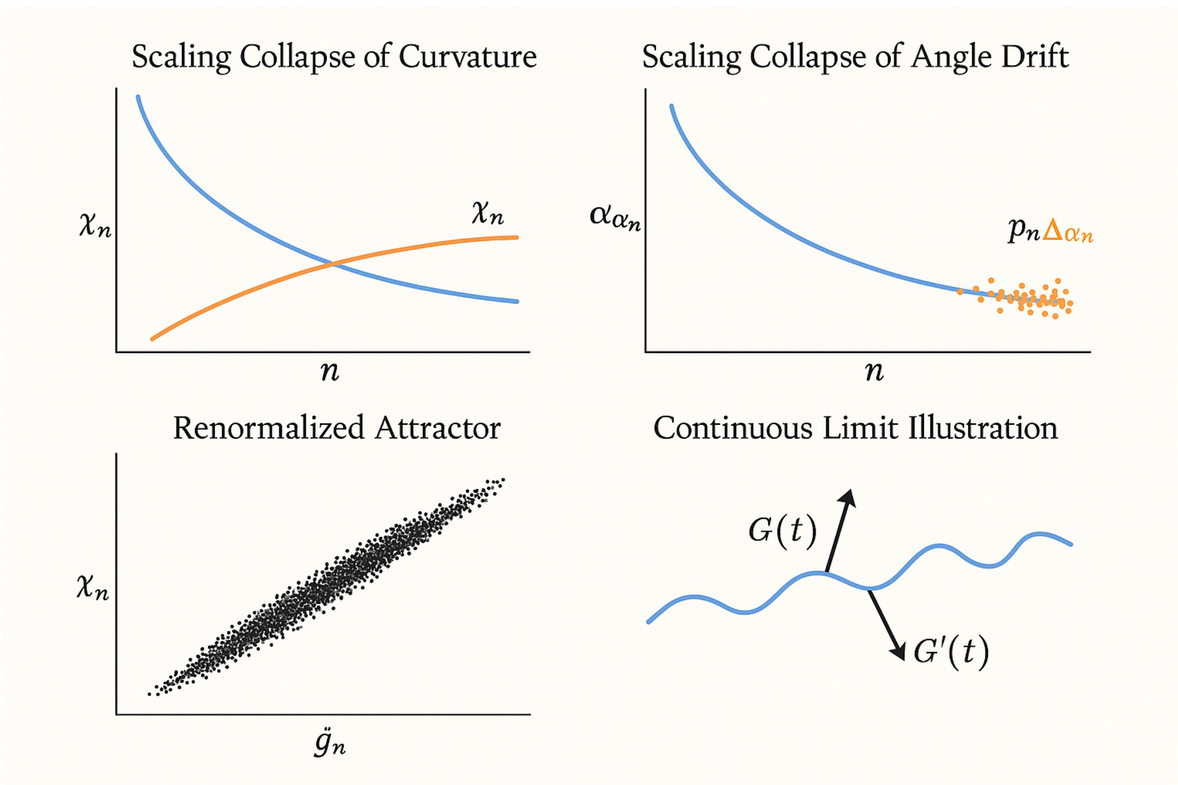

Figure 7: Four-panel conceptual summary of PG10: curvature collapse, angle-drift collapse, renormalized attractor, and continuous-limit interpretation.