Prime Geometry - Explainer

A concise overview of the structure, laws, and evolution equation behind Prime Geometry I–X.

Prime Geometry begins with a simple identity based on right triangles which relates three consecutive primes.

From this identity arise four geometric quantities:

- α_n — the Prime Triangle angle, consistently near 45°
- χ_n — the curvature, a normalized second derivative of the prime gaps
- $\Delta\alpha_n$ — angle drift, behaving as a first derivative
- $S(N) = \sum\chi^2$ — the action, a measure of global roughness

Curvature is almost always tiny, meaning that the prime gaps change only gradually.

Across millions of primes we see:

- small curvature almost everywhere
- long coherent phases where curvature keeps the same sign
- suppressed extremes (rare spikes)
- low total action, far below random permutations
- angle drift tracking curvature's sign structure

In short: prime gaps evolve smoothly, with neighboring gaps often nearly equal in size.

A key structural discovery is the derivative hierarchy:

$$\chi_n \longrightarrow \Delta\alpha_n \longrightarrow \alpha_n.$$

Curvature acts as a second derivative, angle drift as a first derivative, and the angle itself is a cumulative imbalance of curvature. This explains coherence, smoothness, and the remarkable stability of α_n near 45°.

A single proven result anchors the global picture: the Weighted Mean-Zero Curvature Theorem. Its says that the signed curvature cancels when weighted by local gap size, preventing runaway drift.

Two conjectures summarize the empirical structure:

- Curvature Concentration Conjecture — primes overwhelmingly prefer small curvature.
- Low-Action Structure Conjecture — the prime sequence minimizes curvature energy compared to almost all reorderings of the same gaps

These observations suggest a deeper organizing principle: prime evolution is not random but constrained by geometric balance.

This culminates in the Prime Geometry Evolution Equation (PGEE), the empirical dynamical equation coupling the key quantities:

$$g_{n+2} = g_n + (g_n + g_{n+1})\chi_n + 2p_n\Delta\alpha_n + C\Phi'(n) + \varepsilon_n.$$

Here:

- χ_n governs second-order variation (bending of the gap sequence)
- $\Delta\alpha_n$ supplies a first-order correction

- $\Phi'(n)$ is a global potential enforcing low-action behavior
- ε_n is a small structured residual

This evolution law reproduces all major features of the prime gaps:

- coherence phases
- kink transitions
- curvature suppression
- angle stability
- return-map geometry
- and remarkably low action

Plotting the state vector:

$$X_n = (g_n, g_{n+1}, \chi_n)$$

reveals the Prime Geometry Attractor (figure below). The sequence traces a thin one-dimensional manifold in the three-dimensional vector space, sharply distinct from randomized models.

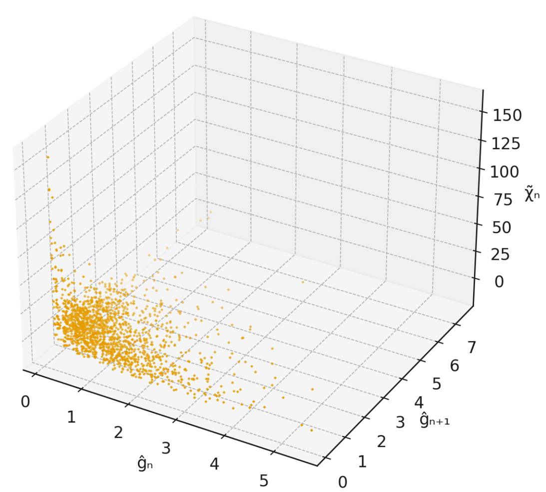
This attractor is the geometric footprint and represents all of the constraints above.

Lastly, the Prime Geometry system is shown to be scale-invariant.

- After renormalizing gaps and curvature by $\log p_n$ and angle drift by p_n, all geometric signals stabilize.
- The PGEE becomes a scale-free law, and the attractor approaches a limiting shape.

The [Prime Geometry I–X papers] therefore presents a unified picture:

> Prime gaps evolve along a smooth, self-correcting, scale-invariant geometric pathway governed by a specific evolution equation and constrained by curvature suppression, balance laws, and minimal action.



PG10 Fig 4: Renormalized Prime Geometry Attractor (3D)