Prime Geometry II: Curvature and Local Variation in the Prime Gap Sequence

Allen Proxmire

December 2025

Abstract

This paper examines empirical behavior associated with the curvature quantity

$$\chi_n = \frac{g_{n+2} - g_n}{g_n + g_{n+1}}, \qquad L_n = \chi_n^2,$$

defined from consecutive prime gaps. We compare (χ_n) and (L_n) for true primes with randomized permutations of the same multiset of gaps and with Crámer-type pseudo-primes. In the ranges examined, the prime data show narrower curvature distributions, reduced extremes, persistent localized patterns in sliding-window averages, and structured clustering in return maps. No theoretical claims are made; results are reported descriptively.

1 Introduction

Let (p_n) denote the sequence of primes and

$$g_n = p_{n+1} - p_n$$

their consecutive gaps.

PG1 introduced the normalized curvature quantity

$$\chi_n = \frac{g_{n+2} - g_n}{g_n + g_{n+1}},$$

which compares forward and backward gaps relative to their total. The derived local measure

$$L_n = \chi_n^2$$

captures curvature magnitude.

This document records numerical patterns in (χ_n), (L_n), and the cumulative measure

$$S(N) = \sum_{n=1}^{N-2} L_n.$$

2 Curvature Framework

The curvature quantity

$$\chi_n = \frac{g_{n+2} - g_n}{g_n + g_{n+1}}$$

is dimensionless and sensitive to the ordering of nearby gaps:

- $\chi_n > 0$ if $g_{n+2} > g_n$,
- $\chi_n < 0$ if $g_{n+2} < g_n$,
- $\chi_n \approx 0$ if the gaps are locally symmetric.

We define

$$L_n = \chi_n^2, \qquad S(N) = \sum_{k=1}^{N-2} L_k.$$

3 Experiment I: Comparison of Total Curvature $S(N)$

We compare $S(N)$ for true primes with the distribution obtained by randomly permuting the same gap multiset.

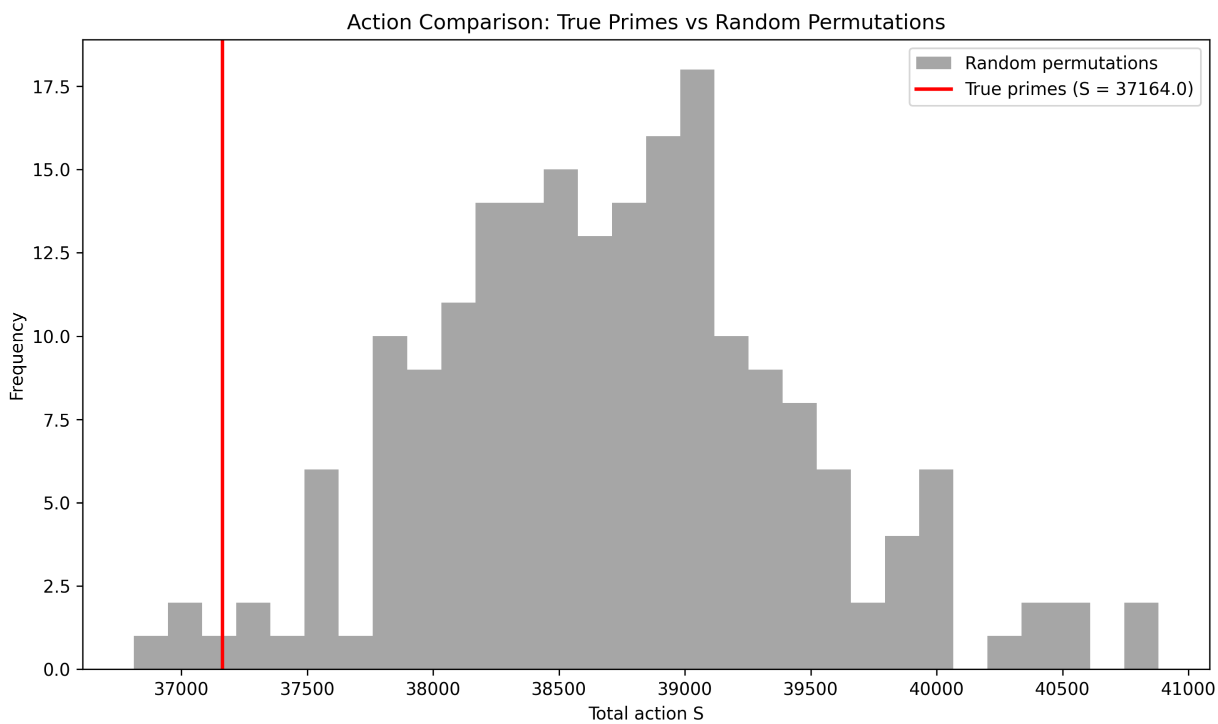

Figure 1: Total curvature measure $S(N)$ for primes compared with the permutation distribution.

In these computations, the prime sequence yields smaller values of $S(N)$ than most permutations of the same gaps. This observation is empirical and limited to the ranges tested.

4 Experiment II: Curvature Distribution

We compare histograms of χ_n across three sequences: true primes, random permutations, and Crámer-type pseudo-primes.

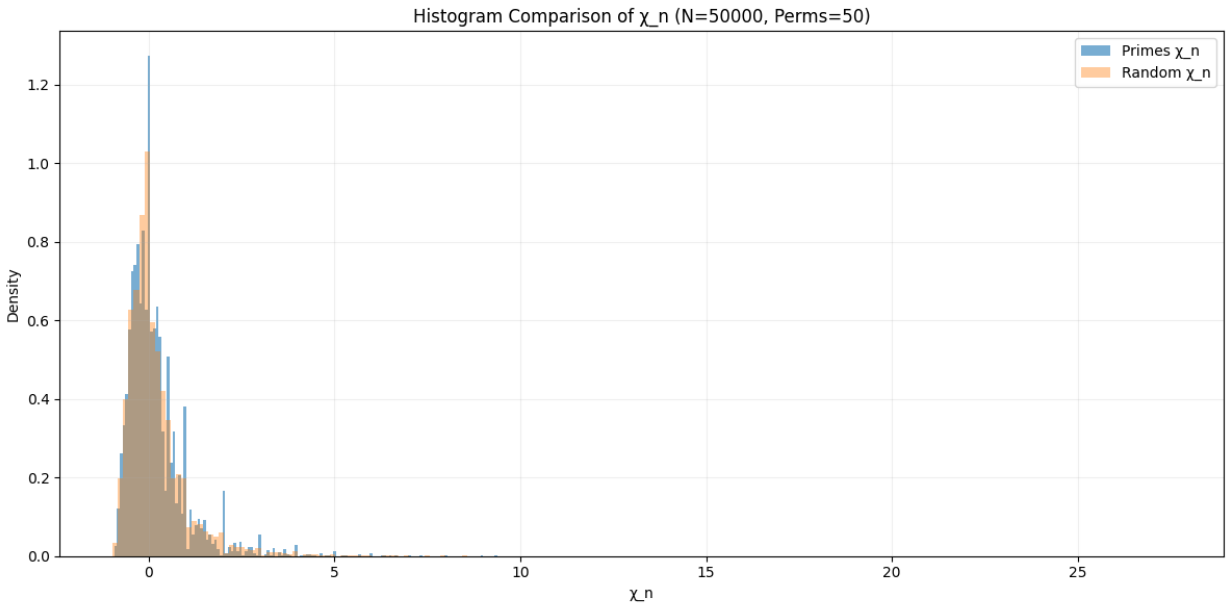

Figure 2: Curvature distribution χ_n for primes and comparison sequences.

Prime curvature values show narrower spread and smaller extremes relative to the comparison models.

5 Experiment III: Sliding-Window Patterns

Define a sliding-window mean of L_n with window size W:

$$\overline{L}_n = \frac{1}{W} \sum_{k=n-W+1}^{n} L_k.$$

Prime data show extended intervals of relatively higher or lower curvature magnitude. Random permutations exhibit less persistent structure.

6 Experiment IV: Multi-Scale Heatmaps

We compute moving averages over several window sizes and visualize results as heatmaps.

Heatmaps reveal localized vertical bands consistent across multiple scales.

6.1 Mean curvature sign

Smoothed curvature values

$$\overline{\chi}_n^{(W)} = \frac{1}{W} \sum_{k=n-W+1}^{n} \chi_k$$

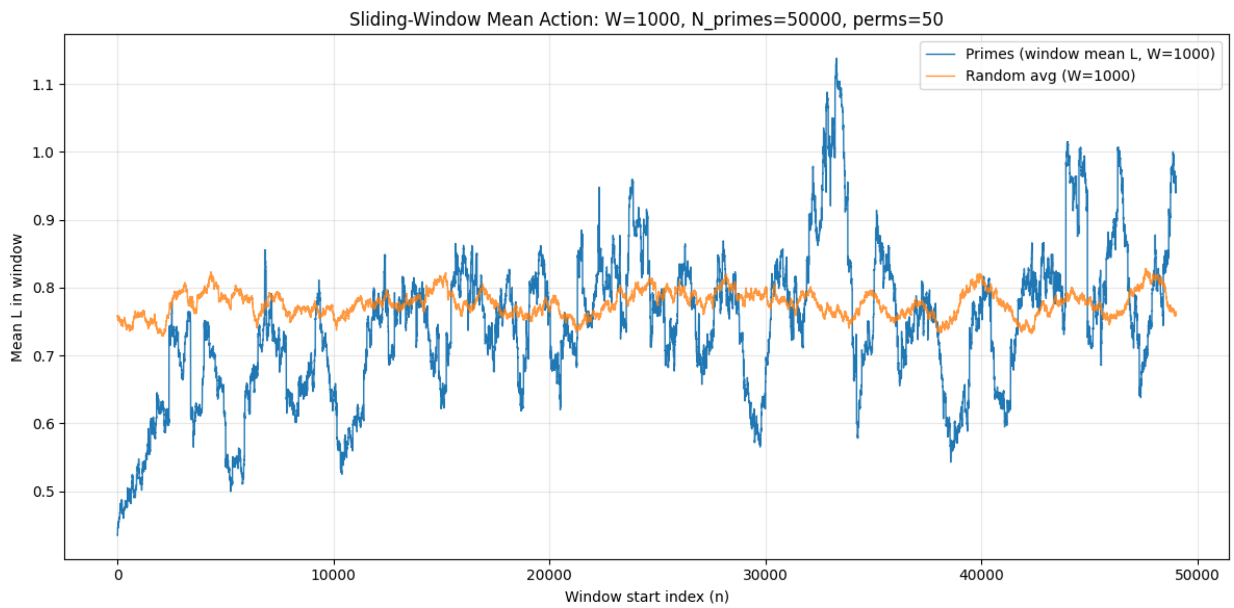

Figure 3: Sliding-window mean $\overline{L}_n$ for $W = 1000$.

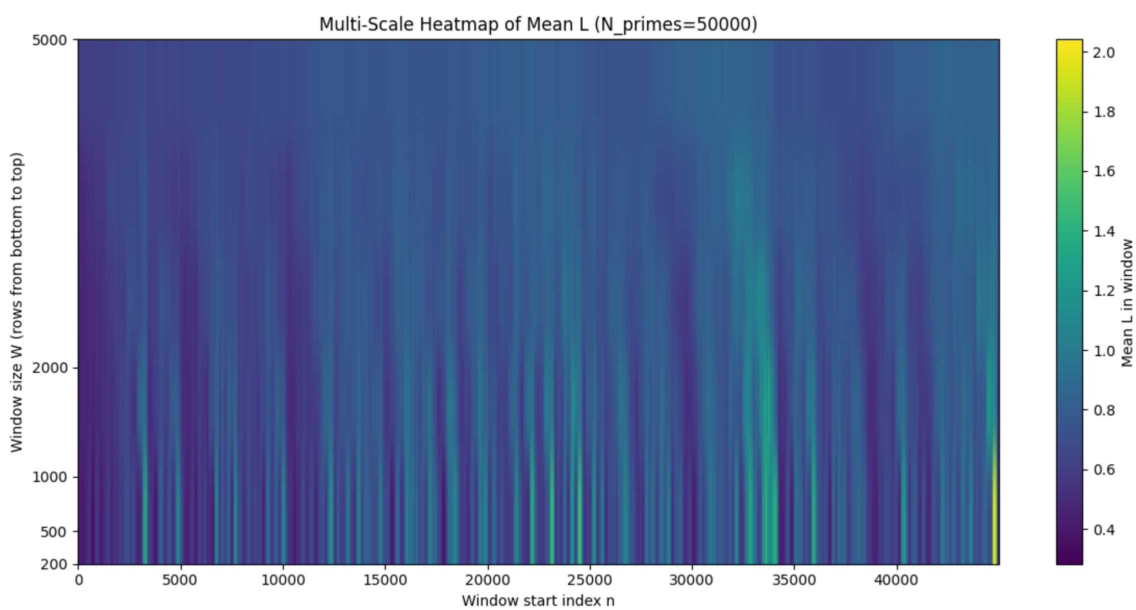

Figure 4: Multi-scale heatmap of averaged L_n across various window sizes.

exhibit long intervals of predominantly positive or negative sign.

7 Experiment V: Return-Map Structure

7.1 Gaps

The return map of consecutive gaps shows a confined geometric region for primes.

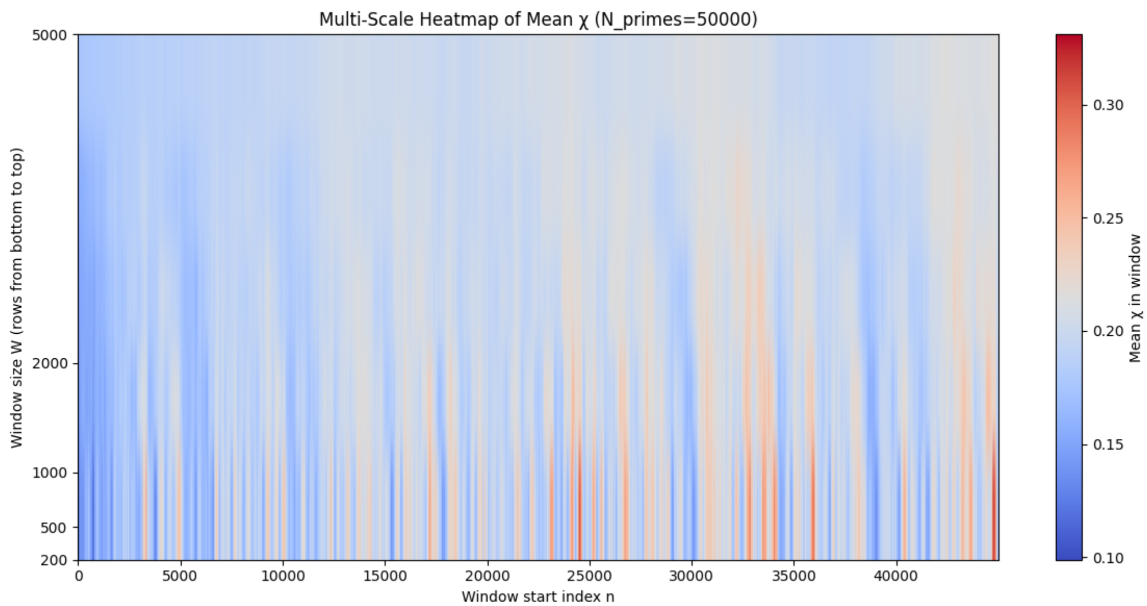

Figure 5: Multi-scale heatmap of averaged curvature $\overline{\chi}_n^{(W)}$.

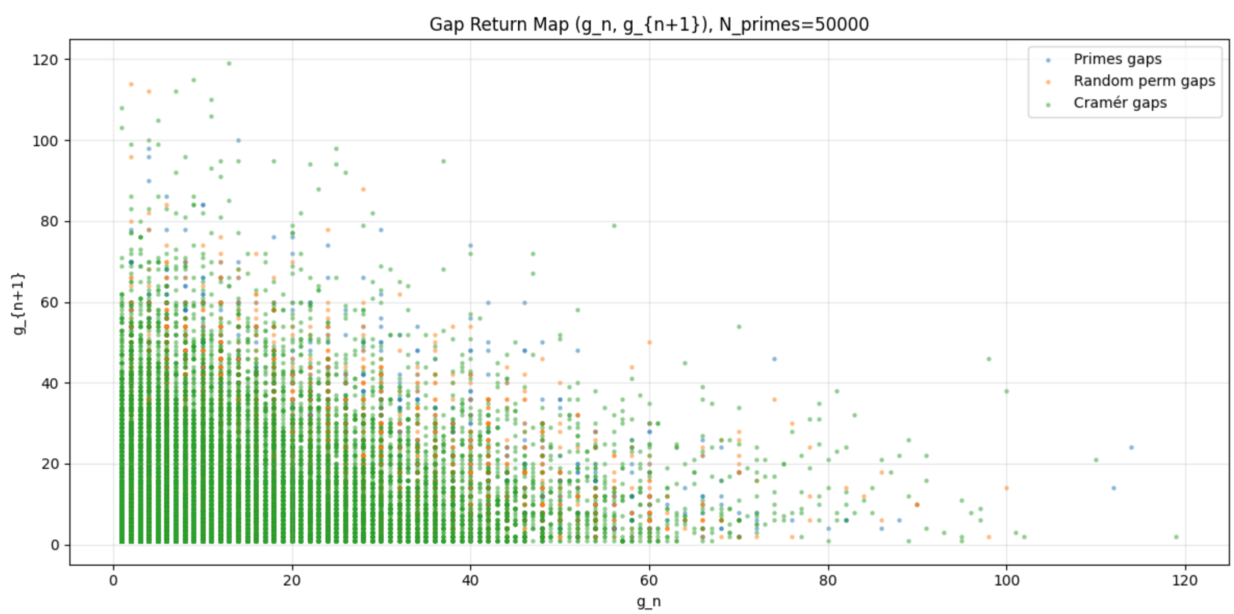

Figure 6: Return map (g_n, g_{n+1}) for primes.

7.2 Curvature

The map of (χ_n, χ_{n+1}) forms a bounded cluster for primes, visibly distinct from randomized or Crámer sequences.

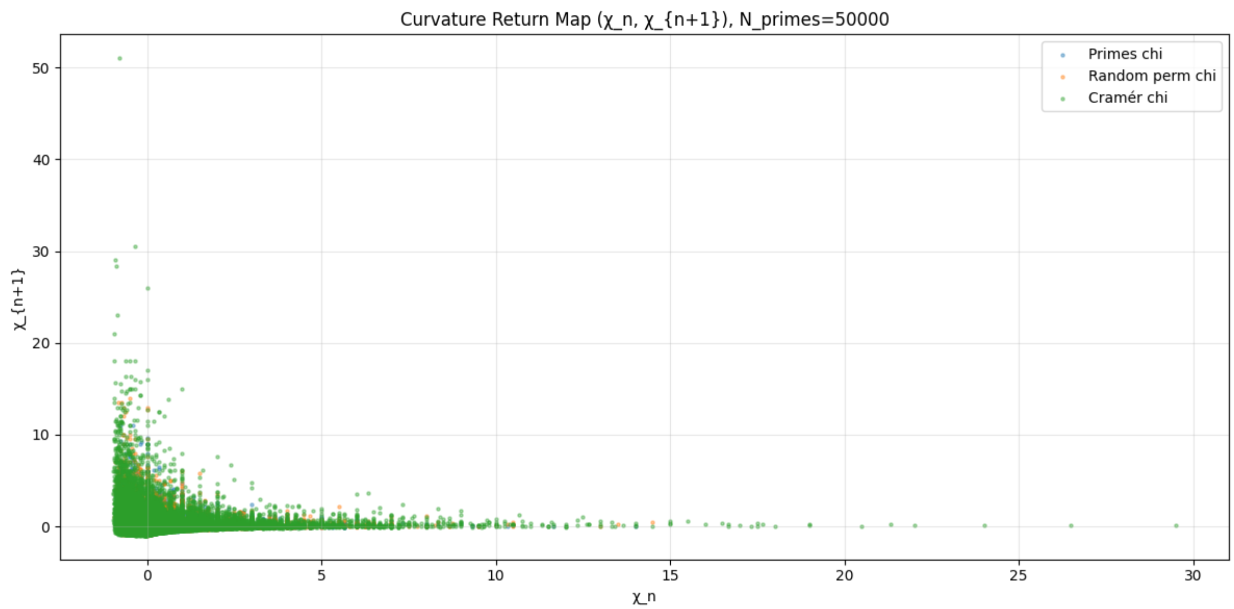

Figure 7: Return map (χ_n, χ_{n+1}) for primes.

8 Synthesis

Across the experiments:

- Prime curvature values are more tightly distributed than randomized or Crámer models.
- The cumulative measure $S(N)$ tends to be smaller for true primes.
- Sliding-window and multi-scale views show persistent localized structure.
- Return maps display distinct geometric clustering.

These findings are empirical observations without theoretical interpretation.

9 Conclusion

This document compiles numerical behavior of the curvature quantities χ_n and L_n for the prime gaps. The data reveal recurring patterns that differ from randomized and independent models. PG3 examines structural questions and recurrences associated with these quantities.