OFFSETTING TWIN PRIMES

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ABSTRACT. Here it is computationally shown that twin primes can be offset by four expressions which generate additional primes with surprising frequency. The offsets are: $q_1 = 2p + 1$, $q_2 = 2p + 7$, $q_3 = 2p - 3$, and $q_4 = 2p + 3$. The results are categorized by twin prime pair endings, (1,3), (7,9), and (9,1). At least one additional prime is generated 63.52%, 64.05%, and 43.08%, respectively, for $p \leq$ 1,000,000.

Introduction

The generation of possible primes and efficiently testing them is relative to several fields. Additionally, the study of the structure of prime number distributions is of particular interest to number theorists as the key to solving several open questions in mathematics.

In this note, we consider twin primes (p, p_1), where $p_1 = p + 2$, as the input for the offset expressions $q_1 = 2p + 1$, $q_2 = 2p + 7$, $q_3 = 2p - 3$, and $q_4 = 2p + 3$.

In the next section results are presented. While the result totals are not insignificant in themselves, the results as categorized by the ending-digits of the twin primes (1,3), (7,9), and (9,1) used to produce the results, reveal surprisingly high prime production rates and interesting patterns to investigate.

The final section will summarize the highlights of the data from using twin primes and these offsets to generate prime numbers.

Definitions, Example, and Results

Let the twin primes be (p, p_1), where $p_1 = p + 2$

Let the offset expressions be:

$$q_1 = 2p + 1$$
$$q_2 = 2p + 7$$
$$q_3 = 2p - 3$$
$$q_4 = 2p + 3$$

Example:

Given (11, 13).

Apply offsets: $q_1 = 2(11) + 1 = 23$

$q_2 = 2(11) + 7 = 29$

$q_3 = 2(11) - 3 = 19$

$q_4 = 2(11) + 3 = 25$

Check offset answers for primality:

23, prime
29, prime
19, prime
25, not prime

Results:

Total twin prime pairs considered: 8167 (all twin primes ≤ 1,000,000)

Number of unique prime numbers produced by the four offsets: 5947

Results from all offsets for all twin prime pairs: (Total pairs: 8167)
  Four primes found: 0 (0.00%)
  Three primes found: 105 (1.29%)
  Two primes found: 1083 (13.26%)
  One prime found: 3463 (42.40%)
  No primes found: 3516 (43.05%)
  At least one prime found: 4651 (56.95%)

Results from all offsets for all (1,3) ending pairs: (Total pairs: 2736)
  Four primes found: 0 (0.00%)
  Three primes found: 46 (1.68%)
  Two primes found: 487 (17.80%)
  One prime found: 1205 (44.04%)
  No primes found: 998 (36.48%)
  At least one prime found: 1738 (63.52%)

Results from all offsets for all (7,9) ending pairs: (Total pairs: 2734)
 Four primes found: 0 (0.00%)
 Three primes found: 59 (2.16%)
 Two primes found: 453 (16.57%)
 One prime found: 1239 (45.32%)
 No primes found: 983 (35.95%)
 At least one prime found: 1751 (64.05%)

Results from all offsets for all (9,1) ending pairs: (Total pairs: 2697)
 Four primes found: 0 (0.00%)
 Three primes found: 0 (0.00%)
 Two primes found: 143 (5.30%)
 One prime found: 1019 (37.78%)
 No primes found: 1535 (56.92%)
 At least one prime found: 1162 (43.08%)

Conclusions

The four offsets, $q_1 = 2p + 1$, $q_2 = 2p + 7$, $q_3 = 2p - 3$, and $q_4 = 2p + 3$, when applied to the first term of twin prime pairs (p, p_1), produce additional prime numbers almost 57% of the time. Almost 64% of twin primes pairs ending in (1, 3) and (7, 9) produce additional primes. Only the q_1 and q_4 offsets produce primes for twin prime pairs ending in (9,1). However, the q_1 and q_4 offsets applied to twin prime pairs ending in (9,1) is the only case in which twin primes can be generated, which occurs at 5.3%.