OFFSETTING TWIN PRIMES

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ABSTRACT. This computational note examines the frequency with which simple linear offsets of twin primes produce additional primes.
For each twin prime pair (p,p+2), p ≤ 1,000,000, four offsets were applied to p: q1=2p+1, q2=2p+7, q3=2p−3, q4=2p+3.
Across all twin prime pairs tested, at least one offset produced a prime in **56.95%** of cases, with higher rates for pairs ending in (1,3) and (7,9) (~64%) than those ending in (9,1) (~43%).
Full data, code, and analysis are available at:
https://github.com/YourUsername/twin-primes-offsets

Introduction

The generation of possible primes and efficiently testing them is relative to several fields. Additionally, the study of the structure of prime number distributions is of particular interest to number theorists as the key to solving several open questions in mathematics.

In this note, we consider twin primes (p, p_1), where $p_1 = p + 2$, as the input for the offset expressions $q_1 = 2p + 1$, $q_2 = 2p + 7$, $q_3 = 2p - 3$, and $q_4 = 2p + 3$.

In the next section, results are presented. While the result totals are not insignificant in themselves, the results as categorized by the ending-digits of the twin primes (1,3), (7,9), and (9,1) used to produce the results, reveal surprisingly high prime production rates and interesting patterns to investigate.

The final section will summarize the highlights of the data from using twin primes and these offsets to generate prime numbers.

Definitions

Let the twin primes be (p, p_1), where $p_1 = p + 2$

Let the offset expressions be:

$$q_1 = 2p + 1$$
$$q_2 = 2p + 7$$
$$q_3 = 2p - 3$$
$$q_4 = 2p + 3$$

Methods

Twin primes were generated up to $p \leq 1{,}000{,}000$ using a standard sieve of Eratosthenes implementation.

Primality tests for offset results q_1 - q_4 were performed using Python's sympy.isprime() function (version X.X).

All computations were run in Python 3.X on a standard desktop system.

Offset results were categorized by the last-digit pattern of the twin prime pair: (1,3), (7,9), and (9,1).

Example

Given (11, 13).

Apply offsets:

$q_1 = 2(11) + 1 = 23$
$q_2 = 2(11) + 7 = 29$
$q_3 = 2(11) - 3 = 19$
$q_4 = 2(11) + 3 = 25$

Check offset answers for primality:

23, prime
29, prime
19, prime
25, not prime

Results

Total twin prime pairs considered: 8167 (all twin primes ≤ 1,000,000)

Number of unique prime numbers produced by the four offsets: 5947

Results from all offsets for all twin prime pairs: (Total pairs: 8167)
Four primes found: 0 (0.00%)
Three primes found: 105 (1.29%)
Two primes found: 1083 (13.26%)
One prime found: 3463 (42.40%)
No primes found: 3516 (43.05%)
At least one prime found: 4651 (56.95%)

Results from all offsets for all (1,3) ending pairs: (Total pairs: 2736)
Four primes found: 0 (0.00%)
Three primes found: 46 (1.68%)
Two primes found: 487 (17.80%)
One prime found: 1205 (44.04%)

No primes found: 998 (36.48%)
At least one prime found: 1738 (63.52%)

Results from all offsets for all (7,9) ending pairs: (Total pairs: 2734)
Four primes found: 0 (0.00%)
Three primes found: 59 (2.16%)
Two primes found: 453 (16.57%)
One prime found: 1239 (45.32%)
No primes found: 983 (35.95%)
At least one prime found: 1751 (64.05%)

Results from all offsets for all (9,1) ending pairs: (Total pairs: 2697)
Four primes found: 0 (0.00%)
Three primes found: 0 (0.00%)
Two primes found: 143 (5.30%)
One prime found: 1019 (37.78%)
No primes found: 1535 (56.92%)
At least one prime found: 1162 (43.08%)

Results Summary:
All twin primes: At least one offset prime in 56.95% of pairs.
(1,3) endings: 63.52% success rate.
(7,9) endings: 64.05% success rate.
(9,1) endings: 43.08% success rate, with only q_1 and q_4 producing primes.

Full numerical tables and CSV data are provided in the GitHub repository.

Discussion

The consistent differences in prime production rates across ending-digit classes suggest subtle structural biases in twin prime distributions. The particularly high rates for (1,3) and (7,9) endings may relate to modular constraints in prime gaps that should be further investigated for larger bounds of *p*.

This note makes no conjecture or association to conjecture but provides a framework and dataset for future statistical and heuristic analysis.

Conclusions

The four offsets, $q_1 = 2p + 1$, $q_2 = 2p + 7$, $q_3 = 2p - 3$, and $q_4 = 2p + 3$, when applied to the first term of twin prime pairs (p, p_1), produce additional prime numbers almost 57% of the time. Almost 64% of twin primes pairs ending in (1, 3) and (7, 9) produce additional primes. Only the

q_1 and q_4 offsets produce primes for twin prime pairs ending in (9,1). However, the q_1 and q_4 offsets applied to twin prime pairs ending in (9,1) is the only case in which twin primes can be generated, which occurs at 5.3%.

Data Availability

The complete dataset, Python code, and additional analysis files are publicly available at: https://github.com/YourUsername/twin-primes-offsets

This includes CSV summaries, per-ending-pattern results, and a template for the full raw data.

References

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Caldwell, C.K. "Sophie Germain Primes." The Prime Pages, https://primes.utm.edu/glossary/page.php?sort=SophieGermainPrime

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