OFFSETTING TWIN PRIMES

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ABSTRACT. This computational note examines the frequency with which simple linear offsets of twin primes produce additional primes.
For each twin prime pair (p,p+2), p < 1,000,000, seven offsets were applied to p: $q_1 = 2p+1$, $q_2 = 2p+3$, $q_3 = 2p-3$, $q_4 = 2p-5$, $q_5 = 2p+7$, $q_6 = 2p+9$, and $q_7 = 2p-9$.
Across all twin prime pairs tested, at least one offset produced a prime in **82.03%** of cases. The success of the individual offsets are within 5% of each other, when categorized by the last digit of the twin prime used to produce the results ((1,3), (7,9), (9,1)).
Full data, code, and analysis are available at:
https://github.com/allen-proxmire/twin-primes-offsets

Introduction

The generation of possible primes and efficiently testing them is relative to several fields. Additionally, the study of the structure of prime number distributions is of particular interest to number theorists as the key to solving several open questions in mathematics.

In this note, we consider twin primes (p, p_1), where $p_1 = p + 2$, as the input for the offset expressions $q_1 = 2p+1$, $q_2 = 2p+3$, $q_3 = 2p-3$, $q_4 = 2p-5$, $q_5 = 2p+7$, $q_6 = 2p+9$, and $q_7 = 2p-9$.

In the following sections, results are presented. Total results, cumulative results, and results categorized by the last digits (residuals) of the twin primes pairs (1,3), (7,9), and (9,1) used as inputs, reveal surprisingly high prime production rates, equal success, and interesting patterns to investigate.

The final sections will summarize the highlights from the results, present discussion items, provide links to data and code for validation, and list references.

Definitions

Let the twin primes be (p, p_1), where $p_1 = p + 2$

Let the offset expressions be:

$$
\begin{aligned}
q_1 &= 2p + 1 \\
q_2 &= 2p + 3 \\
q_3 &= 2p - 3 \\
q_4 &= 2p - 5 \\
q_5 &= 2p + 7 \\
q_6 &= 2p + 9 \\
q_7 &= 2p - 9
\end{aligned}
$$

Methods

The first 8,169 twin primes were generated using a standard sieve of Eratosthenes implementation. There are 8,169 twin primes $p < 1{,}000{,}000$.

Primality tests for offset results q_1 through q_8 were performed using Python's sympy.isprime() function (version X.X).

All computations were run in Python 3.X on a standard desktop system.

Offset results were categorized total, cumulative total, and by the last-digit pattern of the twin prime pair: (1,3), (7,9), and (9,1).

Example

Given (11, 13).

Apply offsets:

$q_1 = 2(11) + 1 = 23$
$q_2 = 2(11) + 3 = 25$
$q_3 = 2(11) - 3 = 19$
$q_4 = 2(11) - 5 = 17$
$q_5 = 2(11) + 7 = 29$
$q_6 = 2(11) + 9 = 31$
$q_7 = 2(11) - 9 = 13$

Check offset answers for primality:

23, prime
25, not prime
19, prime
17, prime
29, prime
31, prime
13, prime

Results

Total twin prime pairs considered: 8,169

Results: Number of total prime numbers produced by the seven offsets: 11,492
Number of unique prime numbers produced by the seven offsets: 11,389

Last unique prime produced: 1,999,223
Number of primes ≤ 1,999,223 = 148,993
Percent of primes found by the seven offsets: 7.65%

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Offsets success totals

Results: Cumulative success count distribution:

at least 1 offsets succeeded:	6701	(82.03%)
at least 2 offsets succeeded:	3494	(42.76%)
at least 3 offsets succeeded:	1078	(13.20%)
at least 4 offsets succeeded:	202	(2.47%)
at least 5 offsets succeeded:	15	(0.18%)
at least 6 offsets succeeded:	2	(0.02%)
at least 7 offsets succeeded:	0	(0.00%)

Results: Exact success count distribution:

0 offsets succeeded:	1468	(17.97%)
1 offsets succeeded:	3207	(39.26%)
2 offsets succeeded:	2416	(29.58%)
3 offsets succeeded:	876	(10.72%)
4 offsets succeeded:	187	(2.29%)
5 offsets succeeded:	13	(0.16%)
6 offsets succeeded:	2	(0.02%)
7 offsets succeeded:	0	(0.00%)

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Offset Success by Last Digit Categories

Results: Offset Success for (1,3) ending pairs: (Total pairs: 2,736)

Offset 2p - 9:	771 successes	(17.35%)
Offset 2p - 5:	674 successes	(15.17%)
Offset 2p - 3:	815 successes	(18.34%)
Offset 2p + 1:	670 successes	(15.08%)
Offset 2p + 3:	0 successes	(0.00%)
Offset 2p + 7:	832 successes	(18.73%)
Offset 2p + 9:	681 successes	(15.33%)

Primes produced: 4,443

Results: Offset Success for (7,9) ending pairs: (Total pairs: 2,734)

Offset 2p - 9:	0 successes	(0.00%)
Offset 2p - 5:	672 successes	(18.38%)
Offset 2p - 3:	837 successes	(22.89%)

Offset 2p + 1:	0 successes	(0.00%)
Offset 2p + 3:	666 successes	(18.21%)
Offset 2p + 7:	819 successes	(22.40%)
Offset 2p + 9:	663 successes	(18.13%)

Primes produced: 3,657

Results: Offset Success for (9,1) ending pairs: (Total pairs: 2,697)

Offset 2p - 9:	746 successes	(22.05%)
Offset 2p - 5:	664 successes	(19.63%)
Offset 2p - 3:	0 successes	(0.00%)
Offset 2p + 1:	649 successes	(19.18%)
Offset 2p + 3:	656 successes	(19.39%)
Offset 2p + 7:	0 successes	(0.00%)
Offset 2p + 9:	678 successes	(19.75%)

Primes produced: 3,383

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Full numerical tables and CSV data are provided in the GitHub repository.

Discussion

The surprisingly high prime production rates of ~82% for the seven offsets suggest subtle biases related to the twin primes. The suggestion of equal success rates for all offsets is made by the results for offset success by last digit categories, which are within ~3% of each other, and further supports a structure related to the twin primes.

The generation of new primes by the seven offsets include other twin primes, suggesting a method for producing large primes, though the efficacy of such a process has not been examined.

Further analysis will include testing the following observation. The seven twin prime offsets produce a set of unique primes, call these U. Call all the primes P. Within P, U will either match or not match some primes, call these accounted primes and unaccounted primes, respectively. Call a run of consecutive unaccounted primes ≥ 4 numbers an unaccounted gap. It appears that each unaccounted gap contains a twin prime pair.

This note makes no conjecture or association to conjecture but provides a framework and dataset for future statistical and heuristic analysis.

Conclusions

The seven twin prime offsets have a surprising prime production rate of almost 82%. Applying the seven offsets to the twin primes generates 7.65% of all prime numbers tested and, on average, produces 1.4 times more primes than are input. The success of the twin prime offsets are within 5% of each other, when categorized by the last digit of the twin primes used to produce the results ((1,3), (7,9), (9,1)).

Data Availability

The complete dataset, Python code, and additional analysis files are publicly available at: https://github.com/allen-proxmire/twin-primes-offsets

References

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