機率期中考試考前猜題 2022/04/26

1. 敘述機率公設

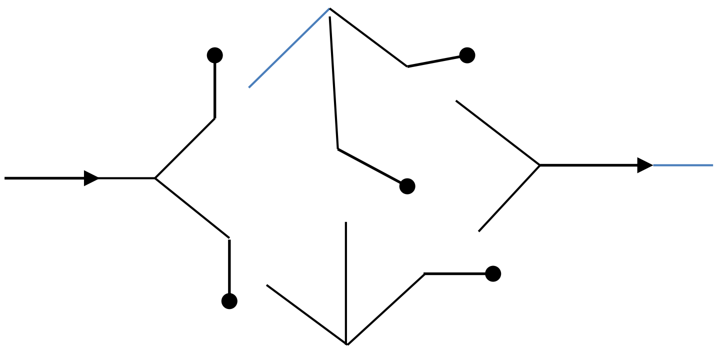
定義. 令S為一隨機現象(試驗)之樣本空間。設對每一事件A都被賦予一數P(A)，使P滿足以下公設(axioms)，則稱P為一機率函數，而稱P(A)為是事件A的機率。

Axiom 1. $P(A) \geq 0$

Axiom 2. $P(S) = 1$

Axiom 3. 若$\{A_1, A_2, A_3, \cdots\}$為一序列互斥事件($A_i \cap A_j = \emptyset, i \neq j$。，則$P(\cup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i)$。

2 .The figure below shows an electric circuit in which each of the switches located at 1,2,3,4 and 5 is independently closed or open with probabilities p and $1-p$, respectively. If a signal is fed to the input, what is the probability that is transmitted to the output?



3.Prove that if A,B, and C are independent, then A and B ∪ C are independent.

4. A factory produces its entire output with three machines. Machines I , II, and III produce 50%, 30%, and 20% of output, but 4%, 2%, and 4% of their outputs are defective, respectively. What fraction of the total output is defective?

5.

The distribution of the random variable X is Given by

$$F(x) = \begin{cases} 0 & , x < 0 \\ \frac{x}{2} & , 0 \leq x \leq 1 \\ \frac{2}{3} & , 1 \leq x < 2 \\ \frac{11}{12} & , 2 \leq x < 3 \\ 1 & , 3 \leq x \end{cases}$$

Compute

(a)$P[X < 3]$

(b)$P[X = 1]$

(c)$P\left[X > \frac{1}{2}\right]$ and

(d)$P[2 < X \leq 4]$

6. If $E[X] = 1$ and $Var(X) = 5$, find

(a) $E[(2+X)^2]$ (b) $Var(4+3X)$

7.

Suppose that X takes on values 0,1,2. If for some constant c, $P[X = i] = cP[X = i-1], = 1,2,$ find $E[X]$.

8. The joint density function of X and Y is given by

$$f(x,y) = \begin{cases} 2e^{-x}e^{-2y}, 0 < x < \infty, 0 < y < \infty \\ 0, \quad \text{otherwise} \end{cases}$$

compute

(a) $P(X > 1, Y < 1)$.

(b)$P(X < Y)$, and (c)$P(X < a)$

9. The joint density of X and Y is given by

$$f(x,y) = C(y-x)e^{-y}, -y < x < y, 0 < y < \infty$$

(a) Find C.

(b) Find the density function of X.

(c) Find the density function of Y.

(d) Find $E[X]$.

(e) Find $E[Y]$.

10.The joint density function of X and Y is

$$f(x,y) = \begin{cases} xy, 0 < x < 1, 0 < y < 2 \\ 0, \quad \text{otherwise} \end{cases}$$

(a) Are X and Y independent?
(b) Find the density function of X.
(c) Find the density function of Y.
(d) Find the joint distribution function of X and Y.
(e) Find $E[Y]$.
(f) Find $P(X + Y < 1)$.
(g)
(h)

11.

On average, 5.2 hurricans hit a certain region in a year. What is the probability that will be 3 or fewer hurricans hitting his year?

12.

If X has distribution function F, what is the distribution of

(a) e^X (b) $\alpha X + \beta$