Q1.

(d) 收集網站用戶過往的消費紀錄和點擊、流覽的頻率，通常來說越常消費於此網站的人，在往後也比較容易在此網站重複進行消費，~~並推測其可能的~~消費頻率，便可預測其於接下來7天內進行消費的可能性並進行排序。

Q2.

(e) $y_{n(t)} \cdot W_{t+1}^T \cdot x_{n(t)} > 0$, $y_{n(t)} \in \{+1,-1\}$

let $W_{t+1} = W_t + y_{n(t)} x_{n(t)} \cdot \eta_t$

$y_{n(t)} (W_t^T + y_{n(t)} \cdot \eta_t \cdot x_{n(t)}^T) \cdot x_{n(t)} > 0$

$y_{n(t)} \cdot W_t^T x_{n(t)} + y_{n(t)}^2 \cdot \eta(t) \cdot x_{n(t)}^T \cdot x_{n(t)} > 0$

$\because y_{n(t)}^2 = (+1)^2 = (-1)^2 = 1$

$\therefore \eta(t) \cdot \|x_{n(t)}\|^2 > -y_{n(t)} \cdot W_t^T \cdot x_{n(t)}$

又 $\because y_{n(t)} \cdot W_t^T \cdot x_{n(t)} \le 0$

$\therefore \eta(t) \cdot \|x_{n(t)}\|^2 > -y_{n(t)} \cdot W_t^T \cdot x_{n(t)} \ge 0$

$$\Rightarrow \eta(t) > \left(\frac{-y_{n(t)} \cdot W_t^T \cdot x_{n(t)}}{\|x_{n(t)}\|^2}\right) \ge 0$$

$$\text{而} \left[\frac{-y_{n(t)} \cdot W_t^T \cdot x_{n(t)}}{\|x_{n(t)}\|^2} + 1\right] \ge \frac{-y_{n(t)} \cdot W_t^T \cdot x_{n(t)}}{\|x_{n(t)}\|^2} \ge 0$$

Q3.

(e) $\because$ Dataset is linear separable

$\therefore$ Exists perfect W_f such that $y_n = \text{sign}(W_f^T x_n)$

① inner product grows fast.

$$\frac{W_f^T W_{t+1}}{\|W_f\|} \ge \frac{W_f^T W_t}{\|W_f\|} + \underbrace{\min_n \frac{(y_n W_f^T x_n)}{\|W_f\|}}_{\rho} \cdot \eta_t$$

② length^2 grows slowly

$$\|W_{t+1}\|^2 \le \|W_t\|^2 + \underbrace{\max_n (y_n^2 \|x_n\|^2)}_{R^2} \cdot \eta_t^2$$

starts from $W_0 = 0$, after T mistake corrections

$$1 \ge \frac{W_f^T W_T}{\|W_f\| \|W_T\|} \ge \frac{\sum_{t=0}^{T-1} (\eta_t \cdot \rho)}{\sqrt{\sum_{t=0}^{T-1} (\eta_t)^2} \cdot R} \Rightarrow 1^2 \ge \left(\frac{\sum_{t=0}^{T-1} (\eta_t) \cdot \rho}{\sqrt{\sum_{t=0}^{T-1} (\eta_t)^2} \cdot R}\right)^2 = f(T) \cdot \frac{\rho^2}{R^2}$$

而若 $f(T) = \frac{\left(\sum_{t=0}^{T-1} (\eta_t)\right)^2}{\sum_{t=0}^{T-1} (\eta_t)^2}$ 為 monotonically increasing 而 $f(T) \cdot \frac{\rho^2}{R^2} = 1$ 則在T夠大時成立，即 Perfect Halt.

(a) $\lim_{T\to\infty} f(T) = \frac{\left(\sum_{t=0}^{\infty} (2^{-t})\right)^2}{\sum_{t=0}^{\infty} (2^{-2t})} = \frac{2}{\frac{4}{3}} = 1.5$

(b) $\lim_{T\to\infty} f(T) = \frac{\left(\sum_{t=0}^{\infty} 0.6211\right)^2}{\sum_{t=0}^{\infty} (0.6211)^2} = \infty$

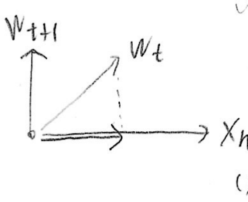
(d) $\lim_{T\to\infty} f(T) = \frac{\left(\sum_{t=0}^{\infty} (\frac{1}{1+t})\right)^2}{\sum_{t=0}^{\infty} (\frac{1}{1+t})^2} = \frac{\infty}{\frac{\pi^2}{6}} = \infty$

(C)(E)在下一頁

(c)

$$W_{t+1} = W_t + \cancel{y_{n(t)}} X_{n(t)} \cdot \left(\frac{-y_{n(t)} W_t^T X_{n(t)}}{\|X_{n(t)}\|^2} \right)$$

$$= W_t - X_{n(t)} \underbrace{\left(\frac{W_t^T X_{n(t)}}{\|X_{n(t)}\|} \right)}_{W_t \text{ project on } X_{n(t)}}$$



⇒ 由於 W_{t+1} 永遠是 $X_{n(t)}$ 之法向量，假若 $sign(0)=+1$，且 $y_{n(t)}=-1$ 而 $X_{n(t+1)}=X_{n(t)}$（即在下一輪中又用同樣的 X 來更新），此時 $W_{t+1}=W_t$

即 W_t 停止更新，但 $sign(W_{t+1}^T X_{n(t+1)}) = sign(0) = 1 \neq -1 = y_{n(t+1)}$

故 (c) 不保證 halt with a perfect line.

(e)

$$W_{t+1} = W_t + y_{n(t)} X_{n(t)} \cdot \left\lfloor \frac{-y_{n(t)} W_t^T X_{n(t)}}{\|X_{n(t)}\|^2} + 1 \right)$$

$$= W_t + y_{n(t)} X_{n(t)} \left\lfloor \frac{-y_{n(t)} W_t^T X_{n(t)}}{\|X_{n(t)}\|^2} \right\rfloor + y_{n(t)} X_{n(t)}$$

$$= W_t + y_{n(t)} X_{n(t)} (\alpha + 1)$$ ，α = 非負整數

$\because -y_{n(t)} W_t^T X_{n(t)} \geq 0$

故 $$\lim_{T \to \infty} f(T) = \frac{\left(\sum_{t=0}^{\infty} (\alpha+1) \right)^2}{\sum_{t=0}^{\infty} (\alpha+1)^2} = \infty$$ $(\because \alpha+1 \geq 1)$ #

故前題之 (b)(d)(e) 之 $f(T)$ 為 monotonically increasing

即有 3 者可 ensure halting with a perfect line.

Q4.

根據題述:

(c) X 為一維度為 $(d+1)\times 1$ 之向量，且

$x_0 = 1$, $x_{1\sim d} \in \{0, 1\}$ 且

同時間 $x_{1\sim d}$ 中最多存在 m 個 1，$m \le d$

另

$$f(x) = \text{sign}(z_+(x) - z_-(x) - 0.5) = \text{sign}(w_f^T x)$$

而 w_f^T 為 $\begin{cases} w_{f_0}^T = -0.5 \\ w_{f_{1\sim d}}^T = \{+1, -1\} \end{cases}$，

其中 $w_{f_{1\sim d}}^T$ 中存在 d_+ 個 1 和 d_- 個 -1

此做法是由於 $(z_+(x) - z_-(x))$ 相當於在計算 spam-like word 和 less spam-like 的個數差，因 X 中的值 $\in \{0, 1\}$，故 $w_{f_{1\sim d}} = \{+1, -1\}$ 和 $x_{1\sim d}$ 的內積和即等於 $(z_+(x) - z_-(x))$。

而

參考 Q3之說明

$$\rho = \min_n \left(\frac{y_n w_f^T x_n}{\|w_f\|} \right) = \frac{0.5}{\sqrt{(0.25 + 1^2 \times d)}}$$

$$R^2 = \max_n \left(y_n^2 \cdot \|x_n\|^2 \right) = 1 + 1^2 \times m = 1 + m$$

($\because x_{1\sim d}$ 中最多存在 m 個 1)

故

$$T \text{ 之 upper bound} = \left(\frac{R}{\rho}\right)^2 = \frac{R^2}{\rho^2} = \frac{m+1}{\frac{0.25}{0.25+d}}$$

$$= 4(d+0.25)(m+1)$$

$$= (4d+1)(m+1) \quad \#$$

Q5.

(b) Consider binary PLA

$$w_{t+1} \leftarrow w_t + y_{n(t)} x_{n(t)}$$

考慮兩種 mistake

① 若 $y_{n(t)} = -1 \ne \text{sign}(w_t^T x_{n(t)})$，則 $w_{t+1} \leftarrow w_t - x_{n(t)}$

而 Multi-Class PLA 會 $\begin{cases} w_1^{(t+1)} \leftarrow w_1^{(t)} + x_{n(t)} \\ w_2^{(t+1)} \leftarrow w_2^{(t)} - x_{n(t)} \end{cases}$

② 若 $y_{n(t)} = 1 \ne \text{sign}(w_t^T x_{n(t)})$，則 $w_{t+1} \leftarrow w_t + x_{n(t)}$

而 Multi-Class PLA 會 $\begin{cases} w_1^{(t+1)} \leftarrow w_1^{(t)} - x_{n(t)} \\ w_2^{(t+1)} \leftarrow w_2^{(t)} + x_{n(t)} \end{cases}$

即 $w_T = \sum_{t=0}^{T-1} y_{n(t)} x_{n(t)}$

$w_1^{(T)} = \sum_{t=0}^{T-1} (-y_{n(t)} x_{n(t)})$

$w_2^{(T)} = \sum_{t=0}^{T-1} (y_{n(t)} x_{n(t)})$

故 $w_{PLA} = -w_1^* = w_2^*$

Q6.

(d)

(a) Train 的過程並無任何向外界的主動探詢，不符 X

(b) 訓練過程中並無明顯的 feedback of goodness 來調整訓練參數，X

(c) 分類結果不為有限數量類別，不符 X

(e) 訓練之輸入為預先錄製的完整資料集，不符 X

(d) 訓練之產出為自我定義的 label，且可結合後續工程，產生更具物理/實際意義之資料，符合 ✓

Q7.

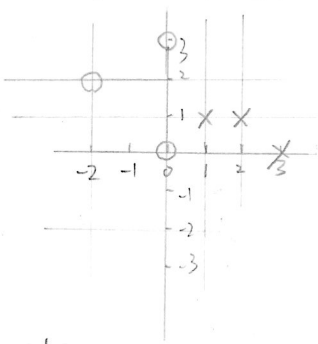
(c) 由於訓練時的input，總數(1126556+1126)筆的資料中，僅有1126筆有target multi-label (事先由專家上好的，label有總數上限，每一筆data有不等數量的label) 而分類目標也為 multi-label 每個Articile 會被分類到複數個tag

故屬於 semi-superised learning 和 multi-label classification。

透過刪去法，僅有(c)符合。

Q8.

(b) 採用 Human-Learning:



○ = +1
× = −1

若抽以 (0,3)，(0,0)，(1,1) 三點進行 PLA，

則可能得出使兩label完美分割的 w_f

例 $w_f^T=[-1,0,0.5]$，其中 w_{f2}^T 是對應常數項

以 (0,3) 為例 $sign(w_f^T \cdot \begin{bmatrix}0\\3\\1\end{bmatrix}) = sign(0.5) = +1$

而若抽選到都是相同label(此處以+1為例)，則可能得到

如 $W^T=[1,0,-3]$，其中 w_2^T 對應常數項

對 +1 label 會全為正確，但對 Out-of-test 的 −1 側而言則是全錯

故 $(\min E_{ots}, \max E_{ots}) = (\frac{1}{3}\times 0, \frac{1}{3}\times 3) = (0,1)$

Q9.

(c)

(a) $E[\hat{\theta}] = E\left[\frac{1}{N}\sum_{n=1}^{N}[\![h(x_n)\neq y_n]\!]\right]$

$= \frac{1}{N}\sum_{n=1}^{N} E\left([\![h(x_n)\neq y_n]\!]\right) = E\left([\![h(x_n)\neq y_n]\!]\right) = \theta$

(∵ P is fixed)

(b) $E[\hat{\theta}] = \frac{1}{N}\sum_{n=1}^{N} E[x_n] = \frac{1}{N}\sum_{n=1}^{N}\left(0\times(1-\theta)+1\times(\theta)\right)$

$= \frac{1}{N}\sum_{n=1}^{N}(\theta) = \theta$

(d) $E[\hat{\theta}] = \frac{1}{N}\sum_{n=1}^{N} E[x_n^2]$ (∵ $Var[x] = E[x_n^2] - (E[x_n])^2$)

$= \frac{1}{N}\sum_{n=1}^{n}\left(Var[x] + (E[x_n])^2\right)$

$= \frac{1}{N}\sum_{n=1}^{n}\left(\theta + (0^2)\right)$ (∵ zero-mean)

$= \theta$

(c) $P(\hat{\theta}\leq x) = P(X_1\leq x, X_2\leq x, \ldots X_N\leq x) = \prod_{i=1}^{n} P(X_i\leq x) = \left(\frac{x}{m}\right)^n$

$PDF(x) = P'(\hat{\theta}\leq x) = \frac{n x^{n-1}}{m^n}$

$E(\hat{\theta}) = \frac{1}{m^n}\int_0^m x\cdot n x^{n-1}\,dx$

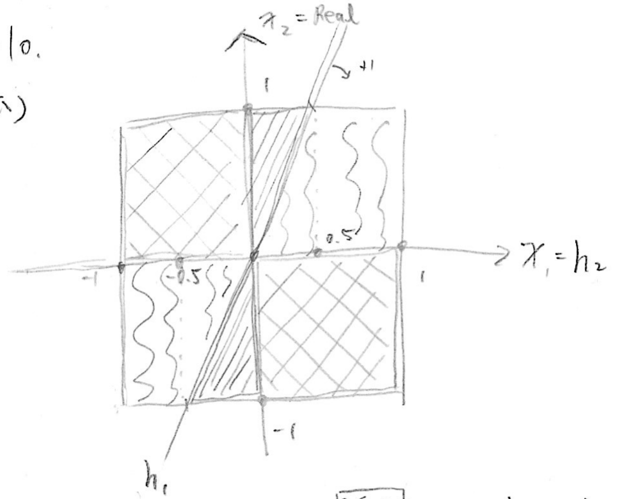
$= \frac{1}{m^n}\int_0^m n x^n\,dx = \frac{1}{m^n}\cdot\frac{n}{n+1}\cdot m^{n+1}$

$= \frac{n}{n+1}\cdot m \neq m = \theta$

#

Q10.

(a)



$\boxed{/\!/}$: $\begin{cases} h_1(x) \neq f(x) \\ h_2(x) = f(x) \end{cases}$ $\boxed{\S\S}$: $h_1(x) = h_2(x) = f(x)$

$\boxed{\times}$: $\begin{cases} h_1(x) = f(x) \\ h_2(x) \neq f(x) \end{cases}$, $\boxed{/\!/} + \boxed{\times} + \boxed{\S\S} = [-1,1]\times[-1,1]$

故 $E_{out}(h_1) = \dfrac{\boxed{/\!/}}{2\times 2} = \dfrac{(1\times 0.5/2)\times 2}{2\times 2} = \dfrac{1}{8}$

$E_{out}(h_2) = \dfrac{\boxed{\times}}{2\times 2} = \dfrac{(1\times 1)\times 2}{2\times 2} = \dfrac{1}{2}$

Q11 根據Q10圖之結果得

(b)

$h_1(x)=f(x)$	$h_2(x)=f(x)$	
T	T	$\dfrac{\boxed{\S\S}}{2\times 2} = \dfrac{1.5}{4} = \dfrac{3}{8}$
T	F	$\dfrac{\boxed{\times}}{2\times 2} = \dfrac{2}{4} = \dfrac{4}{8}$
F	T	$\dfrac{\boxed{/\!/}}{2\times 2} = \dfrac{0.5}{4} = \dfrac{1}{8}$
F	F	$\dfrac{0}{2\times 2} = 0$

若 $E_{in}(h_2) = E_{in}(h_1)$，則僅有 以下三種組合(單組合內順序不拘)

h_1: T T T T, h_2: T T T T ; $E_{in}(h_1)=E_{in}(h_2)=1$

h_1: T T T F, h_2: F T T T ; $E_{in}(h_1)=E_{in}(h_2)=\frac{3}{4}$

h_1: T T F F, h_2: F F T T ; $E_{in}(h_1)=E_{in}(h_2)=\frac{1}{2}$

故 $\left(\frac{3}{8}\right)^4 \times \frac{4!}{4!} + \frac{4}{8}\times\left(\frac{3}{8}\right)^2\times\frac{1}{8}\times\frac{4!}{2!} + \left(\frac{4}{8}\right)^2\times\left(\frac{1}{8}\right)^2\times\frac{4!}{2!\times 2!} = \frac{81}{4096} + \frac{36\times 12}{4096} + \frac{16\times 6}{4096} = \frac{609}{4096}$ ※

Q12.

(b)

	1	2	3	4	5	6
A		G		G		G
B	G	G				G
C						G
D		G	G		G	

1 types only : $(5) \times (A),(B),(C),(D)$ $\left(\frac{5!}{5!}\right)\times 4 = 4$

2 types : $\begin{matrix}(4,1)\\(3,2)\end{matrix} \times \begin{matrix}(A,B),(A,C),(A,D),\\(B,C),(B,D)\end{matrix}$ $\left(\frac{5!}{4!}\times 2 + \frac{5!}{3!2!}\times 2\right)\times 5 = 150$

3 types : $\begin{matrix}(3,1,1)\\(2,2,1)\end{matrix} \times \begin{matrix}(A,B,D),\\(A,B,C)\end{matrix}$ $\left(\frac{5!}{3!}\times\frac{3!}{2!} + \frac{5!}{2!2!}\times\frac{3!}{2!}\right)\times 2 = 300$

∴ The probability that we got some number that is purely green

is : $\left(\frac{1}{4}\right)^5 \times (4+150+300) = \frac{454}{1024}$ ※