2.3, 2.4

HW #1

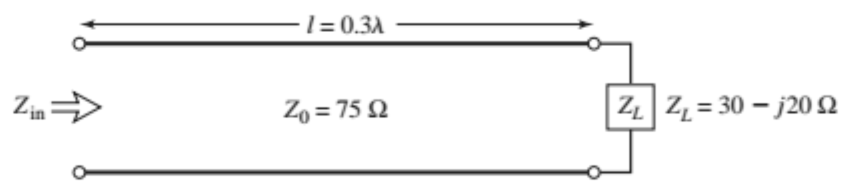
ECE 457, Spring 2024, Due Jan 29th 2024 5pm

2.2 A transmission line has the following per-unit-length parameters: $L = 0.5\ \mu$H/m, $C = 200$ pF/m, $R = 4.0\ \Omega$/m, and $G = 0.02$ S/m. Calculate the propagation constant and characteristic impedance of this line at 800 MHz. If the line is 30 cm long, what is the attenuation in dB? Recalculate these quantities in the absence of loss ($R = G = 0$).

2.3 RG-402U semirigid coaxial cable has an inner conductor diameter of 0.91 mm and a dielectric diameter (equal to the inner diameter of the outer conductor) of 3.02 mm. Both conductors are copper, and the dielectric material is Teflon. Compute the R, L, G, and C parameters of this line at 1 GHz, and use these results to find the characteristic impedance and attenuation of the line at 1 GHz. Compare your results to the manufacturer's specifications of 50 Ω and 0.43 dB/m, and discuss reasons for the difference.

2.4 Compute and plot the attenuation of the coaxial line of Problem 2.3, in dB/m, over a frequency range of 1 MHz to 100 GHz. Use log-log graph paper.

2.8 A lossless transmission line of electrical length $\ell = 0.3\lambda$ is terminated with a complex load impedance as shown in the accompanying figure. Find the reflection coefficient at the load, the SWR on the line, the reflection coefficient at the input of the line, and the input impedance to the line.



2.10 A terminated transmission line with $Z_0 = 60\ \Omega$ has a reflection coefficient at the load of $\Gamma = 0.4\angle 60°$. (a) What is the load impedance? (b) What is the reflection coefficient 0.3λ away from the load? (c) What is the input impedance at this point?

2.11 A 100 Ω transmission line has an effective dielectric constant of 1.65. Find the shortest open-circuited length of this line that appears at its input as a capacitor of 5 pF at 2.5 GHz. Repeat for an inductance of 5 nH.

2.12 A lossless transmission line is terminated with a 100 Ω load. If the SWR on the line is 1.5, find the two possible values for the characteristic impedance of the line.

2.16 The transmission line circuit in the accompanying figure has $V_g = 15$ V rms, $Z_g = 75\ \Omega$, $Z_0 = 75\ \Omega$, $Z_L = 60 - j40\ \Omega$, and $\ell = 0.7\lambda$. Compute the power delivered to the load using three different techniques:

(a) Find Γ and compute

$$P_L = \left(\frac{V_g}{2}\right)^2 \frac{1}{Z_0}(1 - |\Gamma|^2);$$

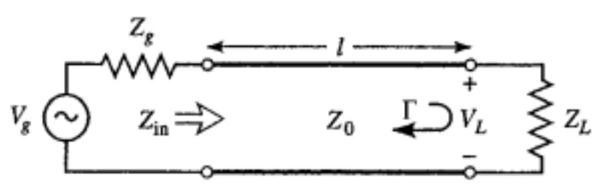
(b) find Z_{in} and compute

$$P_L = \left|\frac{V_g}{Z_g + Z_{\text{in}}}\right|^2 \text{Re}\{Z_{\text{in}}\};$$

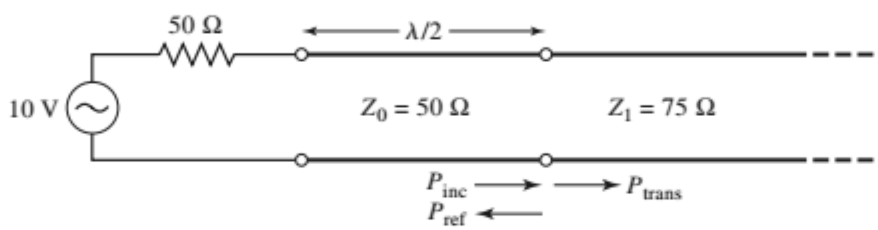
(c) find V_L and compute

$$P_L = \left|\frac{V_L}{Z_L}\right|^2 \text{Re}\{Z_L\}.$$

Discuss the rationale for each of these methods. Which of these methods can be used if the line is not lossless?



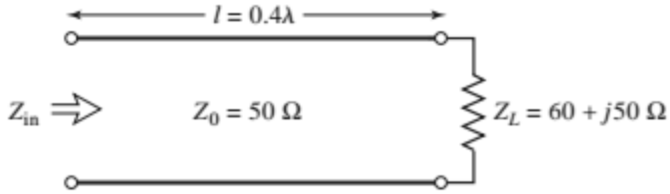
2.18 Consider the transmission line circuit shown in the accompanying figure. Compute the incident power, the reflected power, and the power transmitted into the infinite 75 Ω line. Show that power conservation is satisfied.



2.20 Use the Smith chart to find the following quantities for the transmission line circuit shown in the accompanying figure:

(a) The SWR on the line.
(b) The reflection coefficient at the load.
(c) The load admittance.
(d) The input impedance of the line.
(e) The distance from the load to the first voltage minimum.

(f) The distance from the load to the first voltage maximum.



2.2 A transmission line has the following per-unit-length parameters: $L = 0.5\ \mu$H/m, $C = 200$ pF/m, $R = 4.0\ \Omega$/m, and $G = 0.02$ S/m. Calculate the propagation constant and characteristic impedance of this line at 800 MHz. If the line is 30 cm long, what is the attenuation in dB? Recalculate these quantities in the absence of loss ($R = G = 0$).

Propagation Constant

$$\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{RG + j\omega CR + j\omega LG - \omega^2 LC} = \sqrt{(RG - \omega^2 LC) + j(\omega LG + \omega CR)}$$

$$\gamma = \sqrt{(4 + j(2\pi)(8\times10^8)(0.5\cdot10^{-6}))(0.02 + j(2\pi)(8\times10^8)(200\cdot10^{-12}))}$$

$$\gamma = \sqrt{(4 + j(2\pi)(400))(0.02 + j(2\pi)(16\cdot10^{-2}))}$$

$$\gamma = \sqrt{(4 + j(800\pi))(0.02 + j(32\pi\cdot10^{-2}))}$$

$$\gamma = \sqrt{-2526.5 + 54.287j}$$

$$\gamma = \left(2527.083\, e^{j(3.12)}\right)^{1/2} = 50.27\left(e^{j1.56}\right)$$

$$\gamma = 50.27\cos(1.56) + j\,50.27\sin(1.56)$$

$$\boxed{\gamma = 0.5427 + j50.268 \ \ Np/m}$$

Characteristic Impedance

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{4+j(800\pi)}{0.02+j(32\pi\cdot10^{-2})}}$$

$$\boxed{Z_0 \approx 50.0 + 0.457j\ [\Omega]}$$

if the line is 30cm,

attenuation constant is the real part of γ

$\therefore$ attenuates by factor of $\alpha(0.3m) = 0.5427(0.3) \approx 0.163$ Np

converting to dB:

$$0.163\,\cancel{Np}\cdot\left(\frac{8.686\ dB}{1\,\cancel{Np}}\right)$$

$$\boxed{\text{ratio} \approx 1.4142\,dB}$$

if the line is lossloss ($R = G = 0$)

$$\gamma = j\omega\sqrt{LC} = j\,2\pi\cdot8\cdot10^8\sqrt{0.5\cdot10^{-6}\cdot200\cdot10^{-12}}$$

$$\boxed{\gamma = j50.265 \ \ Np/m}$$

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.5\cdot10^{-6}}{200\cdot10^{-12}}} = \sqrt{\frac{10^6}{400}} = \frac{10^3}{20} = \boxed{50\ \Omega}$$

These values are very close to the lossy line

2.3 RG-402U semirigid coaxial cable has an inner conductor diameter of 0.91 mm and a dielectric diameter (equal to the inner diameter of the outer conductor) of 3.02 mm. Both conductors are copper, and the dielectric material is Teflon. Compute the R, L, G, and C parameters of this line at 1 GHz, and use these results to find the characteristic impedance and attenuation of the line at 1 GHz. Compare your results to the manufacturer's specifications of 50 Ω and 0.43 dB/m, and discuss reasons for the difference.

Skin depth: $\delta_s = \dfrac{1}{\sqrt{\pi f \mu \sigma}}$ $\quad R_s = \dfrac{1}{\sigma \delta_s} = \dfrac{\sqrt{\pi f \mu \sigma}}{\sigma} = \sqrt{\dfrac{2\pi f \mu}{2\sigma}} = \sqrt{\dfrac{\omega \mu}{2\sigma}}$

from online, $\sigma_{copper} = 5.96 \times 10^{7}$ S/m

$\mu_{copper} \propto \mu_0$ (non-ferrous)

From table:
for a coax:

$L = \dfrac{\mu_0}{2\pi} \ln \dfrac{b}{a}$ where $b = \dfrac{3.02}{2}$ mm and $a = \dfrac{0.91\text{mm}}{2}$

$\boxed{L \approx 2.399 \cdot 10^{-7} \text{ H/m}}$

$C = \dfrac{2\pi \varepsilon'}{\ln \frac{b}{a}}$ typically in dielectrics, $\varepsilon' >> \varepsilon''$

$\therefore \varepsilon' \approx \varepsilon_0 \varepsilon_r \approx 2.02 \varepsilon_0 \frac{F}{m}$

$\boxed{C \approx 9.368 \times 10^{-11} \frac{F}{m}}$

$R = \dfrac{R_s}{2\pi}\left(\dfrac{1}{a} + \dfrac{1}{b}\right)$

$\boxed{R \approx 3.705 \ \Omega/m}$

$G = \dfrac{2\pi \omega \varepsilon''}{\ln b/a}$ where $\tan(\delta) = \dfrac{\varepsilon''}{\varepsilon'}$

$\therefore \varepsilon'' = \varepsilon' \tan(\delta)$

$\varepsilon'' = 2.02\varepsilon_0(0.00022)$

$\boxed{G \approx 1.295 \times 10^{-4} \text{ S/m}}$

Char. impedance: $\boxed{Z_0 = \sqrt{\dfrac{L}{C}} \approx 50.605 \ \Omega}$ b/c highfrequency

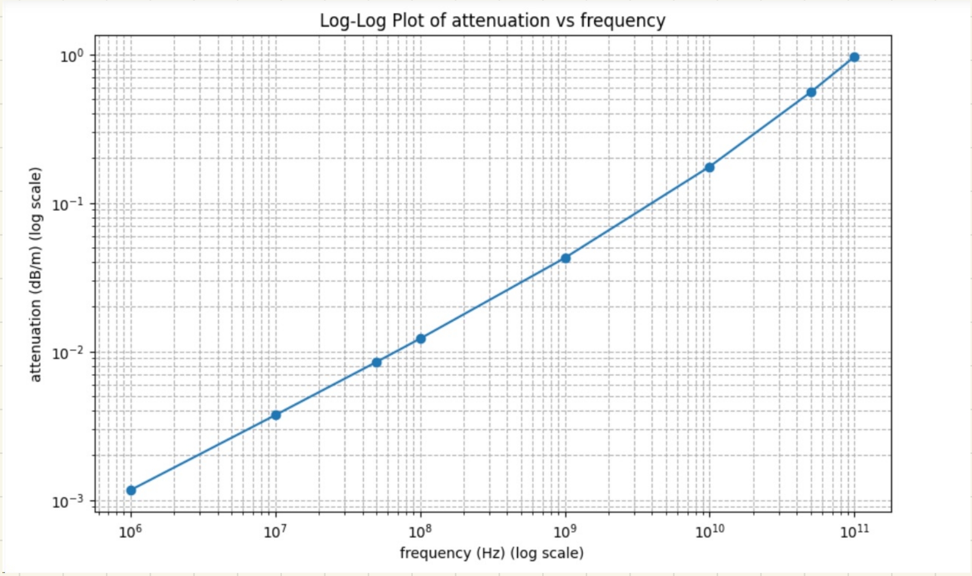
attenuation $\approx \dfrac{1}{2}\left(\dfrac{R}{Z_0} + GZ_0\right)$

$\boxed{\alpha \approx 0.0426 \text{ dB/m}}$

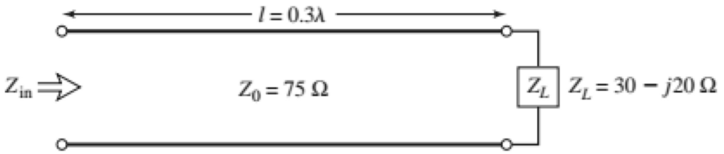
The attenuation and char. impedance are very close, differences arise from my values of σ and ε_r and loss tangent

2.4 Compute and plot the attenuation of the coaxial line of Problem 2.3, in dB/m, over a frequency range of 1 MHz to 100 GHz. Use log-log graph paper.

f	R_s	R	G	α
1MHz	0.000257	0.1172	2.354e-7	0.00116
10MHz	0.00081387	0.37	2.354e-6	0.00372
50MHz	0.00182	0.828	1.177e-5	0.00848
100MHz	0.00257	1.172	2.354e-5	0.01217
1 GHz	0.00814	3.705	2.354e-4	0.04256
10 GHz	0.0257	11.715	2.354e-3	0.17533
50 GHz	0.05755	26.196	1.177e-2	0.55670
100 GHz	0.0814	37.047	2.354e-2	0.96179



2.8 A lossless transmission line of electrical length $\ell = 0.3\lambda$ is terminated with a complex load impedance as shown in the accompanying figure. Find the reflection coefficient at the load, the SWR on the line, the reflection coefficient at the input of the line, and the input impedance to the line.



$Z_L = 30 - j20\,\Omega$

normalize $\rightarrow$ $z_L = \frac{Z_L}{75\Omega} = \frac{2}{5} - \frac{4}{15}j$

From smith chart $|\Gamma_L| \approx 0.46$

$\angle\Gamma_L \approx -144°$

$\therefore \boxed{\Gamma_L = 0.46\, e^{j(-144°)}}$

From smith chart, $\boxed{SWR \approx 2.6}$

From smith chart, $\boxed{\Gamma_G \approx 0.46\, e^{j(-1°)}}$

From smith chart, $z_{in} = 2.6 - 0.01j$

$Z_{in} = (2.6 - 0.01j)75$

$\boxed{Z_{in} = 195 - 0.75j\,\Omega}$

2.10 A terminated transmission line with $Z_0 = 60\ \Omega$ has a reflection coefficient at the load of $\Gamma = 0.4\angle 60°$. (a) What is the load impedance? (b) What is the reflection coefficient 0.3λ away from the load? (c) What is the input impedance at this point?

$Z_0 = 60\Omega \qquad \Gamma_L = 0.4\, e^{j(60°)}$

a) From Smith Chart, $z_L = 1.1 + 0.95j$

$$Z_L = (1.1 + 0.95j)(60\Omega)$$
$$\boxed{Z_L = 66 + 57j\ \Omega}$$

b) From Smith Chart, $\boxed{\Gamma_{0.3\lambda \text{ from load}} = 0.4\, e^{j(-155°)}}$

c) From Smith Chart, $z_{0.3\lambda \text{ from load}} = 0.44 - 0.16j$

$$Z_{0.3\lambda \text{ from load}} = (0.44 - 0.16j)(60)$$
$$\boxed{Z_{0.3\lambda \text{ from load}} = 26.4 - 9.6j\ \Omega}$$

2.11 A 100 Ω transmission line has an effective dielectric constant of 1.65. Find the shortest open-circuited length of this line that appears at its input as a capacitor of 5 pF at 2.5 GHz. Repeat for an inductance of 5 nH.

$Z_0 = 100\Omega$

For open circuit line, $Y_{in} = jY_0 \tan\beta l$ where $\beta = \frac{2\pi}{\lambda}$

$\therefore Z_{in} = \frac{1}{Y_{in}} = -jZ_0 \cot\beta l$

load of a 5pF capacitor @ 2.5 GHz = $Z = \frac{1}{j\omega C}$

$= -j\left(\frac{1}{2.5\cdot10^{-12}\cdot 2\pi\cdot 2.5\cdot10^{9}}\right)$

$= -j\left(\frac{1}{12.5\pi\cdot10^{-3}}\right)$

$= -j(12.732)$

$= -12.732j$

$Z_{in} = \text{load}$

$\cancel{-j}Z_0 \cot(\beta l) = \cancel{-j}(12.732)$

$\cot(\beta l) = \frac{12.732}{100}$

$\beta l = \cot^{-1}\left(\frac{12.732}{100}\right)$

$= \tan^{-1}\left(\frac{100}{12.732}\right)$

$\beta l = 82.744^\circ$

where $\beta = \frac{2\pi}{\lambda}$ $\lambda = \frac{v}{f}$ where $v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{\sqrt{\varepsilon_r}}$

$\beta = \frac{2\pi\cdot f}{v} = \frac{2\pi f\cdot\sqrt{\varepsilon_r}}{c} = 67.2575$

$l = \frac{82.744^\circ\cdot\frac{\pi}{180^\circ}}{67.2575}$

$\boxed{l \approx 0.02147\text{ m or } 2.147\text{ cm}}$

if inductive load => $Z_{in} = j\omega L = j(2\pi)(2.5\cdot\cancel{10^9})(5\cdot\cancel{10^{-9}})$

$= j(25\pi)$

$\cancel{j}25\pi = -\cancel{j}Z_0 \cot\beta l$

$\cot\beta l = \frac{25\pi}{-Z_0}$

$\tan\beta l = \frac{-100}{25\pi}$

$\beta l = \arctan\left(\frac{-4}{\pi}\right)$

$\rightarrow l = \frac{\arctan\left(\frac{-4}{\pi}\right)+\pi}{\beta}$

$\boxed{l \approx 0.03325\text{ m or } 3.325\text{ cm}}$

2.12 A lossless transmission line is terminated with a 100 Ω load. If the SWR on the line is 1.5, find the two possible values for the characteristic impedance of the line.

$Z_L = 100\,\Omega$ $\quad$ SWR = 1.5

$$SWR = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$1.5 = \frac{1+|\Gamma|}{1-|\Gamma|}$$ where off the Smith Chart, SWR = 1.5 $\Rightarrow |\Gamma| \approx 0.2$

confirmed by this ✓

$$|\Gamma| = 0.2 = \left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|$$

$$0.2 = \left|\frac{100 - Z_0}{100 + Z_0}\right|$$

removing absolute value $\Rightarrow$ $\frac{100-Z_0}{100+Z_0} = -0.2$ $\qquad$ $\frac{100-Z_0}{100+Z_0} = 0.2$

$100 - Z_0 = -20 - 0.2Z_0$ $\qquad$ $100 - Z_0 = 20 + 0.2Z_0$

$120 = 0.8Z_0$ $\qquad$ $120 = 1.2Z_0$

$Z_0 = 120 \cdot \frac{5}{4}$ $\qquad$ $\boxed{Z_0 = 100\,\Omega}$

$\boxed{Z_0 = 150\,\Omega}$

2.16 The transmission line circuit in the accompanying figure has $V_g = 15$ V rms, $Z_g = 75\ \Omega$, $Z_0 = 75\ \Omega$, $Z_L = 60 - j40\ \Omega$, and $\ell = 0.7\lambda$. Compute the power delivered to the load using three different techniques:

(a) Find Γ and compute

$$P_L = \left(\frac{V_g}{2}\right)^2 \frac{1}{Z_0}(1 - |\Gamma|^2);$$

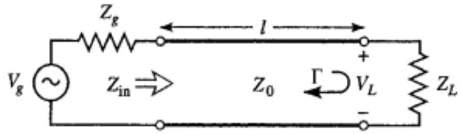
(b) find Z_{in} and compute

$$P_L = \left|\frac{V_g}{Z_g + Z_{in}}\right|^2 \mathrm{Re}\{Z_{in}\};$$

(c) find V_L and compute

$$P_L = \left|\frac{V_L}{Z_L}\right|^2 \mathrm{Re}\{Z_L\}.$$

Discuss the rationale for each of these methods. Which of these methods can be used if the line is not lossless?



$V_g = 15V$ RMS $\quad Z_g = 75\Omega \quad Z_0 = 75\Omega \quad Z_L = 60 - j40\Omega \quad \ell = 0.7\lambda$

a) $\Gamma_g = \frac{Z_g - Z_0}{Z_g + Z_0} = \frac{75-75}{150} = 0$ no reflection at generator

$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{-15-j40}{135-j40}$

$|\Gamma_L| = 0.303$

$\hookrightarrow P_L = \left(\frac{V_g}{2}\right)^2 \frac{1}{Z_0}(1 - |\Gamma|^2)$

$\boxed{P_L = 0.681\ W}$

b) $Z_{in} = ?$

line is 0.7λ long (can reduce down to 0.2λ)

$Z_L = 60 - 40j \quad Z_0 = 75\Omega$

$z_L = \frac{4}{5} - \frac{8}{15}j$

from Smith Chart, $Z_{in} = (0.65 + 0.37j)\,75$

$= 48.75 + 27.75j$

$\mathrm{Re}\{Z_{in}\} = 48.75 \rightarrow P_L = \left|\frac{V_g}{Z_g + Z_{in}}\right|^2 \mathrm{Re}\{Z_{in}\} = \boxed{0.682\ [W]}$

c) $V_L = ?$

$$V_L = V_+ e^{j\beta d} + V_- e^{-j\beta d} \quad \text{where } V_- = V_+ \Gamma_L$$

$$V_L\big|_{d=0} = V_+(1+\Gamma) \qquad V_+ = \frac{V_g}{2} \quad \text{since impedance from gen. is matched} \therefore I_g = \frac{Z_0}{Z_g + Z_0} = \frac{1}{2}$$

$$V_L = 7.5(1+\Gamma) \quad \text{where } \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \approx -0.021 - 0.3j$$

$$V_L = 7.339 - 2.27j$$

$$P_L = \left|\frac{V_L}{Z_L}\right|^2 Re\{Z_L\}$$

$$\boxed{P_L \approx 0.681 [W]}$$

a's method is to find the transmitted power via the transmittance $1 - |\Gamma|^2$ where $P_L = P_{available} \cdot \underbrace{(1-|\Gamma|^2)}_{\text{transmittance}}$

This method applies only to lossless lines since there is an attenuation term inside the waves which carries over to the reflection coefficient.

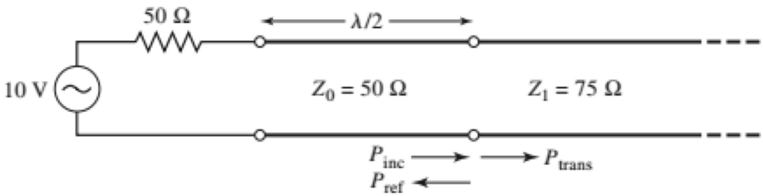
b's method is to find the input impedance, treat that impedance as a load and find power dissipated over it.

This applies only to lossless lines because the length of the line will have a significant impact on the power transmitted to the load.

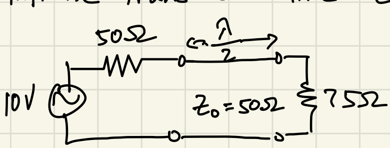
c's method is to find V_L which is the sum of the forward and backwards voltage waves at $d = 0$. Then calculate power dissipated across the load via $|I|^2 R$.

This works for losey lines because it will account for the loss inside the V_L calculation.

2.18 Consider the transmission line circuit shown in the accompanying figure. Compute the incident power, the reflected power, and the power transmitted into the infinite 75 Ω line. Show that power conservation is satisfied.

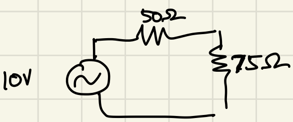


infinite transmission line can be viewed as load.



half wave transformer does "nothing", i.e. $Z_{in} = Z_L$

↳



However $V = \frac{V_g}{2}$ since $Z_g = Z_0$ of the half wave transformer

Power available $= \frac{1}{2}|I|^2 Z_{tot} = \frac{1}{2}\left(\frac{10}{125}\right)^2 (125) = \frac{1}{2}\left(\frac{100}{125}\right) = \frac{2}{5}[W]$
$= 0.4[W]$

Power into 75Ω line $= \frac{1}{2}\left(\frac{10}{125}\right)^2 (75) = \frac{1}{2}\left(\frac{2}{25}\right)^2 (75) = \frac{1}{2}\left(\frac{4}{25\cdot\cancel{25}}\right)(\cancel{25}\cdot 3)$
$= \frac{6}{25}$
$= 0.24[W]$

Power incident $= \frac{1}{2}\frac{|V^+|^2}{Z_0} = \frac{1}{2}\frac{1}{50}(5)^2 = \frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{4} = 0.25[W]$

Power reflected $= \frac{1}{2}\frac{|V^-|^2}{Z_0} = \frac{1}{2}(|\Gamma_L|^2)\frac{|V^+|^2}{Z_0} = \frac{1}{2}\left(\frac{1}{25}\right)\left(\frac{25}{50}\right) = \frac{1}{4}\left(\frac{1}{25}\right)$
$= 0.01[W]$

where $\Gamma_L = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{75-50}{125} = \frac{1}{5}$

Power lost in the 50Ω load $= \frac{1}{2}(50)\left(\frac{10}{125}\right)^2 = \cancel{25}\cdot\frac{100}{5\cdot\cancel{25}\cdot 125} = \frac{100}{625} = \frac{4}{25}$
$= 0.16[W]$

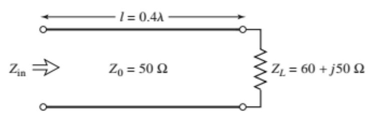
we see that Power available = Power into the 75Ω line + Power lost in the 50Ω load.

AND Power incident = Power into the 75Ω line + Power reflected.

2.20 Use the Smith chart to find the following quantities for the transmission line circuit shown in the accompanying figure:

(a) The SWR on the line.
(b) The reflection coefficient at the load.
(c) The load admittance.
(d) The input impedance of the line.
(e) The distance from the load to the first voltage minimum.

(f) The distance from the load to the first voltage maximum.



a) $Z_L = 60 + j50\,\Omega$ $\quad z_L = \frac{60+j50}{50} = \frac{6}{5} + j$

From Smith Chart, $\boxed{SWR \approx 2.4}$

b) From Smith Chart, $\boxed{\Gamma_L = 0.42\, e^{j(54^\circ)}}$

c) From Smith Chart, $y_L = 0.5 - 0.4j$

$Y_L = (0.5 - 0.4j)\frac{1}{Z_0}$

$= (0.5 - 0.4j)(\frac{1}{50})$

$\boxed{Y_L = 0.01 - 0.008j\ S}$

d) 0.4λ line, $z_{in} = 0.5 + 0.41j$

$Z_{in} = (0.5 + 0.41j)(50)$

$\boxed{Z_{in} = 25 + 20.5j\ \Omega}$

e) first "short" is at $0.076\lambda + 0.25\lambda$

$= \boxed{0.326\lambda \text{ away from load}}$

from the Smith Chart.

f) From Smith chart, distance to the first voltage max

is $\approx \boxed{0.076\lambda \text{ from the load.}}$