LAB 04

1- ${}^{A}p = (3,4)^T$ ${}^{B}p = (-2{,}5,\ 0{,}5)^T$. 30° counterclockwise from ${}^{A}x$ and ${}^{B}y$. a) ${}^{B}A_o = ?$ b) ${}^{A}B_o = ?$ c) ${}^{A}q = ?$ ${}^{B}q = (3,1)^T$

Angle between ${}^{A}x$ and ${}^{B}x$ = $90^\circ - 30^\circ = 60^\circ$

~~Angle between ${}^{B}x$ and ${}^{A}x$ =~~

${}^{A}t_p = (3,4)$ ${}^{A}R_B$ ${}^{B}R_A$ ${}^{p}t_A = (-3,-4)^T$

$${}^{B}R_A = \begin{pmatrix} \cos 60 & -\sin 60 \\ \sin 60 & \cos 60 \end{pmatrix} = \begin{pmatrix} 0{,}5 & -0{,}86 \\ 0{,}86 & 0{,}5 \end{pmatrix} \qquad {}^{p}t_B = (2{,}5,\ -0{,}5)^T$$

a)

$${}^{B}A_o = -\underbrace{{}^{A}p}_{{}^{p}t_A} \cdot {}^{B}R_A + \underbrace{{}^{B}p}_{{}^{B}t_p}$$

$-{}^{p}t_A$ because we want the translat. from p to A

$$\begin{pmatrix} -3 \\ -4 \end{pmatrix} \begin{pmatrix} 0{,}5 & -0{,}86 \\ 0{,}86 & 0{,}5 \end{pmatrix} + \begin{pmatrix} -2{,}5 \\ 0{,}5 \end{pmatrix} = \begin{pmatrix} -4{,}08 \\ \cancel{-5{,}44} \\ -5{,}44 \end{pmatrix} + \begin{pmatrix} -2{,}5 \\ 0{,}5 \end{pmatrix} = \begin{pmatrix} -6{,}58 \\ -4{,}94 \end{pmatrix}$$

$${}^{A}R_B = ({}^{B}R_A)^T = \begin{pmatrix} 0{,}5 & 0{,}86 \\ -0{,}86 & 0{,}5 \end{pmatrix} \qquad \boxed{{}^{B}A_o = \begin{pmatrix} -6{,}58 \\ -4{,}94 \end{pmatrix}}$$

b)

$${}^{A}B_o = {}^{p}t_B \cdot {}^{A}R_B + {}^{A}t_p$$

$$\begin{pmatrix} -0{,}9 \\ -0{,}68 \end{pmatrix}$$

$${}^{A}B_o = \begin{pmatrix} 2{,}5 \\ -0{,}5 \end{pmatrix} \begin{pmatrix} 0{,}5 & 0{,}86 \\ -0{,}86 & 0{,}5 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} \cancel{3{,}4} \\ \cancel{0{,}18} \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} \cancel{6{,}4} \\ \cancel{4{,}18} \end{pmatrix} \rightarrow \begin{pmatrix} 2{,}1 \\ 3{,}32 \end{pmatrix}$$

$$\boxed{{}^{A}B_o = \begin{matrix} 2{,}1 \\ 3{,}32 \end{matrix}}$$

c)

$${}^{A}t_B = {}^{p}t_B \cdot {}^{A}R_B + {}^{A}t_p = {}^{A}B_o$$

$${}^{A}q = {}^{B}q \cdot {}^{A}R_B + {}^{A}t_B$$

$$\begin{pmatrix} 2{,}1 \\ 3{,}32 \end{pmatrix}$$

$${}^{A}t_B = \begin{pmatrix} \cancel{6{,}4} \\ \cancel{4{,}18} \end{pmatrix} \begin{pmatrix} 2{,}1 \\ 3{,}32 \end{pmatrix} \quad {}^{A}q = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \begin{pmatrix} 0{,}5 & 0{,}86 \\ -0{,}86 & 0{,}5 \end{pmatrix} + \begin{pmatrix} \cancel{6{,}4} \\ \cancel{4{,}18} \end{pmatrix} = \cancel{4{,}08} \begin{pmatrix} -1{,}08 \\ 1{,}36 \end{pmatrix} + \begin{pmatrix} 2{,}1 \\ 3{,}32 \end{pmatrix} = \begin{pmatrix} 1{,}02 \\ 4{,}68 \end{pmatrix}$$

$$\boxed{{}^{A}q = \begin{pmatrix} 1{,}02 \\ 4{,}68 \end{pmatrix}}$$

- In this exercise we get the demanded points by applying an affine transformation on the same points seen from another reference frame. To do so we calculate the matrix related to the frame rotation and apply it to the points, adding the corresponding translation to get the new coordinates.

2- ${}^{A}B_o = (3,1,-2)^T$ ${}^{B}C_o = (-3,1,-2)^T$ $A \to B : (\alpha, \beta, \gamma) = (25, 145, 30)^T$

$C \to B$: $\mathring{q} = \frac{1}{7} \cdot (-\sqrt{3} \cdot 3{,}5,\ 3,\ -1,\ -1{,}5)^T$. ${}^{C}v_1 = (0,2,0)^T$ ${}^{C}v_2 = (0,2,5)^T$

a) $\mathring{q} \to$ axis $= (0,0,0)$ angle $= 180^\circ \to {}^{C}R_B = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = {}^{B}R_C$

The translation from C to B is $- {}^{B}C_o \to = (3,-1,2)^T$

$$\boxed{{}^{B}A_{C} = \begin{pmatrix} {}^{C}R_B & {}^{C}t_B \\ 0_{1\times3} & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}}$$

b)

$(\alpha, \beta, \gamma) = (25, 145, 30) \to {}^{A}R_B = \begin{pmatrix} -0{,}71 & -0{,}24 & 0{,}66 \\ -0{,}41 & 0{,}91 & -0{,}11 \\ -0{,}57 & -0{,}35 & -0{,}74 \end{pmatrix}$

${}^{B}R_A = ({}^{A}R_B)^T = \begin{pmatrix} -0{,}71 & -0{,}41 & -0{,}57 \\ -0{,}24 & 0{,}91 & -0{,}35 \\ 0{,}66 & -0{,}11 & -0{,}74 \end{pmatrix}$

$A = \begin{pmatrix} {}^{C}R_A & {}^{C}t_A \\ 0_{1\times3} & 1 \end{pmatrix}$

$V_{F_1} = (R_1 R_2)^T V_{F_2}$

$V_{F_1} = R_2^T R_1^T V_{F_2}$

So ${}^{C}R_A = {}^{B}R_A^T \cdot {}^{C}R_B^T = {}^{A}R_B \cdot {}^{B}R_C$

${}^{C}R_A = \begin{pmatrix} 0{,}71 & 0{,}24 & -0{,}66 \\ 0{,}41 & -0{,}91 & 0{,}11 \\ 0{,}57 & 0{,}35 & 0{,}74 \end{pmatrix} \to {}^{A}R_C = \begin{pmatrix} 0{,}71 & 0{,}41 & 0{,}57 \\ 0{,}24 & -0{,}91 & 0{,}35 \\ -0{,}66 & 0{,}11 & 0{,}74 \end{pmatrix}$

$\cancel{{}^{C}t_A = {}^{C}t_B}$ ${}^{C}t_A = {}^{C}R_B \cdot {}^{C}t_B + ({}^{B}R_A \cdot {}^{B}t_A)$

${}^{C}t_A = {}^{B}R_A\ {}^{C}R_A \cdot {}^{A}t_B + {}^{C}t_B \to$

${}^{C}t_A = {}^{C}R_A \cdot {}^{A}t_B + {}^{C}t_B \to \begin{pmatrix} 0{,}71 & 0{,}24 & -0{,}66 \\ 0{,}41 & -0{,}91 & 0{,}11 \\ 0{,}57 & 0{,}35 & 0{,}74 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} =$

$= \cancel{\begin{pmatrix} -1{,}05 \\ -0{,}54 \end{pmatrix}} \begin{pmatrix} 1{,}05 \\ 0{,}54 \\ 3{,}54 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 4{,}05 \\ -0{,}46 \\ 5{,}54 \end{pmatrix}$

$$\boxed{{}^{C}A_A = \begin{pmatrix} 0{,}71 & 0{,}24 & -0{,}66 & 4{,}05 \\ 0{,}41 & -0{,}91 & 0{,}11 & -0{,}46 \\ 0{,}57 & 0{,}35 & 0{,}74 & 5{,}54 \\ 0 & 0 & 0 & 1 \end{pmatrix}}$$

- The first and second questions of this exercise are solved in a similar way: we get the rotation matrices corresponding to each frame change and use them with the corresponding translations to get the Affine transformations that allow changing from one frame to another.

3- $cc: (1,6,1)^T$ $\beta: -90°$ $\gamma: -20°$ $F_L: \frac{1}{34}m$

$$R_y = \begin{pmatrix} \cos -90 & 0 & \sin -90 \\ 0 & 1 & 0 \\ -\sin -90 & 0 & \cos -90 \end{pmatrix} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$R_z = \begin{pmatrix} \cos -20 & -\sin -20 & 0 \\ \sin -20 & \cos -20 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0{,}94 & 0{,}34 & 0 \\ -0{,}34 & 0{,}94 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$${}^cR_w = R_z \cdot R_y = \begin{pmatrix} 0 & 0{,}34 & -0{,}94 \\ 0 & 0{,}94 & 0{,}34 \\ -1 & 0 & 0 \end{pmatrix}$$ → Rotation from world frame to camera frame

$${}^wt_c = \begin{pmatrix} 1 \\ 6 \\ 1 \end{pmatrix}$$

$${}^cp = {}^cR_w \cdot {}^wp + {}^wt_c$$ → Points in world frame → Points in camera frame

$\cancel{{}^{cp}p = \frac{{}^cp_x}{{}^cp_z}}$ $F_L = \frac{1}{34}m$

$${}^{cp}p = \begin{pmatrix} \dfrac{{}^cp_x}{34 \cdot {}^cp_z} \\ \dfrac{{}^cp_y}{34 \cdot {}^cp_z} \end{pmatrix}$$ → Points in camera frame → Points in camera plane

4- $wc: (4{,}665,\ 3{,}735,\ -0{,}5395)^T$ $\alpha: -170°$ $\vec{u}: (0{,}01, -0{,}2, 1)^T$

$\|\vec{u}\| = 1{,}02 \neq 1$ → $\vec{v} = \frac{\vec{u}}{\|\vec{u}\|}$ $\vec{v}: (0{,}009,\ -0{,}19,\ 0{,}98)$

$${}^cR_w = \begin{pmatrix} -0{,}98 & 0{,}17 & 0{,}05 \\ -0{,}17 & -0{,}91 & -0{,}37 \\ -0{,}02 & -0{,}37 & 0{,}93 \end{pmatrix}$$

${}^ct_w: (4{,}665,\ 3{,}735,\ -0{,}5395)^T$

↓

${}^wt_c: (-4{,}665,\ -3{,}735,\ 0{,}5395)^T$

$${}^cp = {}^cR_w \cdot {}^wp + {}^wt_c$$ → Points in world frame → Points in camera frame

Angle between two vectors (φ)

$$\varphi = \arccos\left(\frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|}\right)$$

$\vec{AB} = (2,804\ ,\ 0,939\ ,\ 1,106)$ $\quad \|\vec{AB}\| = 3,162$

$\vec{CD} = (0,70\ ,\ 2,819\ ,\ 1,250)$ $\quad \|\vec{CD}\| = 3,162$

- Minimum angle:

$$\varphi = \arccos\left(\frac{\vec{AB}\cdot\vec{CD}}{(3,162)^2}\right)$$

$$= \arccos\left(\frac{5,99}{9,99}\right) \qquad \boxed{\varphi = 53,15^\circ}$$

- The angle that the segments form in the image plane:

$$^{cp}\vec{AB} = \begin{pmatrix} 2,54 \\ 0,85 \end{pmatrix}$$

We assume that the focal lenght is 1

$$^{cp}\vec{CD} = \begin{pmatrix} 0,56 \\ 2,25 \end{pmatrix}$$

$\|^{cp}\vec{AB}\| = 2,68$

$\|^{cp}\vec{CD}\| = 2,32$

$$\varphi = \arccos\left(\frac{^{cp}\vec{AB}\cdot{}^{cp}\vec{CD}}{2,68\cdot 2,32}\right)$$

$$= \arccos\left(\frac{3,33}{6,21}\right)$$

$$\boxed{\varphi = 57,57^\circ}$$

3 - $c_c(1,6,1)'$

$w_x \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$w_y \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$w_z \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$C_x = {}^{c}R_w \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 6 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ 6 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \\ 0 \end{pmatrix}$$

$$C_y = \begin{pmatrix} 0,34 \\ 0,94 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 6 \\ 1 \end{pmatrix} = \begin{pmatrix} 1,34 \\ 6,94 \\ 1 \end{pmatrix}$$

$$C_z = \begin{pmatrix} 0,94 \\ -0,34 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 6 \\ 1 \end{pmatrix} = \begin{pmatrix} 1,94 \\ 5,66 \\ 1 \end{pmatrix}$$

$\|C_x\| = 6,08$ $\qquad C_x' = \begin{pmatrix} 0,16 \\ 0,99 \\ 0 \end{pmatrix}$

$\|C_y\| = 7,14$

$$C_y' = \begin{pmatrix} 0,19 \\ 0,97 \\ 0,14 \end{pmatrix}$$

$$C_z' = \begin{pmatrix} 0,32 \\ 0,93 \\ 0,16 \end{pmatrix}$$

$\|C_z\| = 6,07$

4 -

$${}^{c}p = {}^{c}R_w \cdot {}^{w}P + {}^{w}t_c$$

$\vec{AB} = (2,809,\ 0,939,\ 1,706)$

$\vec{CD} = (0,70,\ 2,8791,\ 1,2499)$

$${}^{c}A = \begin{pmatrix} -0,397 \\ -3,15 \\ 2,36 \end{pmatrix} + \begin{pmatrix} -4,665 \\ -3,735 \\ 0,5395 \end{pmatrix} = \begin{pmatrix} -5,062 \\ -6,885 \\ 2,899 \end{pmatrix}$$

$${}^{c}B = \begin{pmatrix} -2,93 \\ -4,89 \\ 2,99 \end{pmatrix} + \begin{pmatrix} -4,665 \\ -3,735 \\ 0,5395 \end{pmatrix} = \begin{pmatrix} -7,595 \\ -8,625 \\ 3,5295 \end{pmatrix}$$

$${}^{c}C = \begin{pmatrix} -1,59 \\ -2,45 \\ 2,62 \end{pmatrix} + \begin{pmatrix} -4,665 \\ -3,735 \\ 0,5395 \end{pmatrix} = \begin{pmatrix} -6,255 \\ -6,185 \\ 3,160 \end{pmatrix}$$

$${}^{c}D = \begin{pmatrix} -1,74 \\ -5,60 \\ 2,73 \end{pmatrix} + \begin{pmatrix} -4,665 \\ -3,735 \\ 0,5395 \end{pmatrix} = \begin{pmatrix} -6,405 \\ -9,335 \\ 3,269 \end{pmatrix}$$

$^{B}C = (-3, 1, -2)$

$^{B}p \to {}^{A}p$ $\qquad$ $^{A}p = {}^{A}R_{B} \cdot {}^{B}p + {}^{B}t_{A}$ $\qquad$ $^{B}p = {}^{B}R_{C} \cdot {}^{C}p + {}^{C}t_{B}$

$$^{A}C_{0} = \begin{pmatrix} 0,57 \\ 2,36 \\ 2,84 \end{pmatrix} + \begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} -2,43 \\ 1,36 \\ 4,84 \end{pmatrix}$$

$$^{C}V_{1} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$$

$$^{B}V_{1} = \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \Rightarrow {}^{B}V_{1} = \begin{pmatrix} 3 \\ -3 \\ 2 \end{pmatrix}$$

$$^{B}V_{2} = \begin{pmatrix} 0 \\ -2 \\ -5 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \Rightarrow {}^{B}V_{2} = \begin{pmatrix} 3 \\ -3 \\ -3 \end{pmatrix}$$

a)

$$^{A}V_{1} = \begin{pmatrix} -0,48 \\ 1,82 \\ -0,7 \end{pmatrix} + \begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix} \Rightarrow {}^{A}V_{1} = \begin{pmatrix} -3,48 \\ 0,82 \\ 1,3 \end{pmatrix}$$

$$^{A}V_{2} = \begin{pmatrix} 2,82 \\ 1,27 \\ -4,1 \end{pmatrix} + \begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix} \Rightarrow {}^{A}V_{2} = \begin{pmatrix} -0,18 \\ 0,27 \\ -2,1 \end{pmatrix}$$

b)

$$^{A}V_{1} = \begin{pmatrix} -0,82 \\ 1,82 \\ -0,22 \end{pmatrix} + \begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix} \Rightarrow {}^{A}V_{1} = \begin{pmatrix} -3,82 \\ 0,82 \\ 2,22 \end{pmatrix}$$

$$^{B}V_{2} = \begin{pmatrix} -3,67 \\ 0,07 \\ -3,42 \end{pmatrix} + \begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix} \Rightarrow {}^{A}V_{2} = \begin{pmatrix} 0,67 \\ -1,07 \\ -1,42 \end{pmatrix}$$

$$ {}^{c}A = \begin{pmatrix} -0,387 \\ -3,23 \\ 2,54 \end{pmatrix} + \begin{pmatrix} -4,665 \\ -3,735 \\ 0,539 \end{pmatrix} = \begin{pmatrix} -5,052 \\ -6,965 \\ 3,079 \end{pmatrix} $$

$$ {}^{c}B = \begin{pmatrix} -2,93 \\ -5,02 \\ 3,23 \end{pmatrix} + \begin{pmatrix} {}^{w}t_{c} \end{pmatrix} = \begin{pmatrix} -7,595 \\ -8,755 \\ 3,769 \end{pmatrix} $$

$$ {}^{c}C = \begin{pmatrix} -1,59 \\ -2,54 \\ 2,83 \end{pmatrix} + \begin{pmatrix} {}^{w}t_{c} \end{pmatrix} = \begin{pmatrix} -6,255 \\ -6,275 \\ 3,369 \end{pmatrix} $$

$$ {}^{c}D = \begin{pmatrix} -1,73 \\ -5,71 \\ 2,95 \end{pmatrix} + \begin{pmatrix} {}^{w}t_{c} \end{pmatrix} = \begin{pmatrix} -6,395 \\ -9,445 \\ 3,489 \end{pmatrix} $$

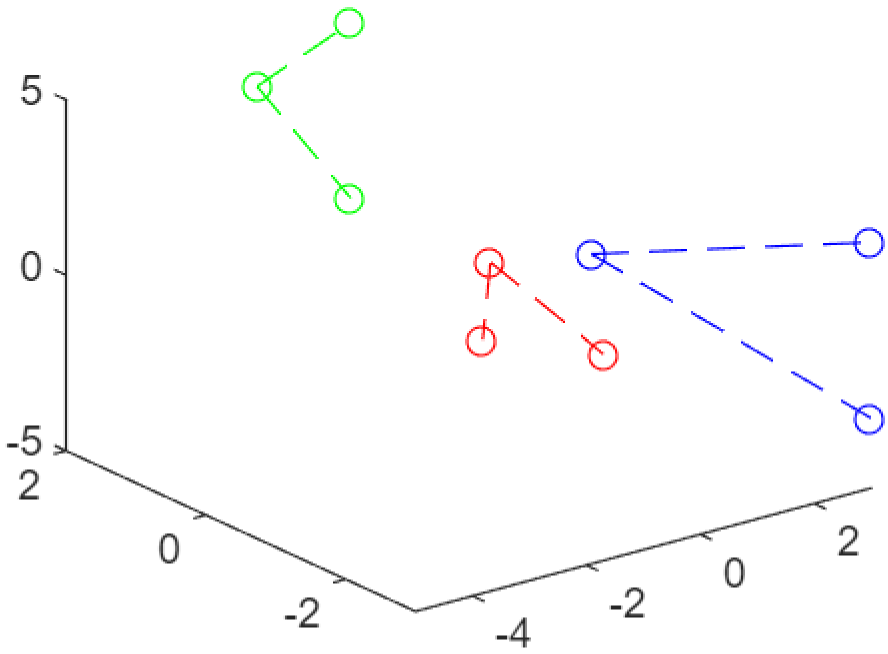
These are exercises 3 and 4.

- In exercise 3 we first get the rotation matrix and translation that allows us to create an affine transformation which can translate coordinates from the world frame to the camera frame. We also get the linear transformation to pass from the camera frame to the image plane of the camera. Finally, we calculate the three vectors that define the camera frame from the world point of view.
- In exercise 4 we get the affine transformation that allows us to pass from the world frame to the camera frame and the linear transformation to get their representation in the camera image plane. Then, we calculate the minimum angle of the segments with the dot product both for the 2D and 3D representations.

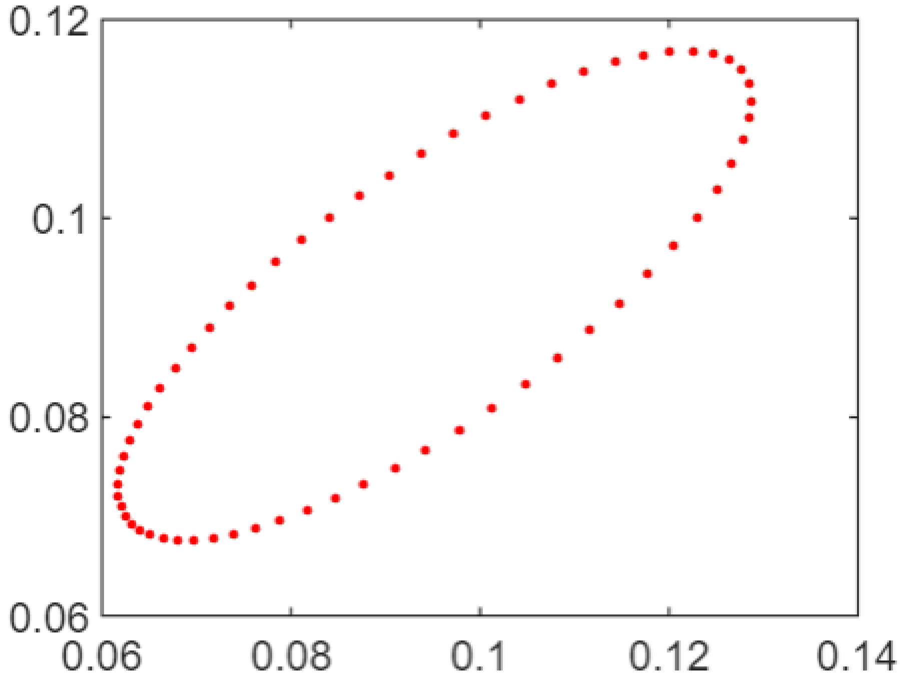
There might be additional operations done in real time inside the programs, these can be found in the scripts that make the graphical representations, added inside the Lab04 folder.

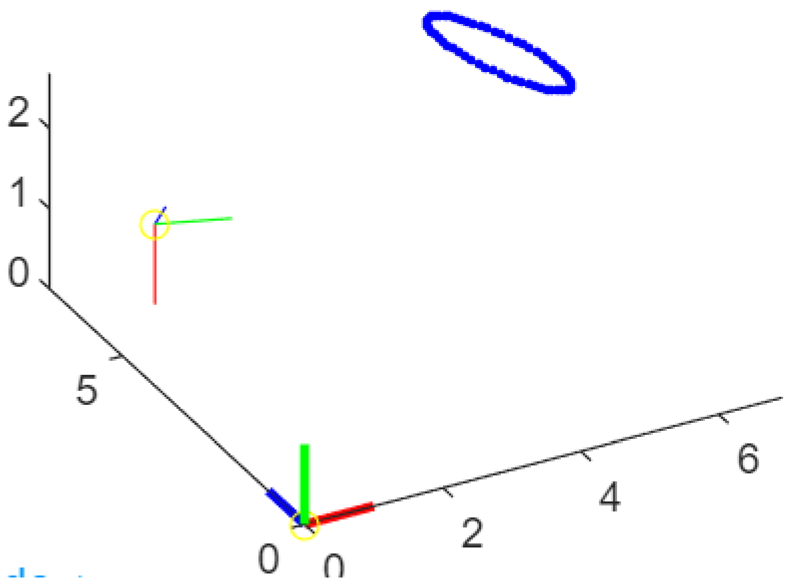
Graphical representations:

Ex 2:



Ex 3:





Ex 4: