



16. 解:

(1)

由题意知, $X \sim N(\mu, \sigma^2)$, 且 $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$, 有,

$$P(\underline{\theta} < \theta < \bar{\theta}) = 1 - \alpha$$

即,

$$P(-z_{\frac{\alpha}{2}} < \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} < z_{\frac{\alpha}{2}}) = 1 - \alpha$$

故,

$$P(\bar{X} - \frac{\sigma}{\sqrt{n}} z_{\frac{\alpha}{2}} < \mu < \bar{X} + z_{\frac{\alpha}{2}}) = 1 - \alpha$$

令 $n = 9, \sigma = 0.6, 1 - \alpha = 0.95$, 得 $\frac{\alpha}{2} = 0.025, z_{0.025} = 1.96, \bar{x} = 6$, 故置信区间为,

$$(6 \pm \frac{0.6}{3} z_{0.025}) = (5.608, 6.392)$$

(2)

由题意, $\frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t(n - 1)$, 故,

$$P(-t_{\frac{\alpha}{2}}(n - 1) < \frac{\bar{X} - \mu}{S/\sqrt{n}} < t_{\frac{\alpha}{2}}(n - 1)) = 1 - \alpha$$

故有,

$$P(\bar{X} - \frac{S}{\sqrt{n}} t_{\frac{\alpha}{2}} < \mu < \bar{X} + \frac{S}{\sqrt{n}} t_{\frac{\alpha}{2}}) = 1 - \alpha$$

令 $n = 9, 1 - \alpha = 0.95, \frac{\alpha}{2} = 0.025, t_{\frac{\alpha}{2}}(8) = 2.306$, 算得 $\bar{x} = 6, s = \sqrt{0.33}$, 故置信区间为,

$$(6 \pm \frac{\sqrt{0.33}}{3} t_{0.025}(8)) = (5.558, 6.442)$$

18. 解:

由题意知, $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$

$$P(\chi_{1-\frac{\alpha}{2}}^2(n-1) < \frac{(n-1)S^2}{\sigma^2} < \chi_{\frac{\alpha}{2}}^2(n-1)) = 1 - \alpha$$

故,

$$P\left(\frac{(n-1)S^2}{\chi_{\frac{\alpha}{2}}^2(n-1)} < \sigma^2 < \frac{(n-1)S^2}{\chi_{1-\frac{\alpha}{2}}^2(n-1)}\right) = 1 - \alpha$$

令 $n = 9, s = 11, 1 - \alpha = 0.95, \frac{\alpha}{2} = 0.025, \chi_{\frac{\alpha}{2}}^2(n-1) = 17.535, \chi_{1-\frac{\alpha}{2}}^2(n-1) = 2.180$, 故,

$$\left(\frac{\sqrt{n-1}S}{\sqrt{\chi_{\frac{\alpha}{2}}^2(n-1)}}, \frac{\sqrt{n-1}S}{\sqrt{\chi_{1-\frac{\alpha}{2}}^2(n-1)}}\right) = (7.4, 21.1)$$

19. 解:

(1)

由题意知, $X_i \sim N(\mu, \sigma^2)$, 故, $\frac{X_i - \mu}{\sigma} \sim N(0, 1)$, 故,

$$\sum_{i=1}^n \left(\frac{X_i - \mu}{\sigma}\right)^2 \sim \chi^2(n)$$

故,

$$P(\chi_{1-\frac{\alpha}{2}}^2 < \sum_{i=1}^n \frac{(X_i - \mu)^2}{\chi_{\frac{\alpha}{2}}^2(n)} < \chi_{\frac{\alpha}{2}}^2(n)) = 1 - \alpha$$

故置信区间为,

$$\left(\frac{\sum_{i=1}^n (X_i - \mu)^2}{\chi_{\frac{\alpha}{2}}^2(n)}, \frac{\sum_{i=1}^n (X_i - \mu)^2}{\chi_{1-\frac{\alpha}{2}}^2(n)}\right)$$