Assignment2

March 3, 2025

1 1.25(b)

$$x(t) = e^{j(\pi t - 1)}$$

1.1 Answer:

Since x(t) is periodic, let T be its fundamental period. Then, we have:

$$x(t) = x(t + T)$$

which implies:

$$e^{j(\pi t - 1)} = e^{j(\pi(t+T) - 1)}$$

Using the periodicity property of the exponential function, we require:

$$\pi t - 1 + 2\pi = \pi(t + T) - 1$$

Solving for T, we get:

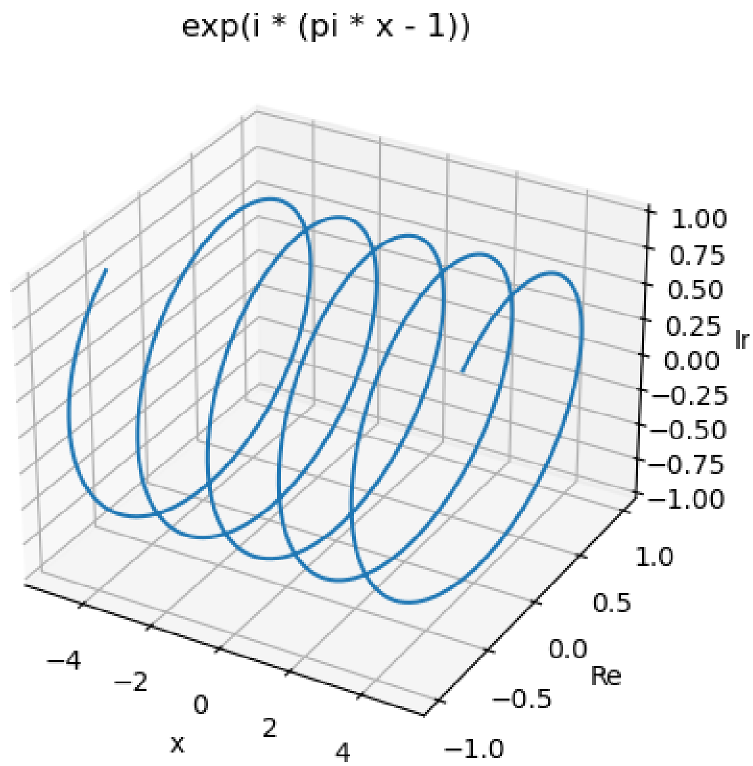
$$\pi T = 2\pi \implies T = 2.$$

Thus, the fundamental period is T = 2.

[51]:
```python
from mpl_toolkits.mplot3d import axes3d
import matplotlib.pyplot as plt
import numpy as np

x = np.linspace(-5, 5, 1000)
f = np.exp(complex(0, 1) * (np.pi * x - 1))
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
ax.plot(x, f.real, f.imag)
ax.set_xlabel('x')
ax.set_ylabel('Re')
ax.set_zlabel('Im')
ax.set_title('exp(i * (pi * x - 1))')
```

```
plt.show()
```



2 1.26(e)

$$x[n] = 2\cos\left(\frac{\pi}{4}n\right) + \sin\left(\frac{\pi}{8}n\right) - 2\cos\left(\frac{\pi}{2}n + \frac{\pi}{6}\right)$$

2.1 Answer:

The overall period is determined by examining each term separately. For the term $2\cos\left(\frac{\pi}{4}n\right)$, the periodicity condition is

$$\frac{\pi}{4}N = 2\pi k \quad \Longrightarrow \quad N = 8k,$$

so the smallest period is 8 (with $k = 1$).

For the term $\sin\left(\frac{\pi}{8}n\right)$, the periodicity condition is

$$\frac{\pi}{8}N = 2\pi k \quad \Longrightarrow \quad N = 16k,$$

so the smallest period is 16 (with $k = 1$).

For the term $-2\cos\left(\frac{\pi}{2}n + \frac{\pi}{6}\right)$, the periodicity condition is

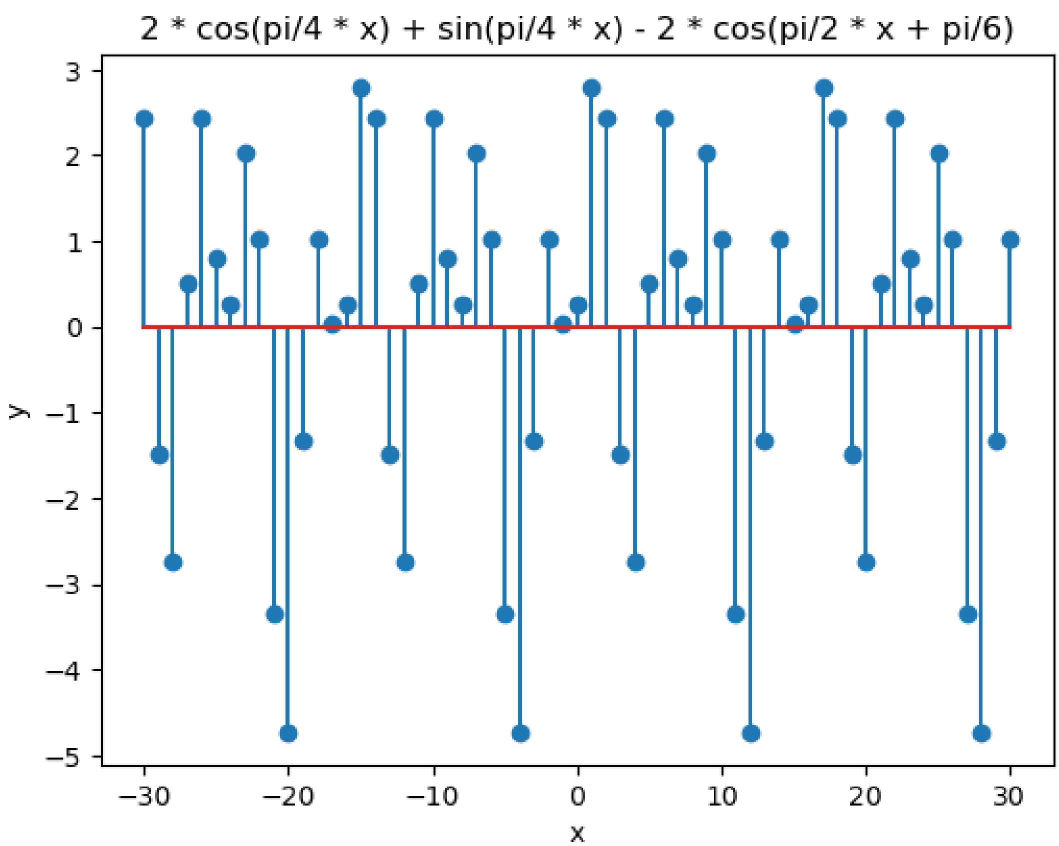
$$\frac{\pi}{2}N = 2\pi k \quad \Longrightarrow \quad N = 4k,$$

so the smallest period is 4 (with $k = 1$).

Thus, the overall fundamental period, being the least common multiple of 8, 16, and 4, is 16.

[52]:
```python
x = np.linspace(-30, 30, 61)
y = 2 * np.cos(np.pi / 4 * x) + np.sin(np.pi / 8 * x) - 2 * np.cos(np.pi / 2 * x + np.pi / 6)
plt.stem(x, y)
plt.xlabel('x')
plt.ylabel('y')
plt.title('2 * cos(pi/4 * x) + sin(pi/4 * x) - 2 * cos(pi/2 * x + pi/6)')
```

[52]:
```
Text(0.5, 1.0, '2 * cos(pi/4 * x) + sin(pi/4 * x) - 2 * cos(pi/2 * x + pi/6)')
```



3 1.21(f)

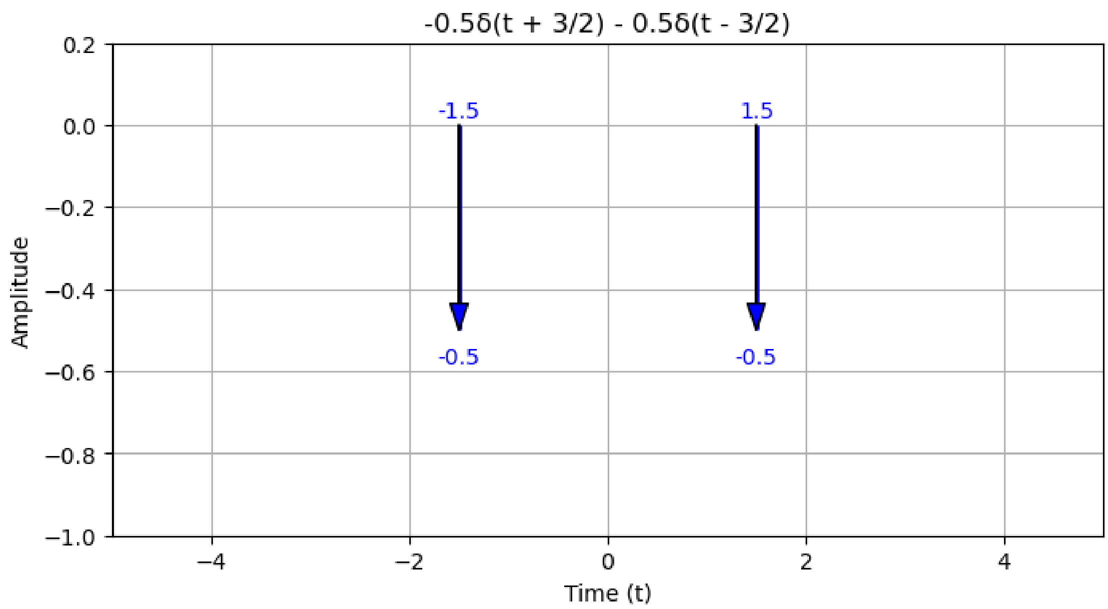
$$x(t)\left[\delta\left(t+\frac{3}{2}\right)-\delta\left(t-\frac{3}{2}\right)\right]$$

3.1 Answer:

$$\begin{aligned} x(t)\left[\delta\left(t+\frac{3}{2}\right)+\delta\left(t-\frac{3}{2}\right)\right] &= x\left(-\frac{3}{2}\right)\delta\left(t+\frac{3}{2}\right)-x\left(\frac{3}{2}\right)\delta\left(t-\frac{3}{2}\right) \\ &= -\frac{1}{2}\delta\left(t+\frac{3}{2}\right)-\frac{1}{2}\delta\left(t-\frac{3}{2}\right) \end{aligned}$$

[53]:
```
plt.figure(figsize=(8, 4))
plt.vlines([-1.5, 1.5], 0, -0.5, colors='b', linewidth=2)

for pos in [-1.5, 1.5]:
    plt.annotate('',
                 xy=(pos, -0.5),
                 xytext=(pos, 0),
                 arrowprops=dict(facecolor='blue', shrink=0.0, width=0.5,␣
 ↪headwidth=8))
    plt.text(pos, -0.54, '-0.5', horizontalalignment='center',␣
 ↪verticalalignment='top', color='blue')
    plt.text(pos, 0.06, pos, horizontalalignment='center',␣
 ↪verticalalignment='top', color='blue')
plt.title('-0.5 (t + 3/2) - 0.5 (t - 3/2)')
plt.xlabel('Time (t)')
plt.ylabel('Amplitude')
plt.xlim(-5, 5)
plt.ylim(-1, 0.2)
plt.grid(True)
plt.show()
```



4 1.22(e)

$$x[n]u[3-n]$$

4.1 Answer:

Since, $u(n) = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$.

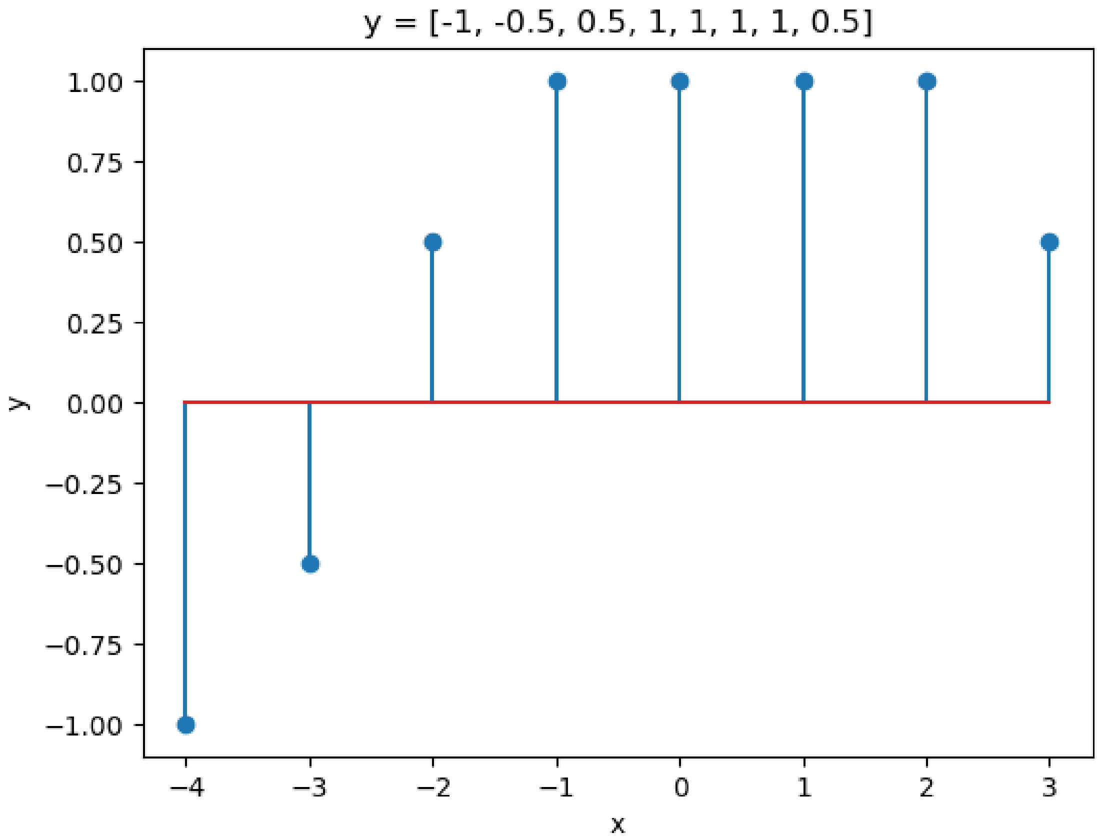
Therefore,

$$x[n]u[3-n] = \begin{cases} x[n] & n \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

[54]:
```
x = np.linspace(-4, 3, 8)
y = np.array([-1, -0.5, 0.5, 1, 1, 1, 1, 0.5])

plt.stem(x, y)
plt.xlabel('x')
plt.ylabel('y')
plt.title('y = [-1, -0.5, 0.5, 1, 1, 1, 1, 0.5]')
```

[54]:
```
Text(0.5, 1.0, 'y = [-1, -0.5, 0.5, 1, 1, 1, 1, 0.5]')
```


y = [-1, -0.5, 0.5, 1, 1, 1, 1, 0.5]
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
y
−4
−3
−2
−1
0
1
2
3
x