9. 解:

(1)

由题意知,

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$$

故,

$$P(\frac{S^2}{\sigma^2} \leq 2.041) = P(\frac{\chi^2(n-1)}{n-1} \leq 2.041) = 1 - P(\chi^2(15) \geq 30.615) \approx 0.99$$

(2)

由 (1) 知,

$$D(\frac{(n-1)S^2}{\sigma^2}) = D(\chi^2(n-1)) = 2(n-1) = 30$$

故,

$$D(S^2) = (\frac{\sigma^2}{n-1})^2 D(\frac{(n-1)S^2}{\sigma^2}) = \frac{2\sigma^4}{15}$$

2. 解:

(2)

$$\mu_1 = E(X) = \int_0^1 xf(x)\mathrm{d}x = \int_0^1 \sqrt{\theta}x^{\sqrt{\theta}}\mathrm{d}x = \frac{\sqrt{\theta}}{\sqrt{\theta}-1} = \overline{X}$$

解得矩估计量

$$\hat{\theta} = \frac{\overline{X}^2}{(\overline{X}-1)^2}$$