

16. 解:

(1)

由题意知, $X \sim N(\mu, \sigma^2)$,且 $rac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$,有,

$$P(\underline{\theta} < \theta < \overline{\theta}) = 1 - \alpha$$

即,

$$P(-z_{rac{lpha}{2}} < rac{\overline{X} - \mu}{\sigma/\sqrt{n}} < z_{rac{lpha}{2}}) = 1 - lpha$$

故,

$$P(\overline{X} - rac{\sigma}{\sqrt{n}} z_{rac{lpha}{2}} < \mu < \overline{X} + z_{rac{lpha}{2}}) = 1 - lpha$$

令 $n=9, \sigma=0.6, 1-lpha=0.95$,得 $rac{lpha}{2}=0.025, z_{0.025}=1.96, \overline{x}=6$,故置信区间为,

$$(6\pm rac{0.6}{3}z_{0.025})=(5.608,6.392)$$

(2)

由题意, $\frac{\overline{X}-\mu}{S/\sqrt{n}}\sim t(n-1)$,故,

$$P(-t_{rac{lpha}{2}}(n-1)<rac{\overline{X}-\mu}{S/\sqrt{n}}< t_{rac{lpha}{2}}(n-1))=1-lpha$$

故有,

$$P(\overline{X} - rac{S}{\sqrt{n}}t_{rac{lpha}{2}} < \mu < \overline{X} + rac{S}{\sqrt{n}}t_{rac{lpha}{2}}) = 1 - lpha$$

令 $n=9,1-\alpha=0.95, \frac{\alpha}{2}=0.025, t_{\frac{\alpha}{2}}(8)=2.306$, 算得 $\overline{x}=6, s=\sqrt{0.33}$,故置信区间为,

$$(6\pm\frac{\sqrt{0.33}}{3}t_{0.025}(8))=(5.558,6.442)$$

18. 解:

由题意知, $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$

$$P(\chi_{1-rac{lpha}{2}}^2(n-1) < rac{(n-1)S^2}{\sigma^2} < \chi_{rac{lpha}{2}}^2(n-1)) = 1-lpha$$

故,

$$P(\frac{(n-1)S^2}{\chi_{\frac{\alpha}{2}}^2(n-1)} < \sigma^2 < \frac{(n-1)S^2}{\chi_{1-\frac{\alpha}{2}}^2(n-1)}) = 1-\alpha$$

令 $n=9, s=11, 1-\alpha=0.95, \frac{\alpha}{2}=0.025, \chi^2_{\frac{\alpha}{2}}(n-1)=17.535, \chi^2_{1-\frac{\alpha}{2}}(n-1)=2.180,$ 故,

$$(rac{\sqrt{n-1}S}{\sqrt{\chi_{rac{lpha}{2}}^2(n-1)}},rac{\sqrt{n-1}S}{\sqrt{\chi_{1-rac{lpha}{2}}^2(n-1)}})=(7.4,21.1)$$

19. 解:

(1)

由题意知, $X_i \sim N(\mu, \sigma^2)$,故, $rac{X_i - \mu}{\sigma} \sim N(0, 1)$,故,

$$\sum_{i=1}^n (rac{X_i - \mu}{\sigma})^2 \sim \chi^2(n)$$

故,

$$P(\chi^2_{1-rac{lpha}{2}} < \sum_{i=1}^n rac{(X_i - \mu)^2}{\chi^2_{rac{lpha}{2}}(n)} < \chi^2_{rac{lpha}{2}}(n)) = 1 - lpha$$

故置信区间为,

$$(rac{\sum\limits_{i=1}^{n}(X_{i}-\mu)^{2}}{\chi_{rac{lpha}{2}}^{2}(n)},rac{\sum\limits_{i=1}^{n}(X_{i}-\mu)}{\chi_{1-rac{lpha}{2}}^{2}(n)})$$