



## 26. 解:

(2)

三种情况下, 均有

$$E(Z) = E(5X - Y + 15) = 5E(X) - E(Y) + E(15) = 29$$

(I)  $X, Y$  相互独立

$$D(Z) = D(5X - Y) = D(5X) + D(-Y) = 25D(X) + D(y) = 109$$

(II)  $X, Y$  不相关

即,  $\text{Cov}(X, Y) = 0$

故,  $D(Z) = 109$

(III)  $\rho_{XY} = 0.25$  则,

$$\begin{aligned}\text{Cov}(X, Y) &= \sqrt{D(X)}\sqrt{D(Y)}\rho_{XY} = 1.5 \\ D(Z) &= D(5X - Y) = 25D(X) + D(Y) - 2\text{Cov}(5X, Y) \\ &= 109 - 10\text{Cov}(X, Y) = 94\end{aligned}$$

## 30.

证明:

已知:

$$\rho_{XY} = 0, X = \begin{pmatrix} 1 & A \\ 0 & \neg A \end{pmatrix}, Y = \begin{pmatrix} 1 & B \\ 0 & \neg B \end{pmatrix}$$

由  $\rho_{XY} = 0$  得,

$$E(XY) = E(X)E(Y)$$

由  $X, Y$  分布律知,

$$\begin{aligned}E(X) &= P(A) \\E(Y) &= P(B) \\E(XY) &= P(AB)\end{aligned}$$

故，

$$P(AB) = P(A)P(B)$$

即  $A, B$  相互独立。

故，

$$\begin{aligned}P(X = 1, Y = 1) &= P(AB) = P(A)P(B) \\&= P(X = 1)P(Y = 1) \\P(X = 1, Y = 0) &= P(A\bar{B}) = P(A)P(\bar{B}) \\&= P(X = 1)P(Y = 0) \\P(X = 0, Y = 1) &= P(\bar{A}B) = P(\bar{A})P(B) \\&= P(X = 0)P(Y = 1) \\P(X = 0, Y = 0) &= P(\bar{A}\bar{B}) = P(\bar{A})P(\bar{B}) \\&= P(X = 0)P(Y = 0)\end{aligned}$$

故， $X, Y$  相互独立。

证毕。