

10. 解:

(1)

$$\diamondsuit \varsigma^2 = c \sum_{i=1}^{n-1} (X_{i+1} - X_i)^2$$

$$egin{aligned} E(arsigma^2) &= c E[\sum_{i=1}^{n-1} (X_{i+1} - X_i)^2] = c \sum_{i=1}^{n-1} E(X_{i+1}^2 - 2X_{i+1}X_i + X_i^2) \ &= c \sum_{i=1}^{n-1} (D(X_{i+1}) + [E(X_{i+1})]^2 - 2E(X_{i+1})E(X_i) + D(X_i) + [E(X_{i+1})]^2) \ &= c(n-1)(\sigma^2 + \mu^2 - 2\mu^2 + \sigma^2 + \mu^2) = \sigma^2 \end{aligned}$$

故有,

$$c = \frac{1}{2(n-1)}$$

(2)

$$E(\overline{X}^2-cS^2)=E(\overline{X}^2)-cE(S^2)=(rac{\sigma^2}{n}+\mu^2)-c\sigma^2=\mu^2$$

故,

$$c = \frac{1}{n}$$

13. 解:

(1)

由题知,
$$E(\hat{\theta}) = \theta, D(\hat{\theta}) > 0, \hat{\theta}^2 = (\hat{\theta})^2$$

故,

$$E(\hat{ heta}^2) = E((\hat{ heta})^2) = D(\hat{ heta}) + E(\hat{ heta})^2 = D(\hat{ heta}) + heta^2 > heta^2$$

似然函数为,

$$L(heta) = egin{cases} rac{1}{ heta^n} & 0 < x_1, x_2, ..., x_n \leq heta \ 0 & ext{otherwise} \end{cases}$$

故最大似然估计量 $\hat{\theta}$ 为,

$$\hat{ heta}=\max\{x_1,x_2,...,x_n\}$$

总体 X 分布函数为,

$$F(X) = egin{cases} 0 & x < 0 \ rac{x}{ heta} & 0 \leq x < heta \ 1 & x \geq heta \end{cases}$$

故,

$$F_{\hat{ heta}}(x) = F_{ ext{max}}(x) = (F(x))^n = egin{cases} 0 & x < 0 \ (rac{x}{ heta})^n & 0 \leq x < heta \ 1 & x \geq heta \end{cases}$$

故,

$$f_{\hat{ heta}}(x) = egin{cases} rac{n}{ heta} (rac{x}{ heta})^{n-1} & 0 \leq z \leq heta \ 0 & ext{otherwise} \end{cases}$$

故,

$$E(\hat{ heta}) = \int_{-\infty}^{+\infty} x f_{\hat{ heta}}(x) \mathrm{d}x = \int_{0}^{ heta} n (rac{x}{ heta})^n \mathrm{d}x = rac{n heta}{n+1}
eq heta$$

15. 解:

$$E(\hat{ heta}) = E(\sum_{i=1}^n a_i X_i) = \sum_{i=1}^n a_i E(X_i) = heta \sum_{i=1}^n a_i$$

故有,

$$\sum_{i=1}^n a_i = 1$$
 $D(\hat{ heta}) = \sum_{i=1}^n a_i^2 D(X_i) = \sum_{i=1}^n a_i^2 \sigma_i^2$

作函数

$$g(a_1,a_2,...,a_n,\lambda) = \sum_{i=1}^n a_i^2 \sigma_i^2 + \lambda (\sum_{i=1}^n a_i - 1)$$

$$egin{aligned} rac{\partial g}{\partial a_i} &= 2\sigma_i^2 a_i + \lambda = 0 \ rac{\partial g}{\partial \lambda} &= \sum_{i=1}^n a_i - 1 = 0 \end{aligned}$$

故有 $a_i = -rac{\lambda}{2\sigma_i^2}$,代入得,

$$-rac{\lambda}{2}\sum_{i=1}^nrac{1}{\sigma_i^2}=1$$

故,

$$\lambda = -rac{2}{\sum_{i=1}^nrac{1}{\sigma_i^2}} \ a_i = rac{1}{\sigma_i^2\sum_{j=1}^nrac{1}{\sigma_j^2}}$$