



## 21. 解:

由题意知,

$$\frac{(\bar{X} - \bar{Y}) - (\mu_1 - \mu_2)}{S_w \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t(n_1 + n_2 - 2)$$

其中,

$$S_w^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}, S_w = \sqrt{S_w^2}$$

故置信区间为,

$$\left( (\bar{X} - \bar{Y}) \mp t_{\frac{\alpha}{2}}(n_1 + n_2 - 2) S_w \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \right)$$

由题知,  $n_1 = 4, n_2 = 5, 3S_1^2 = 2.475 \times 10^{-5}, 4S_2^2 = 2.08 \times 10^{-5}, S_w = \sqrt{\frac{3S_1^2 + 4S_2^2}{7}} = 2.55 \times 10^{-3}, 1 - \alpha = 0.95, \frac{\alpha}{2} = 0.025, t_{0.025}(7) = 2.3646$

故置信区间为,

$$\left( 0.00205 \mp 2.3646 \times 0.00255 \times \sqrt{\frac{1}{4} + \frac{1}{5}} \right) = (0.002 \mp 0.004) = (-0.002, 0.006)$$

## 22. 解:

由题意知,

$$\frac{S_1^2/S_2^2}{\sigma_1^2/\sigma_2^2} \sim F(n_1 - 1, n_2 - 1)$$

故,

$$P(F_{1-\frac{\alpha}{2}}(n_1 - 1, n_2 - 1) < \frac{S_1^2/S_2^2}{\sigma_1^2/\sigma_2^2} < F_{\frac{\alpha}{2}}(n_1 - 1, n_2 - 1)) = 1 - \alpha$$

故置信区间为,

$$\left( \frac{S_1^2}{S_2^2} \frac{1}{F_{\frac{\alpha}{2}}(n_1 - 1, n_2 - 1)}, \frac{S_1^2}{S_2^2} \frac{1}{F_{1 - \frac{\alpha}{2}}(n_1 - 1, n_2 - 1)} \right)$$

由题知， $n_1 = n_2 = 10, s_A^2 = 0.5419, s_B^2 = 0.6065, 1 - \alpha = 0.95$

故解得置信区间为，

$$(0.222, 3.601)$$