\vdash

3. 解:

$$f(x) = egin{cases} \sqrt{ heta} x^{\sqrt{ heta}-1} & 0 \leq x \leq 1 \ 0 & ext{otherwise} \end{cases}$$

故,

$$L(x; heta) = \prod_{i=1}^n \sqrt{ heta} x_i^{\sqrt{ heta}-1} \ \ln L(x; heta) = rac{n}{2} \ln heta + \sqrt{ heta} \sum_{i=1}^n \ln x_i \ rac{\mathrm{d}}{\mathrm{d} heta} \ln L(x; heta) = rac{n}{2 heta} + rac{\sum_{i=1}^n \ln x_i}{2\sqrt{ heta}}$$

令 $\frac{\mathrm{d}}{\mathrm{d} heta} \ln L(x; heta) = 0$, 解得

$$\theta = \frac{n^2}{(\sum\limits_{i=1}^n \ln x_i)^2}$$

容易证明: 当 $\theta > \frac{n^2}{(\sum\limits_{i=1}^n \ln x_i)^2}$ 时, $\frac{\mathrm{d}}{\mathrm{d}\theta} \ln L(x;\theta)$ 恒小于 0;当 $\theta < \frac{n^2}{(\sum\limits_{i=1}^n \ln x_i)^2}$ 时, $\frac{\mathrm{d}}{\mathrm{d}\theta} \ln L(x;\theta)$ 恒大于 0。

故极大似然估计,

$$\hat{ heta} = rac{n^2}{(\sum\limits_{i=1}^n \ln x_i)^2}$$

6. 解:

由题意知,

$$X \sim B(m,p), P(X=x) = inom{m}{x} p^x (1-p)^{m-x}$$

设 $k = \prod_{i=0}^{m} {m \choose i}^{\operatorname{cnt}_i}$,其中 cnt_i 表示 $x_i = i$ 的样本个数。

故,

$$L(x;p) = \prod_{i=1}^n inom{m}{x_i} p^{x_i} (1-p)^{m-x_i} = k p^{\sum\limits_{i=1}^n x_i} (1-p)^{nm-\sum\limits_{i=1}^n x_i} \ \ln L(x;p) = \ln k + (\sum\limits_{i=1}^n x_i) \ln p + (nm - \sum\limits_{i=1}^n x_i) \ln (1-p)$$

$$rac{\mathrm{d}}{\mathrm{d} p} \ln L(x;p) = (\sum_{i=1}^n x_i) rac{1}{p} - (nm - \sum_{i=1}^n x_i) rac{1}{1-p} = 0$$

解得最大似然估计,

$$\hat{p}=rac{1}{nm}\sum_{i=1}^n x_i=rac{\overline{x}}{m}=0.499$$

矩估计量:

$$\mu_1 = E(X) = mp = \overline{x}$$

故,

$$\hat{p} = \frac{\overline{x}}{m} = 0.499$$