



18. 解:

$$f(x) = \begin{cases} \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

其中, $\sigma > 0$

故,

$$\begin{aligned} E(X) &= \int_{-\infty}^{\infty} x f(x) dx \\ &= \int_0^{\infty} \frac{x^2}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} dx \\ &= \int_0^{\infty} -x de^{-\frac{x^2}{2\sigma^2}} \\ &= -xe^{-\frac{x^2}{2\sigma^2}} \Big|_0^{\infty} + \int_0^{\infty} e^{-\frac{x^2}{2\sigma^2}} dx \\ &= 0 + \sqrt{2\pi}\sigma \int_0^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} dx \\ &= \sqrt{\frac{\pi}{2}}\sigma \\ D(X) &= E(X^2) - E^2(X) \\ &= \int_0^{\infty} \frac{x^3}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} dx - \frac{\pi\sigma^2}{2} \\ &= \int_0^{\infty} -x^2 de^{-\frac{x^2}{2\sigma^2}} - \frac{\pi\sigma^2}{2} \\ &= -x^2 e^{-\frac{x^2}{2\sigma^2}} \Big|_0^{\infty} + \int_0^{\infty} e^{-\frac{x^2}{2\sigma^2}} dx^2 - \frac{\pi\sigma^2}{2} \\ &= 0 + (-2\sigma^2 e^{-\frac{x^2}{2\sigma^2}}) \Big|_0^{\infty} - \frac{\pi\sigma^2}{2} \\ &= \sigma^2(2 - \frac{\pi}{2}) \end{aligned}$$