23. 解：

(1)

记两周需求量为 $Z=X+Y$，则有

$$f_Z(z)=\int_{-\infty}^{\infty}f_X(x)f_Y(z-x)dx$$

故，

$$\begin{aligned}
f_Z(z)&=\begin{cases}\int_0^z f(x)f(z-x)dx & z>0\\ 0 & z\le 0\end{cases}\\
&=\begin{cases}e^{-z}\int_0^z x(z-x)dx & z>0\\ 0 & z\le 0\end{cases}\\
&=\begin{cases}\frac{z^3e^{-z}}{6} & z>0\\ 0 & z\le 0\end{cases}
\end{aligned}$$

(2)

记三周需求量为 $W=Z+X$，则有

$$f_W(w)=\int_{-\infty}^{\infty}f_Z(x)f_X(w-x)dx$$

故，

$$\begin{aligned}
f_W(w)&=\begin{cases}\int_0^w f_Z(x)f_X(w-x)dx & w>0\\ 0 & w\le 0\end{cases}\\
&=\begin{cases}\int_0^w \frac{x^3e^{-z}}{6}(w-x)e^{x-w}dx & w>0\\ 0 & w\le 0\end{cases}\\
&=\begin{cases}\frac{w^5e^{-w}}{5!} & w>0\\ 0 & w\le 0\end{cases}
\end{aligned}$$

```
import sympy as sym
import numpy as np
import matplotlib.pyplot as plt
from sympy.abc import x, theta, sigma, r, z

pi = sym.pi

f_Z = 2*pi * 1/(2 * pi * sigma**2) * sym.exp(-r**2/(2 * sigma**2)) * r
sym.integrate(f_Z, (r, 0, z)).diff(z)
```

28. 解：

设 Z 的分布函数为 F_Z，则，

$$
\begin{aligned}
F_Z(Z \le z) &= \begin{cases} P(X^2+Y^2 \le z^2) & z \ge 0 \\ 0 & z < 0 \end{cases} \\
&= \begin{cases} \iint\limits_{x^2+y^2 \le z^2} f(x,y)dxdy & z \ge 0 \\ 0 & z < 0 \end{cases} \\
&= \begin{cases} \iint\limits_{x^2+y^2 \le z^2} \frac{1}{2\pi\sigma^2} e^{\frac{-x^2-y^2}{2\sigma^2}} dxdy & z \ge 0 \\ 0 & z < 0 \end{cases} \\
&= \begin{cases} \int_0^{2\pi} d\theta \int_0^z \frac{1}{2\pi\sigma^2} e^{\frac{-r^2}{2\sigma^2}} rdr & z \ge 0 \\ 0 & z < 0 \end{cases} \\
&= \begin{cases} 1 - e^{-\frac{z^2}{2\sigma^2}} & z \ge 0 \\ 0 & z < 0 \end{cases}
\end{aligned}
$$

故，

$$
f_Z(z) = \frac{d}{dz} F_Z(z) = \frac{z e^{-\frac{z^2}{2\sigma^2}}}{\sigma^2}
$$

6. 解:

(2)

$$
\begin{aligned}
E(\frac{1}{X+1}) &= \sum_{k=0}^{\infty}\frac{1}{k+1}P(X=k) \\
&= \sum_{k=0}^{\infty}\frac{\lambda^k e^{-k}}{(k+1)!} \\
&= \frac{e^{-k}}{\lambda}(\sum_{k=0}^{\infty}\frac{\lambda^k}{k!}-1) \\
&= \frac{e^{-\lambda}}{\lambda}(e^{\lambda}-1) \\
&= \frac{1}{\lambda}(1-e^{-\lambda})
\end{aligned}
$$

7. 解:

(2)

设 U 分布函数为 $F_U(U\leq u)$，则有

$$
F_U(u)=\prod_{i=1}^{n}P(X\leq u)=u^n
$$

故

$$
f_U=\frac{d}{du}F_U(u)=nu^{n-1}
$$

故

$$
\begin{aligned}
E(U) &= \int_0^1 f_U(x)udu \\
&= \int_0^1 nu^n du \\
&= \frac{nu^{n+1}}{n+1}\bigg|_0^1=\frac{n}{n+1}
\end{aligned}
$$