



9. 解:

(1)

$$f(x, y) = \begin{cases} 12y^2 & 0 \leq y \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

设 $G : \{(x, y) \mid 0 \leq y \leq x \leq 1\}$

$$\begin{aligned} E(x) &= E(g(X, Y)) = \iint_G x f(x, y) dx dy \\ &= \iint_G x \cdot 12y^2 dx dy \\ &= \int_0^1 dx \int_0^x 12xy^2 dy \\ &= \frac{4}{5} \end{aligned}$$

同理,

$$\begin{aligned} E(Y) &= \iint_G y \cdot 12y^2 dx dy = \int_0^1 dx \int_0^x 12y^3 dy = \frac{3}{5} \\ E(XY) &= \iint_G xy \cdot 12y^2 dx dy = \int_0^1 dx \int_0^x 12xy^3 dy = \frac{1}{2} \\ E(X^2 + Y^2) &= \iint_G (x^2 + y^2) \cdot 12y^2 dx dy = \int_0^1 dx \int_0^x 12(x^2 y^2 + y^4) dy = \frac{16}{15} \end{aligned}$$