Assignment3

March 10, 2025

1 1.46

1.1 Answer

Consider the following system:

$$y[n] = \begin{cases} e[n-1] & n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$e[n] = x[n] - y[n]$$

1.1.1 (1) For $x[n] = \delta[n]$

Let $x[n] = \delta[n] = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases}$. We derive:

$$\begin{aligned} y[0] &= x[-1] - y[-1] = 0 \\ y[1] &= x[0] - y[0] = 1 \\ &\vdots \\ y[n] &= -y[n-1] \quad (n \geq 2) \end{aligned}$$

Therefore, the final result is:

$$y[n] = \begin{cases} (-1)^{n-1} & n \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

1.1.2 (2) For $x[n] = u[n]$

Let $x[n] = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases}$. We derive:

$$\begin{aligned} y[0] &= x[-1] - y[-1] = 0, \\ y[1] &= x[0] - y[0] = 1, \\ y[2] &= x[1] - y[1] = 1 - 1 = 0, \\ y[3] &= x[2] - y[2] = 1 - 0 = 1, \\ &\vdots \end{aligned}$$

This pattern generalizes to:

$$y[2k-1] = 1 \quad \text{and} \quad y[2k] = 0 \quad \text{for } k \geq 1.$$

Therefore, the result is:

$$y[n] = \frac{1-(-1)^n}{2} u[n-1]$$

This expression is equivalent to the piecewise definition:

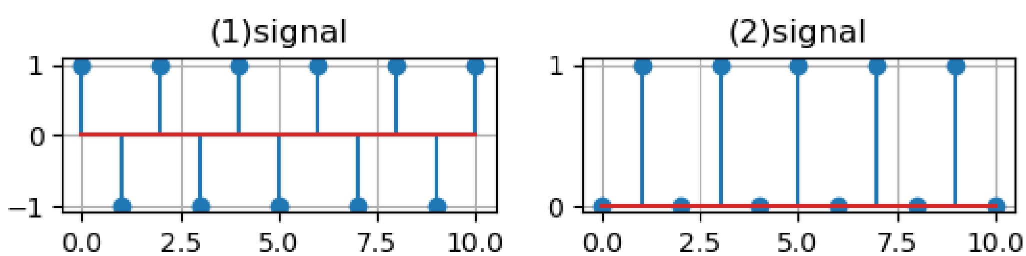
$$y[n] = \begin{cases} 0, & n < 0 \text{ or } n \text{ even (including } n = 0) \\ 1, & n \geq 1 \text{ and } n \text{ odd} \end{cases}$$

[12]:
```
import matplotlib.pyplot as plt
import numpy as np

def draw(x, y, title, a):
    plt.subplot(4, 2, a)
    plt.title(title)
    plt.grid(True)
    plt.stem(x, y)

x = np.linspace(0, 10, 11)
y = (-1)**x
draw(x, y, '(1)signal', 1)

y = (1 - (-1)**x) // 2
draw(x, y, '(2)signal', 2)
```



2 1.27(b)

$$y(t) = \cos(3t)x(t)$$

2.1 Answer

This continuous-time system is analyzed as follows:

1. **Memoryless:** Yes. The output y(t) depends solely on the input x(t) at the same time instant t.
2. **Time-Invariant:** No. Since the multiplier $\cos(3t)$ explicitly depends on time, shifting the input does not simply shift the multiplier. In detail, if the input is shifted by t_0 to give $x(t-t_0)$, the output becomes $y(t) = \cos(3t)x(t-t_0)$, which is different from $y(t-t_0) = \cos(3(t-t_0))x(t-t_0)$.
3. **Linear:** Yes. To verify linearity, we check the additivity and homogeneity properties:
   - *Additivity:* Suppose for inputs $x_1(t)$ and $x_2(t)$ the responses are
   $$y_1(t) = \cos(3t)x_1(t) \quad \text{and} \quad y_2(t) = \cos(3t)x_2(t).$$
   For the input $x_1(t) + x_2(t)$, the output is
   $$y(t) = \cos(3t)[x_1(t) + x_2(t)] = \cos(3t)x_1(t) + \cos(3t)x_2(t) = y_1(t) + y_2(t).$$
   - *Homogeneity:* For any scalar a, if $x(t)$ produces $y(t) = \cos(3t)x(t)$, then the input $ax(t)$ produces
   $$y(t) = \cos(3t)[ax(t)] = a\cos(3t)x(t) = a\,y(t).$$

   Since both additivity and homogeneity hold, the system is linear.
4. **Causal:** Yes. The output at time t depends only on $x(t)$ and $\cos(3t)$ at the same time t, with no dependence on future values.
5. **Stable:**
   For a system to be BIBO (Bounded-Input Bounded-Output) stable, every bounded input must result in a bounded output. Suppose that for all time $ t $ the input satisfies
   $$|x(t)| \leq M,$$
   where (M) is a finite number. Then the output is given by
   $$y(t) = \cos(3t)x(t).$$
   Notice that since
   $$|\cos(3t)| \leq 1 \quad \text{for all } t,$$
   it follows that
   $$|y(t)| = |\cos(3t)x(t)| \leq |\cos(3t)| \cdot |x(t)| \leq 1 \cdot M = M.$$
   This shows that the output remains bounded by the same constant M (or possibly by a scaled constant if a different norm is considered).

   Additionally, even though the multiplier $\cos(3t)$ varies with time, its bounded nature ensures that no unbounded amplification of the input occurs. Therefore, the system is BIBO stable.

3 1.28(b)

$$y[n] = x[n-2] - 2x[n-8]$$

3.1 Answer

This discrete-time system is analyzed as follows:

1. **Memoryless:** No.
   The output depends on past input values $x[n-2]$ and $x[n-8]$ rather than solely on $x[n]$.

2. **Time-Invariance:** Yes.
   A time shift of the input by n_0 results in:

   $$y[n] = x[(n-n_0)-2] - 2x[(n-n_0)-8],$$

   which is equivalent to shifting the entire output by n_0. Thus, the system is time-invariant.

3. **Linear:** Yes.
   The system is a linear combination of delayed inputs with constant coefficients; hence, it satisfies both the additivity and homogeneity properties.

4. **Causal:** Yes.
   The output at time n depends only on $x[n-2]$ and $x[n-8]$, which are past (or present) values relative to n. Therefore, the system is causal.

5. **Stable:** Yes.
   The system is a finite impulse response (FIR) filter. Its impulse response is

   $$h[n] = \delta[n-2] - 2\delta[n-8],$$

   and the sum of the absolute values of the impulse response is

   $$\sum_{n=-\infty}^{\infty} |h[n]| = |1| + |2| = 3,$$

   which is finite. Therefore, the system is BIBO (Bounded-Input Bounded-Output) stable.