Assignment1

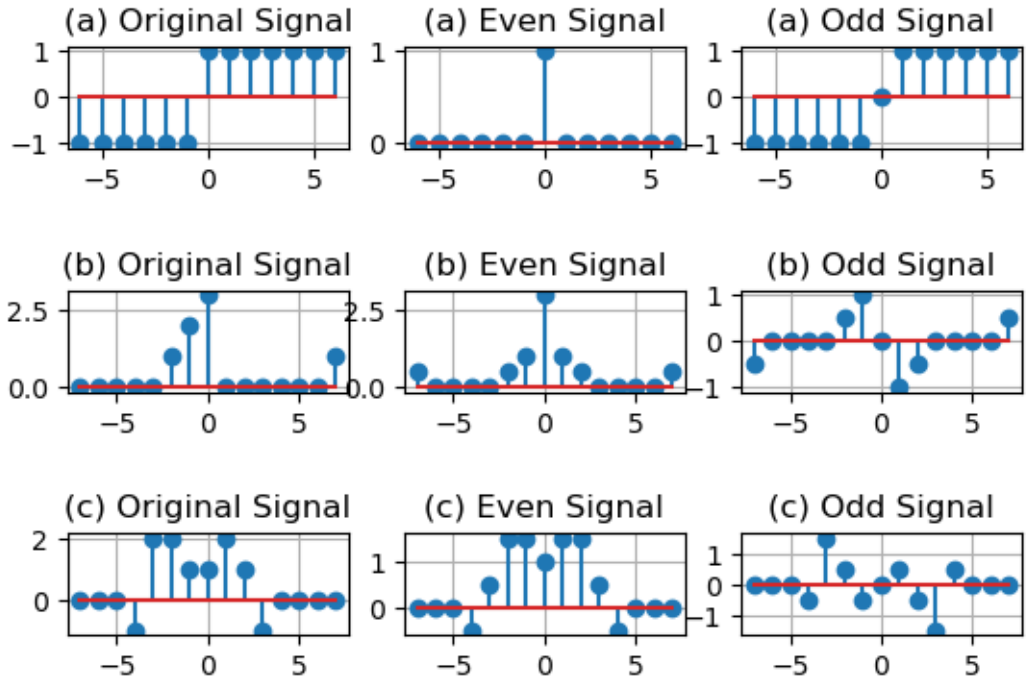
March 4, 2025

1 1.24

1.1 Answer:

[2]:
```
import numpy as np
import matplotlib.pyplot as plt

def draw(x, y, title, a):
    plt.subplot(6, 3, a)
    plt.title(title)
    plt.grid(True)
    plt.stem(x, y)

x = np.linspace(-6, 6, 13)
y = [-1, -1, -1, -1, -1, -1, 1, 1, 1, 1, 1, 1, 1]
draw(x, y, '(a) Original Signal', 1)

Ey = [(y[i] + y[12 - i])/2 for i in range(0, 13)]
draw(x, Ey, '(a) Even Signal', 2)

Oy = [(y[i] - y[12 - i])/2 for i in range(0, 13)]
draw(x, Oy, '(a) Odd Signal', 3)

x = np.linspace(-7, 7, 15)
y = [0] * 15
y[5], y[6], y[7], y[14] = 1, 2, 3, 1
draw(x, y, '(b) Original Signal', 7)

Ey = [(y[i] + y[14 - i])/2 for i in range(0, 15)]
draw(x, Ey, '(b) Even Signal', 8)

Oy = [(y[i] - y[14 - i])/2 for i in range(0, 15)]
draw(x, Oy, '(b) Odd Signal', 9)

y = [0] * 15
y[3], y[4], y[5], y[6], y[7], y[8], y[9], y[10] = -1, 2, 2, 1, 1, 2, 1, -1
draw(x, y, '(c) Original Signal', 13)
```

```
Ey = [(y[i] + y[14 - i])/2 for i in range(0, 15)]
draw(x, Ey, '(c) Even Signal', 14)

Oy = [(y[i] - y[14 - i])/2 for i in range(0, 15)]
draw(x, Oy, '(c) Odd Signal', 15)
```



2 1.26(c)

2.1 Answer:

$$x[n] = \cos\left(\frac{\pi n^2}{8}\right)$$

x[n] is periodic if there exists a T such that

$$\cos\left(\frac{\pi(n+T)^2}{8}\right) = \cos\left(\frac{\pi n^2}{8}\right) \quad \forall n.$$

Using the identity for cosine periodicity, we require

$$\frac{\pi(n+T)^2}{8} - \frac{\pi n^2}{8} = \frac{\pi(2nT+T^2)}{8} = 2\pi k, \quad k \in \mathbb{Z}.$$

Dividing by π gives

$$\frac{2nT + T^2}{8} = 2k \quad \Longrightarrow \quad n\frac{T}{4} + \frac{T^2}{8} = 2k.$$

For this to hold for every integer n the coefficient of n must be an integer

1. $\frac{T}{4}$ must be an integer. Let $T = 4m$ with $m \in \mathbb{Z}$.
2. Substituting $T = 4m$ we have:

$$nm + \frac{(4m)^2}{8} = nm + \frac{16m^2}{8} = nm + 2m^2 = 2k.$$

Since k can be an integer that depends on n, the term nm must always be even. This forces m to be even. Let $m = 2p$ where $p \in \mathbb{Z}$. Therefore,

$$T = 4(2p) = 8p.$$

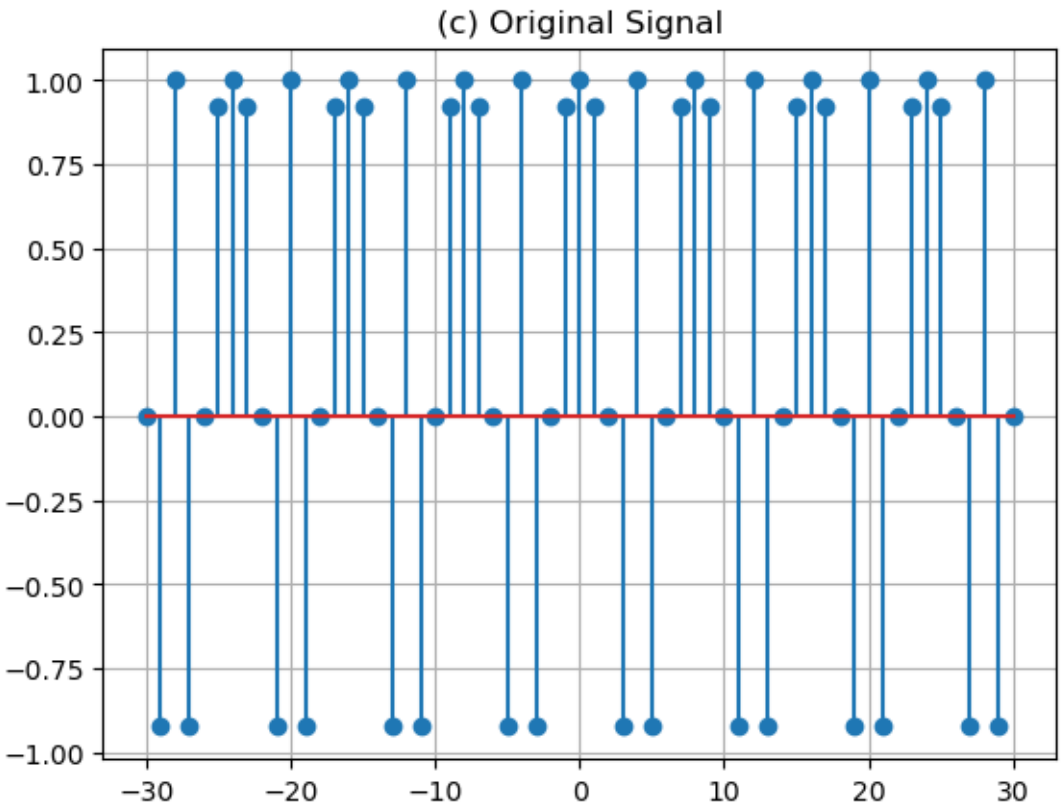
The smallest nonzero period is obtained for $p = 1$, hence

$$T = 8.$$

Thus, the signal is periodic with period $T = 8$.

[10]:
```
x = np.linspace(-30, 30, 61)
y = np.cos(np.pi * x * x / 8)
plt.title('(c) Original Signal')
plt.grid(True)
plt.stem(x, y)
```

[10]:
```
<StemContainer object of 3 artists>
```


(c) Original Signal
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−30
−20
−10
0
10
20
30