Proposed Origins of Magnetic Fields of Neutron Stars

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1 Introduction

When the remains of a star that has burned all of its light nuclei is of mass greater than the Chandrasekhar limit (approximately 1.4 $M_{\odot}$), the remnants undergo gravitational collapse. This collapse results in a neutron star, or a black hole in the case of the mass being sufficiently large. In some instances a neutron star is observed to have a strong magnetic field, in which case the star is referred to as a pulsar or magnetar, depending on the field strength. Unlike magnetars, pulsars emit characteristic beams of electromagnetic radiation, with pulsars emitting in the radio wave spectrum having rotational periods of .02 to .5 seconds.[1] The magnetic fields of these stars are some of the strongest in the known universe, although the mechanism that produces them is currently an open question. In this paper we will examine three proposed models for the generation of these magnetic fields.

2 Fossil Fields

If we assume the magnetic flux of a star undergoing gravitational collapse is conserved, we can construct a simple model for the magnetic field of a neutron star. To check the functionality of this model, if we consider the strongest observed magnetic fields of large mass ($10M_{\odot}$) main sequence stars we have a field on the order of 10^4 Gauss. After gravitational collapse, the field is on the order of 10^{15} Gauss. This result is consistent with the magnetic field of a magnetar, however, there are some flawed assumptions made in this model.[2]

The above result demands that the gravitational collapse leaves the core with the vast majority of magnetic flux. If we replace this with the more reasonable assumption that the conservation of the magnetic flux is uniform across the mass of the star, since the remaining core has less than 15% of the main sequence star's mass, the resulting magnetic field is of order of magnitude 10^{14} Gauss. This result is inconsistent with the observed fields of magnetars, however it fits pulsars.[2]

This model further unravels when we consider that the observed rate at which magnetars are formed is also inconsistent with the observed proportion of main sequence stars, of sufficient mass, with a field of order 10^4 Gauss. It would seem that this variety of main sequence stars are exceedingly rare, whereas it is suggested that magnetars form at a similar rate to neutron stars.[2]

In addition to the complications associated with the formation of magnetars, there is a further complication with the rotation of pulsars. Since this hypothesis demands that the magnetic field is a result of conservation of magnetic flux, a strong magnetic field must be present for the entire process of gravitational collapse. The presence of the field demands that the rotation of the inert core of the star would rotate with the envelope, where most of the angular momentum is contained, thus as the star collapses, the angular frequency would remain relatively constant, resulting in an angular frequency inconstant with that of pulsars. Hence, if this process is responsible for the magnetic field of pulsars, the angular momentum of pulsars would have to originate from a source other than conservation of angular momentum.[2]

3 Convection Driven Dynamos

A common theory for the magnetic field generation of stellar objects is convection within the star. Typically, the magnetic field energy is assumed to to given by:

$$B^2 = 4\pi\rho v^2$$

Where v is convectional velocity and ρ is density. In addition, heat flux can be approximated as:

$$\Phi \approx \rho v^3$$

Since heat flux transports the luminosity of the star, if we assume that the average density is an appropriate approximation for density then:

$$B \approx M^{1/6} L^{1/3} r^{-7/6}$$

This results in a field strength of order 10^{13} Gauss, which is sufficient for a pulsar. While the result is on the correct order of magnitude for a pulsar, it may not be the case that the approximations are appropriate, since the same calculation for the sun results in a field that is 300 times weaker than what is observed.

If we examine this model during the gravitational collapse, then the convection is driven by neutrino flux which have an extremely high mean free path through the star, thus the convection occurs at a much more rapid rate. Using this, we get a magnetic field strength on order of 10^{15} Gauss, which is within the range of a magnetar. If we assume that this calculation is off by the same factor as with the sun, then the field strength is of order of magnitude of 10^{14}, which is within range of a pulsar.

4 Gravito-Magnetic Induced Field

It has been proposed that while a pulsar is made primarily of neutrons that some charged particles are also present. In the presence of an induced gravito-magnetic field, thought to be a consequence of general relativity, can add to a total observed magnetic field. The induced gravito-magnetic field, $\mathbf{B}_{(gm)}$ can be calculated using the following simplified equations:

$$\nabla \times \mathbf{B}_{(gm)} = -4\pi\beta c^{-1}G^{1/2}\rho\mathbf{v}$$
$$\nabla \cdot \mathbf{B}_{(gm)} = 0$$

Where β is dimensionless and assumed to be +1, $\mathbf{v}$ is the angular velocity at a given radius within the pulsar, and ρ is the mass density, assumed to be constant throughout the pulsar. Since this is only valid within the the radius of the star, if we assume the magnetic field to be given by a magnetic dipole, we can use the following equation to determine $\mathbf{B}_{(gm)}$ at point $\mathbf{r}$ outside the pulsar:

$$\mathbf{B}_{(gm)} = \frac{3\mathbf{M}\cdot\mathbf{r}}{r^5}\mathbf{r} - \frac{\mathbf{M}}{r^3}$$

Here, the magnetic dipole moment, M, is given by the Wilson-Blackett formula:

$$\mathbf{M} = -\frac{1}{2}\beta c^{-1}G^{1/2}\mathbf{L}$$

Furthermore, since we are treating $\mathbf{B}_{(gm)}$ as a dipole outside the pulsar, in spherical coordinates:

$$\mathbf{B}_{(gm)} = \frac{2Mcos(\theta)}{r^3}\hat{r} + \frac{Msin(\theta)}{r^3}\hat{\theta}$$

These equations can be solved outside the star, however within, in order to find the solutions we must assume that the gravito-magnetic field is an ideal magnetic dipole at the center of the star, to arrive at an approximate solution. This gives a current density of:

$$\mathbf{J} = \sigma(\mathbf{E} + c^{-1}\mathbf{v} \times \mathbf{B}_(tot))$$

Here $\mathbf{B}_{tot} = \mathbf{B}_{(gm)} + \mathbf{B}_{(em)}$, where $\mathbf{B}_{(em)}$ is the field induced by by moving charges in the $\mathbf{B}_{(gm)}$ field and $\mathbf{v}$ is defined as before. An approximation for $\mathbf{E}$ can be calculated assuming that $\mathbf{B}_{(em)} = 0$, giving the following approximation; however, we must be cautious when making such an approximation since it is invalid if there is accretion present in the system:

$$\mathbf{E} = \frac{-\omega M sin^2(\theta}{cr^2}\hat{r} + \frac{2\omega M sin(\theta) cos(\theta}{cr^2}\hat{\theta}$$

If we consider the following Maxwell Equation, we can solve for the charge distribution ρ_c:

$$\nabla \cdot \mathbf{E} = 4\pi\rho_c$$

First if we assume a quadropole expansion for the electric field, giving the following:

$$\rho_c = \frac{-\omega M}{2\pi cr^3}(3cos^2(\theta) - 1)$$

From this charge density we can integrate over a small radius and obtain an expression for positive and negative charges, equal in magnitude to one another, thus, $Q = 0$. Furthermore, if we adopt a monopole expansion for $\mathbf{E}$, again using the Maxwell Equation, we can find $Q_{monopole}$ using Gauss's law:

$$Q_{monopole} = -\frac{2\omega M}{3c}$$

This gives rise to a a magnetic dipole moment, thus allowing us to find $\mathbf{M}_{em}$ as follows:

$$\mathbf{M}_{em} = \frac{1}{2c}\int_V \rho_c \mathbf{r} \times \mathbf{v} d\tau = \frac{2\omega^{22}}{15c^2}\mathbf{M}_{gm}$$

This allows us to find $\mathbf{M}_{tot}$ in terms of $\mathbf{M}_{gm}$:

$$\mathbf{M}_{tot} = (1 + \frac{2\omega^{22}}{15c^2})\mathbf{M}_{gm}$$

When compared with observational data of many known pulsars, the order of magnitude of the magnetic field from the Wilson-Blackett formula was successful. The discrepancies in observational data could be due to effects of the electromagnetic induction, however, this result is highly contingent on several assumptions made on the observational parameters as well as the validity of the of the gravito-magnetic hypothesis. As a brief note of interest, this result can be applied to all rotating, large neutral bodies of matter.[3]

Since this theory is dependent on the validity of the gravito-magnetic hypothesis, there are some serious shortcomings to the model. A test on this was run in a laboratory setting and the results were unsatisfactory when compared with the result in astrophysical bodies. Interestingly, measurements of the magnetic fields of Mercery, Venus, and the Moon have been consistent with this effect, however, this is not necessarily a result of the gravito-magnetic hypothesis.[4]

5 Conclusion

Of the three methods for generation of the magnetic fields of pulsars and magnetars, each one had significant drawbacks and assumptions. Arguably the gravito-magnetic generation of the electric field had the best comparison to the observations, however it is built on the largest assumption; in this case that an unlikely hypothesis is an accurate reflection of reality. Overall, the origins of the magnetic fields of neutron stars is still very much an open question with no conclusive theories.

6 References

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