Testing Assumptions in Models of Pulsar Magnetic Fields

Grant Allen

Background:

Currently, the origins of the magnetic fields of pulsars are an open question. In this project I examined the validity of some assumptions made in developing potential theoretical models. It has been proposed that the magnetic field is due to a combination of a magnetic field induced by adding up quantum mechanical effects of neutrons under a strong gravitational force undergoing rapid rotation and the existence of a small proportion of protons remaining within the core of the collapsed star.

Model:

The adding of neutron spins under the influence of the strong gravitational field gives a magnetic dipole moment as described by the Wilson-Blackett formula:

$$\mathbf{M} = -\tfrac{1}{2}\beta c^{-1} G^{1/2} \mathbf{L}$$

Here, β is known to be 1, and L is the angular momentum. The equation for the field is that of a standard magnetic dipole, with the above magnetic moment. With no displacement current, using Ohm's law, the electric field can be solved for, satisfying:

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Here there are two components to the magnetic field: from the rotating charge within the star itself and from the Wilson-Blackett magnetic dipole. From here, if we assume that the contribution from the rotating charge is 0, then the Electric field can be solved for fairly simply, and the total magnetic field can easily be computed from there. Since this assumption seems extremely large, I tested it across multiple parameters within the bounds for neutron stars.

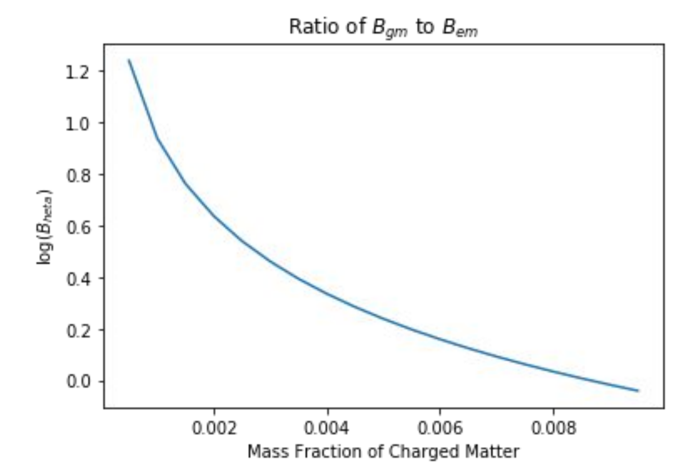
Algorithm:

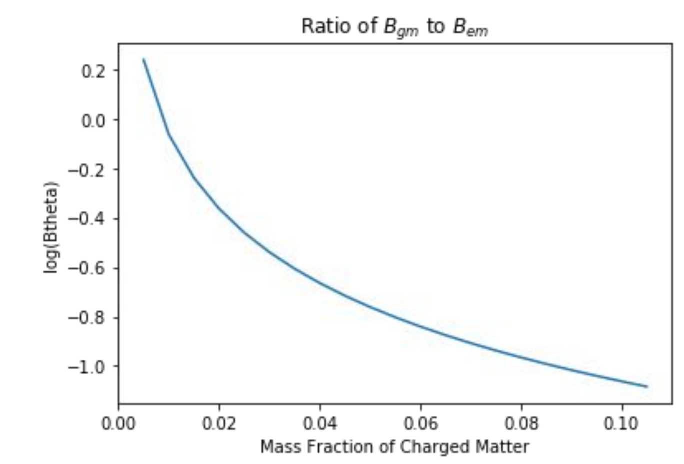
To construct a model of rotating charge within the neutron star, first I assumed all the charge was due to protons since it seems unlikely that a neutron star would be able to hold on to electrons and accounting for their acceleration is not viable computation. From here if we approximate the neutrons as rotating with the same frequency of the neutron star, and assume that they are evenly distributed we can computationally approach the problem using loops drawn around the central axis of rotation for the star. Furthermore, since each loop is symmetric, we can calculate the contributions from an individual proton in the loop and then get the full contribution from the loop by multiplying by the number of protons. When Treated as a moving point charge each proton contributes:

$$\mathrm{B} = \frac{\mu_0 e}{4\pi r^3} \mathbf{v} \times \mathbf{r}$$

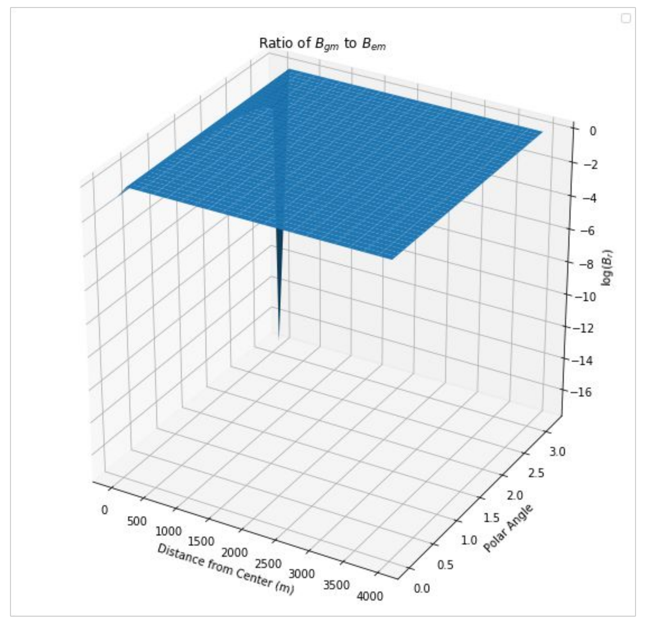
Here, r denotes the separation vector from the point where the field is being evaluated at and the proton.

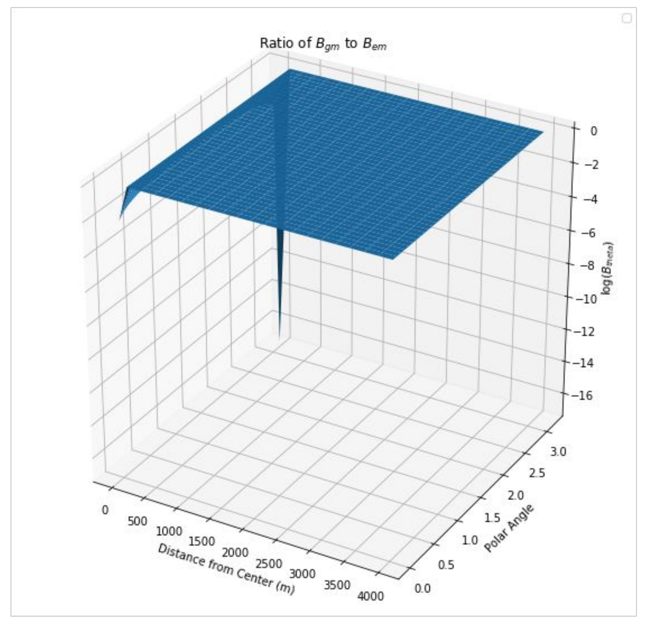
Results:



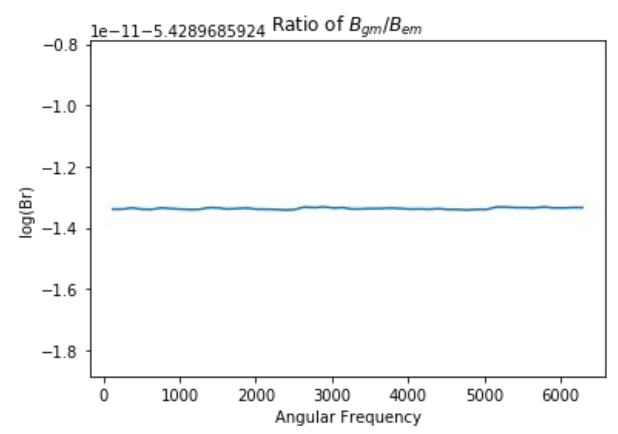


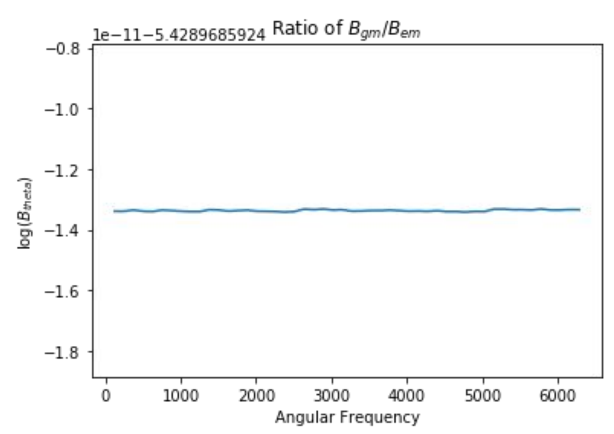
When tested with constant mass, radius, and angular frequency, the approximation is valid up to a mass fraction of about .002, until the contribution from the rotating charge starts to have a significant contribution to the total field.





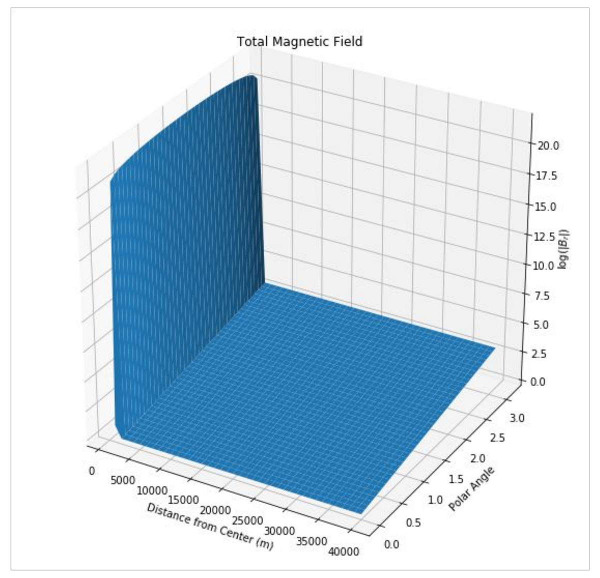
Above is the result of computing the ratio of the two magnetic fields within the neutron star. In this test, the assumptions made in the model are clearly invalid, since the magnetic field generated by the rotating charged particles is larger than the field given by the Wilson-Blackett formula.

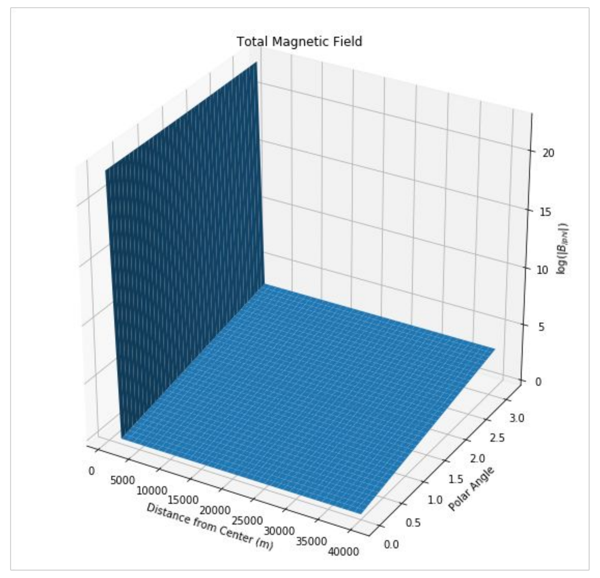




To test the dependence on angular frequency, the differences in the magnetic fields were compared across frequencies corresponding to a second pulsar at the low end and a millisecond pulsar at the high end. Note that the time given refers to the time between pulses, or equivalently, the period of rotation. As expected, since both fields are proportional to the frequency, the model is valid at all frequencies.

Total Magnetic Field





Since the algorithm does not have the same assumption that are built in to the mathematical model, the magnetic field in all space can be calculated, even in cases where the assumptions don't hold. The above fields were calculated for a neutron star of 1.4 solar masses, with an angular frequency of 400 Hz, and a radius of 20 km. Due to symmetry about the axis of rotation, the magnitude of the magnetic field has no dependence on the azimuthal angle or a directional component in the azimuthal direction.

Conclusion:

If the computational model constructed is valid for neutron stars, then the mathematical model only works for a small proportion of observed neutron stars as the assumptions made were demonstrated to be invalid. While the simulation is a good first look at the problem, it doesn't model effects of the Theory of General Relativity, nor does it attempt to model the behavior of a neutron star thermodynamically. If all of these were taken into account, the results could easily be different. In order to attain a solution involving these higher order effects far more computational power would be required than was accessible for this project. Notably, the magnetic fields produced by the model were significantly higher than those observed, likely due to a combination of the assumptions of the algorithm and a not accounting for effects from specific stellar structures.

References:

1. Birth kicks as the origin of pulsar rotation, Spruit H., Phinney E. S., Nature volume 393, pages 139–141 (14 May 1998).
2. Origin of neutron star magnetic fields, Spruit, H.C., Max-Planck-Institut fu¨r Astrophysik, Postfach 1317, 85741 Garching Germany, April 28, 2008.
3. The magnetic field of pulsars and the gravito-magnetic theory, Biemond, Jacob, Vrije Universiteit, Amsterdam, Section: Nuclear magnetic resonance, 1971-1975.
4. Magnetic Field of Celestial Objects, Eid, A., Babatin, M.M., Adv. Studies Theor. Phys., Vol. 8, 2014, no. 4, 151 - 161, Department of Physics, College of Science, Al Imam Mohammad Ibn Saud Islamic University (IMSIU))
5. Gravitational Magnetism: an Update, Sirag, Saul-Paul, International Space Sciences Organization 3220 Sacramento Street, San Francisco, CA 94115