T-line lossless Summary

V, I, Z depend on Position $V(x)$, $I(x)$, $Z(x)$

$$V(x) = V^+(x) + V^-(x)$$

$$I(x) = I^+(x) + I^-(x)$$

$$Z(x) = \frac{V(x)}{I(x)} = Z_0 \frac{1+\Gamma(x)}{1-\Gamma(x)}$$

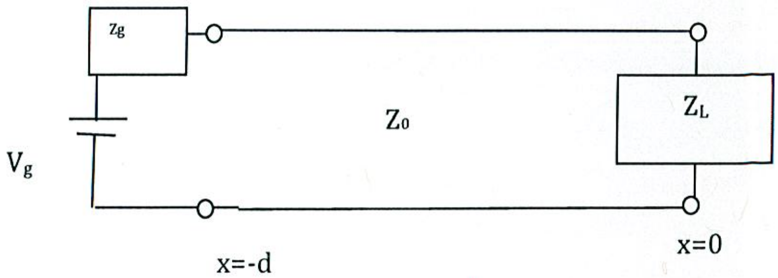
$$V^+(x) = V^+(0)\, e^{-j\beta x}$$

$$V^-(x) = V^-(0)\, e^{+j\beta x}$$

$$I^+(x) = I^+(0)\, e^{-j\beta x}$$

$$I^-(x) = I^-(0)\, e^{+j\beta x}$$

$$I^+(x) = \frac{V^+(x)}{Z_0}, \quad I^-(x) = -\frac{V^-(x)}{Z_0}$$



① $\Gamma(x) = \frac{V^-(x)}{V^+(x)} = \frac{\text{Reflected}}{\text{incident}} = \frac{V_0^- e^{+j\beta x}}{V_0^+ e^{-j\beta x}} = \underbrace{\Gamma(0)\, e^{2j\beta x}}$

How voltage Reflection Coef moves on the line!

② $\Gamma(x=0) = \frac{Z_L - Z_0}{Z_L + Z_0} = \Gamma(0)$

$$\boxed{\Gamma(x=-d) = \Gamma(0)\, e^{-2j\beta d}}$$

This is voltage Reflection coef at the input of the line

③ $Z(x=-d) = Z_0 \frac{1+\Gamma(x=-d)}{1-\Gamma(x=-d)} = Z_0 \frac{1+\Gamma_0 e^{-2j\beta d}}{1-\Gamma_0 e^{-2j\beta d}} \xrightarrow[\text{shown}]{\text{can be}} Z = Z_0 \frac{Z_L + jZ_0 \tan(\beta d)}{Z_0 + jZ_L \tan(\beta d)}$

④ $V(x=-d) = ?$

$$-V_g + I(x=-d) Z_g + V(x=-d) = 0$$

$$-V_g + \frac{V(x=-d)}{Z(x=-d)} Z_g + V(x=-d) = 0 \Rightarrow V(x=-d) = \frac{V_g}{\frac{Z_g}{Z(x=-d)} + 1}$$

$$\boxed{V(x=-d) = \frac{Z(x=-d)}{Z(x=-d) + Z_g} V_g}$$

Voltage divider

⑤ $V^+(x) = \frac{V(x)}{1+\Gamma(x)}$ why $V(x) = V^+(x) + V^-(x) = V^+(x) + \Gamma(x) V^+(x) = V^+(x)\left(1+\Gamma(x)\right)$

$V^-(x) = \Gamma(x)\, V^+(x)$

$I^+(x) = \frac{I(x)}{1-\Gamma(x)}$

$I^-(x) = -\Gamma(x)\, I^+(x)$

$\frac{V^+(x)}{Z_0} = I^+(x)$

$-\frac{V^-(x)}{Z_0} = I^-(x)$

$P = VI^*$

average Power

$P_{AV} = \frac{1}{2} VI^*$ ← take the real part