

**How about Steady-State (Phasor Form) for T-lines**

These are the  
General Coupled  
Telegraphy  
Equations in  
PHASOR Form

$$\begin{cases} \frac{d}{dz} \underline{V}_z = -(j\omega L + R) \underline{I}_z = -(j\omega L + R) \underline{I}_z \\ \frac{d}{dz} \underline{I}_z = -(G + j\omega C) \underline{V}_z \end{cases}$$

**\*\*In here  $\underline{I}_z$  and  $\underline{V}_z$  are phasors\*\***

$$\frac{\partial^2}{\partial z^2} \begin{Bmatrix} \underline{V}_z \\ \underline{I}_z \end{Bmatrix} = \gamma^2 \begin{Bmatrix} \underline{V}_z \\ \underline{I}_z \end{Bmatrix} \quad \text{Both satisfy the same equation}$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$$

Where  $\gamma$  = Propagation constant

$\alpha$  = Attenuation constant

$\beta$  = Phase constant

$$\underline{V}(z) = \underline{V}^+(z) + \underline{V}^-(z)$$

$$\underline{I}(z) = \underline{I}^+(z) + \underline{I}^-(z)$$

$$\underline{V}(z) = \underline{V}_0^+ e^{-\gamma z} + \underline{V}_0^- e^{+\gamma z} \xrightarrow{\text{lossless}} \underline{V}_0^+ e^{-j\beta z} + \underline{V}_0^- e^{+j\beta z}$$

$$\underline{I}(z) = \underline{I}_0^+ e^{-\gamma z} + \underline{I}_0^- e^{+\gamma z} \xrightarrow{\text{lossless}} \underline{I}_0^+ e^{-j\beta z} + \underline{I}_0^- e^{+j\beta z}$$

**Development of Steady-State/Phasor Formulation**

## 1. Phasor Issues:

We assume there is  $\omega$  rad/sec =  $2\pi f$  as frequency of operation. This means that the phasor notation is done for one frequency.

2. The question arises => but what if we have a  $\Delta\omega$ ? In that case, we need to look at an integrating effect of a typical phasor solution => The Fourier Treatment

## 3. In a single frequency phase, we assume all waves to be cos form.

$$A \cos(\omega t \pm \beta z) \text{ for lossless}$$

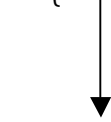
$$Ae^{-\alpha z} \cos(\omega t - \beta z) \text{ for lossy}$$

4.  $Ae^{\pm j\beta z}$  are the phasor representations of the two cases  
 $Ae^{-\alpha z} e^{\pm j\beta z} = Ae^{-\gamma z}$

$$A \cos(\omega t \pm \beta z + \phi) \rightarrow Ae^{(\pm j\beta z + \phi)}$$

5. In phasors, we assume  $e^{j\omega t}$  as the time form to get back to time domain

$$\text{Re}\{e^{j\omega t} Ae^{\pm j\beta z}\} \quad \text{or} \quad \text{Re}\{e^{j\omega t} Ae^{-\gamma z}\}$$



$$A \cos(\omega t \pm \beta z) \quad \text{or} \quad Ae^{-\alpha z} \cos(\omega t - \beta z)$$

**Note:** In the equation

$$\text{Re}\{e^{j\omega t} Ae^{-\gamma z}\}, e^{-\gamma z} = e^{-\alpha z} e^{-j\beta z}$$

6. Based on our previous discussion:

$$e^{+j\beta z} \text{ is a } +z \text{ traveling lossless propagation} \rightarrow \cos(\omega t - \beta z)$$

$$e^{-j\beta z} \text{ is a } -z \text{ traveling lossless propagation} \rightarrow \cos(\omega t + \beta z)$$

$$e^{-\gamma z} \text{ is a } +z \text{ traveling lossy propagation} \rightarrow e^{-\alpha z} \cos(\omega t - \beta z) \quad \text{**dies down as } z \rightarrow +z \text{**}$$

$$e^{+\gamma z} \text{ is a } -z \text{ traveling lossy propagation} \rightarrow e^{+\alpha z} \cos(\omega t - \beta z) \quad \text{**dies down as } z \rightarrow -z \text{**}$$