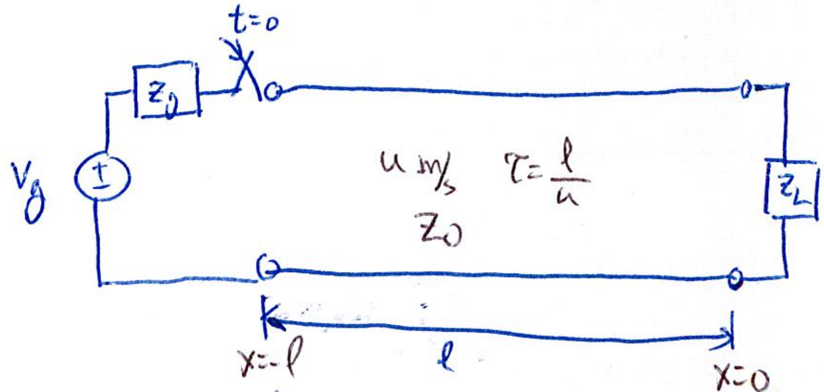
Setting up the problem



- Assuming V_g is as $V_g \cos(\omega t)$ V
- t=0 the switch closes &
- $t \gg \tau$ the bouncing of the signal going to the load reflecting → from Z_L back ← reflecting from Z_g ⋮

⟹ We reach "steady state"

$t \gg \tau$ " there standing waves of V & I "

- $V(x,t) \to \underline{V}(x)$; $I(x,t) \to \underline{I}(x)$ — we know these are "phasors"

- at any position on the line

$x = -d \qquad \underline{V}(-d) = \underline{V}^+(-d) + \underline{V}^-(-d)$

Important

Assume lossless line

$\gamma = j\beta \qquad \beta = \frac{2\pi}{\lambda}$ of the "wave" on the line $\frac{\omega}{\beta} = u$

$$\boxed{\underline{V}(x) = \underline{V}_0^+ e^{-j\beta x} + \underline{V}_0^- e^{+j\beta x}}$$

$$\boxed{\underline{I}(x) = \underline{I}_0^+ e^{-j\beta x} + \underline{I}_0^- e^{j\beta x}}$$

$Z(x) = \frac{V(x)}{I(x)}$

$\Gamma_V(x)$ = Voltage reflection coef at x $= \frac{V^-(x)}{V^+(x)}$

$I^+(x) = \frac{V^+(x)}{Z_0} \qquad I^-(x) = -\frac{V^-(x)}{Z_0}$

$\Gamma_I(x) = -\Gamma_V(x)$

* what if we know $\underline{V}_g = V_g \Rightarrow V_g \cos\omega t$ V

To see how to know V & I all along the line we need to have systematic formulation

We can alway "play"

$$\boxed{Z(x) = \frac{V(x)}{I(x)} = \frac{V_o^+ e^{-j\beta x} + V_o^- e^{+j\beta x}}{I_o^+ e^{-j\beta x} + I_o^- e^{+j\beta x}} = \frac{V_o^+ e^{-j\beta x}}{I_o^+ e^{-j\beta x}} \frac{1 + \Gamma_{(0)} e^{2j\beta x}}{1 - \Gamma_{(0)} e^{j\beta x}}}$$

But let us be more systematic

$$\Gamma_v(x) = \frac{V_i^-}{V_i^+} = \text{voltage Reflection coeff} = \frac{V_o^- e^{+j\beta x}}{V_o^+ e^{-j\beta x}} = \Gamma_{(0)} e^{2j\beta x}$$

$$\boxed{\Gamma_v(x) = \Gamma_{(0)} e^{2j\beta x}}$$

$$Z(x) = \frac{V(x)}{I(x)} = \frac{V_o^+ e^{-j\beta x} + V_o^- e^{+j\beta x}}{I_o^+ e^{-j\beta x} + I_o^- e^{+j\beta x}} = Z_o \frac{1 + \Gamma(x)}{1 - \Gamma(x)}$$

examine this at the load

$$Z(x=0) = Z_L = Z_o \frac{1 + \Gamma_L}{1 - \Gamma_L} \Rightarrow \Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$ We know that

So $\Gamma(x=-\ell) = \Gamma_{(0)} e^{2j\beta(-\ell)}$

$$\boxed{\Gamma_{(x=-\ell)} = \Gamma_{(0)} e^{-2j\beta\ell}}$$

$$Z(x=-\ell) = Z_0 \frac{1 + \Gamma(x=-\ell)}{1 - \Gamma(x=-\ell)} = Z_o$$