4/24/20 T-line voltage and current

For the following T-line system, you know $Z_L=100$ ohms $Z_0=50$ ohms $Z_g=150$ ohms and the length of the line is $\frac{300\lambda}{4}$

If you know that $V^-(x_{in}=-\frac{300\lambda}{4})$ is 10 volts

- a) Find V^+ and total I at the input of the line
- b) Find I^- at the load positions
- c) Find the average power at the load (Note: In stead sate average power is realpart of $\frac{1}{2}VI^*$)
- Can you find V₈? Explain and show if you can or cannot and find it if you think you can



- a) To do this the easiest way is to find Zin
 - a. $Z_{in} = Z(x_{in} = -\frac{300\lambda}{4}) = Z_0 \frac{1 + \Gamma_L e^{j2\beta\left(-\frac{300\lambda}{4}\right)}}{1 \Gamma_L e^{j2\beta\left(-\frac{300\lambda}{4}\right)}} = 100 \ \Omega \ NOTE \ \frac{300\lambda}{4} = \frac{150\lambda}{2}$ this is integer number of half wavelength

b.
$$\Gamma\left(x_{in} = -\frac{300\lambda}{4}\right) = \Gamma_{in} = \Gamma_L e^{j2\beta\left(-\frac{300\lambda}{4}\right)} \quad \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{1}{3} \qquad e^{j2\beta\left(-\frac{300\lambda}{4}\right)} = e^{-j2\frac{2\pi}{\lambda}\left(-\frac{300\lambda}{4}\right)} = 1 \rightarrow \Gamma_{in} = \frac{1}{3}$$

c.
$$\Gamma_{in} = \frac{V_{in}^-}{V_{in}^+}$$
 so. $V_{in}^+ = 10(\frac{1}{\frac{1}{3}}) = 30 \text{ V}$

d.
$$I_{in} = \frac{V_{in}}{Z_{in}} = \frac{40}{100} = 0.4 A = I_{in}$$

$$\begin{aligned} \text{d.} \quad &I_{in} = \frac{v_{in}}{z_{in}} = \frac{40}{100} = 0.4 \ A = I_{in} \\ \text{b)} \quad &I_{in}^{-} = -\frac{v_{in}^{-}}{z_{0}} = -\frac{10}{50} = -0.2 \ A = I_{in}^{-} \quad &I_{in}^{+} = \frac{v_{in}^{+}}{z_{0}} = \frac{30}{50} = 0.6 \ \text{A} = I_{in}^{+} \end{aligned}$$

c) Since this is lossles should it be different that the one at the input? Let us see. As you can see they are the same. However, for tests, games etc. we need to show what the question is asking for.

a.
$$P_{in\ ave} = \frac{1}{2}V_{in}I_{in}^* = \frac{1}{2}(40)(0.4) = 8 \text{ W} = P_{in\ ave}$$

b.
$$P_{L ave} = \frac{1}{2} V_L I_L^* = \frac{V_L V_L^*}{2Z_L^*} = \frac{(40)(0.4)}{2} = 8 \text{ W}$$

d) We can find V_g how? $V_{in} = V_g \frac{Z_{in}}{Z_g + Z_{in}} \rightarrow V_g = V_{in} \frac{Z_g + Z_{in}}{Z_{in}} = 40 \frac{150 + 100}{100} = 100V = V_g$