EE311

Fall 2020

Plane waves and BC and power review -doing a complete incident, reflected, transmitted (A Solution)

A boundary between two dielectric media is at z=0 (z<0 is air $\eta_1 = 120\pi\ ohms$, z>0 $\eta_2 = 80\pi\ ohms$). The incident field is impinging on the boundary from z<0

$\vec{\boldsymbol{H}}_{\boldsymbol{inc}} = \boldsymbol{e^{-j6z}}\ \hat{\boldsymbol{x}}\ \frac{\boldsymbol{mA}}{\boldsymbol{m}}$ given in z<0 we can assume that the second media has $\boldsymbol{\beta_2}$

a) In the first media, can we find the frequency of operation? Explain/show.
b) Is the frequency of operation be the same in the second media?
c) Find $\vec{E}_{inc}, \vec{E}_{ref}, \vec{H}_{ref}, \vec{E}_{trans}, \vec{H}_{trans}$
d) Find the incident, reflected, and transmitted average powers ($\frac{1}{2}\ \vec{E}x\vec{H}^*$).
e) Discuss if the powers balance. (you should **show** this part, indicating what is the total of Incident and reflected and if it is the same as transmitted, and not just say that it balances)

Here is a solution

a) The answer is yes. The first media is air. Speed of EM in air is $c \cong 3x10^8\frac{m}{s} = \frac{1}{\sqrt{\mu_0\epsilon_0}}$
We also know that $Phase\ speed = u_p = \frac{\omega}{\beta} \to \omega = \beta u_p = (6)c = 18x10^8\frac{rad}{s}$
So frequency in Hz, which means number of cycles per second is $f = \frac{3x10^8}{2\pi}$ Hz
Angular frequency is $18x10^8\frac{rad}{s}\ and\ frequency\ is \frac{3x10^8}{2\pi}$ Hz
b) The frequency of operation will not change (this is a linear system input and output have the same frequencies). So, $f\ and\ \omega$ would be the same in both media
c) Here is who to find them all, we need to identify, magnitudes, direction of propagations and direction of oscillations

$\vec{E}_{inc} = \eta_1(\boldsymbol{e^{-j6z}})\ \hat{\boldsymbol{a}}_{\boldsymbol{Ei}}\ \frac{\boldsymbol{mV}}{\boldsymbol{m}}$ **we know** $\hat{\boldsymbol{a}}_{\boldsymbol{Ei}}\ \boldsymbol{x}\ \hat{\boldsymbol{a}}_{\boldsymbol{Hi}} = \hat{\boldsymbol{z}} \to \hat{\boldsymbol{a}}_{\boldsymbol{Ei}}\ \boldsymbol{x}\ \hat{\boldsymbol{x}} = \hat{\boldsymbol{z}} \to \hat{\boldsymbol{a}}_{\boldsymbol{Ei}} = -\hat{\boldsymbol{y}}$

$$\vec{E}_{inc} = 120\pi(\boldsymbol{e^{-j6z}})\ (-\hat{\boldsymbol{y}})\ \frac{\boldsymbol{mV}}{\boldsymbol{m}}$$

$$\vec{H}_{inc} = e^{-j6z}\ \hat{x}\ \frac{mA}{m}$$

$$\text{Reflection coefficient} = \Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \frac{80\pi-120\pi}{80\pi+120\pi} = -\frac{1}{5} = \frac{E_r}{E_i}$$

So.

$$\vec{E}_{ref} = -\frac{1}{5}(\eta_1)(e^{+j6z})\ (-\hat{y})\ \frac{mV}{m} = 24\pi e^{+j6z}\hat{y}\ \frac{mV}{m} = \vec{E}_{refl}$$

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- **Note: Since reflection coeff is negative, $\vec{E}_{inc}$ *is in* $\hat{y}$ *direction and* is in opposite direction (is in $\hat{y}$) *compared to* $\vec{E}_{refl}$ which is in -$\hat{y}$.**

$$\vec{H}_{ref} = \frac{\mathbf{1}}{5} e^{+j6z}\,\hat{x} \quad \frac{mA}{m}$$

Please note that. $\vec{E}_{inc}\ x\ \vec{H}_{inc}$ $is\ in\ \hat{z}$ z direction and $\vec{E}_{refl}\ x\ \vec{H}_{refl}$ $is\ in - \hat{z}$

$$Transmission\ Coeff = T = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{160\pi}{200\pi} = \frac{4}{5} = \frac{E_{trans}}{E_{inc}}$$

$$\vec{E}_{trans} = \frac{4}{5}\eta_1(\boldsymbol{e^{-j\beta_2 z}})\,(\boldsymbol{-\hat{y}}) = \boldsymbol{-96\pi e^{-j\beta_2 z}\hat{y}} \quad \frac{\boldsymbol{mV}}{\boldsymbol{m}}$$

$$\vec{H}_{trans} = \frac{96\boldsymbol{\pi}}{80\boldsymbol{\pi}}(\boldsymbol{e^{-j\beta_2 z}})\,(\hat{\boldsymbol{x}}) \quad \frac{mA}{m}$$

$$\vec{H}_{trans} = 1.2(\boldsymbol{e^{-j\beta_2 z}})\,(\hat{\boldsymbol{x}}) \quad \frac{mA}{m}$$

$$\vec{H}_{trans} = 1.2\boldsymbol{e^{-j\beta_2 z}\hat{x}} \quad \frac{mA}{m}$$

Now the energy, is it balanced?

If so $\vec{S}_{inc} + \vec{S}_{ref} \overset{?}{=} \vec{S}_{trnas}$ $\quad \vec{S}_x = \frac{1}{2}\vec{E}_x\, x\vec{H}_x^*$

$\frac{1}{2}\vec{E}_{inc}\, x\vec{H}_{inc}^* + \frac{1}{2}\vec{E}_{refl}\, x\vec{H}_{refl}^*$ and $\frac{1}{2}\vec{E}_{trans}\, x\vec{H}_{trans}^*$

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$\vec{S}_{inc} = \frac{120\pi}{2}(\boldsymbol{e^{-j6z}})\,(-\boldsymbol{\hat{y}})\ \boldsymbol{x}\ e^{+j6z}\,\hat{x} = 60\pi\,\hat{z}\ \frac{\mu W}{m^2}$

$\vec{S}_{ref} = \frac{1}{2}\ 24\pi\left(e^{+j6z}\right)\hat{y}\ \ x\frac{\mathbf{1}}{5}e^{-j6z}\,\hat{x} = -\frac{\mathbf{12}}{5}\pi\hat{z}\ \frac{\mu W}{m^2}$

$\vec{S}_{trans} = \frac{1.2}{2}\left((96)\pi e^{-j\beta_2 z}\,\hat{y}\right)\,x\ \ e^{+j\beta_2 z}\,\hat{x} = 57.6\,\pi\hat{z}\,\frac{\mu W}{m^2}$

$$\vec{S}_{inc} + \vec{S}_{ref} = \pi\hat{z} = \frac{288}{5}\pi\hat{z} = 57.6\pi\hat{z}\ \frac{\mu W}{m^2}$$

so $\vec{S}_{inc} + \vec{S}_{ref} = \vec{S}_{trans}$