How about Steady-State (Phasor Form) for T-lines

These are the General Coupled Telegraphy Equations in PHASOR Form

$$\begin{cases} \dfrac{d}{dz}\underline{V}_z = -(j\omega LI + RI) = -(j\omega L + R)\underline{I}_z \\ \dfrac{d}{dz}\underline{I}_z = -(G + j\omega C)\underline{V}_z \end{cases}$$

In here $\underline{I}_z$ and $\underline{V}_z$ are phasors

$$\frac{\partial^2}{\partial z^2}\begin{Bmatrix} \underline{V}_z \\ \underline{I}_z \end{Bmatrix} = \gamma^2 \begin{Bmatrix} \underline{V}_z \\ \underline{I}_z \end{Bmatrix}$$ Both satisfy the same equation

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$$

Where γ = Propagation constant
α = Attenuation constant
β = Phase constant

$$\underline{V}(z) = \underline{V}^+(z) + \underline{V}^-(z)$$

$$\underline{I}(z) = \underline{I}^+(z) + \underline{I}^-(z)$$

$$\underline{V}(z) = \underline{V}_0^+ e^{-\gamma z} + \underline{V}_0^- e^{+\gamma z} \xrightarrow{\text{lossless}} \underline{V}_0^+ e^{-j\beta z} + \underline{V}_0^- e^{+j\beta z}$$

$$\underline{I}(z) = \underline{I}_0^+ e^{-\gamma z} + \underline{I}_0^- e^{+\gamma z} \xrightarrow{\text{lossless}} \underline{I}_0^+ e^{-j\beta z} + \underline{I}_0^- e^{+j\beta z}$$

Development of Steady-State/Phasor Formulation

1. Phasor Issues:
   We assume there is ω rad/sec = 2πf as frequency of operation. This means that the phasor notation is done for one frequency.

2. The question arises => but what if we have a Δω? In that case, we need to look at an integrating effect of a typical phasor solution => The Fourier Treatment

3. In a single frequency phase, we assume all waves to be cos form.

$$A\cos(\omega t \pm \beta z) \text{ for lossless}$$
$$Ae^{-\alpha z}\cos(\omega t - \beta z) \text{ for lossy}$$

4. $Ae^{\pm j\beta z}$, $Ae^{-\alpha z}e^{\pm j\beta z} = Ae^{-\gamma z}$ are the phasor representations of the two cases

$$A\cos(\omega t \pm \beta z + \phi) \rightarrow Ae^{(\pm j\beta z + \phi)}$$

5. In phasors, we assume $e^{j\omega t}$ as the time form to get back to time domain

$$\mathrm{Re}\left\{e^{j\omega t}Ae^{\pm j\beta z}\right\} \quad or \quad \mathrm{Re}\left\{e^{j\omega t}Ae^{-\gamma z}\right\}$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

$$A\cos(\omega t \pm j\beta z) \quad or \quad Ae^{-\alpha t}\cos(\omega t - \beta z)$$

Note: In the equation $\mathrm{Re}\left\{e^{j\omega t}Ae^{-\gamma z}\right\}$, $e^{-\gamma z} = e^{-\alpha z}e^{-j\beta z}$

6. Based on our previous discussion:

$e^{-j\beta z}$ is a +z traveling lossless propagation $\rightarrow$ $\cos(\omega t-\beta z)$

$e^{+j\beta z}$ is a -z traveling lossless propagation $\rightarrow$ $\cos(\omega t+\beta z)$

$e^{-\gamma z}$ is a +z traveling lossy propagation $\rightarrow$ $e^{-\alpha z}\cos(\omega t-\beta z)$ **dies down as z $\rightarrow$ + z**

$e^{+\gamma z}$ is a -z traveling lossy propagation $\rightarrow$ $e^{+\alpha z}\cos(\omega t-\beta z)$ **dies down as z $\rightarrow$ -z**