Transient Issues:

1. We know how to derive the coupled wave equations for V and I. For the lossless case we have:

$$\frac{\partial \mathrm{V}}{\partial z} = \frac{-L\partial I}{\partial t} \text{ and } \frac{\partial \mathrm{I}}{\partial z} = \frac{-C\partial V}{\partial t}$$

Please note that V = V(z,t) and I = I(z,t)

2. We can decouple these and get the wave equation

$$\frac{\partial^2}{\partial z^2}\begin{Bmatrix} V \\ I \end{Bmatrix} = LC\frac{\partial^2}{\partial t^2}\begin{Bmatrix} V \\ I \end{Bmatrix}$$

3. We assume:

   $V(z,t) = V^+(z,t) + V^-(z,t)$
   $I(z,t) = I^+(z,t) + I^-(z,t)$

   Which means:

   a) Each V and I at any point on the line at any time is generally a linear combination superposition of the positive and negative traveling waves.
   b) A positive traveling wave means that the wave is going in the +z direction. V^+ means that voltage is going to +z and it can be negative or positive in magnitude.
   c) Always be careful about I. Generally speaking I^+ is a current going to the +z direction and I^- is a current going to the –z direction.
   d) While I is a scalar, it has a kind of direction associated with it.
      If 2A goes to +z and 2A goes to -z at (z_1,t_1), we know that the next current at (z_1,t_1) is 0. So while I is a scalar, the direction +z vs. –z needs to be considered.

4.

$$\left.\begin{aligned} V(z,t) &= V^+(t-\frac{z}{u}) + V^-(t+\frac{z}{u}) \\ I(z,t) &= I^+(t-\frac{z}{u}) + I^-(t+\frac{z}{u}) \end{aligned}\right\}$$ These are the general definitions assuming no functional form.

$$\frac{\partial V}{\partial z} = -\frac{1}{u}V^{+}(t-\frac{z}{u}) + \frac{1}{u}V^{-'}(t+\frac{z}{u})$$
$$\frac{\partial V}{\partial t} = V^{+'}(t-\frac{z}{u}) + V^{-'}(t+\frac{z}{u})$$

u = phase speed → speed for constant phase

$$\frac{\partial}{\partial t}(t-\frac{z}{u})=0 \;\rightarrow\; \frac{+1}{u}\frac{\partial z}{\partial t}=1 \;\rightarrow\; \frac{\partial z}{\partial t}=u \;\rightarrow\; +\text{ traveling}$$

$$\frac{\partial}{\partial t}(t+\frac{z}{u})=0 \;\rightarrow\; \frac{\partial z}{\partial t}=-u \;\rightarrow\; -\text{ traveling}$$

$$\frac{\partial I}{\partial z}=-\frac{1}{u}I^{+'}(t-\frac{z}{u})+\frac{1}{u}I^{-'}(t+\frac{z}{u})$$

$$\frac{\partial I}{\partial t}=I^{+}(t-\frac{z}{u})+I^{-'}(t+\frac{z}{u})$$

Note 1:

$$\frac{\partial V^{+}(t-\frac{z}{u})}{\partial z}=\frac{\partial V^{+}(t-\frac{z}{u})}{\partial(t-\frac{z}{u})}\frac{\partial(t-\frac{z}{u})}{\partial z}=-\frac{1}{u}V^{+'}(t-\frac{z}{u})$$

$$\frac{\partial V^{-}(t+\frac{z}{u})}{\partial z}=\frac{\partial V^{-}(t+\frac{z}{u})}{\partial(t+\frac{z}{u})}\frac{\partial(t+\frac{z}{u})}{\partial z}=\frac{1}{u}V^{-'}(t+\frac{z}{u})$$

This note defines $V^{+'}$, $V^{-'}$, $I^{+'}$, and $I^{-'}$

5. $$\frac{\partial V}{\partial z}=-L\frac{\partial I}{\partial t} \;\rightarrow\; \frac{-1}{u}V^{+'}+\frac{1}{u}V^{-'}=-L(I^{+'}+I^{-'}) \;\rightarrow\; \boxed{I^{+}+I^{-}=\frac{V^{+'}}{uL}-\frac{V^{-'}}{uL}}$$

Note: The equation written above is important!

a) The + traveling items $I^{+'}=\frac{V^{+'}}{uL}$ should match

b) The – traveling items $I^{-'}=-\frac{V^{-'}}{uL}$ should match

By integration one can see (note: at t = 0, I and V don't exist), it is the same relation as $I^{+}=\frac{V^{+}}{uL}$ and $I^{-}=-\frac{V^{-}}{uL}$.

6.

$$\frac{\partial I}{\partial z}=-C\frac{\partial V}{\partial t}$$

$$-\frac{1}{u}I^{+'}+\frac{1}{u}I^{-'}=-C(V^{+'}+V^{-'})$$

$$\rightarrow \quad I^{+'} + I^{-'} = uCV^{+'} - uCV^{-'}$$

Again, we can see that: $I^{+} = uCV^{+}$ and $I^{-} = -uCV^{-}$.

7. So a) $uC = \dfrac{1}{uL} \rightarrow u^2 = \dfrac{1}{LC} \rightarrow u = \dfrac{1}{\sqrt{LC}}$

   b) $uC = \sqrt{\dfrac{C}{L}}$ and $uL = \sqrt{\dfrac{L}{C}}$

   $\therefore I^{+} = \dfrac{V^{+}}{\sqrt{\dfrac{L}{C}}}$ and $I^{-} = -\dfrac{V^{-}}{\sqrt{\dfrac{L}{C}}} \rightarrow \sqrt{\dfrac{L}{C}} = R_o$ (Characteristic Impedance)

   => $I^{+} = \dfrac{V^{+}}{R_o}$ and $I^{-} = -\dfrac{V^{-}}{R_o}$

8. Two important items:

   a) $R_o = \sqrt{\dfrac{L}{C}}$ comes out of the equation and is characteristic impedance

   ** R_o is a special ratio of L and C and relates V to I on positive and negative traveling waves**

   b) $I^{-} = -\dfrac{V^{-}}{R_o}$

   This – comes out of the formulation and makes physical sense.

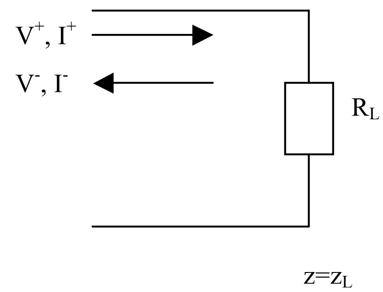
   If I^{+} is going +z, it will be > 0. The same going to –z will be < 0.

   As you can see, while current is a scalar, there's a direction (direction of moving charges) associated with it.

Note: By unit analysis, $R_o = \sqrt{\dfrac{L}{C}}$ => ohms; this is like a load. Even in a lossless line, the way we balance energy between series inductance and shunt capacitance acts like a loading factor R_o which is called characteristic impedance.

9. Reflection Coefficients:

   a) At the load:



At $z=z_L$ and $t = \tau$,

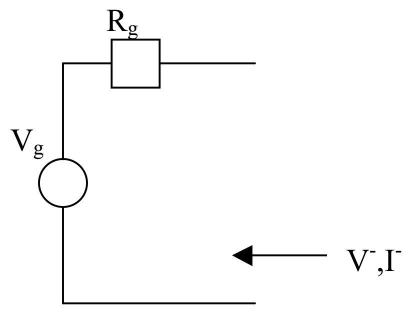
$$\frac{V_{Total}}{I_{Total}} = R_L = \frac{V^+(z_L,\tau)+V^-(z_L,\tau)}{I^+(z_L,\tau)+I^-(z_L,\tau)} \rightarrow \overset{R_o}{\overset{\uparrow}{\frac{V^+(z_L,\tau)}{I^+(z_L,\tau)}}} \frac{1+\frac{V^-(z_L,\tau)}{V^+(z_L,\tau)}}{1+\frac{I^-(z_L,\tau)}{I^+(z_L,\tau)}}$$

$$\text{Reflection Coefficient} = \Gamma = \frac{Reflected}{Transmitted} = \frac{V^-}{V^+}$$

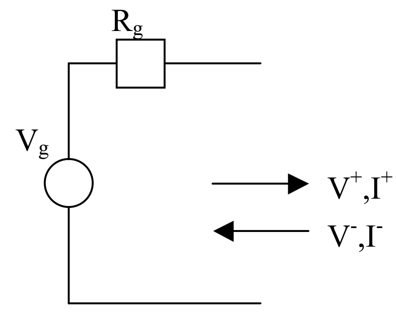
$$\Gamma_L = \Gamma_{at\,load} = \frac{V_L^-}{V_L^+} \text{ and } \frac{I_L^-}{I_L^+} = \frac{-\frac{V_L^-}{R_0}}{\frac{V_L^+}{R_0}} = -\Gamma_L$$

$$R_L = R_0\frac{1+\Gamma_L}{1-\Gamma_L} \rightarrow \boxed{\Gamma_L = \frac{R_L - R_0}{R_L + R_0}} = \frac{V^-}{V^+} \text{ at } z=z_L$$

What about $t = 2\tau$, when the wave reaches the load and bounces back to get back to the $z=z_{in}$ position?



$t = 2\tau^-$



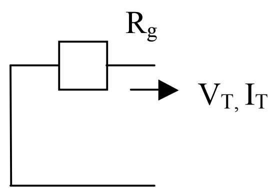
$z = z_{in}$
$t = 2\tau^{-+}$ (ref. happens)

$\Gamma g = \dfrac{V^+(z = z_{in})}{V^-(z = z_{in})}$ for transient the inc. is V^- and reflected is V^+

Note: If $V_g \rightarrow 0$ at $t = 2\tau$ => the source was a pulse

Then, at $z = z_{il} + \dfrac{V(z_{in}, 2\tau^+)}{I(z_{in}, 2\tau^+)} = R_g$

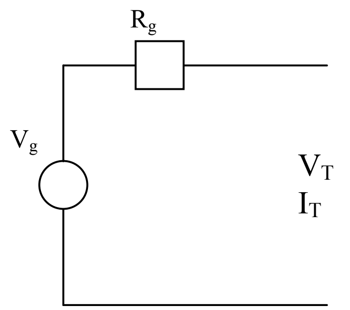
$$+\frac{V^+ + V^-}{\dfrac{V^+}{R_O} - \dfrac{V^-}{R_O}} = R_g \qquad\qquad + \frac{V(\Gamma_g + 1)}{\dfrac{V}{R_O}(\Gamma_g - 1)} = R_g$$



$$R_O\left(\frac{\Gamma_g + 1}{\Gamma_g - 1}\right) = R_g \quad => \quad \Gamma_g = \frac{R_g - R_O}{R_g + R_O} = \frac{V^+}{V^-}$$

at $z = z_{in}$

The difficulty of the conceptual picture arises when we need to include V_g.



$z=z_{in}$

$z=z_{in}$
$t = 2\tau^+$

$$V_g - V_T = I_T R_g$$

$$V_g - (V^+ + V^-) = (I^+ + I^-)R_g$$

$$V_g - V^+(1 + \frac{1}{\Gamma_g}) = \frac{V^+}{R_O}(1 - \frac{1}{\Gamma_g})R_g$$

$$V^+\left[\frac{R_g}{R_O} - \frac{R_g}{R_O\Gamma_g} + 1 + \frac{1}{\Gamma_O}\right] = V_g$$

$$V^+\left[\frac{R_g}{R_O} + 1 - \frac{1}{\Gamma_O}(\frac{R_g}{R_O} - 1)\right] = V_g$$

$$V^+\left[1 - \frac{1}{\Gamma_O}\frac{\frac{R_g}{R_O} - 1}{\frac{R_g}{R_O} + 1}\right] = \frac{V_g}{\frac{R_g}{R_O} + 1}$$

$$V^+ - V^+\frac{V^-}{V^+}\frac{R_g - R_O}{R_g + R_O}$$

This is the contribution due to V^- (coming toward) the generator.

At $t = 2\tau^+$, $z=z_{in}$

$$V^+ = V_g\frac{R_O}{R_O + R_g} + \frac{R_g - R_O}{R_g + R_O}V^-$$

Total + traveling voltage at $t = 2\tau^+$. This is going towards the load.

This is the contribution to V^+ due to V_g (which is on at $t = 2\tau^+$). V_{1a}^+ is at $t = 0^+$

Let us focus on:

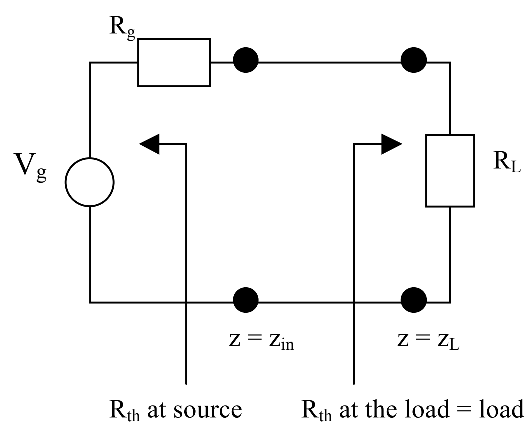
$$\frac{R_g - R_O}{R_g + R_O} V^-$$

But we know, $\Gamma_g = \dfrac{V^+}{V^-}$ → $V^+_{due\ to\ reflection} = \Gamma_g V^-$

→ $\boxed{\Gamma_g = \dfrac{R_g - R_O}{R_g + R_O}} = \dfrac{V^+}{V^-}\Bigg|_{z=z_{in}}$ This is a transient phenomenon!

Transient Thinking and Concepts Continued…

Note: Reflection coefficient at the load = $\Gamma_L = \dfrac{R_L - R_0}{R_L + R_0} = \dfrac{V^-}{V^+}$



Γ_g = *reflection coefficient at the generator* $= \dfrac{V^+_{z=z_{in}}}{V^-_{z=z_{in}}} = \dfrac{R_g - R_O}{R_g + R_O}$

R_L = is the effective load at $z = z_L$ looking toward the load (to z^+)

R_g = is the Thevenin Equivalent R_{th} of the generator. This means at $z = z_{in}$ to find R_g, one needs to look into the source (generator) at the input (to z^-).

Power Discussion:

At any point in time and space (z_1,t_1), we can calculate power as:

$$P(z_1,t_1) = V_{Total}(z_1,t_1)I_{Total}(z_1,t_1)$$

$$= [V^+(z_1,t_1) + V^-(z_1,t_1)][I^+(z_1,t_1) + I^-(z_1,t_1)]$$

$$= \underbrace{V^+(z_1,t_1)I^+(z_1,t_1)}_{1} + \underbrace{V^-(z_1,t_1)I^-(z_1,t_1)}_{2} + \underbrace{V^+(z_1,t_1)I^-(z_1,t_1)}_{3} + \underbrace{V^-(z_1,t_1)I^+(z_1,t_1)}_{4}$$

1) This is + traveling power = $V^+I^+ = \dfrac{V^+V^+}{R_O} = \dfrac{{V^+}^2}{R_O} = R_O {I^+}^2$ always positive

2) This is – traveling power = $V^-I^- = V^-\left(-\dfrac{V^-}{R_O}\right) = -\dfrac{{V^-}^2}{R_O} = -R_O {I^-}^2$ always negative

3) & 4) are mixed terms:

$V^+I^- + V^-I^+ = V^+\left(-\dfrac{V^-}{R_O}\right) + V^-\left(\dfrac{V^+}{R_O}\right) = 0$ The cross terms add to 0!

$$=> P_{Total} = P^+ + P^- \text{ at all } (z,t)$$

$$P^+ = \frac{{V^+}^2}{R_O} \qquad P^- = -\frac{{V^-}^2}{R_O}$$

As you can see, $P^+ > 0$ and $P^- < 0$ which means power also has direction.