EE 330

Homework 7 Solutions

Fall 2020

Solution 1

From the process parameters:

$$V_{TH} = 0.79V, \frac{W}{L} = \frac{3}{0.6}, \frac{\mu Cox}{2} = 56.4 * 10^{-6}\frac{A}{V^2}$$

I will start by assuming saturation, meaning:

$$I_D = \frac{\mu Cox}{2}\frac{W}{L}(V_{GS} - V_T)^2 = 2.82 * 10^{-4}(3 - V_S - 0.79)^2 = 2.82 * 10^{-4}(2.21 - V_S)^2$$

Since $I_D = I_S = \frac{V_S}{2*10^3} \Rightarrow \frac{V_S}{2*10^3} = 2.82 * 10^{-4}(2.21 - V_S)^2$

So $1.773V_S = 4.8841 - 4.42V_s + V_s^2 \Rightarrow V_s^2 - 6.193V_s + 4.8841 = 0$

$\therefore V_S = \begin{cases} 5.265V \\ 0.928V \end{cases}$ and since $V_{DS} > V_{GS} - V_T$ it has to be 0.928V, because

for $V_S = 5.265V, V_{GS} < V_T$

So for $V_S = 0.928V$, $I_D = 0.463mA$ and $V_{out} = 9.07V$ and $V_{DS} = 9.07 - 0.928 > V_{GS} - V_T$

Solution 2

From the process parameters:

$$V_{TH} = 0.79V, \frac{W}{L} = \frac{3}{0.6}, \frac{\mu Cox}{2} = 56.4 * 10^{-6}\frac{A}{V^2}$$

$$V_G = V_{dd} * \frac{90}{90 + 10} = 9V$$

I will start by assuming saturation, meaning:

$$I_D = \frac{\mu Cox}{2}\frac{W}{L}(V_{GS} - V_T)^2 = 2.82 * 10^{-4}(9 - V_S - 0.79)^2 = 2.82 * 10^{-4}(8.21 - V_S)^2$$

Since $I_D = I_S = \frac{V_S}{2*10^3} \Rightarrow \frac{V_S}{2*10^3} = 2.82 * 10^{-4}(2.21 - V_S)^2$

So $0.564V_S = 67.4 - 16.42V_s + V_s^2 \Rightarrow V_s^2 - 16.984V_s + 67.4 = 0$

$\therefore V_S = \begin{cases} 10.66V \\ 6.32V \end{cases}$. The answer cannot be 10.66V since Vdd is only 10V. For Vs = 6.32V

$I_D = 1mA$ and $V_{out} = 5V$ but $V_{DS} = 5 - 6.32 < V_{GS} - V_T$

This means it is not saturation, so it has to be in the triode region.

$$I_D = \mu Cox\frac{W}{L}\left(V_{GS} - V_T - \frac{V_{DS}}{2}\right)^2 = 5.64 * 10^{-4}\left(9 - V_S - 0.79 - \frac{V_{DS}}{2}\right)V_{DS}$$

$$I_D = \frac{V_S}{2*10^3} = \frac{10 - V_D}{5*10^3} \Rightarrow V_s = 4 - 0.4V_D \Rightarrow V_{DS} = V_D - 4 + 0.4V_D = 1.4V_D - 4$$

From the above 2 equations:

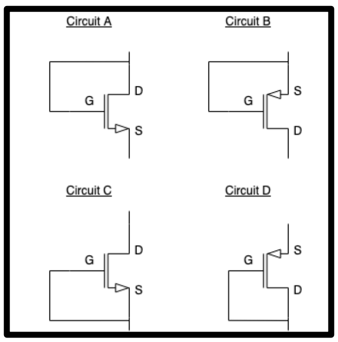
$$\frac{10 - V_D}{5*10^3} = 5.64*10^{-4}\left(9 - (4 - 0.4V_D) - 0.79 - \frac{1.4V_D - 4}{2}\right)(1.4V_D - 4)$$

$\Rightarrow V_D = \begin{cases} 21.22V \\ 3.186V \end{cases}$, V_D of $21.22V$ is impossible since Vdd is 10V, so the answer must be $V_{out} = 3.186V$

If we check for $V_{DS} < V_{GS} - VT = 9 - 2.726 = 6.27V$ then we confirm we are in the triode region.

Solution 3

The terminals of each device are presented below.



Recall that the operating mode of an NMOS or PMOS device can be characterized using the equations provided below.

N-Channel MOSFET Operating Modes	
Cutoff	$V_{GS} < V_{Tn}$
Triode	$V_{GS} \geq V_{Tn}$ and $V_{DS} < V_{GS} - V_{Tn}$
Saturation	$V_{GS} \geq V_{Tn}$ and $V_{DS} \geq V_{GS} - V_{Tn}$
P-Channel MOSFET Operating Modes	
Cutoff	$V_{SG} < \lvert V_{Tp} \rvert$
Triode	$V_{SG} \geq \lvert V_{Tp} \rvert$ and $V_{SD} < V_{SG} - \lvert V_{Tp} \rvert$
Saturation	$V_{SG} \geq \lvert V_{Tp} \rvert$ and $V_{SD} \geq V_{SG} - \lvert V_{Tp} \rvert$

We can now figure out the operating mode for each circuit. We begin with Circuit A. Note that the gate and drain of Circuit A are tied together, meaning that $V_{GS} = V_{DS}$. As a result of this, because $V_{DS} \geq V_{Tn}$, we can say that the device is not in the cutoff operating mode. We can further observe that the saturation condition is always true for this configuration, meaning that the device is in saturation: $V_{DS} \geq V_{GS} - V_{Tn} \rightarrow 0 \geq -V_{Tn} \rightarrow True$

Circuit D can be approached similarly. Here, the gate and drain of Circuit B are tied together, meaning that $V_{SG} = V_{SD}$. As a result of this, because $V_{SD} \geq |V_{Tp}|$, we can say that the device is not operating in cutoff. We can further observe that the saturation condition is always true for this configuration, meaning that the device is in saturation: $V_{SD} \geq V_{SG} - |V_{Tp}| \rightarrow 0 \geq -|V_{Tp}| \rightarrow True$

Circuit B and Circuit C are even easier than Circuit A and D. Here, we note that the gate and source in both circuits are shorted together. As a result of this, $V_{GS} = 0$ for Circuit C and $V_{SG} = 0$ for Circuit B. So, both devices are in cutoff.

Solution 4

From the process parameters:

$$V_{TH} = 0.79V, \frac{W}{L} = \frac{3}{0.6}, \frac{\mu Cox}{2} = 56.4 * 10^{-6} \frac{A}{V^2}$$

Since the early voltage is $100V$ that mean $\lambda = \frac{1}{100} = 0.01$

$$V_{TH} = V_{TH0} + \gamma\left(\sqrt{\varphi - V_{BS}} - \sqrt{\varphi}\right) = 0.79 + 0.4(\sqrt{\varphi} - \sqrt{\varphi})$$

I will start by assuming saturation:

Assuming saturation, $I_{DS} = \frac{\mu Cox}{2}\frac{W}{L}(V_{GS} - V_T)^2 = 56.4 * 10^{-6} * \frac{3}{0.6} * (1 - 0.79)^2(1 + \lambda V_{DS})$

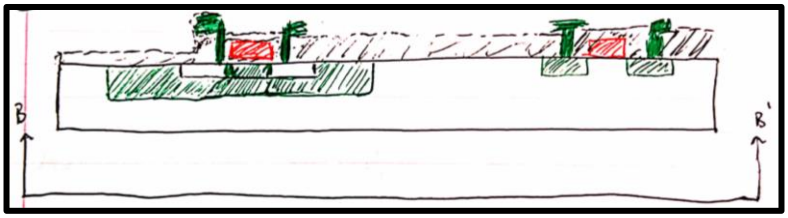
$\Rightarrow I_D = 1.244 * 10^{-5}(1 + 0.01V_{DS})$and $I_D = \frac{VDD - VD}{R} = \frac{10 - V_D}{5000}$

$$\frac{10 - V_D}{5000} = 1.244 * 10^{-5}(1 + 0.01V_{DS}) \Rightarrow V_{DS} = V_D = V_{out} = 9.93V > V_{GS} - VT$$

Solution 5

The left-most device is composed of an n-active region and a polysilicon layer placed on top of it, with interconnects used to connect to either side of the polysilicon and the polysilicon itself. This forms an NMOS device. The right-most device is composed of a serpentine polysilicon layer with interconnects on either side of it, forming a resistor.

Solution 6



Solution 7

Guessing F.A:

$I_C = \beta I_B \Rightarrow 100 I_B = I_C$ *and* $V_{BE} = V_B - V_E = 0.7V$

$$I_C + I_B = I_E = 101 I_B \Rightarrow \frac{V_E}{1} = 101 \frac{2 - V_B}{100} \Rightarrow V_B - 0.7 = 2.02 - 1.01 V_B \Rightarrow V_B = 1.35V$$

$\Rightarrow V_E = 0.65V$

$I_C = 0.65mA \Rightarrow V_C = 4.35V > 0.2$ so it was in F.A.

Solution 8

$$V_t = \frac{kT}{q} = 8.62 * 10^{-5} * 273 = 0.0235$$

$$J_S = J_{SX}(T^m e^{-\frac{V_{G0}}{V_t}}) 20 * 10^{-3} \left(273^{2.3} e^{-\frac{1.17}{0.0235}}\right) = 1.914 * 10^{-18}$$

Guessing F.A:

$$I_B = \frac{J_S A_e}{\beta} e^{\frac{V_{BE}}{V_t}}\ ;\ I_C = J_S A_e e^{\frac{V_{BE}}{V_t}},\ I_E = I_B$$

$$\frac{2 - V_{BE}}{100 * 10^3} = \frac{1.914 * 10^{-18} 100 * 10^{-6}}{100} e^{\frac{V_{BE}}{0.0235}} \Rightarrow V_{BE} = 1V$$

Used Excel Goal seek to solve

$$I_C = J_S A_e e^{\frac{V_{BE}}{V_t}} = \beta I_B = 100 * \frac{2 - 1}{100 * 10^3} = 1mA \Rightarrow V_{out} = 4 > 0.2$$

Solution 9

To begin, note that M_1 and M_2 have the same base-emitter voltage as each other since their bases and emitters are shorted. Then, recall that current through a forward active BJT can be modeled using the equation shown below.

$$I_C = J_S A_E e^{\frac{V_{BE}}{V_t}}$$

The currents through each device can then be modelled as follows:

$$I_{M1} = I_{IN} = J_S A_{E1} e^{\frac{V_{BE1}}{V_t}}$$

$$I_{M2} = J_S A_{E2} e^{\frac{V_{BE2}}{V_t}}$$

By dividing the two currents, we can obtain the following expression:

$$\frac{I_{IN}}{I_{M2}} = \frac{A_{E1}}{A_{E2}}$$

Which can then be solved for to find the current through the load resistor:

$$I_{M2} = \frac{A_{E2}}{A_{E1}} I_{IN}$$