EE 330

Homework 8 Solutions

Fall 2020

Solution 1

The goal of a small-signal analysis is to take a nonlinear mathematical expression, such as the square-law model of a MOSFET, and to linearize it at a specific point. The point that the circuit's expression is linearized at is known by a number of names, such as the "quiescent point" or "operating point" of the circuit. This is particularly useful for circuits which use nonlinear components in them, such almost every semiconductor device, since it allows us to easily analyze these circuits under specific conditions. This is in comparison to doing math with a large number of nonlinear expressions in it, which is difficult and time consuming.

Solution 2

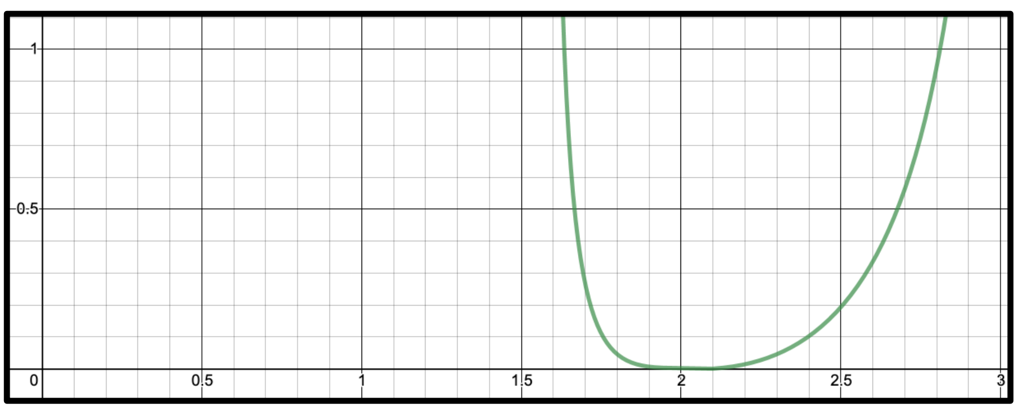
To linearize the relationship between current and input voltage at $V_{in} = 2V$, we need to first find the slope of the function at $2V$. To do so, we take the derivative of the expression:

$$\frac{d}{dV_{in}}[I_{out}(V_{in})] = V_{in}(2V_{in}\cos^2(V_{in}) + 2\sin(V_{in})\cos(V_{in}) - V_{in})$$

Now, substituting $2V$ for V_{in}, we find the slope of the expression to be -4.13. We then need to find the output current of the expression at $2V$. Doing so is as simple as plugging $V_{in} = 2$ into the exact output equation. This yields an I_{out} of -1.51. Thus, our linearized relationship is given as follows:

$$I_{approx}(V_{in}) + 1.51 = -4.13(V_{in} - 2) \rightarrow I_{approx}(V_{in}) = -4.13V_{in} + 6.75$$

Plotting the error of the approximated output yields the following:



We can see that, near the operating point, the error increases as the input voltage moves away from $2V$. However, in the region around $2V$, the error is minimal. For example, the error at $V_{in} = 1.9V$ is only about 0.6% and the error at $V_{in} = 2.1V$ is only about 0.06%. Thus, we can accurately model this device using the linearized expression, which is less computationally expensive.

Solution 3

The operating point, or Q-point, of a circuit is the steady-state DC voltage and current at a device's terminals when the input signal is zero. This is done by replacing small voltage inputs with shorts and small current inputs with open circuits. Some other devices are also replaced depending on their behavior at steady state. All of this is important because it is the "biasing" of the circuit which is something that you will want to determine when designing a circuit.

Solution 4

Vcc=10V and β=150

Starting from the left one:

$I_B = \frac{10-0.7}{500*10^3} = 18.6\mu A \Rightarrow I_C = 150 * I_B = 2.79mA \Rightarrow V_{out1} = 10 - 5k\Omega * 2.79mA = -3.95$ which is impossible. So, the BJT must be in the Saturation region meaning Vce =0.2V

On the left :

VB=0.7V meaning current would flow out of the base which is impossible. So the 2nd BJT is off meaning Vout is 10V

For Vcc=5V and β=100

Starting from the left one:

$I_B = \frac{5-0.7}{500*10^3} = 8.6\mu A \Rightarrow I_C = 50 * I_B = 0.43mA \Rightarrow V_{out1} = 5 - 5k\Omega * 0.43mA = 2.85V > 0.2V$ *so we are in F.A.*

On the left :

$I_B = \frac{2.85-0.7}{50*10^3} = 43\mu A \Rightarrow I_C = 50 * I_B = 2.15mA \Rightarrow V_{out1} = 5 - 20k\Omega * 2.15mA = -38V < 0.2V$ *so we must be in Saturation so* $V_{CE} = 0.2$ *meaning Vout=0.2V*

Solution 5

$I_B = \frac{2-0.7}{100*10^3} = 13\mu A \Rightarrow I_C = 100 * I_B = 1.3mA \Rightarrow V_C = 10 - 5k\Omega * 1.3mA = 3.5V > 0.2$ So we are in F.A.

For the second BJT

$$I_B = \frac{3.5 - 0.7}{100 * 10^3} = 28\mu A \Rightarrow I_E = (\beta + 1) * I_B = 2.83mA \Rightarrow V_E = 2.83mA * 3k\Omega = 8.48V$$

$V_{CE} = 10 - 8.48 = 1.52V > 0.2$ so we are in F.A. and Vout = 8.48V

Solution 6

To begin, note that the AMI06 datasheet lists the V_T for a minimally sized NMOS device as $0.79V$ and the $\frac{\mu_n C_{ox}}{2}$ to be $56.4\mu A/V^2$. Now, recall from lecture that the small-signal gain of this circuit can be expressed as follows:

$$A_V = -\frac{2I_{DQ}R_1}{V_{GSQ} - V_T}$$

To find the quiescent current draw, we can start by assuming that the MOSFET is operating in the saturation region:

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{Tn})^2 = 56.4\mu * 5 * (1.21)^2 = 413\mu A$$

Substituting this in, we find that we are left with only one parameter to solve for, R_1:

$$-1.5 = \frac{-2 * 413\mu * R_1}{2 - 0.79} \rightarrow R_1 = \frac{-1.5 * 1.21}{-2 * 413\mu} = 2.2k\Omega$$

However, we are not done yet! Earlier, we assumed that the MOSFET was operating in the saturation region. Before stating that our answer is correct, we need to ensure that this is true:

$$V_{DS} = 5 - I_{DQ}R_1 = 4.09V \geq V_{GSQ} - V_T$$

$$V_{GSQ} = 2 \geq V_T$$

Thus, the device is saturated. So, the resistance needed to generate a small-signal gain of -1.5 is $2.2k\Omega$.