EE 330

Homework 6 Solutions

Fall 2020

Solution 1

Recall that the current through a diode can be modeled using the following formula:

$$I_D = I_S(e^{\frac{V_D}{nV_t}} - 1)$$

Wherein V_t and I_S are both given as follows (with all temperatures in units of Kelvin):

$$I_S = J_S A = AJ_{SX}T^m e^{-\frac{V_{G0}}{nV_t}}$$

$$V_t = \frac{kT}{q} \approx (8.62 \times 10^{-5})T$$

Given this, we can begin by calculating the device's thermal voltage and junction current:

$$V_t \approx (8.62 \times 10^{-5})T = \left(8.62 \times 10^{-5} \frac{V}{°K}\right) 125°C = \left(8.62 \times 10^{-5} \frac{V}{°K}\right) 398.15°K \approx 34.3mV$$

$$I_S = J_{SX}T^m e^{-\frac{V_{G0}}{Vt}} = [100][0.5][398.15^{2.3}]\left[e^{-\frac{1.17}{34.3mV}}\right] \approx 73.27nA$$

Now, we can calculate the device's current when biased with a forward voltage of $1V$:

$$I_D = [73.27nA]\left(e^{\frac{0.6}{34.3mV}} - 1\right) = 2.89A$$

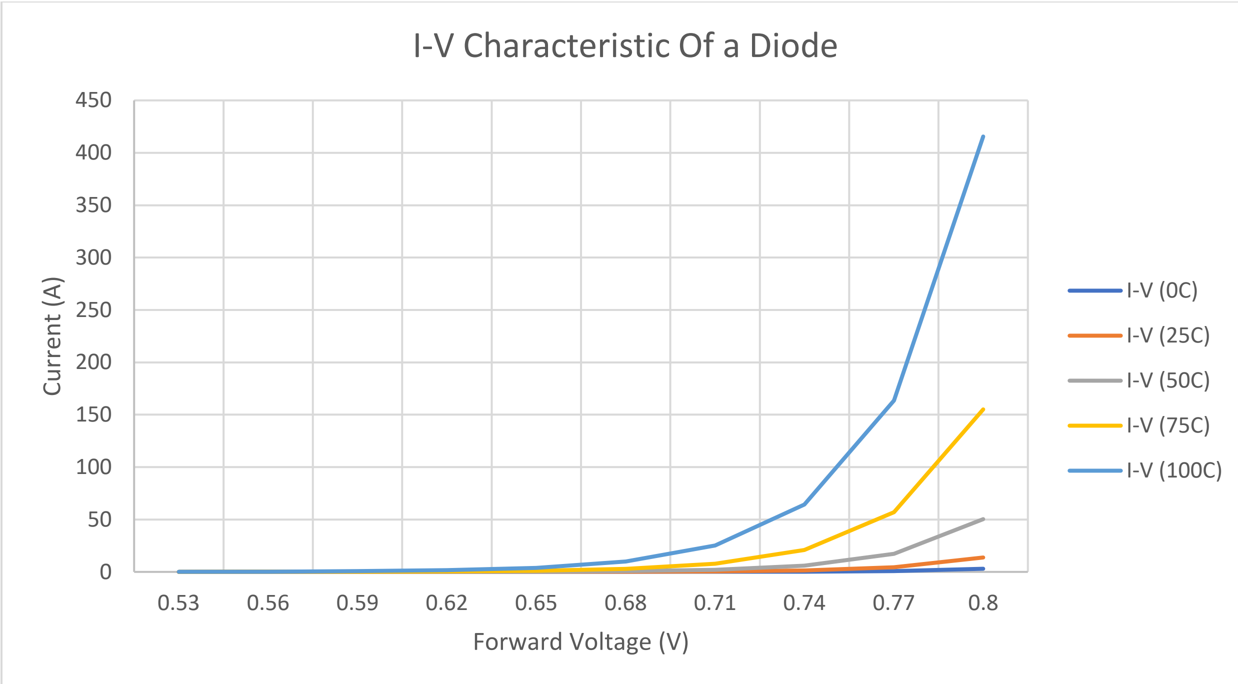
This process can be repeated with the temperature of the device set at $0°C$ and $27°C$, obtaining the following values:

Temperature	$\mathbf{125°C}$	$\mathbf{0°C}$	$\mathbf{27°C}$
Current	$2.89A$	$615.3\mu A$	$6.7mA$

Clearly, diodes are extremely temperature dependent.

Solution 2

The plot below was generated using Excel. It used five temperature values, indicated on the right-side of the plot by the legend.



As expected, the current of the diode increases exponentially as its forward voltage increases. Further, the device's current increases dramatically as temperature increases. This is because, as temperature increases, the carriers in the silicon have a greater energy. The increased energy of the carriers makes them more likely to diffuse through the pn-junction.

Solution 3

V_D should be greater than 0.6V, so assume on.

$I_D = \frac{12V-0.6V}{5k\Omega} = 2.28mA$, solution makes sense because I_D should be greater than 0 if V_D = 0.6V (Diode is "on")

Solution 4

V_D will be less than 0.6V, therefore the diode will be in the "off" state and I_D will be ~ 0A

Solution 5

(a) Assuming saturation, $I_{DS} = \mu Cox \frac{W}{2L}(V_{GS} - V_T)^2 = 300 * 10^{-6} * \frac{2}{2} * (0.5)^2 = 0.75 * 10^{-4}$

$I_{DS} = \frac{VDD-Vout}{R} \Rightarrow V_{out} = 9.9925V$

(b) Assuming saturation, $I_{DS} = \mu Cox \frac{W}{2L}(V_{GS} - V_T)^2 = 300 * 10^{-6} * \frac{1}{2} * (2.5)^2 = 0.938 * 10^{-3}$

$I_{DS} = \frac{VDD-Vout}{R} \Rightarrow V_{out} = 9.06V$

(c) Assuming saturation, $I_{DS} = \mu Cox \frac{W}{2L}(V_{GS} - V_T)^2 = 300 * 10^{-6} * \frac{1}{4} * (4.5)^2 = 1.5 * 10^{-3}$

$I_{DS} = \frac{VDD - Vout}{R} \Rightarrow V_{out} = 6.2V$

Solution 6

Recall from lecture that the current through a saturated n-channel MOSFET can be approximated using the square-law model as follows:

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{Tn})^2$$

To find the current flowing through M_2, we somehow need to find the gate-to-source voltage (V_{GS}) of the device. We note that the gates of M_1 and M_2 are tied together, meaning that the two devices have the same V_{GS}. Solving for the V_{GS} of M_1 will then also yield the V_{GS} of M_2:

$$I_{IN} = \frac{1}{2}\mu_n C_{ox}\frac{W_1}{L_1}(V_{GS} - V_{Tn})^2$$

$$V_{GS} = \sqrt{\frac{2I_{IN}L_1}{\mu_n C_{ox} W_1}} + V_{Tn}$$

We can substitute this V_{GS} into the saturation current equation for M_2:

$$I_R = \frac{1}{2}\mu_n C_{ox}\frac{W_2}{L_2}\left(\sqrt{\frac{2I_{IN}L_1}{\mu_n C_{ox} W_1}} + V_{Tn} - V_{Tn}\right)^2 = \frac{1}{2}\mu_n C_{ox}\frac{W_2}{L_2}\left(\sqrt{\frac{2I_{IN}L_1}{\mu_n C_{ox} W_1}}\right)^2$$

Now, we simplify:

$$I_R = \frac{1}{2}\mu_n C_{ox}\frac{W_2}{L_2}\left(\frac{2I_{IN}L_1}{\mu_n C_{ox} W_1}\right) = \frac{W_2 L_1}{L_2 W_1} I_{IN} = \frac{W_2/L_2}{W_1/L_1} I_{IN}$$

One way that this circuit might be used is as a way of reflecting, or mirroring, a current from one source to another part of a circuit, while also being able to scale the magnitude of the current. For example, let's say you are working on an IC which only has two current sources available to you: one which can source $100\mu A$ and one which can source $22\mu A$. If you need $200\mu A$ for some part of your circuit, you might be able to get away with using this handy circuit to "reflect" the $100\mu A$ source's current to another part, while also scaling it up by a factor of 2 by setting $\frac{W_2}{L_2} = 2\frac{W_1}{L_1}$.

Solution 7

Part A:

Recall that the current through a saturated n-channel MOSFET can be modelled as follows:

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{Tn})^2$$

Because both devices have the same gate-source voltage, V_{GS}, as well as the same width-length ratios, we can see that both devices *must* have the same current flowing through them:

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{Tn})^2 = I_{D2}$$

To get the current through each device as a ratio of I_{IN}, we must observe that the current through both devices, by necessity, sum to I_{IN}:

$$I_{IN} = I_{D1} + I_{D2} = 2I_{D1,2}$$

Therefore, we can conclude that $I_{D1} = I_{D2} = \frac{I_{IN}}{2}$. If I_{IN} is equal to $100\mu A$, it is easy to see that $I_{D1} = I_{D2} = 50\mu A$.

Part B:

We can solve for V_1 and V_2 by noting that $V_{GS} = V_1 - 0 = V_2 - 0$ for both devices. We can use the saturation current equation to solve for V_{GS}. We have already concluded that the currents through the two devices is the same, and that they are sized identically, so we only need to do this equation once:

$$I_D = 50\mu A = \frac{1}{2}[300\mu][2](V_{GS} - 0.5)^2$$

$$V_{GS} = V_1 = V_2 = \sqrt{\frac{100\mu}{600\mu}} + 0.5 = 0.816mV$$

Solution 8

$V_{DS} = V_{GS} - V_T$. This is where the nmos will switch. $V_{DS} = 0.5V$

Solving with both the triode and the saturation equation.

First, find the current trough the nmos and diode.

Triode: $I_D = \frac{300\mu A}{V^2} * 5V(1V - 0.5V - 0.25V)0.25V = 187.5\mu A$

Saturation: $I_D = \frac{300\mu A}{V^2} * \frac{5}{2}(1V - 0.5V)^2 = 187.5\mu A$

Second, solve for the voltage across the diode.

$V_D = 3V - (0.5V + 187.5\mu A * 10K\Omega) = 0.625V$

Third, use the diode equation to find the area.

$187.5\mu A = 10^{-15} A/\mu^2 * A * e^{0.625/0.0258}$

$A = 5.98\mu^2$

Solution 9

For V_D=0.6.	R1: I=0.6mA.	R2: I=1.2mA.	R3: I=0.6mA.	R4: I=0.6mA
For V_D=0.7	R1: I=0.2mA.	R2: I=1.4mA.	R3: I=0.2mA.	R4: I=0.2mA

Solution 10

Left Circuit:

Begin by assuming that the diode is in the off state. In this state, the unknown current is equal to $2.5mA$ and the voltage across the diode is $-10V$. The fact that the voltage across the diode is $-10V$ supports the assumption that the diode is in the off state.

Right Circuit:

Begin by assuming that the diode is in the off state. In this state, the unknown current is equal to $3.75mA$ and the voltage across the diode is $-1.5V$. The fact that the voltage across the diode is $-1.5V$ supports the assumption that the diode is in the off state.

Solution 11

Let us assume that the middle diode is in the off-state and that the right diode is in the on state. This would imply that the middle diode is an open-circuit and the right diode is a closed-circuit, yielding the following current:

$$I_2 = \frac{4}{20k\Omega} = 200\mu A$$

We need to check that the voltages across the diodes are consistent with our guesses. Because no current flows through the middle $10k\Omega$ resitor if the middle diode is off, the voltage at the anode of the diode is $2V$. This reverse-biases the diode by $8V$, indicating that it is off. Further, the voltage drop across the right diode is $2V$ if it is assumed to be ideal, which means that it is forward-biased.

Solution 12

Recall that the current through a pn-junction can be modelled as follows:

$$I_D = J_S A(e^{\frac{V_D}{V_T}} - 1)$$

The current through the two diodes can thus be summed together as:

$$I_D = I_{D1} + I_{D2} = J_S 5A\left(e^{\frac{V_D}{V_T}} - 1\right) + J_S A\left(e^{\frac{V_D}{V_T}} - 1\right) = J_S 6A(e^{\frac{V_D}{V_T}} - 1)$$

The voltage across the diodes can be written in terms of the total current drawn:

$$V_D = V_x - 1k\Omega * I_D$$

This forms a system of equations with two unknown values, as seen below:

$$I_D = [5f][900\mu m^2][e^{\frac{V_D}{0.0259}} - 1]$$

$$V_D = 1.5 - 1k\Omega I_D$$

Simplifying:

$$I_D = [5f][900\mu m^2][e^{\frac{1.5-1k\Omega I_D}{0.0259}} - 1]$$

This can be solved by hand, but that's a lot of work which you would *never* do in real life. Using a calculator (or WolframAlpha) to solve this equation for I_D yields $I_D \approx 1mA$. Now, we need to figure out how much of this current flow belongs to I_{D1}:

$$I_D = I_{D1} + I_{D2} = I_{D1} + 5I_{D1} = 6I_{D1}$$

$$I_{D1} = \frac{1mA}{6} = 166\mu A$$