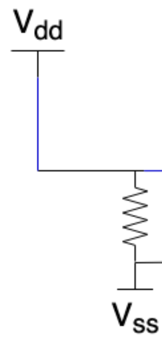
EE 330

Homework 9 Solutions

Fall 2020

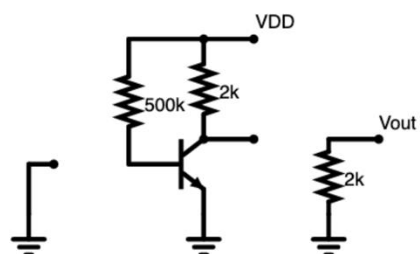
Solution 1

In small-signal, large capacitors become short-circuits and large inductors become open-circuits. As a result, the small-signal equivalent circuit looks like this:



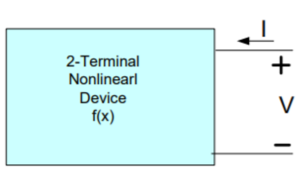
Solution 2

Begin by drawing the large-signal equivalent circuit. Recall that, when doing large-signal analysis, large capacitors become open-circuits.



Here, it is seen that V_{OUT} is connected to ground through a $2k\Omega$ resistor, and that there is no other path for current to enter or leave the V_{OUT} node. Thus, the quiescent value of V_{OUT} is $0V$.

Solution 3



Thinking of the NMOS in these terms, it is a 2-port device with its VS as the reference. So the "conductance" will be $y = \frac{dI_D}{dV_{GS}}$ evaluated at the quiescent voltage.

$$y = \frac{d}{dV}\left(\frac{1}{2}uCox\frac{W}{L}(V_{GS} - V_T)^2\right) = uCox\frac{W}{L}(V_{GS} - V_T)$$

$$R = \frac{1}{uCox\frac{W}{L}(V_{GS} - V_T)}$$

As the voltage on the input increases, so does VGS. As VGS increases, R decreases in an inverse analogous fashion. So as voltage across the device increases, resistance drops and current increases. This is similar to what a diode does. A diode does this much better though, since its relationship is exponential, so the decrease happens much faster.

Solution 4

Part A:

Recall that the current through a saturated NMOS can be approximated using the following equation:

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_T)^2(1 + \lambda V_{DS})$$

The transconductance of a MOSFET, g_m, is defined as the measure of how much the device's drain current changes with respect to a changing V_{GS}. Thus:

$$g_m = \frac{\partial \mathrm{I_D}}{\partial \mathrm{V_{GS}}} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(1 + \lambda V_{DS}) * \frac{\partial}{\partial V_{GSQ}}[(V_{GS} - V_T)^2] = \frac{1}{2}\mu_n C_{ox}\frac{W}{L} * 2(V_{GSQ} - V_T) * (1 + \lambda V_{DS})$$
$$\approx \mu_n C_{ox}\frac{W}{L}(V_{GSQ} - V_T)$$

The output conductance, g_o, of a MOSFET is defined as the measure of how much the device's drain current changes with respect to a changing V_{DS}. Thus:

$$g_o = \frac{\partial I_D}{\partial V_{DS}} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_T)^2\frac{\partial}{\partial V_{DS}}[1 + \lambda V_{DS}] = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GSQ} - V_T)^2(\lambda) \approx \lambda I_D$$

Part B:

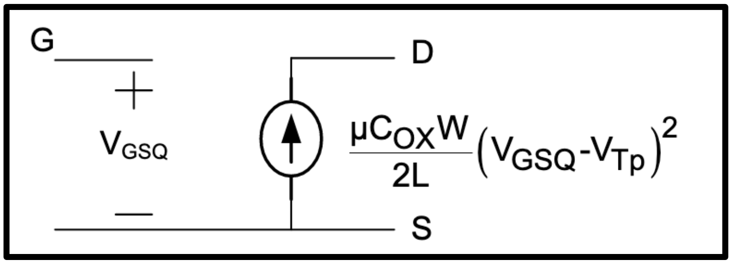
Recall that the current through a saturated PMOS can be approximated using the following equation:

$$I_S = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(V_{GS} - |V_T|)^2(1 + \lambda V_{SD})$$

The same process used in Part A to find g_m and g_o for an NMOS can be used to find the g_m and g_o. Thus, g_m and g_o for a PMOS are as follows:

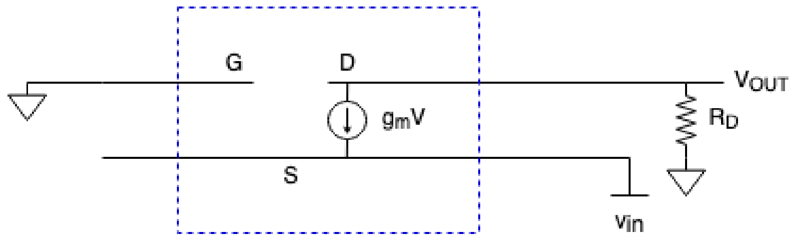
$$g_m \approx \mu_p C_{ox}\frac{W}{L}(V_{GSQ} - V_T)$$

$$g_o \approx \lambda I_S = 0$$



Solution 5

To begin, redraw the circuit, but replace the NMOS device with its two-port model:



Now, simply solve for v_{out}/v_{in}:

$$\frac{v_{out}}{R_D} + g_m(-v_{in}) = 0 \rightarrow \frac{v_{out}}{R_D} = v_{in}g_m \rightarrow \frac{v_{out}}{v_{in}} = g_m R_D$$

This answer is not acceptable because it is in terms of g_m. Recalling that $g_m = \frac{2I_{DQ}}{V_{GSQ}-V_T}$ allows us to correct for this:

$$A_v = \frac{v_{out}}{v_{in}} = \frac{2I_{DQ}}{V_{GSQ} - V_T} R_D$$

Solution 6

In lab, it was seen that λ is extremely dependent upon device length, particularly at small lengths, as well as on V_{DS}. Because g_o is approximately equal to λI_{DQ}, g_o is also dependent upon device length and V_{DS}.

Solution 7

$V_T = 0.79\ \frac{W}{L} = \frac{3}{0.6}\ \frac{\mu Cox}{2} = 56.4 * 10^{-6}\ \frac{A}{V^2}$

Small input $\Rightarrow V_G = 0V$

$$V_D = V_{DD} - I_D * R = 10 - \frac{1}{2}\mu Cox\frac{W}{L}(V_{GS} - V_T)^2 R = 10 - 8.246 * 10^{-3} R$$

Saturation means $V_{DS} \geq V_{GS} - V_T$

$V_{DS} \geq V_{GS} - V_T \Rightarrow V_D + 2.5 \geq 2.5 - 0.79 \Rightarrow 10 - 8.246 * 10^{-3} R \geq -0.79$

$R \leq 1308\Omega$

So for part 2 if R=436Ω

$$A_V = \mu Cox\frac{W}{L}(V_{SS} + V_T)R = -0.809$$

Solution 8

$V_T = 0.79 \; \frac{W}{L} = \frac{3}{0.6} \; \frac{\mu Cox}{2} = 56.4 * 10^{-6}\frac{A}{V^2}$

$$I_D = \frac{V_{DD} - V_{out}}{R} = 1.5mA$$

$$I_D = \frac{1}{2}\mu Cox\frac{W}{L}(V_{GS} - V_T)^2 \Rightarrow 1.5 * 10^{-3} = 56.4 * 10^{-6}\frac{3}{0.6}(V_{GS} - 0.79)^2$$

$5.32 = (V_{GS} - 0.79)^2 \Rightarrow V_{GS} = \begin{cases} 3.097 \\ -1.517 \end{cases}$ *Since VGS =* -1.517 *would mean the NMOS is off, the answer must be* $3.097V$

Since we are in saturation, $V_{DS} \geq VGS - VT \Rightarrow 2.5 - VS \geq VG - VS - 0.7 \Rightarrow VG \leq 3.2$

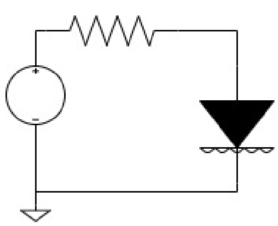
If VG is higher than 3.2V, it would bean VS would be higher than -0.1V meaning VDS would be less than 2.6V which means VDS would not be greater than or equal to VGS-VT

Solution 9

a) In a DC analysis the capacitor becomes an open circuit.

$$I_D = I_B = J_S A_D\left(e^{\frac{V_D}{Vt}} - 1\right) \Rightarrow V_{out} = V_D = V_t \ln\left(\frac{I_B}{J_S A_D} + 1\right) = 0.56V$$

b)



c)$R_D = \frac{V_t}{I_{DQ}} = \frac{0.026}{0.002} = 13\Omega$

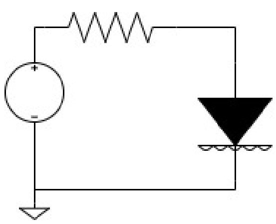
$$A_V = \frac{R_D}{R_D + R} = \frac{13}{13 + 25} = 0.342$$

Solution 10

a) In a DC analysis the capacitor becomes an open circuit.

$$I_D = I_B = J_S A_D \left(e^{\frac{V_D}{Vt}} - 1\right) \Rightarrow V_{out} = V_D = V_t \ln\left(\frac{I_B}{J_S A_D} + 1\right) = 0.539V$$

b)



c)$R_D = \frac{V_t}{I_{DQ}} = \frac{0.26}{0.001} = 26\Omega$

$$A_V = \frac{R_D}{R_D + R} = \frac{26}{26 + 50} = 0.34$$

Solution 11

The operating point, or Q-point, of a circuit is the steady-state DC voltage and current at a device's terminals when the input signal is zero. This is done by replacing small voltage inputs with shorts and small current inputs with open circuits. Some other devices are also replaced depending on their behavior at steady state. All of this is important because it is the "biasing" of the circuit which is something that you will want to determine when designing a circuit.

Solution 12

We need to factor things so we have the equation in the form $V_{out} = V_{outQ} + A_V * V_{in}$

$$V_{out} = \left[V_{dd} - \frac{\lambda\psi}{\rho}(V_{ss} - V_C) + 10\delta\right] + \left[\frac{\lambda\psi}{\rho}(V_{ss} - V_C)^2 R + 10\right] V_{in}$$

From this it is now clear that:

$$V_{outQ} = V_{dd} - \frac{\lambda\psi}{\rho}(V_{ss} - V_C) + 10\delta$$

$$A_v = \frac{\lambda\psi}{\rho}(V_{ss} - V_C)^2 R + 10$$