EE240: Pattern Recognition and Machine Learning
Homework 3

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May 19, 2018

1 H3.1

1.1 a

$$
\begin{aligned}
J(W,b) &= \frac{1}{2}||y - Wx - b||^2 \\
&= \frac{1}{2}(y - Wx - b)^T(y - Wx - b) \\
&= \frac{1}{2}(y^Ty - y^TWx - y^Tb - W^Tx^Ty + W^Tx^TxW + W^Tx^Tb - b^Ty + b^TWx + b^Tb)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial J}{\partial W_{ij}} &= \frac{1}{2}(-2x^Ty + 2x^Tb - 2x^TxW) \\
\text{Set } \frac{\partial J}{\partial W_{ij}} &= 0 \Rightarrow xW = y + b
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial J}{\partial b} &= \frac{1}{2}(-2y + 2W^Tx^T + 2b) \\
\text{Set } \frac{\partial J}{\partial b} &= 0 \Rightarrow b = y - Wx
\end{aligned}
$$

1.2 b

$$J(W,b) = \frac{1}{2}||y - w * x - b||^2$$

$$\frac{\partial J}{\partial w_i} = 0$$

$$\Rightarrow w_i * x = y + b$$

$$\Rightarrow \sum_{j=1}^{d} x_i w_{j-i+1} = y + b$$

$$\Rightarrow \begin{bmatrix} w_1 x_1 \\ w_2 x_1 + w_1 x_2 \\ w_3 x_1 + w_2 x_2 + w_1 x_3 \\ \vdots \\ w_k x_1 + w_{k-1} x_2 + \ldots + w_1 x_k \\ \vdots \\ w_k x_k \end{bmatrix} = \begin{bmatrix} y_1 \\ \vdots \\ y_k \end{bmatrix} + \begin{bmatrix} b_1 \\ \vdots \\ b_k \end{bmatrix}$$

$$\Rightarrow w_1 x_1 = y_1 + b_1$$

$$, w_2 x_1 + w_1 x_2 = y_2 + b_2$$

$$, w_3 x_1 + w_2 x_2 + w_1 x_3 = y_3 + b_3$$

$$\vdots$$

$$, w_k x_k = y_k + b_k$$

$$\frac{\partial J}{\partial b} = 0$$

$$\Rightarrow b = y - Wx$$

$$\Rightarrow \begin{bmatrix} b_1 \\ \vdots \\ b_k \end{bmatrix} = \begin{bmatrix} y_1 \\ \vdots \\ y_k \end{bmatrix} - \begin{bmatrix} w_1 x_1 \\ w_2 x_1 + w_1 x_2 \\ w_3 x_1 + w_2 x_2 + w_1 x_3 \\ \vdots \\ w_k x_1 + w_{k-1} x_2 + \ldots + w_1 x_k \\ \vdots \\ w_k x_k \end{bmatrix}$$