EE240: Pattern Recognition and Machine Learning
Homework 1

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1 H1.1

1.1 a

Probability that some coin out of N that has no head in T coin tosses:
$1-(1-(\frac{1}{2})^T)^N$

1.2 b

Probability that two randomly chosen students have birthday on the same day:
$1-\frac{330!}{365!}$

2 H1.2

Present in h1_2.ipynb

3 H1.3

3.1 a

phase 1 (*prior*)	**phase 2** (*likelyhood*)	**phase 3** (*posterior*)
$P(\text{car at } 1 \mid 1) = \frac{1}{3}$	$P(\text{open } 3 \mid \text{car at } 1) = \frac{1}{2}$	$P(\text{car at } 1 \mid \text{open } 3) = \frac{\frac{1}{2}\cdot\frac{1}{3}}{\frac{1}{2}\cdot\frac{1}{3}+1\cdot\frac{1}{3}+0\cdot\frac{1}{3}} = \frac{1}{3}$
$P(\text{car at } 2 \mid 2) = \frac{1}{3}$	$P(\text{open } 3 \mid \text{car at } 2) = 1$	$P(\text{car at } 2 \mid \text{open } 3) = \frac{1\cdot\frac{1}{3}}{\frac{1}{2}\cdot\frac{1}{3}+1\cdot\frac{1}{3}+0\cdot\frac{1}{3}} = \frac{2}{3}$
$P(\text{car at } 3 \mid 3) = \frac{1}{3}$	$P(\text{open } 3 \mid \text{car at } 3) = 0$	$P(\text{car at } 1 \mid \text{open } 3) = \frac{0\cdot\frac{1}{3}}{\frac{1}{2}\cdot\frac{1}{3}+1\cdot\frac{1}{3}+0\cdot\frac{1}{3}} = 0$

The difference between open door No.1 and door No.2 is that in phase 2, the likelyhood of opening door No.3 given the car is at door No.1 becomes $\frac{1}{2}$; while the likelyhood of opening door No.3 given the car is at door No.2 becomes 1. Therefore, by Bayes Theorem the probability of the car is at door No.1 given that door No.3 is opened is still $\frac{1}{3}$; meanwhile, the probability of the car is at door No.2 given that door No.3 is opened becomes $\frac{2}{3}$. Thus switching the choice to open door No.2 obtain high probability to get a car rather than a goat.

3.2 b

Present in h1_3.ipynb

4 H1.4

4.1 a

(i) $P(A|red) = \frac{P(red|A) \cdot P(A)}{P(red)} = \frac{0.3 \cdot 0.4}{0.3 \cdot 0.4 + 0.7 \cdot 0.6} = \frac{2}{9}$

(ii) $P(A|red) = \frac{0.1 \cdot 0.4}{0.1 \cdot 0.4 + 0.4 \cdot 0.6} = \frac{1}{7}$

4.2 b

$P(A|\text{red and blue}) = \frac{P(\text{red and blue}|A) \cdot P(A)}{P(\text{red and blue})} = \frac{0.3 \cdot 0.7 \cdot 0.4}{0.3 \cdot 0.7 \cdot 0.4 + 0.3 \cdot 0.7 \cdot 0.6} = 0.4$

4.3 c

$P(A|\text{red and blue}) = \frac{0.1 \cdot 0.3 \cdot 0.4}{0.1 \cdot 0.3 \cdot 0.4 + 0.4 \cdot 0.2 \cdot 0.6} = 0.2$

5 H1.5

Assume that within N training samples, there are m samples in class $y_n = +1$ and $n = N - m$ samples in class $y_n = -1$.

$$
\begin{aligned}
w &= 2(\mu_+ - \mu_-) \\
&= 2(\frac{1}{m} \sum_{y_+} \boldsymbol{x}_n - \frac{1}{n} \sum_{y_-} \boldsymbol{x}_n) \\
\boldsymbol{w}^T \boldsymbol{x} + b &= < \boldsymbol{w}, \boldsymbol{x} > + b \\
&= < 2(\frac{1}{m} \sum_{y_+} \boldsymbol{x}_n - \frac{1}{n} \sum_{y_-} \boldsymbol{x}_n), \boldsymbol{x} > + b \\
&= \alpha_n < \boldsymbol{x}_n, \boldsymbol{x} > + b \\
&\text{, where } \alpha_n = \begin{cases} \frac{2}{m} \text{ , if } y_n = +1 \\ \frac{-2}{n} \text{ , if } y_n = -1 \end{cases}
\end{aligned}
$$

6 H1.6

Present in h1_6.ipynb

7 H1.7

Present in h1_7.ipynb