- Problem 1.
  - Exercise 7.2:

    approximation pyramid: $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} \rightarrow \begin{bmatrix} 3.5 & 5.5 \\ 11.5 & 13.5 \end{bmatrix} \rightarrow [8.5]$

    prediction residual pyramid:

    - level 0: $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} - \begin{bmatrix} 3.5 & 3.5 & 5.5 & 5.5 \\ 3.5 & 3.5 & 5.5 & 5.5 \\ 11.5 & 11.5 & 13.5 & 13.5 \\ 11.5 & 11.5 & 13.5 & 13.5 \end{bmatrix} = \begin{bmatrix} -2.5 & -1.5 & -2.5 & -1.5 \\ 1.5 & 2.5 & 1.5 & 2.5 \\ -2.5 & -1.5 & -2.5 & -1.5 \\ 1.5 & 2.5 & 1.5 & 2.5 \end{bmatrix}$
    - level 1: $\begin{bmatrix} 3.5 & 5.5 \\ 11.5 & 13.5 \end{bmatrix} - \begin{bmatrix} 8.5 & 8.5 \\ 8.5 & 8.5 \end{bmatrix} = \begin{bmatrix} -5 & -3 \\ 3 & 5 \end{bmatrix}$
    - level 0 to level 2: $\begin{bmatrix} -2.5 & -1.5 & -2.5 & -1.5 \\ 1.5 & 2.5 & 1.5 & 2.5 \\ -2.5 & -1.5 & -2.5 & -1.5 \\ 1.5 & 2.5 & 1.5 & 2.5 \end{bmatrix} \rightarrow \begin{bmatrix} -5 & -3 \\ 3 & 5 \end{bmatrix} \rightarrow [8.5]$

  - Exercise 7.3:

    All the levels are expansions except the 0^{th} one, and the expansion ratio is bounded by 4/3.

J	Expansion ratio
0	1
1	$1 + {}^{1}/_{4^1} = 1.25$
2	$1 + {}^{1}/_{4^1} + {}^{1}/_{4^2} = 1.31$
J+1	$1 + {}^{1}/_{4^1} + {}^{1}/_{4^2} + \cdots + {}^{1}/_{4^{J+1}}$
∞	$1 + {}^{1}/_{4^1} + {}^{1}/_{4^2} + \cdots = {}^{4}/_{3} = 1.33$

  - Exercise 7.15:

    While j_0=1,

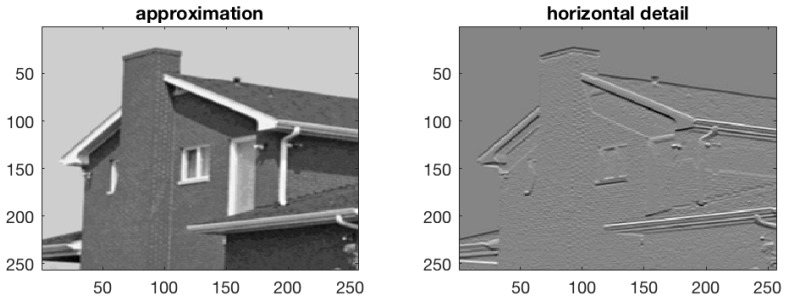
$$h_1(0) = \int_0^{0.5} x^2\sqrt{2}dx = \left(\frac{\sqrt{2}x^3}{3}\right)\Big|_0^{0.5} = \frac{\sqrt{2}}{24}$$

$$h_1(1) = \int_{0.5}^{1} x^2\sqrt{2}dx = \left(\frac{\sqrt{2}x^3}{3}\right)\Big|_{0.5}^{1} = \frac{7\sqrt{2}}{24}$$

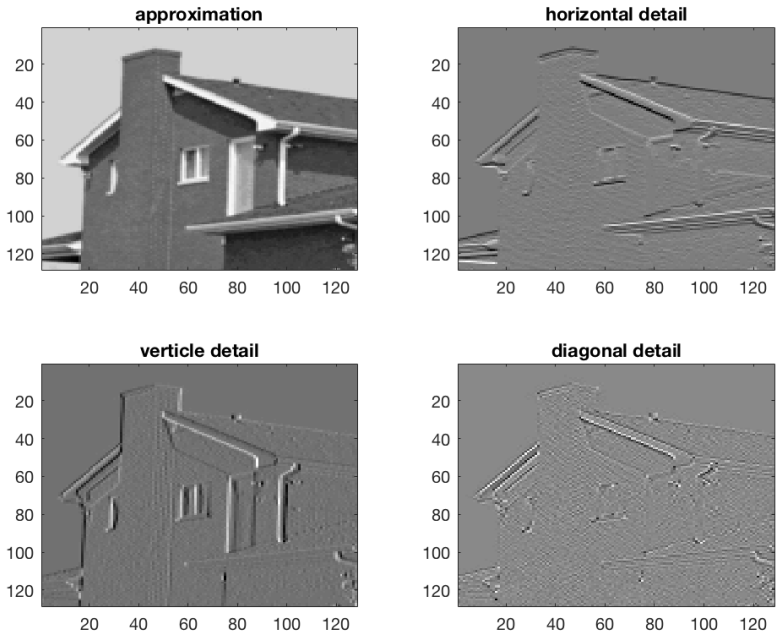
$$\Rightarrow V_1 = \frac{\sqrt{2}}{24}\varphi_{1,0}(x) + \frac{7\sqrt{2}}{24}\varphi_{1,1}(x)$$

$$expansion = \frac{\sqrt{2}}{24}\varphi_{1,0}(x) + \frac{7\sqrt{2}}{24}\varphi_{1,1}(x) + \left[\frac{-\sqrt{2}}{32}\psi_{1,0}(x) - \frac{3\sqrt{2}}{32}\psi_{1,1}(x)\right] + \cdots$$

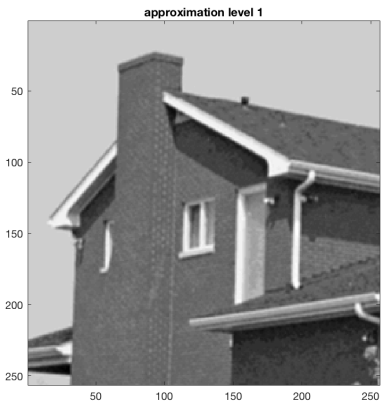
- Problem 2.

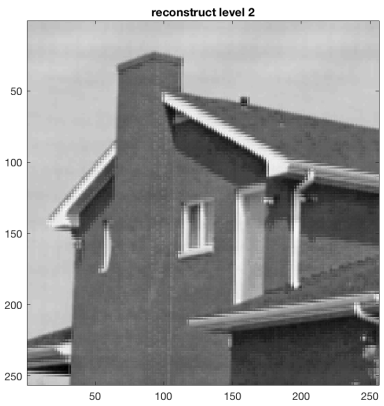


(level 1)

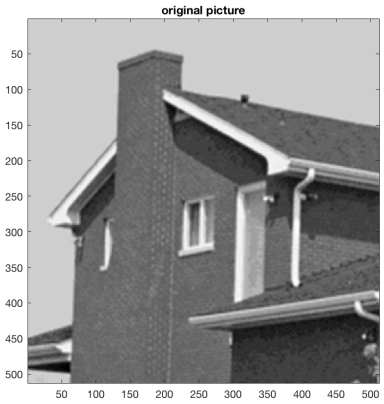


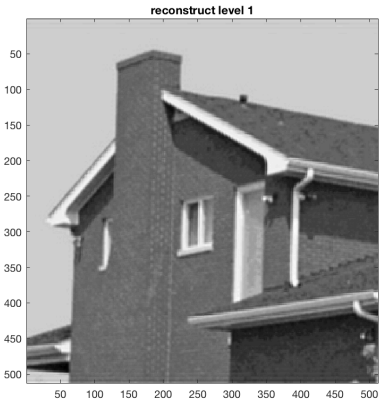
(level 2)





Reconstruction by level 2 with error rate 212.8





Reconstruction by level 2 with error rate 61.4

*note: I used DFT to transform the approximation and detail of the wavelet transformation, and then extract the peaks in those DFTs for removing the high frequency components. As a result, the error rate is reduced because the high frequency noise had been dropped out from the wavelets while reconstruction.

- Problem 3.
  - (a) Choose the threshold with the most complete edges and the least details in the inner part of the image.

LoG: threshold 0.003000

Canny: threshold[0.030000, 0.150000]

LoG: threshold 0.004000

Canny: threshold[0.040000, 0.200000]

LoG: threshold 0.005000

Canny: threshold[0.050000, 0.250000]

LoG: threshold 0.006000

Canny: threshold[0.060000, 0.300000]

LoG: threshold 0.007000

Canny: threshold[0.070000, 0.350000]

LoG: threshold 0.002500

Canny: threshold[0.025000, 0.125000]

LoG: threshold 0.003500

Canny: threshold[0.035000, 0.175000]

LoG: threshold 0.004500

Canny: threshold[0.045000, 0.225000]

LoG: threshold 0.005500

Canny: threshold[0.055000, 0.275000]

LoG: threshold 0.006500

Canny: threshold[0.065000, 0.325000]

- Problem 4.
  - >>getCorner(house.tif, ncorners=50);