Rules of Inference:

Valid argument or valid conclusion:

If a conclusion is derived from a set of premises by using the accepted rules of reasoning, then the conclusion is called valid argument or valid conclusion.

Defn:

We say that from a set of premises $\{H_1, H_2, \ldots H_m\}$ a conclusion 'C' follows if $H_1 \wedge H_2 \wedge \ldots \wedge H_m \Rightarrow C$

I) Truth table technique:

$(H_1 \wedge H_2 \wedge \ldots \wedge H_m) \rightarrow C$ is a tautology

1) Determine whether the conclusion C follows logically from the premises H_1 and H_2

a) $H_1 : P \rightarrow Q$ $H_2 : P$ $C : Q$ T.P $((P \rightarrow Q) \wedge P) \rightarrow Q$ is tautology

P	Q	$P \rightarrow Q$	$(P \rightarrow Q) \wedge P$	$((P \rightarrow Q) \wedge P) \rightarrow Q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Valid conclusion

b) $H_1: P \rightarrow Q$ $H_2: \neg P$ $C: Q$

To verify, $[(P \rightarrow Q) \wedge \neg P] \rightarrow Q$ is tautology or not

P	Q	$(P \rightarrow Q)$	$\neg P$	$(P \rightarrow Q) \wedge \neg P$	$(P \rightarrow Q) \wedge \neg P \rightarrow Q$
T	T	T	F	F	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	F

Not tautology

Not valid conclusion

Validity (of Verbal arguments)

2) Determine the validity of the following argument.

premises

If two sides of a triangle are equal then opposite angles are equal. Two sides of a triangle are not equal,

conclusion

therefore the opposite angles are not equal

Soln P : Two sides of a triangle are equal

Q : Opposite angles are equal

$H_1 : P \rightarrow Q \quad H_2 : \neg P \quad C : \neg Q$

To verify that $((P \to Q) \wedge \neg P) \to \neg Q$ is a tautology.

P	Q	$P \to Q$	$\neg P$	$(P \to Q) \wedge \neg P$	$\neg Q$	$(P \to Q) \wedge \neg P \to \neg Q$
T	T	T	F	F	F	T
T	F	F	F	F	T	T
F	T	T	T	T	F	F
F	F	T	T	T	T	T

Not tautology
Not valid argument.

3) If you invest in stock market, then you will get rich. If you get rich then you will be happy

premises

therefore, If you invest in stock market then you will be happy.

conclusion

P : Invest in stock market   Q : you will get rich

R : you will be happy

$H_1 : P \rightarrow Q \qquad H_2 : Q \rightarrow R \qquad C : P \rightarrow R$

$((P \rightarrow Q) \wedge (Q \rightarrow R)) \rightarrow (P \rightarrow R)$ is a tautology (verify)

hence given argument is valid.

II) Without using Truth table

Implications

Simplification : I_1: $P \wedge Q \Rightarrow P$

I_2: $P \wedge Q \Rightarrow Q$

Addition I_3 $P \Rightarrow P \vee Q$

I_4 $Q \Rightarrow P \vee Q$

I_5 : $\neg P \Rightarrow P \rightarrow Q$

I_6 : $Q \Rightarrow P \rightarrow Q$

I_7: $\neg(P \rightarrow Q) \Rightarrow P$

I_8: $\neg(P \rightarrow Q) \Rightarrow \neg Q$

I_9 $(P) \wedge (Q) \Rightarrow (P \wedge Q)$

I_{10} $\neg P,\ P \vee Q \Rightarrow Q$ disjunctive syllogism

I_{11} $P,\ P \rightarrow Q \Rightarrow Q$ modus ponens

I_{12} $\neg Q,\ P \rightarrow Q \Rightarrow \neg P$ modus tollens

I_{13} $P \rightarrow Q \rightarrow R \Rightarrow P \rightarrow R$ hypothetical syllogism

I_{14} $P \vee Q,\ P \rightarrow R,\ Q \rightarrow R \Rightarrow R$ dilemma.

Rules of Inference

Name	Tautology	Rule of Inference
Modus ponens	$[P \wedge (P \rightarrow Q)] \rightarrow Q$	$\begin{array}{l} P \\ P \rightarrow Q \\ \hline \therefore\ Q \end{array}$

Name	Tautology	Rule of Inference
Modus tollens	$[\neg Q \land (P \to Q)] \to \neg P$	$\neg Q$ $P \to Q$ ——— $\therefore \neg P$
Hypothetical syllogism	$[(P \to Q) \land (Q \to R)] \to (P \to R)$	$P \to Q$ $Q \to R$ ——— $\therefore P \to R$
Disjunctive Syllogism	$[(P \lor Q) \land \neg P] \to Q$	$P \lor Q$ $\neg P$ ——— $\therefore Q$

Name	Tautology	Rule of Inference
Addition	$P \rightarrow (P \vee Q)$	$\dfrac{P}{P \vee Q}$
Simplification	$(P \wedge Q) \rightarrow P$	$\dfrac{P \wedge Q}{P}$
Conjunction	$[(P) \wedge (Q)] \rightarrow (P \wedge Q)$	$\dfrac{\begin{array}{c}P\\Q\end{array}}{P \wedge Q}$
Resolution	$((P \vee Q) \wedge [\neg P \vee R]) \rightarrow (Q \vee R)$	$\dfrac{\begin{array}{c}P \vee Q\\ \neg P \vee R\end{array}}{Q \vee R}$

1) State which rule is the basis of the following argument

i) It is below freezing now.

Therefore, it is either below freezing or raining now

<u>Soln</u> P : It is below freezing now

Q : It is raining now

$$\frac{P}{\therefore P \vee Q}$$

Addition Rule is used.

ii) It is below freezing and raining now

Therefore, it is below freezing now.

$$\frac{P \land Q}{\therefore P}$$

Simplification rule is used.