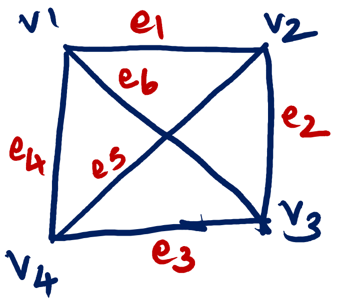
Paths, cycles and Connectivity:

Defns:

1) A path in a graph is a finite alternating sequence of vertices and edges begining and ending with vertices, such that each edge is incident on the vertices preceding and following it.



$v_1 e_1 v_2 e_5 v_4$ is a path.

$v_1 e_1 v_2 e_5 v_4 e_4 v_1$ is a circuit or cycle.

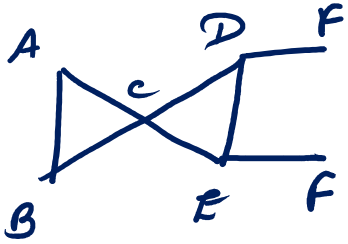
2) An undirected graph is said to be connected if there is path between every

pair of distinct vertices of the graph.

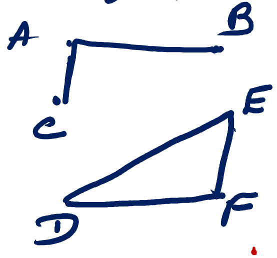
A graph that is not connected is called disconnected

G_1:

ex)



G_2:



G_1 is connected

G_2 is not connected (two components are there)

Result: The maximum number of edges in a simple disconnected graph 'G' is $\dfrac{(n-K)(n-K+1)}{2}$

ex) Find max. number of edges in a simple disconnected graph with 5 vertices and 2 components

Ans: $n = 5$, $k = 2$

$$\text{max. number of lines} = \frac{(n-k)(n-k+1)}{2} = \frac{(5-2)(5-2+1)}{2}$$

$$= \frac{3 \times 4}{2} = 6$$

Eulerian and Hamiltonian Graphs:

Defns:

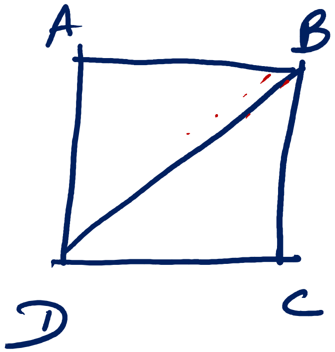
1) A path of graph G is called Eulerian path, if it includes every edge exactly (eee) once

2) A circuit (cycle, closed path) of a graph G is called Eulerian circuit, if it includes each edge of G exactly once.

3) A graph containing an Eulerian circuit is called Eulerian graph.

ex)

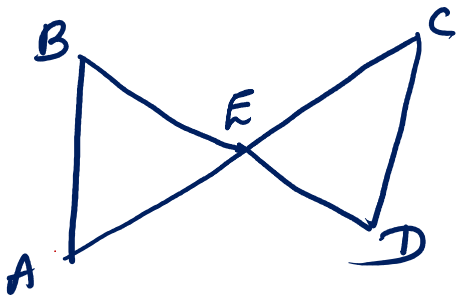
G_1:



Eulerian path: B – D – C – B – A – D
(every edge exactly once)

No Euler circuit ∴ not Euler graph

G_2:



Euler circuit : A – E – C – D – E – B – A

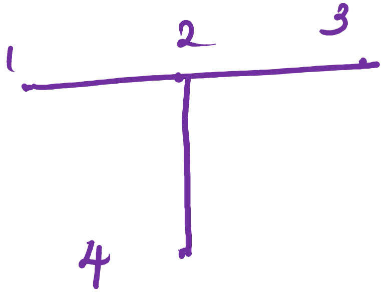
G_2 is Euler graph.

Result: * A connected graph contains an Euler circuit, if and only if each of its vertices is of even degree

* A connected graph contains an Euler path, if and only if it has exactly two vertices of odd degree.

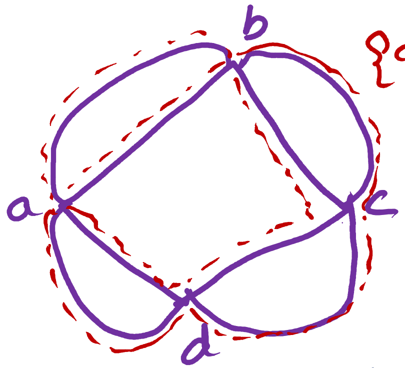
pbm: Identify Euler Path/ Euler circuit

i)



No Euler path
No Euler circuit

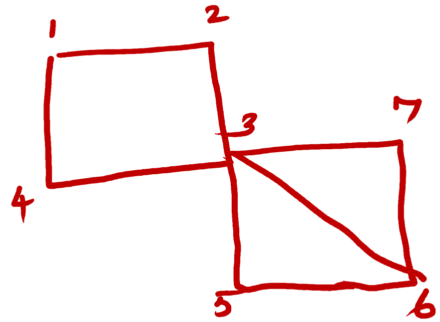
(ii)



{all vertices with even deg

Euler path = Euler circuit = abcdabcda

iii)



EP : 6 7 3 6 5 3 4 1 2 3

No euler circuit

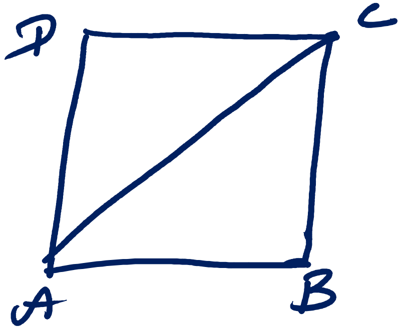
Defns :

A path of a graph G is called a Hamiltonian path, if it includes each vertex of 'G' exactly once {eve}

A circuit of a graph G is called a Hamiltonian circuit, if it includes each vertex of G, exactly once {closed H. path - starting + end vertices are the same

A graph containing a Hamiltonian circuit is called Hamiltonian graph

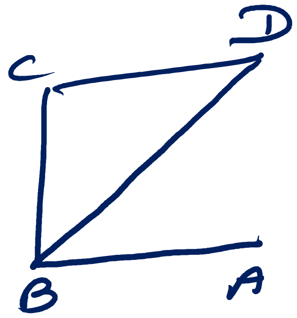
(ex)



Hamiltonian path: A - B - C - D

Hamiltonian circuit: A - B - C - D - A

(ex)



Hamiltonian path: A - B - C - D

NO Hamiltonian circuit.

Note:

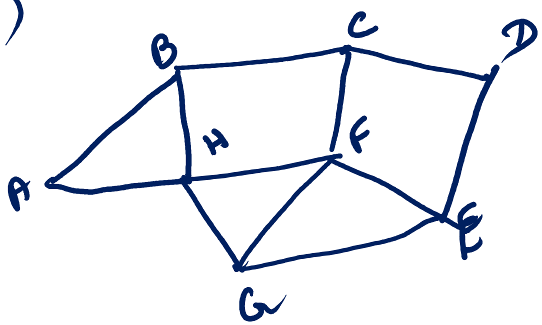
* From Hamiltonian circuit, we obtain H. path by deleting one edge.
* A H. circuit contains the Hamiltonian path but H. path does not contain H. circuit.

* A complete graph K_n, will always a a Hamiltonian circuit when $n \geq 3$.

* A given graph may contain more than one Hamiltonian circuit.

ex) Find Hamiltonian path, Hamiltonian circuit:

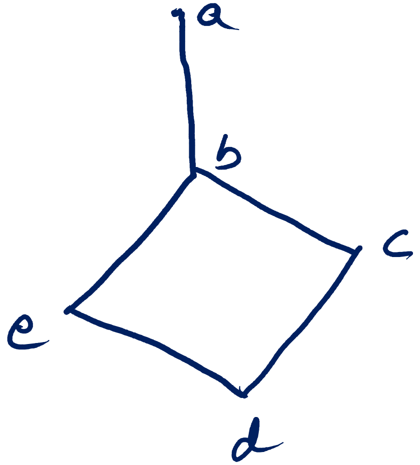
i)



HP : A, B, C, D, E, F, G, H

HC : A, B, C, D, E, F, G, H, A

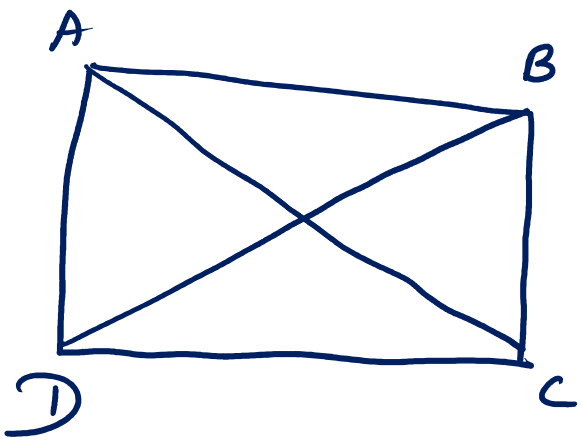
ii)



H. Path : a, b, c, d, e

No Hamiltonian circuit.

iii)



H. Path : A, B, C, D

H. circuit : A, B, C, D, A