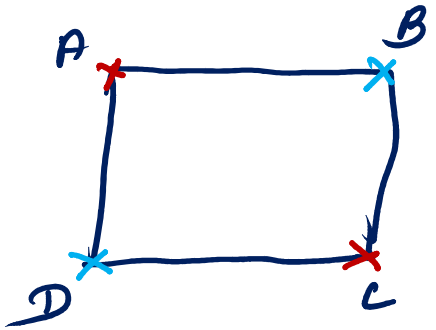
Graph Colouring:

Defn:

A colouring of a simple graph 'G' is the assignment of a colour to each vertex of the graph so that no two adjacent vertices are assigned the same colour.

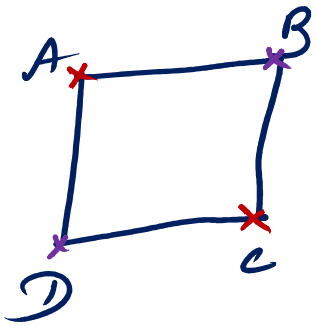
(ex) G:



Two colours are required for proper colouring of 'G'.

Defn: The smallest number of colours needed to produce a proper colouring of a graph 'G' is called chromatic number of G. denoted by $\chi(G)$

ex)



chromatic number = 2
$= \chi(G)$

Defn: The number of ways of colouring a graph G, with x or fewer colours

is a function of 'x'. $P_G(x)$, called the chromatic polynomial of 'G'.

Defn: The smallest positive value of 'x' for which $P_G(x) \neq 0$ is the chromatic number of G.

1) For any linear graph, L_n :

$$P_{L_n}(x) = x(x-1)^{n-1} \leftarrow \text{chro. poly}$$

$$\chi(L_n) = \text{chromatic number} = 2$$

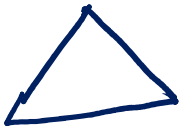
$$\begin{cases} 2(2-1)^{2-1} \\ = 2 \neq 0 \\ \text{'Smallest' value} \end{cases}$$

2) For any complete graph, K_n :

$$P_{K_n}(x) = x(x-1)\dots(x-(n-1)) = \text{Cho. Poly.}$$

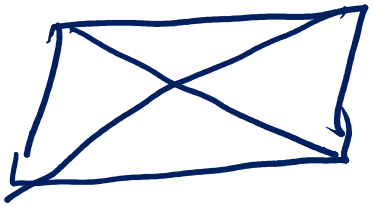
$\chi_{K_n} = n$ { Smallest value for which $P_{K_n}(n) = n(n-1)\dots n-(n-1) \neq 0$

K_3



Cho. number $\chi_{K_3} = 3$

K_4



Cho. number = 4

3) Write the chromatic polynomial of K_3. Find the number of ways of colouring K_3

Soln $P_{K_3}(x) = x(x-1)(x-2)$

Total number of ways of colouring K_3 with 3 colours, $(N_{cn}=3)$

$$P_{K_3}(3) = 3(3-1)(3-2) = 6$$

4) For any discrete graph [with n vertices only

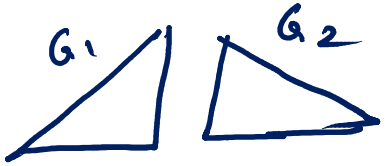
$$P_{U_n}(x) = x^n$$

$P_{U_n}(x) \neq 0$ for $x = 1$ $\therefore$ $\psi_{U_n}(x) = 1$

5) For any disconnected graph with components $G_1, G_2, \ldots G_n$

$$P_G(x) = P_{G_1}(x)\, P_{G_2}(x) \ldots P_{G_n}(x)$$

6) Find number of proper colouring of G.

G: 

$P_G(x) = P_{G_1}(x)\, P_{G_2}(x) = P_{K_3}(x)\, P_{K_3}(x)$

$= [x(x-1)(x-2)]^2$

$P_G(x) \neq 0$ for $x = 3$ $\therefore \chi(G) = 3$

Number of ways of proper colouring of G

$= P_G(3) = [3(3-1)(3-2)]^2 = 6^2 = 36.$

7) State the chromatic polynomial of complete graph K_4 and hence find its chromatic number.

Soln $P_{K_4}(x) = x(x-1)(x-2)(x-3)$

The smallest value of 'x', for which

$P_{K_4}(x) \neq 0$ is $(x = 4)$

$\chi(K_4) = 4$

8) State the chromatic polynomial of linear graph L_4

Ans : $P_{L_4}(x) = x(x-1)^{4-1} = x(x-1)^3$

Defn: Let $G(V, E)$ be a graph with no multiple edges. Let $\{c_1, c_2, \ldots c_n\}$ be any set of colors. A function $f: V \to C$ is called coloring of G using 'n' colors.