Defn: A relation R on a set A is called a Partial ordering (or) Partial Order relation if R is reflexive, antisymmetric and transitive.

(ie) (i) aRa for every $a \in A$

(ii) if aRb and bRa then $a = b$

iii) if aRb and bRc then aRc

(ex) Less than or equal to relation $\leq$, greater than or equal to relation $(\geq)$ on set of integers are Partial Order relations.

$a \geq a$ $\therefore$ reflexive, if $a \geq b$ and $b \geq a$
then $a = b$
$\therefore$ antisymmetric

if $a \geq b$ and $b \geq c$ then $a \geq c$ $\therefore$ transitive.

$(\mathbb{Z}, \geq)$ is a PO Set.

|||ly $(\mathbb{Z}, \leq)$ is a PO Set.

Pbm: 1) Prove that the relation $\subseteq$ of set inclusion is a Partial ordering on any collection of sets.

Soln $A \subseteq A$ $\therefore$ reflexive

if $A \subseteq B$ and $B \subseteq A$ then $A = B$
hence antisymmetric

if $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$

hence transitive.

$\therefore \subseteq$ is a Partial ordering relation.

2) If R is the relation on the set of integers such that $(a, b) \in R$ if $b = a^m$ for some positive integer 'm' then R is a Partial ordering.

Soln $a = a^1 \therefore a R a$ (where $m=1$, $b=a$)

R is reflexive

if $(a, b) \in R$ then $b = a^m$ for some 'm'

if $(b, a) \in R$ then $a = b^n$ for some 'n'

$\therefore a^1 = (a^m)^n \therefore mn = 1$ hence $m=n=1 \therefore \boxed{a=b}$

if $(a, b) \in R$ and $(b, c) \in R$

$b = a^m$ and $c = b^n$

then $c = (a^m)^n = a^{mn}$

$(a, c) \in R$ $\therefore$ R is transitive

R is a Partial ordering.

Hasse diagram the simplified form of digraph of partial ordering on a finite set that contains sufficient information about partial ordering is called a Hasse diagram.

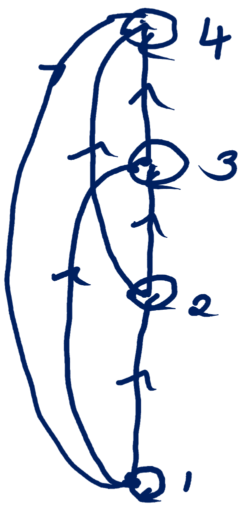
The Hasse diagram representing a partial ordering can be obtained from its digraph by:

* Removing all loops at vertices
* Removing all edges that are implied by transitive property
* Draw digraph with all edges pointing upwards so that arrows may be omitted from the lines

(ex) i) construct Hasse diagram for partial ordering $\{(a,b) / a \le b\}$ on $\{1, 2, 3, 4\}$

Soln $R = \{(1,1)\ (1,2)\ (1,3),\ (1,4)\ (2,2)\ (2,3),\ (2,4)\ (3,3)\ (3,4)\ (4,4)$

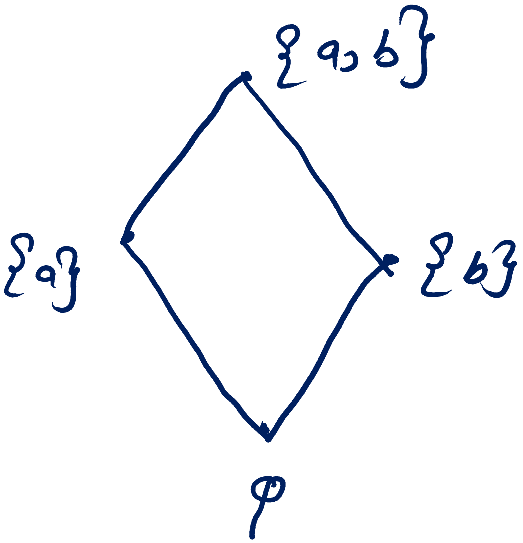
Digraph of R



Hasse Diagram of R

2) Let $S = \{a, b\}$ and $A = P(S)$, power set of S the Hasse diagram of PO-set A with partial order $\subseteq$, set inclusion is given by:

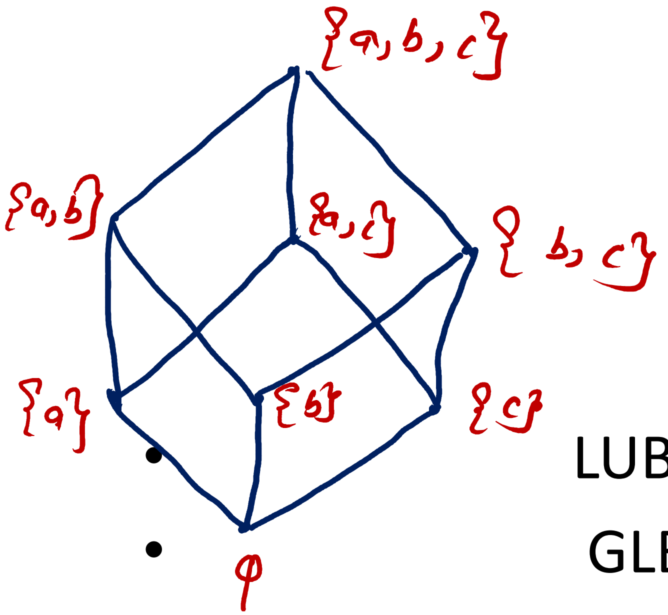
$$A = \{\varphi, \{a\}, \{b\}, \{a, b\}\}$$



$$\begin{cases} \varphi \subseteq \{a\} \subseteq \{a, b\} \\ \varphi \subseteq \{b\} \subseteq \{a, b\} \end{cases}$$

3) Let $S = \{a, b, c\}$; $A = P(S)$, power set of S.
The Hasse diagram of 'A' with partial order $\subseteq$, set inclusion.

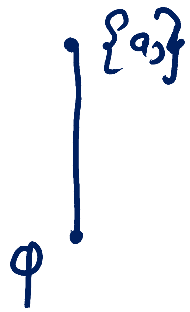
Ans: $A = \{\varphi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}, \{a,b,c\}\}$



$$\varphi \begin{cases} \subseteq \{a\} \subseteq \{a,b\}, \{a,c\} \subseteq \{a,b,c\} \\ \subseteq \{b\} \subseteq \{b,c\}, \{a,b\} \subseteq \{a,b,c\} \\ \subseteq \{c\} \subseteq \{a,c\}\ \{b,c\} \subseteq \{a,b,c\} \end{cases}$$

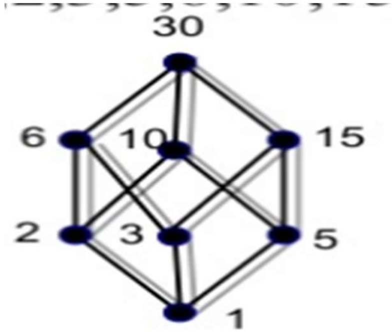
- LUB of {a},{a,b}= {a}U{a,b}= {a,b}
- GLB of {a},{a,b}= {a}∩{a,b}= {a}

4) Let $S = \{a\}$ and $A = P(S)$ then the Hasse diagram of $(A, \subseteq)$ is



5) Draw the Hasse Diagram of D_{30}, divisiors of 30.

- $D_{30} = \{1,2,3,5,6,10,15,30\}$



The LUB of 3 and 6 = LCM(3,6)=6

The GLB of 3 and 6 =GCD(3,6)=3