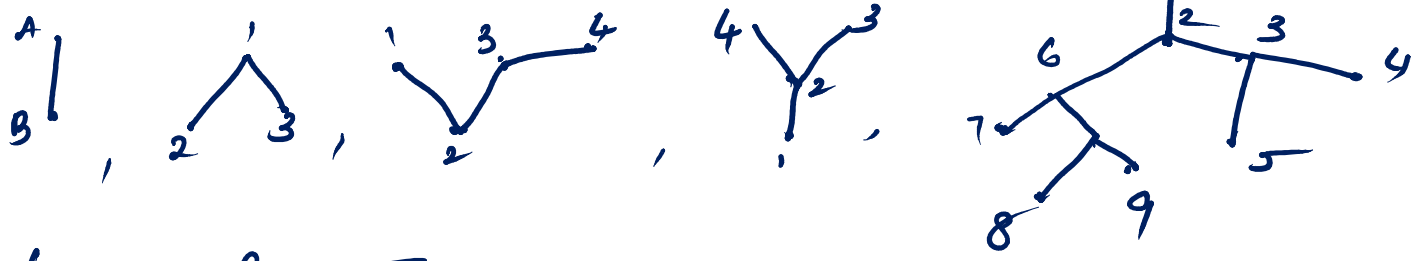
Trees:

Defn:

A connected graph without any circuits is called a tree.

(ex) 

Properties of Trees:

1. An undirected graph is a tree, if and only if, there is a unique path

between every pair of vertices.

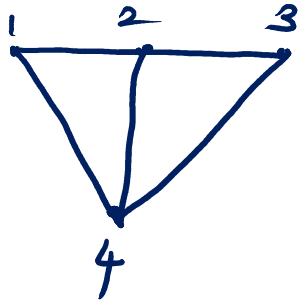
2. A tree with 'n' vertices has (n-1) edges.

3. Any circuitless graph with 'n' vertices and (n-1) edges is a tree.

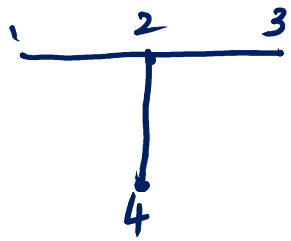
Spanning Trees:

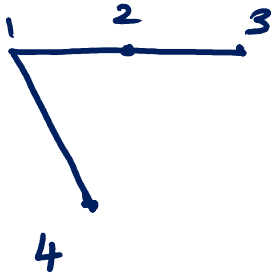
Defn: If the subgraph T of a connected graph 'G' is a tree containing all vertices of G is called a **Spanning Tree** of G.

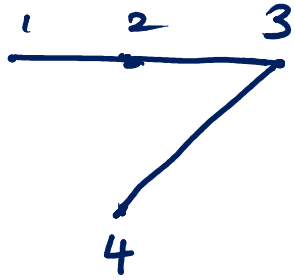
(ex) Consider the connected graph G :

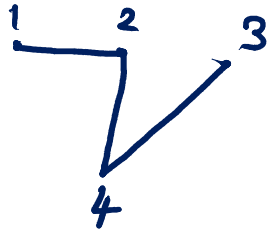


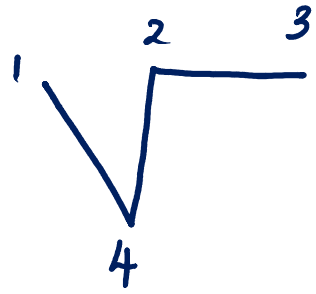
Spaning Trees of G :

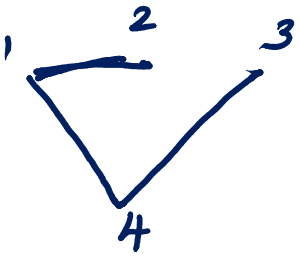
① 

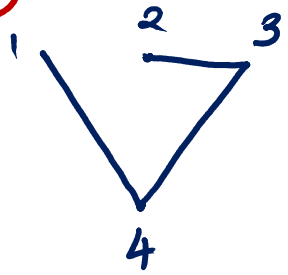
② 

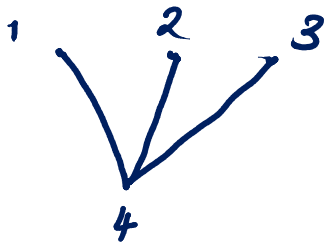
③ 

④ 

⑤ 

⑥ 

⑦ 

⑧ 

Minimum Spanning Tree:

Defn: If G is connected weighted graph, the spanning tree of G with the smallest total weight [sum of wgts of edges] is called minimum spanning Tree of G.

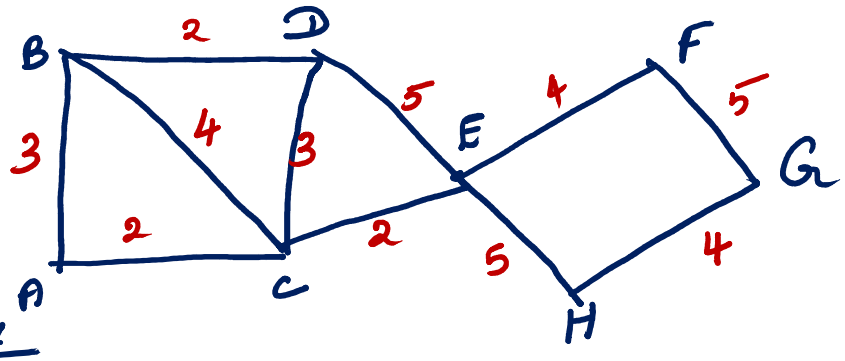
Kruskal's Algorithm:

1. The edges of the given graph G are arranged in order of increasing weights

2. An edge 'a' with minimum weight is selected as an edge of required Spanning Tree.

3. Edges with minimum weight that do not form a circuit are succesively added.

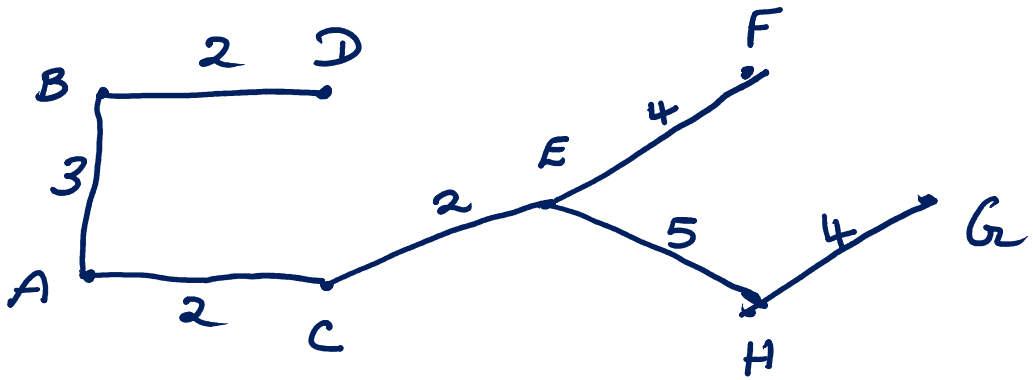
1) Find minimal spanning Tree using Kruskal's Algorithm.



Ans:

Arrange the edges in ascending order of weight

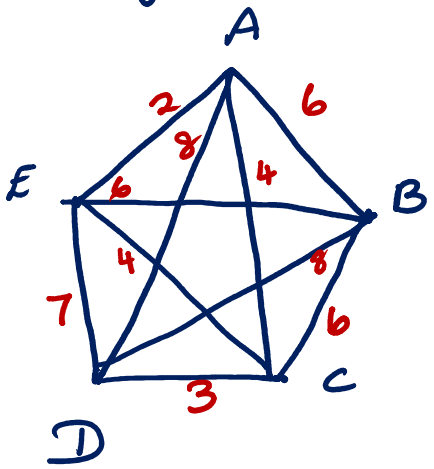
edges	weight	
AC	2 ✓	
BD	2 ✓	
CE	2 ✓	
AB	3 ✓	
CD	3	× forms circuit
BC	4	× forms circuit
EF	4 ✓	
HG	4 ✓	
DE	5	× forms circuit
EH	5 ✓	
FG	5	× forms circuit



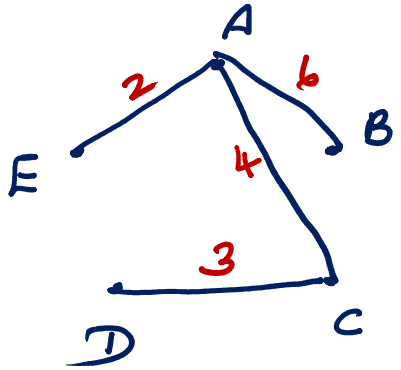
minimum weight : (2+3+2+2+4+5+4)

= 22

2) Find minimum Spanning tree for the weighted graph using Kruskal's algorithm:



Ans:

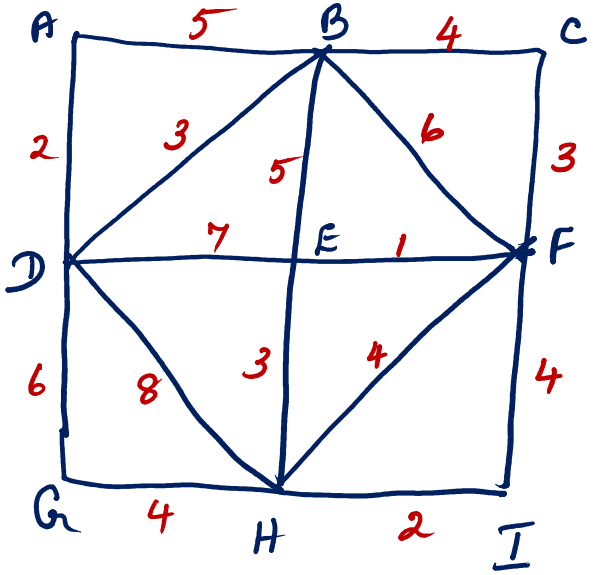


minimum weight = 15

Ans Arrange the edges in increasing order of weight

Edges	weight
AE	2 ✓
CD	3 ✓
AC	4 ✓
CE	4 ×
AB	6 ✓
CB	6 ×
EB	6 ×
DE	7 ×
AD	8 ×
BD	8 ×

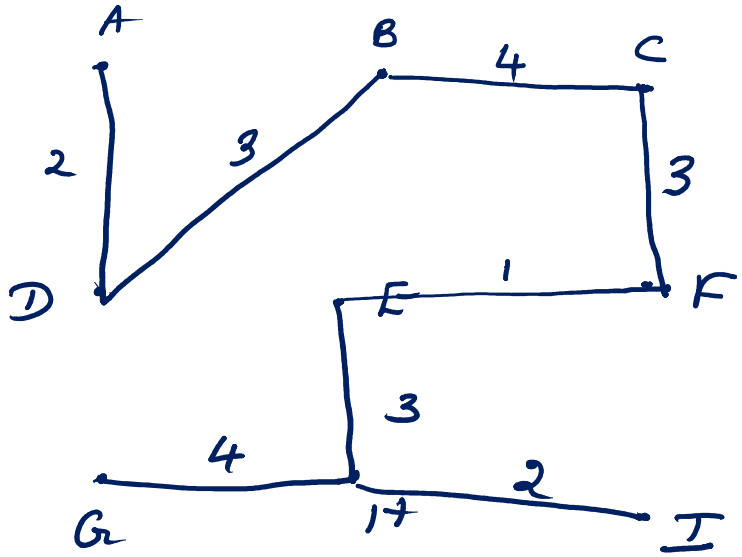
3) Find minimum Spanning Tree using Kruskal's algorithm:



Arrange the edges in ascending order of weight.

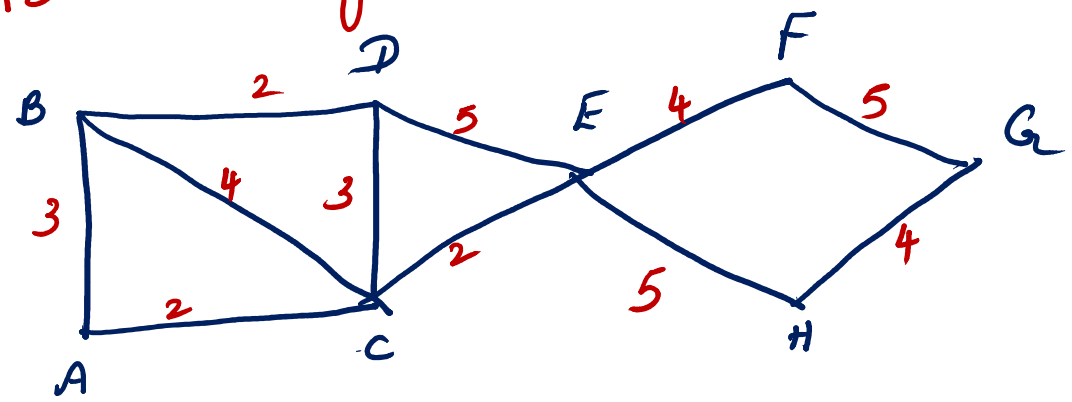
Edge	weight
E F	1 ✓
A D	2 ✓
H I	2 ✓
B D	3 ✓
C F	3 ✓
E H	3 ✓
B C	4 ✓
F H	4 X
F I	4 X
G H	4 ✓
A B	5
B E	5
B F	6
D G	6
D E	7
D H	8

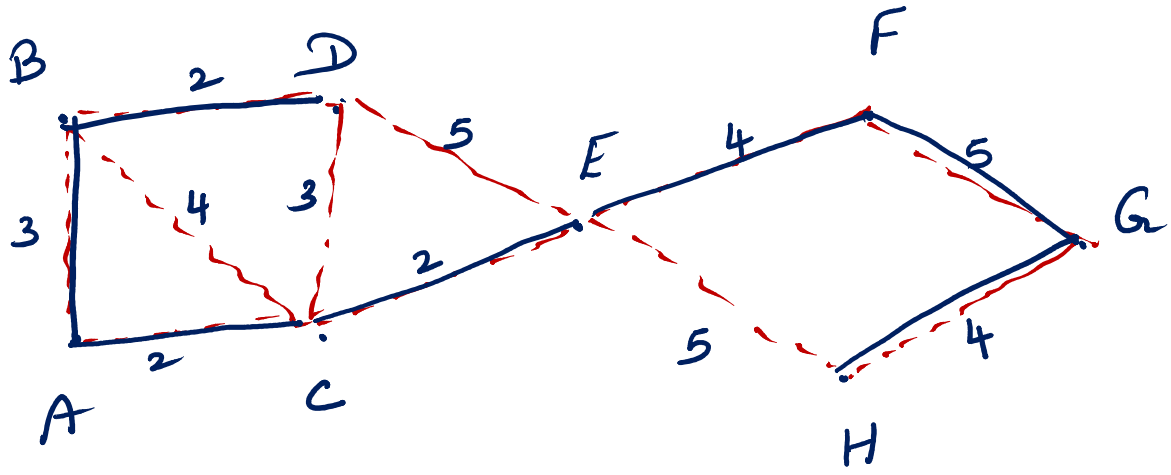
weight of minimum spanning tree = 22



Prim's Algorithm: Edges of minimum weight that are incident on a vertex already in a spanning tree and not forming a circuit are selected.

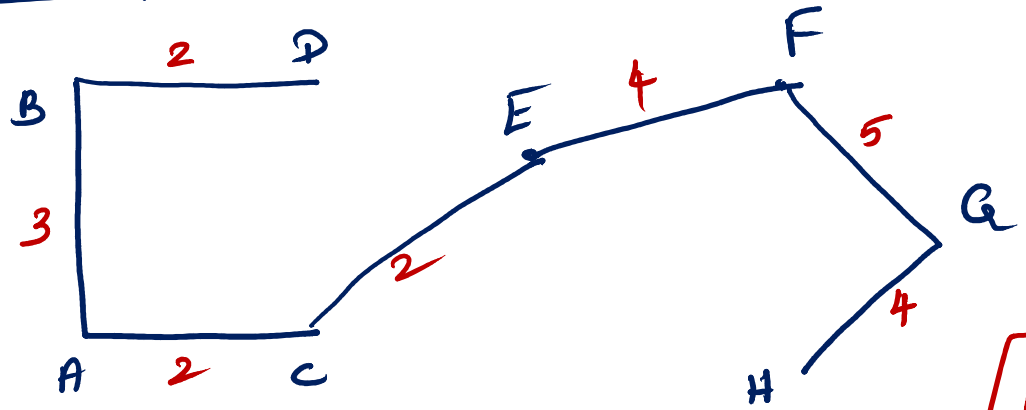
Pbm: Find minimum spanning Tree using Prim's Algorithm:





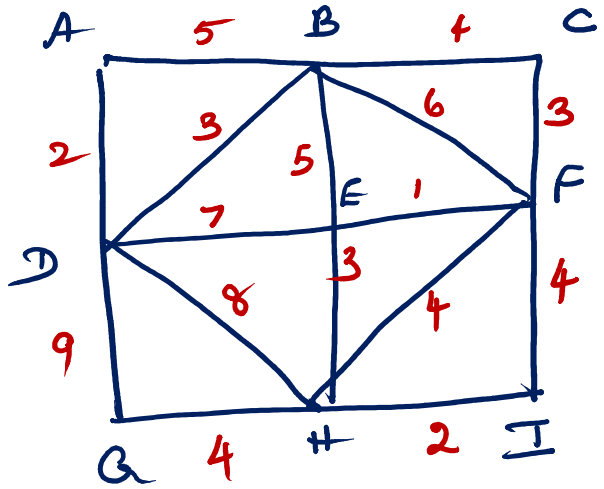
Selected Edges	wgt
A C	2
C E	2
E F	4
F G	5
G H	4
A B	3
B D	2
	22

Spanning Tree:

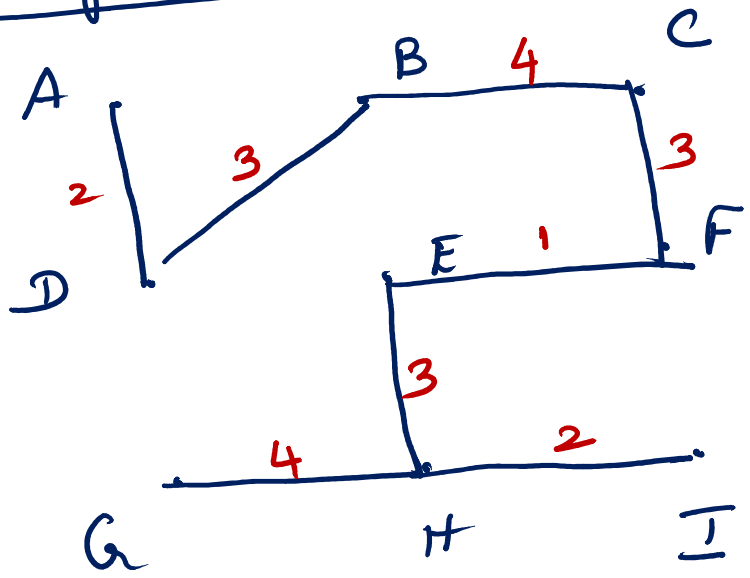


mini wgt = 22

ii)



Spanning Tree



mini wgt = 22

Selected Edges	: weight
A D	2
D B	3
B C	4
C F	3
F E	1
E H	3
H I	2
H G	4
	22

Application:

Minimum Spanning trees have applications in the design of networks, including computer networks, telecommunication network, transportation network, water supply network and electrical grid.