Relations:

Defn Let A and B be two nonempty sets, a binary relation R from A to B is a subset of $(A \times B)$ and $(a, b) \in R$ if 'a' is related to 'b' (where $a \in A$, $b \in B$)

if $a R b$ then $(a, b) \in R$
(related to)

The domain of R is the set of all elements of A that are related to some elements in B

$$\text{Domain} = \{a \in A \,/\, a R b \text{ for some } b \in B\} = D(R)$$

Range of R is the set of all elements in 'B' that are related to some elements in 'A'

Range of 'R' = $R(R) = \{ b \in B / aRb$ for some $a \in A \}$

(ex) Find the Relation 'R', Domain, Range (or Image of R)

1) $A = \{0, 1, 2, 3, 4\}$; $B = \{0, 1, 2, 3\}$ and aRb if $a + b = 4$.

Soln $R = \{(1, 3) \ (2, 2) \ (3, 1) \ (4, 0)\}$

Domain $= \{1, 2, 3, 4\}$; Range $= \{0, 1, 2, 3\}$

2) Let R be the relation on $A = \{1, 2, 3, 4\}$ define aRb if $a \leq b$, $a, b \in A$

Soln $R = \{(1,1)\ (1,2)\ (1,3),\ (1,4)\ (2,2)\ (2,3)\ (2,4)\ (3,3),\ (3,4)\ (4,4)\}$

Domain = Range = $A = \{1, 2, 3, 4\}$

3) $A = \{1, 3, 5, 7\}$; $B = \{2, 4, 6\}$

$a R b$ if $a < b$.

$R = \{(1,2)\ (1,4)\ (1,6)\ (3,4)\ (3,6),\ (5,6)\}$

Domain $= \{1, 3, 5\}$ Range $= \{2, 4, 6\}$

4) $A = \{1, 2, 3, 4\}$ $B = \{2, 3, 4, 5\}$ aRb if a and b are both odd.

<u>Soln</u> $R = \{(1, 3)\ (1, 5)\ (3, 3)\ (3, 5\}$

Domain $= \{1, 3\}$ Range $= \{3, 5\}$

5) $A = \{1, 2, 3, 4\}$ $B = \{1, 4, 6, 8, 9\}$ aRb if $b = a^2$

$R = \{(1, 1)\ (2, 4)\ (3, 9)\}$

Domain $= \{1, 2, 3\}$ Range $= \{1, 4, 9\}$

6) $A = \{0, 1, 2, 3, 4\}$ $B = \{0, 1, 2, 3\}$ aRb if 'a' divides 'b'

$R = \{(1,0), (1,1), (1,2), (1,3), (2,2), (2,0), (3,0), (3,3), (4,0)\}$

Domain $= \{1, 2, 3, 4\}$ Range $= \{0, 1, 2, 3\}$.

<u>Note:</u> when a set has 'n' elts, $A \times A$ has n^2 elements; the number of subsets of $(A \times A) = 2^{n^2}$ and hence there are 2^{n^2} relations on a set with 'n' elements.

<u>Operations on Relations:</u>

If R and S denote two relations, then

1) $a(R \cap S)\,b = (aRb) \underset{\text{meet}}{\overset{\text{and}}{\wedge}} (aSb)$

ii) $a(R \cup S)\, b = (aRb) \underset{\text{joint}}{\overset{\text{or}}{\vee}} (aSb)$

iii) $a(R-S)\, b = (aRb) \wedge (a \not{S} b)$

iv) Complement of R, R' or R^c or $\bar{R}$ or $\sim R$

$(a\bar{R}b)$ if $(a \not{R} b)$

v) Inverse relation R^{-1}:

if aRb then $bR^{-1}a$

(ie) $(a,b) \in R$ then $(b,a) \in R^{-1}$

vi) $R \oplus S = (R \cup S) - (R \cap S)$

Pbm: 1 Let $A = \{x, y, z\}$, $B = \{1, 2, 3\}$, $C = \{x, y\}$

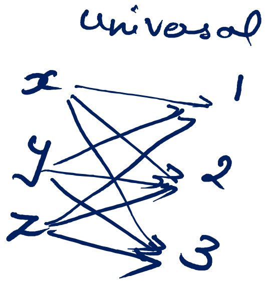
$D = \{2, 3\}$. Let R be a relation from A to B defined by $R = \{(x,1), (x,2), (y,3)\}$

Let S be a relation from C to D defined by

$$S = \{(x,2), (y,3)\}$$

Find 1) $R \cap S$ (2) $R \cup S$ (3) $\bar{R}$ (4) R^{-1} (5) $R - S$ (6) $S - R$

7) $R \oplus S$



Soln

1) $R \cap S = \{(x,2),\ (y,3)\}$

2) $R \cup S = \{(x,1)\ \ (x,2)\ \ (y,3)\}$

3) $\bar{R} = \{(x,3)\ \ (y,1)\ \ (y,2)\ \ (z,1)\ \ (z,2)\ \ (z,3)\}$

4) $R^{-1} = \{(1,x)\ \ (2,x)\ \ (3,y)\}$

5) $R - S = \{(x,1)\}$

6) $S - R = \{\ \}$

7) $R \oplus S = (R \cup S) - (R \cap S) = \{(x,1)\}$

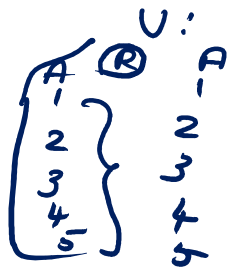
2) Let R be a relation on set $A = \{1, 2, 3, 4, 5\}$ is defined by the rule $(a, b) \in R$ if 3 divides $(a-b)$ [or $(a-b)$ is a multiple of 3 (or) $a \equiv b \pmod 3$]

List R, R^{-1}, $\bar{R}$

Soln

$$R = \{(1,1), (1,4), (2,2), (2,5), (3,3), (4,1), (4,4), (5,2), (5,5)\}$$

$$R^{-1} = \{(1,1)\ (4,1)\ (2,2)\ (5,2)\ (3,3), (1,4)\ (4,4)\ (2,5), (5,5)\}$$



$R = R^{-1}$

$$\bar{R} = \{(1,2)\ (1,3)\ (1,5)\ (2,1)\ (2,3)\ (2,4), (3,1)\ (3,2)\ (3,4)\ (3,5)\ (4,2)\ (4,3)\ (4,5)\ (5,1)\ (5,3)\ (5,4)\}$$