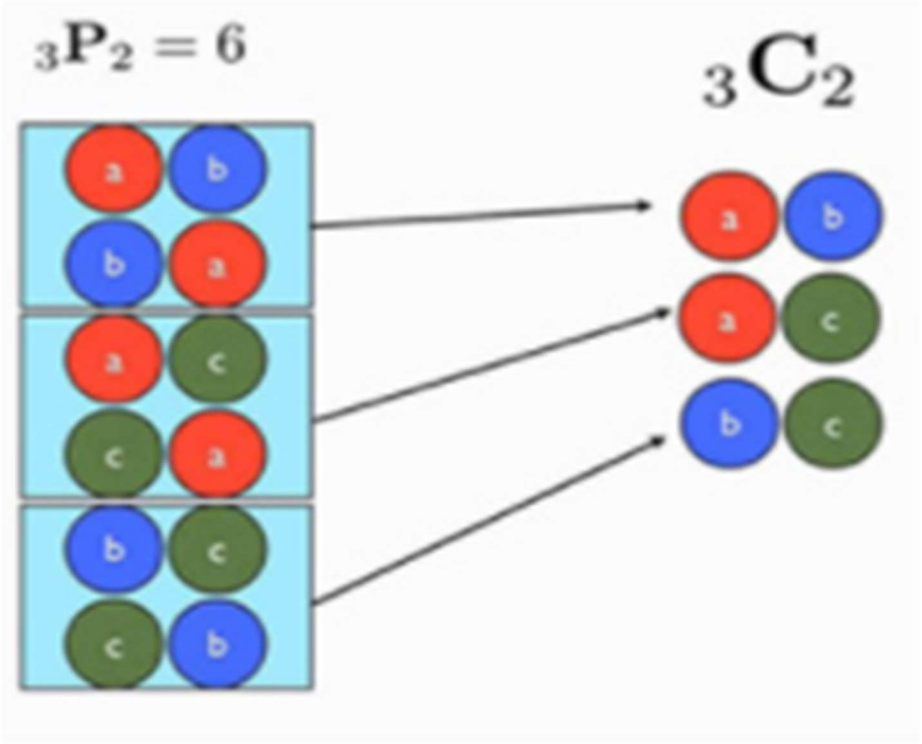
Permutation, combination



Results: 1) $nC_n = nC_0 = 1$; $nC_1 = n$ (ie) $C(n,1) = n$

$C(n,n) = C(n,0) = 1$

2) $nC_r = nC_{n-r}$; $C(n,n) = C(n,n-r)$

3) $\boxed{nC_r = \frac{P(n,r)}{r!}}$

1) Compute (a) $C(5,3)$ (b) $C(8,0) = {}^8C_0$

a) $C(5,3) = \frac{5!}{3!(5-3)!} = \frac{5!}{3!\,2!} = \frac{5\times4\times3\times2\times1}{(3\times2\times1)(2\times1)} = 10$

b) $C(8,0) = \frac{8!}{0!(8-0)!} = \frac{8!}{8!} = 1$

c) $C(5,5) = 1 = {}^5C_5$ (d) $C(5,1) = 5 = {}^5C_1$

2) Find 'n' if $20C_{n+2} = 20C_{2n-1}$

Soln if $nC_x = nC_y$ then $x = y$

$$20C_{n+2} = 20C_{2n-1}$$

$$\therefore n+2 = 2n-1$$

$$2+1 = 2n-n \quad \therefore \boxed{3 = n}$$

3) In how many ways can a set of five letters to be selected from English alphabet.

Soln Total no. of letter = 26 ; Selecting 5 out of 26

$$26C_5 \text{ ways} = \frac{26!}{5!(26-5)!} = 65,780$$

4) Suppose that there are 9 faculty members in the mathematics department and 11 in CS dept. How many ways are there to select a committee to develop a discrete mathematics course at a school if the committee consists of three from mathematics and four from CS dept.

Soln No. of ways to select the committee

$$= (9C_3)(11C_4)$$

$$= \frac{9!}{3!(9-3)!} \quad \frac{11!}{4!(11-4)!} = (84)(330) = 27,720$$

6) From a club consisting of 6 men and 7 women in how many ways can we select a committee of

a) 3 men and 4 women

$$6C_3 \times 7C_4 = \left[\frac{6!}{3!(3!)}\right] \times \left[\frac{7!}{4!\,3!}\right] = 700 \text{ ways.}$$

b) 4 persons which has atleast one woman.

i) 1w, 3m ii) 2w 2m iii) 3w 1m (iv) 4w 0m

$$= 7C_1\,6C_3 + 7C_2\,6C_2 + 7C_3\,6C_1 + 7C_4\,6C_0$$

$$= 140 + 315 + 210 + 35 = 700 \text{ ways}$$

c) 4 persons that has atmost one man.

i) 1m, 3w (ii) 0m, 4w

$= {}^6C_1 \cdot {}^7C_3 + {}^6C_0 \cdot {}^7C_4 = 6(35) + 1(35) = 245$

d) 4 persons that has persons of both Sexes.

i) 1m 3w ii) 2m 2w iii) 3m 1w

$= {}^6C_1\,{}^7C_3 + {}^6C_2\,{}^7C_2 + {}^6C_3\,{}^7C_1 = 210 + 315 + 140 = 665$ ways

e) 4 persons so that two specific members are not included

$= {}^{13}C_4 - {}^{11}C_2$

$= 715 - 55 = 660.$

{ two specific members : 13 − 2 = 11
two members selected from remaing 11 in $^{11}C_2$ ways.