Chapter 2

GAs: How They Work?

- For generality:

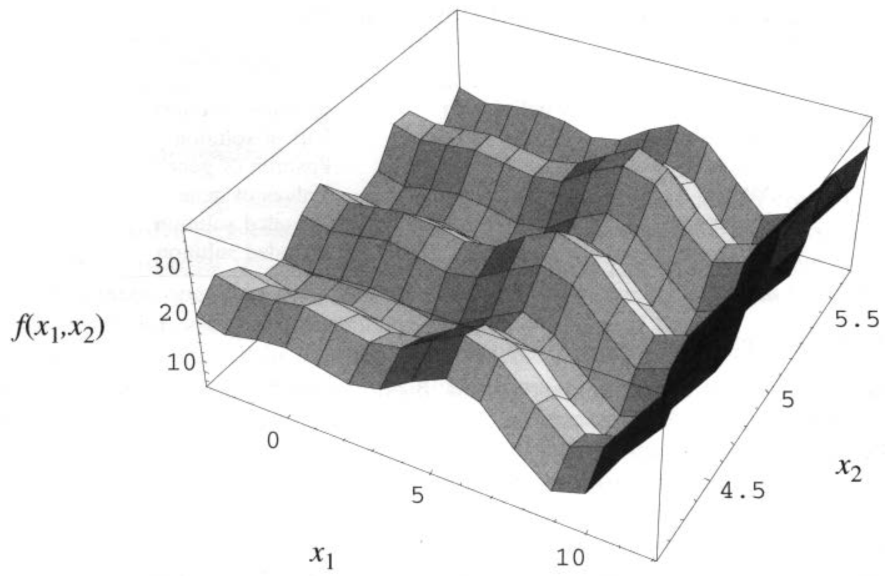
$$\min f(x) = \max g(x) = \max\{-f(x)\}$$

$$\max g(x) = \max\{g(x) + C\}$$

Example

(multi-variable optimization problem)

$$\max\ f(x_1, x_2) = 21.5 + x_1 \sin(4\pi x_1) + x_2 \sin(20\pi x_2)$$
$$-3.0 \le x_1 \le 12.1$$
$$4.1 \le x_2 \le 5.8$$



General Scheme of an GA

1. An **evaluation function**
2. Values for various **parameters**
3. A genetic **representation**
4. A way to create an **initial population**
5. Genetic **operators**
6. A way to **terminate** the algorithm
7. GA **algorithm**

SGA

1. An **evaluation function**
   - *Fitness Function : Problem Requirement*
2. Values for various **parameters**
   - *population _size = N*
   - *probability_crossover = Pc*
   - *probability_mutation = Pm*
3. A genetic **representation**
   - *Binary encoding*
4. An i**nitial population**
   - *Randomly*
5. Genetic **operators**
   - *Crossover: one-point crossover*
   - *Mutation: bit reverse mutation*
   - *Reproduction*
   - *Survivor Selection : Roulette Wheel*
6. A way to **terminate** algorithm
   - *t = MaxT*
7. **GA algorithm**

Genetic Algorithm

Procedure **Genetic Algorithm**
Begin
    t ← 0
    Initialize P(t)
    Evaluate P(t)
    While (not termination-condition) do
    Begin
        t ← t + 1
        Select P(t+1) from P(t)
        Alter P(t)
        Evaluate P(t)
    End
End

Results

- ***Step 1***: *t=0*
- ***Step 2***: Initialize P(*t*) (Randomly)

$$x_j \in [a_j..b_j] \rightarrow 2^{m-1} \le (b-a+1)\times 10^P \le (2^m-1)$$

18 bits | 15 bits

000001010100101001 101111011111110

33 bits

$$v_1 = [000001010100101001101111011111110]$$
$$v_2 = [001110101110011000000010101001000]$$
$$v_3 = [111000111000001000010101001000110]$$
$$v_4 = [100110110100101101000000010111001]$$
$$v_5 = [000010111101100010001110001101000]$$
$$v_6 = [111110101011011000000010110011001]$$
$$v_7 = [110100010011111000100110011101101]$$
$$v_8 = [001011010100001100010110011001100]$$
$$v_9 = [111110001011101100011101000111101]$$
$$v_{10} = [111010011101010100000101011101010]$$

$$v_1 = [x_1, x_2] = [-2.687969, 5.361653]$$
$$v_2 = [x_1, x_2] = [\ 0.474101, 4.170144]$$
$$v_3 = [x_1, x_2] = [10.419457, 4.661464]$$
$$v_4 = [x_1, x_2] = [\ 6.159951, 4.109598]$$
$$v_5 = [x_1, x_2] = [-2.301286, 4.477282]$$
$$v_6 = [x_1, x_2] = [11.788084, 4.174346]$$
$$v_7 = [x_1, x_2] = [9.342067,\ 5.121702]$$
$$v_8 = [x_1, x_2] = [-0.330256, 4.694977]$$
$$v_9 = [x_1, x_2] = [11.671267, 4.873501]$$
$$v_{10} = [x_1, x_2] = [11.446273, 4.171908]$$

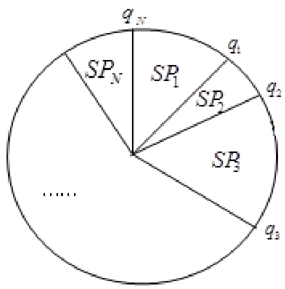
- *Step 3*: Evaluate P(t)

$$eval(v) = f(x_1, x_2, \cdots x_n)$$
$$= 21.5 + x_1 \sin(4\pi x_1) + x_2 \sin(20\pi x_2)$$

$$\mathrm{eval}(v_1) = f(-2.687969, 5.361653) = 19.805119$$
$$\mathrm{eval}(v_2) = f(\ 0.474101, 4.170144) = 17.370896$$
$$\mathrm{eval}(v_3) = f(10.419457, 4.661464) = 9.590546$$
$$\mathrm{eval}(v_4) = f(\ 6.159951, 4.109598) = 29.406122$$
$$\mathrm{eval}(v_5) = f(-2.301286, 4.477282) = 15.686091$$
$$\mathrm{eval}(v_6) = f(11.788084, 4.174346) = 11.900541$$
$$\mathrm{eval}(v_7) = f(9.342067,\ 5.121702) = 17.958717$$
$$\mathrm{eval}(v_8) = f(-0.330256, 4.694977) = 19.763190$$
$$\mathrm{eval}(v_9) = f(11.671267, 4.873501) = 26.401669$$
$$\mathrm{eval}(v_{10}) = f(11.446273, 4.171908) = 10.252480$$

- *Step 4*: $t=t+1$

- ***Step 5***: Select P(t) from P(t-1)



$SP_1 = 0.11180$	$SP_2 = 0.097515$	$SP_3 = 0.053839$
$SP_4 = 0.165077$	$SP_5 = 0.088057$	$SP_6 = 0.066806$
$SP_7 = 0.100815$	$SP_8 = 0.110945$	$SP_9 = 0.148211$
$SP_{10} = 0.057554$		

$q_1 = 0.11180$	$q_2 = 0.208695$	$q_3 = 0.262534$
$q_4 = 0.427611$	$q_5 = 0.515668$	$q_6 = 0.582475$
$q_7 = 0.683290$	$q_8 = 0.794234$	$q_9 = 0.942446$
$q_{10} = 1.000000$		

$r_1 = 0.301431$	$r_2 = 0.322062$	$r_3 = 0.766503$
$r_4 = 0.881892$	$r_5 = 0.350871$	$r_6 = 0.583392$
$r_7 = 0.177618$	$r_8 = 0.343242$	$r_9 = 0.032685$
$r_{10} = 0.197577$		

$$v_1' = [100110110100101101000000010111001] \quad (v_4)$$
$$v_2' = [100110110100101101000000010111001] \quad (v_4)$$
$$v_3' = [001011010100001100010110011001100] \quad (v_8)$$
$$v_4' = [111110001011101100011101000111101] \quad (v_9)$$
$$v_5' = [100110110100101101000000010111001] \quad (v_4)$$
$$v_6' = [110100010011111000100110011101101] \quad (v_7)$$
$$v_7' = [001110101110011000000010101001000] \quad (v_2)$$
$$v_8' = [100110110100101101000000010111001] \quad (v_4)$$
$$v_9' = [000001010100101001101111011111110] \quad (v_1)$$
$$v_{10}' = [001110101110011000000010101001000] \quad (v_2)$$

- *Step 6*: Crossover (One point crossover)

N = 10; position = 1 (randomly, [0..m-1]); $p_c = 0.25$

$r_1 = 0.625721$ $r_2 = 0.266823$ $r_3 = 0.288644$

$r_4 = 0.295114$ $r_5 = 0.163274$ $r_6 = 0.567461$

$r_7 = 0.085940$ $r_8 = 0.392865$ $r_9 = 0.770714$

$r_{10} = 0.548656$

Parents:

v_5 = [100110110100101101000000010111001]

v_7 = [001110101110011000000010101001000]

Offsprints:

v_5' = [101110101110011000000010101001000]

v_7' = [000110110100101101000000010111001]

- *Step 7*: Mutation $(p_m = 0.01)$

bit_pos	$r_k \leq p_c$	chrom_num	bit_no
105	0.009857	4	6
164	0.003113	5	32
199	0.000946	7	1
329	0.001282	10	32

$$v_1' = [100110110100101101000000010111001]$$
$$v_2' = [100110110100101101000000010111001]$$
$$v_3' = [001011010100001100010110011001100]$$
$$v_4' = [11111\underline{1}001011101100011101000111101]$$
$$v_5' = [1011101011100110000000101010010\underline{1}0]$$
$$v_6' = [110100010011111000100110011101101]$$
$$v_7' = [\underline{1}00110110100101101000000010111001]$$
$$v_8' = [100110110100101101000000010111001]$$
$$v_9' = [000001010100101001101111011111110]$$
$$v_{10}' = [0011101011100110000000101010010\underline{1}0]$$

- *Step 8*: Evaluation P(*t*)

$$eval(v) = f(x_1, x_2, \cdots x_n)$$
$$= 21.5 + x_1 \sin(4\pi x_1) + x_2 \sin(20\pi x_2)$$

$$eval(v_1) = f(6.159951, 4.109598) = 29.406122$$
$$eval(v_2) = f(6.159951, 4.109598) = 29.406122$$
$$eval(v_3) = f(-0.330256, 4.694977) = 19.763190$$
$$eval(v_4) = f(11.907206, 4.873501) = 5.702781$$
$$eval(v_5) = f(8.024130, 4.170248) = 19.91025$$
$$eval(v_6) = f(9.342067, 5.121702) = 17.958717$$
$$eval(v_7) = f(6.159951, 4.109598) = 29.406122$$
$$eval(v_8) = f(6.159951, 4.109598) = 29.406122$$
$$eval(v_9) = f(-2.687969, 5.361653) = 19.805119$$
$$eval(v_{10}) = f(0.474101, 4.170248) = 17.370896$$

- *Step 9*: Go to Step 4 while (not termination-condition)

- **C. Solution**

$$v^* = (1111100000\ 0011100011\ 1101001010\ 110)$$

$$\text{eval}(v^*) = f(11.631407, 5.724824) = 38.818208$$

$$x_1^* = 11.631407$$

$$x_2^* = 5.724824$$

$$f(x_1^*, x_2^*) = 38.818208$$