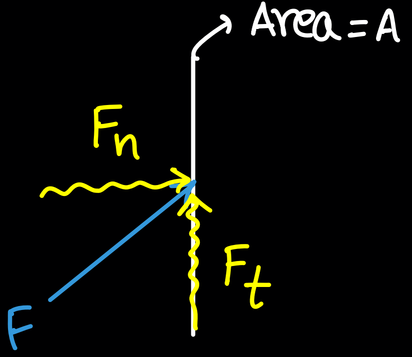
Pressure:-



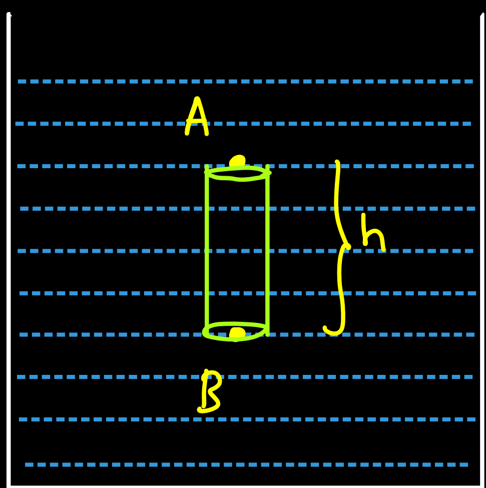
$$\text{Pressure} = \frac{F_n}{A}.$$

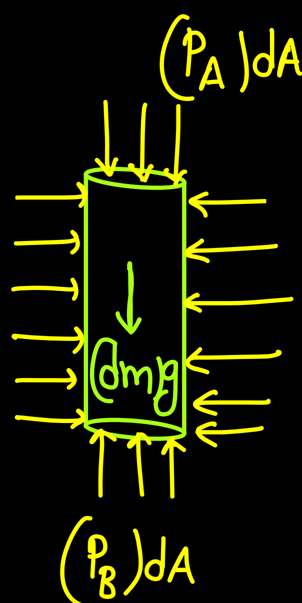
S.I = N/m^2
= Pa.

⇒ P = absolute pressure

⇒ P_0 = atmospheric pressure ⇒ at mean sea level $P_0 = 1.01 \times 10^5$ Pa.

⇒ $P - P_0$ = guage pressure.





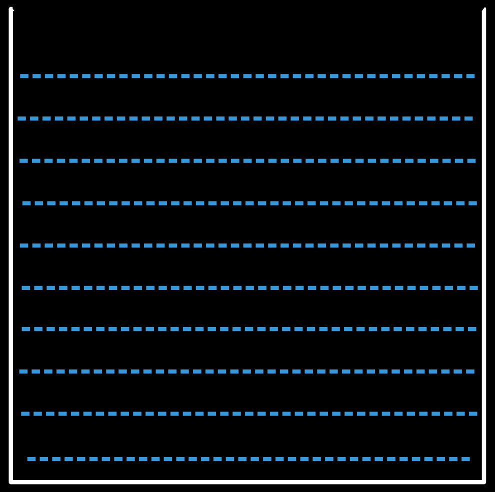
Vertical direction:-

$$F_{net} = 0$$

$$(P_B - P_A)dA = (dm)g$$

$$(P_B - P_A)\cancel{dA} = \rho[(\cancel{dA})h]g$$

$$P_B - P_A = \rho g h.$$



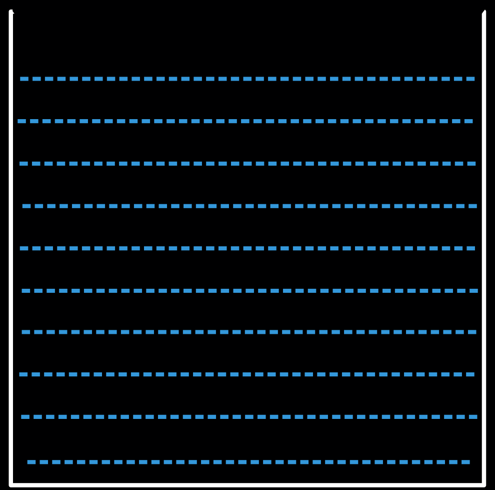
$\uparrow a$

$$F_{net} = (dm)a.$$

$$(P_B - P_A)dA - (dm)g = (dm)a.$$

$$(P_B - P_A)dA = dm(g+a)$$

$$P_B - P_A = \rho(g+a)h.$$



$\downarrow a$

$$(dm)g + P_A dA - P_B dA = (dm)a.$$

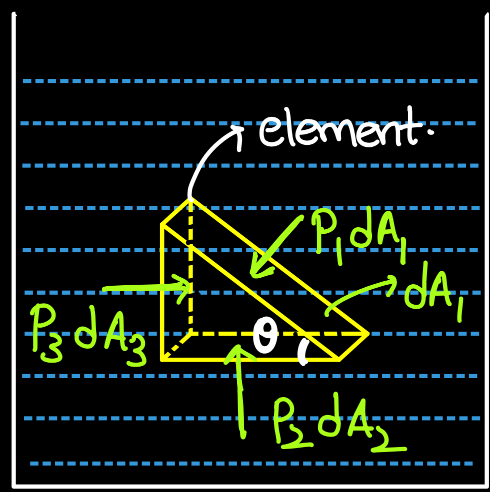
$$(P_B - P_A) = \rho(g-a)h.$$

if container is in free fall $\quad P_B - P_A = 0 \Rightarrow P_B = P_A.$

$$\boxed{P_B - P_A = \rho g_{eff} h}$$

$\Rightarrow$ Observation:- pressure is due to the normal force b/w layers. Normal will cancel weight of liquid above it and that Normal force per unit area is called pressure.

Pascal's law:-



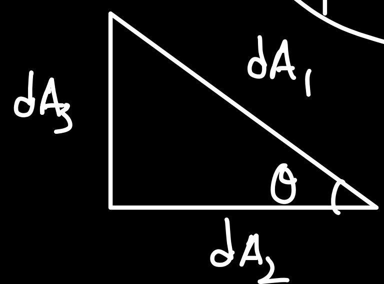
$$\Sigma F_{net} = 0$$

horizontal

$$P_3 dA_3 = P_1 dA_1 \sin\theta$$

Vertical

$$P_2 dA_2 = P_1 dA_1 \cos\theta.$$

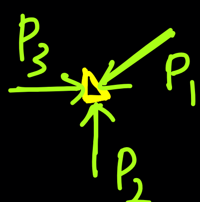


$$dA_1 \sin\theta = dA_3$$

$$dA_1 \cos\theta = dA_2$$

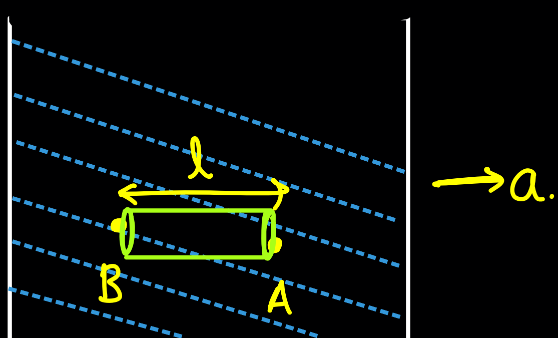
$$P_3 = P_1 \qquad P_2 = P_1$$

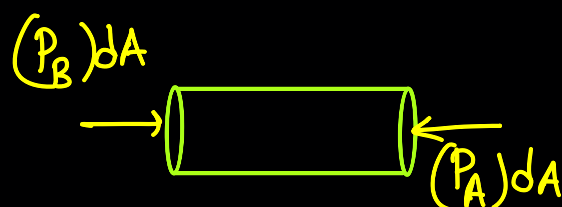
$$\therefore \quad P_1 = P_2 = P_3.$$



Observation:- pressure acting on a point from all the directions are same.

Horizontally accelerated container:-

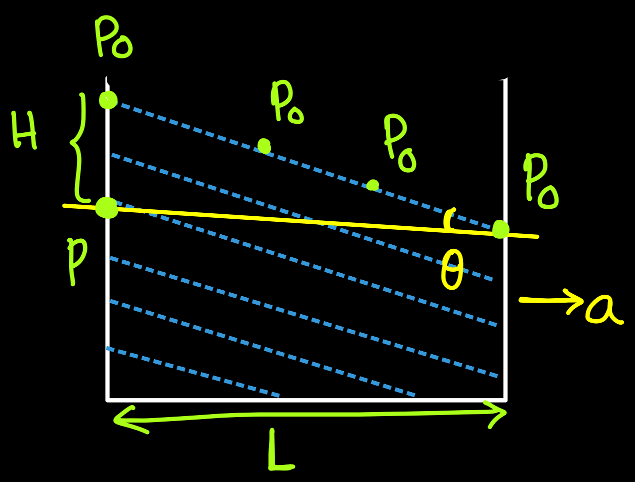




$$F_{net} = (dm)a.$$

$$(P_B - P_A)\cancel{dA} = \rho[(\cancel{dA})l]a$$

$$P_B - P_A = \rho l a$$



Horizontal :-

$$P - P_0 = \rho L a \quad -(1)$$

Vertical :-

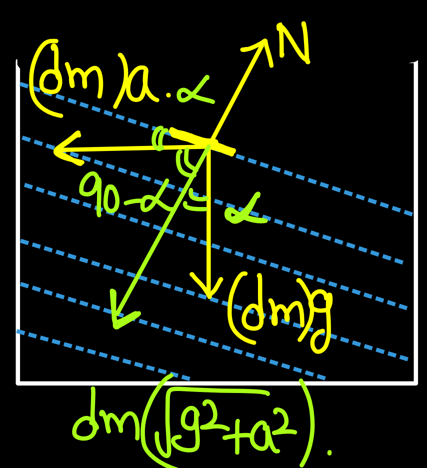
$$P - P_0 = \rho g H \quad -(2)$$

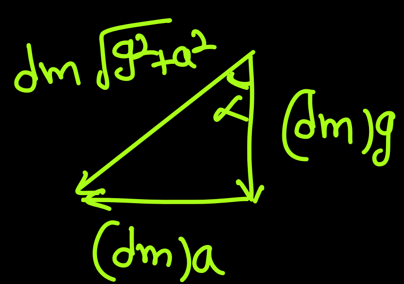
$$\frac{(1)}{(2)} \Rightarrow \frac{La}{gH} = 1$$

$$\frac{H}{L} = \frac{a}{g}.$$

$$\boxed{\tan\theta = \frac{a}{g}.}$$

Let's look from the liquid point of view.





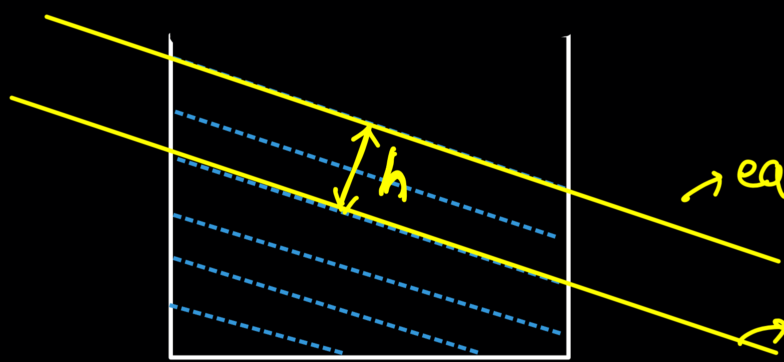
$$\tan\alpha = \frac{a}{g} = \tan\theta$$

$$\alpha = \theta.$$

$dm\sqrt{g^2+a^2}$ acts normal to the surface.

⇒ liquid always take a shape where net force apart from normal is always $\perp$ to surface.

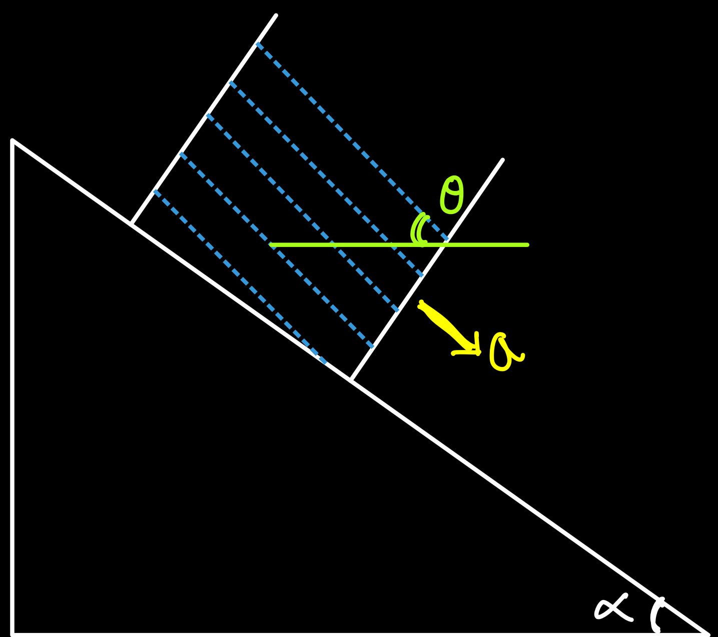
w.r.t liquid.



→ equipressure line $= P_0$.

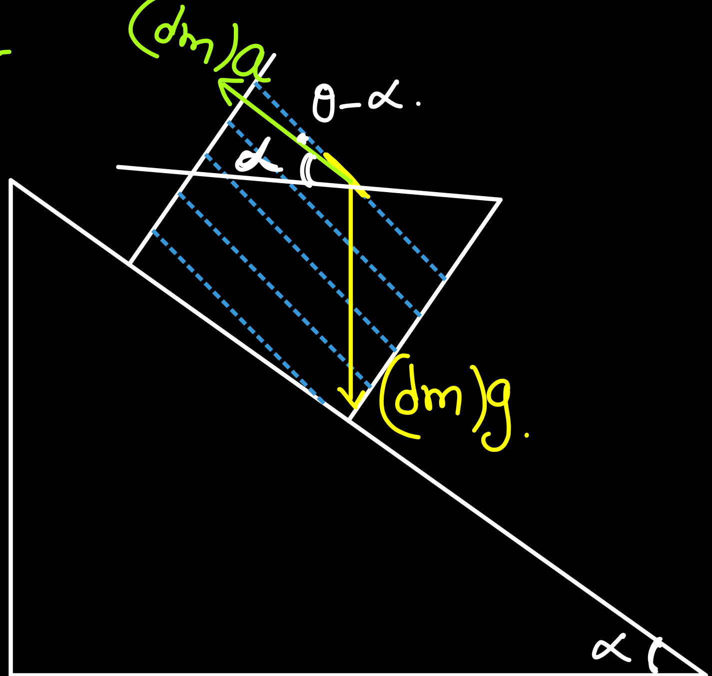
→ equipressure line $= P_0 + \rho\sqrt{g^2+a^2}\,h$.

Q)



find θ ?

Sol :-



Net tangential force = 0.

$$(\cancel{dm})\,a\cos(\theta-\alpha) = (\cancel{dm})\,g\sin\theta$$

$$a[\cos\theta\cos\alpha + \sin\theta\sin\alpha] = g\sin\theta.$$

$$\sin\theta[g - a\sin\alpha] = a\cos\theta\cos\alpha$$

$$\tan\theta = \frac{a\cos\alpha}{g - a\sin\alpha}.$$

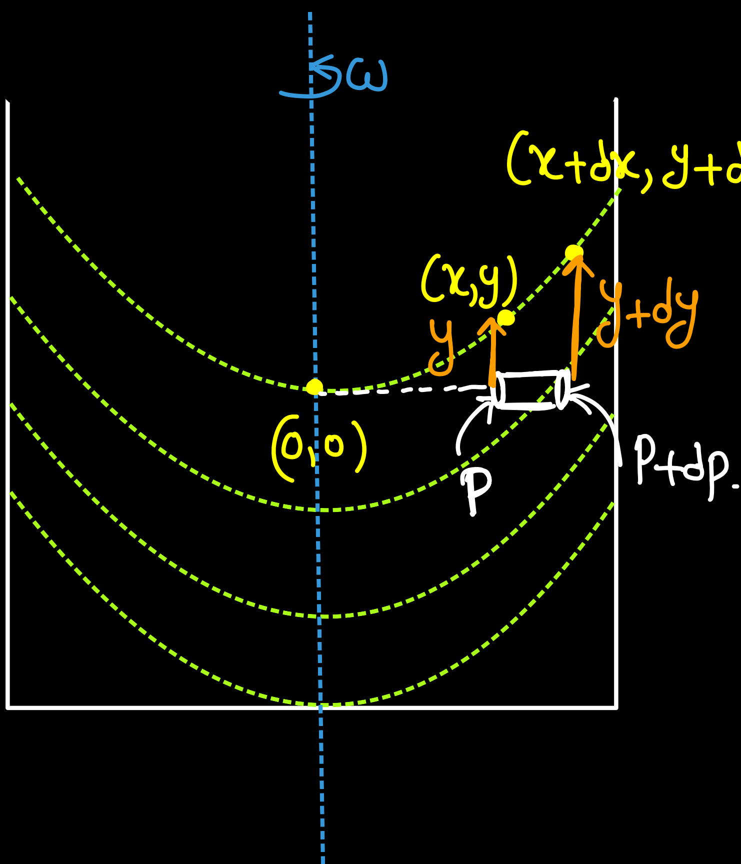
if surface is frictionless $a = g\sin\alpha$.

$$\tan\theta = \frac{(g\sin\alpha)\cos\alpha}{g - g\sin^2\alpha} = \tan\alpha$$

* $\theta = \alpha$.

$\Rightarrow$ surface of the liquid is parallel to inclined plane

Liquid rotated with angular velocity ω :-



$$(dP)dA = (dm)x\omega^2$$

$$(\cancel{\rho} g\,dy)\cancel{dA} = (\cancel{dA}\,dx)\cancel{\rho}\,x\omega^2$$

$$\int_0^y dy = \frac{\omega^2}{g}\int_0^x x\,dx$$

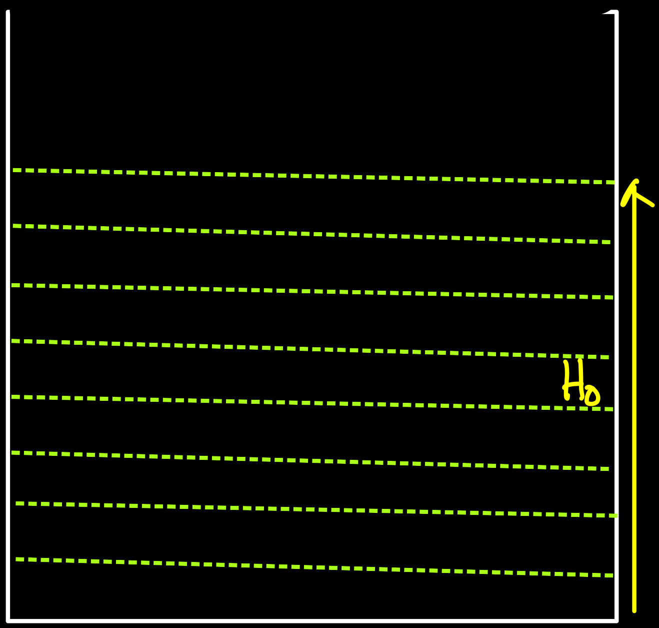
$$y = \frac{\omega^2}{2g}x^2.$$

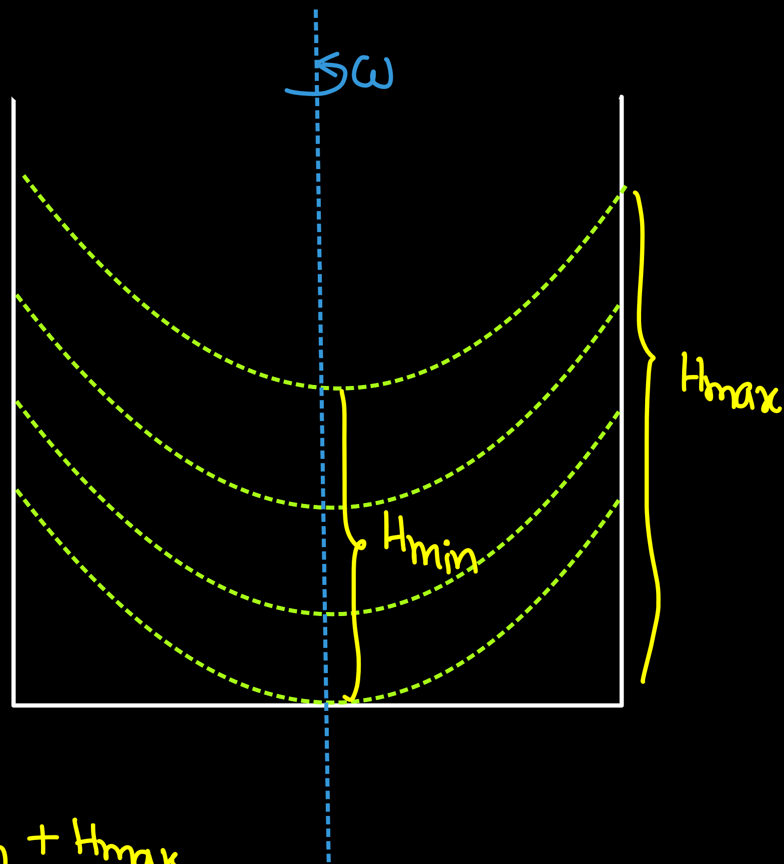
$\Rightarrow$ surface of liquid is parabola.

3D

Paraboloid.

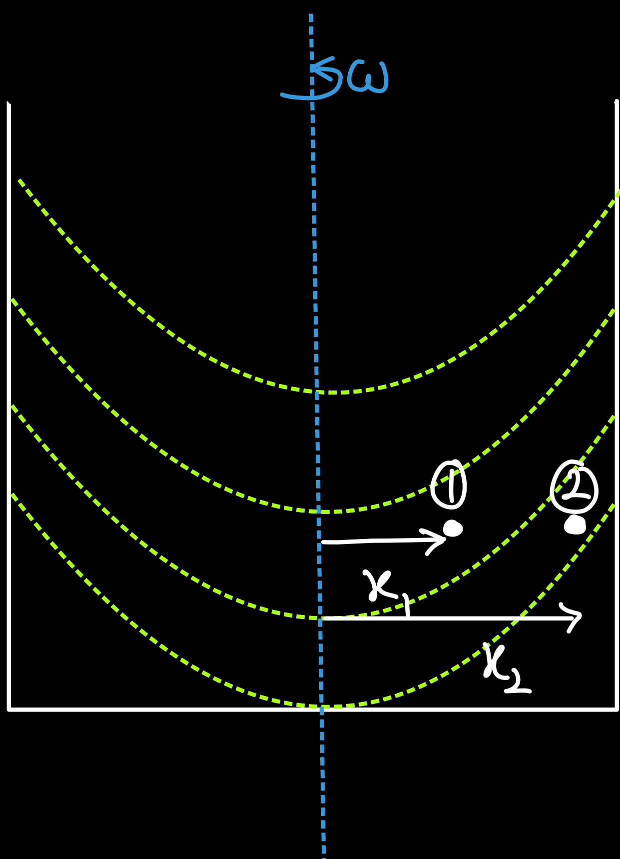
HW





$$H_0 = \frac{H_{min} + H_{max}}{2}.$$

Let's do pressure analysis:-



find $P_2 - P_1$?

Sol:-

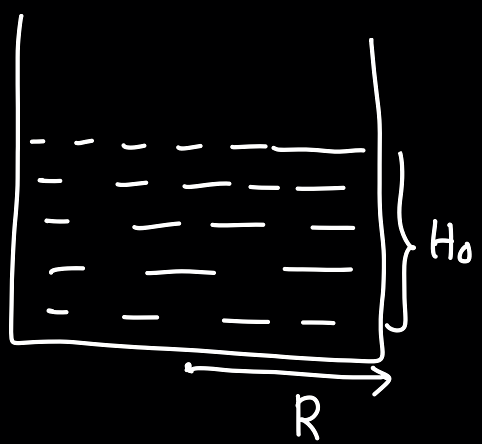
$$P_2 - P_1 = \rho g (y_2 - y_1)$$

$$P_2 - P_1 = \rho g\left[\frac{\omega^2 x_2^2}{2g} - \frac{\omega^2 x_1^2}{2g}\right]$$

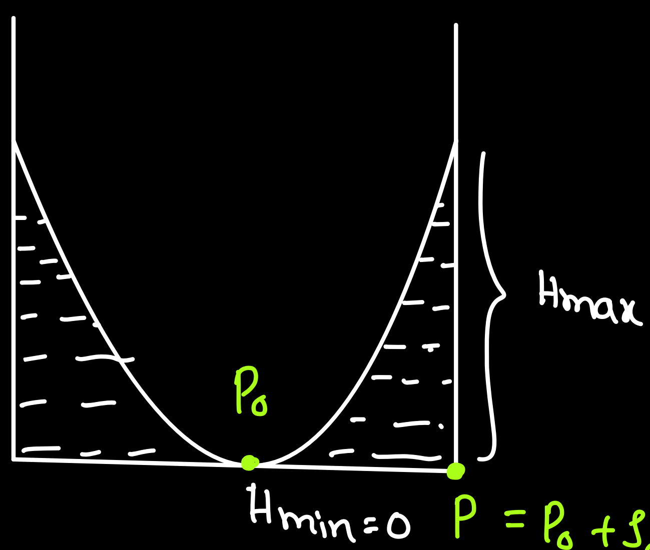
$$P_2 - P_1 = \frac{\rho\omega^2}{2}\left[x_2^2 - x_1^2\right].$$

Q) for what angular velocity, surface of liquid touches base of container?

take.



Sol :-



$$H_0 = \frac{H_{min} + H_{max}}{2}$$

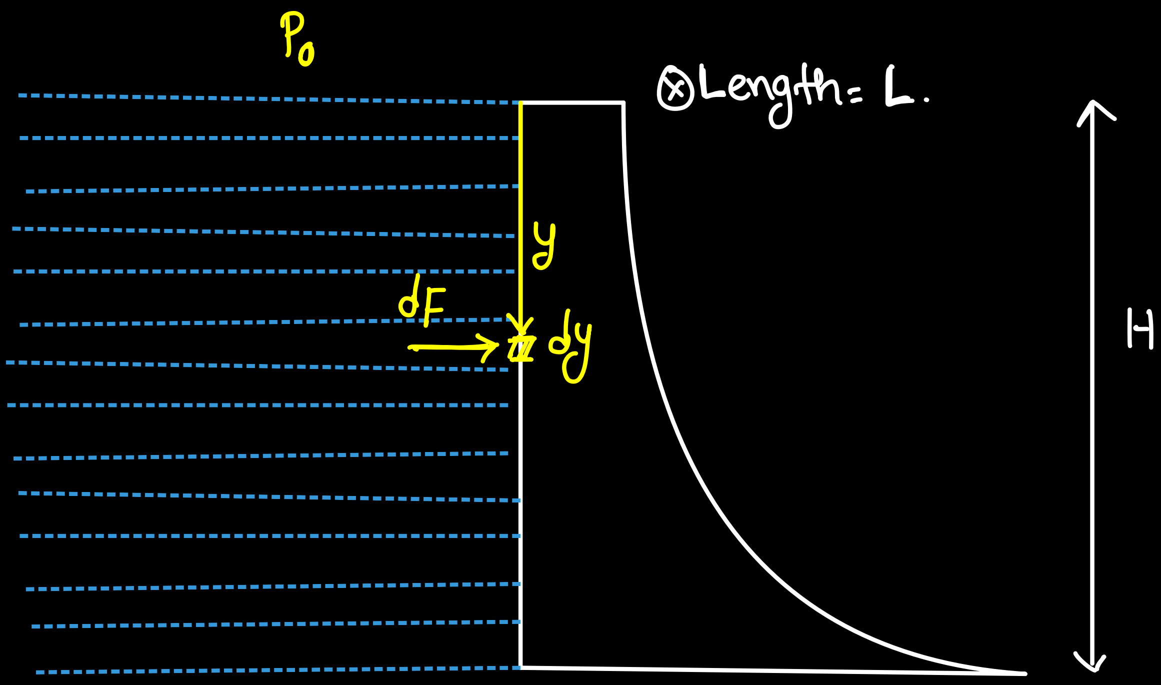
$$H_{max} = 2H_0.$$

$P = P_0 + \rho g H_{max} \Rightarrow P = P_0 + 2\rho g H_0.$

$$P - P_0 = \frac{\rho\omega^2}{2}\left[R^2 - 0^2\right].$$

$$2\cancel{\rho} g H_0 = \frac{\cancel{\rho}\omega^2}{2} R^2 \Rightarrow \omega^2 = \frac{4gH_0}{R^2} \Rightarrow \omega = \frac{2\sqrt{gH_0}}{R}.$$

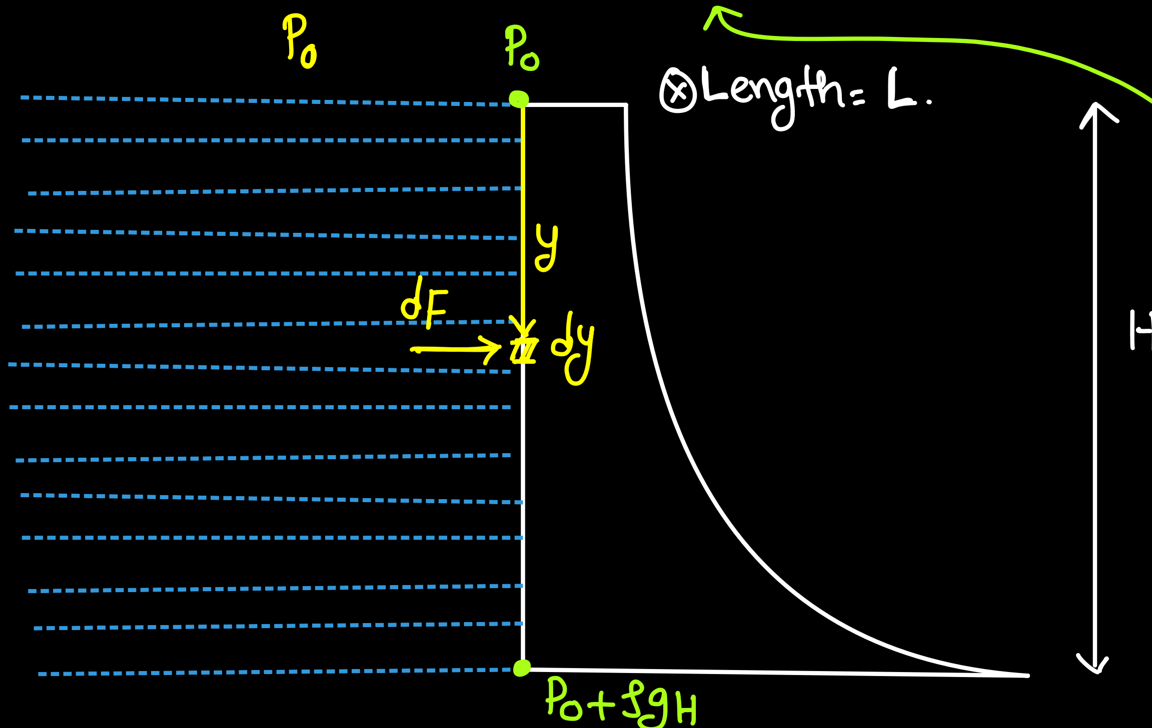
Force acting on a dam:-



$$dF = (P)(dA) = (P_0 + \rho g y)(L\,dy).$$

$$F = \int_0^H P_0 L\,dy + \int_0^H \rho g L\,y\,dy$$

$$F = \left(P_0 + \frac{\rho g H}{2}\right)(LH) \rightarrow \text{Area of dam.}$$

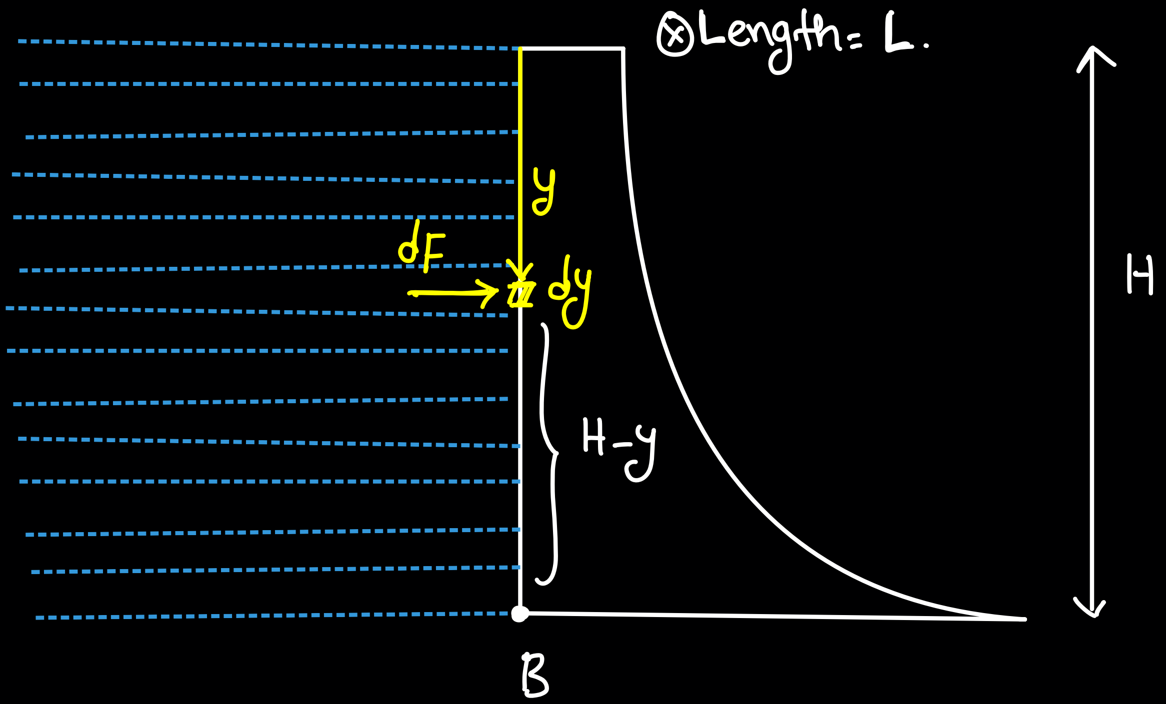


$$P_{avg} = \frac{P_0 + (P_0 + \rho g H)}{2} = P_0 + \frac{\rho g H}{2}$$

$$\boxed{F = (P_{avg})\,\text{Area}}$$

if atm is ignored then $F = \frac{\rho g L H^2}{2}$.

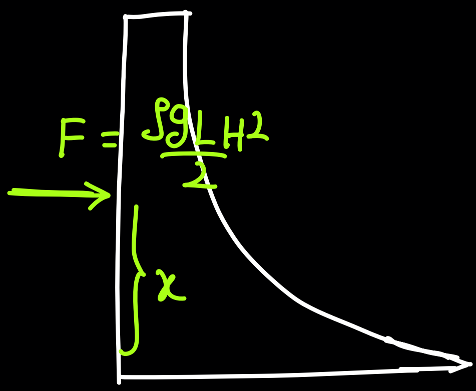
find point of application of force by ignoring atm ?



$$d\tau_B = (r_\perp)\,dF = (H-y)(\rho g y)\,L\,dy.$$

$$\tau_B = \int_0^H (H-y)(\rho g y)\,L\,dy$$

$$\tau_B = \frac{\rho g L H^3}{6}.$$



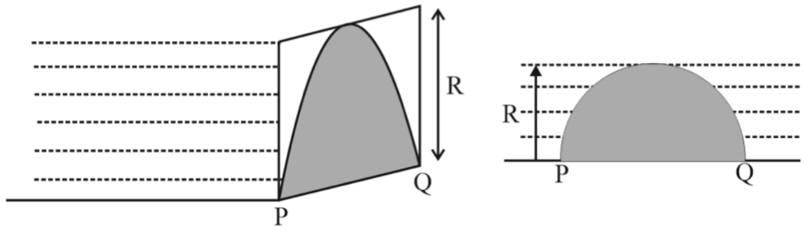
$$\tau_B = Fx = \frac{\rho g L H^3}{6}$$

$$\left(\frac{\rho g L H^2}{2}\right)x = \frac{\rho g L H^3}{6}$$

$$x = \frac{H}{3}.$$

HW :-

9.



The height of fluid of density ρ is R. There is a gate in shape of half circular plate as shown in figure having radius R. Find force exerted by the fluid on the gate and its torque about the side PQ. (Neglect atmospheric pressure)

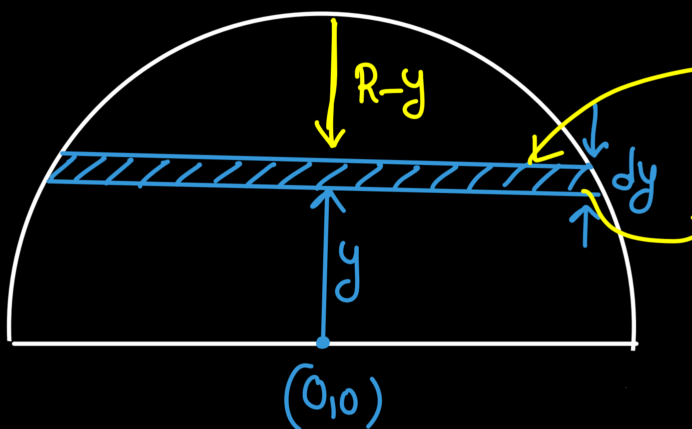
(A) $F = 2\rho g R^3\left(\frac{1}{3} - \frac{\pi}{16}\right)$

(B) $F = 2\rho g R^3\left[\frac{\pi}{4} - \frac{1}{3}\right]$

(C) $\tau = 2\rho g R^4\left[\frac{\pi}{4} - \frac{1}{3}\right]$

(D) $\tau = 2\rho g R^4\left[\frac{1}{3} - \frac{\pi}{16}\right]$

Sol :-



$P = \rho g(R-y).$

$dA = (l)dy$

$= \left(2\sqrt{R^2-y^2}\right)dy.$

$$dF = (P)(dA) = \rho g(R-y)\,2\sqrt{R^2-y^2}\,dy$$

$$F = \int_0^R \rho g R\left(2\sqrt{R^2-y^2}\,dy\right) - \int_0^R \rho g\, y\left(2\sqrt{R^2-y^2}\right)dy$$

$$= \rho g R\int_0^R 2\sqrt{R^2-y^2}\,dy + \rho g\int_0^R (-2y)\sqrt{R^2-y^2}\,dy$$

$$= \rho g R\left(\frac{\pi R^2}{2}\right) + \rho g\left[\frac{\left[R^2-y^2\right]^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^R$$

$$= \rho g R^3 \frac{\pi}{2} - \rho g R^3 \left(\frac{2}{3}\right)$$

$$F = 2\rho g R^3 \left[\frac{\pi}{4} - \frac{1}{3}\right].$$

** if we use average concept

$$F = (P_{avg})A = \left(\frac{\rho g R}{2}\right)\frac{\pi R^2}{2} = \rho g R^3 \frac{\pi}{4}$$

if it was full circle

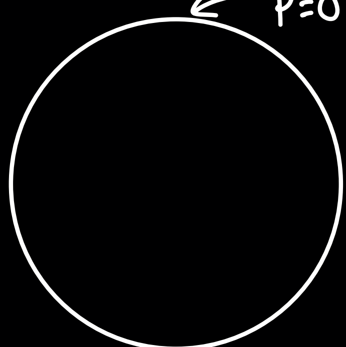
$$F = \int_{-R}^{R} \rho g R \left(2\sqrt{R^2 - y^2}\,dy\right) - \int_{-R}^{R} \rho g y \left(2\sqrt{R^2-y^2}\right)dy$$

$$= \rho g R \int_{-R}^{R} 2\sqrt{R^2-y^2}\,dy - 2\rho g \underbrace{\int_{-R}^{+R} y\sqrt{R^2-y^2}\,dy}_{\to 0}$$

$$= \rho g R \left(\pi R^2\right)$$

if we use average concept.

$P = 0$

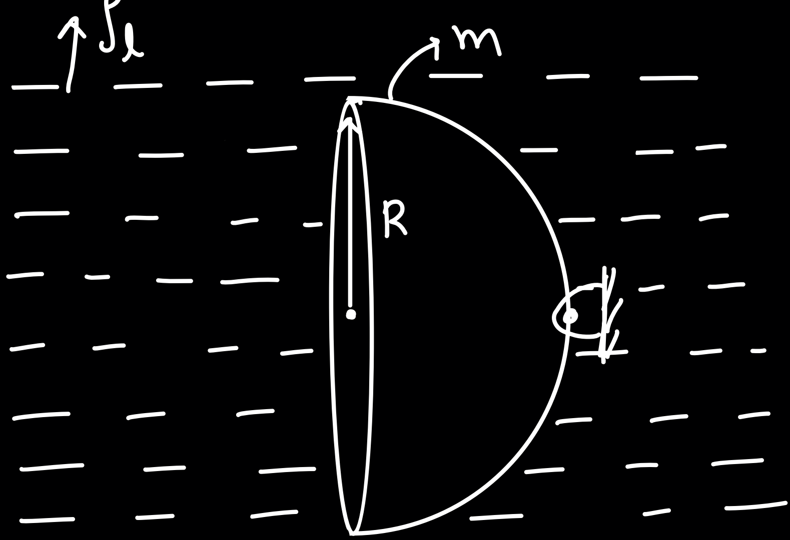


$P = \rho g (2R)$

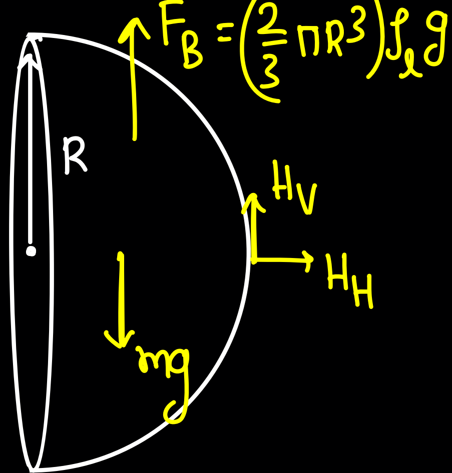
$$P_{avg} = \frac{0 + 2\rho g R}{2} = \rho g R.$$

$$F = P_{avg} A = (\rho g R)\pi R^2.$$

Q)



find hinge force ?

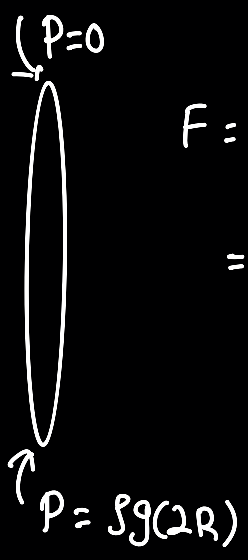


$$H_H = 0$$

$$H_V + \frac{2}{3}\pi R^3 \rho_l g = mg$$

$$H_V = mg - \frac{2}{3}\pi R^3 \rho_l g.$$

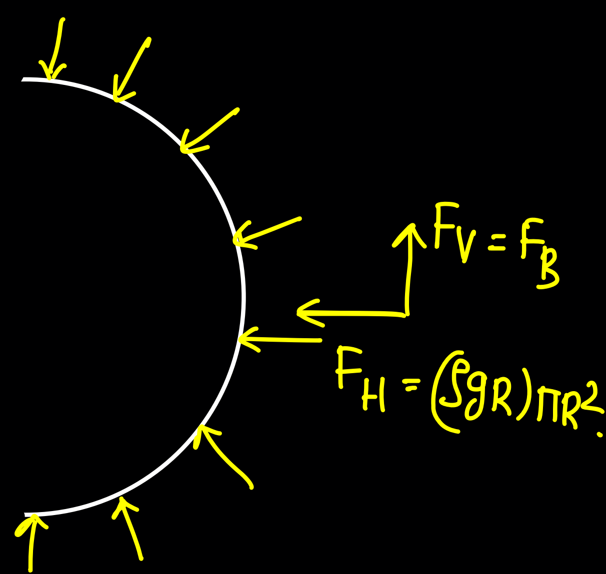
$\Rightarrow$ force on flat area.



$$F = P_{avg} A$$

$$= (\rho g R)\pi R^2$$

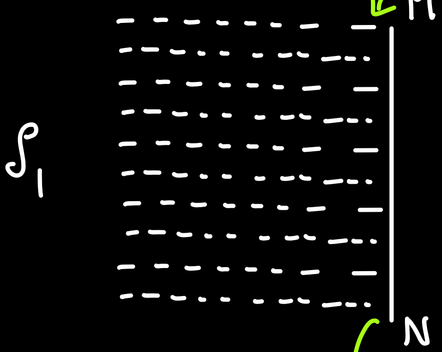
$\Rightarrow$ force on curved area.



Ex: 4A

Q11)

$P = 0$



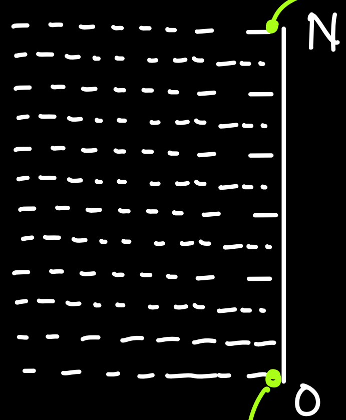
$P = \rho_1 g (5)$

$P_{avg} = \dfrac{0 + 5\rho_1 g}{2} = \dfrac{5\rho_1 g}{2}$.

$F = (P_{avg})\,\text{area}$.

$F_{MN} = \left(\dfrac{5\rho_1 g}{2}\right)(25)$.

$P = 5\rho_1 g$



$P = 5\rho_1 g + 5\rho_2 g$

$= 15\rho_1 g$

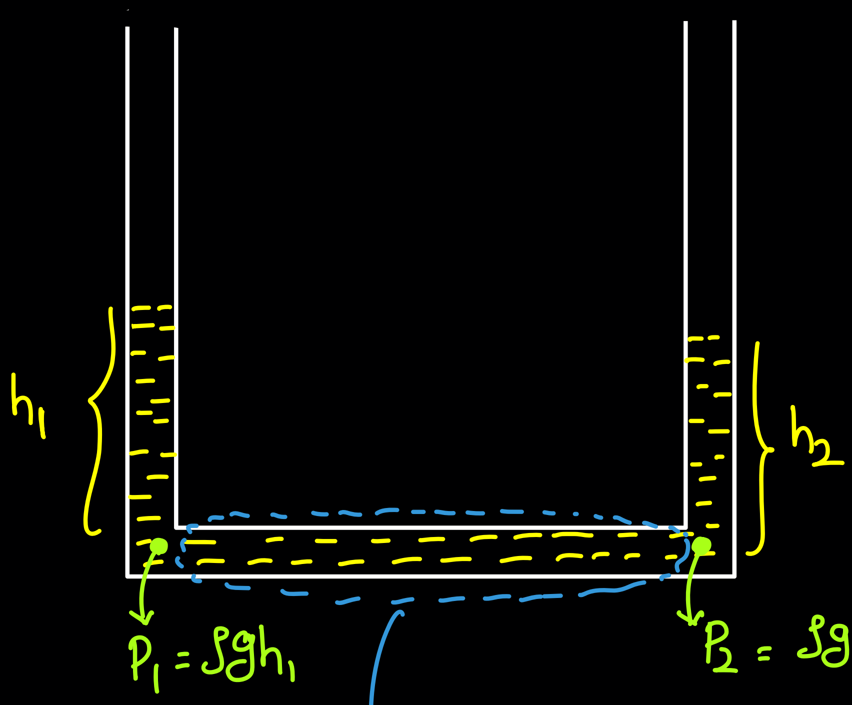
$P_{avg} = \dfrac{5\rho_1 g + 15\rho_1 g}{2} = 10\rho_1 g$.

$F_{NO} = (P_{avg})\,\text{area}$

$= (10\rho_1 g)(25)$.

$\dfrac{F_{MN}}{F_{NO}} = \dfrac{1}{4}$.

U tube :-

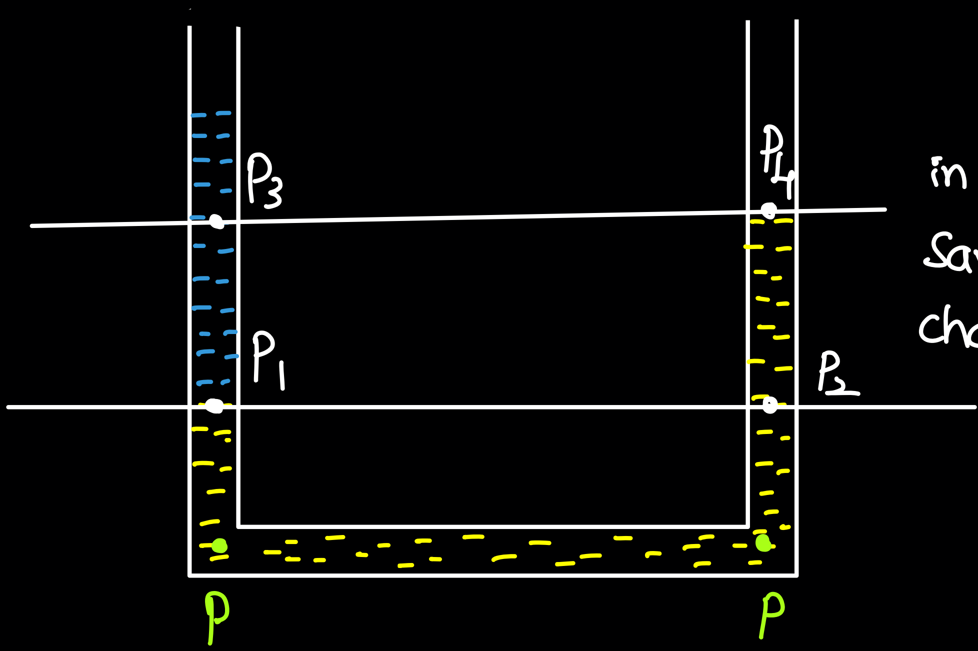


$P_1 = \rho g h_1$ $\quad P_2 = \rho g h_2$.

↓

$F_{net} = 0$

$P_1 = P_2 \Rightarrow h_1 = h_2$.



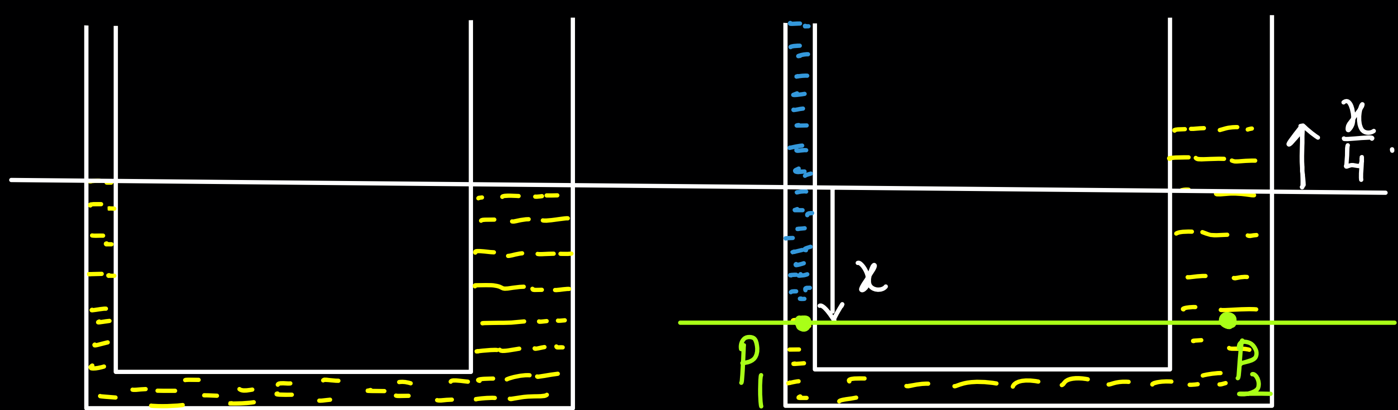
$P_1 = P_2$ as we move in same liquid by same height so pressure changes will be same.

$P_3 \neq P_4$ as we move in different liquids by same height.

** draw a horizontal line passing through the interface. and equate pressures.

BB:2

Q4)



$$P_1 = P_0 + \rho_W g(36+x) \quad \Big| \quad P_2 = \rho_M g\left(x+\frac{x}{4}\right)$$

$$P_1 = P_2$$

$$\rho_W \cancel{g}(36+x) = (13.6\,\cancel{\rho_W})\cancel{g}\left(\frac{5x}{4}\right)$$

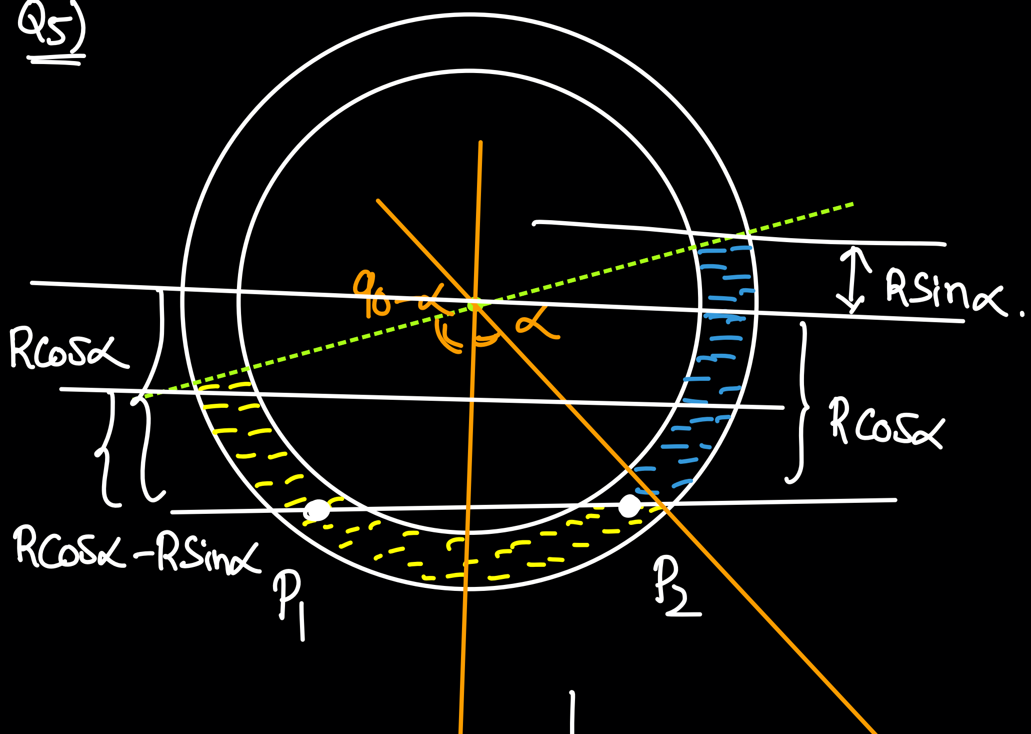
$$36+x = 3.4 \times 5x$$

$$36 = 16x \implies x = \frac{36}{16}$$

$$\text{Ans}: \quad \frac{x}{4} = \frac{36}{4\times 16} = \frac{9}{16}$$

Ex: 4A

Q5)



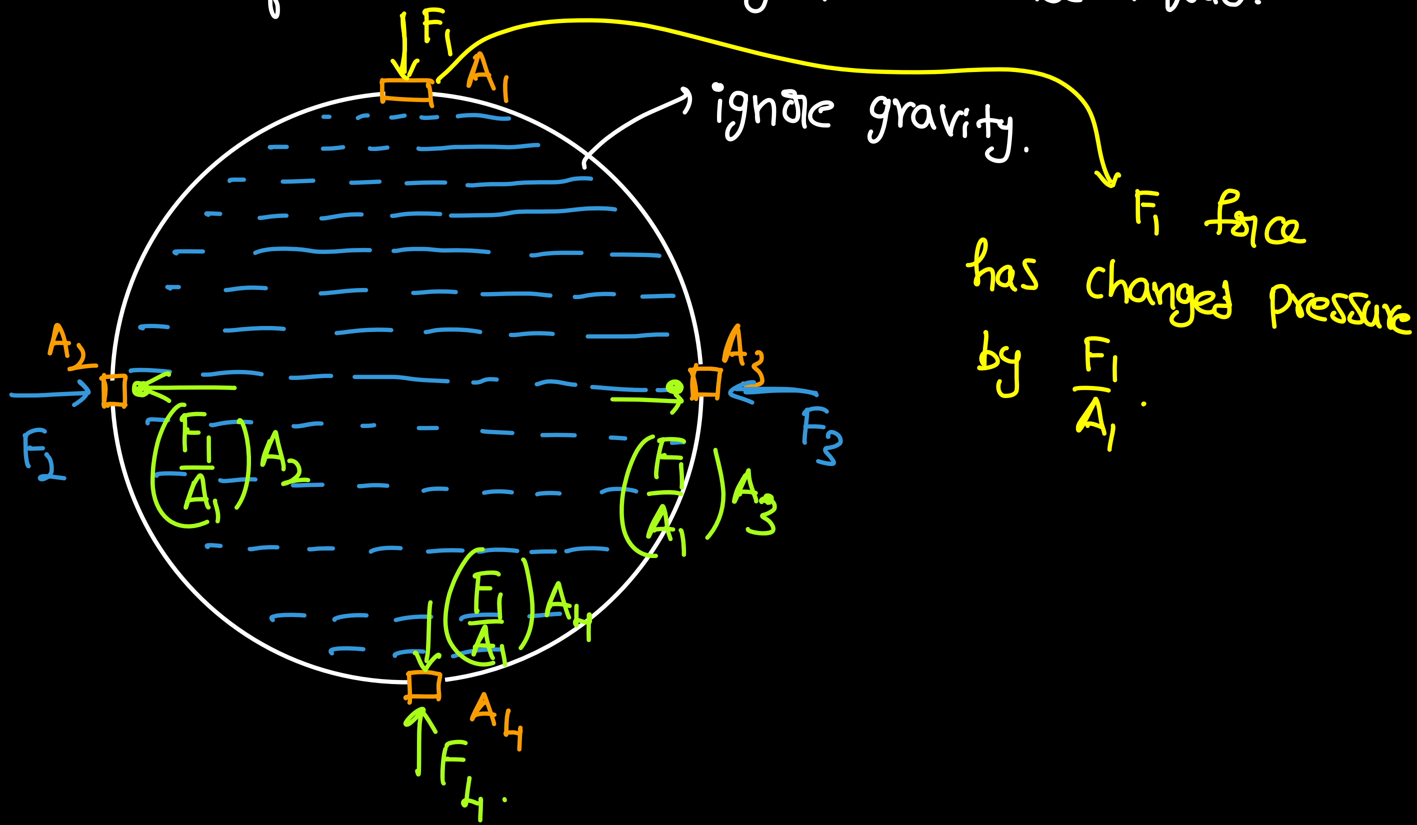
$$P_1 = \rho_1 g R(\cos\alpha - \sin\alpha) \quad \Big| \quad P_2 = \rho_2 g\, R(\sin\alpha + \cos\alpha)$$

$$P_1 = P_2$$

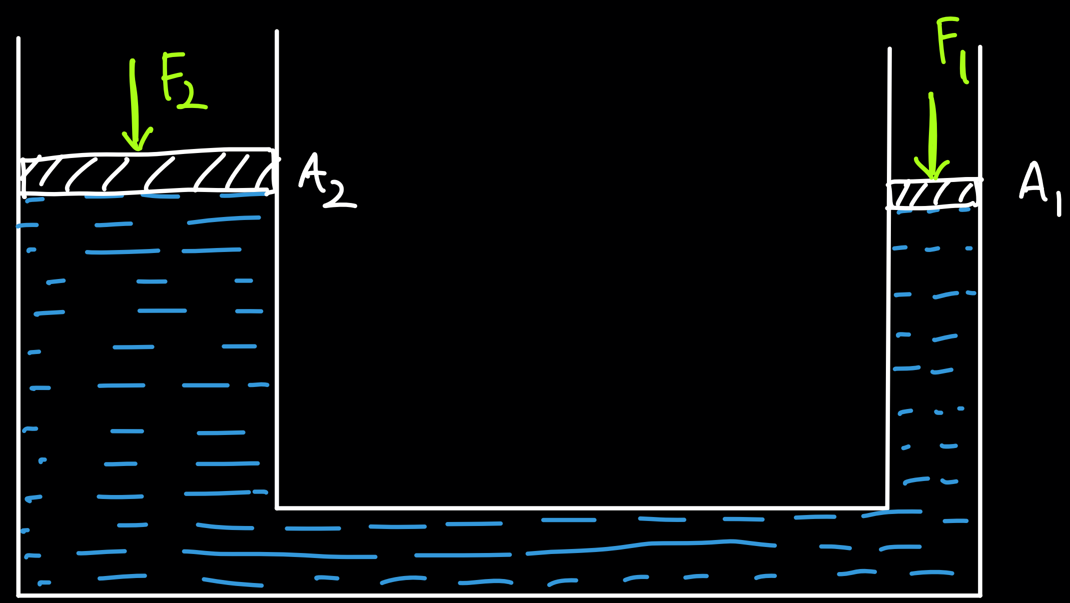
$$\rho_1 g R(\cos\alpha - \sin\alpha) = \rho_2 g R[\sin\alpha + \cos\alpha]$$

$$\frac{\rho_1}{\rho_2} = \frac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha} = \frac{1+\tan\alpha}{1-\tan\alpha}$$

Pascal's law:- increase in pressure at a point will result same amount of increase at every point inside liquid.



⇒ hydralic lift:-



$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow F_1 = \left(\frac{F_2}{A_2}\right)A_1.$$

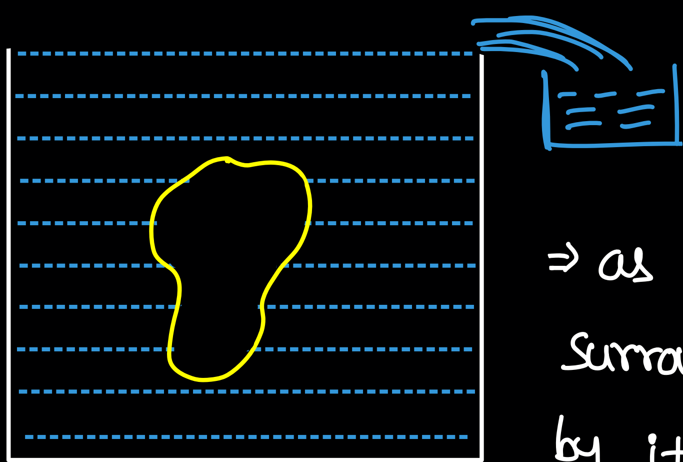
Archimede's principle :- Weight loss of the body is equal to weight of liquid displaced by body.

let's understand this :-

$$F_{net} = 0$$

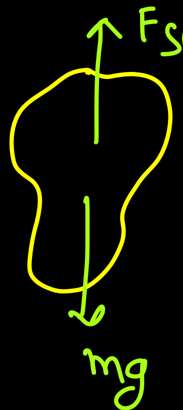
$$\{F_{\text{surrounding liquid}}\} = \text{Weight of liquid.}$$

$$F_{\text{Buoyant}} = \text{Weight of liquid.}$$



the body has displaced liquid.

⇒ as nothing has changed for surrounding liquid force applied by it will not change.



$$F_{\text{surrounding liquid}} = \text{Weight of liquid displaced}$$

$$= V_l \rho_l g.$$

$$F_B = V_l \rho_l g$$

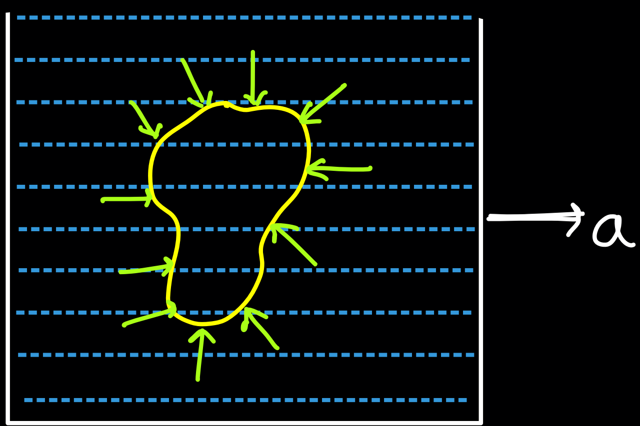
* acts up.

⇒ Buoyant force acts at C.M. of the liquid displaced.

* generally buoyant force terminology will be used for force due to surrounding liquid when liquid is at rest.

It is not mandatory that force due to surrounding liquid to act vertically up.

eg:-

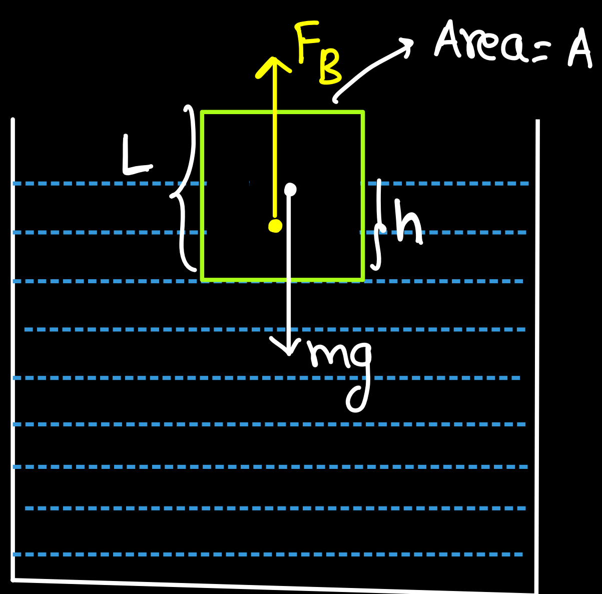


$$(F_s)_V = \underbrace{m}_{}g \quad \text{weight of liquid}$$

$$(F_s)_H = \underbrace{m}_{\text{mass of liquid}}a$$

$\Rightarrow$ F_B is due to difference in pressure

Floating:- $\rho_B < \rho_l$.



$$F_{net} = 0$$

$$F_B = mg.$$

$$V_l \rho_l g = V_B \rho_B g$$

$$V_l \rho_l = V_B \rho_B.$$

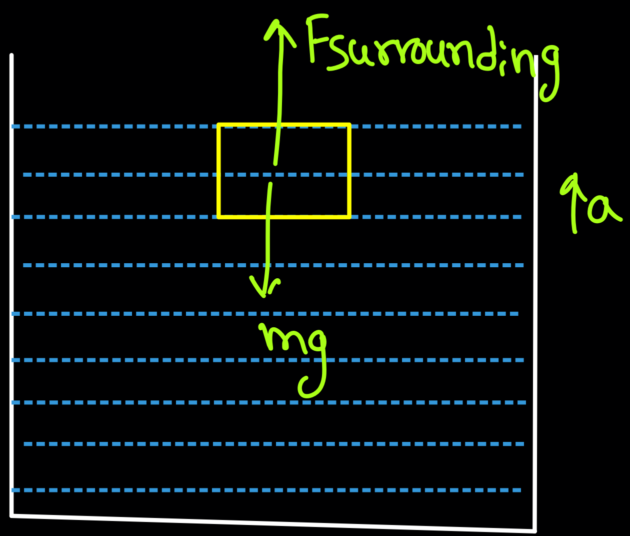
$$(Ah)\rho_l = (AL)\rho_B.$$

$$h = \left(\frac{\rho_B}{\rho_l}\right)L.$$

what happens to "h" when container is accelerated up with "a" ?

Sol:- Recap

$$F_{\text{surrounding liquid}} = F_{\text{Buoyancy}}.$$

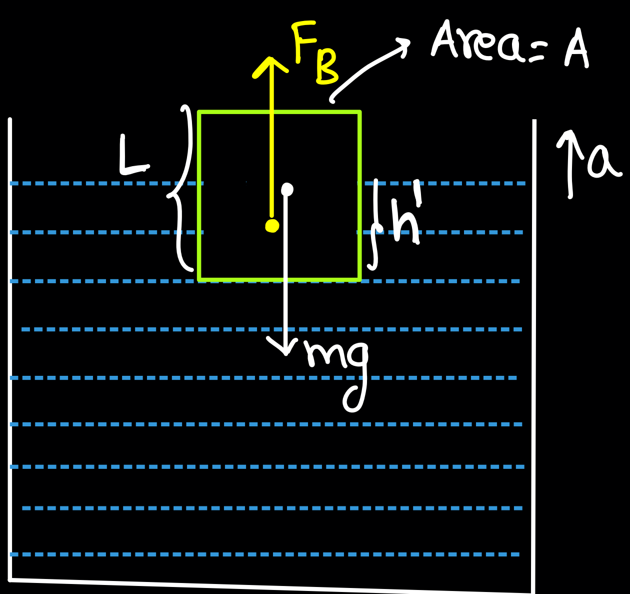


$$F_S - mg = ma$$

$$F_S = m(g+a)$$

$$F_B = V_l \rho_l (g+a)$$

$$* \quad F_B = V_l \rho_l g_{eff}.$$



$$F_B - mg = ma.$$

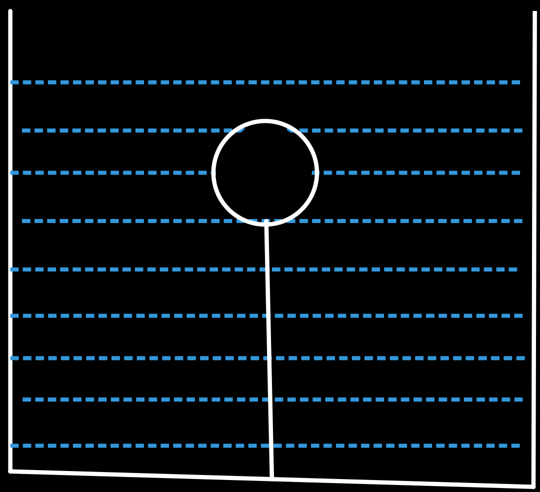
$$V_l \rho_l \cancel{(g+a)} = m\cancel{(g+a)}$$

$$V_l \rho_l = V_B \rho_B$$

$$(Ah')\rho_l = (AL)\rho_B$$

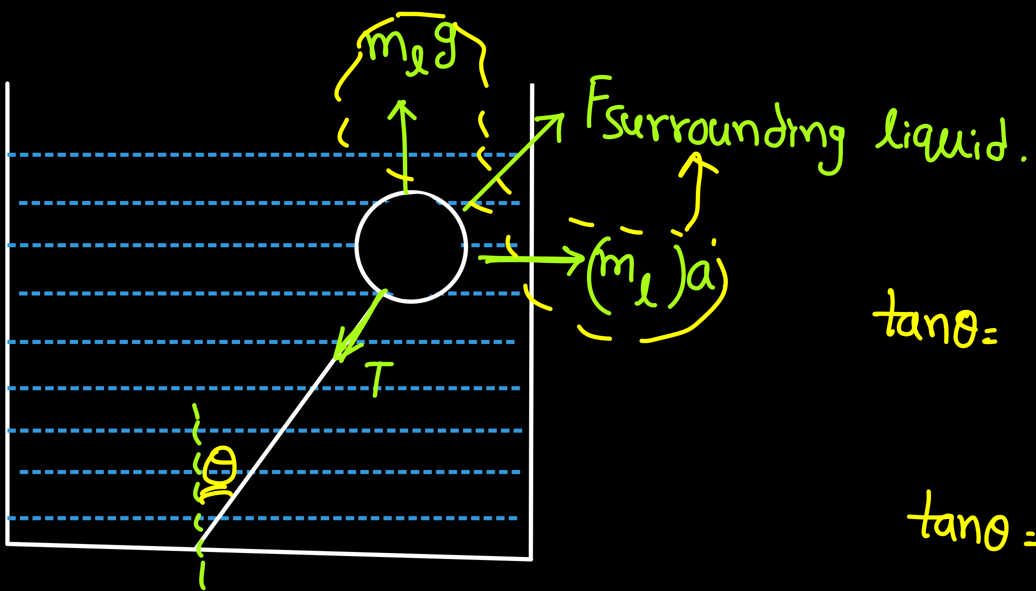
$$h' = \left(\frac{\rho_B}{\rho_l}\right)L = h.$$

Q)



if the container is accelerated to right with "a" find the new position of balloon? (ignore weight of balloon)

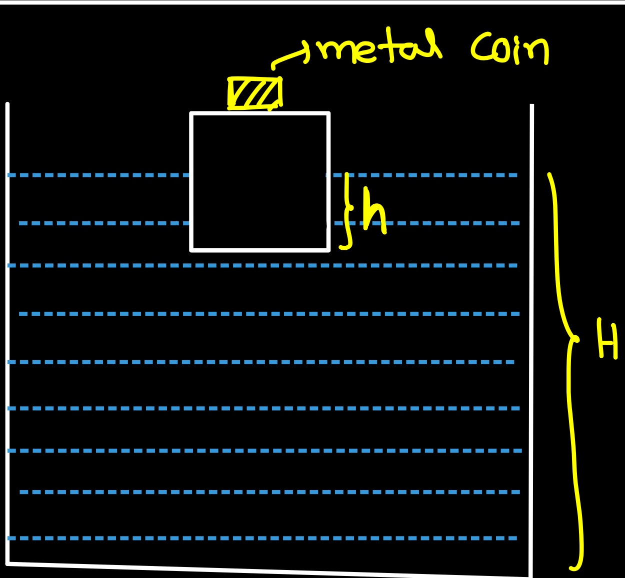
sol :-



$$\tan\theta = \frac{m_l a}{m_l g}$$

$$\tan\theta = \frac{a}{g}.$$

Q)



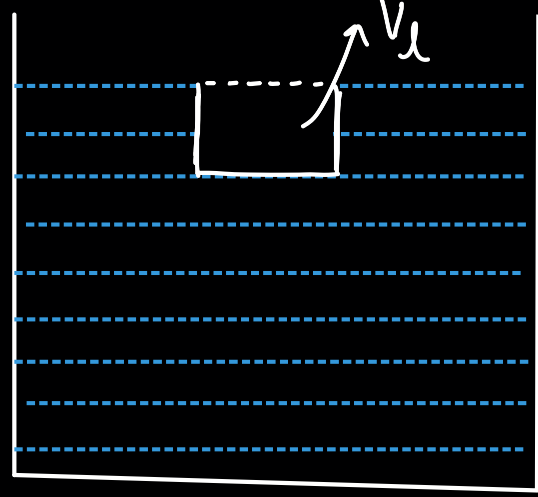
if coin is made to fall into the liquid what happens to h and H?

Sol :-

$$F_B = W_{block} + W_{coin}$$

$$V_l \rho_l g = V_B \rho_B g + V_C \rho_C g$$

$$V_l = \frac{V_B \rho_B}{\rho_l} + V_C\left(\frac{\rho_C}{\rho_l}\right).$$

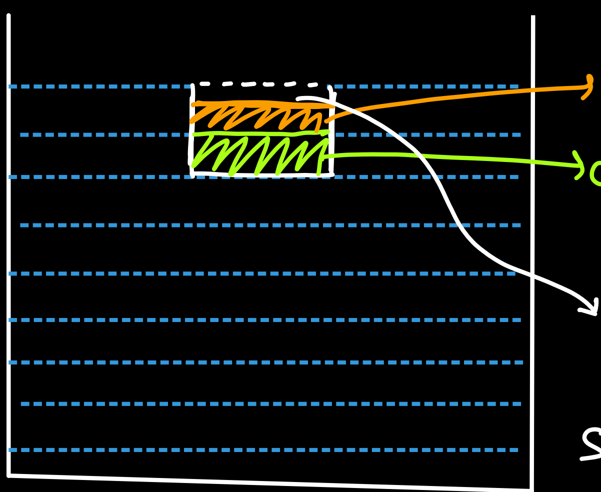


when coin falls into liquid

$$V_{\text{displaced by coin}} = V_C.$$

$$V_{\text{displaced by block}} = \frac{V_B \rho_B g}{\rho_l g} = \frac{V_B \rho_B}{\rho_l}.$$

overall height of liquid in container

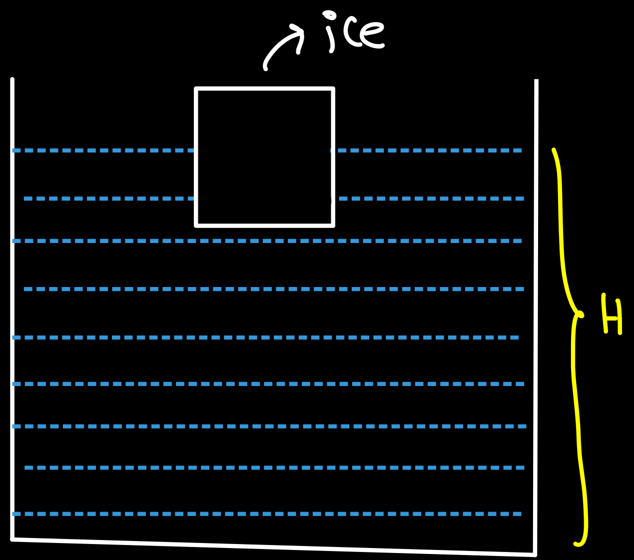


$V_C < V_C\left(\frac{\rho_C}{\rho_l}\right)$

due to block $= \frac{V_B \rho_B}{\rho_l}$

this place will be filled by surrounding liquid as a result overall height falls.

Q)



What happens to H when ice melts to water?

Case (i) ice floating in water

Case (ii) ice floating in liquid $\rho_l < \rho_W$

Case (iii) : $\rho_l > \rho_W$.

Sol:-

$$F_B = mg.$$

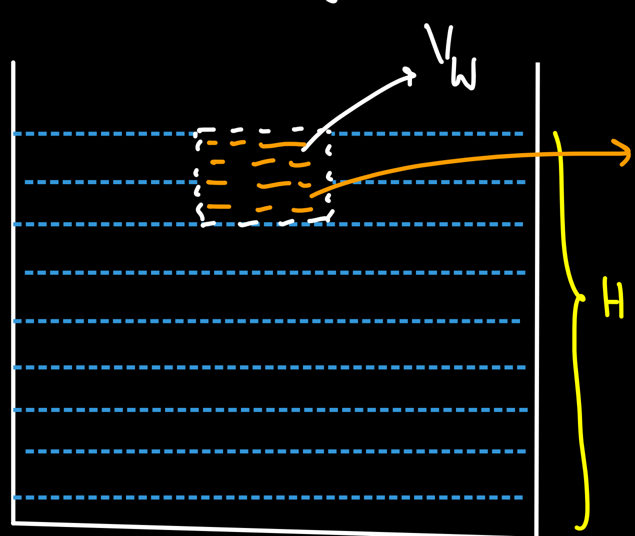
$$V_l \rho_l g = V_W \rho_W g.$$

$$\boxed{V_l = V_W \frac{\rho_W}{\rho_l}}$$

* we got V_W Volume got created

if $\rho_l = \rho_W$

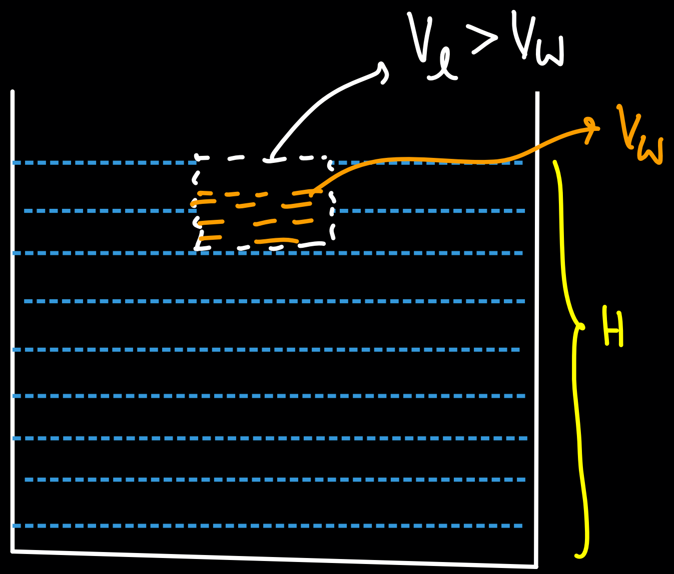
$$V_l = V_W$$



V_W got created which exactly fills so H won't change.

if $\rho_\ell < \rho_W$

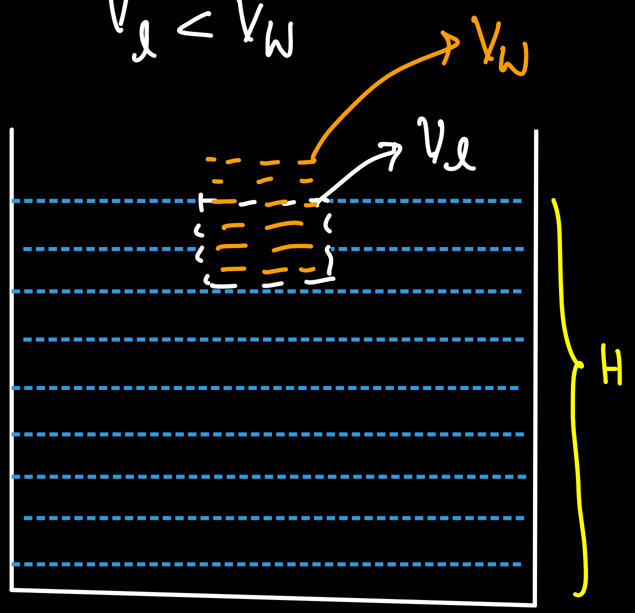
$V_\ell > V_W$



H $\downarrow$.

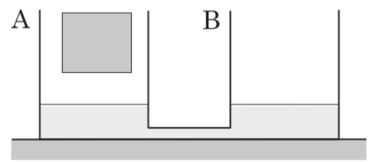
if $\rho_\ell > \rho_W$

$V_\ell < V_W$



H $\uparrow$.

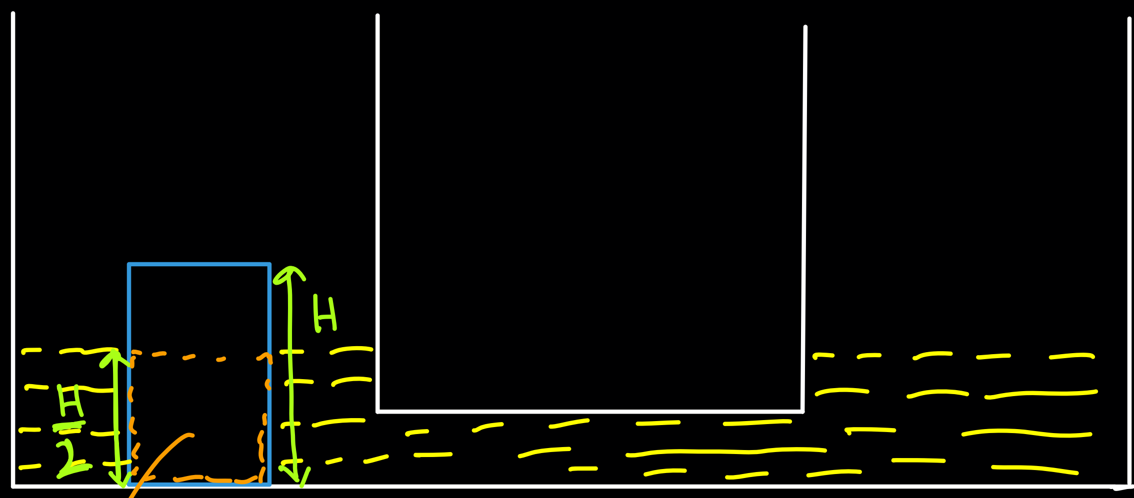
1. On a horizontal floor, two identical cylindrical vessels A and B connected by a thin tube near their bottoms contain some water. When an ice block of volume 100 cm^3 is gently put inside the vessel A, it gets half - submerged in the water. How much mass of water will flow through the connecting tube during the process of melting of the ice cube?



Density of water is 1000 kg/m^3 and density of ice is 900 kg/m^3.

(*a*) 20 g (*b*) 25 g
(*c*) 45 g (*d*) 50 g

Sol :-



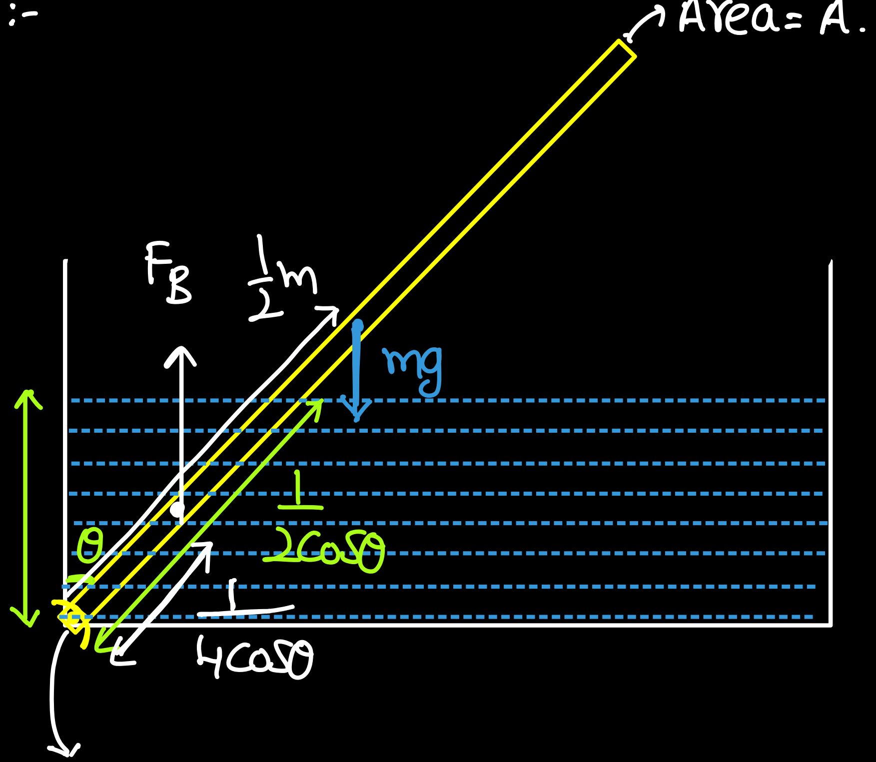
$$V_W = \left(\frac{900}{1000}\right) V_{ice} = 90 cm^3.$$

$$V_l = 50 cm^3.$$

We got extra 40 cm^3 of water out of which 20 cm^3 goes to other side.

$$\text{mass} = (20 cm^3)(1000) = 20 g.$$

ill:14 :-



$$\tau_{Hinge} = 0.$$

$$S.G = \frac{\rho_{Body}}{\rho_{Water}}.$$

$$\tau_{F_B} = \tau_{mg}.$$

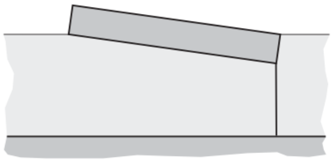
$$\left(\frac{1}{4\cos\theta}\right)\cancel{\sin\theta}\, F_B = \left(\frac{1}{2}\cancel{\sin\theta}\right)mg$$

$$2\left(\frac{1}{\cancel{4}\cos\theta}\right)\left(\cancel{A}\frac{1}{\cancel{2}\cos\theta}\right)\rho_{\cancel{W}}\cancel{g} = \frac{1}{\cancel{2}}\left[\cancel{A}(1)\right]\frac{1}{\cancel{2}}\cancel{\rho_W}\cancel{g}.$$

$$\cos^2\theta = \frac{1}{2}$$

$$\cos\theta = \frac{1}{\sqrt{2}}$$

$$\underline{\underline{\theta = 45^\circ.}}$$

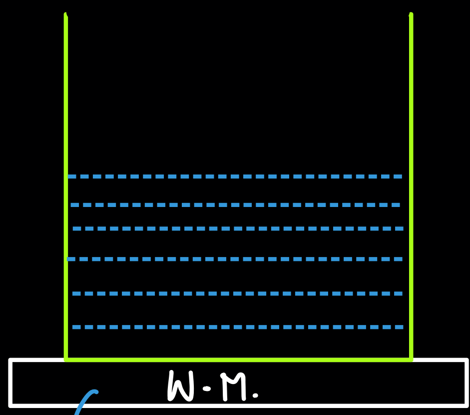


9. A wooden plank is floating on water in a pool. It is tethered to the bottom of the pool by a string attached at mid-point of an edge of its bottom face, which causes it to float with a diagonal of one of its vertical cross-section coinciding with the level surface of the water, as shown in the figure. What is the specific gravity of the wood?

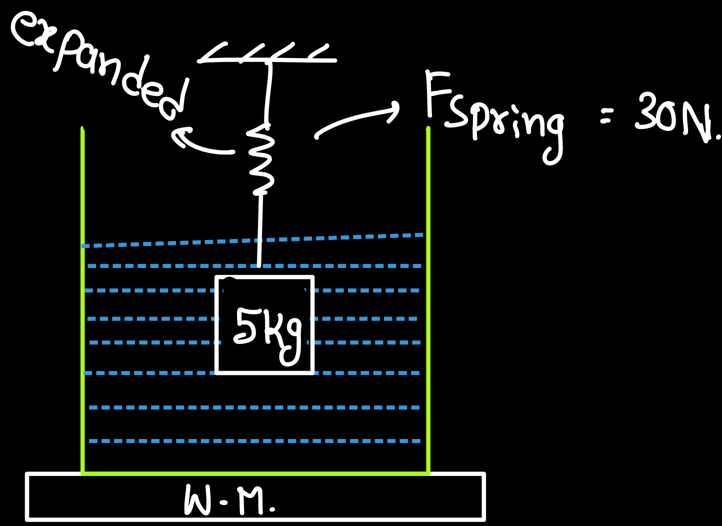
(a) 1/5 *(b)* 1/4

(c) 1/3 *(d)* 1/2

Q)

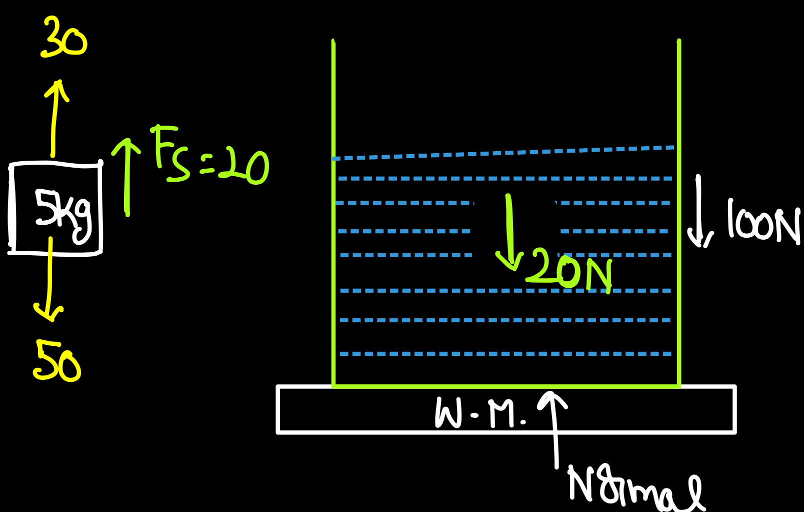


Reading = 10kg.



find reading shown by W.M. ?

Sol :-

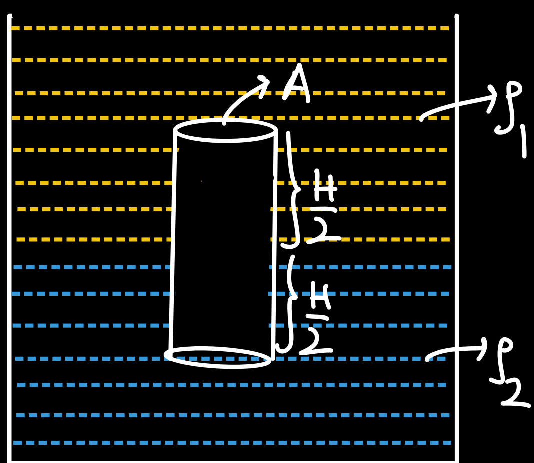


Normal = 120

$$\text{Reading} = \frac{N}{g}$$

$$= 12\text{kg.}$$

Buoyancy when multiple liquids are there :-



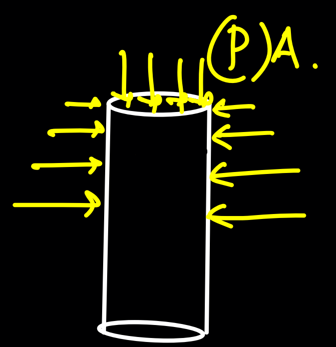
$$F_{\text{surrounding}} = \text{Weight of liquid}$$

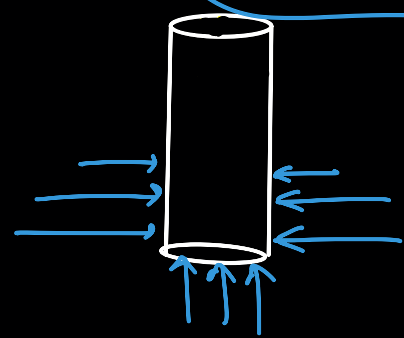
$$= \left(\frac{AH}{2}\right)\rho_1 g + \frac{AH}{2}\rho_2 g.$$

$$F_B = \frac{AH}{2}(\rho_1 + \rho_2)g.$$

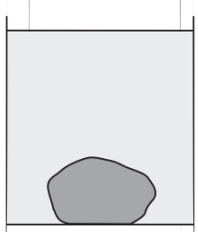
⇒ Common misconception is that yellow liquid will apply $\frac{AH}{2}\rho_1 g$ force up on cylinder which is non-sense.

Actually yellow liquid will apply a force downwards.





$$F = \left[\frac{AH}{2}(\rho_1+\rho_2)g\right] + PA.$$

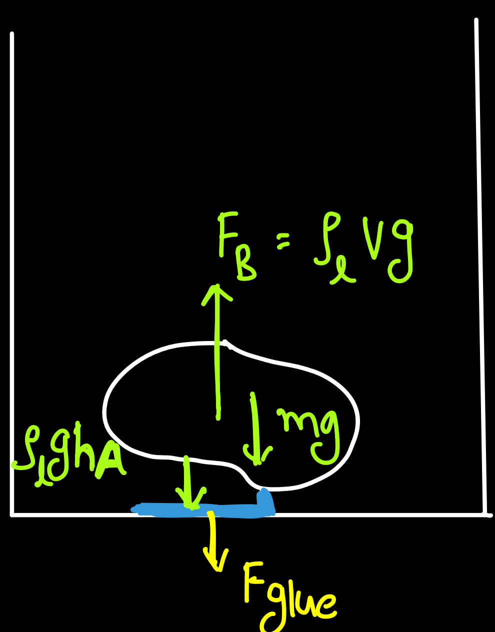


6. An object of volume 0.5 m³ and density 500 kg/m³ is glued to the flat bottom of a vessel containing water. The flat base of the object glued to the bottom has an area 100 cm² and density of water is 1.0 g/cm³. If the maximum force that the glue can withstand is 2000 N, to what minimum ~~maximum~~ height above the base can water be filled in the vessel so that the object is not torn apart from the base?

(a) 5 m *(b)* 10 m

(c) Up to any height *(d)* Insufficient information

Sol:-



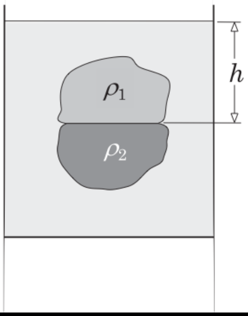
$$F_{glue} = F_B - \rho_l g h A - mg$$

$$2000 = \rho_l g V - \rho_l g h A - mg$$

$$h = 5m.$$

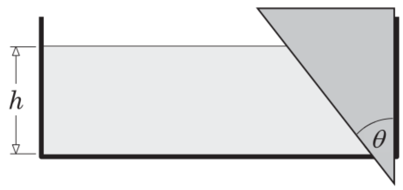
Ans : a

HW :-



19. Two objects of equal volume $V = 1.0\ \mathrm{m^3}$ and densities $\rho_1 = 400\ \mathrm{kg/m^3}$ and $\rho_2 = 600\ \mathrm{kg/m^3}$ have identical flat portions of area $S = 100\ \mathrm{cm^2}$ on their surfaces. These flat portions are glued to each other. The composite body thus formed, floats fully submerged in a liquid with the common flat portion horizontal as shown in the figure. If the glue can withstand a maximum force $F = 500$ N, at what minimum depth h in the liquid can the common flat portion be in equilibrium keeping the objects intact? Acceleration of free fall is g

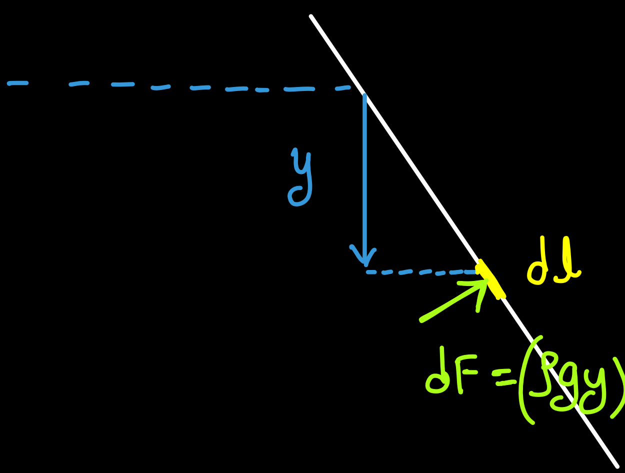
3. A slit is cut along the right bottom edge of a rectangular tank. The slit is closed by a wooden wedge of mass m and apex angle θ as shown in the figure. The vertical plane surface of the wedge is in contact with the right vertical wall of the tank. Coefficient of static friction between these surfaces in contact is μ. To what maximum height, can water be filled in the tank without any leakage through the slit? The width of tank is b and density of water is ρ.



(a) $\sqrt{\dfrac{2m}{\rho b(\tan\theta-\mu)}}$ (b) $\sqrt{\dfrac{4m}{\rho b(\tan\theta-\mu)}}$

(c) $\sqrt{\dfrac{2m}{\rho b(\sin\theta-\mu\cos\theta)}}$ (d) $\sqrt{\dfrac{2m\cos\theta}{\rho b(\tan\theta-\mu\cos\theta)}}$

Sol :-

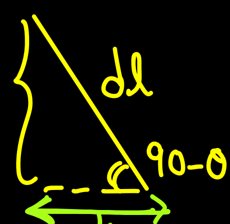


$dF = (\rho g y)\, b\, (dl)$

$dF_H = dF\cos\theta$

$= \rho g y\, b (dl\cos\theta).$

$dy = dl\cos\theta$



$dy\tan\theta = dl\sin\theta$

$dF_V = dF\sin\theta$

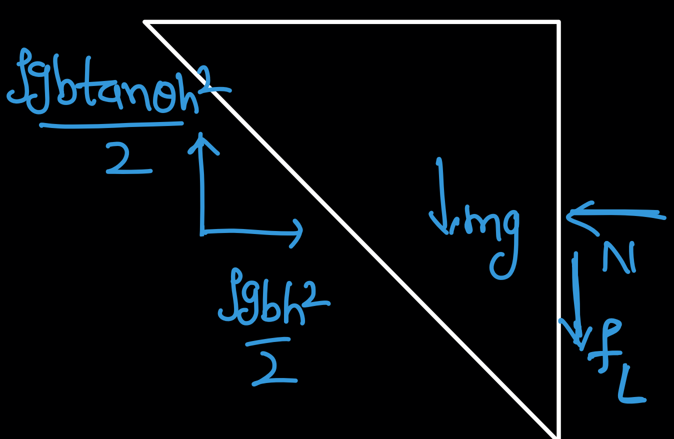
$= \rho g y\, b (dl\sin\theta)$

$= \rho g b\tan\theta\,\dfrac{h^2}{2}$

$dF_H = \rho g y\, b\, dy$

$F_H = \rho g b\displaystyle\int_0^h y\,dy$

$F_H = \dfrac{\rho g b h^2}{2}$



$N = \dfrac{\rho g b h^2}{2}.$

$f_L + mg = \dfrac{\rho g b\tan\theta\, h^2}{2}.$

$$\mu \frac{\rho g b h^2}{2} + mg = \frac{\rho g b \tan\theta \, h^2}{2}$$

$$\frac{\rho \cancel{g} b}{2}(\tan\theta - \mu)\, h^2 = m\cancel{g}$$

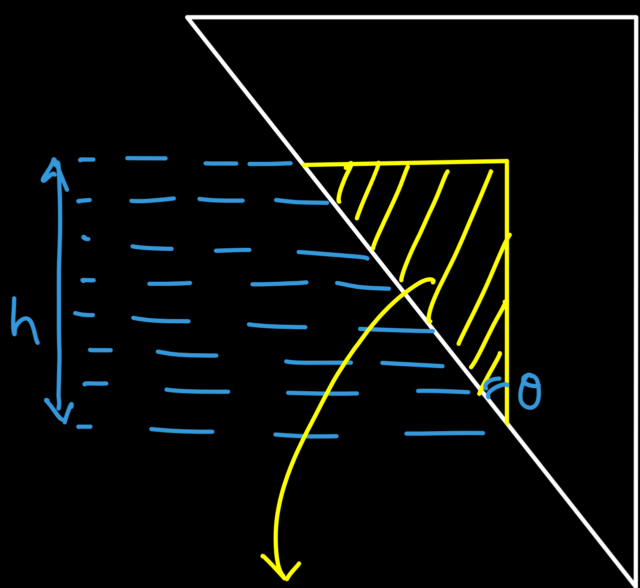
$$h = \sqrt{\frac{2m}{\rho b(\tan\theta - \mu)}}$$

observation:-

$$F_B = V_l \rho_l g$$

$$= \frac{b \tan\theta h^2 \rho g}{2}$$

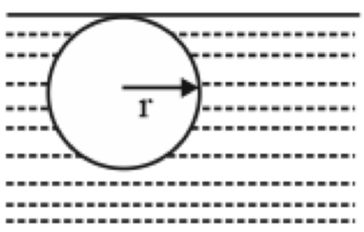
$$F_B = \frac{\rho g b h^2 \tan\theta}{2}.$$



$$V_l = \frac{1}{2}(h)(h\tan\theta)b.$$

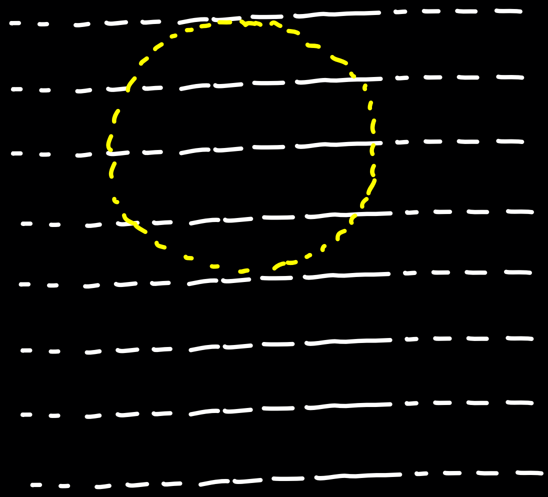
$$= \frac{b\tan\theta h^2}{2}.$$

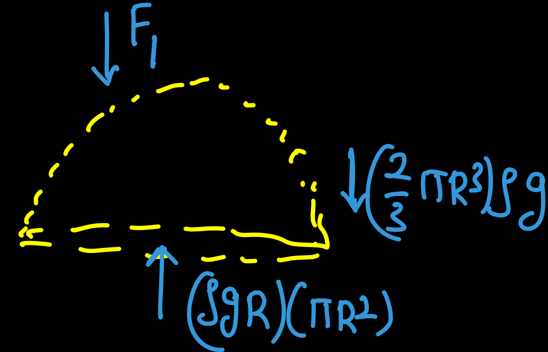
6. Figure shows a solid sphere positioned just below the surface of a fluid and held there. Let the density of the fluid be σ, force applied by fluid on the upper half of the sphere be F_1 and the force on lower half be F_2 (excluding the contribution due to atmospheric pressure). Pick correct options(s) :-



(A) $F_1 = \frac{1}{3}\pi r^3 \sigma g$ (B) $F_2 = \frac{5}{3}\pi r^3 \sigma g$ (C) $F_1 = \frac{2}{3}\pi r^3 \sigma g$ (D) $F_2 = \frac{7}{3}\pi r^3 \sigma g$

Sol :-

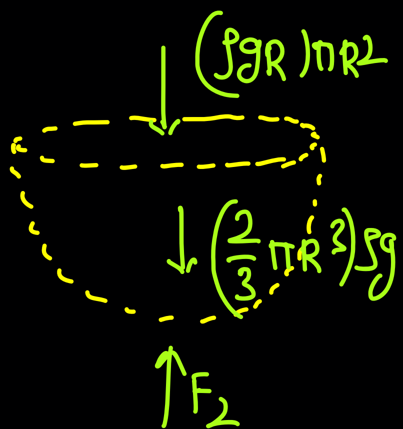




$$F_1 + \frac{2}{3}\pi R^3 \rho g = \pi R^3 \rho g$$

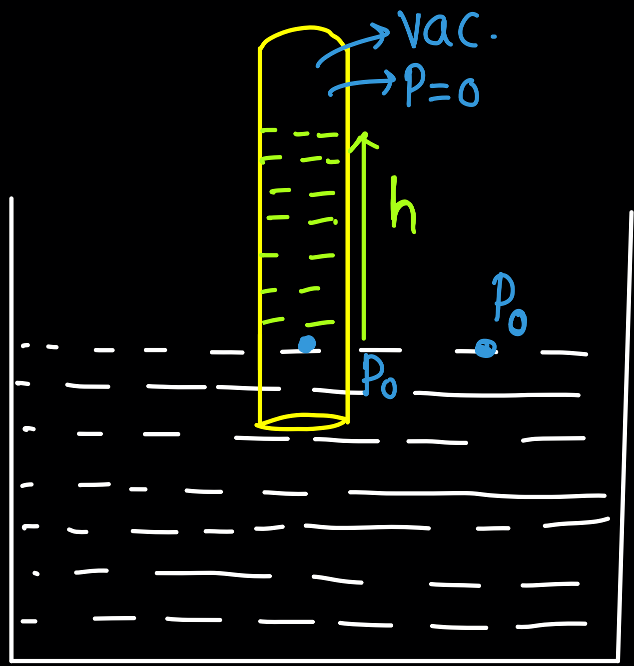
$$F_1 = \frac{1}{3}\pi R^3 \rho g.$$

HW get same result with integration.



$$F_2 = \frac{5}{3}\pi R^3 \rho g.$$

Barometer :- It's an instrument which measures atmospheric pressure.



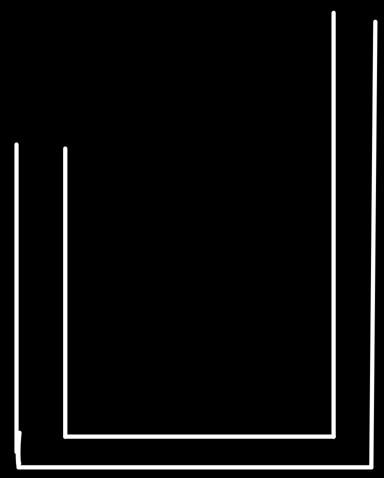
$$P_0 - 0 = \rho_M g h.$$

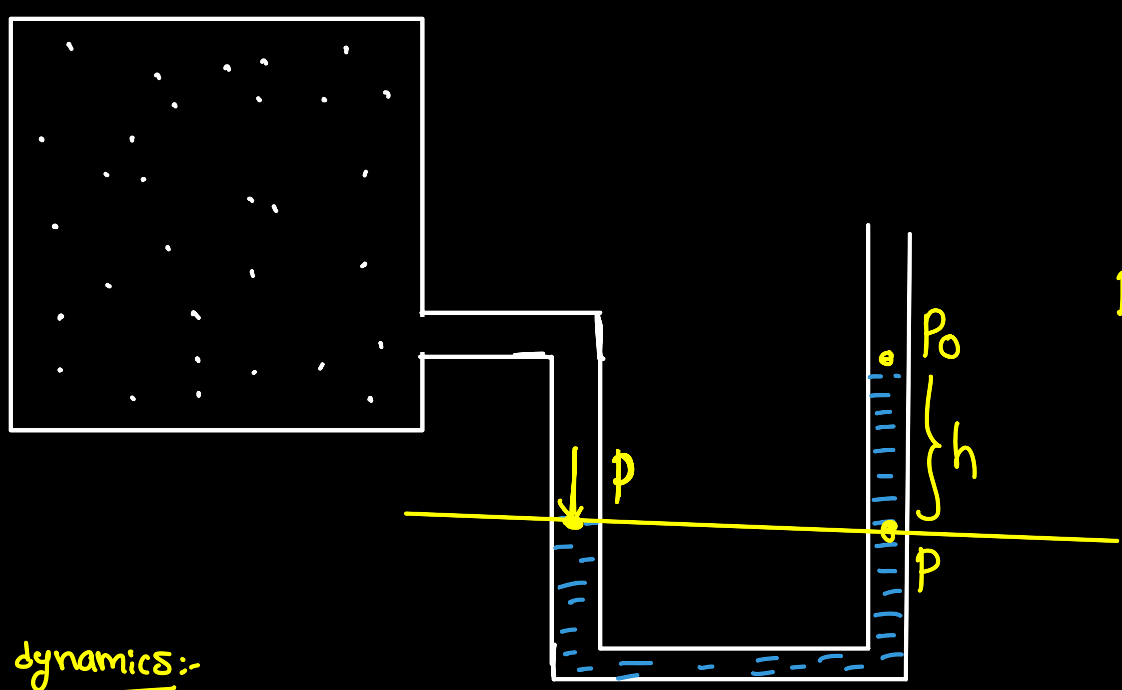
$$P_0 = \rho_M g h$$

at sea-level that

$h = 76$ cm.

Manometer:- used to find pressure inside Container.



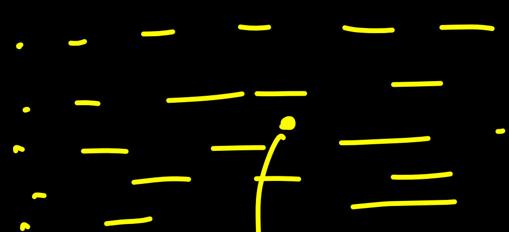


$$P - P_0 = \rho_l g h.$$

Fluid dynamics:-

Types of flow:-

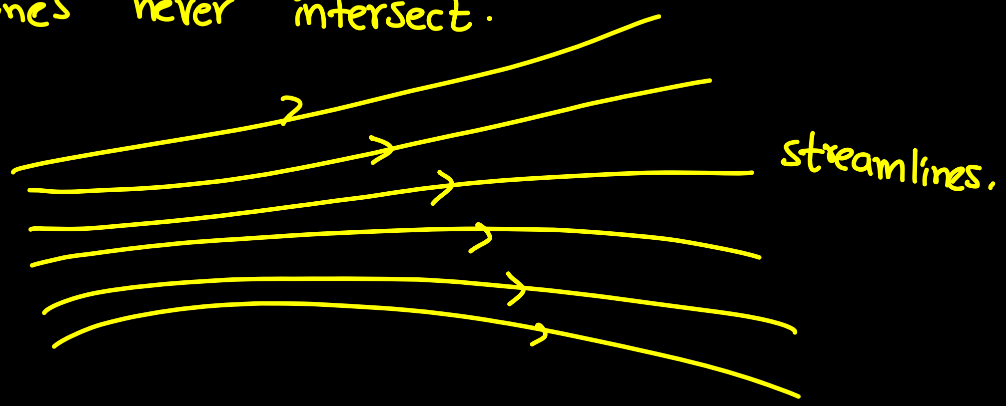
(i) **steady flow:** fluid characteristics like velocity, pressure and density doesn't change w.r.t time.



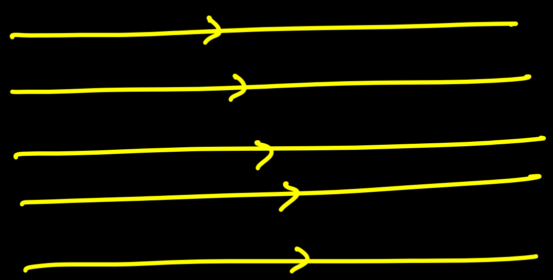
at this point $\vec{v}$, P, ρ are constant.

(ii) **streamline flow:** in steady flow all the particles passing through a given point follow same path and hence a unique line of flow called streamline.

⇒ streamlines never intersect.



Laminar flow:-



well defined streamlines which are straight and parallel.

⇒ at different points mag. of vel can change but not direction

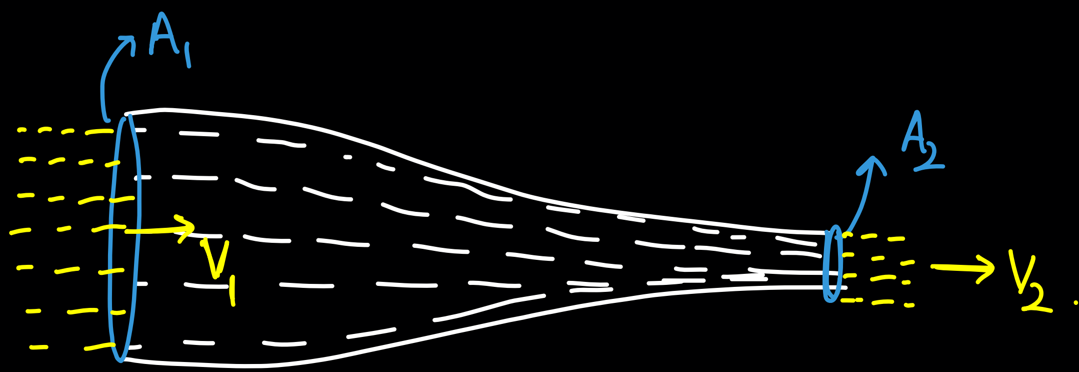
Turbulent flow:- irregular flow in which particles move randomly.

Compressible liquid : Volume can be changed, so density can be changed.

incompressible liquid : Volume can't be changed so density of liquid at any point will remain constant.

In fluid mechanics we take ideal liquids which are not compressible

Equation of continuity :- (Conservation of mass).



as liquid is incompressible

$$m_{entering} = m_{leaving}$$

in dt time

$$\rho\left[A_1 V_1 dt\right] = \rho\left[A_2 V_2 dt\right]$$

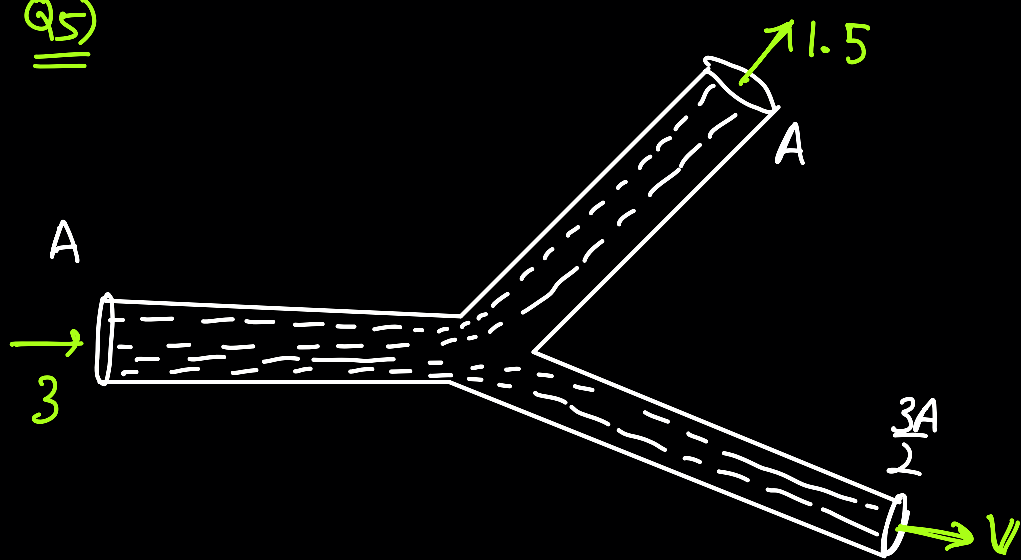
$$\boxed{A_1 V_1 = A_2 V_2}$$

$$AV = \text{constant}$$

m^2 m/s

$AV \rightarrow m^3/s \Rightarrow$ Volume rate flow of liquid.

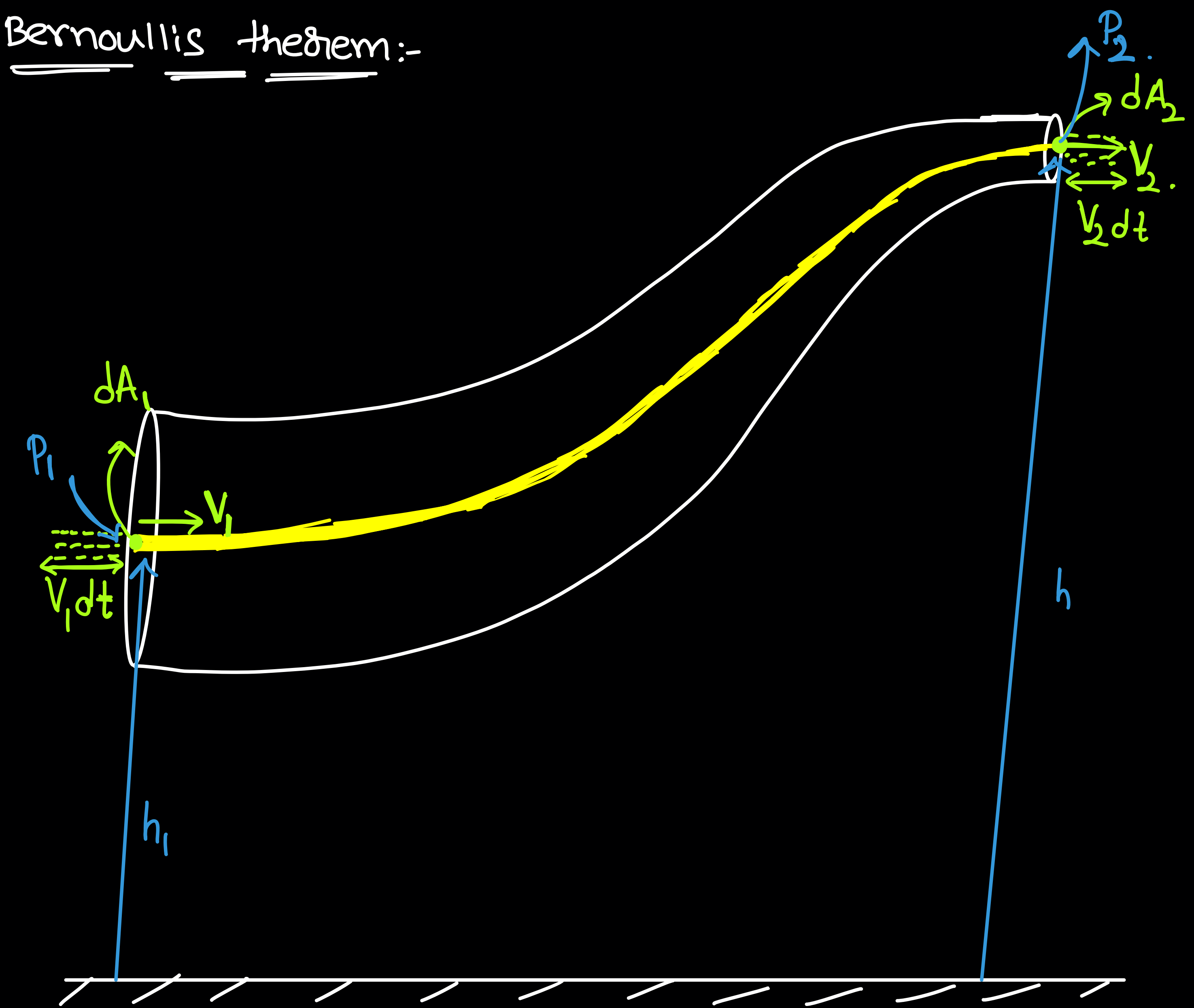
BB:5 Q5)



$3A = (1.5)A + (1.5A)V$

$V = 1 m/s.$

Bernoullis theorem:-



$W_{all\ forces} = \Delta K.E.$

$W_{surrounding\ liquid} + W_{gravity} = K.E_f - K.E_i$

$(P_1 dA_1)V_1 dt - (P_2 dA_2)(V_2 dt) - (dm)g(h_2 - h_1) = \frac{1}{2}(dm)V_2^2 - \frac{1}{2}dm V_1^2$

from equation of continuity

$$(dA_1)V_1 dt = (dA_2)V_2 dt = dV \xrightarrow{} \text{small volume/sec}$$

$$P_1 \cancel{dV} - P_2 \cancel{dV} - \rho \cancel{dV} g(h_2 - h_1) = \frac{1}{2}\rho \cancel{dV}(V_2^2 - V_1^2)$$

$$P_1 + \rho g h_1 + \frac{1}{2}\rho V_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho V_2^2.$$

$$\boxed{P + \rho g h + \frac{1}{2}\rho V^2 = \text{Constant}} \rightarrow \text{Bernoullis theorem.}$$

P = pressure energy = $\dfrac{W_{\text{surrounding}}}{\text{Volume}}$

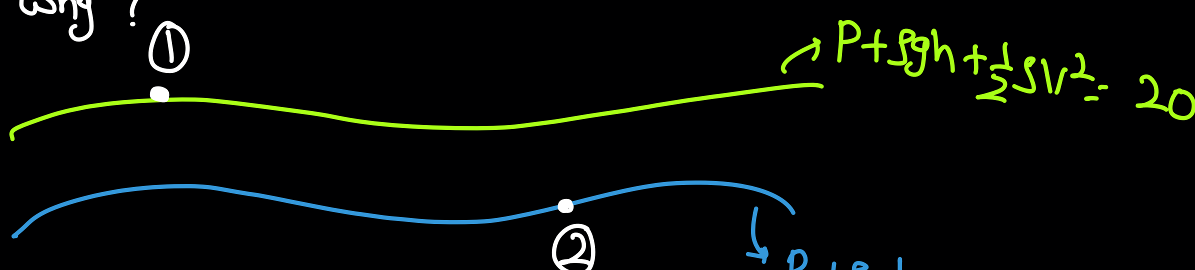
$\rho g h$ = P.E / Volume

$\frac{1}{2}\rho V^2$ = K.E / Volume.

Note :-

① B.T should be used for two points on same streamline.

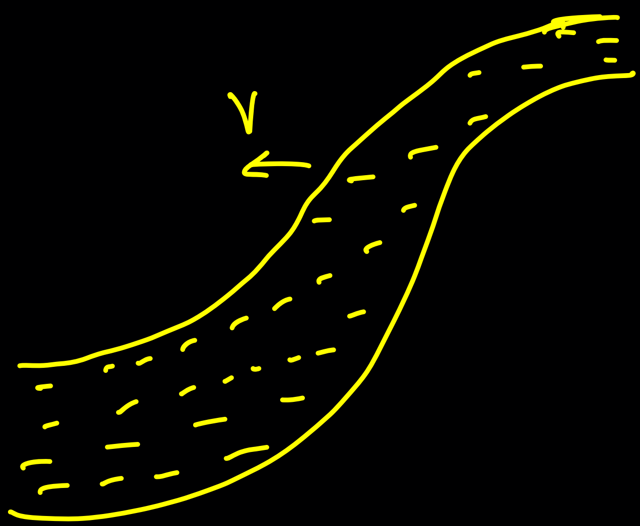
why ?



$20 \neq 10$

② B.T is valid only when surrounding liquid and gravity are the only things doing work. But conservation of energy / work energy theorem is always valid.

what if pipe moves??



$$W_{pipe} + W_{surrounding} + W_{gravity} = \Delta K.E.$$

we won't get $P + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}$

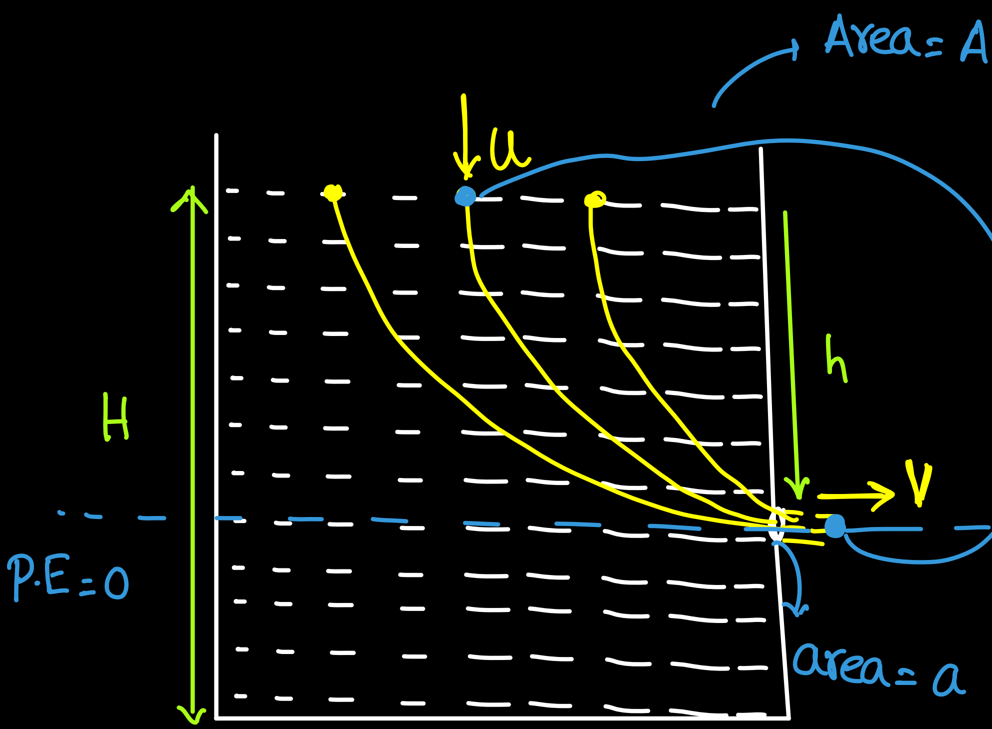
then bernoullis theorem is not valid.

if we take w.r.t pipe then bernoullis theorem can be used as pipe is moving with constant velocity.

⇒ conservation of energy is valid in any frame but bernoullis theorem is valid when pipe doesn't move.

Torricelli's law of efflux:-

given $A >>> a$.



$$Au = av \quad -(1)$$

$$P_0 + \rho g h + \frac{1}{2}\rho u^2$$
$$\parallel$$
$$P_0 + 0 + \frac{1}{2}\rho v^2$$

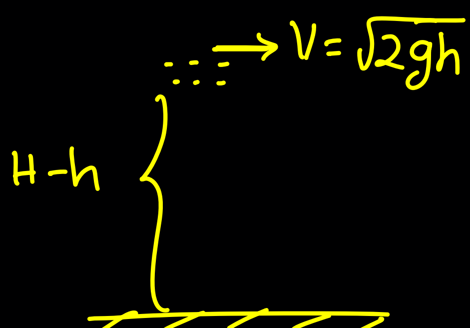
$$P_0 + \rho g h + \frac{1}{2}\rho u^2 = P_0 + \frac{1}{2}\rho v^2$$

$$2gh + \left(\frac{a}{A}v\right)^2 = v^2$$

$$V = \sqrt{\frac{2gh}{1 - \left(\frac{a}{A}\right)^2}}.$$

as $A >>> a \Rightarrow 1 - \frac{a^2}{A^2} \approx 1.$

$$\boxed{V = \sqrt{2gh}}$$



$$t = \sqrt{\frac{2(H-h)}{g}}$$

$$R = (V)t = 2\sqrt{(H-h)(h)}.$$

for what value of h, R is max ?

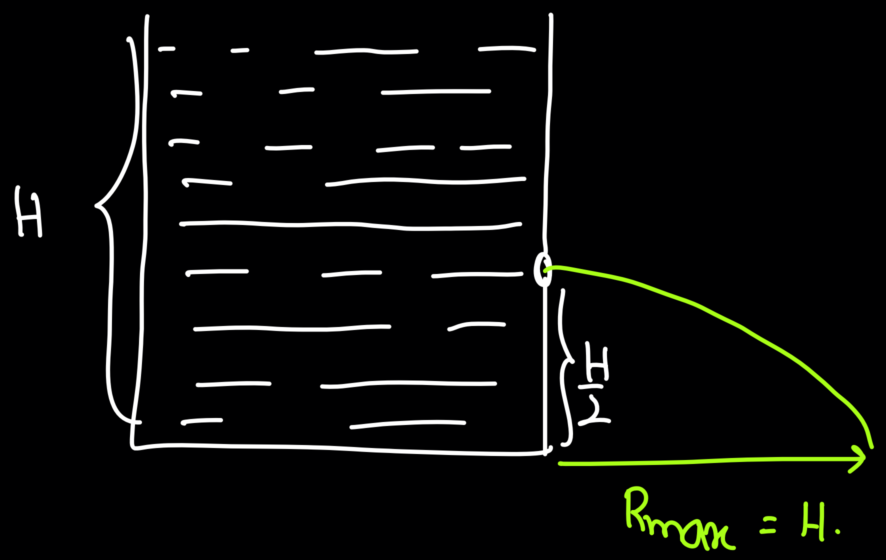
$$R = 2\sqrt{(H-h)h}$$

$$\frac{d[(H-h)h]}{dh} = 0$$

$$h = \frac{H}{2}.$$

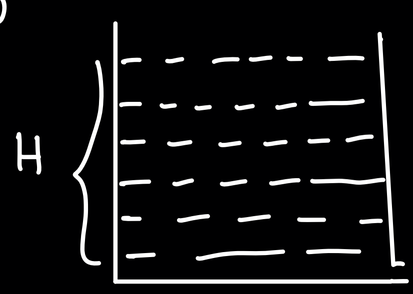
at $h = \frac{H}{2}$ $\Rightarrow$ Range is max.

$$R_{max} = H.$$

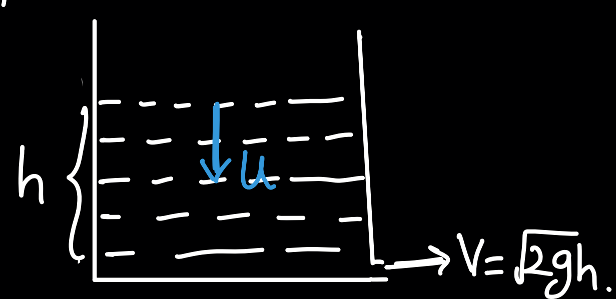


$\Rightarrow$ if the hole is made at bottom find time taken to empty the tank ?

at $t=0$



time 't'



$$Au = av$$

$$u = \frac{a}{A}\sqrt{2gh}.$$

$$-\frac{dh}{dt} = \sqrt{2g}\,\frac{a}{A}\,h^{\frac{1}{2}}$$

$$\int_H^h -h^{-\frac{1}{2}}\,dh = \int_0^t \sqrt{2g}\,\frac{a}{A}\,dt$$

$$2\left(\sqrt{H}-\sqrt{h}\right) = \sqrt{2g}\,\frac{a}{A}\,t$$

$$t = \frac{A}{a}\sqrt{\frac{2}{g}}\left(\sqrt{H}-\sqrt{h}\right)$$

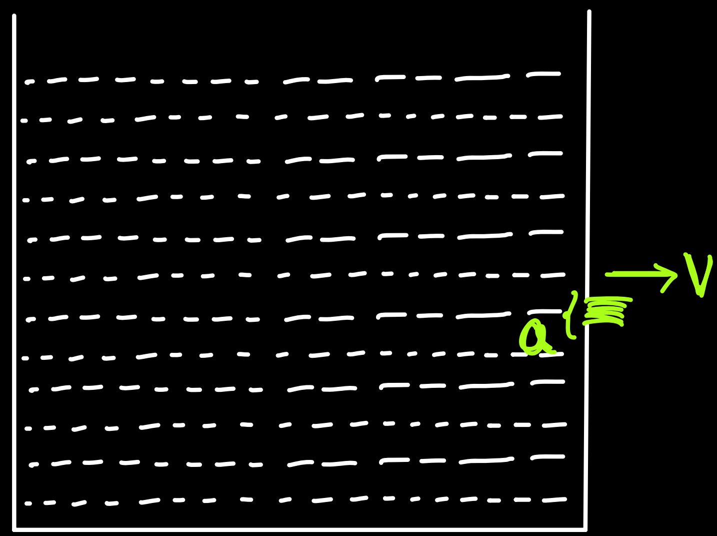
$$\boxed{t \propto \sqrt{H}-\sqrt{h}}$$

time taken for water level to go from $H \to h$.

$\Rightarrow$ find ratio of times to empty 1st half to 2nd half?

$$\frac{t_1}{t_2} = \frac{\sqrt{H}-\sqrt{\frac{H}{2}}}{\sqrt{\frac{H}{2}}-\sqrt{0}} = \frac{1-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = \sqrt{2}-1$$

Thrust force on container:-



in next dt time

$dl = v\,dt$

$dm = a(v\,dt)\rho$

$\vec{F} = \dfrac{d\vec{p}}{dt}$

$\vec{F} = \dfrac{(dm)v_f - dm\,v_i}{dt}$

$= \dfrac{(a\,v\,dt)\rho\,v}{dt}$

$F = \rho a v^2$

$\vec{F}_{thrust} = \left(\dfrac{dm}{dt}\right)\vec{v}_{rel}$

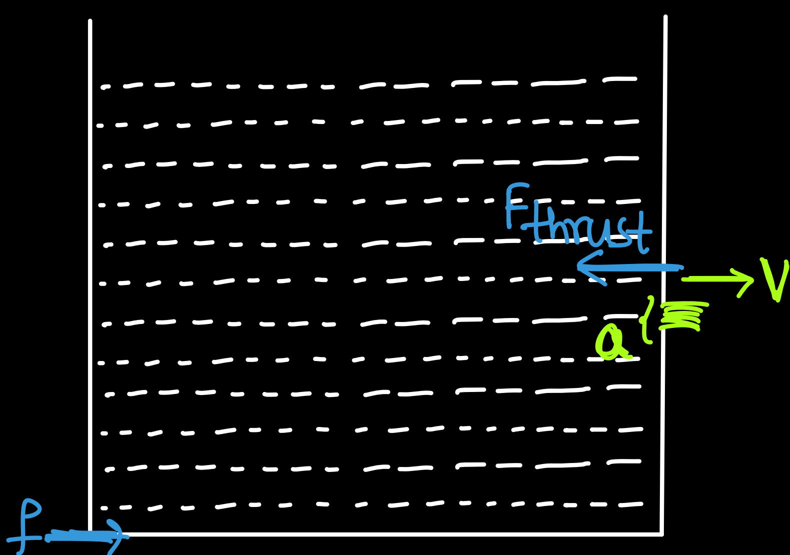
w.r.t container

$\vec{v}_{rel} = \vec{v} - \vec{v}_{container}$

$= \vec{v}$

$\dfrac{dm}{dt} = -\dfrac{a(v\,dt)\rho}{dt} = -\rho a v$

$\vec{F}_{thrust} = -\rho a v^2$



friction balances the thrust force.

Ex: 2

Comp: 1 :-

Q11) in dt time

$$dl = (10)dt$$

$$dm = \rho\left[2\times10^{-4}\right]10dt \Rightarrow \frac{dm}{dt} = 25\times10^{-3}\ \text{kg/s}.$$

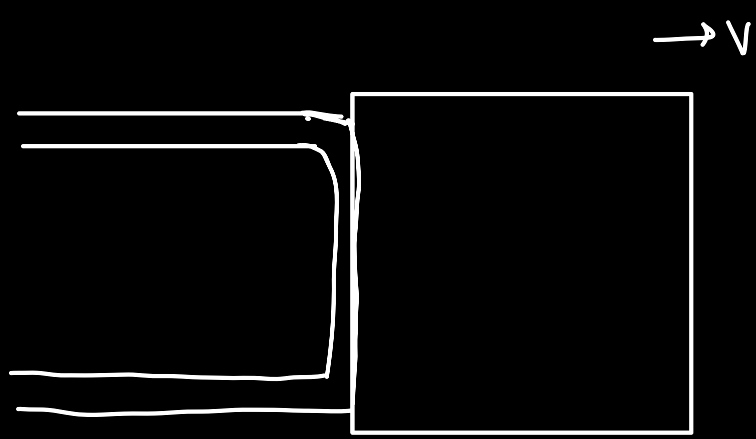
$$F = \frac{dm\vec{V_f} - dm\vec{V_i}}{dt} \Rightarrow \vec{F} = -20\frac{dm}{dt}\hat{i}$$

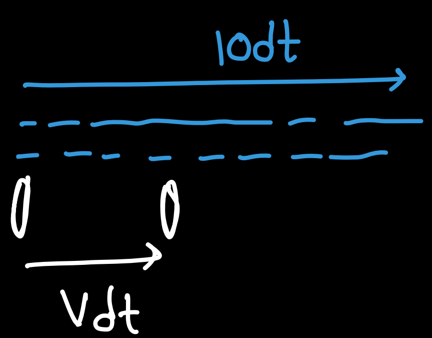
$$= -40\,N.$$

$$F_{cart} = 40$$

---

at some time 't'





length that enters

$$dl = (10-V)dt$$

$$\frac{dm}{dt} = A(10-V)\rho.$$

$$\vec{F} = \frac{dm\vec{V}_f - dm\vec{V}_i}{dt} = 2\rho A(10-V)^2$$

$$(10)a = 2\times10^3 \times 2\times10^{-4}\,[10-V]^2$$

$$\frac{dV}{dt} = (4\times10^{-2})\,[10-V]^2$$

$$\int_0^V (10-V)^{-2}\,dV = \int_0^t 4\times10^{-2}\,dt$$

$$\left[(10-V)^{-1}\right]_0^V = (4\times10^{-2})t$$

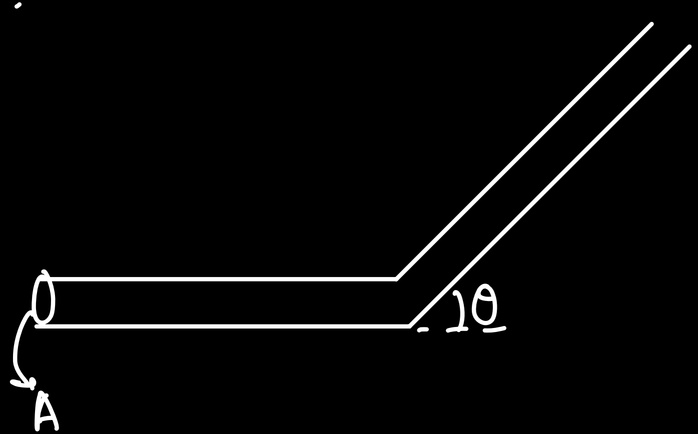
$$\boxed{\frac{1}{10-V} - \frac{1}{10} = (4\times10^{-2})t}$$

$t = 10$

$$\frac{1}{10-V} = 0.5 \Rightarrow 10 - V = 2$$

$$\underline{V = 8}.$$

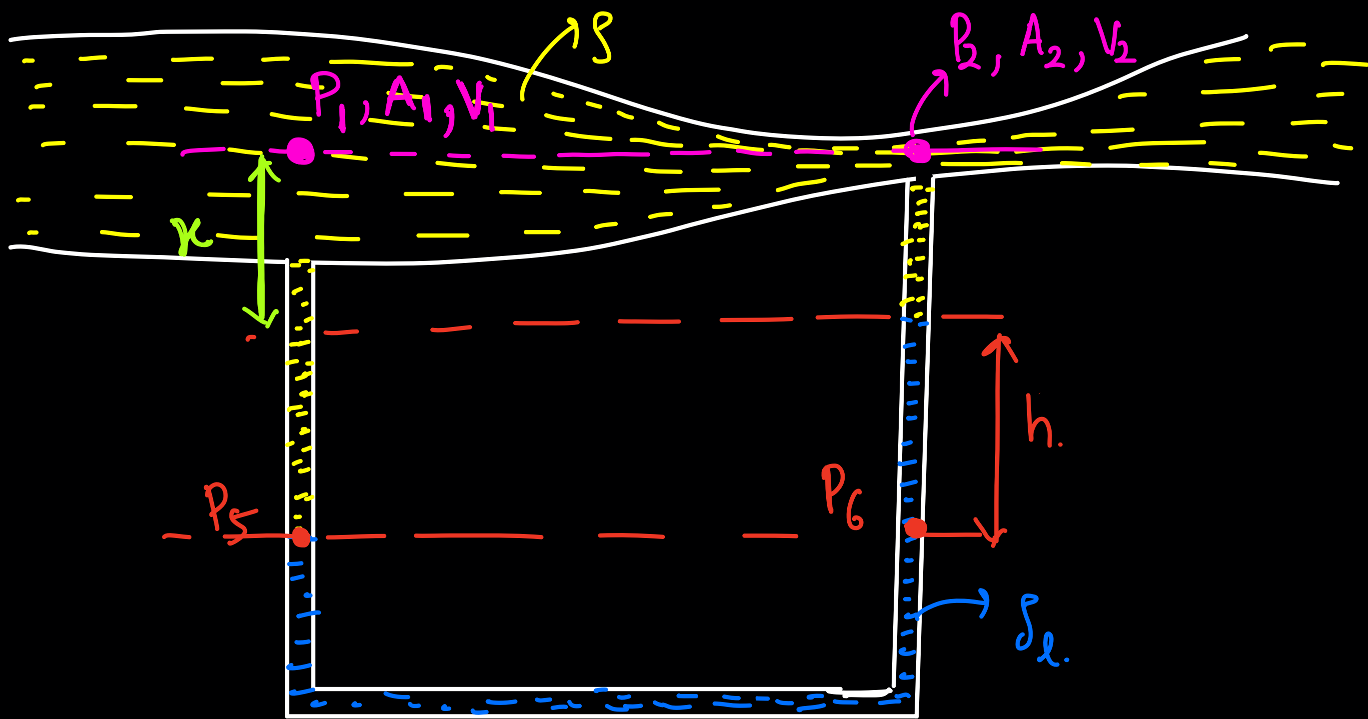
Shortcut:-



$$\boxed{F_{tube} = \rho A V_{rel}^2\,[1-\cos\theta]}$$

V_{rel} → Vel. of liquid w.r.t tube.

Venturimeter:- it's a device used to measure speed of liquid inside a pipe / volume rate flow of liquid



$$P_5 = P_1 + \rho g x + \rho g h$$

$$P_6 = P_2 + \rho g x + \rho_l g h$$

$$P_5 = P_6$$

$$P_1 - P_2 = \rho_l g h - \rho g h.$$

generally $\rho << \rho_l$

$$P_1 - P_2 = \rho_l g h.$$

Bernau:- ① = ②

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2.$$

$$A_1 V_1 = A_2 V_2$$

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho\left(\frac{A_1 V_1}{A_2}\right)^2.$$

$$P_1 - P_2 = \frac{1}{2}\rho\left[\frac{A_1^2}{A_2^2} - 1\right]V_1^2.$$

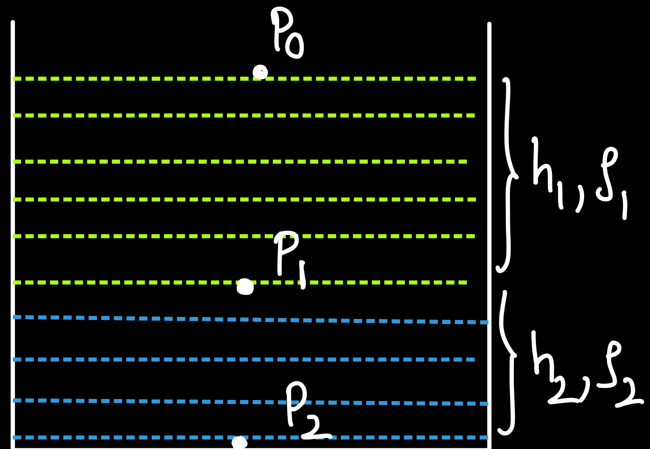
$$\rho_l g h = \frac{1}{2}\rho\left[\frac{A_1^2}{A_2^2} - 1\right]V_1^2$$

$$V_1 = \sqrt{\frac{2\rho_l g h}{\rho\left[\frac{A_1^2}{A_2^2} - 1\right]}}.$$

Volume rate flow $= A_1 V_1$.

$$= A_1 A_2 \sqrt{\frac{2\Delta P}{\rho\left[A_1^2 - A_2^2\right]}}.$$

What if we have multiple liquids ?

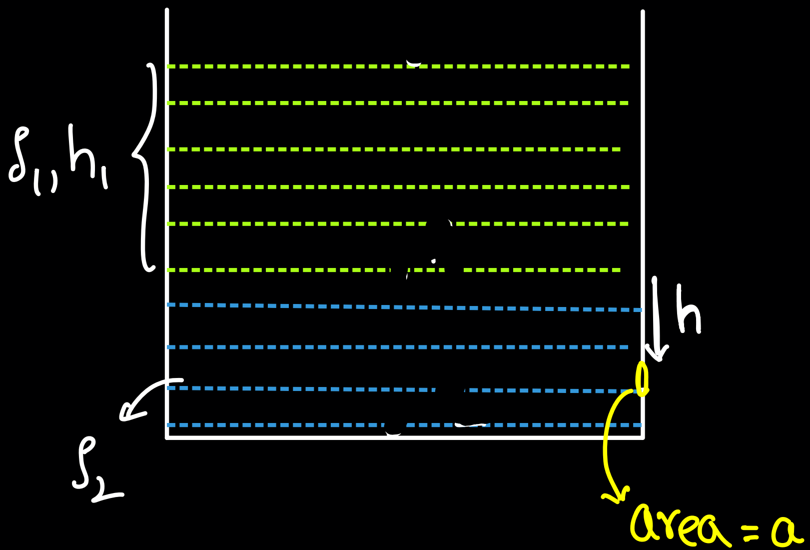


$$P_1 - P_0 = \rho_1 g h_1.$$

$$P_2 - P_1 = \rho_2 g h_2$$

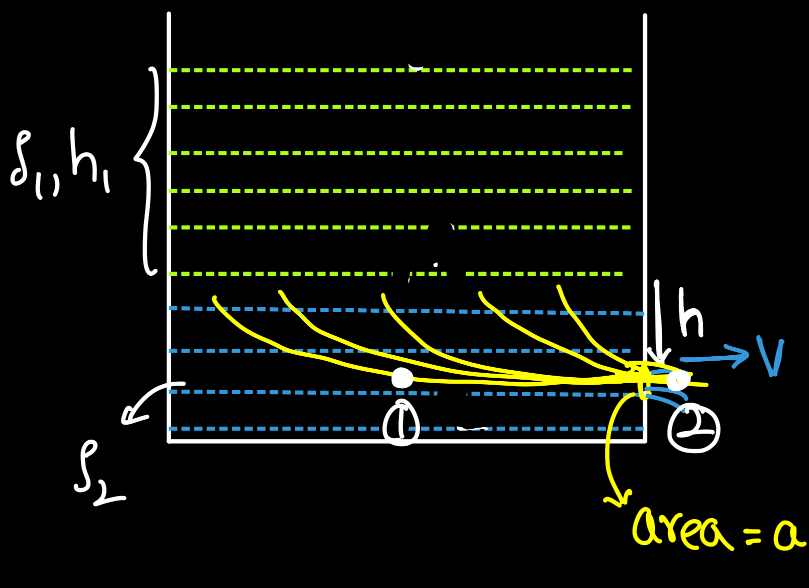
+ve

$$P_2 - P_0 = \rho_1 g h_1 + \rho_2 g h_2.$$



find speed with which liquid comes out?

Sol :-



Point 1

take P.E = 0

$$(P_0+\rho_1 g h_1+\rho_2 g h)+0+0$$

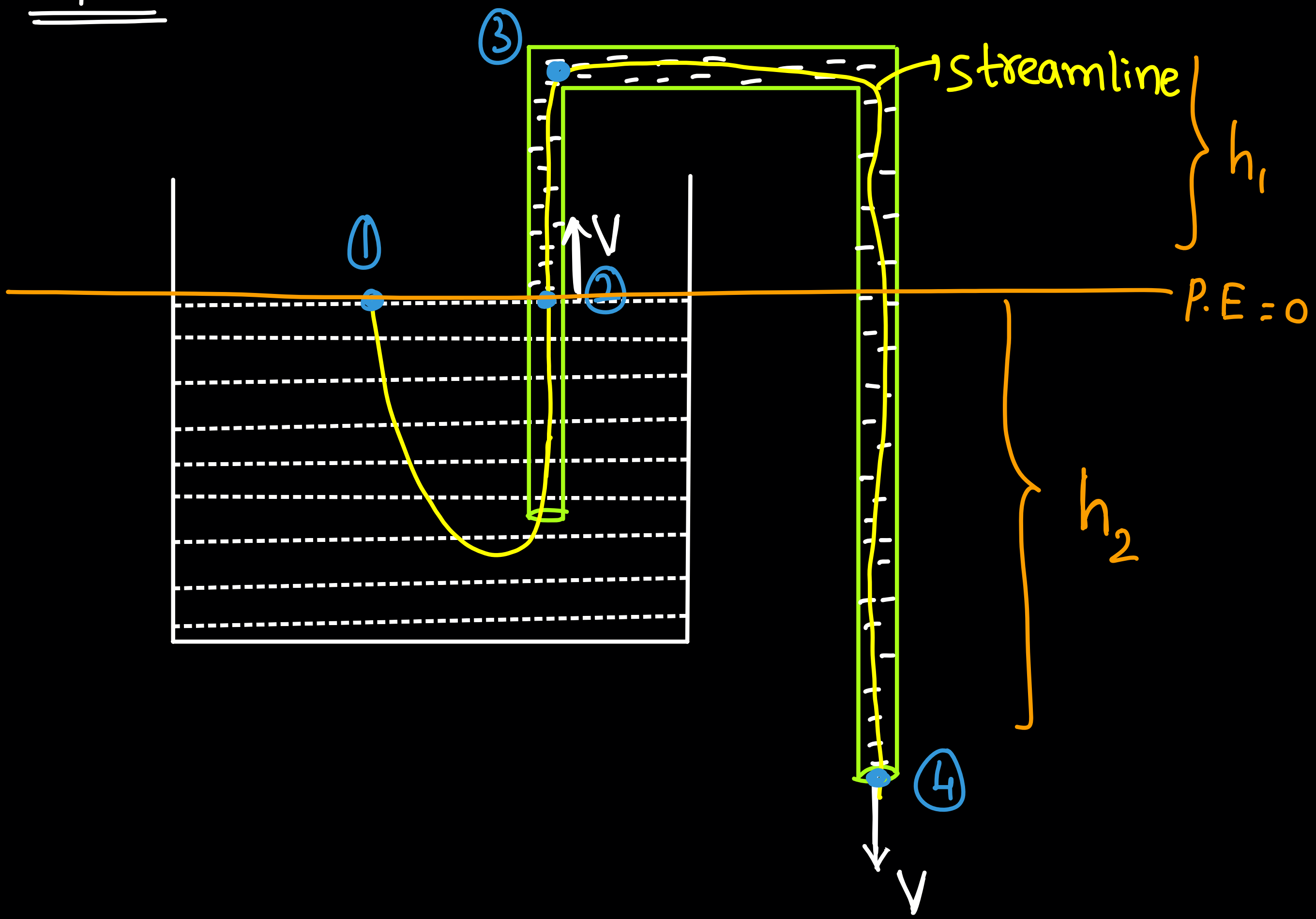
Point 2 :-

$$P_0+0+\frac{1}{2}\rho_2 v^2$$

$$\cancel{P_0}+\rho_1 g h_1+\rho_2 g h = \cancel{P_0}+\frac{1}{2}\rho_2 V^2$$

$$V = \sqrt{\frac{2(\rho_1 h_1+\rho_2 h)g}{\rho_2}}$$

Siphon:-



Point ① ⇒ $P_0 + 0 + 0$

Point ② ⇒ $P_2 + 0 + \frac{1}{2}\rho v^2$

Point ③ ⇒ $P_3 + \rho g h_1 + \frac{1}{2}\rho v^2$

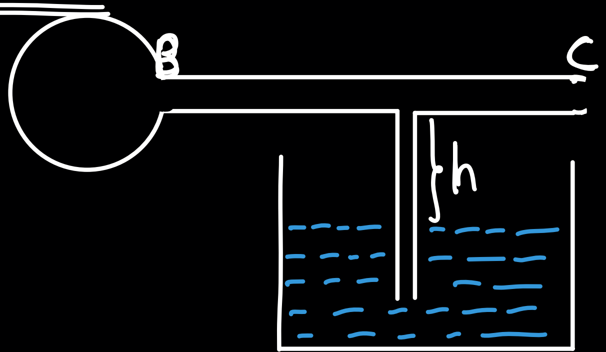
Point ④ ⇒ $P_0 - \rho g h_2 + \frac{1}{2}\rho v^2$.

Speed :- ① = ④

$$P_0 = P_0 - \rho g h_2 + \frac{1}{2}\rho v^2$$

$$v = \sqrt{2gh_2}.$$

Atomizer:-

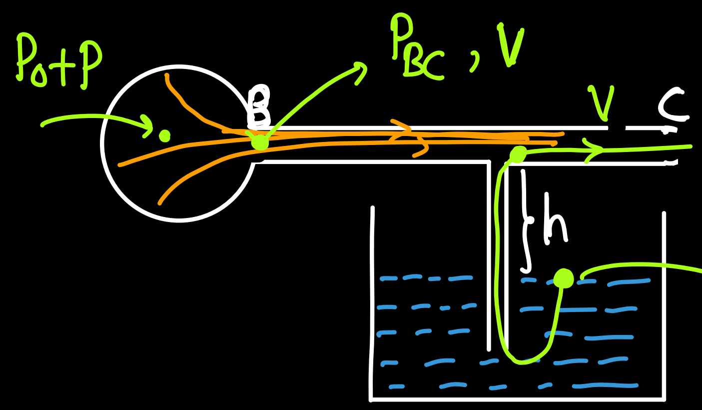


density of air = ρ_a

density of liquid = ρ_l.

if guage pressure of P is created in the bulb, if

V is the speed of air in flat tube find approx. pressure in BC and Velocity of liquid in BC ?



$$(P_0+P)+0+0 = P_{BC}+\frac{1}{2}\rho_a V^2$$

$$P_0+P = P_{BC}+\frac{1}{2}\rho_a V^2 \quad -(1)$$

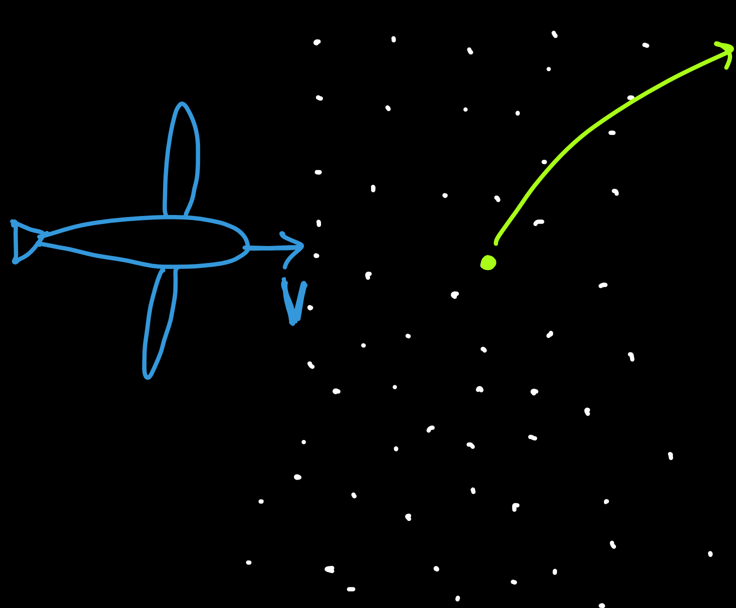
$$P_0-\rho_l gh+0 = P_{BC}+\frac{1}{2}\rho_l v^2$$

$$P_0-P_{BC} = \rho_l gh+\frac{1}{2}\rho_l v^2$$

$$P_0-P_{BC} = \frac{1}{2}\rho_a V^2-P$$

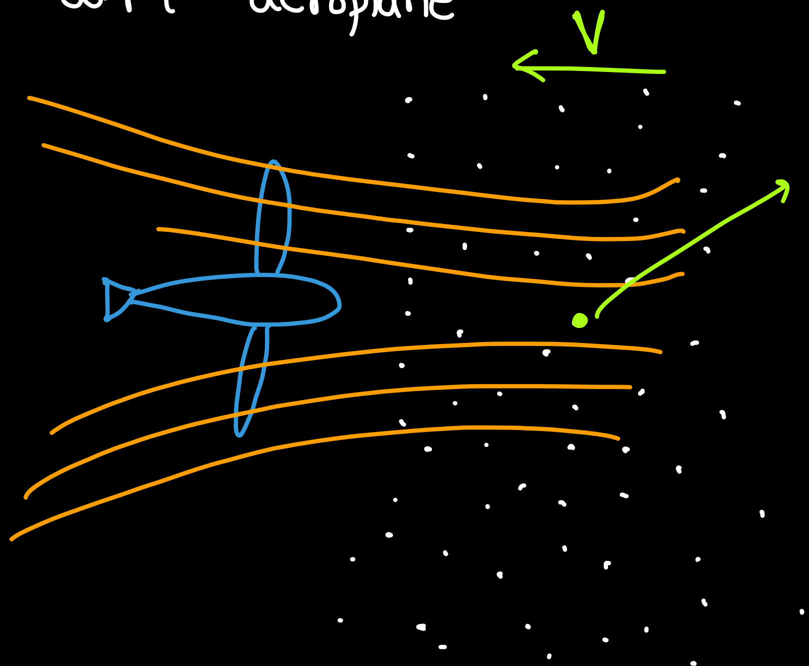
$$\frac{1}{2}\rho_a V^2-P = \rho_l gh+\frac{1}{2}\rho_l v^2$$

Aeroplane:-



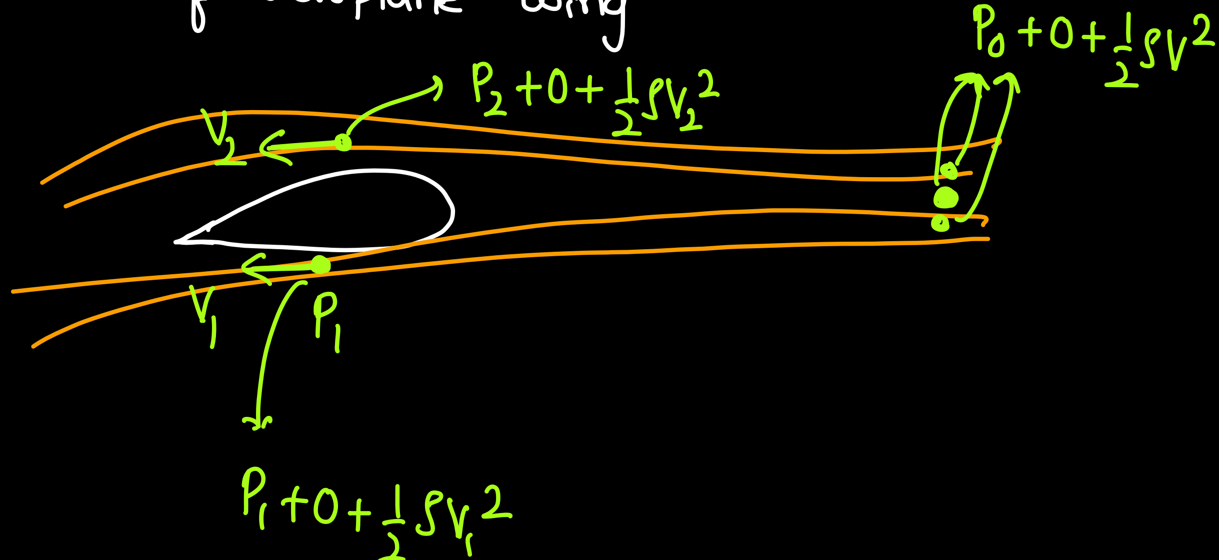
point under observation in ground frame speed of air molecules is going to change with time. so flow in unsteady. we can't apply Bernoulli's theorem.

w.r.t aeroplane



at this point speed is always V, so w.r.t aeroplane flow is steady.

Side view of aeroplane wing



$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 = P_0 + \frac{1}{2}\rho V^2$$

$$P_1 - P_2 = \frac{1}{2}\rho\left[v_2^2 - v_1^2\right]$$

$$F_{upward} = (P_1 - P_2)(\text{area of wing})$$

Ex: 4B

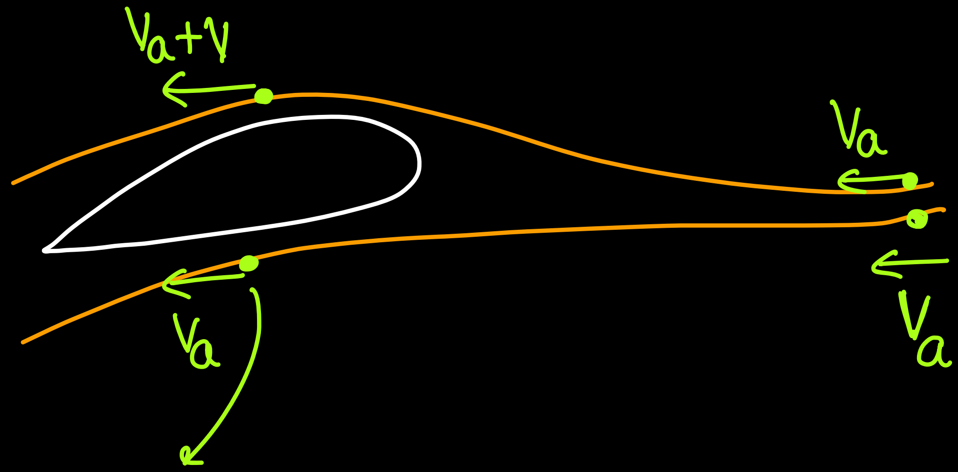
Q10)

Ex: 5

Q9)

Q10)

w.r.t aeroplane



$$P_0+0+\frac{1}{2}\rho V_a^2 = P+0+\frac{1}{2}\rho(V_a+V)^2$$

$$P_0-P = \frac{1}{2}\rho\left[(V_a+V)^2-V_a^2\right] \quad -(1)$$

$$(P_0-P)A = mg$$

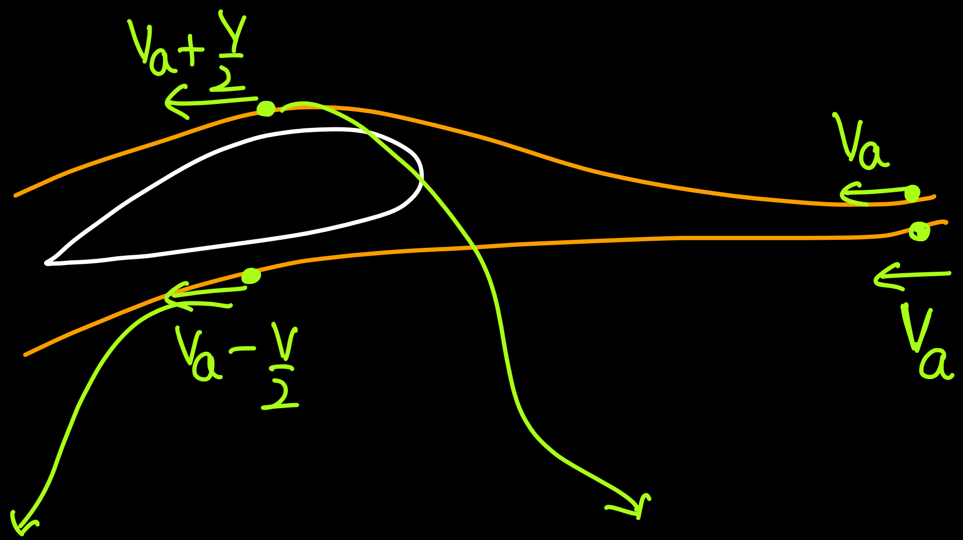
$$P_0-P = \frac{mg}{A} = \frac{1}{2}\rho\left[V_a^2\left[\left[1+\frac{V}{V_a}\right]^2-1\right]\right]$$

$$\frac{mg}{A} = \frac{1}{2}\rho V_a^2\left(1+\frac{2V}{V_a}-1\right)$$

$$\frac{\cancel{2}mg}{A} = \rho V_a^{\cancel{2}}\left(\frac{\cancel{2}V}{\cancel{V_a}}\right)$$

$$V = \frac{mg}{A\rho V_a} = \frac{(5.4\times10^5)(10)}{(500)(1.2)(300)}$$

NCERT :-



$$P_1 + 0 + \frac{1}{2}\rho\left(V_a - \frac{V}{2}\right)^2 = P_2 + 0 + \frac{1}{2}\rho\left(V_a + \frac{V}{2}\right)^2$$

$$P_1 - P_2 = \frac{1}{2}\rho V_a^2\left[\left(1 + \frac{V}{2a}\right)^2 - \left(1 - \frac{V}{2a}\right)^2\right].$$

$$\frac{mg}{A} = \frac{1}{2}\rho V_a^2\left[\frac{2V}{V_a}\right]$$

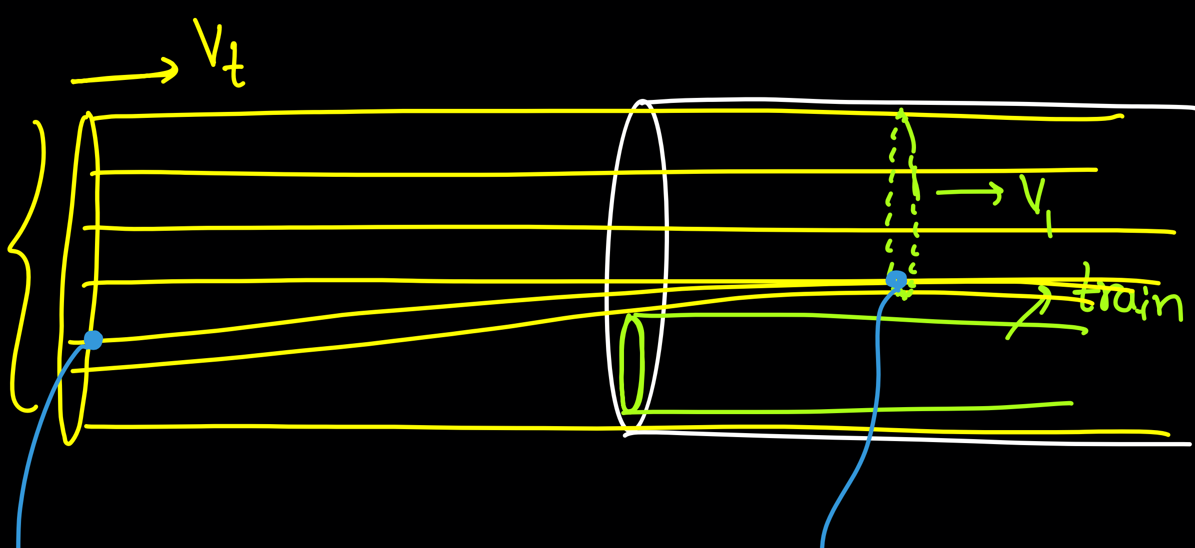
$$V = ?$$

HW :- All questions from 1.315 to 1.329

Ex:5

Q9)

w.r.t train



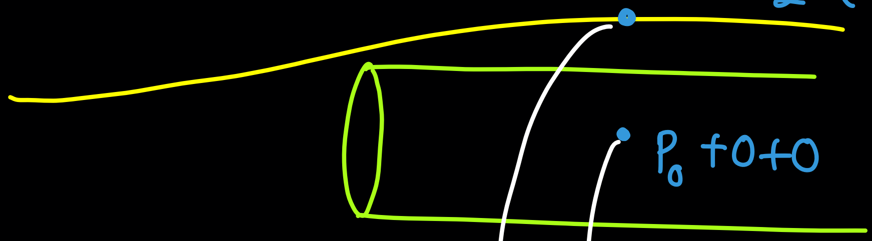
$$(4S_t)v_t = (3S_t)v_1 \Rightarrow v_1 = \frac{4v_t}{3}.$$

$$P_0 + 0 + \frac{1}{2}\rho v_t^2 = P + 0 + \frac{1}{2}\rho v_1^2$$

$$P - P_0 = \frac{1}{2}\rho v_t^2 - \frac{1}{2}\rho \frac{16}{9} v_t^2$$

$$P_0 - P = \frac{7\rho v_t^2}{18}.$$

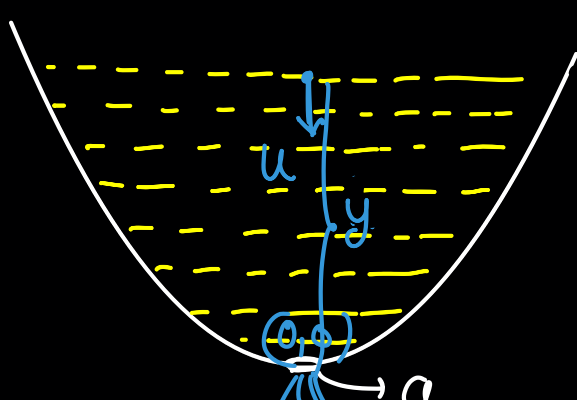
$$P + 0 + \frac{1}{2}\rho\left(\frac{16}{9}v_t^2\right)$$



$$P + \frac{8\rho v_t^2}{9} = P_0 \Rightarrow P - P_0 = \frac{8\rho v_t^2}{9}.$$

---

hour glass:-



$V = \sqrt{2gy}$

$$Au = av.$$

$$(\pi x^2)u = a\sqrt{2gy}.$$

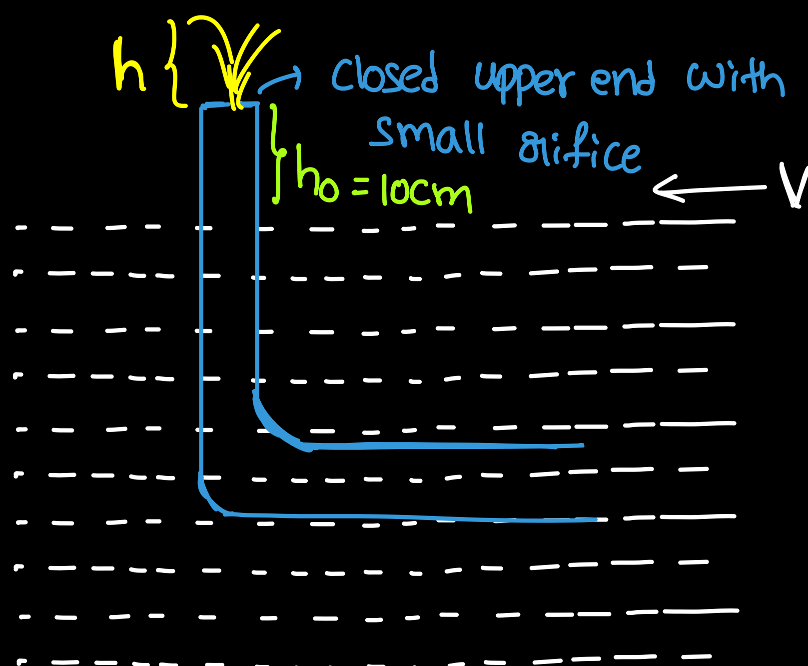
to measure time, u should be constant.

for u to be independent of y

$$x^2 \propto \sqrt{y}$$

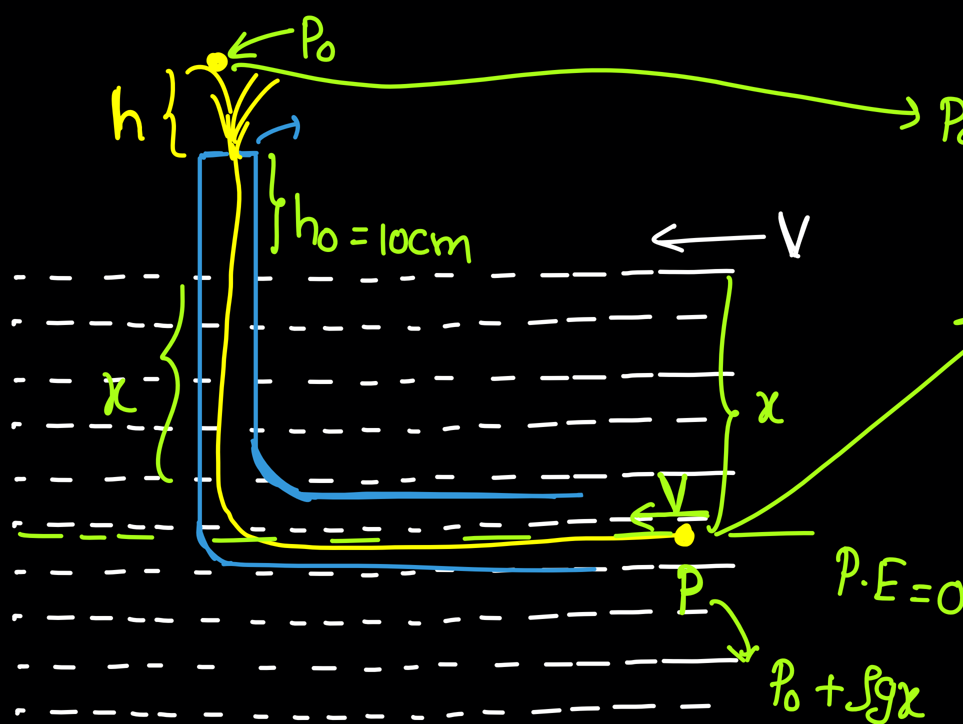
$\boxed{y \propto x^4}$ → if shape is like this then level of sand falls at a constant rate.

Q)



Vel. of stream relative to tube $V = 2m/s$

find h?



$P_0 + \rho g(h + h_0 + x) + 0$

⇕

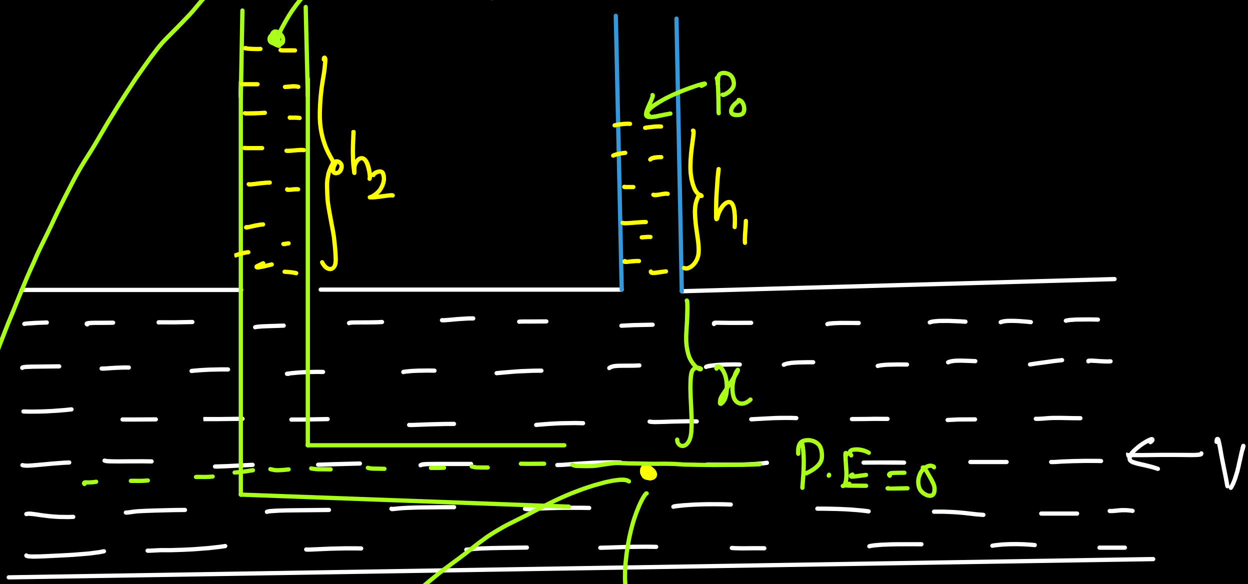
$(P_0 + \rho g x) + 0 + \frac{1}{2}\rho V^2$

on equating

$h = 10cm.$

Observation:-

$P_0 + \rho g(h_2+x) + 0$



$P = P_0 + \rho g(h_1+x)$

$P + 0 + \frac{1}{2}\rho v^2$

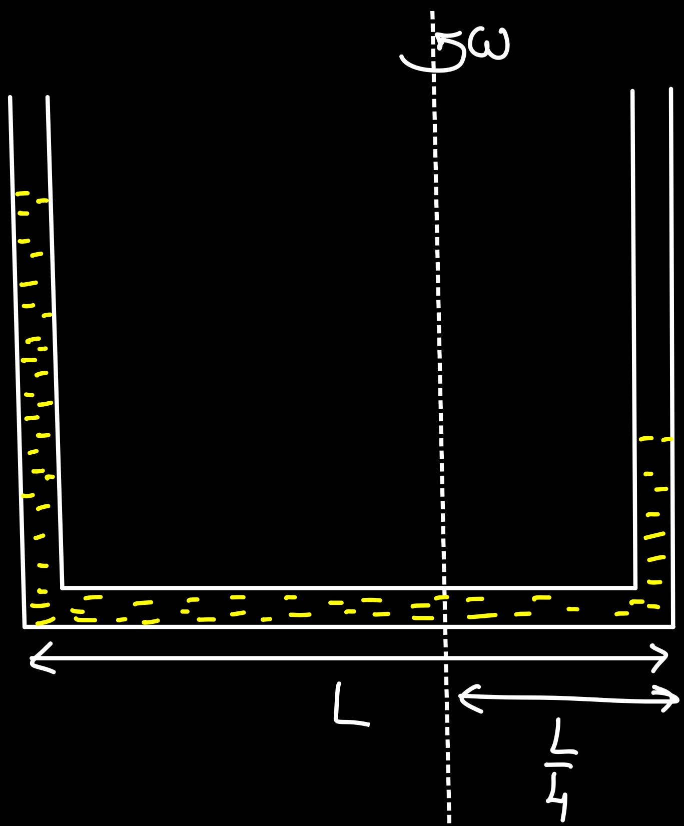
$$P + \frac{1}{2}\rho v^2 = P_0 + \rho g(h_2+x)$$

$$\cancel{P_0} + \rho g(h_1+x) + \frac{1}{2}\rho v^2 = \cancel{P_0} + \rho g(h_2+x)$$

$$\rho g h_1 + \frac{1}{2}\rho v^2 = \rho g h_2 .$$

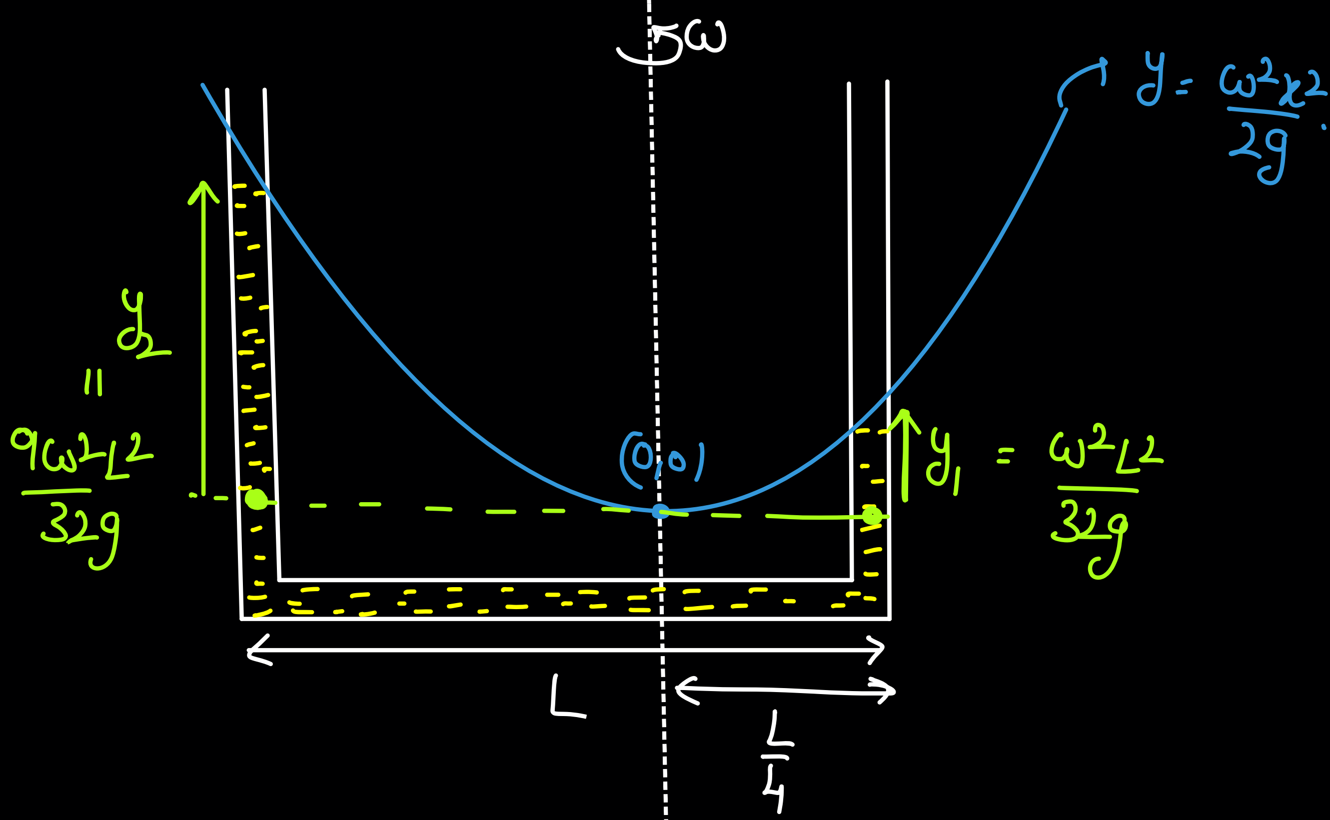
$$\boxed{h_2 = h_1 + \frac{v^2}{2g}}$$

Q)



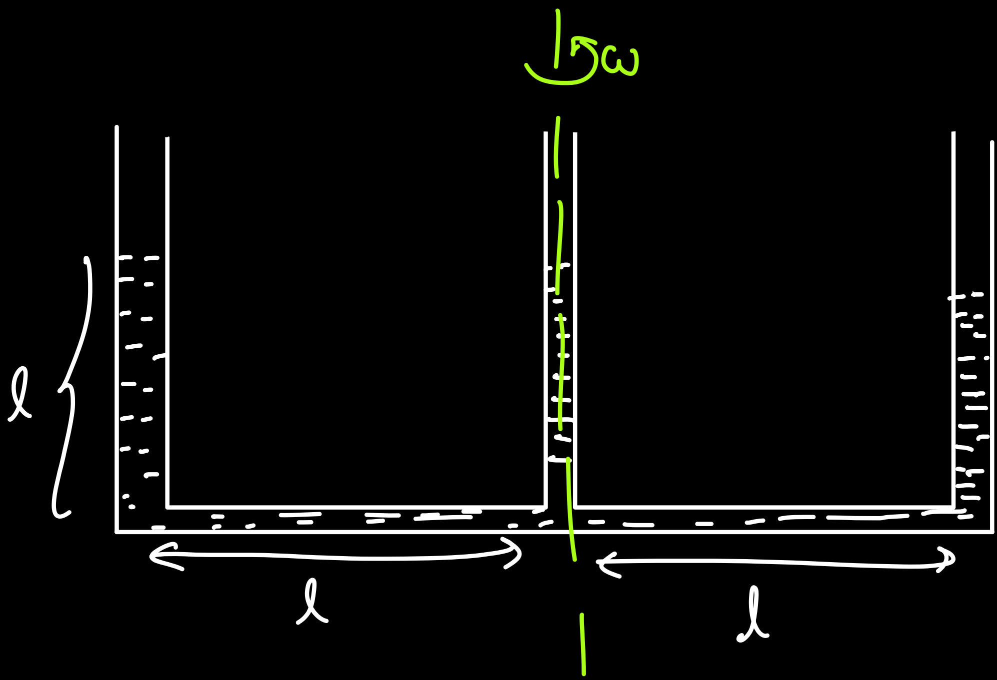
find the difference in height ?

Sol:



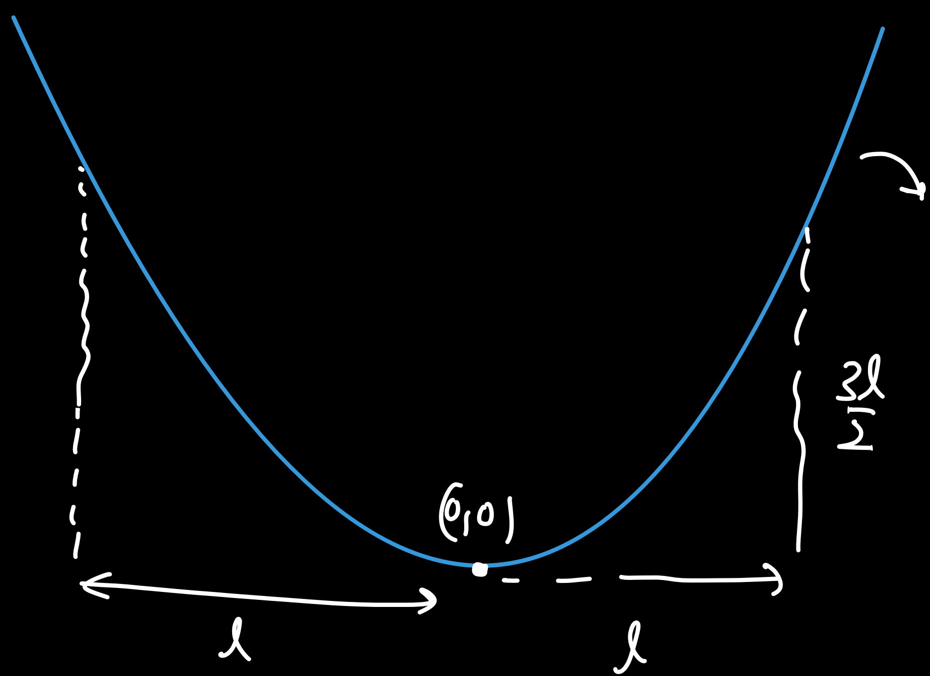
difference in height $= y_2 - y_1 = \frac{\omega^2 L^2}{4g}$.

Q)



find ω for which level of liquid in middle arm will be "0"?

Sol :-



$$y = \frac{\omega^2 x^2}{2g}$$

$$\frac{3l}{\cancel{2}} = \frac{\omega^2 (l^2)}{\cancel{2}g}$$

$$\omega = \sqrt{\frac{3g}{l}}$$