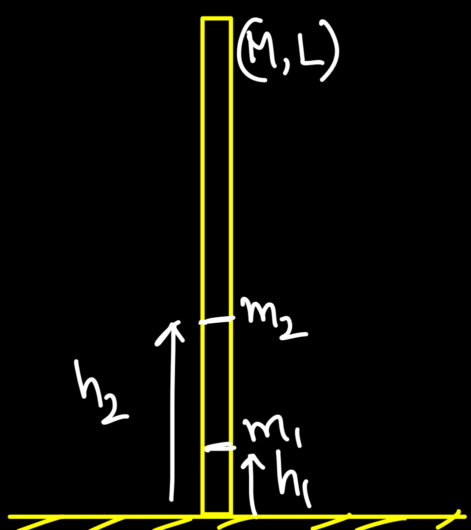
⇒ work energy power chapter made analysis of motion easy by directly looking at flow of energy rather than cal. forces then acc. then velocity.

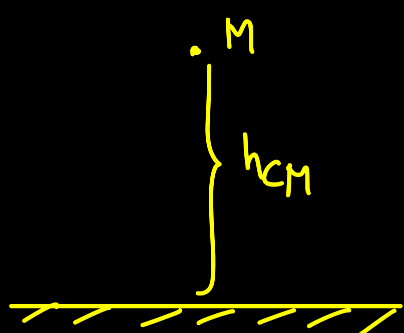
⇒ Concept of potential energy has been introduced to cal. work done by conservative forces easily.

⇒ in the same way, concept of centre of mass will make us understand the complicated motion easily.

C.M concept has been introduced to deal with physical quantities having mass x distance in them.

eg :- P.E $= mgh. = (mh)g.$





$$P.E = m_1gh_1 + m_2gh_2 + \dots$$
$$= (m_1h_1 + m_2h_2 + \dots)g$$

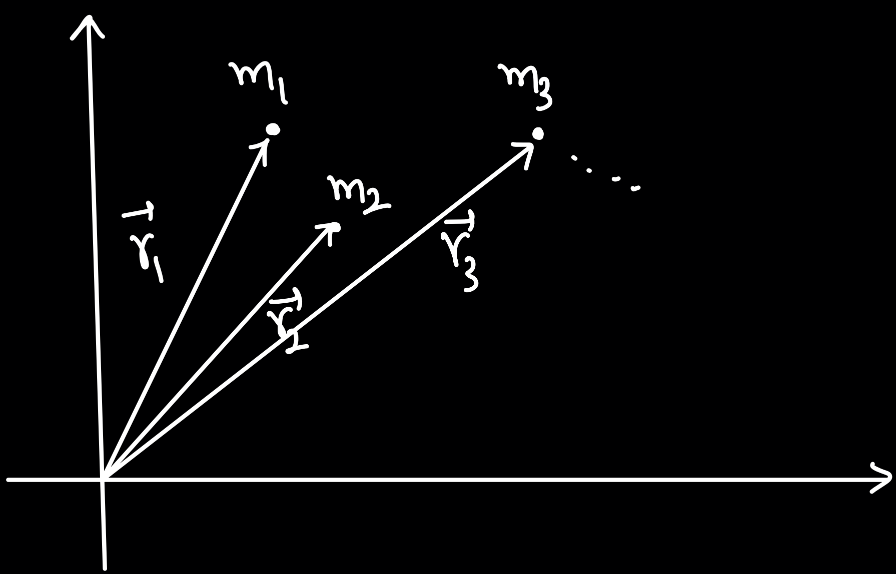
$$P.E = Mgh_{CM}$$
$$= (Mh_{CM})g$$

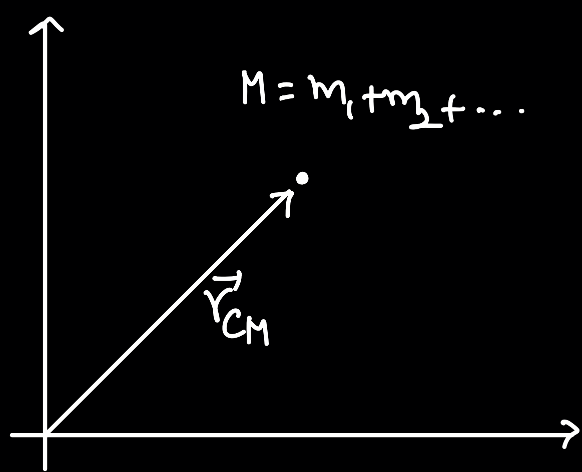
$$h_{CM} = \frac{m_1h_1 + m_2h_2 + \dots}{M} = \frac{m_1h_1 + m_2h_2 + \dots}{m_1 + m_2 + \dots}$$

C.M :- It's the point where the entire mass of body is assumed to be present.

$\Rightarrow$ it can be inside & outside the body.

for system of particles :-





$$\vec{r}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \dots}{m_1 + m_2 + \dots}$$ $\Rightarrow$ the moment C.M. location is given as above, we can replace all masses with a single mass only when dealing with physical quantity has (m)(distance) in it.

P.E = mgh ✓ $F_c = mr\omega^2$ ✓ $F = \frac{Gm_1m_2}{r^2}$ ✗

$$x_{CM}\hat{i} + y_{CM}\hat{j} + z_{CM}\hat{k} = \frac{m_1(x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) + m_2(x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) + \dots}{m_1 + m_2 + \dots}$$

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2 + \cdots}{m_1 + m_2 + \cdots}$$

$$y_{CM} = \frac{m_1 y_1 + m_2 y_2 + \cdots}{m_1 + m_2 + \cdots}$$

$$Z_{CM} = \frac{m_1 Z_1 + m_2 Z_2 + \cdots}{m_1 + m_2 + \cdots}$$

When we change position of particles, position of CM should change.

$$\Delta x_{CM} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2 + \cdots}{m_1 + m_2 + \cdots}$$

$$\Delta y_{CM} = \frac{m_1 \Delta y_1 + m_2 \Delta y_2 + \cdots}{m_1 + m_2 + \cdots}$$

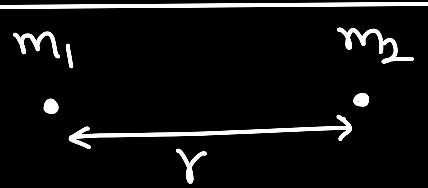
$$\Delta Z_{CM} = \frac{m_1 \Delta Z_1 + m_2 \Delta Z_2 + \cdots}{m_1 + m_2 + \cdots}$$

Velocity and acceleration of C.M :-

$$\vec{V}_{CM} = \frac{m_1 \vec{V}_1 + m_2 \vec{V}_2 + \cdots}{m_1 + m_2 + \cdots}$$

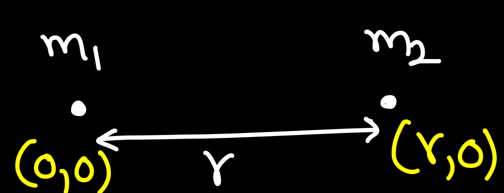
$$\vec{a}_{CM} = \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2 + \cdots}{m_1 + m_2 + \cdots}$$

Q)



find location of C.M from m_1 and m_2 ?

Sol:-



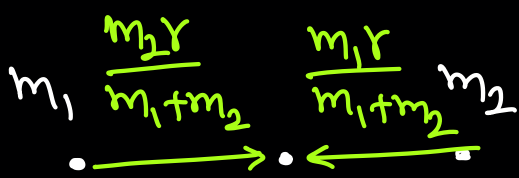
$$x_{CM} = \frac{m_1(0) + m_2 r}{m_1 + m_2}$$

$$= \frac{m_2 r}{m_1 + m_2}.$$

m_1 $(-r, 0)$ $\qquad$ m_2 $(0, 0)$

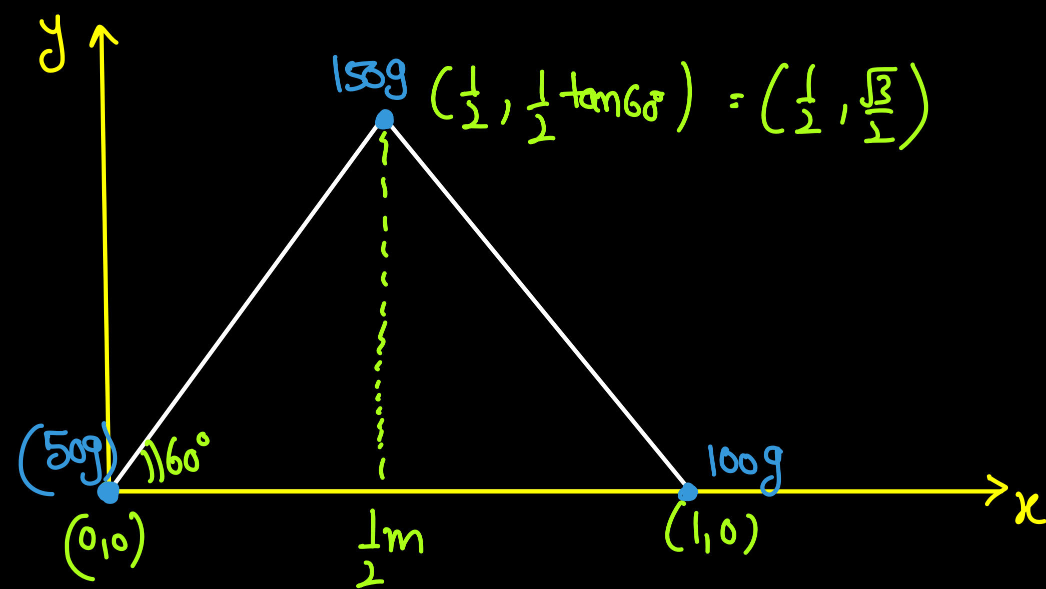
$$x_{CM} = \frac{m_1(-r) + m_2(0)}{m_1 + m_2}$$

$$= -\frac{m_1 r}{m_1 + m_2}.$$



C.M. → location is fixed.

Ex: 4A Q18) :-



$$x_{CM} = \frac{(50)(0) + (150)\left(\frac{1}{2}\right) + (100)(1)}{300} = \frac{\cancel{350}^{7}}{2 \times \cancel{300}_{6}} = \frac{7}{12}.$$

$$y_{CM} = \frac{(50)(0) + (150)\left(\frac{\sqrt{3}}{2}\right) + 100(0)}{300} = \frac{\sqrt{3}}{4}.$$

Q) 2kg $(1,2) \rightarrow (3,1)$

5kg $(2,4) \rightarrow (?\ ?)$ such that position of C.M doesn't change?
x, y.

Sol:- $\Delta x_{CM} = \dfrac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2} \Rightarrow 0 = \dfrac{2(3-1) + 5(x-2)}{2+5}$

$0 = 4 + 5x - 10$

$5x = 6 \Rightarrow x = \dfrac{6}{5}$.

$\Delta y_{CM} = \dfrac{m_1 \Delta y_1 + m_2 \Delta y_2}{m_1 + m_2} \Rightarrow 0 = \dfrac{2(1-2) + 5(y-4)}{2+5}$

$0 = -2 + 5y - 20$

$5y = 22$

$y = \dfrac{22}{5}$.

$\left(\dfrac{6}{5}, \dfrac{22}{5}\right)$

C.M. of continuous mass distribution:-

for point masses $\vec{r}_{CM} = \dfrac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots}{m_1 + m_2 + \dots}$

$$\boxed{\vec{r}_{CM} = \dfrac{\int (dm)\vec{r}}{\int dm}}$$

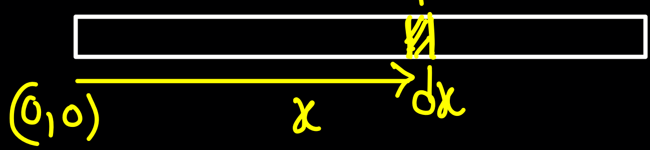
its C.M. of dm element

$$x_{CM} = \frac{\int (dm)x}{\int dm} \quad \Bigg| \quad y_{CM} = \frac{\int (dm)y}{\int dm} \quad \Bigg| \quad z_{CM} = \frac{\int (dm)z}{\int dm}$$

Rod:-

(i) Uniform rod (M, L) :- $\lambda = \text{constant} \rightarrow \lambda = \frac{M}{L}$.

$dm = \lambda dx = \frac{M}{L} dx$



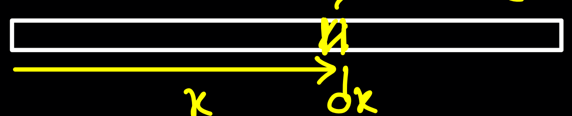
$$x_{CM} = \frac{\int (dm)x}{\int dm} = \frac{\int (\lambda dx)x}{\int \lambda dx} = \frac{\cancel{\lambda}\int_0^L x\,dx}{\cancel{\lambda}\int_0^L dx}$$

$$x_{CM} = \frac{\left[\frac{x^2}{2}\right]_0^L}{\left[x\right]_0^L} = \frac{L}{2}.$$

We can replace

(ii) Non-uniform rod :- $\lambda = \lambda_0 x$.

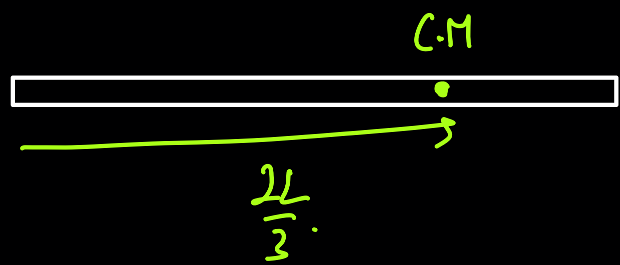
$dm = (\lambda)dx$



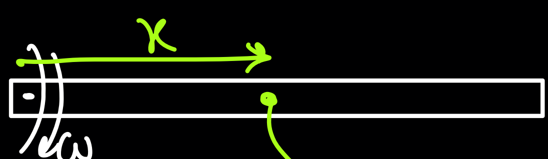
$$x_{CM} = \frac{\int (dm)x}{\int dm} = \frac{\int (\lambda dx)x}{\int \lambda dx} = \frac{\int (\lambda_0 x\,dx)x}{\int \lambda_0 x\,dx}$$

$$= \frac{\cancel{\lambda_0}\int x^2\,dx}{\cancel{\lambda_0}\int x\,dx}$$

$$x_{CM} = \frac{\left[\frac{x^3}{3}\right]_0^L}{\left[\frac{x^2}{2}\right]_0^L} = \frac{\frac{L^3}{3}}{\frac{L^2}{2}} = \frac{2L}{3}.$$

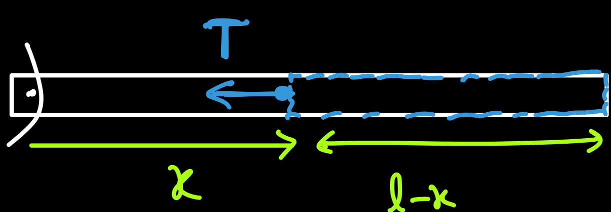


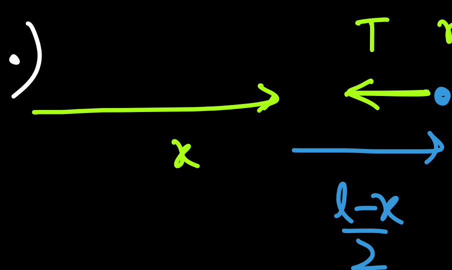
⇒ let's talk about 'T' in rod given in previous chapter



$$T = \frac{m\omega^2(l^2 - x^2)}{2l}.$$

using. C.M. concept we get this easily.





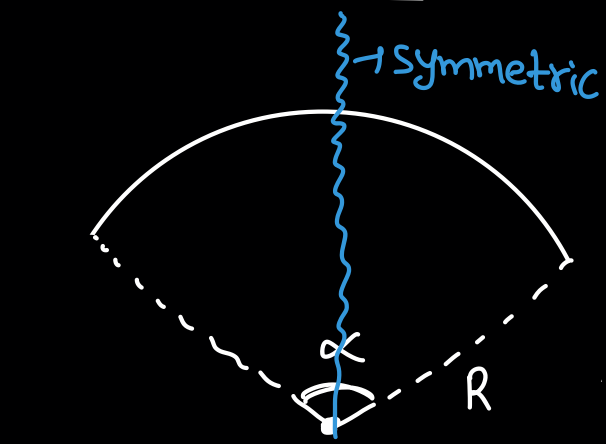
$m = (\lambda)(l-x)$

$$T = mr\omega^2$$

$$T = \lambda(l-x)\left(\frac{l+x}{2}\right)\omega^2$$

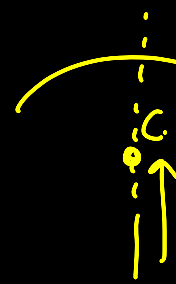
$$T = \frac{\lambda\omega^2(l^2-x^2)}{2}.$$

uniform circular arc:- (λ).

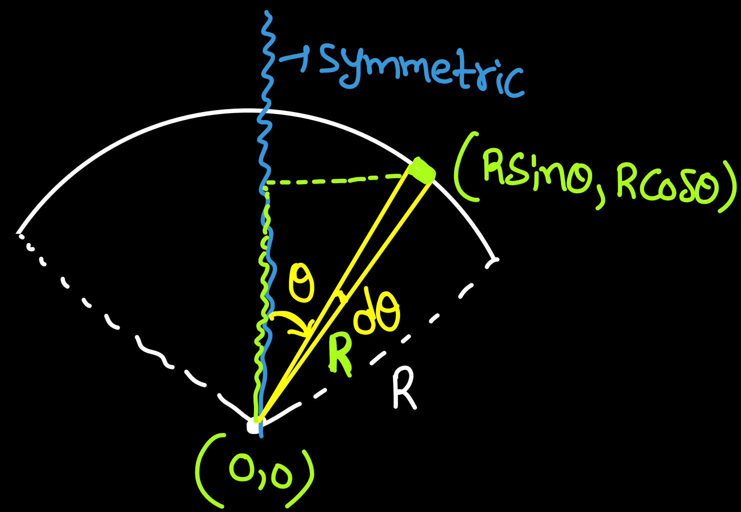


find location of C.M ?

Ans:



$\frac{R\sin\left(\frac{\alpha}{2}\right)}{\frac{\alpha}{2}}$.



$$x_{CM} = \frac{\int dm\, x}{\int dm}$$

$$dm = (\lambda)(R\,d\theta).$$

$$x_{CM} = \frac{\int_{-\frac{\alpha}{2}}^{+\frac{\alpha}{2}} (\lambda R\,d\theta)\, R\sin\theta}{\int_{-\frac{\alpha}{2}}^{+\frac{\alpha}{2}} \lambda R\,d\theta}$$

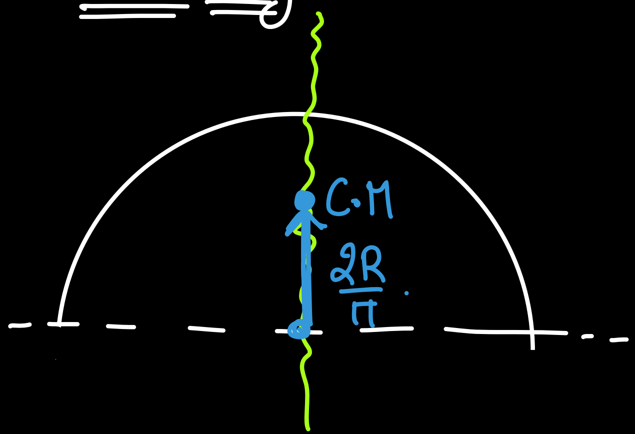
$$x_{CM} = \frac{R\left[-\cos\theta\right]_{-\frac{\alpha}{2}}^{+\frac{\alpha}{2}}}{\left[\theta\right]_{-\frac{\alpha}{2}}^{+\frac{\alpha}{2}}}$$

$$= 0.$$

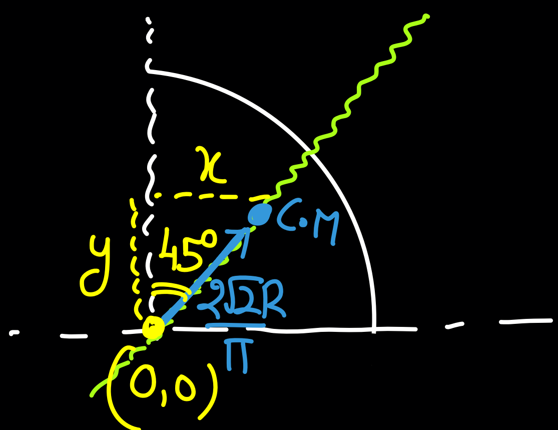
$$y_{CM} = \frac{\int (dm)\, y}{\int dm} = \frac{\int (\lambda R\, d\theta)(R\cos\theta)}{\int \lambda R\, d\theta} = \frac{R\int_{-\frac{\alpha}{2}}^{+\frac{\alpha}{2}} \cos\theta\, d\theta}{\int_{-\frac{\alpha}{2}}^{\frac{\alpha}{2}} d\theta}$$

$$y_{CM} = \frac{R\left[\sin\frac{\alpha}{2} - \sin\left(-\frac{\alpha}{2}\right)\right]}{\frac{\alpha}{2} - \left(-\frac{\alpha}{2}\right)} = \frac{R\sin\frac{\alpha}{2}}{\frac{\alpha}{2}}.$$

Semi ring :-



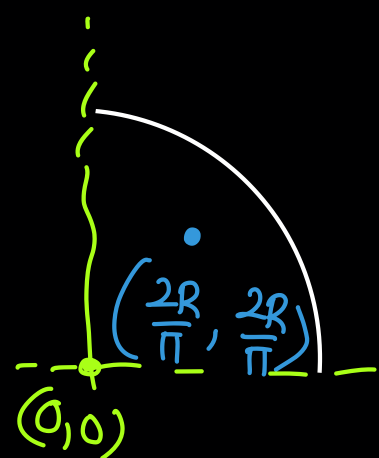
$$\frac{R\sin\left(\frac{\pi}{2}\right)}{\frac{\pi}{2}} = \frac{2R}{\pi}.$$



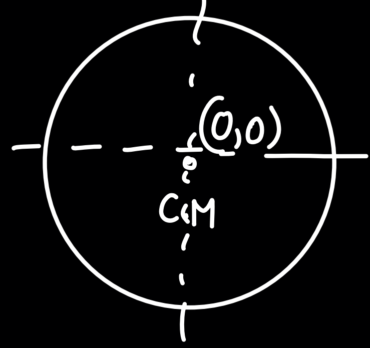
$$\frac{R\sin\left(\frac{\frac{\pi}{2}}{2}\right)}{\left(\frac{\frac{\pi}{2}}{2}\right)} \Rightarrow \frac{2\sqrt{2}R}{\pi}.$$

$$x = \left(\frac{2\sqrt{2}R}{\pi}\right)\sin 45^\circ = \frac{2R}{\pi}$$

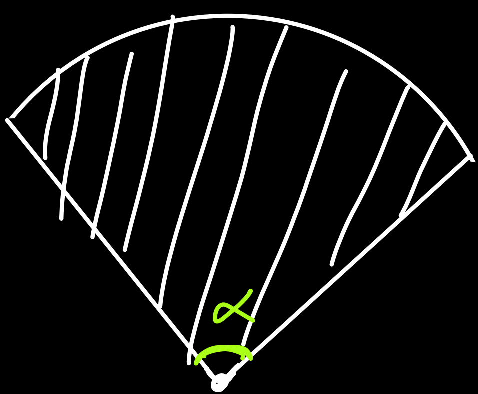
$$y = \frac{2\sqrt{2}R}{\pi}\cos 45^\circ = \frac{2R}{\pi}.$$



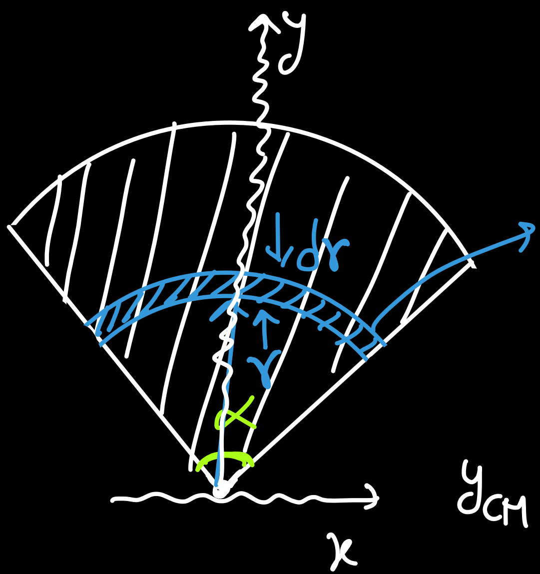
Complete ring



Disc :-



find location of C.M ?



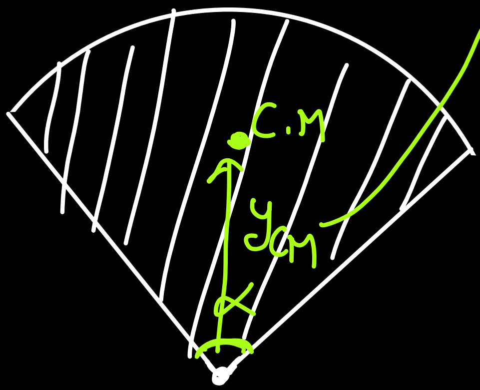
$$dm = (\sigma)(dA)$$

$$= (\sigma)(r\alpha)dr.$$

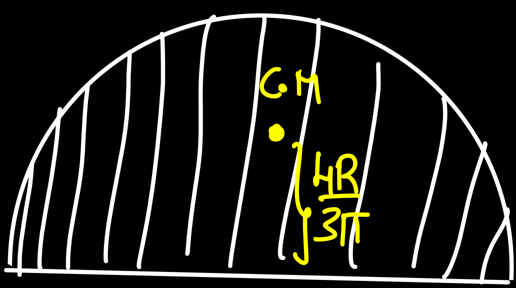
$$y_{CM} = \frac{\int (dm)\, y}{\int dm} = \frac{\int (\cancel{\sigma}\, r \cancel{\alpha}\, dr)\left[\dfrac{r\sin\left(\frac{\alpha}{2}\right)}{\frac{\alpha}{2}}\right]}{\int \cancel{\sigma}\, r \cancel{\alpha}\, dr}$$

$$= \frac{\sin\left(\frac{\alpha}{2}\right)}{\left(\frac{\alpha}{2}\right)} \frac{\int_0^R r^2 dr}{\int_0^R r\, dr}$$

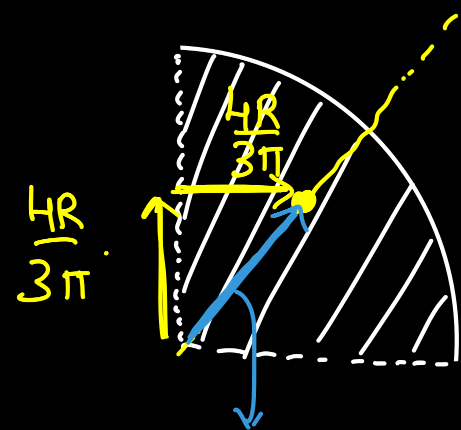
$$y_{CM} = \frac{2}{3}\left[\frac{R\sin\left(\frac{\alpha}{2}\right)}{\frac{\alpha}{2}}\right].$$



Semi disc :-

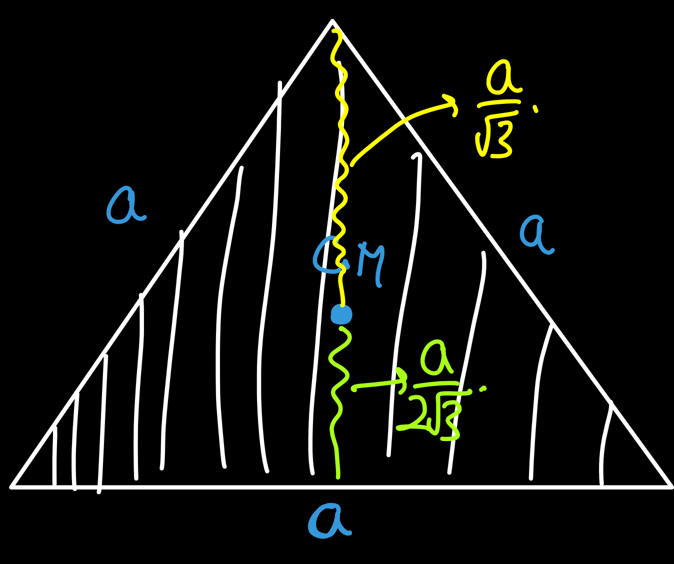


Quarter disc :-

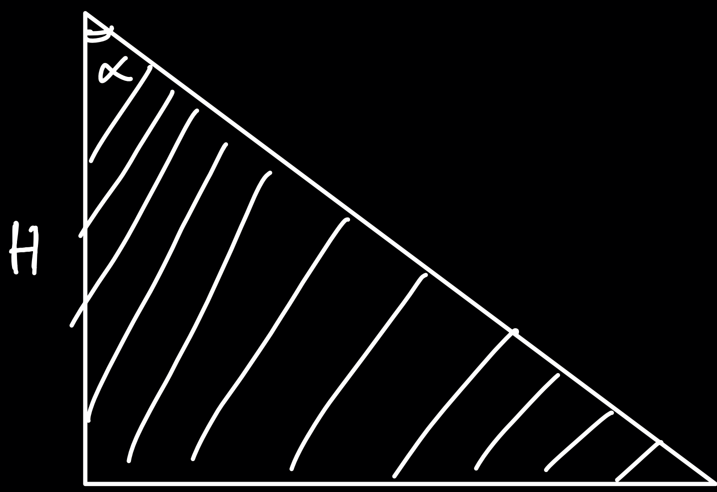


$$\frac{2}{3}\left[\frac{2\sqrt{2}R}{\pi}\right] = \frac{4\sqrt{2}R}{3\pi}.$$

Equilateral triangle :-

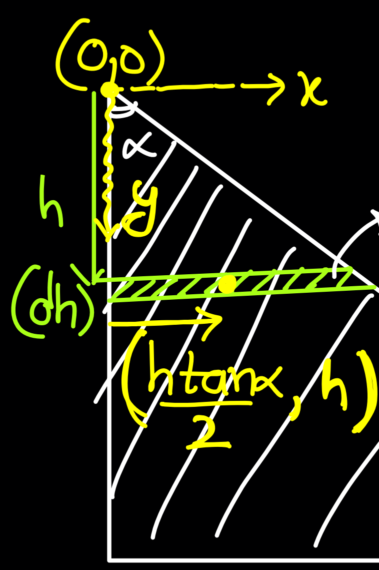


Right angled triangle:-



find C.M ?

Sol:-



$dm = (\sigma)[h\tan\alpha\, dh]$

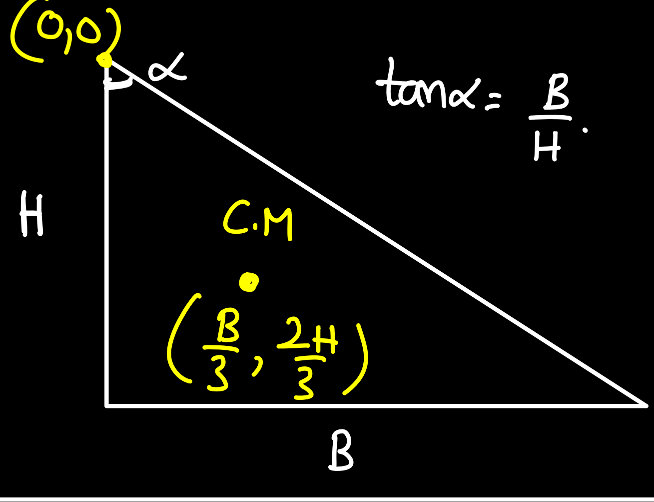
$$x_{CM} = \frac{\int dm\, x}{\int dm}$$

$$= \frac{\int (\cancel{\sigma}\, \cancel{\tan\alpha}\, h\, dh)\left(\frac{h\tan\alpha}{2}\right)}{\int \cancel{\sigma}\, \cancel{\tan\alpha}\, h\, dh}$$

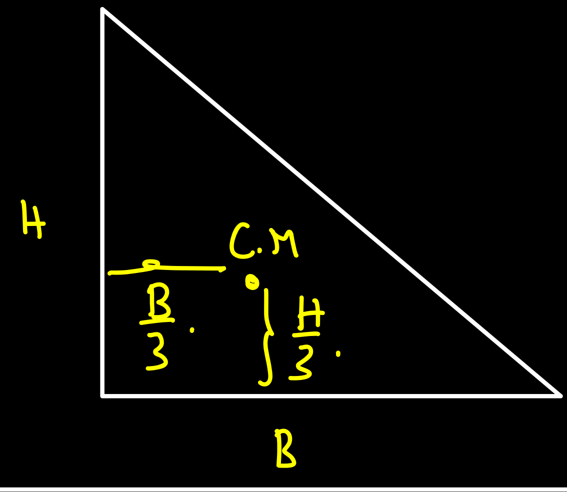
$$= \frac{\frac{\tan\alpha}{2}\int_0^H h^2 dh}{\int_0^H h\, dh}$$

$$x_{CM} = \frac{H}{3}\tan\alpha.$$

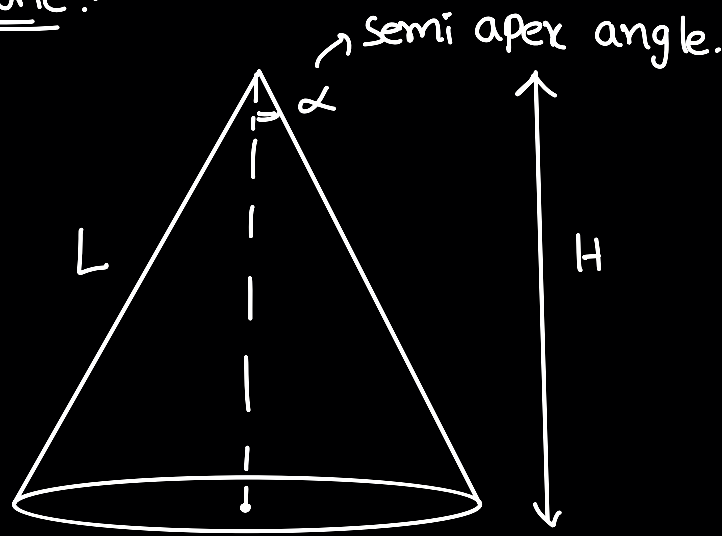
$$y_{CM} = \frac{\int dm\, y}{\int dm} = \frac{\int_0^H (\cancel{\sigma}\, \cancel{\tan\alpha}\, h\, dh)\, h}{\int_0^H \cancel{\sigma}\, \cancel{\tan\alpha}\, h\, dh} = \frac{2H}{3}.$$



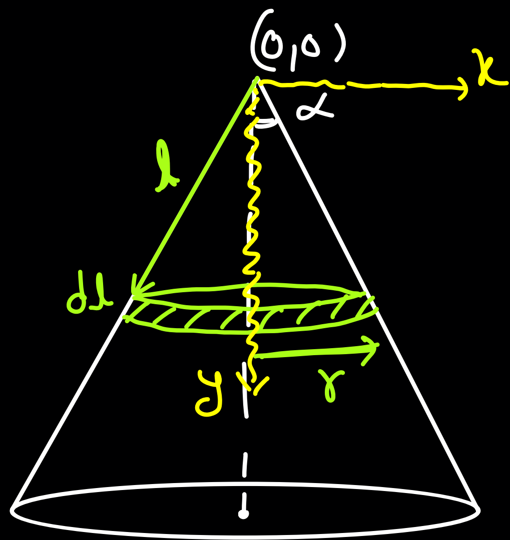
$\tan\alpha = \frac{B}{H}.$



hollow cone:-



find C.M. ?



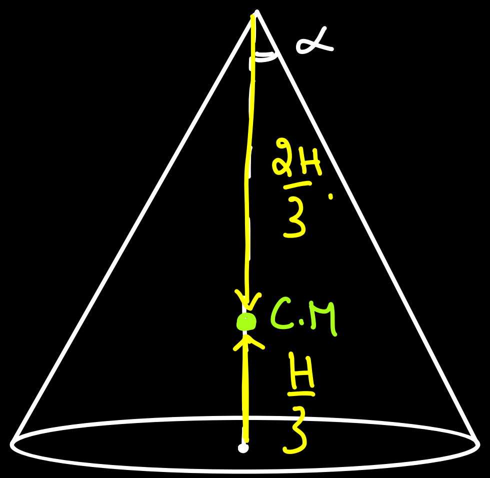
$\sin\alpha = \frac{r}{l} \Rightarrow r = l\sin\alpha.$

$dA = (2\pi r)(dl) = 2\pi\sin\alpha\, l\, dl.$

$dm = (\sigma)\, 2\pi\sin\alpha\, l\, dl.$

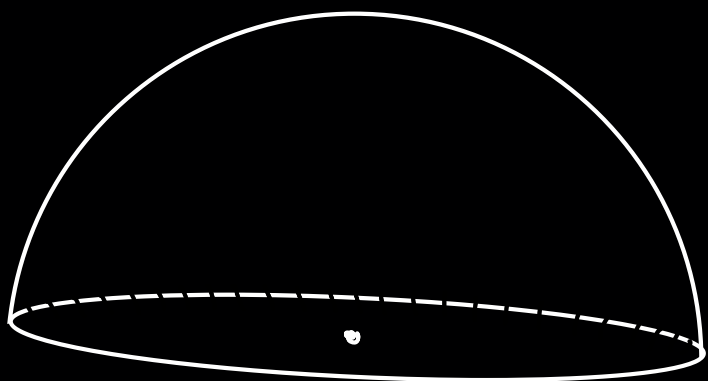
$$y_{CM} = \frac{\int_0^L (\cancel{\sigma}\,\cancel{2\pi}\,\cancel{\sin\alpha}\, l\, dl)(l\cos\alpha)}{\int_0^L \cancel{\sigma}\,\cancel{2\pi}\,\cancel{\sin\alpha}\, l\, dl}$$

$$= \cos\alpha \frac{\int l^2 dl}{\int l\, dl}.$$



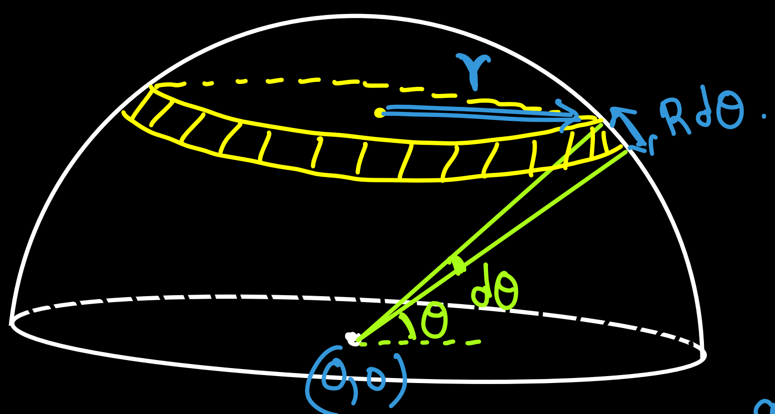
$$= \frac{2L\cos\alpha}{3} = \frac{2H}{3}.$$

hollow hemi sphere:-



find C.M. ?

sol:-



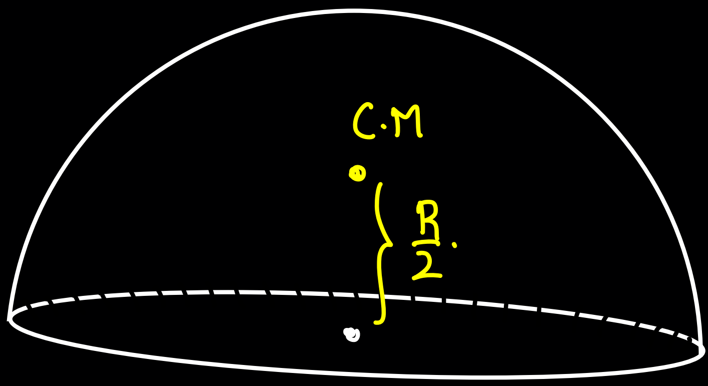
$r = R\cos\theta$

$dA = (2\pi r)(Rd\theta)$

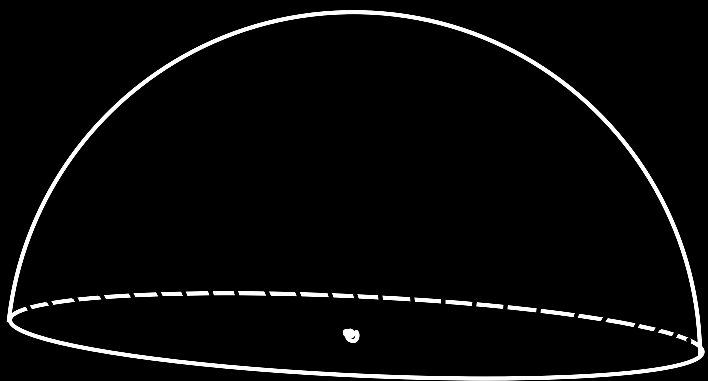
$= 2\pi R^2 \cos\theta \, d\theta$

$dm = (\sigma)(2\pi R^2 \cos\theta \, d\theta)$.

$$y_{CM} = \frac{\int dm\, y}{\int dm} = \frac{\int_0^{90^\circ} (\cancel{\sigma}\, \cancel{2\pi R^2} \cos\theta \, d\theta)(R\sin\theta)}{\int_0^{90^\circ} \cancel{\sigma}\, \cancel{2\pi R^2} \cos\theta \, d\theta} = \frac{R}{2}.$$

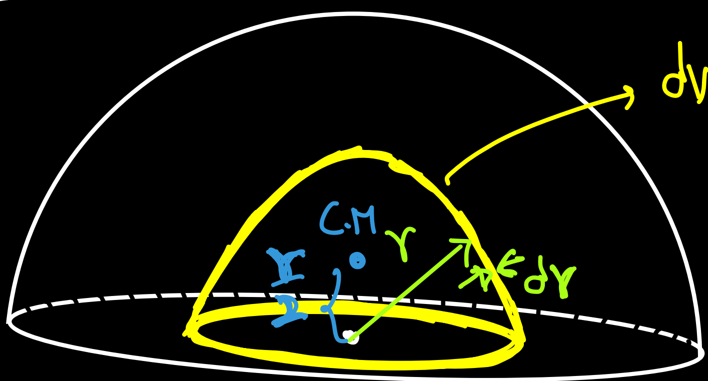


Hemi-Solid sphere:-



find C.M. ?

Sol:-

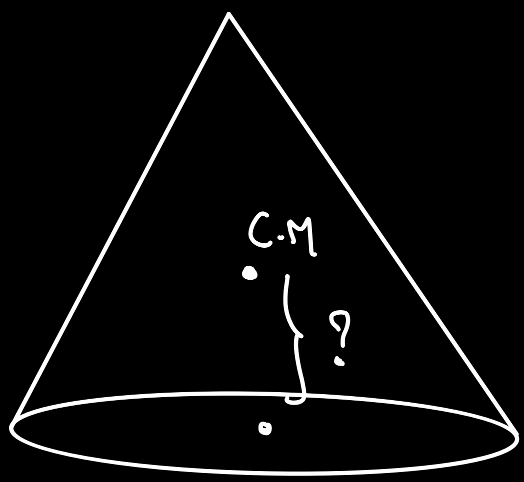


$$dV = (2\pi r^2)dr \quad \Big| \quad dm = (\rho)dV$$
$$= \rho 2\pi r^2 dr.$$

$$y_{CM} = \frac{\int dm\, y}{\int dm} = \frac{\int (\cancel{\rho}\cancel{2\pi} r^2 dr)\frac{r}{2}}{\int \cancel{\rho}\cancel{2\pi} r^2 dr} = \frac{1}{2}\frac{\int_0^R r^3 dr}{\int_0^R r^2 dr}$$

$$y_{CM} = \frac{3R}{8}.$$

HW Solid Cone

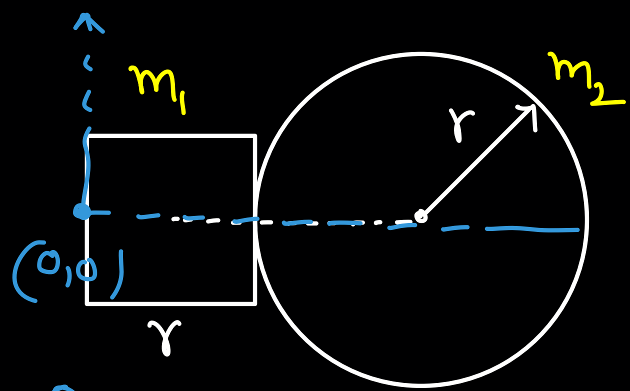


Ans :

$$? = \frac{H}{4}.$$

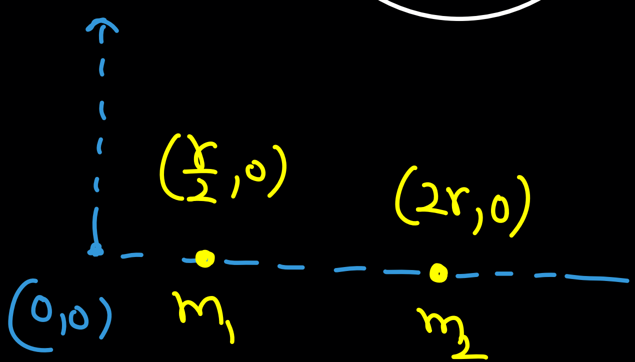
Centre of mass of Composite bodies :-

ill:I :-



$$m_1 = (\sigma) r^2.$$

$$m_2 = (\sigma)\pi r^2.$$



$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

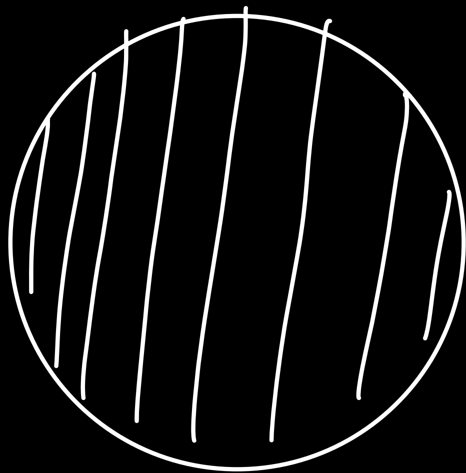
$$= \frac{(\cancel{\sigma} r^2)\left(\frac{r}{2}\right) + (\cancel{\sigma}\pi r^2)(2r)}{\cancel{\sigma} r^2 + \cancel{\sigma}\pi r^2}$$

$$= \frac{\frac{r}{2} + 2\pi r}{1+\pi} = \frac{(1+4\pi)r}{2(1+\pi)}$$

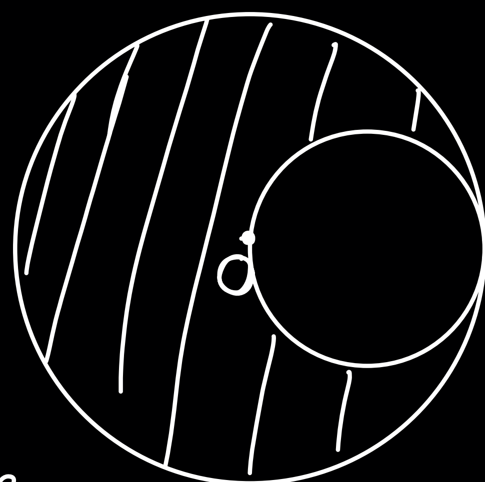
C.M. of bodies with cavity:-

eg:-

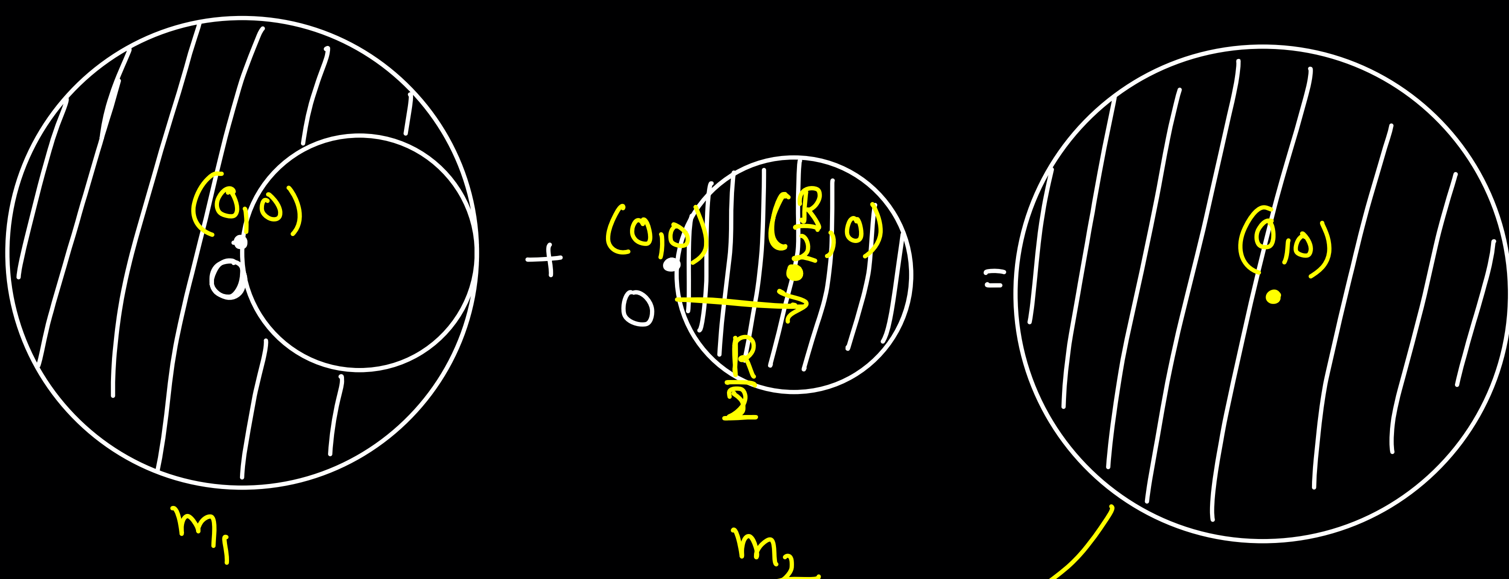
disc



from the disc, a small disc of radius $\frac{R}{2}$ is cut out as shown



find C.M. of body with cavity?



$$\frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = x_{CM} = 0$$

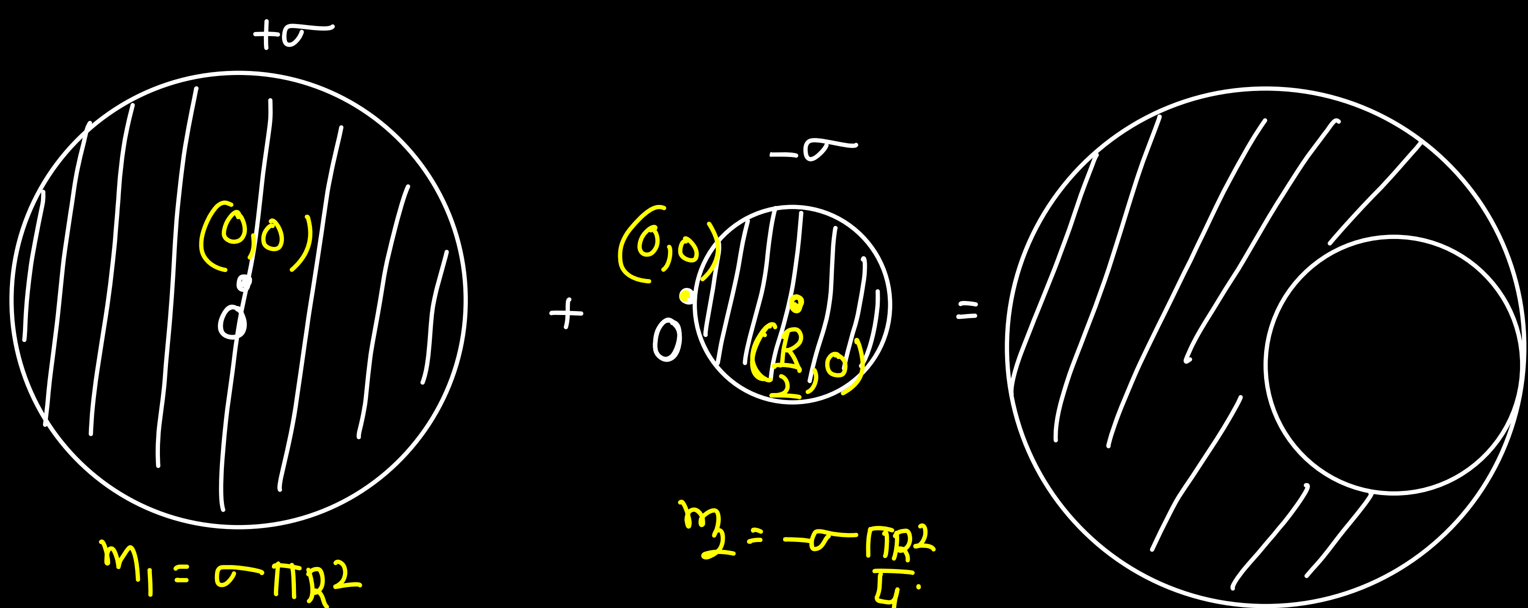
$$m_1 x_1 + m_2 x_2 = 0$$

$$\cancel{\sigma}\left[\pi R^2 - \frac{\pi R^2}{4}\right]x_1 + \left[\cancel{\sigma}\left(\frac{\pi R^2}{4}\right)\right]\frac{R}{2} = 0$$

$$\frac{3\pi R^2}{4}x_1 + \frac{\pi R^3}{8} = 0$$

$$x_1 = -\frac{R}{6}.$$

2nd method :-



$m_1 = \sigma\pi R^2$

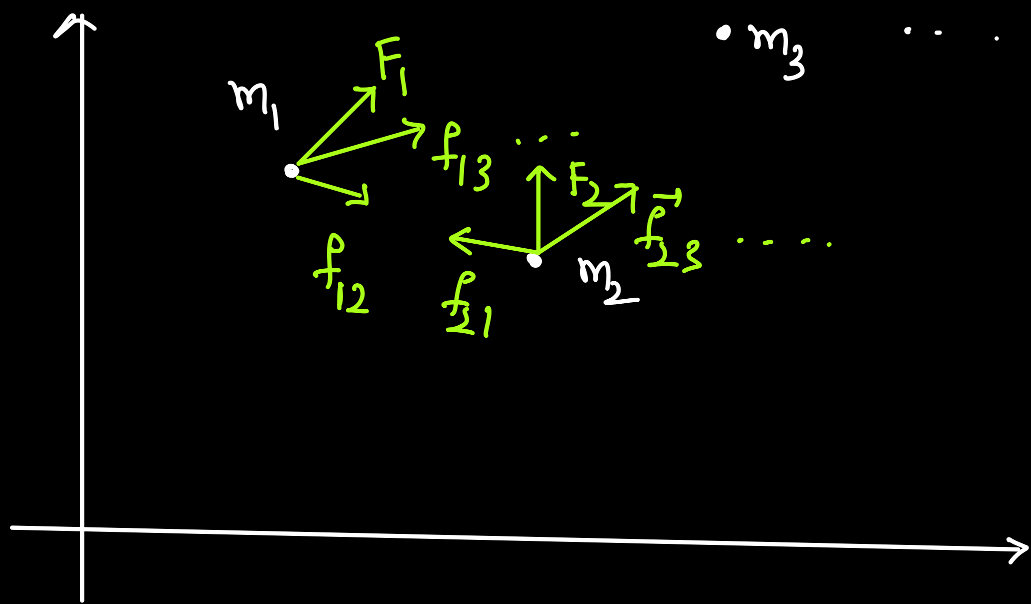
$m_2 = -\sigma\frac{\pi R^2}{4}.$

$$\frac{m_1x_1 + m_2x_2}{m_1+m_2} = x_{CM}.$$

$$\frac{(\cancel{\sigma\pi R^2})0 + \left[-\cancel{\sigma\frac{\pi R^2}{4}}\right]\frac{R}{2}}{\cancel{\sigma\pi R^2} - \cancel{\sigma\frac{\pi R^2}{4}}} = x_{CM}$$

$$\frac{-\frac{R}{8}}{\frac{3}{4}} = x_{CM} \Rightarrow x_{CM} = -\frac{R}{6}.$$

Newton's laws of motion for system of particles :-



$$\vec{a}_{CM} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + \cdots\cdots}{m_1+m_2+\cdots}$$

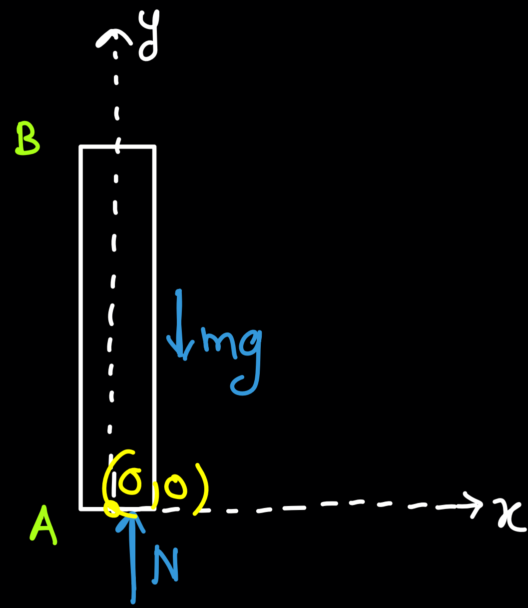
$$= \frac{\left[\vec{F}_1 + \cancel{\vec{f}_{12}} + \cancel{\vec{f}_{13}} + \cdots\right] + \left[\vec{F}_2 + \cancel{\vec{f}_{21}} + \cancel{\vec{f}_{23}} + \cdots\right] + \cdots}{m_1+m_2+\cdots\cdots}$$

$$\vec{a}_{CM} = \frac{\vec{F}_1 + \vec{F}_2 + \cdots\cdots}{m_1+m_2+\cdots} \quad m_1+m_2+\cdots \rightarrow M$$

$$\boxed{\left(\vec{F}_{net}\right)_{ext} = \vec{F}_1 + \vec{F}_2 + \cdots = M\vec{a}_{CM}}$$

BB:2

Q5)



$(\Sigma F)_{net} \rightarrow y$ direction

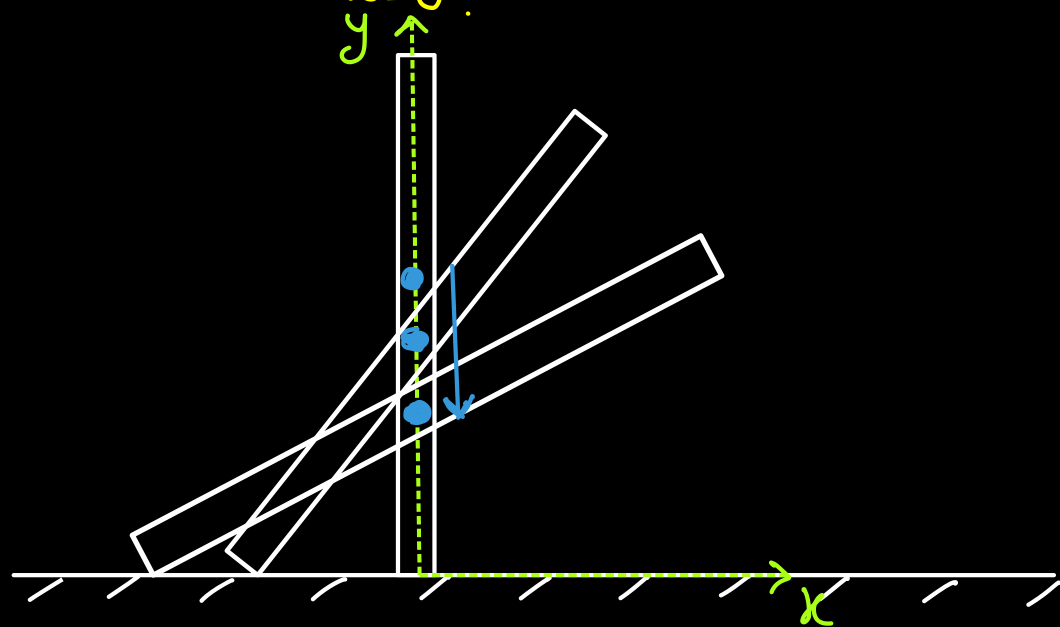
$(\Sigma F_{net})_x = 0$

$(a_{CM})_x = 0$

$(\Delta V_{CM})_x = 0$

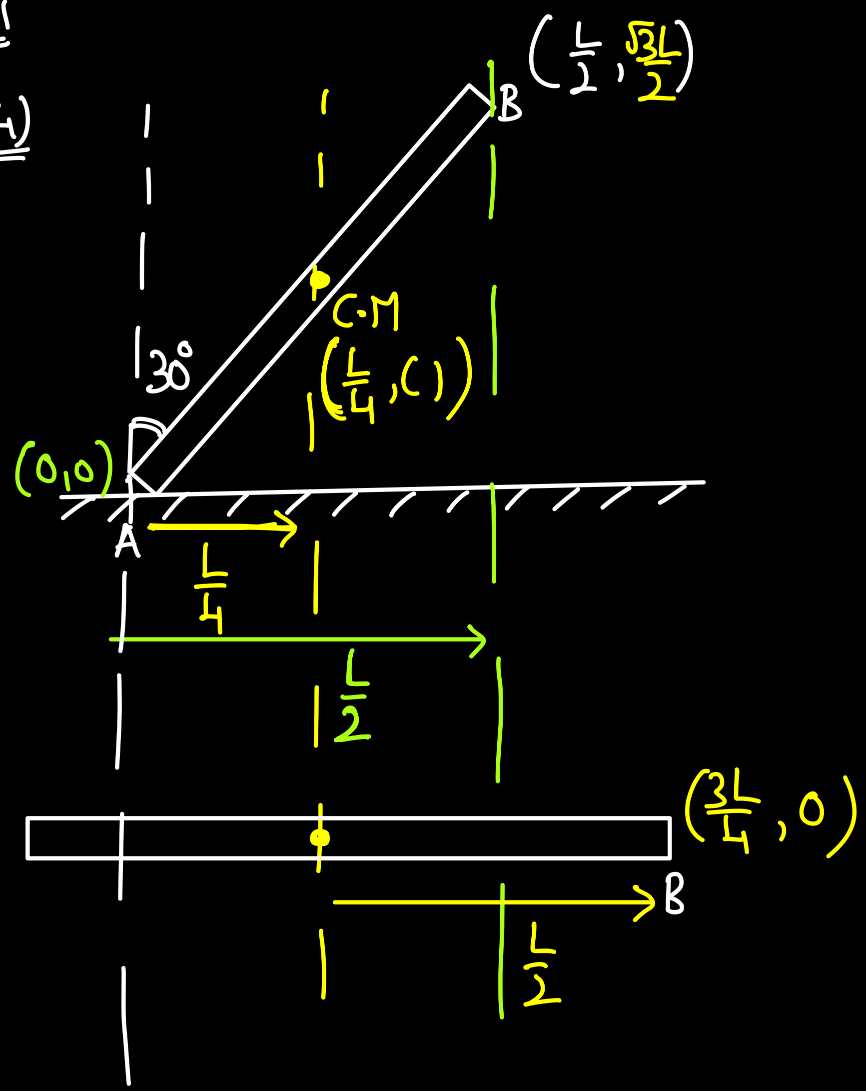
$(V_{CM})_x = (u_{CM})_x = 0.$

$\Rightarrow$ So x coordinate doesn't change i.e. $x = 0$



Ex:1

Q24)



displacement of B

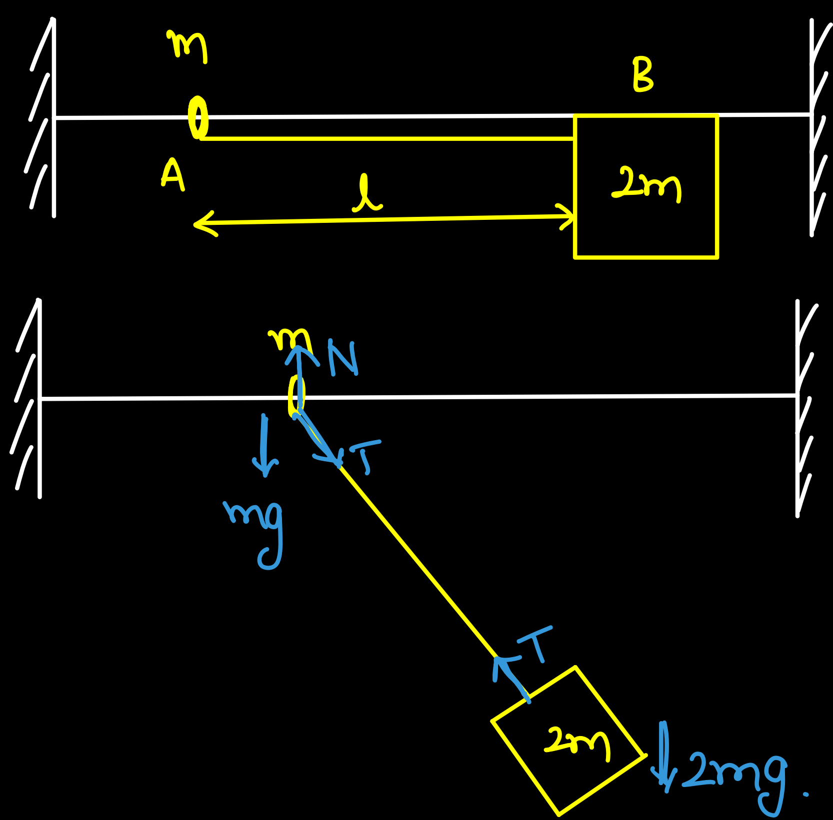
$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{\left(\frac{L}{4}\right)^2 + \left(\frac{-\sqrt{3}L}{2}\right)^2}$$

$$= \frac{\sqrt{13}\,L}{4}.$$

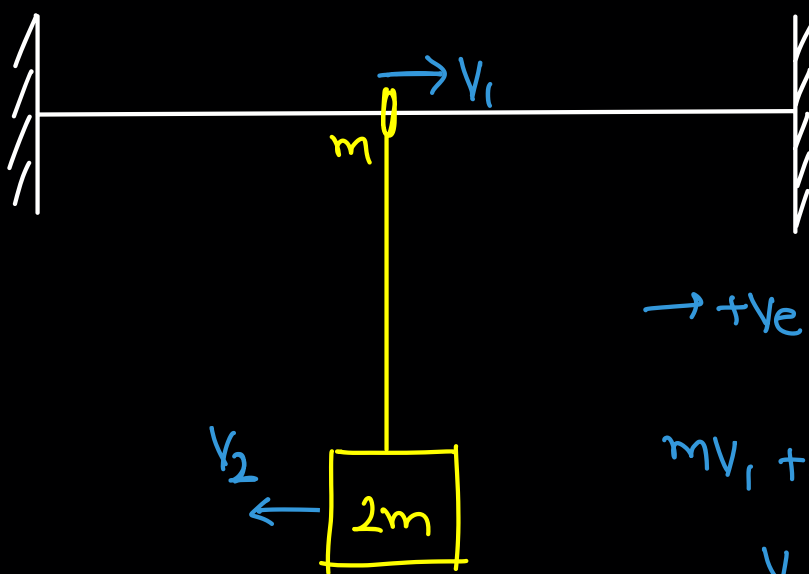
Ex:3

Q11)



Net force is only in vertical direction.

$$(\Sigma F)_x = 0 \Rightarrow (a_{CM})_x = 0 \Rightarrow (V_{CM})_x = (u_{CM})_x$$
$$= 0.$$
$$\left(\frac{m_1 V_1 + m_2 V_2}{m_1 + m_2}\right)_x = 0$$
$$(m_1 V_1 + m_2 V_2)_x = 0.$$



$$mV_1 + 2m(-V_2) = 0$$
$$V_1 = 2V_2 \quad -(1).$$

AS only gravity does work on system

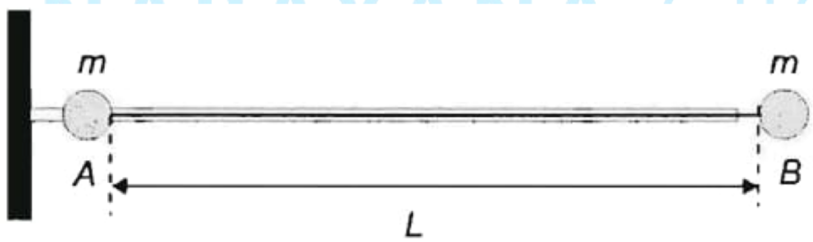
loss in P.E = gain in K.E

$$(2m)gl = \frac{1}{2}mV_1^2 + \frac{1}{2}(2m)V_2^2 \quad -(2)$$

$$2\cancel{m}gl = \frac{1}{2}\cancel{m}V_1^2 + \frac{1}{2}(\cancel{2}\cancel{m})\frac{V_1^2}{\cancel{4}2}$$

$$2gl = \frac{3V_1^2}{4} \Rightarrow V_1 = \sqrt{\frac{8gl}{3}}.$$

An iron ball (A) of mass m can slide without friction on a fixed horizontal rod, which is led through a diametric hole across the ball. There is another ball (B) of the same mass m attached to the first ball by a thin thread of length L. Initially the balls are at rest, the thread is horizontally stretched to its total length and coincides with the rod, as is shown in the figure. Then the ball B is released with zero initial velocity. At the time, when the thread is vertical



A) The force exerted by rod on ball A is 4mg.

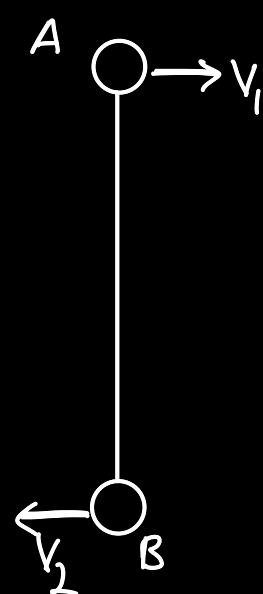
B) Tension in the string is 5mg.

C) Speed of ball B is $\sqrt{2gL}$.

D) Radius of curvature of path of ball B at that moment is $\frac{L}{4}$.

Sol :-

rest



$P_i = 0$

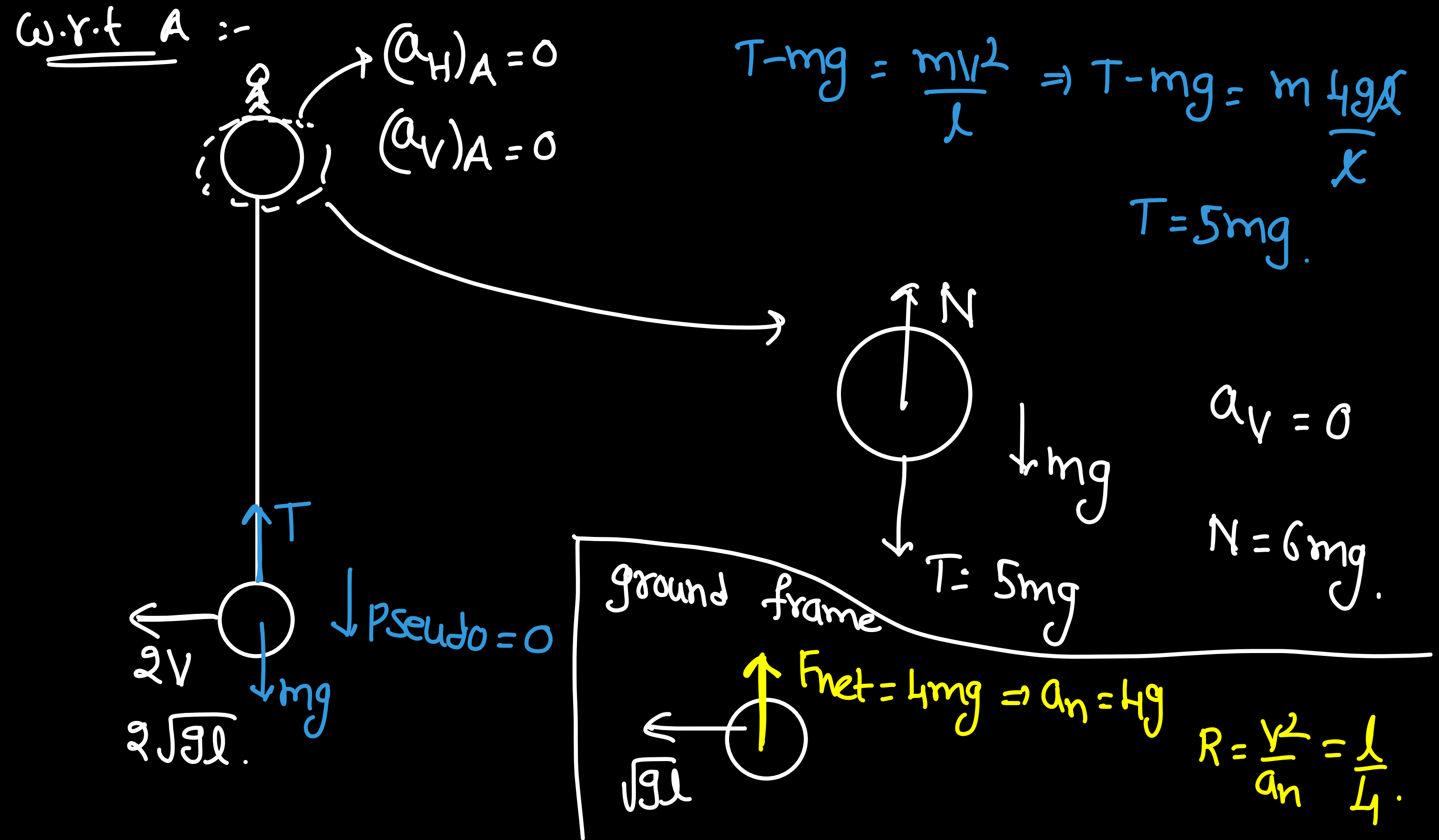
$P_f = mV_1 - mV_2$

$P_i = P_f \Rightarrow V_1 = V_2$ — (1)

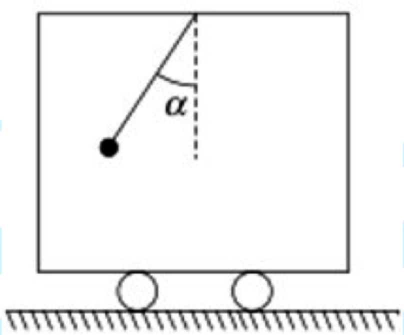
$\Rightarrow$ loss in P.E = gain in K.E

$mgl = \cancel{2}\left[\frac{1}{\cancel{2}} m V_1^2\right]$

$V_1 = V_2 = \sqrt{gl}$.



A wagon of mass M can move without friction along horizontal rails. A simple pendulum consisting of a bob of mass m is suspended from the ceiling by a string of length l. At the initial moment, the wagon and pendulum are at rest and the string is deflected through an angle α from the vertical. The system is released from this position. The velocity of wagon, when the string forms an angle $\beta\,(\beta < \alpha)$ with vertical is v. The velocity of wagon (v) is



Before releasing the bob

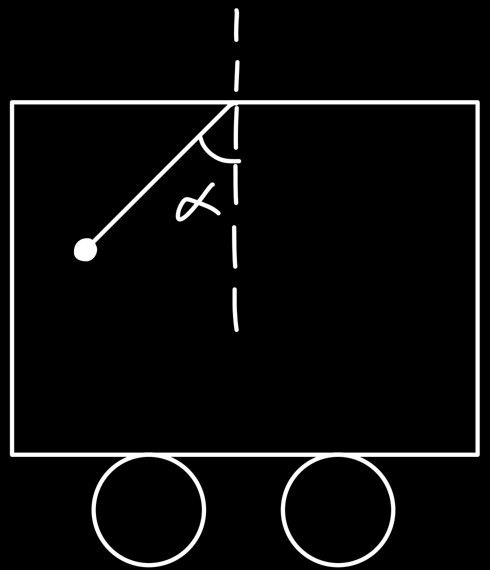
A) $v = \sqrt{\frac{2mMgl}{M+m}\left[\frac{(\cos\beta - \cos\alpha)\cos^2\beta}{M + m\sin^2\beta}\right]}$

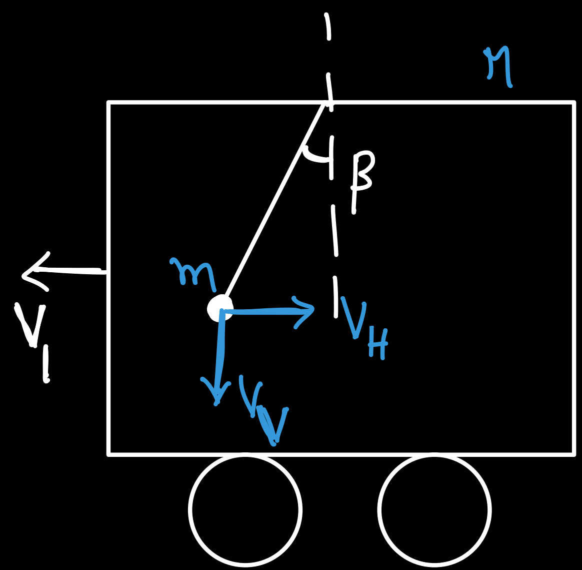
B) $v = \sqrt{\frac{2m^2gl}{M+m}\left[\frac{(\cos\beta - \cos\alpha)\cos^2\beta}{M + m\sin^2\beta}\right]}$

C) $v = \sqrt{\frac{2mMgl}{M+m}\left[\frac{(\cos\beta - \cos\alpha)\cos\beta}{M + m\sin\beta}\right]}$

D) $v = \sqrt{\frac{2m^2gl}{M+m}\left[\frac{(\cos\beta - \cos\alpha)\cos 2\beta}{M + m\sin 2\beta}\right]}$

Sol :-





$\vec{P_i} = 0$

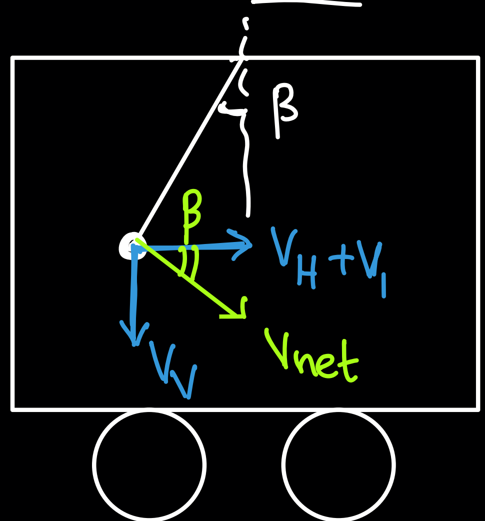
$P_f = mV_H - MV_1$

$$mV_H = MV_1 \quad -①$$

loss in P.E = gain in K.E.

$$mg\,l(\cos\beta - \cos\alpha) = \frac{1}{2}MV_1^2 + \frac{1}{2}m\left(V_H^2 + V_V^2\right) \quad -②.$$

w.r.t wagon :-

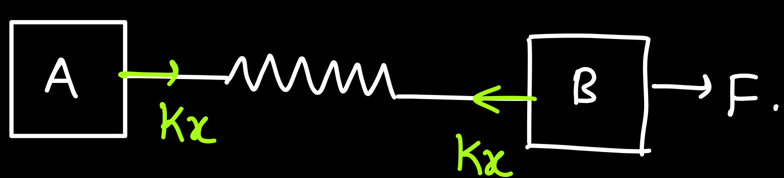


$$\tan\beta = \frac{V_V}{V_H + V_1}$$

$$V_V = (V_H + V_1)\tan\beta \quad -③.$$

ill:17 :-

lets say spring expanded by x.



$$(F_{net})_{ext} = F$$

$$(M)a_{CM} = F \Rightarrow \boxed{a_{CM} = \frac{F}{2m}.}$$

$$u_{CM} = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2} = 0.$$

$$s_{CM} = u_{CM}t + \frac{1}{2}a_{CM}t^2$$

$$s_{CM} = \frac{1}{2}\left(\frac{F}{2m}\right)t^2$$

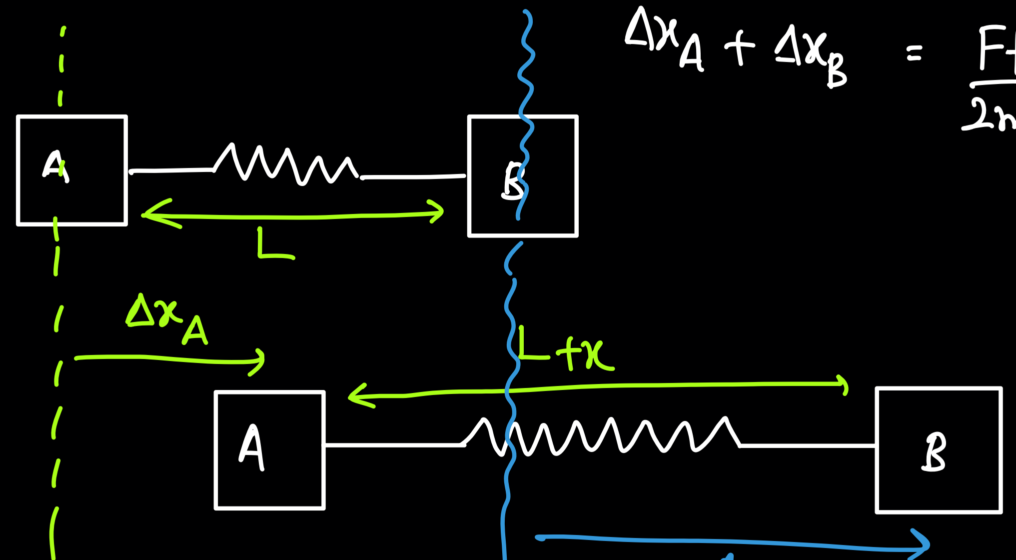
$$\boxed{s_{CM} = \frac{F}{4m}t^2} \Rightarrow \Delta x_{CM} = \frac{F}{4m}t^2.$$

$$\frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2} = \frac{F}{4m}t^2$$

$$\frac{m\Delta x_A + m\Delta x_B}{2m} = \frac{F}{4m}t^2$$

$$\Delta x_A + \Delta x_B = \frac{Ft^2}{2m}. \quad (1)$$

$t=0$



$$x + \Delta x_A = \Delta x_B.$$

$$\Delta x_B - \Delta x_A = x \quad -②$$

$$①+② \Rightarrow 2\Delta x_B = \frac{Ft^2}{2m} + x$$

$$\Delta x_B = \frac{\frac{Ft^2}{2m} + x}{2}.$$

$$\Delta x_A = \frac{\frac{Ft^2}{2m} - x}{2}.$$

Man-plank :-

$$(F_{net})_x = 0.$$

$$(a_{CM})_x = 0$$

$$(\Delta V_{CM})_x = 0 \Rightarrow (V_{CM})_x = (u_{CM})_x$$

$$= 0.$$

$$\Delta x_{CM} = 0$$

$$m\Delta x_{man} + M\Delta x_{plank} = 0$$

$m + M$

$$m\vec{S}_m + M\vec{S}_p = 0 \quad - (1).$$

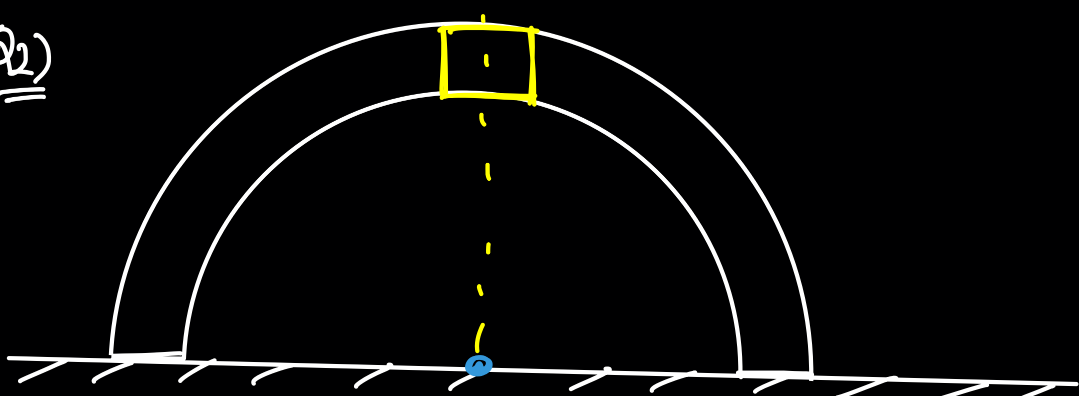
$$\vec{S}_{mp} = \vec{S}_m - \vec{S}_p \quad - (2)$$

$$\boxed{\begin{aligned}\vec{S}_m &= \frac{M\vec{S}_{mp}}{M+m} \\ \vec{S}_p &= \frac{m(-\vec{S}_{mp})}{M+m}\end{aligned}}$$

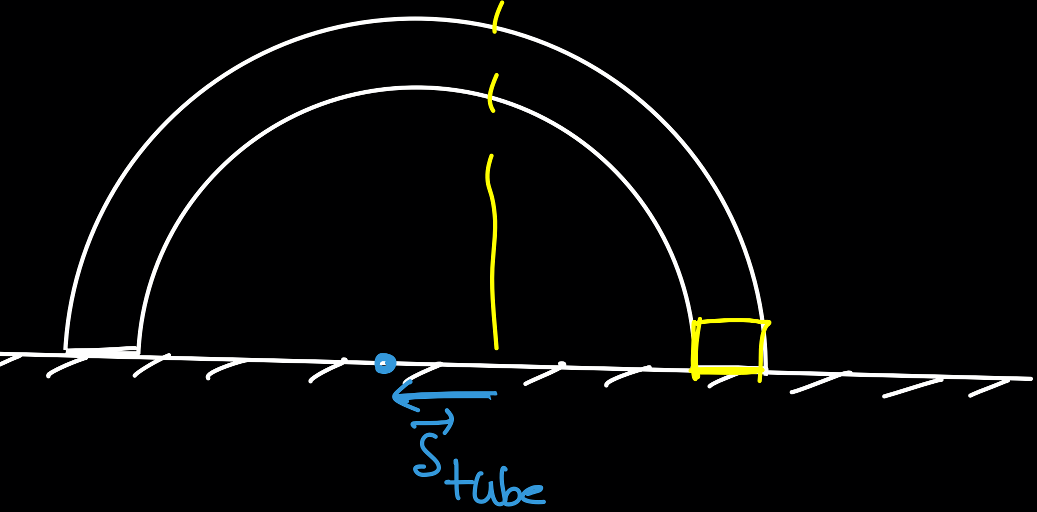
although derived for man-plank situation but valid for all situations when $\vec{F}_{net} = 0$ and $\vec{u}_{CM} = 0$

BB:3

Q2)



$$\Sigma F_x = 0 \Rightarrow (a_{CM})_x = 0 \Rightarrow (u_{CM})_x = 0$$



$$\vec{S}_{tube} = \frac{(m)(-R\hat{i})}{m+m}$$

$$= -\frac{R}{2}\hat{i}.$$

Q4)

$$\vec{S}_{balloon} = \frac{m[-L\hat{j}]}{m+M}$$

$$= -\frac{mL}{m+M}\hat{j} = \frac{mL}{m+M} \text{ downwards}$$

additional info :-

Point of contact of monkey moves and it is equal to disp. of balloon.

What if we have more than one person ?

⇒ make one move at a time and take rest of them as luggage on plank and mass of plank in formula should be put as [mass of plank + luggage].

Ex:5

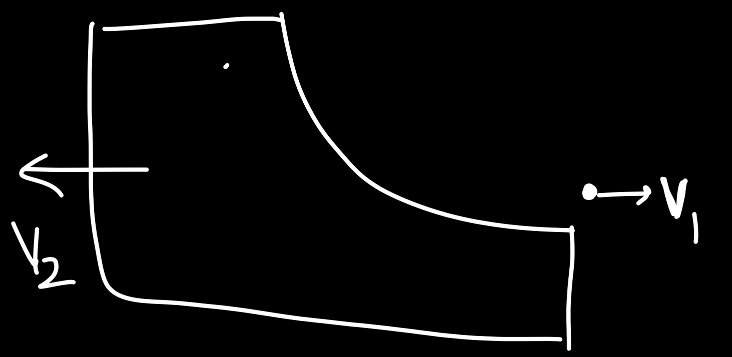
Q14)

$$\vec{S}_{mass} = \frac{M}{M+m}(R\hat{i})$$

$$\Delta x_{mass} = \frac{MR\hat{i}}{M+m}.$$

$$\vec{x}_f - (-R\hat{i}) = \frac{MR\hat{i}}{M+m} \Rightarrow \vec{x}_f = -\frac{mR}{M+m}.$$

$$\vec{S}_{block} = -\frac{m(R\hat{i})}{M+m} = \frac{-mR\hat{i}}{M+m}.$$



$$(\vec{P}_i)_x = (\vec{P}_f)_x$$

$$0+0 = mV_1 - MV_2$$

$$mV_1 = MV_2 \quad -(1)$$

loss in P.E = gain in K.E

$$mgR = \frac{1}{2}mV_1^2 + \frac{1}{2}MV_2^2 \quad -(2).$$

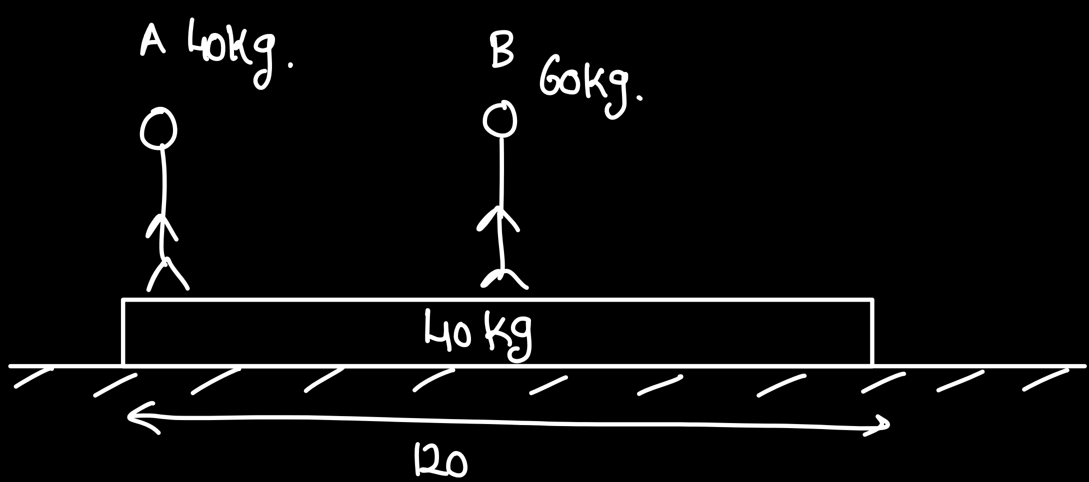
Solve eq -(1) and (2) to get V_1 and V_2.

$$V_1 = \sqrt{\frac{2gR}{1+\frac{m}{M}}}$$

$$V_2 = \frac{m}{M}\sqrt{\frac{2gR}{1+\frac{m}{M}}}.$$

Ex:1

Q18)



$$\vec{S}_A = \frac{(100)\vec{S}_{AP}}{140} \Rightarrow \vec{S}_B = -\frac{(40)\vec{S}_{AP}}{140}$$

$$\vec{S}_B = \frac{(80)\vec{S}_{BP}}{140} \Rightarrow \vec{S}_A = -\frac{60\,\vec{S}_{BP}}{140}$$

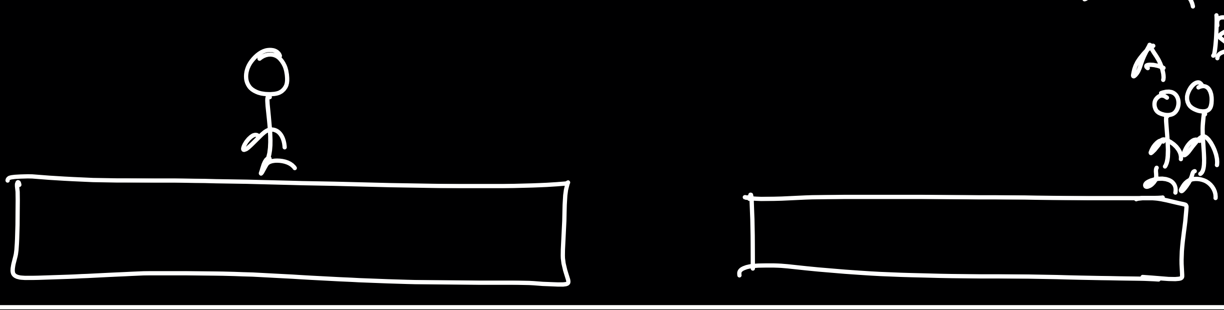
$$-\frac{40\vec{S}_{AP}}{140} + \frac{80\vec{S}_{BP}}{140} = 0 \Rightarrow \boxed{\vec{S}_{AP} = 2\vec{S}_{BP}}$$

$$\frac{100\vec{S}_{AP}}{140} - \frac{60\vec{S}_{BP}}{140} = 60\hat{i}$$

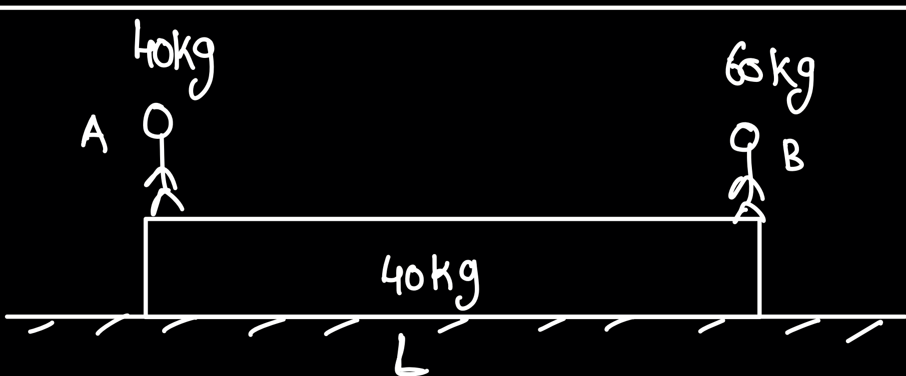
$$\frac{200\vec{S}_{BP}}{140} - \frac{60\vec{S}_{BP}}{140} = 60\hat{i}$$

$$\vec{S}_{BP} = 60\hat{i}$$

w.r.t plank



Q)



find disp. of plank and A if they meet at middle of plank?

Sol :- let A move to centre first

$$\vec{S}_A = \frac{100\left(\frac{L}{2}\hat{i}\right)}{140} \quad \Big| \quad \vec{S}_B = \vec{S}_P = -\frac{40\left(\frac{L}{2}\hat{i}\right)}{140}$$

Now B moves to centre

$$\vec{S}_B = \frac{80\left(-\frac{L}{2}\hat{i}\right)}{140} \quad \Big| \quad \vec{S}_A = \vec{S}_P = \frac{-60\left[-\frac{L}{2}\hat{i}\right)}{140}.$$

$$\left(\vec{S}_A\right)_{total} = \frac{100L}{280}\hat{i} + \frac{60}{280}L\hat{i} = \frac{16}{28}L\hat{i}$$

$$\left(\vec{S}_B\right)_{total} = -\frac{40L}{280}\hat{i} - \frac{80L}{280}\hat{i} = -\frac{12}{28}L\hat{i}$$

$$\left(\vec{S}_P\right)_{total} = \frac{20L\hat{i}}{280}.$$

HW :-

3. A boy of mass 50 kg is standing at one end of a 9.0 m long boat of mass 100 kg that is floating motionless on a calm lake. The man walks to the other end of the boat and stops there. The boat also moves and finally stops due to water resistance. If force of water resistance is proportional to velocity of the boat relative to the water. What is the magnitude of the net displacement s of the boat?

(a) $s = 0.0$ m | (b) $s = 3.0$ m
(c) $s < 3.0$ m | (d) $s > 3.0$ m

Sol :- Initially man-boat system is at rest

$$\vec{P}_i = 0.$$

finally man-boat system is at rest

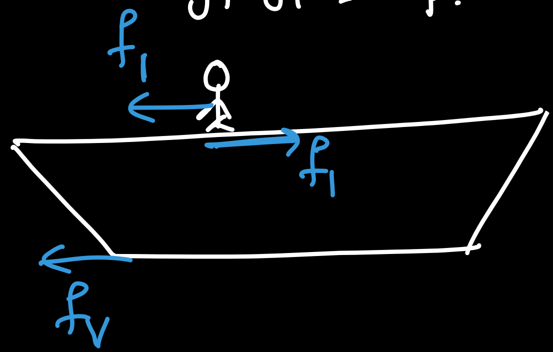
$$\vec{p}_f = 0.$$

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\int \vec{F}\,dt = \Delta\vec{p}.$$

$\vec{f}_v \propto -\vec{v}_b$

$\vec{f}_v = -c\,\vec{v}_b$



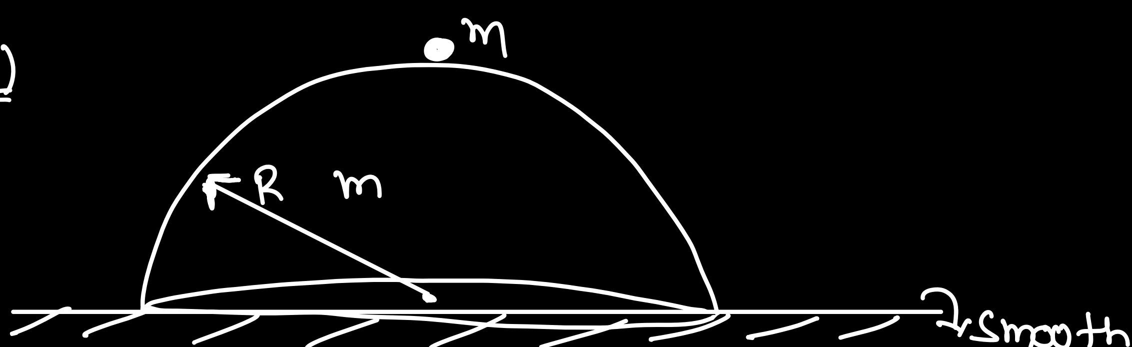
on man-boat system $\vec{F} = \vec{f}_{viscous} = -c\vec{v}_b$

$$\int -c\,\vec{v}_b\,dt = \vec{p}_f - \vec{p}_i$$

$$(-c)\int \vec{v}_b\,dt = 0-0$$

$$\vec{S}_b = 0.$$

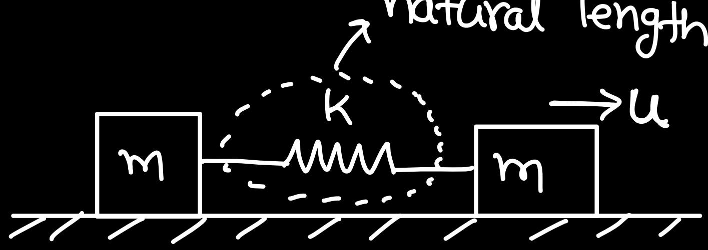
Q)



if particle is given a gentle push
find
i) angle at which it leaves contact
ii) speed of bowl and particle at the above instant?

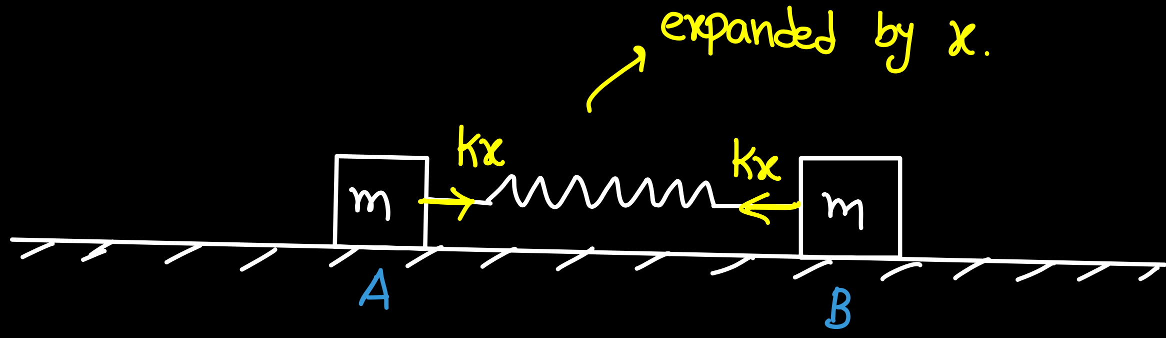
Maximum expansion/compression in springs:-

eg :-



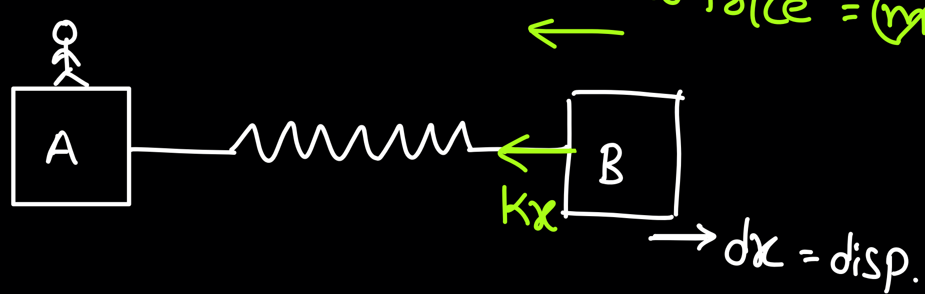
find max. expansion in the spring?

Sol :- at some time 't'



$a_A = \frac{kx}{m}$.

w.r.t A



Pseudo force $= (m)\frac{kx}{m} = kx$.

$W_{spring} + W_{pseudo\ force} = \Delta K.E.$

$$\int kx\,dx + \int -kx\,dx = 0 - \frac{1}{2} m u_{rel}^2$$

$$+\cancel{\frac{1}{2}} kx^2 + \cancel{\frac{1}{2}} kx^2 = +\cancel{\frac{1}{2}} m u^2$$

$$2kx^2 = mu^2$$

$$x = \sqrt{\frac{m}{2k}}\, u.$$

Observation:- at max. expansion / compression

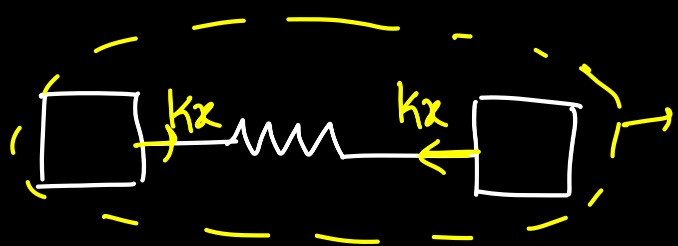
$$V_{BA} = 0.$$

$$\vec{V}_B - \vec{V}_A = 0.$$

$$\vec{V}_B = \vec{V}_A.$$

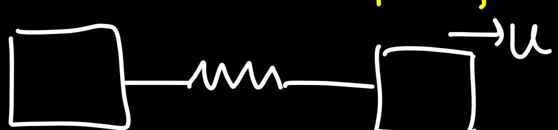
→ both particles should have same velocity along the line joining them.

2nd method:-



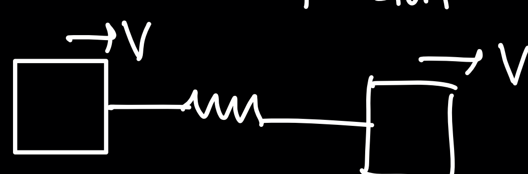
→ $F_{net} = 0.$

$P_i = P_f.$



$\vec{P}_i = mu\,\hat{i}$

at max. expansion



$\vec{P}_f = mv\,\hat{i} + mv\,\hat{i}$

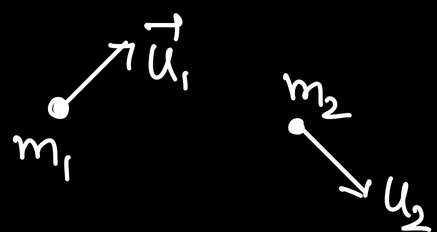
$$mu = 2mV \Rightarrow V = \frac{u}{2} \quad \text{—①}$$

$$T.E_i = T.E_f$$

$$\frac{1}{2}mu^2 + 0 = 2\left[\frac{1}{2}mV^2\right] + \frac{1}{2}kx^2$$

$$x = \sqrt{\frac{m}{2k}}\,u.$$

3rd approach:- for two particle system.



ground frame:

$$K.E = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2.$$

$$K.E = \frac{1}{2}(m_1+m_2)V_{CM}^2 + \frac{1}{2}\underbrace{\left(\frac{m_1m_2}{m_1+m_2}\right)}_{\mu = \text{reduced mass.}}\overbrace{(\vec{u}_1-\vec{u}_2)}^{\vec{u}_{rel}}{}^2.$$

$$K.E = \frac{1}{2}(m_1+m_2)V_{CM}^2 + \frac{1}{2}\mu(\vec{u}_{rel})^2$$

if $F_{ext} = 0 \Rightarrow a_{CM} = 0 \Rightarrow V_{CM} = \text{constant}.$

$$K.E = \text{Constant} + \text{Variable}$$

in general

$$\boxed{K.E = \frac{1}{2}(m_1+m_2)V_{CM}^2 + \frac{1}{2}\mu(\vec{V}_{rel})^2}$$

in the above question

$$K.E_i = \frac{1}{2}(m_1+m_2)u_{CM}^2 + \frac{1}{2}\mu u^2$$

at max. expansion

$$\vec{V}_{rel} = 0$$

$$K.E_f = \frac{1}{2}(m_1+m_2)V_{CM}^2$$

as $F_{ext} = 0$

$$V_{CM} = u_{CM}.$$

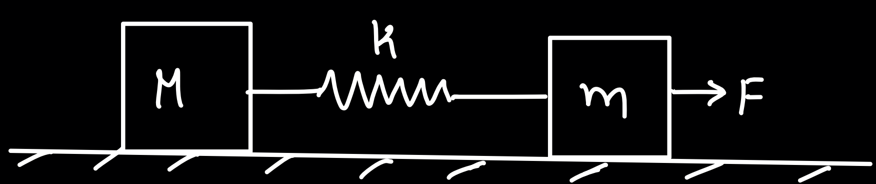
$$\text{loss in } K.E = \frac{1}{2}\mu u^2$$

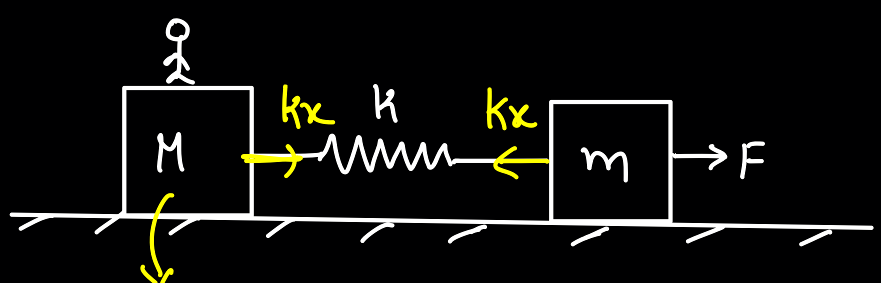
$$\text{gain in } P.E = \frac{1}{2}\mu u^2$$

$$\frac{1}{2}kx^2 = \frac{1}{2}\frac{(m)(m)}{m+m}u^2$$

$$x = \sqrt{\frac{m}{2k}}\,u.$$

ill: 33 :-

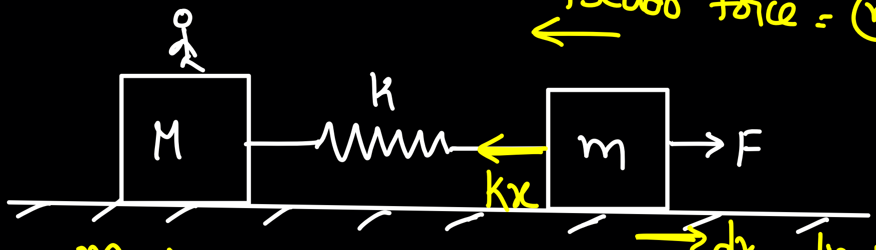




$a = \frac{Kx}{M}$.

w.r.t M :-

Pseudo force $= (m)\left(\frac{Kx}{M}\right) = \frac{m}{M}Kx$



dx = displacement.

m :-

Wall forces = $\Delta K.E$

$$\int Kx\,dx - \int \frac{m}{M}Kx\,dx + \int F\,dx = 0 - 0.$$

$$-\frac{1}{2}Kx^{\cancel{2}} - \frac{1}{2}Kx^{\cancel{2}}\frac{m}{M} + F\cancel{x} = 0$$

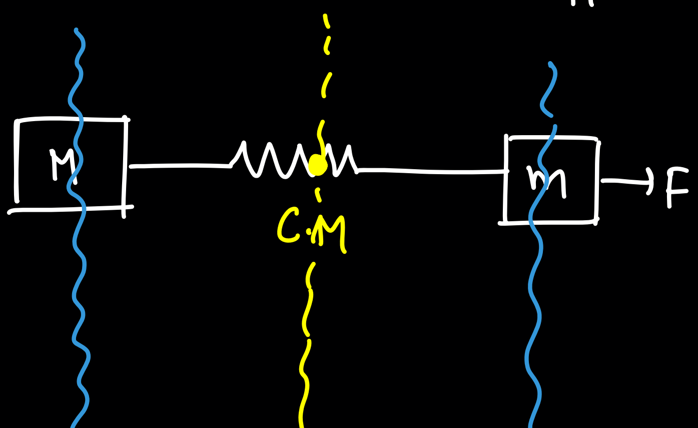
$$\frac{Kx}{2}\left[1 + \frac{m}{M}\right] = F$$

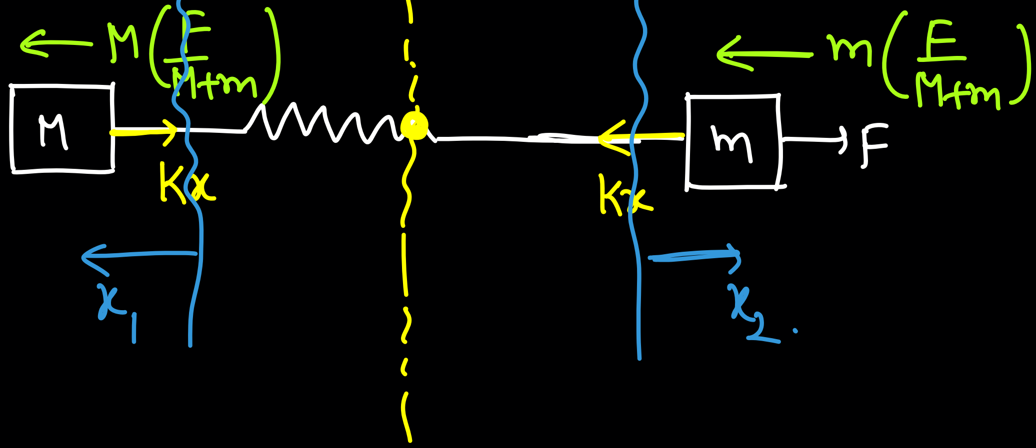
$$x = \frac{2MF}{(M+m)K}.$$

2nd approach:-

$$a_{CM} = \frac{F}{M+m}.$$

w.r.t C.M $\Rightarrow$ $\Delta x_{CM} = 0$.





$$\Delta x_{CM} = 0.$$

$$\frac{-Mx_1 + mx_2}{M+m} = 0 \quad \Rightarrow \quad Mx_1 = mx_2.$$

$$W_{\text{pseudo force on system}} = \frac{MF}{M+m}x_1 - \frac{mF}{M+m}x_2$$

$$= \frac{F}{M+m}(Mx_1 - mx_2)$$

$$= 0.$$

in centroidal frame, pseudo force due to acc. of C.M does no work.

at max. expansion $\vec{V}_{rel} = 0$ in ground frame

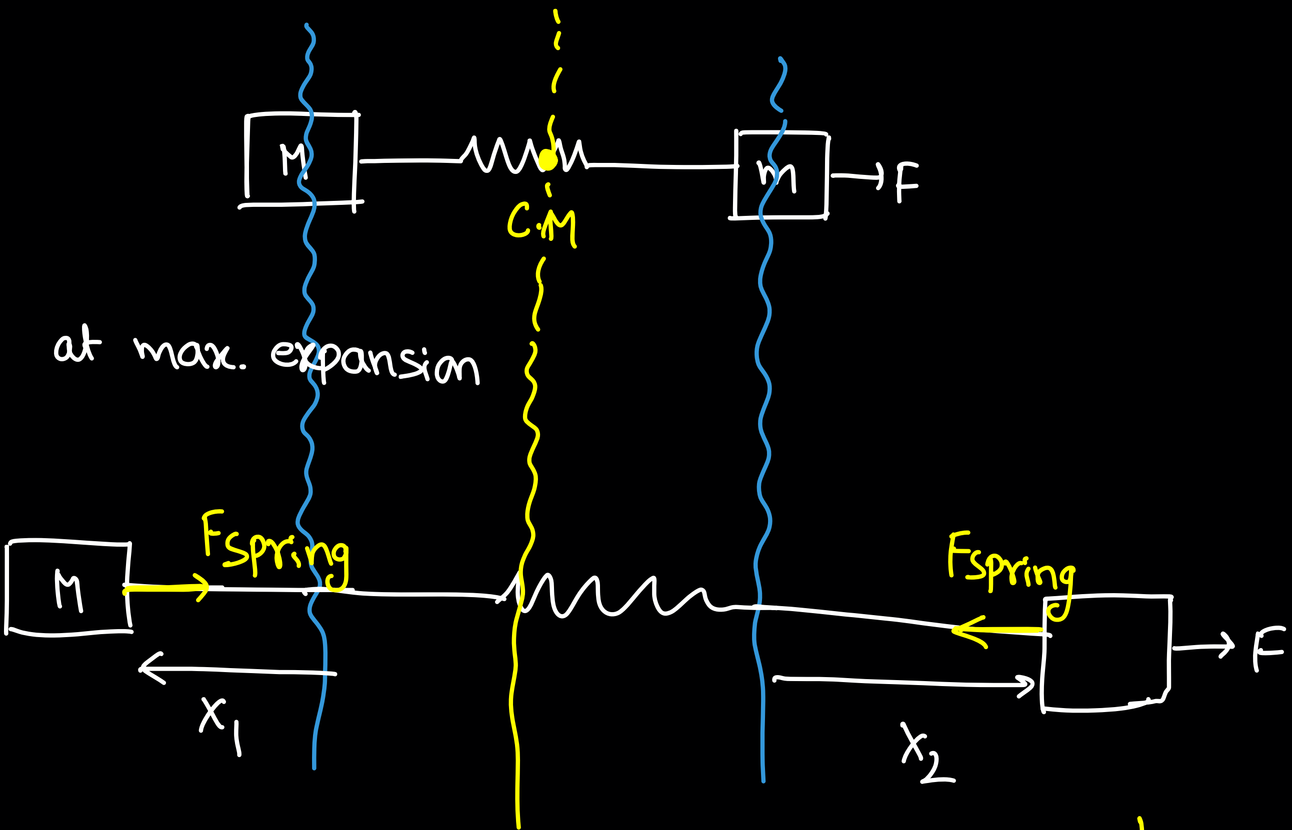
let $\vec{V}_1 = \vec{V}_2 = V$

$$V_{CM} = \frac{MV + mV}{M+m} = V.$$

at max. expansion w.r.t C.M.

$$\vec{V}_{1CM} = 0$$

$$\vec{V}_{2CM} = 0.$$



w.r.t C.M

$$W_{spring} + W_F = \Delta K.E.$$

$$\frac{1}{2}kx^2 + Fx_2 = 0.$$

$$\frac{1}{2}kx^{\cancel{2}} + F\left[\frac{M\cancel{x}}{M+m}\right] = 0$$

$$x = \frac{2FM}{(M+m)K}.$$

$$x_1 + x_2 = x.$$

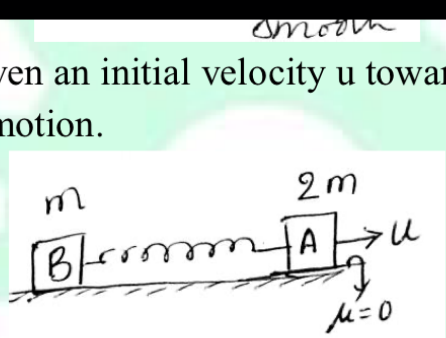
$$Mx_1 = mx_2$$

$$x_1 = \frac{mx_2}{M}$$

$$x_2 = \frac{Mx}{M+m}.$$

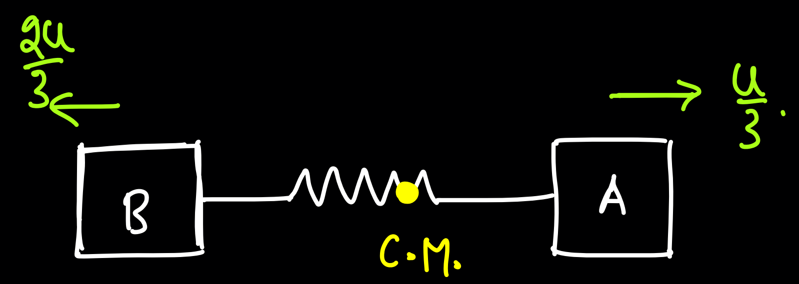
HW :-

In the system shown, A is given an initial velocity u towards right. Find their range of velocities in the subsequent motion.



Sol :- $V_{CM} = \frac{2mu}{3m} = \frac{2u}{3}$.

w.r.t C.M

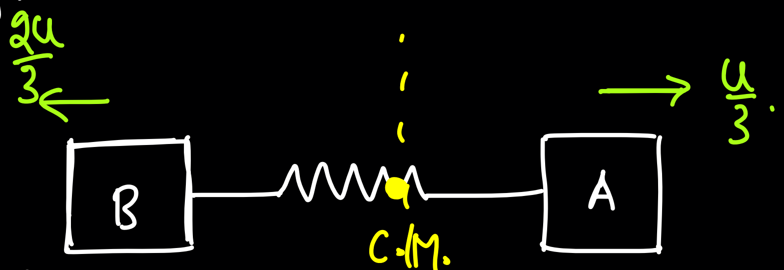


$(K.E_{CM})_i = \frac{1}{2}\left(\frac{2m}{3}\right)u^2$

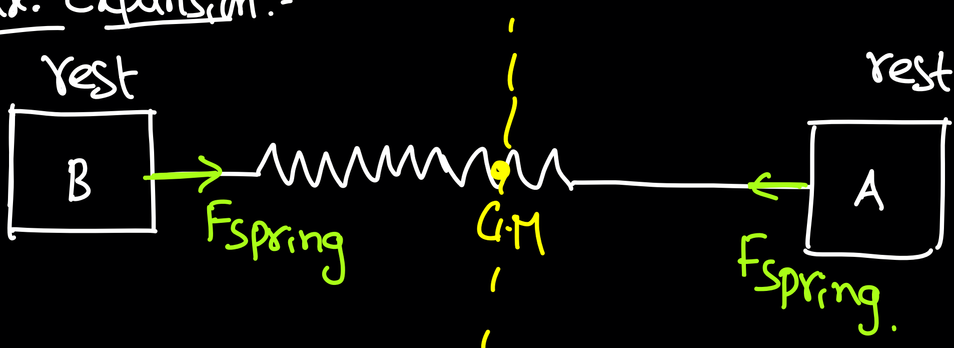
$= \frac{mu^2}{3}$.

at max. expansion entire $\frac{mu^2}{3}$ gets converted to elastic P.E.

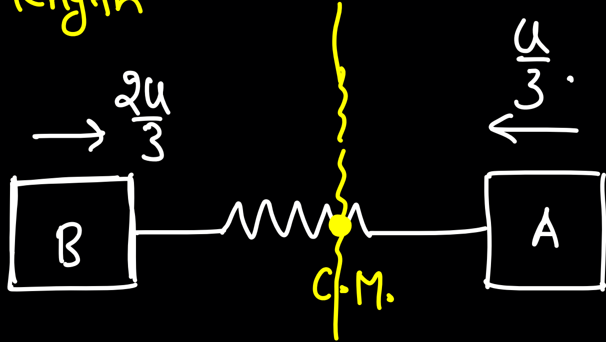
initially :-



Max. expansion :-



natural length



after this compression starts and speed of B as well as A start decreasing.

$$\vec{V}_{ACM} = \vec{V}_A - \vec{V}_{C.M} \Rightarrow \vec{V}_A = \vec{V}_{AC.M} + \vec{V}_{CM}.$$

$\longrightarrow +ve$

max $= |\vec{V}_A| = V_A = \frac{u}{3} + \frac{2u}{3} = u.$

min $= -\frac{u}{3} + \frac{2u}{3} = \frac{u}{3}.$

A will never come to rest in ground frame.

if min somehow came out −ve then −ve indicates that particle in b/w has come to rest then min. speed is "0".

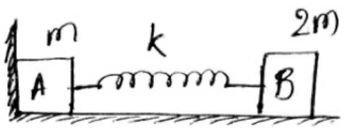
Same way $\rightarrow \vec{V}_B = \vec{V}_{BC.M} + \vec{V}_{C.M.}$

min $V_B = \frac{2u}{3} - \frac{2u}{3} = 0$

max $V_B = \frac{2u}{3} + \frac{2u}{3} = \frac{4u}{3}$

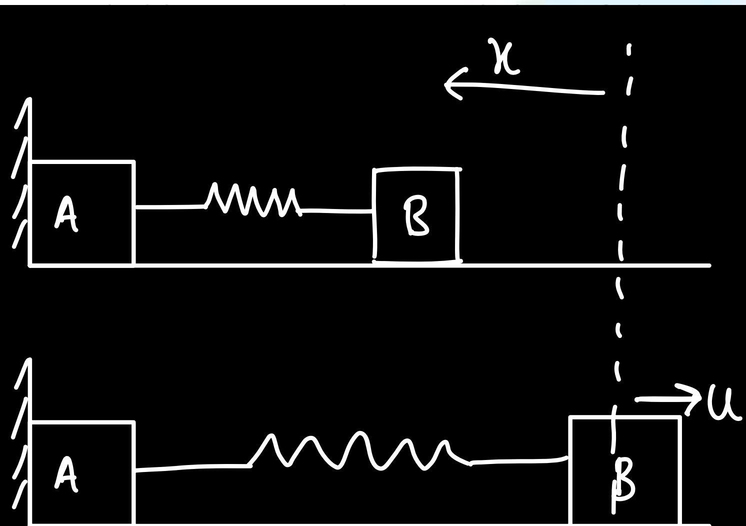
range of speeds $0 - \frac{4u}{3}$.

In the figure shown, the blocks are initially at rest and the spring is at natural length. B is pressed towards wall by a distance 'x' and then released. (t=0). If friction is neglected, find



(i) maximum speed acquired by A
(ii) the time at which A acquires its maximum speed

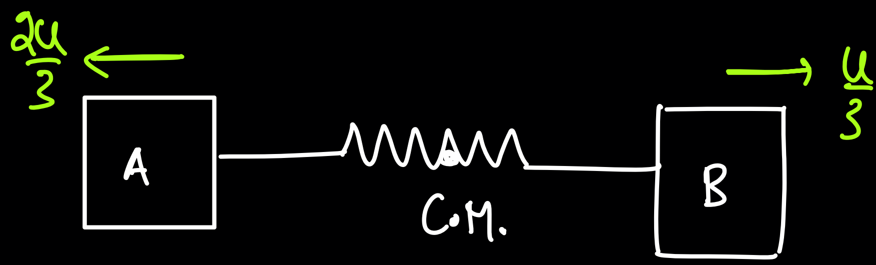
S :-



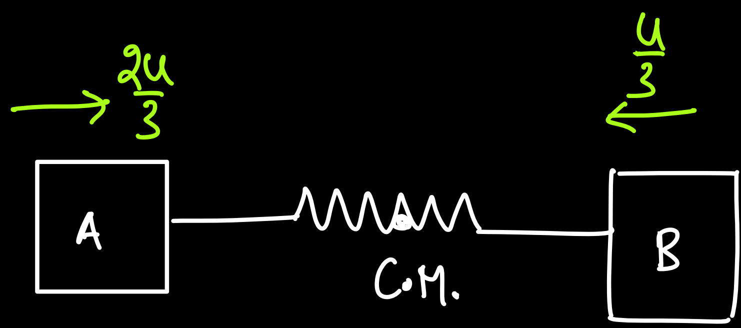
$\frac{1}{2}(2m)u^2 = \frac{1}{2}kx^2$

$u = \sqrt{\frac{k}{2m}}\, x.$

$V_{CM} = \frac{(2m)u}{3m} = \frac{2u}{3}.$



again attains natural length

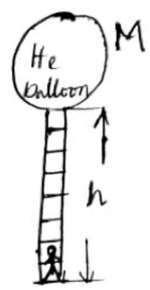


$$\vec{V}_A = \vec{V}_{A\,C.M} + \vec{V}_{CM}$$

$$= \frac{2u}{3} + \frac{2u}{3}.$$

$$= \frac{4u}{3}.$$

Mass of balloon + ladder system is M and that of the man is ‘m’. Initially, the system is floating in air at rest. Find the work done by internal forces of the man in raising himself up the ladder.



Sol :- $W_{\text{internal forces}} = (\Delta P.E)_{\text{system}}.$

$$\vec{S}_B = \frac{(m)(-h\hat{j})}{M+m}.$$

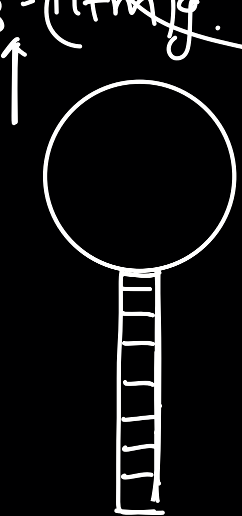
$$\vec{S}_m = \frac{(M)h\hat{j}}{M+m}.$$

$$\Delta P.E = Mg\left(-\frac{mh}{M+m}\right) + mg\left(\frac{Mh}{M+m}\right)$$

$$= 0.$$

this only work by gravity but F_B also does work.

$F_B = (M+m)g.$



so $-(F_B)\left(\frac{mh}{M+m}\right).$

Wall forces = $\Delta K.E = 0$

$$W_{F_B} + W_{internal} + \underbrace{W_{gravity}}_{\to 0} = 0$$

$$W_{internal} = -W_{F_B} = mgh.$$

<u>Impulse of a force</u> ($\vec{I}$):-

It is equal to change in momentum caused by the force.

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\int_{\vec{p}_i}^{\vec{p}_f} d\vec{p} = \int \vec{F}\, dt$$

$$\boxed{\vec{I} = \vec{p}_f - \vec{p}_i = \int \vec{F}\, dt}$$

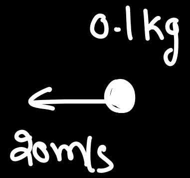
area under $\vec{F}, t$ graph.

⇒ **Impulsive force** :- A force which changes momentum of mass by a finite value in 'dt' interval of time.

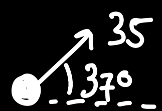
eg :- force on ball when bat hits it.

* gravity is not an impulsive force

111:23 :-



$$\vec{P}_i = (0.1)(-20\hat{i}) = -2\hat{i}$$



$$\vec{V} = 35\cos 37\,\hat{i} + 35\sin 37\,\hat{j}$$

$$= 28\hat{i} + 21\hat{j}$$

$$\vec{P}_f = 2.8\hat{i} + 2.1\hat{j}$$

$$\vec{I}_{Total} = \vec{P}_f - \vec{P}_i$$

$$= 4.8\hat{i} + 2.1\hat{j}.$$

Case (i)

$$\vec{I}_{gravity} + \vec{I}_{bat} = 4.8\hat{i} + 2.1\hat{j}$$

$$(-\hat{j})\Delta t + \vec{I}_{bat} = 4.8\hat{i} + 2.1\hat{j}$$

$$\vec{I}_{bat} = 4.8\hat{i} + 2.4\hat{j}$$

case (iii)

$$(-\hat{j})(0.003) + \vec{I}_{bat} = 4.8\hat{i} + 2.1\hat{j}$$

$$\vec{I}_{bat} = 4.8\hat{i} + 2.103\hat{j}$$

$$\boxed{\vec{I}_{bat} \approx \vec{I}_{Total}}$$

when Δt is very small impulse by gravity $\to 0$

⇒ we call gravity as non-impulsive force.

⇒ Forces which change momentum in very short span of time are called impulsive forces.

* so whenever we deal with Just before and Just after cases we can conserve momentum even though we have ext. forces but they should not be impulsive in nature.

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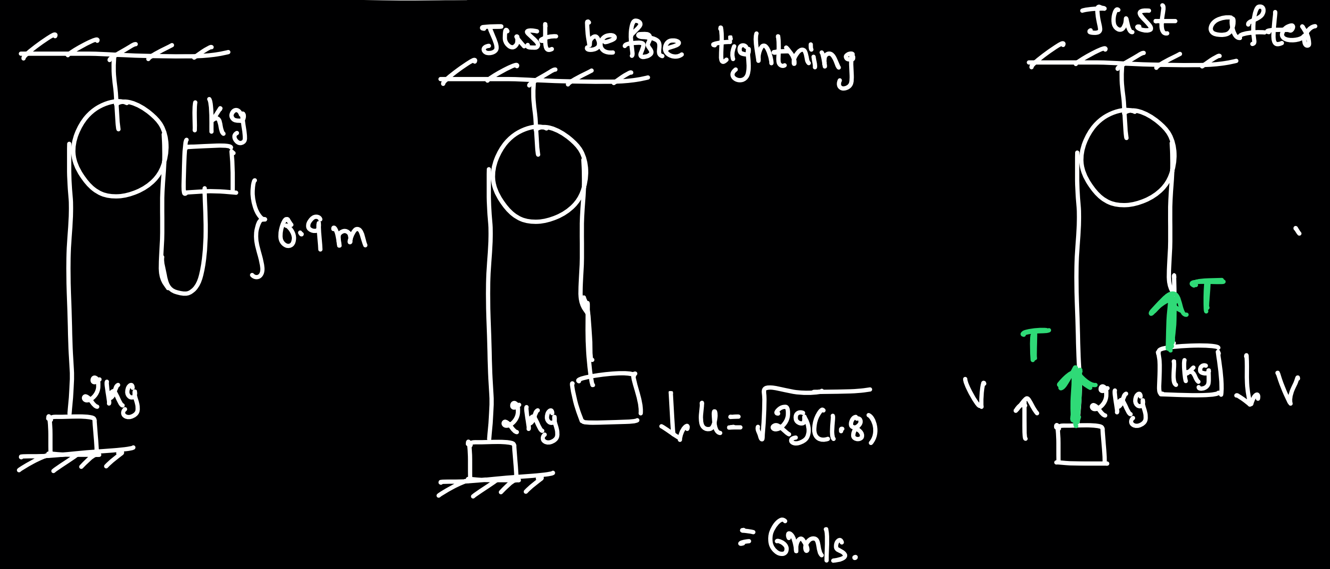
some worked out illustrations :-

ill: 22



1 kg :-

$\vec{I} = \Delta\vec{p}$

$\int(+T\hat{j})dt = (1)(-v\hat{j}) - (1)(-6\hat{j})$

2 kg

$\int(+T\hat{j})dt = 2(v\hat{j}) - 2(0)$

$-v + 6 = 2v \Rightarrow v = 2\ m/s.$

2nd approach:-

if we move along the length of string Net tension force comes out to be "0".

so momentum along the string can be conserved.

Just before taut

$\vec{P_i} = (1)(6)$

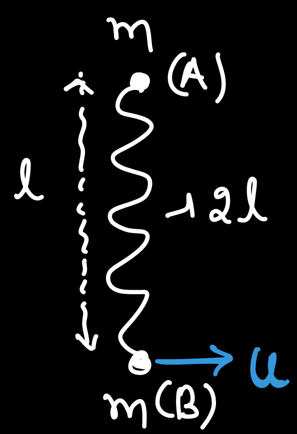
Just after taut

$\vec{P_f} = (2)v + 1v.$

$6 = 3v$

$v = 2\ m/s.$

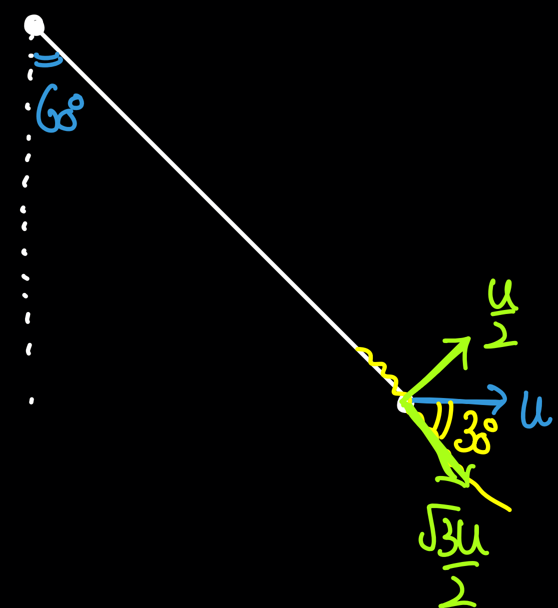
Q)



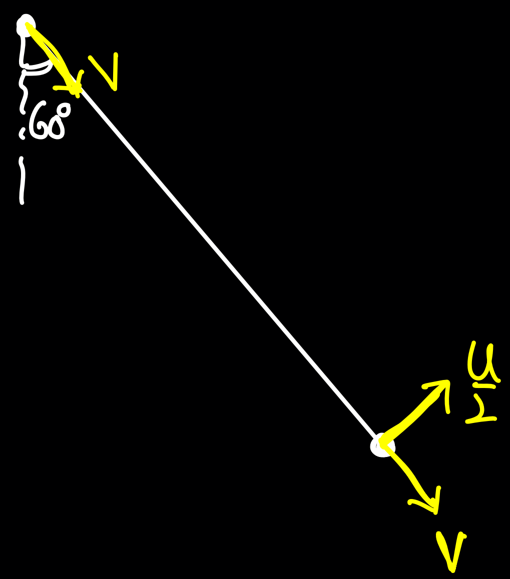
whole setup is on a horizontal frictionless table. find speed of A, B just after string gets taut?

Sol :-

Just before

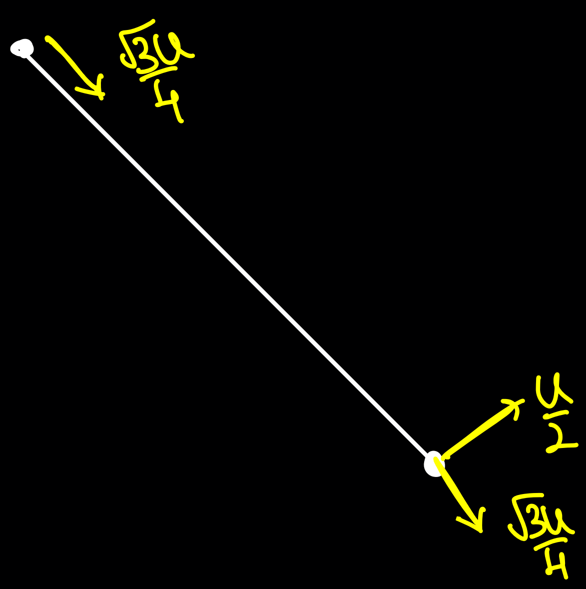


Just after



Conserve momentum along length

$$m\frac{\sqrt{3}u}{2} = mv + mv \Rightarrow v = \frac{\sqrt{3}u}{4}.$$

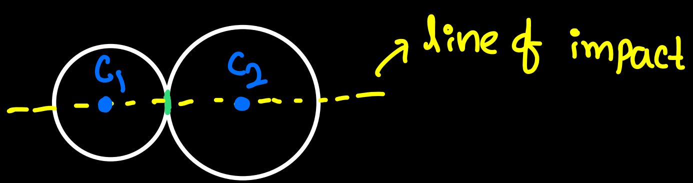


$$\Rightarrow V = \sqrt{\frac{u^2}{4} + \frac{3u^2}{16}} = \frac{\sqrt{7}u}{4}.$$

Collisions :-

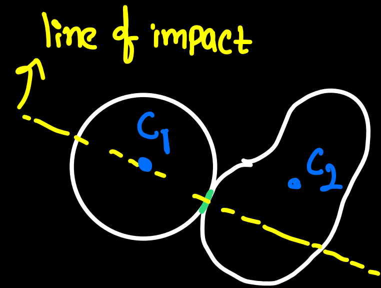
⇒ Types of impacts :-

Central impact



⇒ No rotational energy is generated after collision

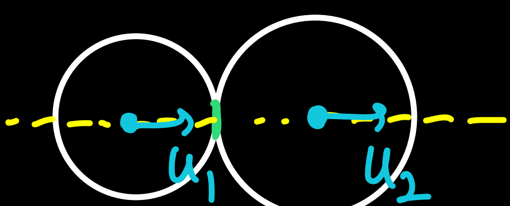
eccentric impact



⇒ rotational energy gets generated after collision

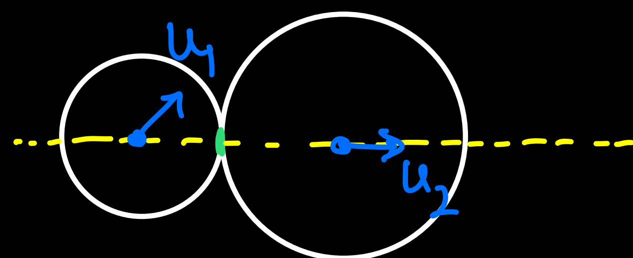
Central impact

Head-on collision



⇒ if vel. of both bodies are along line of impact then its called head on collision.

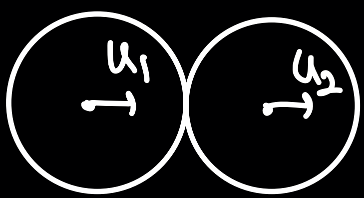
oblique collision

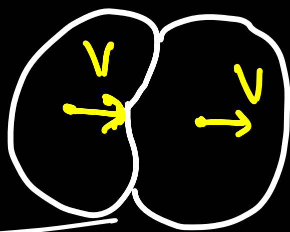


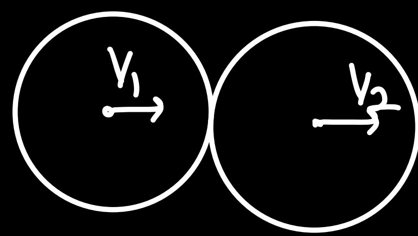
⇒ if vel. of any one & both are not along the line of impact then its called oblique collision.

Head on collision :-

$u_1 > u_2$



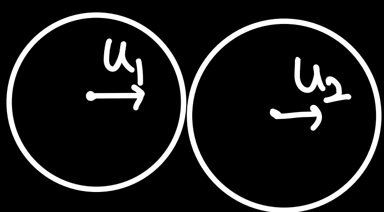


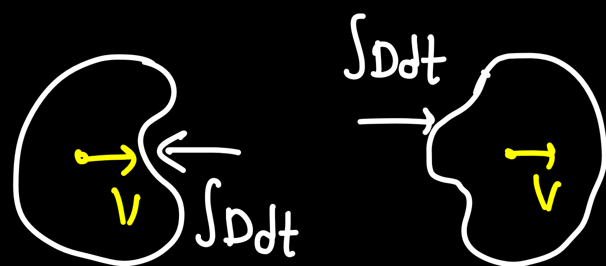


Deformation stage

restitution stage.

Deformation stage :-





m_1

$\rightarrow$ +ve

$$-\int D\,dt = m_1 V - m_1 u_1 \quad -(1)$$

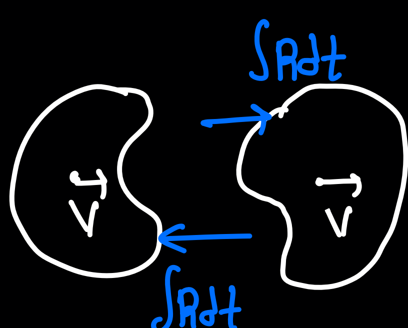
m_2

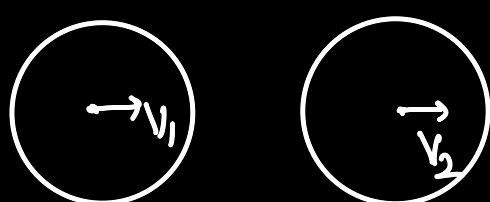
$$\int D\,dt = m_2 V - m_2 u_2 \quad -(2)$$

from (1)+(2)

$$m_1 u_1 + m_2 u_2 = m_1 V + m_2 V$$

Restituition stage:-





$\underline{\underline{m_1}}$

$$-\int R\,dt = m_1 v_1 - m_1 v \quad \text{--- (3)}$$

$\underline{\underline{m_2}}$

$$\int R\,dt = m_2 v_2 - m_2 v \quad \text{--- (4)}$$

(3) + (4)

$$m_2 v + m_1 v = m_1 v_1 + m_2 v_2$$

$$\boxed{m_1 u_1 + m_2 u_2 = m_1 v + m_2 v = m_1 v_1 + m_2 v_2}$$

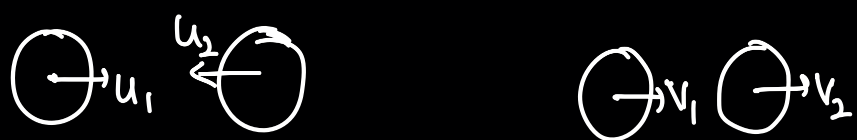
<u>Coefficient</u> of <u>restitution</u> (e) :-

$$e = \frac{\int R\,dt}{\int D\,dt} = \frac{\text{impulse of restitution}}{\text{impulse of deformation}}.$$

using (1), (2), (3), (4)

$$\boxed{e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{\text{vel. of seperation}}{\text{vel. of approach}}}$$

eg:-

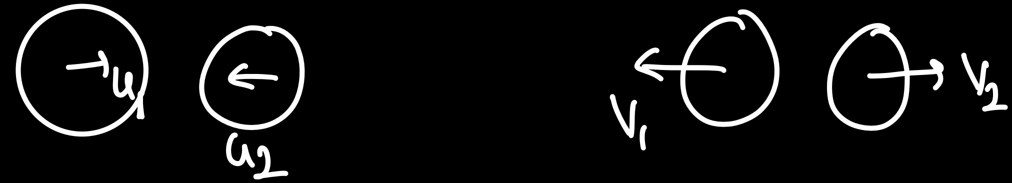


vel. of seperation $= +v_2 - v_1$

vel. of approach $= +u_1 + u_2$

$$e = \frac{v_2 - v_1}{u_1 + u_2}.$$

eg

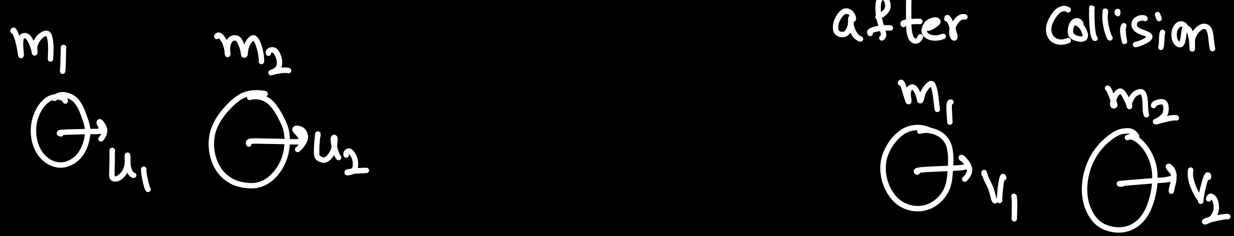


vel. of seperation $= +v_2 + v_1$

vel. of approach $= +u_1 + u_2$.

$$e = \frac{v_1 + v_2}{u_1 + u_2}.$$

Perfect elastic collision [elastic collision]:- $e = 1$.



$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \quad -①$$

$$e = 1 = \frac{v_2 - v_1}{u_1 - u_2}$$

$$v_2 - v_1 = u_1 - u_2 \quad -②$$

on solving eq ①,②

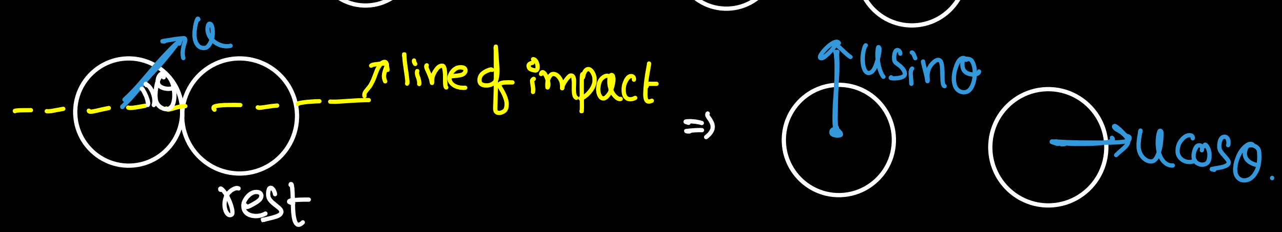
$$v_1 = \frac{(m_1 - m_2)u_1 + 2m_2 u_2}{m_1 + m_2} \quad \Bigg| \quad v_2 = \frac{(m_2 - m_1)u_2 + 2m_1 u_1}{m_1 + m_2}$$

observation:- if $m_1 = m_2$

$v_1 = u_2$ and $v_2 = u_1$.

⇒ In a perfect elastic collision if masses are same then velocities along the line of impact get exchanged.

eg



$$K.E_f = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

on solving

$$= \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2.$$

in perfect elastic collision $K.E_f = K.E_i$

K.E is conserved.

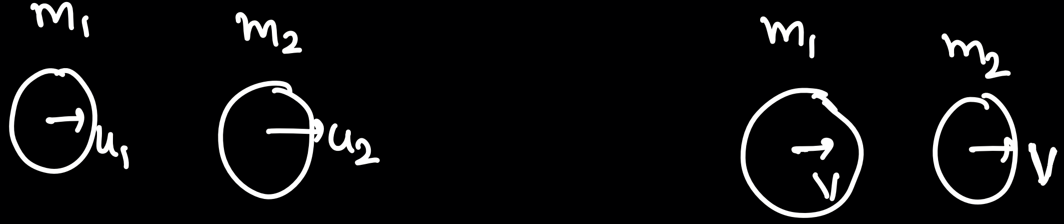
Perfect inelastic collision:- $e = 0$

Vel. of seperation $= 0$

$$v_2 - v_1 = 0$$

$$v_2 = v_1$$

⇒ Just after both bodies should have same vel. along line of impact.



$$m_1u_1 + m_2u_2 = m_1V + m_2V$$

$$V = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}.$$

$$K.E_f = \frac{1}{2}\frac{(m_1u_1 + m_2u_2)^2}{m_1 + m_2}$$

$$K.E_{loss} = K.E_i - K.E_f$$

$\bullet\!\!\rightarrow u_1 \quad \bullet\!\!\rightarrow u_2$

$$K.E_{loss} = \frac{1}{2}\frac{m_1m_2}{m_1+m_2}(u_1 - u_2)^2$$

if $\bullet\!\!\rightarrow u_1 \quad u_2 \leftarrow\!\!\bullet$

$$K.E_{loss} = \frac{1}{2}\frac{m_1m_2}{m_1+m_2}(u_1 + u_2)^2$$

$$\boxed{K.E_{loss} = \frac{1}{2}\frac{m_1m_2}{m_1+m_2}(\vec{u}_{rel})^2} = \frac{1}{2}\mu(u_{rel})^2.$$

⇒ in perfect inelastic collision, the entire variable K.E part is lost i.e. $K.E_{loss}$ is maximum in perfect inelastic collision.

---

inelastic collisions :-

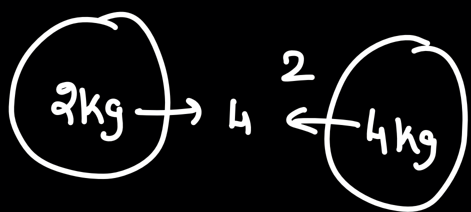
$$0 < e < 1$$

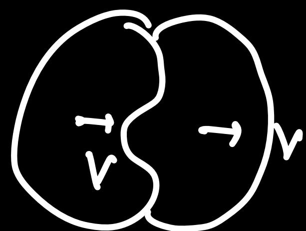
⇒ momentum is conserved but not K.E

$$K.E_{loss} < \frac{1}{2}\frac{m_1 m_2}{m_1+m_2}(\vec{u}_{rel})^2 \rightarrow (K.E_{loss})_{max}$$

Two smooth spheres are moving towards each other. Both have same radius but their masses are 2kg and 4kg. If the velocities are 4m/s and 2m/s, respectively, and coefficient of restitution is e=1/3. Their lines of motion are along same line [redacted] Maximum potential energy of deformation (in joules) is_______J.

sol





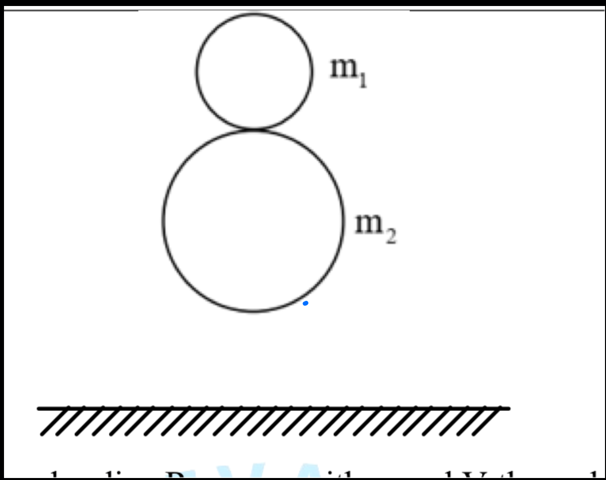
$$+8 - 8 = (6)V \Rightarrow V = 0.$$

$$K.E_{loss} = \text{deformation energy}$$

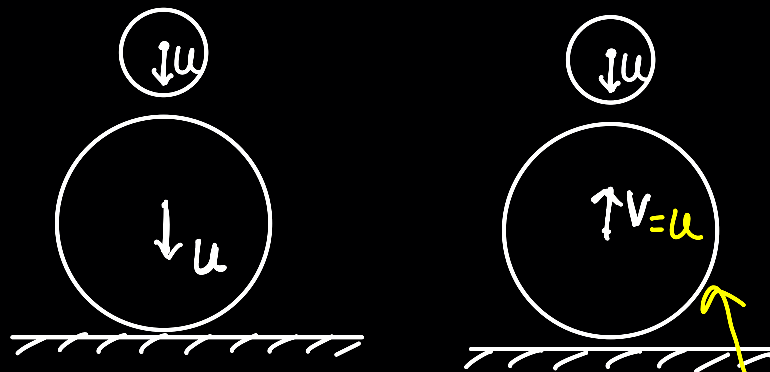
$$\frac{1}{2}(2)4^2 + \frac{1}{2}(4)2^2 = \text{energy}$$

$$24 = \text{energy.}$$

Two elastic balls of masses m_1 and m_2 are placed on top of each other (with a small gap between them) and then dropped onto the ground. What is the ratio m_2/m_1, for which the upper ball ultimately receives the largest possible fraction of the total energy in the collision between m_1 and m_2? (Neglect air resistance and assume all collisions to be elastic).



Sol:-

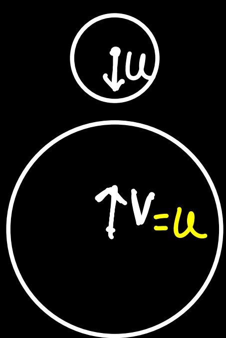


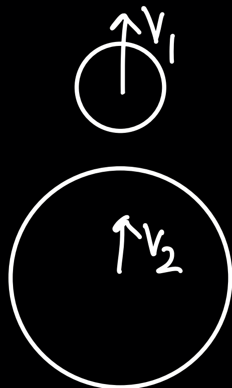
$e = \dfrac{V-0}{u-0} \Rightarrow V = eu$

$V = (1)u = u$

Just before

Just after





$e = \dfrac{V_1 - V_2}{u+u}$

$V_1 - V_2 = 2u$ —(1).

↑+ve $\vec{P}_i = m_2u - m_1u$

$\vec{P}_f = m_2V_2 + m_1V_1$.

$m_2V_2 + m_1V_1 = (m_2 - m_1)u$.

$m_2V_2 + m_1(V_2 + 2u) = (m_2 - m_1)u$.

$(m_1 + m_2)V_2 = (m_2 - 3m_1)u$

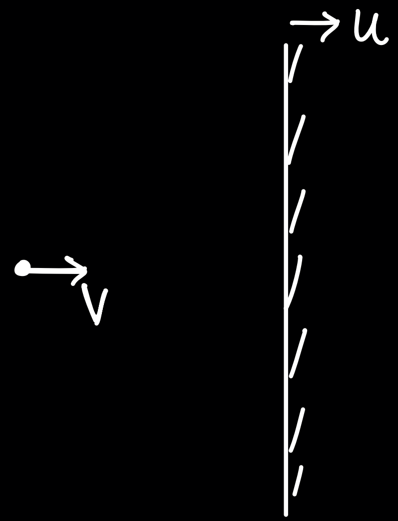
$V_2 = \left(\dfrac{m_2 - 3m_1}{m_1 + m_2}\right)u$.

if $V_2 = 0$ then entire K.E is with m_1

$$0 = \left(\frac{m_2 - 3m_1}{m_1+m_2}\right)u$$

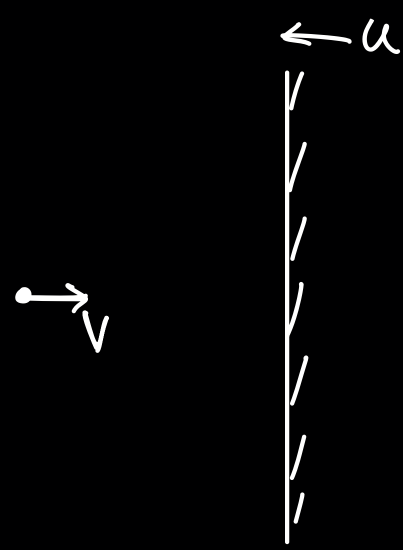
$$\frac{m_2}{m_1} = 3.$$

Q) (i)



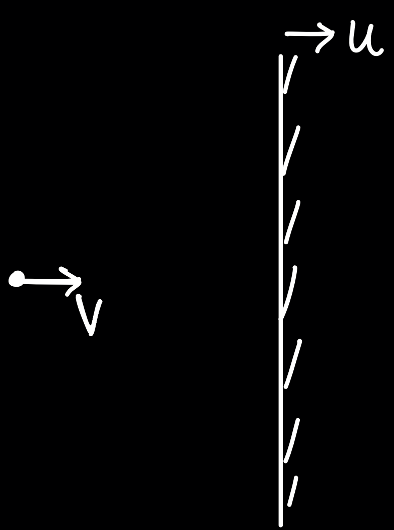
makes elastic collision find speed of particle after collision ?

(ii)

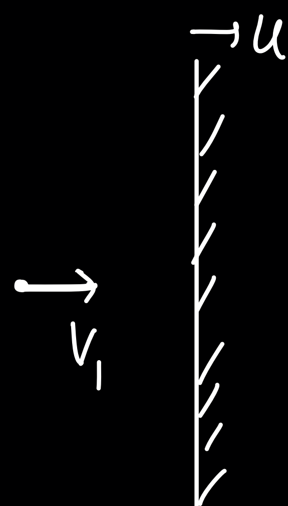


makes elastic collision find speed of particle after collision ?

Sol:-

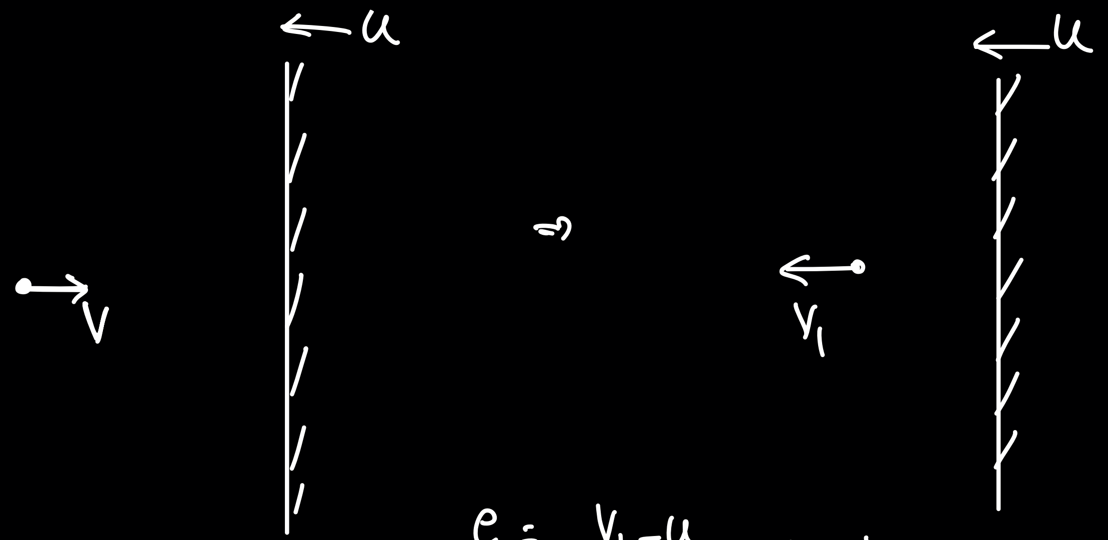


$\Rightarrow$



$$e = \frac{+u - V_1}{+V - u} \Rightarrow V - u = u - V_1$$

$$V_1 = 2u - V.$$



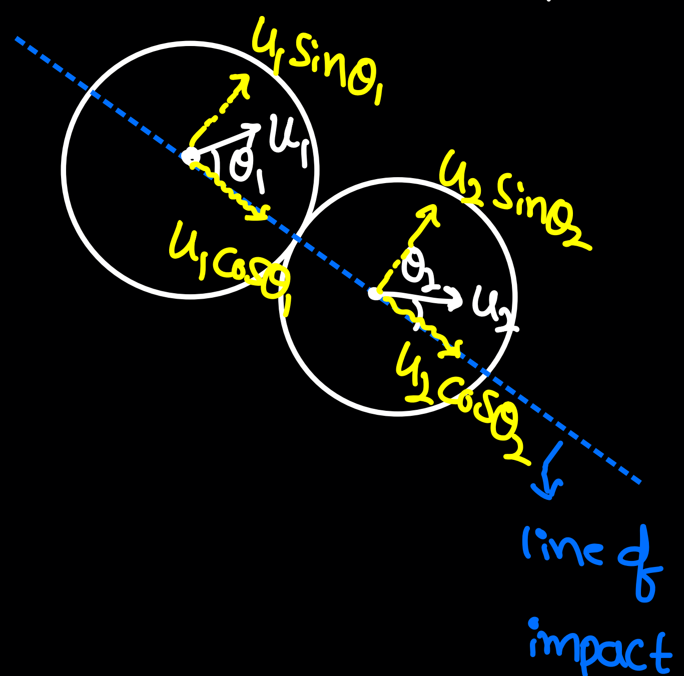
$$e = \frac{v_1 - u}{v + u} \Rightarrow v_1 = 2u + v.$$

Oblique Collisions:-

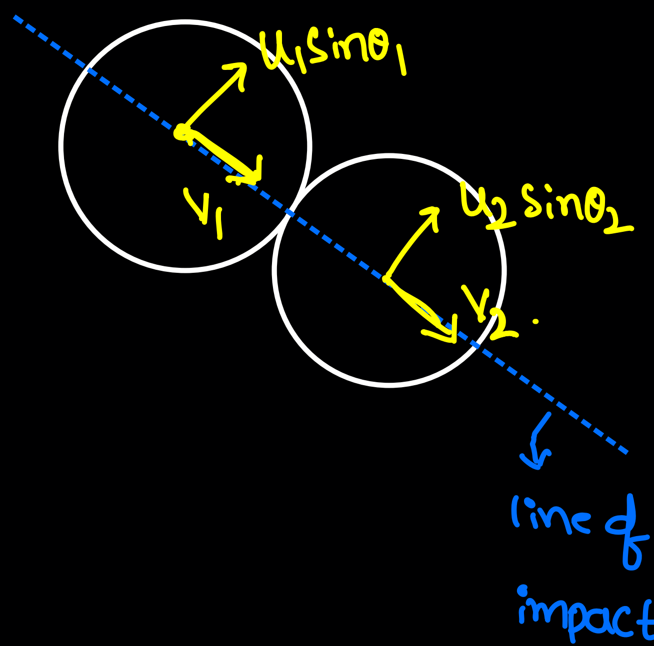
⇒ we find it difficult to get vel. of seperation and approach and sometimes conserving momentum too.

how to get vel. of seperation and approach.

Just before Collision



Just after collision

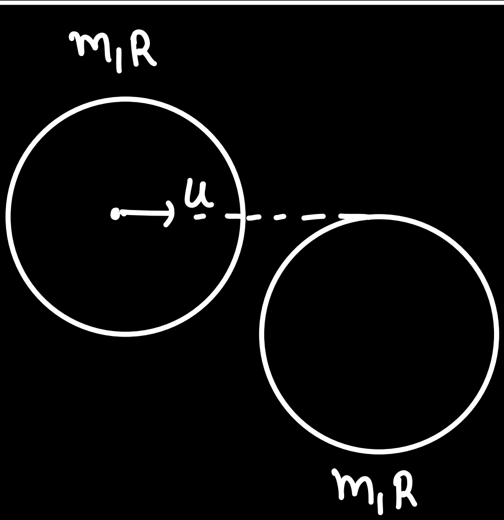


vel. of seperation $= v_2 - v_1$.

Vel. of approach $= u_1\cos\theta_1 - u_2\cos\theta_2$.

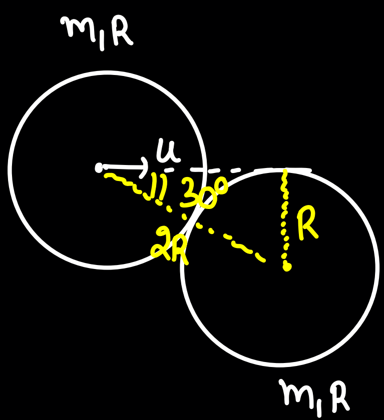
$\Rightarrow$ for conservation of momentum, we have to see the direction in which there is no impulsive force

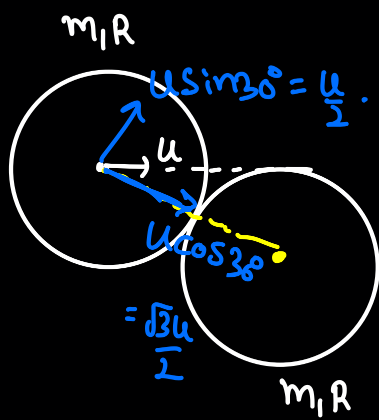
---

Q) 

find speeds of both discs after collision? Take coe. of restituition to be "e".

Sol:- Just before collision

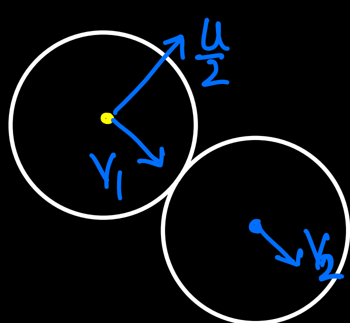




$$e = \frac{V_2 - V_1}{\frac{\sqrt{3}u}{2}}$$

$$V_2 - V_1 = \frac{\sqrt{3}eu}{2} \quad -①$$

Just after collision



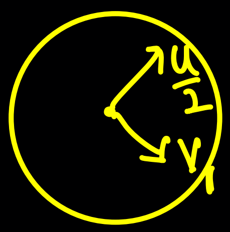
Conservation of linear momentum along line of impact

$$m\frac{\sqrt{3}u}{2} = mV_1 + mV_2$$

$$V_1 + V_2 = \frac{\sqrt{3}u}{2} \quad -②$$

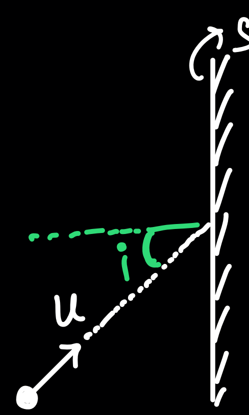
$$V_2 = \frac{\sqrt{3}u}{4}(1+e).$$

$$V_1 = \frac{\sqrt{3}u}{4}(1-e).$$



$$V_{net} = \sqrt{V_1^2 + \frac{u^2}{4}}$$

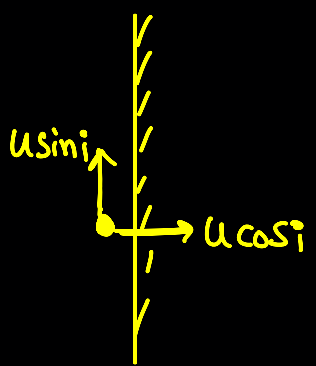
$$= \sqrt{\frac{3u^2}{16}(1-e)^2 + \frac{u^2}{4}} = \frac{u}{2}\sqrt{1 + \frac{3}{4}(1-e)^2}.$$

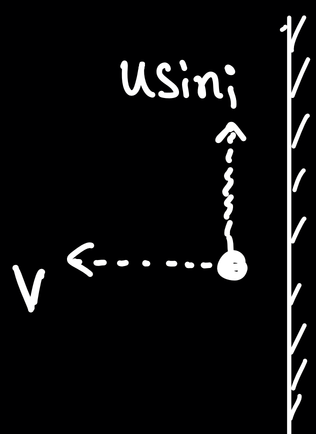
Q) 

taking coe. of restitution as e

find angle of reflection and speed of rebound?

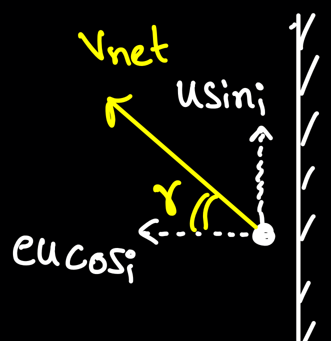
Sol :-





$$e = \frac{V}{u\cos i} \Rightarrow V = eu\cos i$$

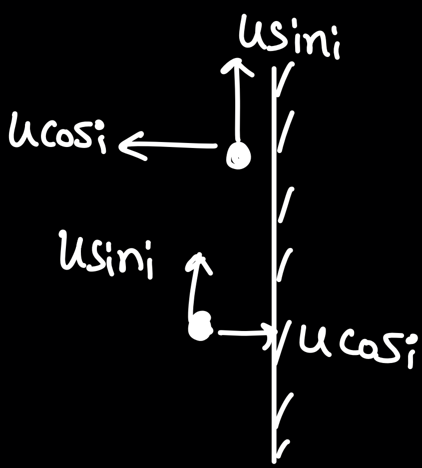
$$V_{net} = \sqrt{V^2 + u^2\sin^2 i} = \sqrt{e^2u^2\cos^2 i + u^2\sin^2 i}$$



$$\tan r = \frac{u\sin i}{eu\cos i} \Rightarrow \boxed{\tan r = \frac{1}{e}\tan i}$$

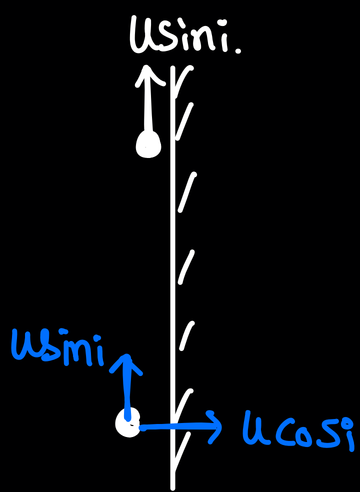
if $e=1$

$\tan r = \tan i \Rightarrow \angle r = \angle i.$

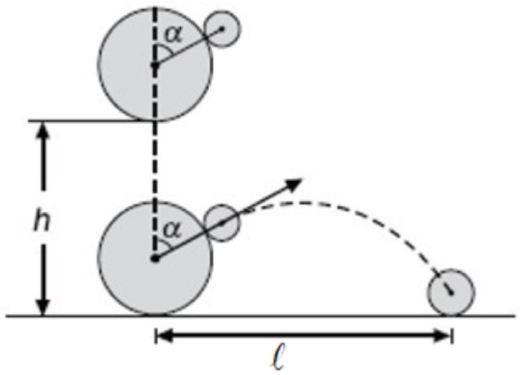


if $e=0$

$\tan r = \frac{1}{0}\tan i \Rightarrow \tan r = \infty \Rightarrow r = 90^\circ$



A large heavy sphere and a small light sphere are dropped onto a flat surface from a height h. The radii of both spheres is much smaller than height h. The large sphere collides with the surface with velocity v_0 and immediately thereafter with the small sphere. The spheres are dropped so that all motion is vertical before the second collision and the small sphere hits the larger sphere at an angle α from its upper most point, as shown in figure. If all collisions are perfectly elastic and there is no surface friction between the spheres. Then,



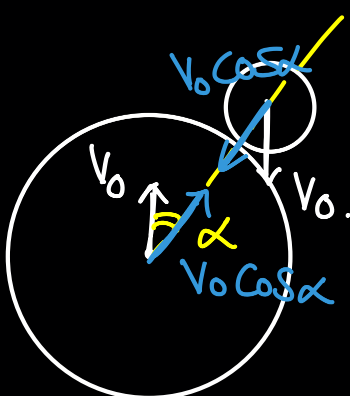
A) The angle made by velocity vector of small sphere with the vertical just after the second collision in the frame of large sphere is α

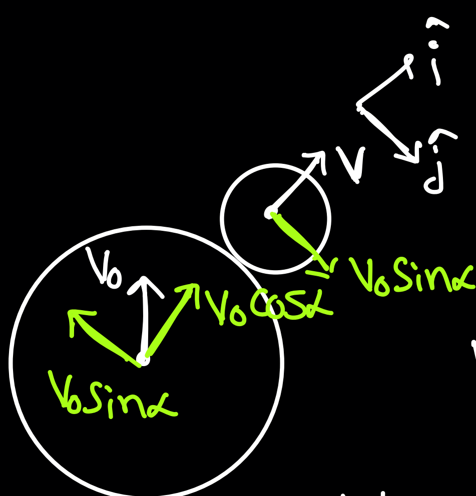
B) The angle made by velocity vector of small sphere with the vertical just after the second collision in the frame of large sphere is 2α

C) The vertical velocity of smaller sphere just after the collision with respect to ground is $v_0 \cos\alpha + v_0$

D) The vertical velocity of smaller sphere just after the collision with respect to ground is $2v_0 \cos 2\alpha + v_0$

Sol :-





$$e = \frac{V - V_0 \cos\alpha}{2V_0 \cos\alpha}$$

$$\boxed{V = 3V_0 \cos\alpha}$$

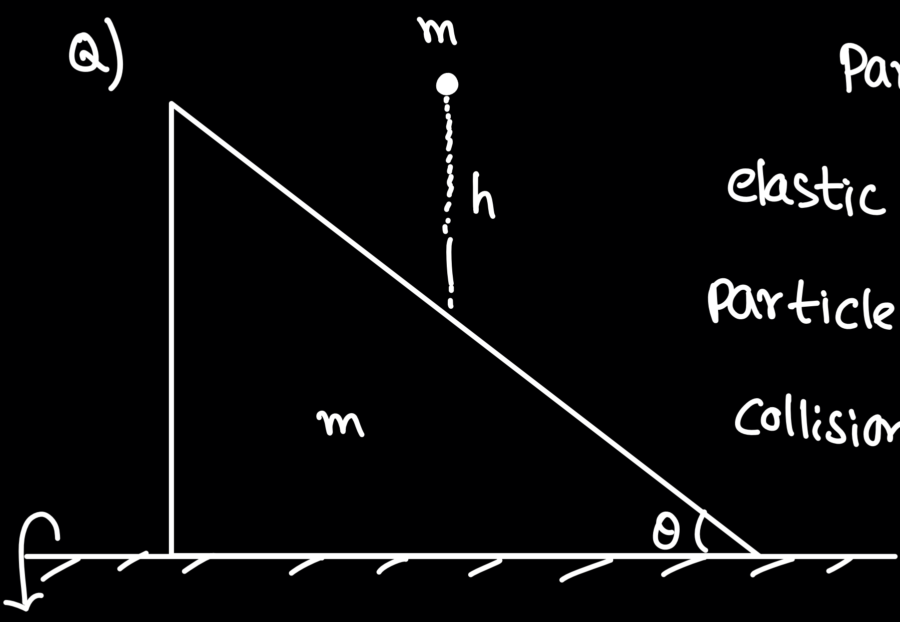
$$\vec{V}_{SB} = \vec{V}_S - \vec{V}_B$$

$$= 2V_0 \cos\alpha\,\hat{i} + 2V_0 \sin\alpha\,\hat{j}$$

$$|\vec{V}_{SB}| = 2V_0.$$

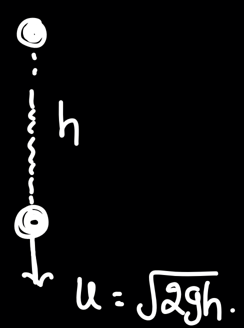
Collisions with inclined plane:-

Q)

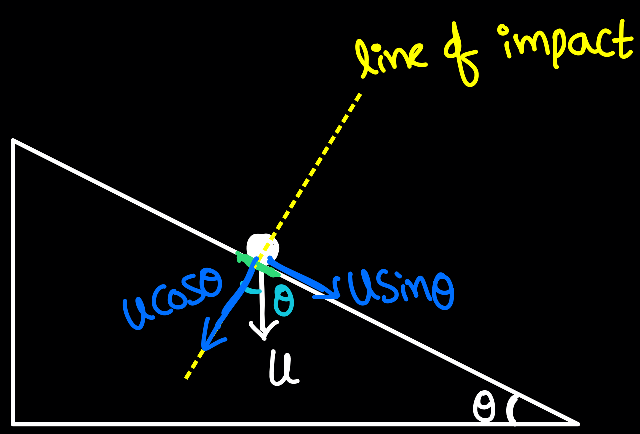


Particle makes perfectly elastic collision. find speed of particle and wedge just after collision ?

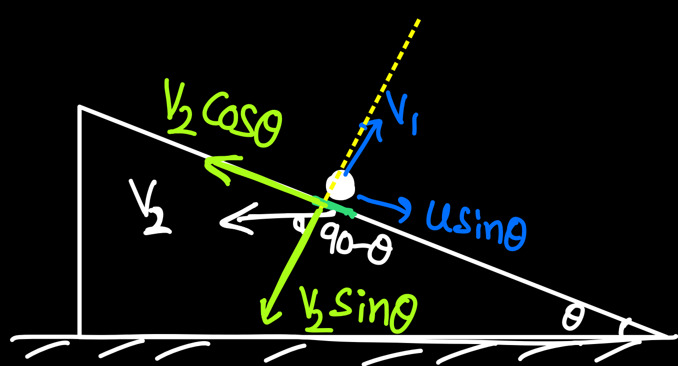
Sol:-



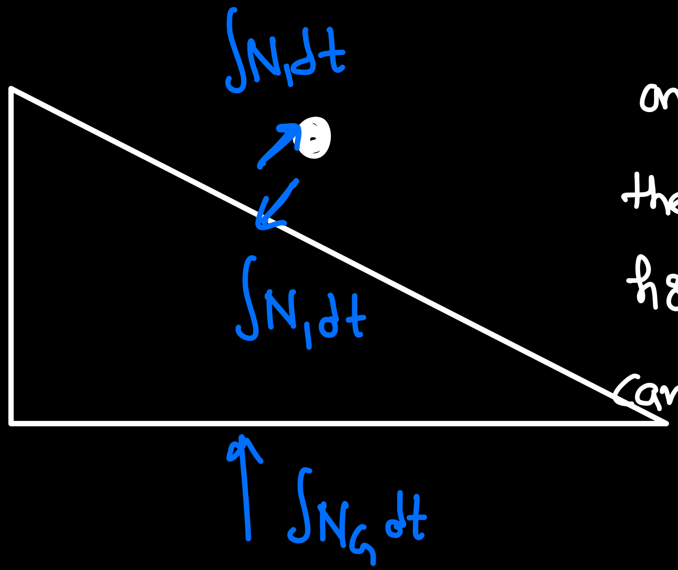
Just before collision



Just after collision:-

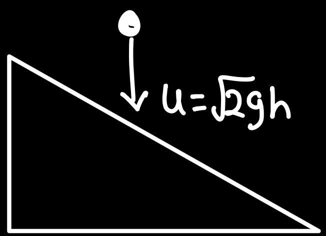


$$e = \frac{V_1 + V_2 \sin\theta}{u\cos\theta} \Rightarrow V_1 + V_2 \sin\theta = u\cos\theta \quad -(1)$$



on the wedge and particle system there is no impulsive ~~force~~ force in horizontal direction so momentum can be conserved in horizontal direction.

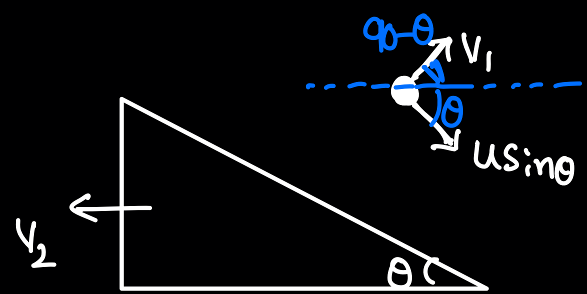
Just before :-



horizontal

$\vec{P}_i = m(0) + m(0)$

Just after collision :- $\rightarrow$ +ve



horizontal

$\vec{P}_f = m(-V_2) + mu\sin\theta\cos\theta + mV_1\sin\theta$

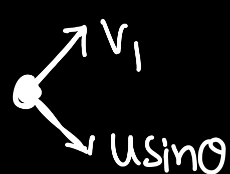
$\vec{P}_i = \vec{P}_f$

$0 = -mV_2 + mu\sin\theta\cos\theta + mV_1\sin\theta$ — ②.

on solving ①, ②

$$\boxed{V_1 = \frac{u(1+\sin^2\theta)}{\cos\theta} \quad \Big| \quad V_2 = 2u\tan\theta.} \rightarrow \text{Verify.}$$

Speeds :-

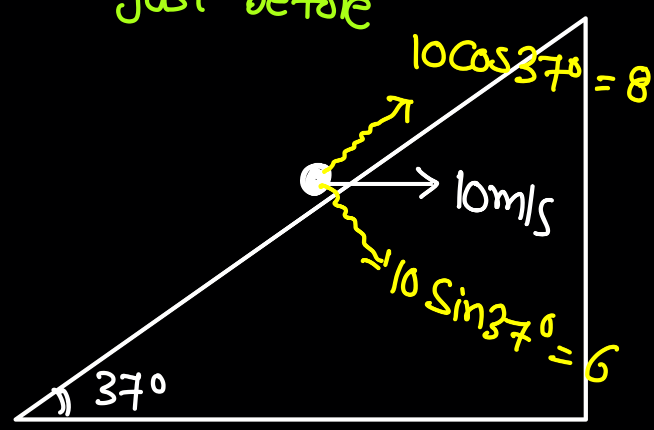


$\text{Speed} = \sqrt{V_1^2 + u^2\sin^2\theta}$

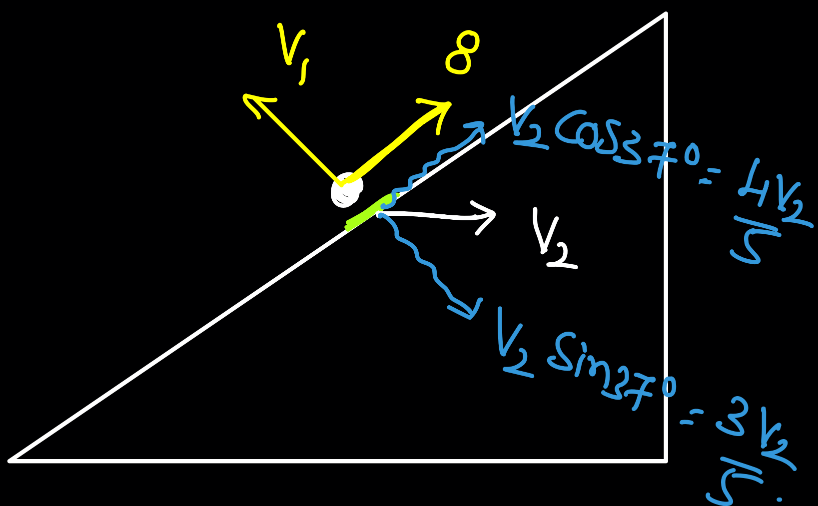
$\text{Speed} = V_2$.

ill:39 :-

Just before



Just after.



$$c = \frac{\text{Vel. of Sep}}{\text{Vel. of app.}}$$

$$0.8 = \frac{\left(V_1 + \frac{3V_2}{5}\right)}{6}$$

$$V_1 + 0.6V_2 = 4.8 \quad -①$$

Conservation of momentum in horizontal direction.

$\rightarrow +ve$ $\quad P_i = (0.25)10$

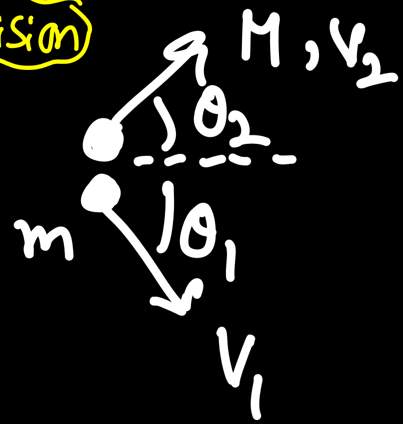
$$P_f = (0.72)V_2 + (0.25)(8\cos 37^\circ) - (0.25)(V_1\cos 53^\circ).$$

$$(0.25)(10) = (0.72)V_2 + (0.25)\left(\frac{32}{5}\right) - (0.25)\frac{3V_1}{5} \quad -②.$$

* Sometimes we can't conserve momentum in any direction then use $\vec{I} = \int \vec{F}\,dt = \Delta\vec{p}$ to get equations.

Q) "Elastic Collision"

M $\bullet \rightarrow u$ $\qquad$ m $\bullet$ (rest) $\qquad$ $M > m \Rightarrow$



find max. Value of θ_2?

sol :-

x-direction:-

$$Mu = MV_2\cos\theta_2 + mv_1\cos\theta_1 \quad -(1)$$

y-direction

$$MV_2\sin\theta_2 = mv_1\sin\theta_1 \quad -(2)$$

$$\frac{1}{2}Mu^2 = \frac{1}{2}MV_2^2 + \frac{1}{2}mv_1^2 \quad -(3) \Rightarrow Mu^2 = MV_2^2 + mv_1^2$$

$$M(u - V_2\cos\theta_2) = mv_1\cos\theta_1$$

$$MV_2\sin\theta_2 = mv_1\sin\theta_1$$

squaring and adding.

$$M^2u^2 + M^2V_2^2\cos^2\theta_2 - 2M^2uV_2\cos\theta_2 + MV_2^2\sin^2\theta_2 = m^2v_1^2$$

$$M^2u^2 + M^2V_2^2 - 2M^2uV_2\cos\theta_2 = m^2v_1^2$$

$$M^2u^2 + M^2V_2^2 - 2M^2uV_2\cos\theta_2 = mMu^2 - mMV_2^2$$

$$Mu^2 + MV_2^2 - 2MuV_2\cos\theta_2 = mu^2 - mV_2^2.$$

$$(M+m)v_2^2 - (2Mu\cos\theta_2)v_2 + (M-m)u^2 = 0.$$

for v_2 to be real

$$b^2-4ac \geq 0.$$

$$4M^2\cancel{u^2}\cos^2\theta_2 - 4(M+m)(M-m)\cancel{u^2} \geq 0$$

$$4M^2\cos^2\theta_2 - 4(M^2-m^2) \geq 0$$

$$M^2\cos^2\theta_2 \geq M^2-m^2$$

$$m^2 \geq M^2(1-\cos^2\theta_2)$$

$$m^2 \geq M^2\sin^2\theta_2$$

$$\sin\theta_2 \leq \frac{m}{M}. \quad \Rightarrow (\sin\theta_2)_{max} = \frac{m}{M}.$$

$$(\theta_2)_{max} = \sin^{-1}\left[\frac{m}{M}\right].$$

HW: SBT level 3

Q 109).

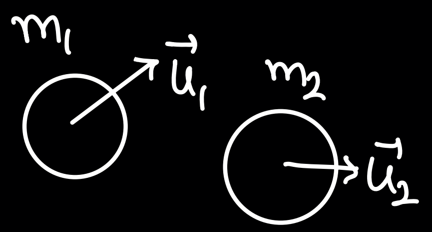
Let's look at Centroidal frame.

w.r.t C.M

$a_{C.M \text{ w.r.t } C.M} = 0 \Rightarrow \vec{F}_{net \text{ w.r.t } C.M} = 0$

$\Delta V_{C.M \text{ w.r.t } C.M} = 0$

$V_{C.M \text{ w.r.t } C.M} = u_{C.M \text{ w.r.t } C.M} = 0.$



$$\vec{u}_{CM} = \frac{m_1\vec{u}_1 + m_2\vec{u}_2}{m_1+m_2}$$

$$\vec{P}_{1CM} = m_1[\vec{u}_1 - \vec{u}_{CM}]$$

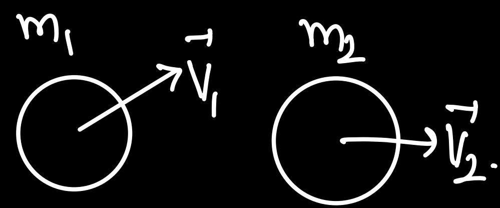
$$\vec{P}_{2CM} = m_2[\vec{u}_2 - \vec{u}_{CM}]$$

$$\vec{P}_{1CM} = \frac{m_1m_2}{m_1+m_2}(\vec{u}_1 - \vec{u}_2) = \mu\vec{u}_{12}$$

$$\vec{P}_{2CM} = \frac{m_1m_2}{m_1+m_2}(\vec{u}_2 - \vec{u}_1) = \mu\vec{u}_{21}$$

$$\vec{P}_{1CM} = -\vec{P}_{2CM} \Rightarrow * \boxed{\vec{P}_{net\ C.M} = 0}$$

$$|\vec{P}_{1CM}| = |\vec{P}_{2CM}|$$



$$\vec{V}_{CM} = \frac{m_1\vec{V}_1 + m_2\vec{V}_2}{m_1+m_2}.$$

Same way

$$\vec{P}'_{1CM} = \mu\vec{V}_{12}$$

$$\vec{P}'_{2CM} = \mu\vec{V}_{21}$$

$$\vec{P}'_{1CM} = -\vec{P}'_{2CM}$$

$$|\vec{P}'_{1CM}| = |\vec{P}'_{2CM}|$$

$$* \boxed{\vec{P}'_{net\ C.M} = 0}.$$

$$K.E \text{ w.r.t C.M} = \frac{P_{1CM}^2}{2m_1} + \frac{P_{2CM}^2}{2m_2}$$

$$= \frac{1}{2}\mu \vec{u}_{rel}^{\,2}$$

$$K.E \text{ w.r.t CM} = \frac{P_{1CM}'^2}{2m_1} + \frac{P_{2CM}'^2}{2m_2}.$$

$$= \frac{1}{2}\mu (\vec{V}_{rel})^2$$

if collision is elastic

$$K.E_{initial} = K.E_{final}$$

$$P_{1CM} = P_{2CM} = P_{1CM}' = P_{2CM}'.$$

if collision is inelastic

$$K.E_{initial} > K.E_{final}$$

$$P_{1CM} = P_{2CM} > P_{1CM}' = P_{2CM}'.$$

*****

$$\vec{P}_{1CM}' = m_1[\vec{V}_1' - \vec{V}_{CM}]$$

$$\vec{P}_{1CM}' = m_1\vec{V}_1' - m_1\vec{V}_{CM}$$

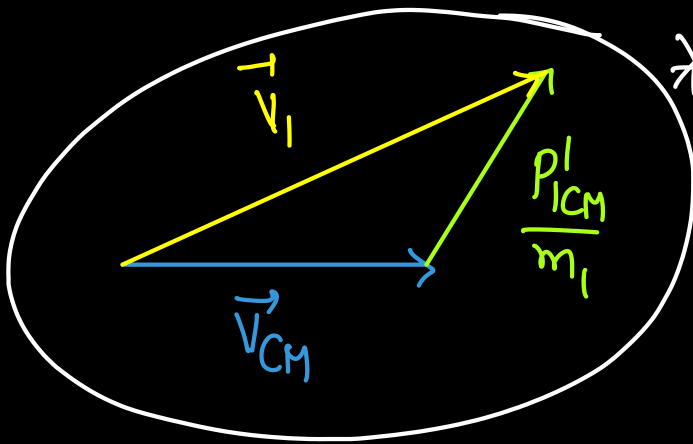
$$\vec{P}_{1CM}' = \vec{P}_1' - m_1\vec{V}_{CM}$$

$$\vec{P}_1' = \vec{P}_{1CM}' + m_1\vec{V}_{CM}$$

$$\vec{V}_1 = \frac{\vec{P}_{1CM}'}{m_1} + \vec{V}_{CM}$$

same way

$$\vec{V}_2 = \frac{\vec{P}_{2CM}'}{m_2} + \vec{V}_{CM}.$$



****.

lets revisit the question we did yesterday.

$M > m$

$M \longrightarrow u$ $\quad m$ (rest)

$\Rightarrow$ elastic collision

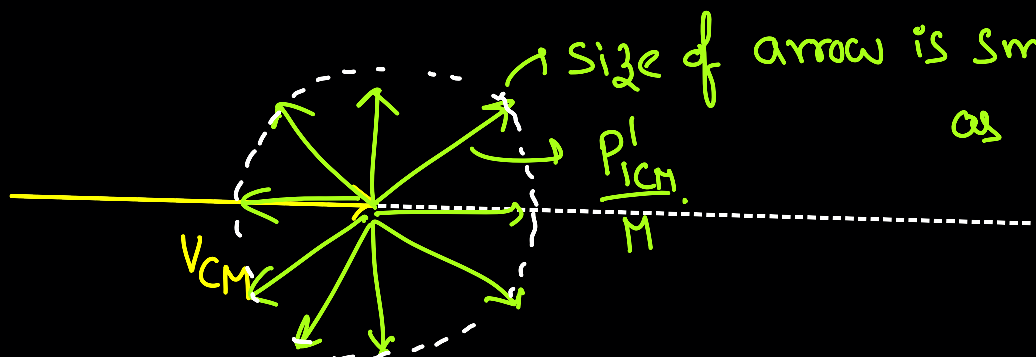
find max. deflection of M ?

Sol:- initial velocity direction of M and direction of V_{CM} are same.

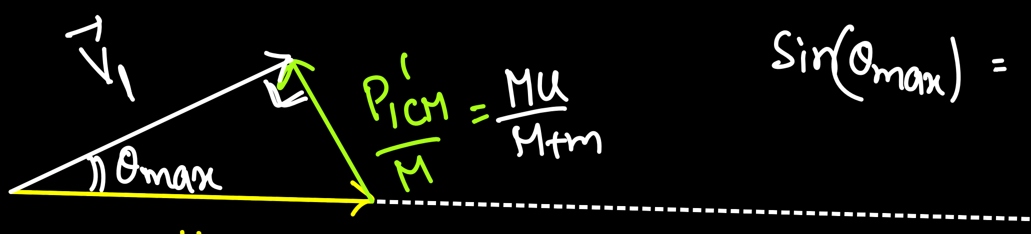
as collision is elastic

$$P'_{1CM} = P_{1CM} = \mu u_{rel} = \left(\frac{Mm}{M+m}\right)u. \quad \Bigg| \quad \frac{P'_{1CM}}{M} = \frac{mu}{M+m}.$$

$$V_{CM} = u_{CM} = \frac{Mu}{M+m}.$$



max. deflection is when V_1 is tangent to circle.

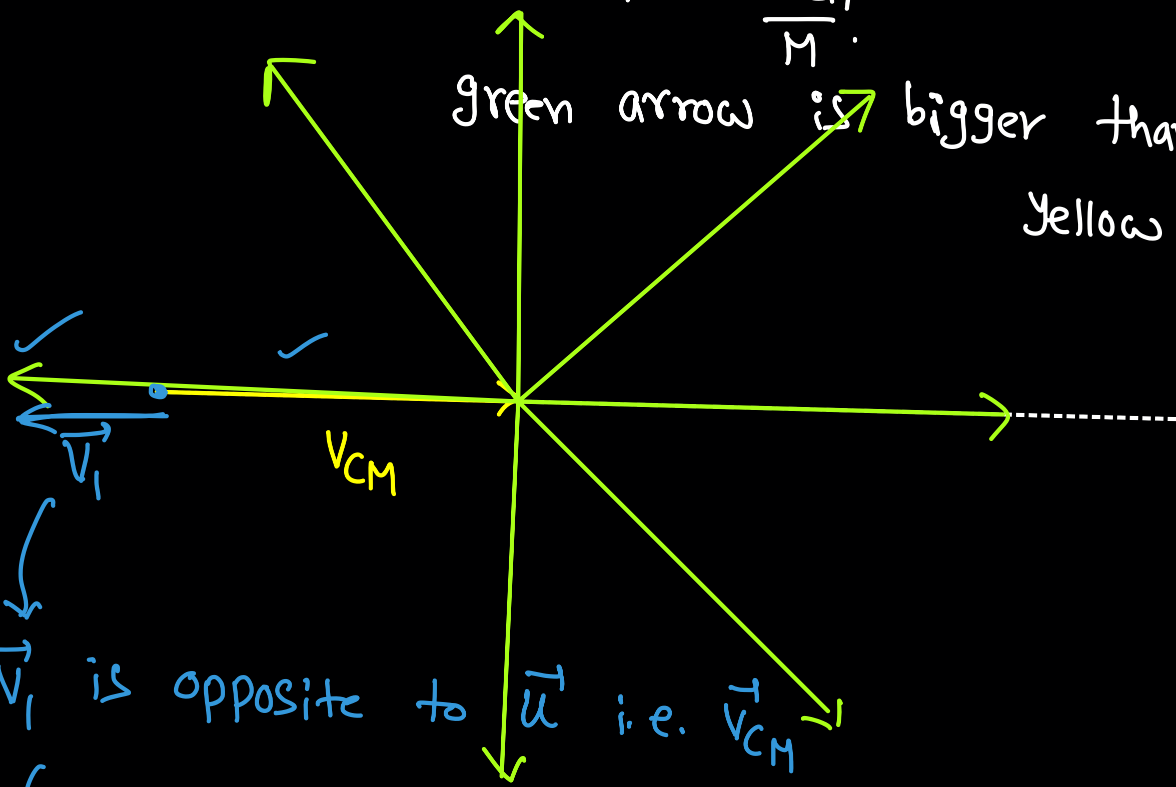


$$\sin(\theta_{max}) = \frac{\frac{mu}{M+m}}{\frac{Mu}{M+m}} \Rightarrow \theta_{max} = \sin^{-1}\left(\frac{m}{M}\right).$$

V_{CM}

if $M < m$ then $V_{CM} < \frac{P'_{1CM}}{M}$.

green arrow is bigger than yellow arrow



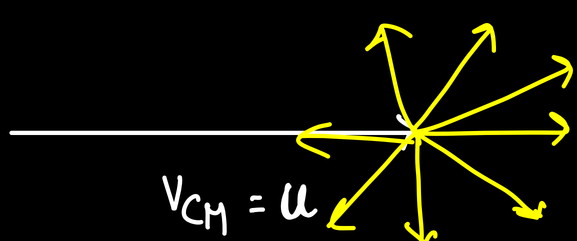
$\vec{V_1}$ is opposite to $\vec{u}$ i.e. $\vec{V_{CM}}$

max. deflection is 180°.

Path finder Build your understanding Q 33) :-

33. In free space, a shell is flying with a speed u towards a large stationary plane along its normal. When the shell is at a distance l from the plane, it explodes into an infinitely large number of fragments that fly apart in all directions such that speed of each fragment relative to the mass centre is v. Find area on the plane in which all the fragments strike.

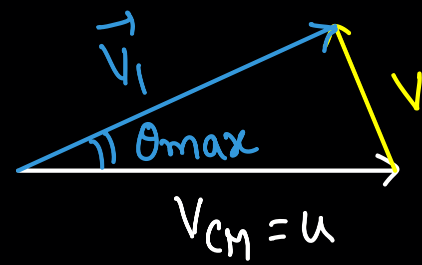
Sol :- if $u > V$



length of arrow $= \frac{P'_{1CM}}{m_1}$

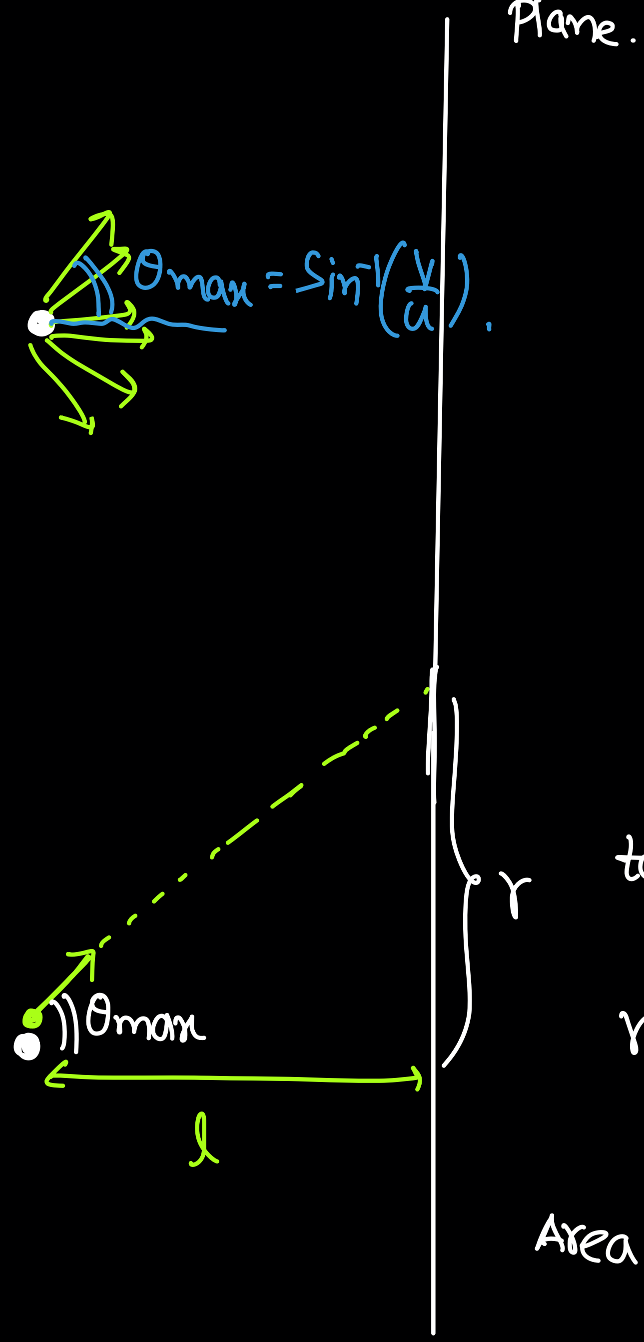
$= \frac{\cancel{m_1}(V)}{\cancel{m_1}}$

$= V.$



$$\sin\theta_{max} = \frac{v}{u}$$

$$\theta_{max} = \sin^{-1}\left[\frac{v}{u}\right].$$



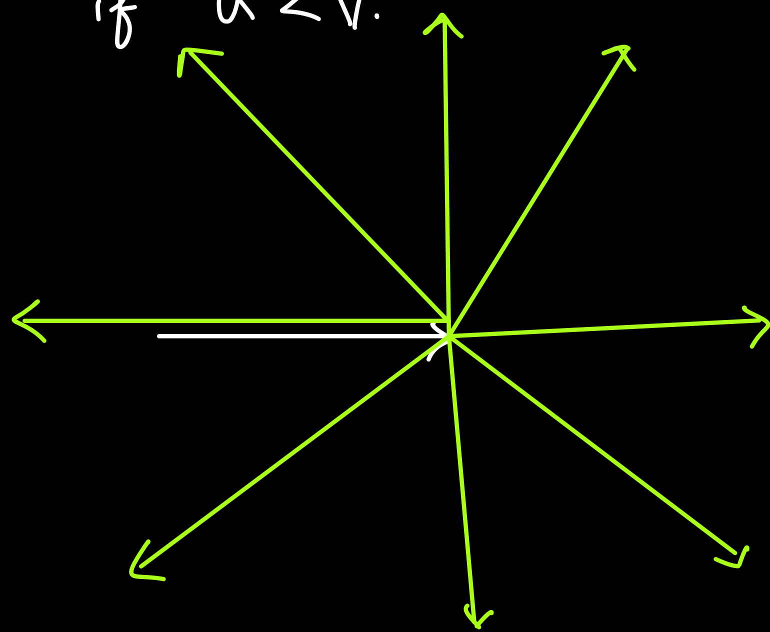
$$\tan\theta_{max} = \frac{r}{l}.$$

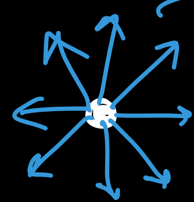
$$r = l\frac{v}{\sqrt{u^2-v^2}}$$

$$\text{Area} = \pi r^2$$

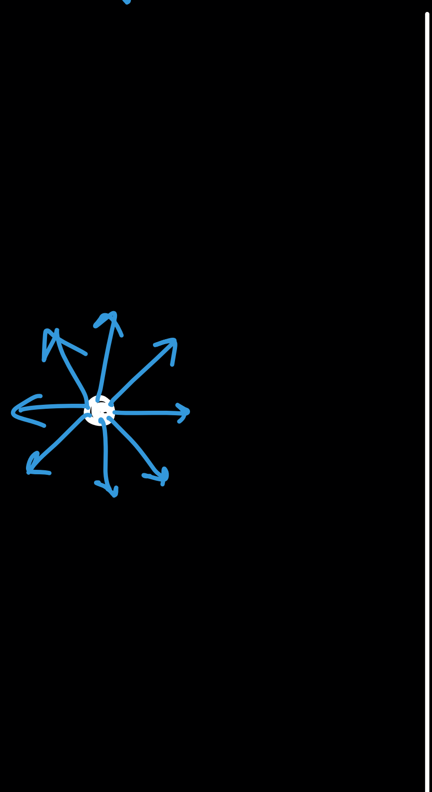
$$= \frac{\pi v^2 l^2}{u^2-v^2}.$$

Case (ii) if $u < v$.





fragments can go in all the directions.



whole area of plane gets fragments.

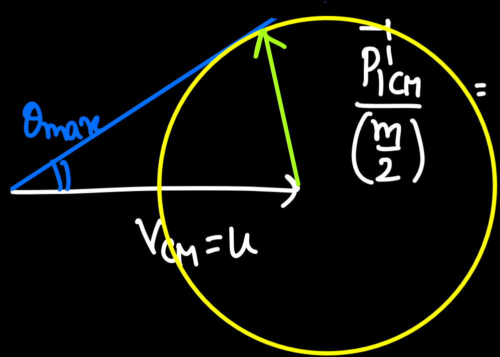
21. A shell of mass m moving with a velocity u in free space explodes into two identical fragments. If mass of the explosive material is negligible as compared to the mass of the shell and due to the explosion total kinetic energy increases by an amount ΔW, find the maximum angle between the directions of motion of the fragments.

$V_{CM} = u$

$K.E_i = \frac{1}{2}\mu(u-u)^2 = 0$

$K.E_f = 0 + \Delta W = \frac{(\vec{P}'_{1CM})^2}{2\times\frac{m}{2}} + \frac{(P'_{2CM})^2}{2\times\frac{m}{2}}$

$|\vec{P}'_{1CM}| = \sqrt{\frac{m\Delta W}{2}} = |P'_{2CM}|.$



$\frac{\vec{P}'_{1CM}}{\left(\frac{m}{2}\right)} = \sqrt{\frac{2\Delta W}{m}}$

$\sin\theta_{max} = \frac{\left(\sqrt{\frac{2\Delta W}{m}}\right)}{u}$

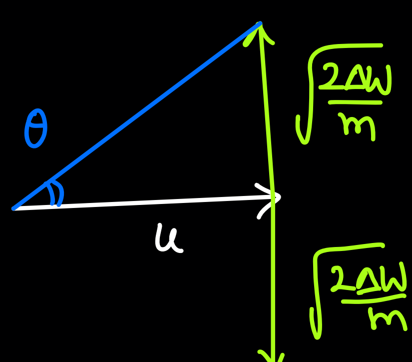
$\sin\theta_{max} = \sqrt{\frac{2\Delta W}{mu^2}}$

$\theta_{max} = \sin^{-1}\left(\sqrt{\frac{2\Delta W}{mu^2}}\right)$

Max. angle between direction of fragments $= 2\theta_{max}$.

but the answer doesn't match why?

$\Rightarrow$ they said two fragments so $\vec{P}'_{1CM}$ and $\vec{P}'_{2CM}$ should be exactly opposite. so

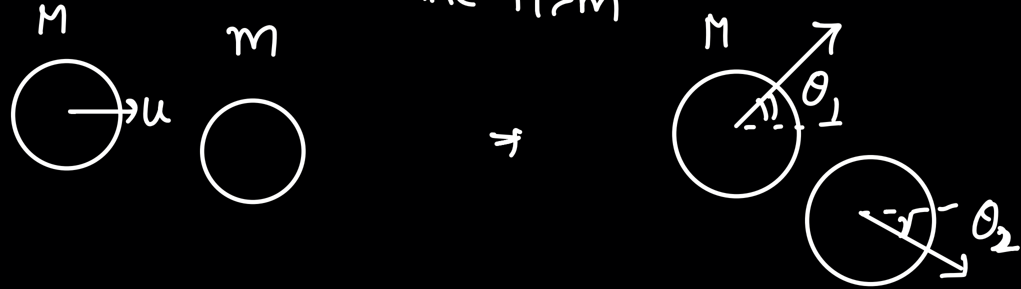


$\tan\theta = \frac{\sqrt{\frac{2\Delta W}{m}}}{u} = \sqrt{\frac{2\Delta W}{mu^2}}$

Ans $\Rightarrow 2\theta$

$\Rightarrow 2\tan^{-1}\sqrt{\frac{2\Delta W}{mu^2}}.$

Q) Take $M > m$



find max θ_1 and θ_2 ?

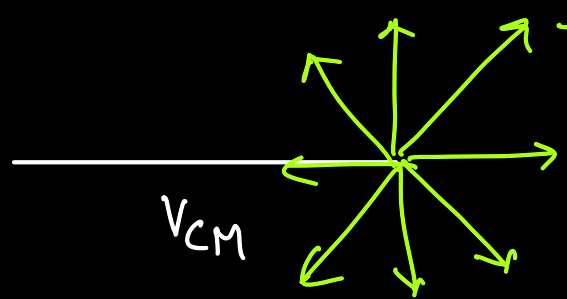
Sol: Whenever we deal with max. θ and type of collision is not mentioned then take it to be elastic as momentum after collision is max in elastic collision only.

we already know for M $\Rightarrow (\theta_1)_{max} = \sin^{-1}\left(\frac{m}{M}\right)$.

$$p'_{1CM} = p'_{2CM} = \frac{Mmu}{M+m}.$$

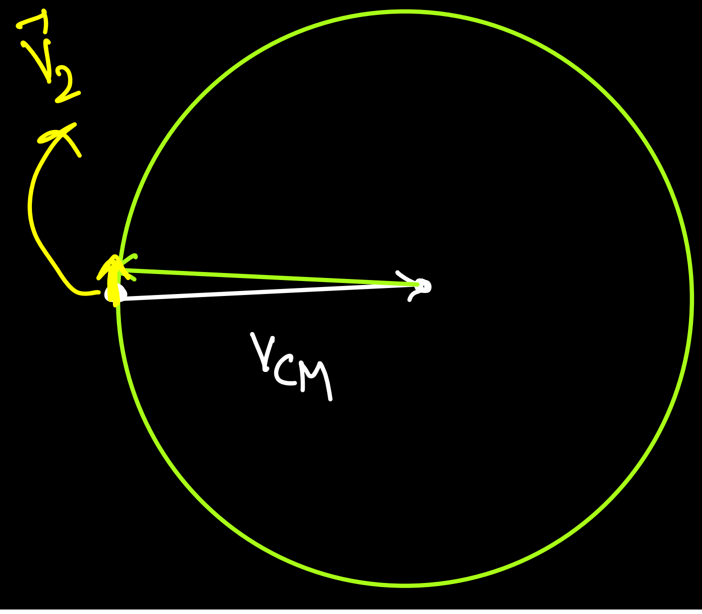
$$V_{CM} = \frac{Mu}{M+m}.$$

m



length of arrow $= \frac{p'_{2CM}}{m}$

$$= \frac{Mu}{M+m}.$$



max. angle with initial line of motion $\rightarrow 90^\circ$.

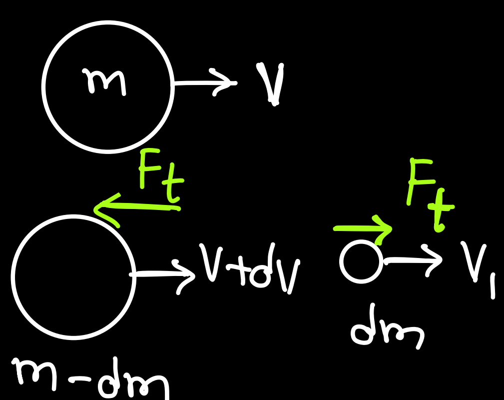
HW :-

35. Two balls of unequal masses, moving in opposite directions with equal speeds collide elastically. Thereafter, the heavier particle is observed deviated from its original direction of motion by an angle $\alpha = 30°$ in the laboratory frame and by an angle $\beta = 60°$ in the centre of mass frame. How many times of the mass of lighter ball is the mass of heavier ball?

$\Rightarrow$ can also be done with normal equation approach.

24. A shell of mass $m = 100$ kg explodes at some point on its trajectory into two fragments that fly with momenta $p_1 = 36 \times 10^4$ kg·m/s and $p_2 = 24 \times 10^4$ kg·m/s making angle $\theta = 60°$ from each other. Determine ratio of masses of the fragments for which kinetic energy released in the explosion will be minimal. How much is this kinetic energy?

Variable mass system :-



$\rightarrow$ +ve

$$\vec{V}_{rel} = \vec{V}_1 - (\vec{V} + d\vec{V})$$

$$\vec{V}_1 = \vec{V}_{rel} + (\vec{V} + d\vec{V})$$

$$V_1 = V_{rel} + V + dV.$$

$$P_i = mV$$

$$P_f = (m - dm)(v + dv) + (dm)v_1$$

$$P_i = P_f \Rightarrow \cancel{mv} = \cancel{mv} + m\,dv - \cancel{dm\,v} - \cancel{(dm)\,dv}$$

$$+ dm\, v_{rel} + \cancel{(dm)\,v} + \cancel{(dm)\,dv}$$

$$-m\,dv = (dm)\, v_{rel}$$

$$m\left(-\frac{dv}{dt}\right) = \left(\frac{dm}{dt}\right) v_{rel}$$

$$ma = \left(\frac{dm}{dt}\right) v_{rel}$$

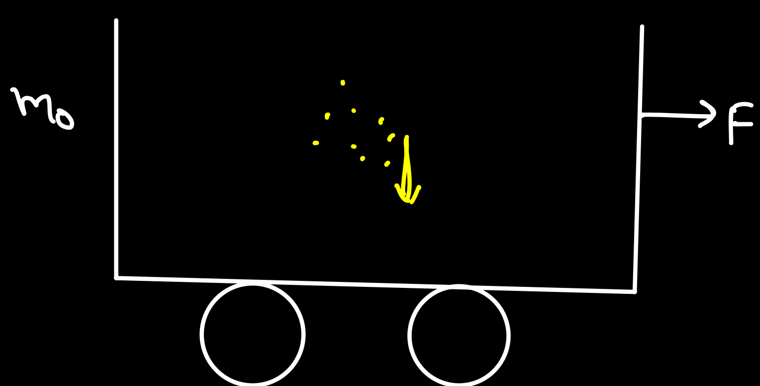
Vector

$$\boxed{\vec{F}_t = \left(\frac{dm}{dt}\right) \vec{v}_{rel}}$$

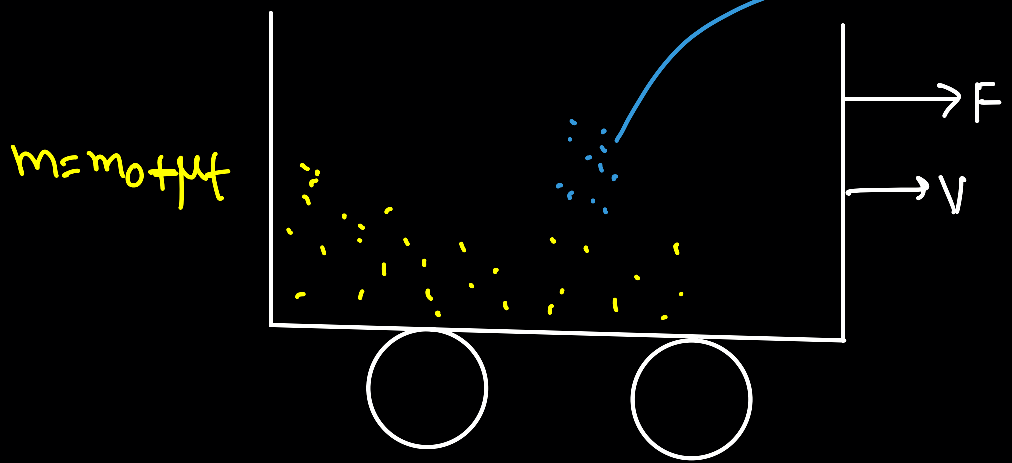
vel. rel. to system.

rate of change of mass of system

11:40



at some time 't'.



next dt time dm mass of sand gets added.

$(v_{sand})_H = 0$

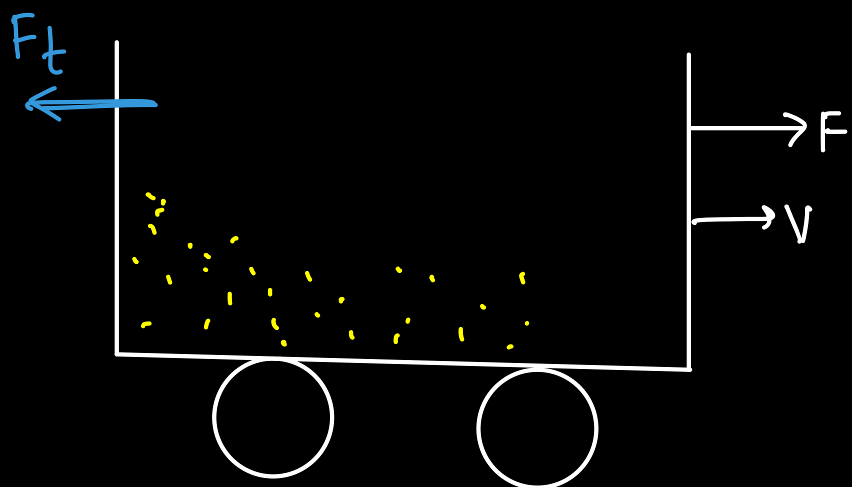
$(v_{flat\ car})_H = v\hat{i}$

$v_{SF} = -v\hat{i}$

$\vec{v}_{rel} = -v\hat{i}$

$$\vec{F}_{thrust} = \left(\frac{dm}{dt}\right)\vec{v}_{rel}$$

$$= (\mu)(-v\hat{i})$$



$$F_{net} = F - F_t$$

$$m\frac{dv}{dt} = F - \mu v$$

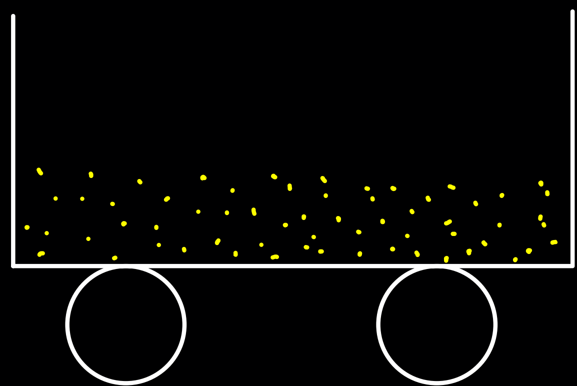
$$\int_0^V \frac{dv}{F-\mu v} = \int_0^t \frac{dt}{m} \Rightarrow \int_0^V \frac{dv}{F-\mu v} = \int_0^t \frac{dt}{m_0+\mu t}$$

$$V = \frac{Ft}{m_0 + \mu t}.$$

$$a = \frac{dv}{dt} \quad \& \quad a = \frac{F - F_t}{m}$$

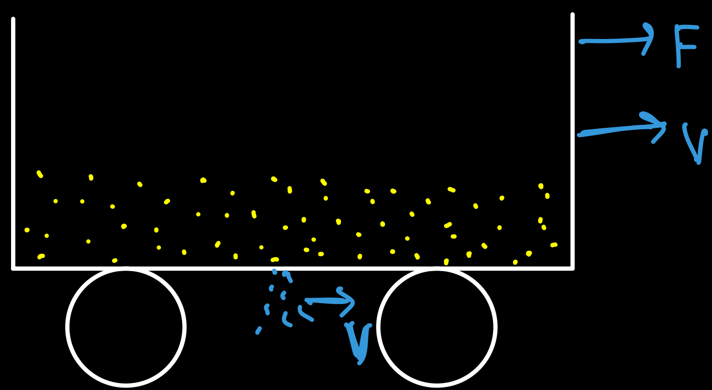
ill:41 at $t = 0$

m_0



at sometime 't'

$m_0 - \mu t$



$$\vec{V}_{rel} = V_S - V_T$$
$$= 0$$
$$F_t = 0.$$

$$F_{net} = ma$$

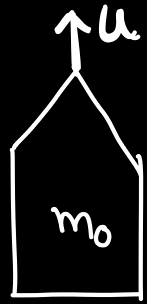
$$F = (m_0 - \mu t)\frac{dv}{dt}$$

$$\int_0^V dv = \int_0^t \frac{F}{m_0 - \mu t}\, dt$$

$$V = \frac{F}{\mu} \ln\left(\frac{m_0}{m_0 - \mu t}\right).$$

rocket propulsion:-

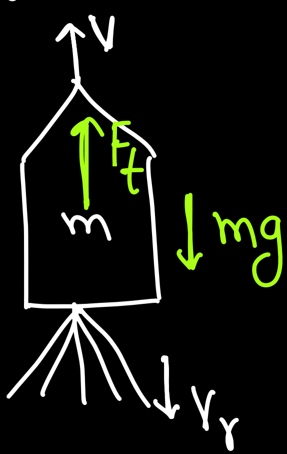
at $t=0$



lets take that rate of consumption of fuel is constant, as a result mass of rocket changes at constant rate (μ).

$\Rightarrow$ let's assume that exhaust velocity of gases w.r.t rocket to be V_r and constant with time.

time $= t$



$$\vec{F_t} = \left(\frac{dm}{dt}\right)\vec{V}_{rel}$$

$\downarrow$ $-ve$

F_t and V_r are in opposite direction.

$$F_{net} = F_t - mg$$

$$m\frac{dv}{dt} = \mu V_r - mg$$

$$\frac{dv}{dt} = \frac{\mu V_r}{m} - g$$

$$\frac{dv}{dt} = \frac{\mu V_r}{m_0 - \mu t} - g$$

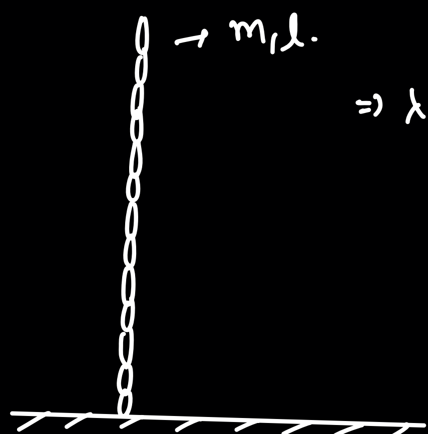
$$\int_u^v dv = \int_0^t \frac{\mu V_r}{m_0 - \mu t}\, dt - \int_0^t g\, dt.$$

$$v - u = \mu V_r \left[\frac{\ln(m_0 - \mu t)}{-\mu}\right]_0^t - g\left[t\right]_0^t$$

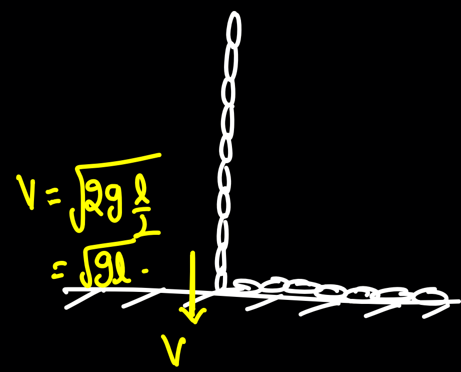
$$V - u = V_r\left[\ln(m_0) - \ln(m_0 - \mu t)\right] - gt$$

$$V = u + V_r \ln\left(\frac{m_0}{m_0 - \mu t}\right) - gt.$$

ill: 43 :-



$\Rightarrow \lambda = \dfrac{m}{l}.$



in next 'dt' time

(Vdt) length gets added to earth.

change in mass of earth $(dm) = (\lambda) v\,dt.$

$$\vec{V}_{ce} = \vec{V}_{rel} = \vec{V}_c - \vec{V}_{earth}$$

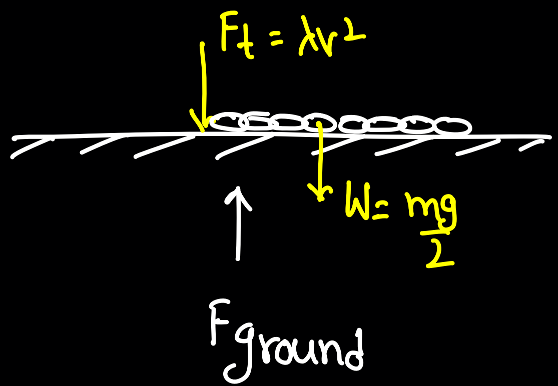
$$\vec{V}_{rel} = V(-\hat{j}) - 0$$

$$\vec{V}_{rel} = -V\hat{j}.$$

$$\vec{F}_t = \vec{V}_{rel}\left(\frac{dm}{dt}\right)$$

$$= (-v\hat{j})(\lambda v)$$

$$\boxed{\vec{F}_t = -\lambda v^2 \hat{j}}$$

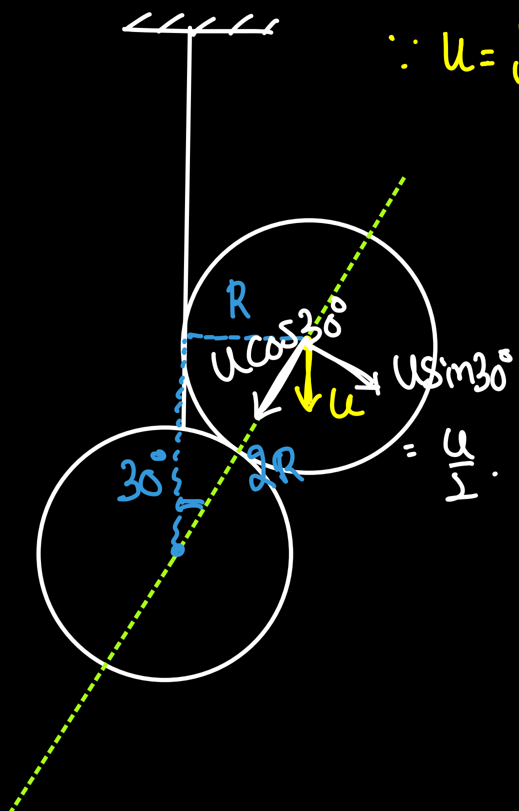


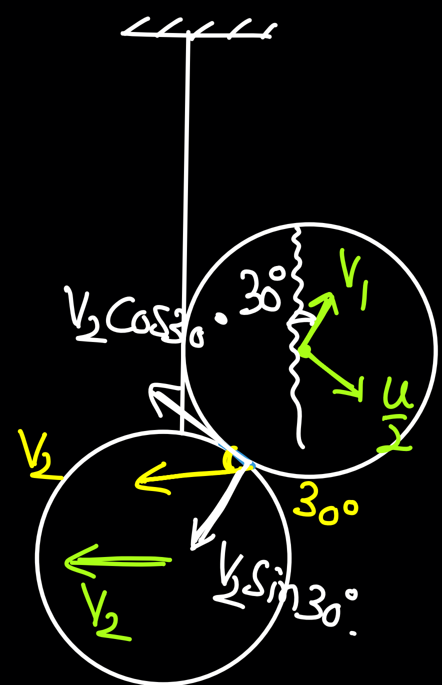
$$F_{ground} = \frac{mg}{2} + \lambda v^2$$
$$= \frac{mg}{2} + \lambda g l$$
$$= \frac{mg}{2} + \frac{m}{\cancel{l}} g \cancel{l}$$
$$= \frac{3mg}{2}.$$

Ex:3

Q16)

$\because u = \sqrt{2gh}.$





$$e = \frac{V_1 + \frac{V_2}{2}}{\frac{\sqrt{3}u}{2}} \Rightarrow V_1 + \frac{V_2}{2} = \frac{\sqrt{3}}{2} ue \quad -(1).$$

Conservation of momentum in horizontal direction

$$P_i = 0$$

$\rightarrow +ve$ $\quad P_f = m(-V_2) + m(V_1 \sin 30^\circ) + m(\frac{u}{2}\cos 30^\circ).$

$$-\cancel{m}V_2 + \frac{\cancel{m}V_1}{2} + \cancel{m}\frac{\sqrt{3}u}{4} = 0$$

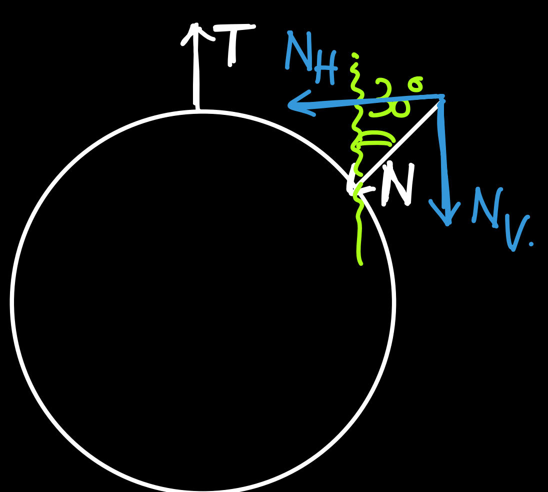
$$V_2 - \frac{V_1}{2} = \frac{\sqrt{3}u}{4}. \quad — ②$$

$\frac{1}{2}① + ②$

$$\frac{V_1}{2} + \frac{V_2}{4} = \frac{\sqrt{3}}{4}ue$$

$$V_2 - \frac{V_1}{2} = \frac{\sqrt{3}u}{4}$$

$$\frac{5V_2}{\cancel{4}} = \frac{\sqrt{3}u}{\cancel{4}}(1+e) \Rightarrow V_2 = \frac{\sqrt{3}(1+e)u}{5}.$$



$N_H = N\cos 60^\circ = \frac{N}{2}.$

$N_V = N\sin 60^\circ = \frac{\sqrt{3}N}{2}.$

$$\boxed{T = N_V = \frac{\sqrt{3}N}{2}.}$$

$\overleftarrow{\ \ }$ +ve $\quad \int N_H\,dt = P_f - P_i = mV_2 - m(0)$

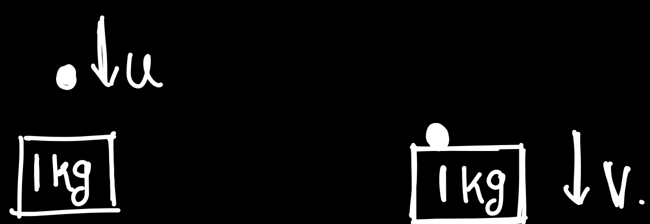
$$\int \frac{N}{2}\,dt = mV_2 \Rightarrow \int N\,dt = 2m\left(\frac{\sqrt{3}(1+e)u}{5}\right).$$

$$\int T\,dt = \int N_V\,dt = \int \frac{\sqrt{3}N}{2}\,dt = \frac{\sqrt{3}}{2}\int N\,dt$$

$$= \frac{3mu(1+e)}{5}.$$

Ex:3

Q15)

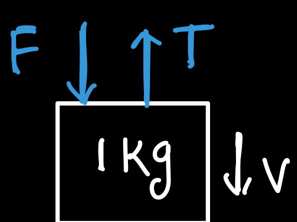


Ball :-

$\uparrow$ +ve $\quad \int F\,dt = J = (1)(-V) - (1)(-u)$

$$J = u - V \quad \text{—①}$$

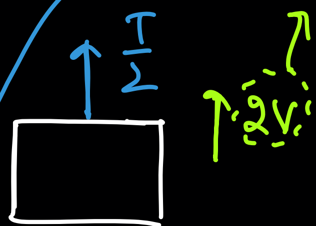
A



$\downarrow$ +ve $\quad \int F\,dt - \int T\,dt = (1)(V) - (1)(0)$

$J - \int T\,dt = V$ $-②$

B



↑ from constraint

↑ +ve $\qquad \int \frac{I}{2}\,dt = (2)(2V) - (2)0$

$\int T\,dt = 8V$ $-③$.

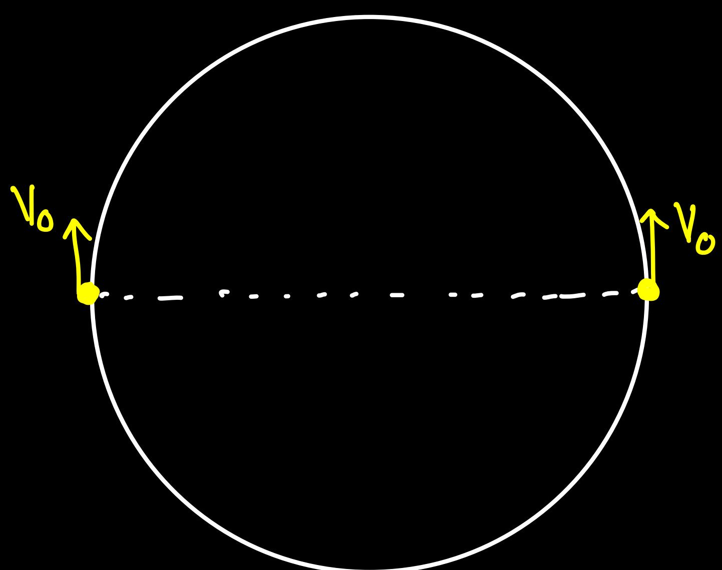
$J - 8V = V \Rightarrow 9V = J \Rightarrow V = \frac{J}{9}$.

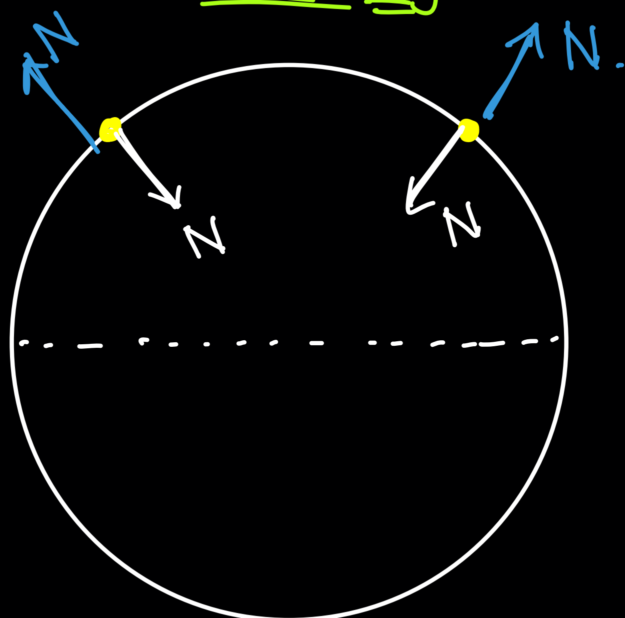
impulse on A is $\Delta p = V = \frac{J}{9}$.

" on B is $\Delta p = 4V = \frac{4J}{9}$.

Q16)

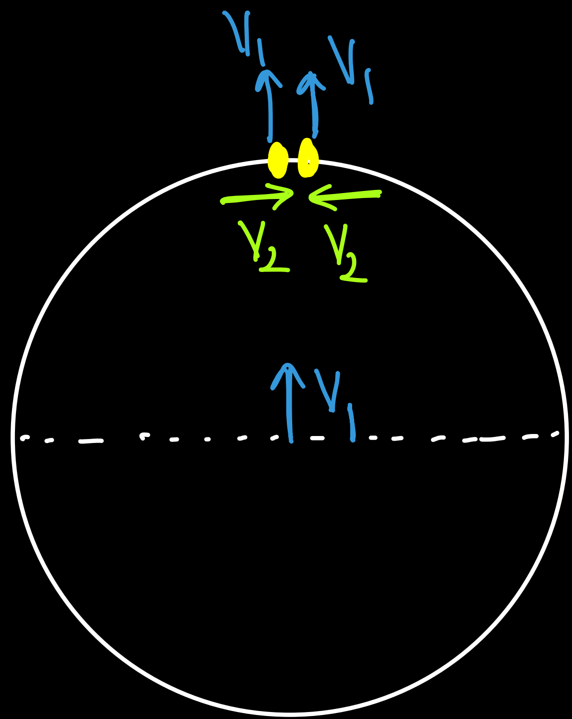
w.r.t ring :-





in vertical direction

$\vec{p}_i = mV_0 + mV_0 = 2mV_0\,\hat{j}$



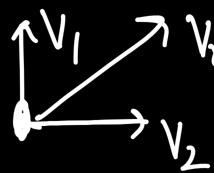
in vertical direction

$$\vec{P}_f = mV_1 + mV_1 + mV_1$$

$$= 3mV_1\,\hat{j}$$

$$\vec{P}_i = \vec{P}_f$$

$$2mV_0 = 3mV_1 \Rightarrow V_1 = \frac{2V_0}{3}$$



$$V_{net} = \sqrt{V_1^2 + V_2^2}.$$

$$K.E_i = \frac{1}{2}mV_0^2 + \frac{1}{2}mV_0^2$$

$$K.E_f = \frac{1}{2}mV_1^2 + \frac{1}{2}m\left(\sqrt{V_1^2+V_2^2}\right)^2 + \frac{1}{2}m\left(\sqrt{V_1^2+V_2^2}\right)^2$$

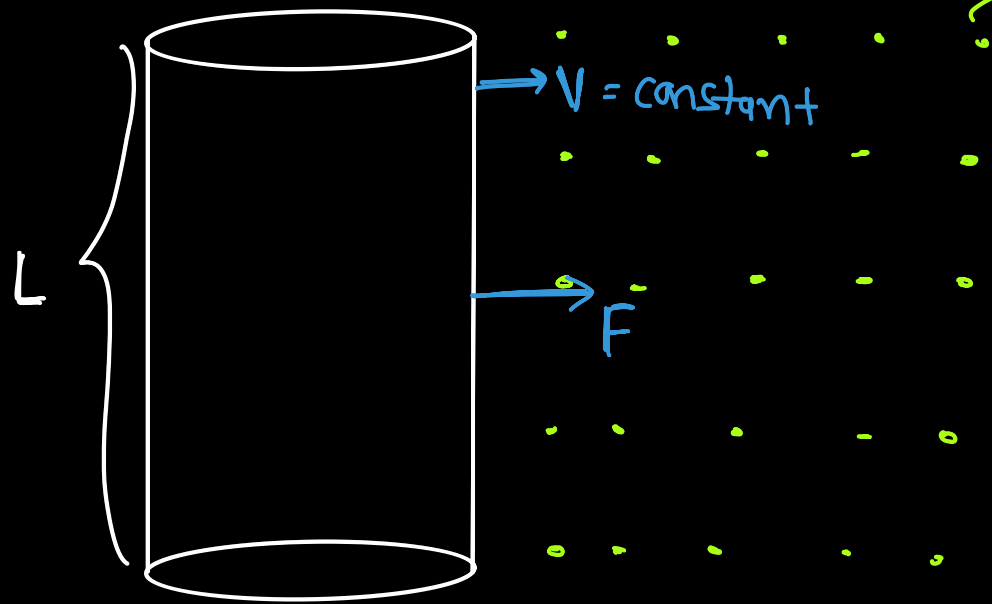
$$K.E_i = K.E_f$$

$$V_2 = \frac{V_0}{\sqrt{3}}.$$

$$\text{Speed} = \sqrt{V_1^2 + V_2^2} = \sqrt{\frac{4V_0^2}{9} + \frac{V_0^2}{3}} = \sqrt{\frac{7V_0^2}{9}}$$

$$= \frac{\sqrt{7}\,V_0}{3}.$$

HW



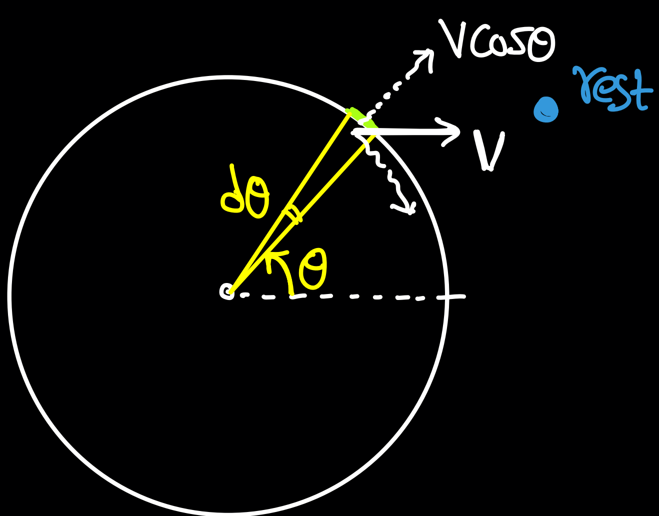
↗ rest (mass "m")

density of space = ρ.

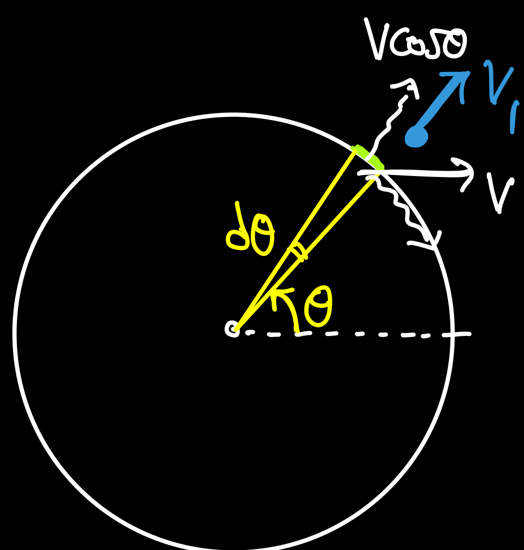
find F needed to keep cylinder move with constant speed ? (collision (i) elastic
(ii) perfect inelastic)

Sol :- Top View :-

Before Collision



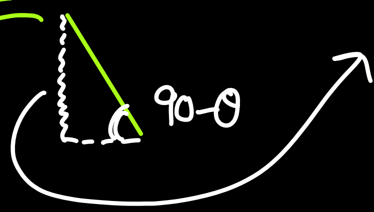
After collision



$$e = \frac{V_1 - V\cos\theta}{V\cos\theta}$$

$$1 = \frac{V_1 - V\cos\theta}{V\cos\theta} \Rightarrow V_1 = 2V\cos\theta \quad -(1).$$

$dA = (Rd\theta)L$



$dA_\perp = (dA)\sin(90-\theta) = dA\cos\theta.$

Volume swept in dt time $= (dA\cos\theta)\,V\,dt.$

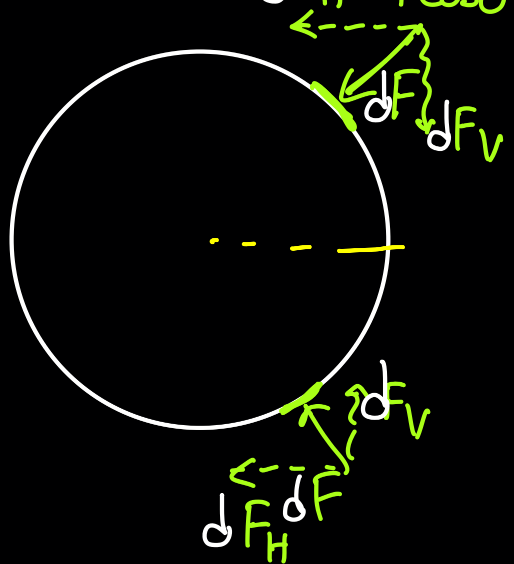
mass $(dm) = (\rho)(dA\cos\theta)\,V\,dt.$

Total impulse on particles

$(dF)\,dt = (dm)(2V\cos\theta)$

$(dF)\cancel{dt} = (V\rho\, dA\cos\theta\,\cancel{dt})\,2V\cos\theta$

$(dF) = 2\rho V^2 (dA)\cos^2\theta.$



We can see that for net force only horizontal contributes

$dF_{net} = dF\cos\theta.$

$F_{net} = \int dF\cos\theta$

$= \int 2\rho V^2 (Rd\theta L)\cos^3\theta$

$$= 2\rho V^2 RL \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \cos^3\theta \, d\theta$$

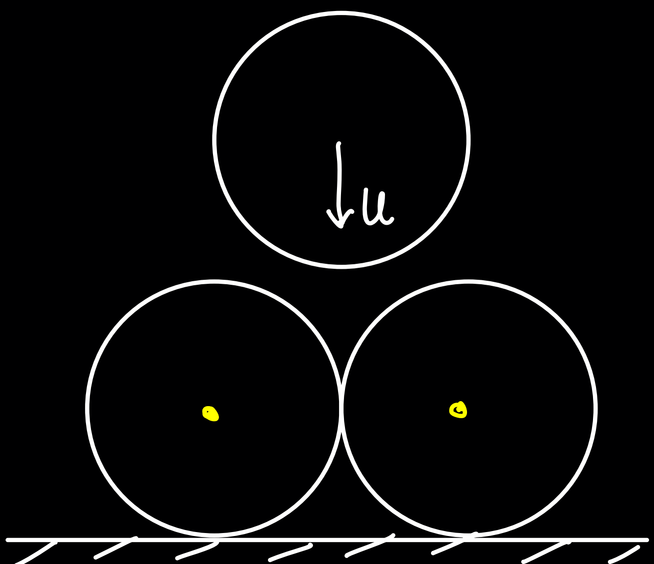
$$= \frac{8\rho V^2 RL}{3}.$$

As particles apply $\frac{8\rho V^2 RL}{3}$ force, to move it with constant velocity $\Rightarrow F_{net} = 0$ so external agent should apply $\frac{8\rho V^2 RL}{3}$ in opposite direction to cancel it.

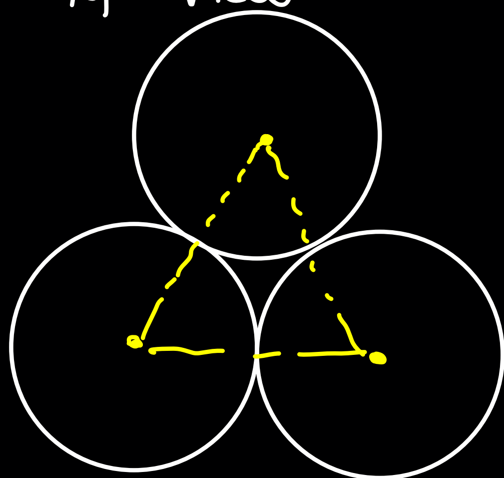
for inelastic, do in the same way

we will get $\frac{4\rho V^2 RL}{3}$.

Q)



Top View



after collision:-