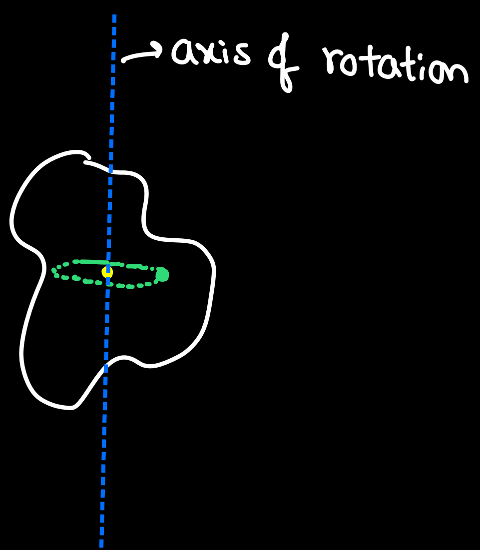
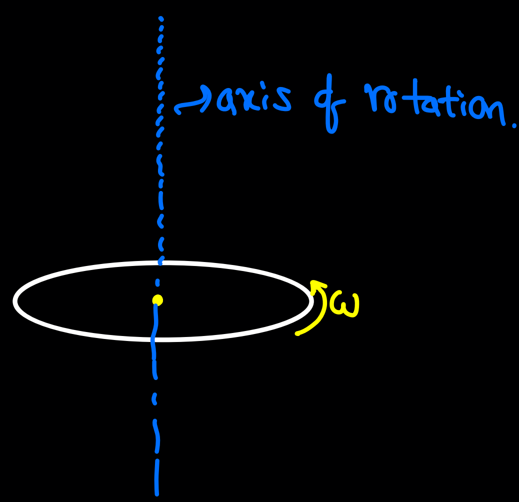
Rigid body:- under any circumstance relative seperation b/w any two points inside the body will not change.

⇒ stone, metal sphere these are close to rigid bodies.

Rotational motion:-

Axis of rotation:- its an imaginary line passing through centres of circular paths taken by particles of body and ⊥ to the plane of circular motion.





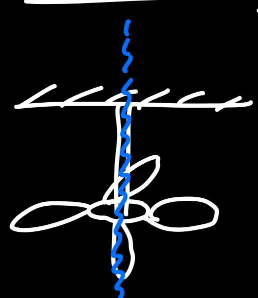
observations:-

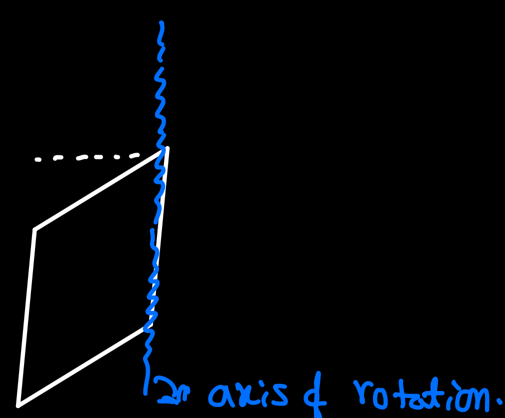
→ No two particles in same plane of circular motion will have same velocity and acceleration.

Types of rotational motion:-

① **Rotation about fixed axis:-**

eg:-

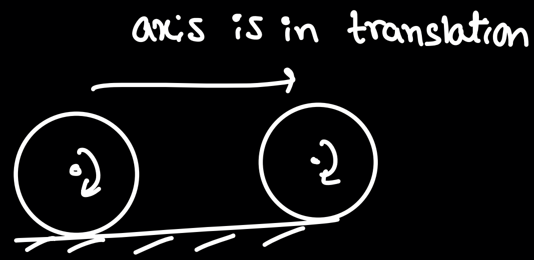




fixed axis

② Rotation about an axis in translation:- (rolling).

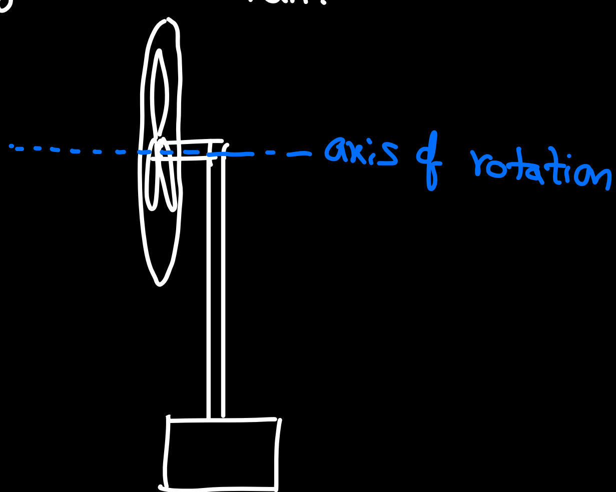
eg:-



⇒ This motion can be taken as superposition of pure rotation about axis and translation of axis.

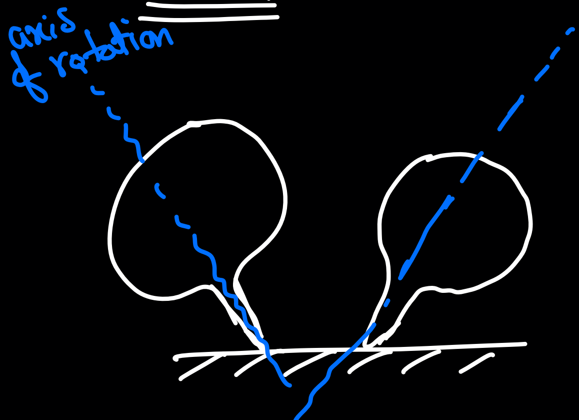
③ Rotation about an axis in rotation:-

eg:- Table fan.

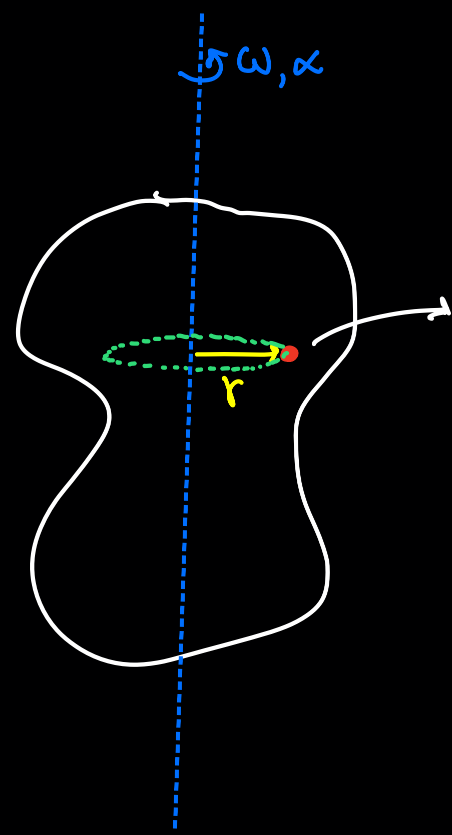


if swing mode is on then axis rotates.

spin top:-



Kinetics of body rotating about fixed axis :-



$$\vec{v} = \vec{\omega} \times \vec{r}$$

⊥ distance of point from axis.

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d\omega}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}$$

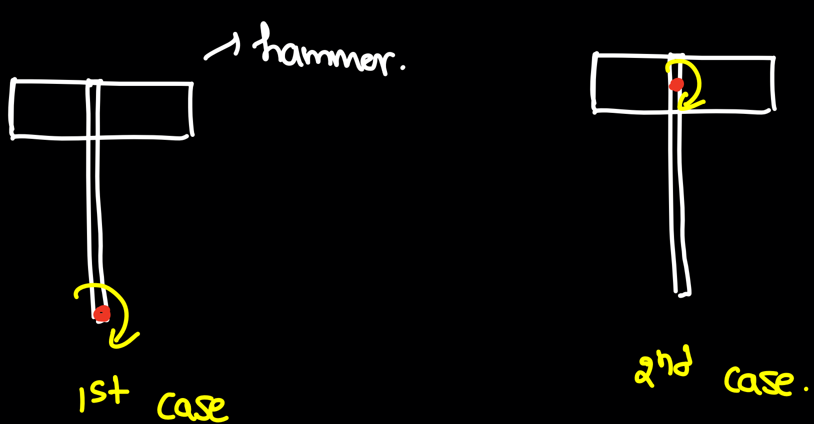
$$= \underbrace{\vec{\alpha} \times \vec{r}}_{\vec{a}_t} + \underbrace{\vec{\omega} \times \vec{v}}_{\vec{a}_n}.$$

$$\vec{a}_t = \vec{\alpha} \times \vec{r}$$

$$\vec{a}_n = \vec{\omega} \times \vec{v}$$

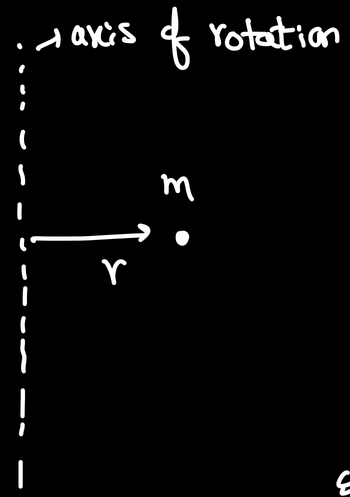
Moment of inertia (I) :- it is a measure of resistance offered for change in rotational motion.

eg:-



resistance offered in 1st case is more. this indicates that not only mass but distribution of mass also effects the resistance.

__Point mass:-__



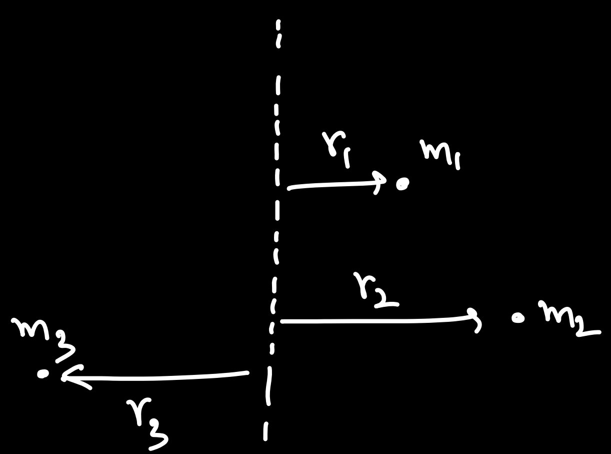
$I \propto m$

$I \propto r^2$

$I \propto mr^2$

$$\boxed{I = mr^2}$$

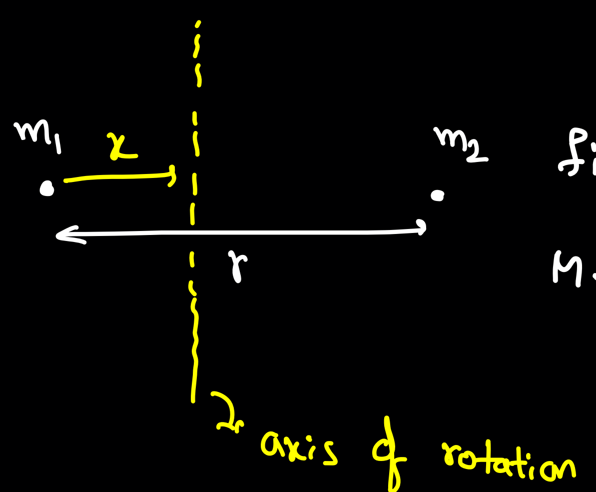
we can't use concept of C.M. here as we have (mass)(distance)^2. but not (mass)(distance).



$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots$

$$I = \Sigma mr^2$$

Q)

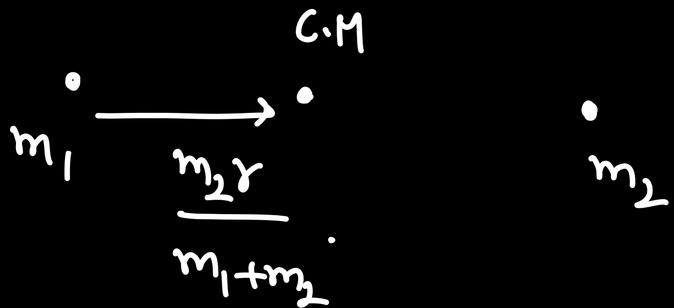


find the value of x for which M.I. is minimum?

sol:- $I = m_1 x^2 + m_2 (r-x)^2$.

$$\frac{dI}{dx} = 0$$

$$2m_1 x + 2m_2 (r-x)(-1) = 0 \Rightarrow x = \frac{m_2 r}{m_1+m_2}$$

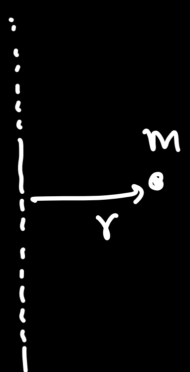


$\Rightarrow$ we got to know that M.I. is minimum about an axis passing through C.M.

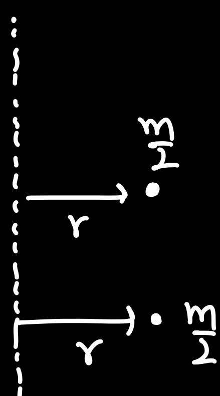
$\Rightarrow$ Any body which is free to choose its axis of rotation will choose an axis passing through C.M. as M.I is least.

---

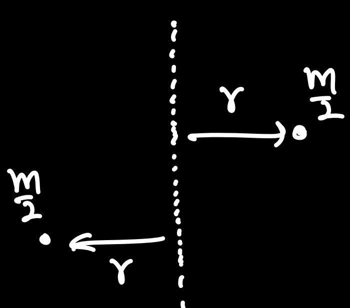
observation:-



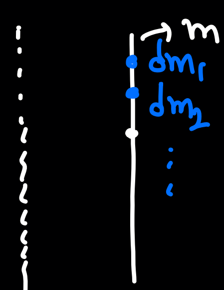
$I = mr^2$



$$I = \frac{m}{2}r^2 + \frac{m}{2}r^2$$
$$= mr^2.$$



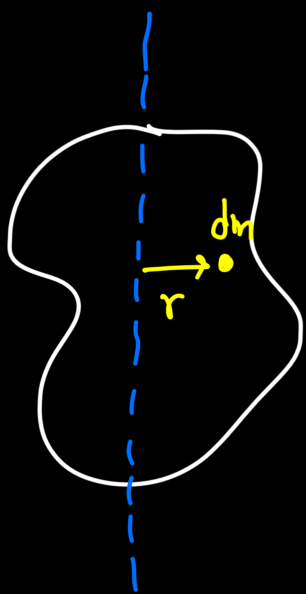
$$I = \frac{m}{2}r^2 + \frac{m}{2}r^2$$
$$= mr^2$$



$$I = dm_1 r^2 + dm_2 r^2 + \dots$$
$$= (dm_1 + dm_2 + \dots) r^2$$
$$= mr^2.$$

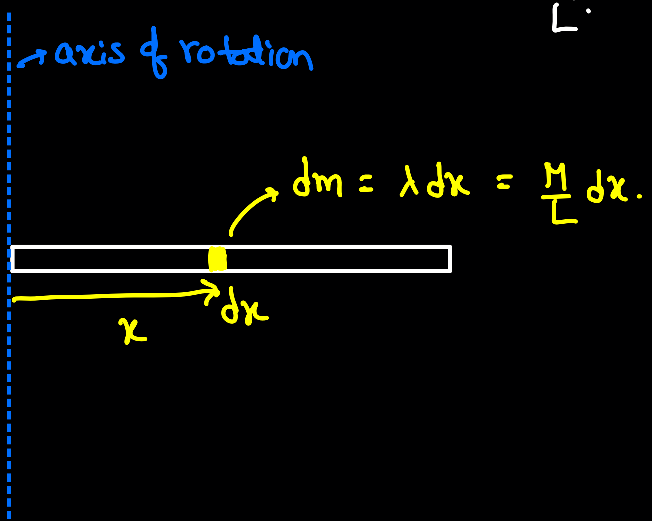
* ⇒ As long as total mass and distribution of mass doesn't change M.I. doesn't change.

M.I. of continuous mass distribution:-



$$I = \int (dm) r^2.$$

Uniform Rod :- (M, L) $\Rightarrow \lambda = \dfrac{M}{L}.$



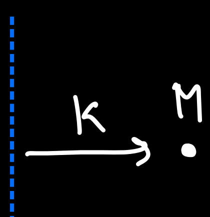
$$I = \int dm\, r^2$$

$$= \int \left(\frac{M}{L} dx\right)(x^2)$$

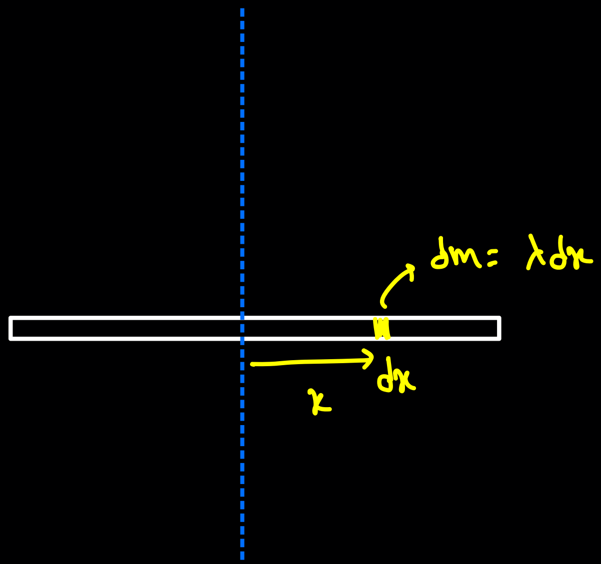
$$= \frac{M}{L}\int_0^L x^2\, dx$$

$$= \frac{ML^2}{3}.$$

if rod has to be replaced with point mass where the mass has to be kept is called radius of gyration (K).



$$I = MK^2 = \frac{ML^2}{3} \Rightarrow K = \frac{L}{\sqrt{3}}.$$



$$I = \int (dm) r^2$$

$$= \int (\lambda dx)\, x^2$$

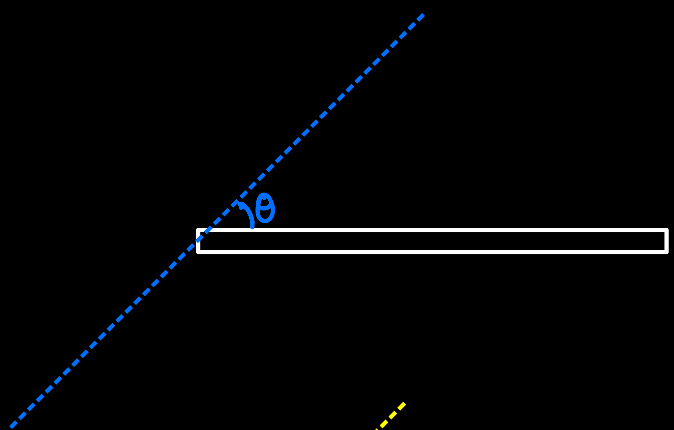
$$= \lambda \int_{-\frac{L}{2}}^{+\frac{L}{2}} x^2\, dx$$

$$= \frac{ML^2}{12}.$$

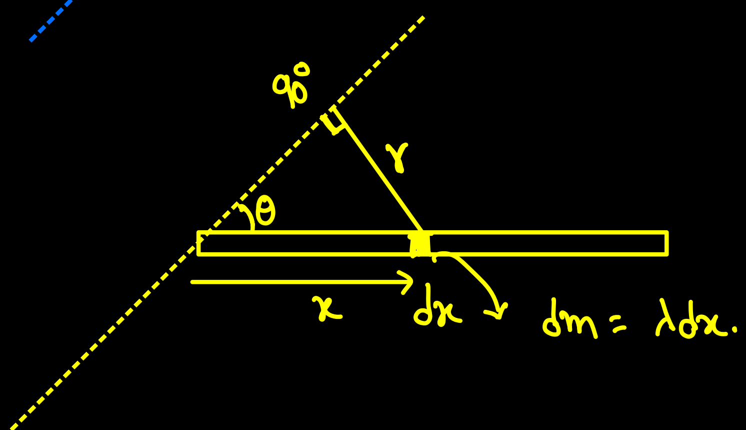
$\xrightarrow[K]{} \overset{M}{\cdot}$

$$MK^2 = \frac{ML^2}{12}$$

$$K = \frac{L}{\sqrt{12}}.$$



find M.I ?



$$\sin\theta = \frac{r}{x}$$

$$r = x\sin\theta.$$

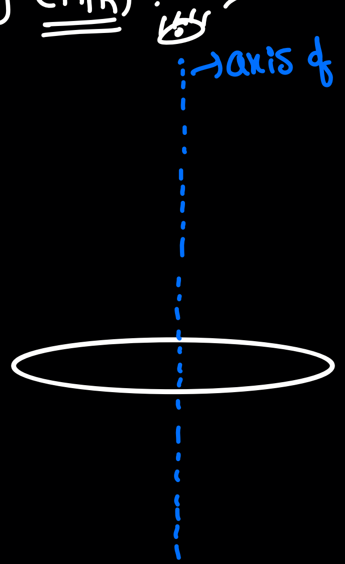
$$I = \int (dm) r^2$$

$$= \int (\lambda dx)(x\sin\theta)^2$$

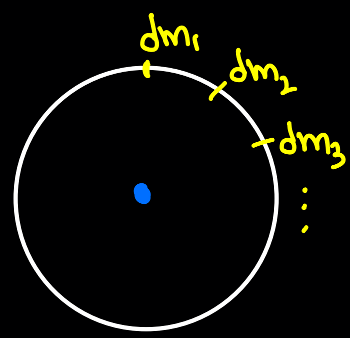
$$= \lambda \sin^2\theta \int_0^L x^2\, dx$$

$$= \frac{ML^2 \sin^2\theta}{3}.$$

Ring (M, R) :-



from top view

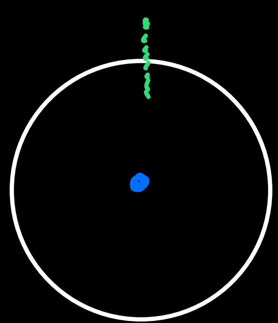


$$I = (dm_1)R^2 + (dm_2)R^2 + \dots\dots$$

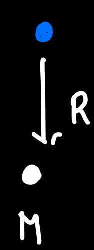
$$= (dm_1 + dm_2 + \dots\dots)R^2$$

$$= MR^2.$$

otherway of looking at it:

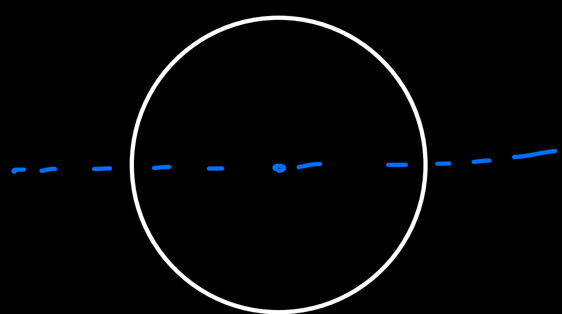


equivalent to

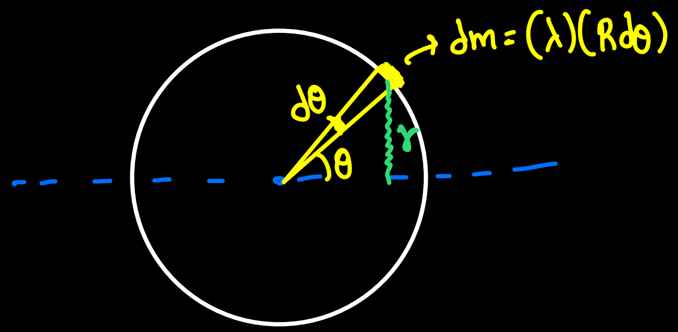


$I = \underline{MR^2}.$

Ring (M, R)



find M.I. about diameter?



$r = R\sin\theta$

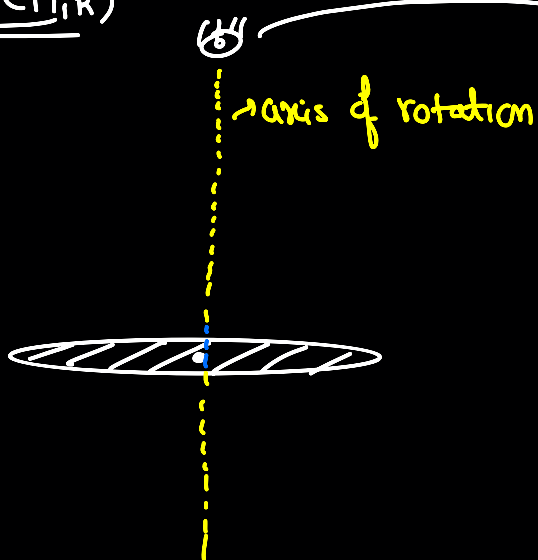
$$I = \int (dm) r^2$$

$$= \int (\lambda R\, d\theta) R^2 \sin^2\theta$$

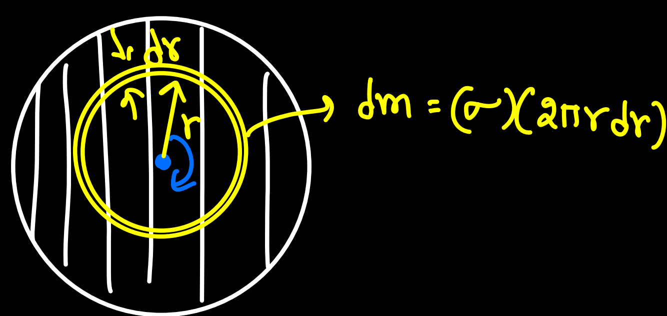
$$= \lambda R^3 \int_0^{2\pi} \sin^2\theta \, d\theta$$

$$I = \frac{MR^2}{2}.$$

Disc (M, R)



from top view



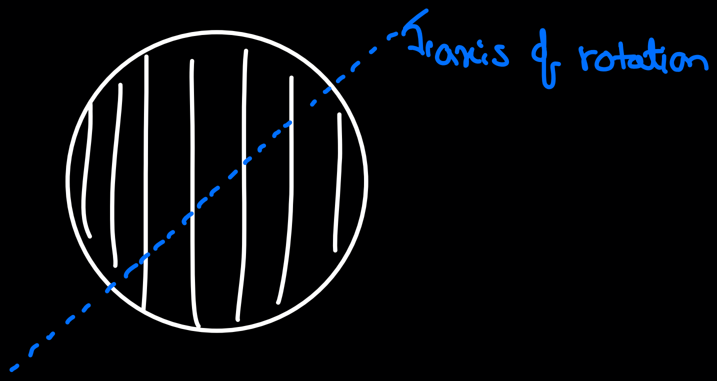
$$dI_{ring} = (dm) r^2$$

$$I = \int (dm) r^2$$

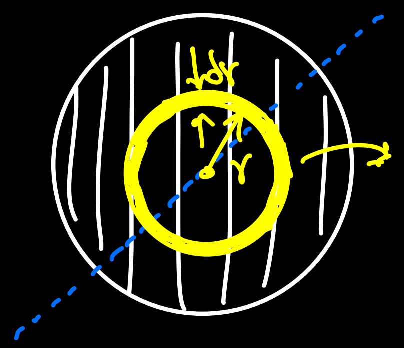
$$= \int_0^R (\sigma 2\pi r\, dr) r^2$$

$$I = \frac{MR^2}{2}.$$

disc (M, R) :-



find M.I ?

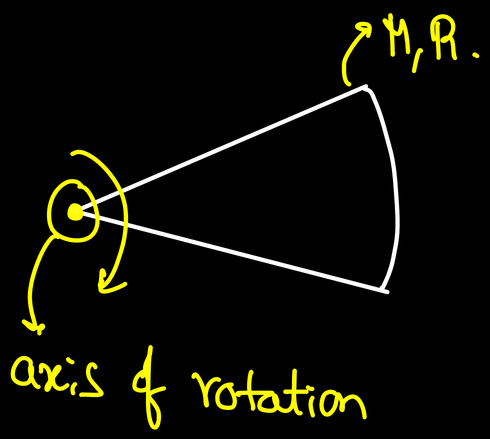


$dm = (\sigma)(2\pi r\, dr)$

$$dI_{ring} = \frac{(dm)r^2}{2}$$

$$I = \int \frac{(dm)r^2}{2} = \frac{\int (dm) r^2}{2} = \frac{\frac{MR^2}{2}}{2} = \frac{MR^2}{4}.$$

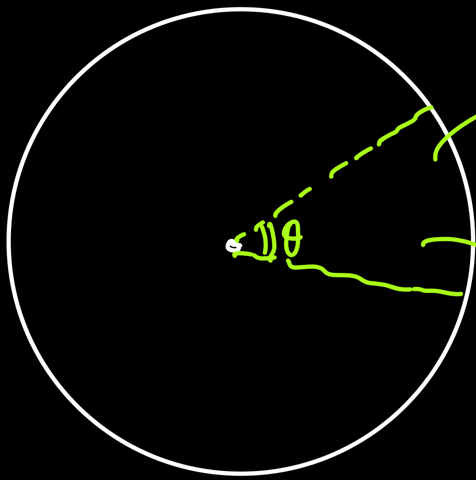
Q)



find M.I ?

Sol :-

m



θ

$m' = \frac{m\theta}{2\pi}$.

$I' = (I)\frac{\theta}{2\pi}$.

$= \frac{mR^2}{2}\frac{\theta}{2\pi}$

$= \left(\frac{m\theta}{2\pi}\right)\frac{R^2}{2}$

$= (m')\frac{R^2}{2}$.

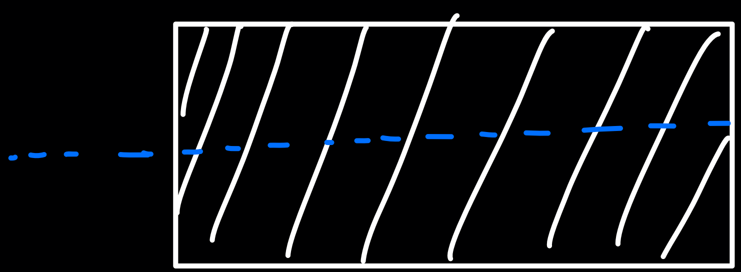
$I = (\text{mass of piece})\frac{R^2}{2}$

above question

$= (M)\frac{R^2}{2}$.

Sheet :- (Lamina).

Mass "M".

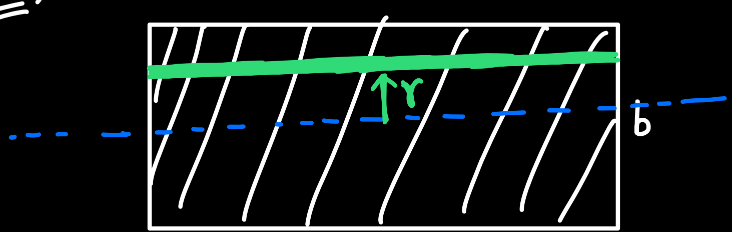


b

$I = ?$

l

Sol :-



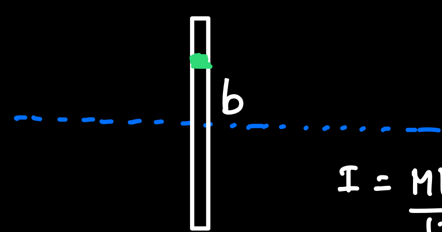
r

b

$I = ?$

l

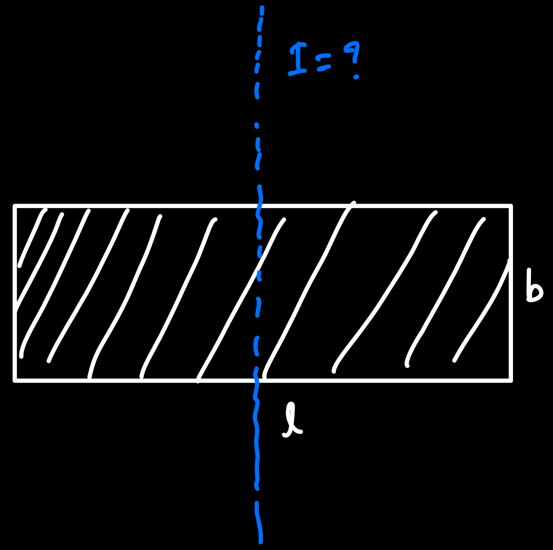
$\Rightarrow$

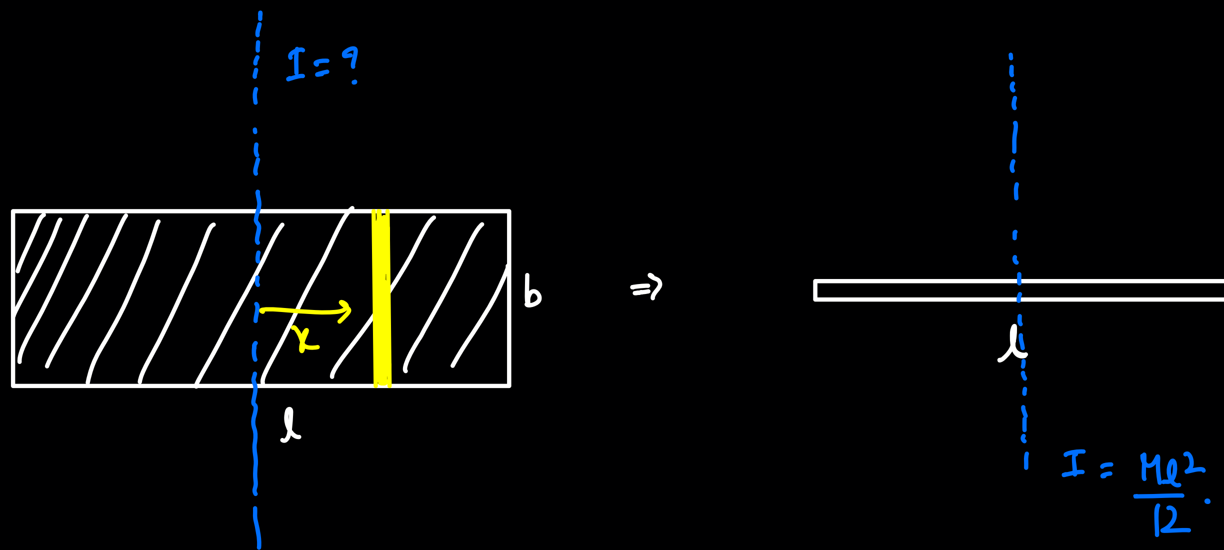


b

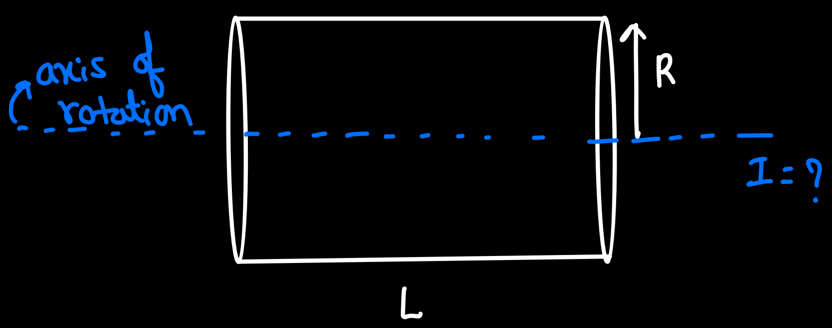
$I = \frac{Mb^2}{12}$.

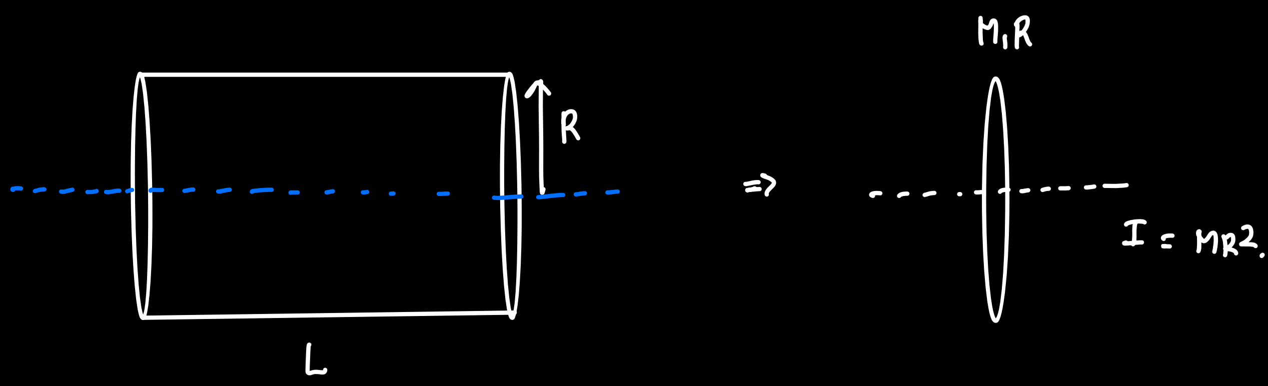
Q)



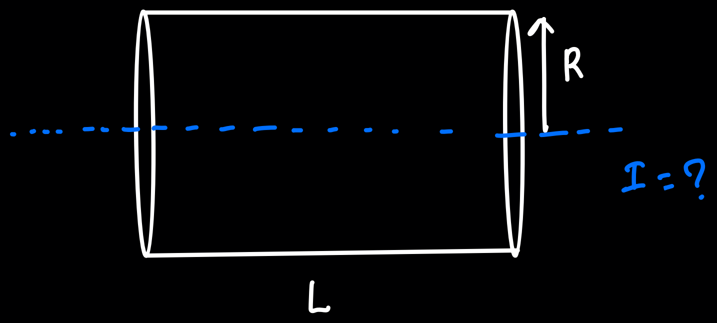


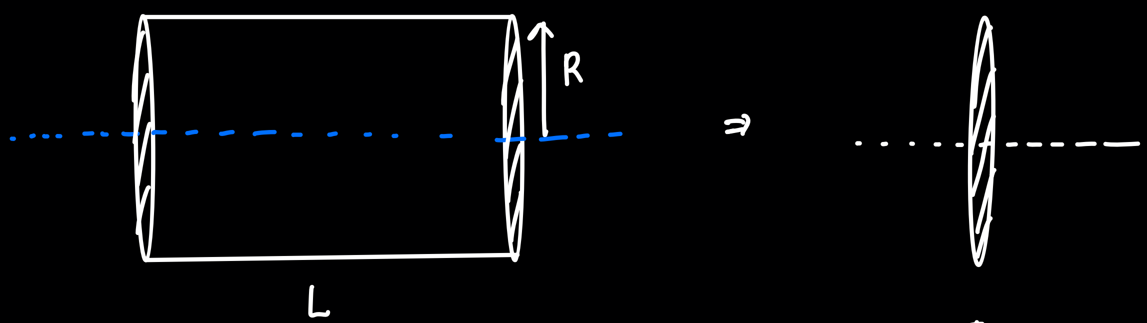
hollow cylinder (M, R):-



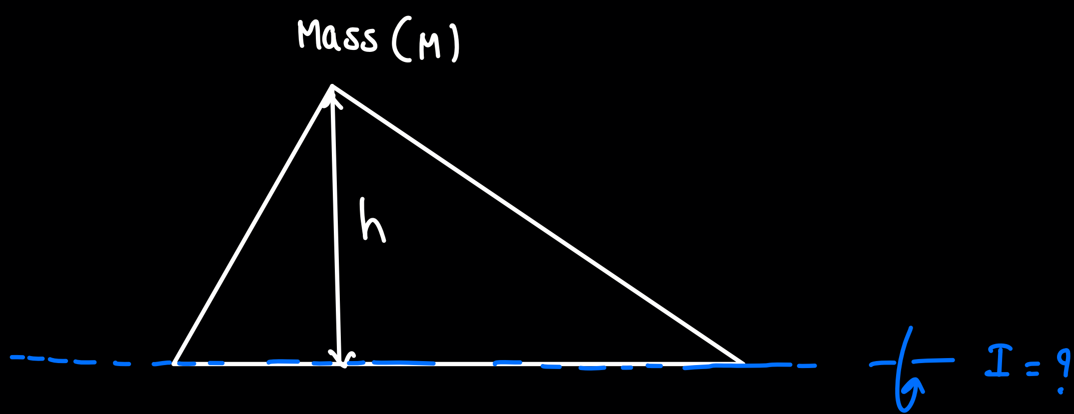


Solid cylinder (M, R, L) :-



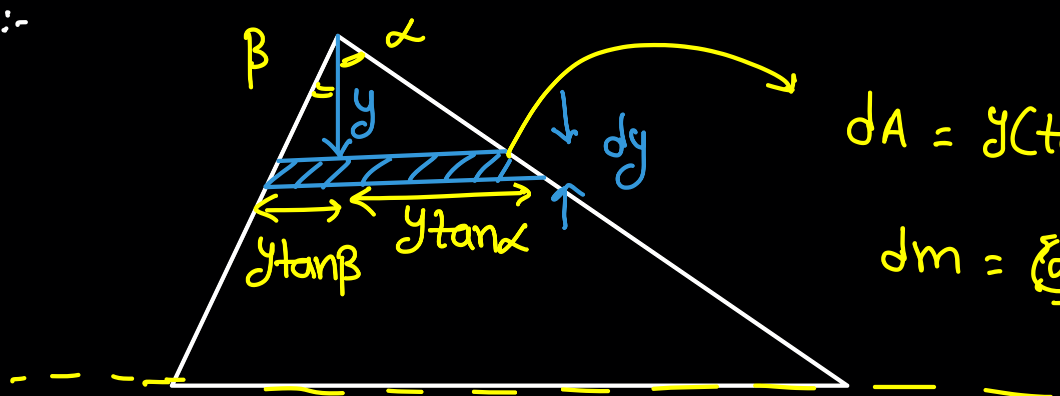


$$I = \frac{MR^2}{2}.$$



Ans : $\frac{Mh^2}{6}$.

Sol :-



$$dA = y(\tan\alpha + \tan\beta)\,dy.$$

$$dm = \underbrace{(\sigma)(\tan\alpha + \tan\beta)}_{C}\,y\,dy.$$

$$dI_{rod} = (dm)(h-y)^2$$

$$I = \int (dm)(h-y)^2$$

$$= \int c\, y\, dy\, (h^2+y^2-2hy)$$

$$= c\left[\int_0^h h^2 y\, dy + \int_0^h y^3 dy - 2h\int_0^h y^2 dy\right]$$

$$= c\left[\frac{h^4}{2}+\frac{h^4}{4}-\frac{2h^4}{3}\right]$$

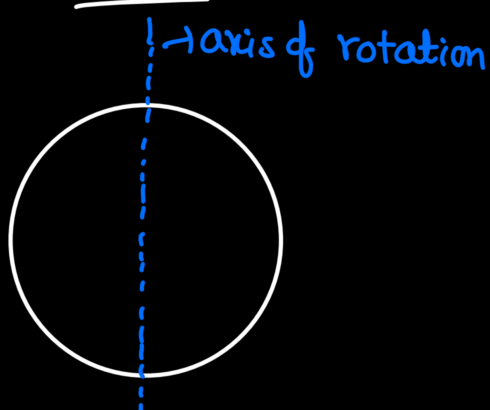
$$= \sigma(\tan\alpha+\tan\beta)\frac{h^4}{12}.$$

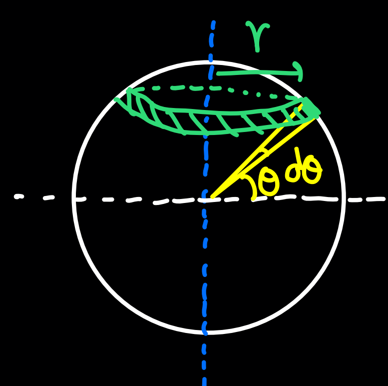
$$= \left(\sigma\left[\frac{1}{2}\underbrace{h(\tan\alpha+\tan\beta)}_{\text{base}}\underbrace{h}_{\text{height.}}\right]\right)\frac{h^2}{6}$$

$$= \frac{Mh^2}{6}.$$

---

hollow sphere (M, R) :-





$r = R\cos\theta.$

$dm = (\sigma)(2\pi r)(R\,d\theta)$

$= \sigma\, 2\pi R^2 \cos\theta\, d\theta$

$$dI_{ring} = (dm)r^2 = (dm)(R^2\cos^2\theta)$$

$$I = \int (dm)r^2 = \int \sigma\, 2\pi R^4 \cos^3\theta\, d\theta$$

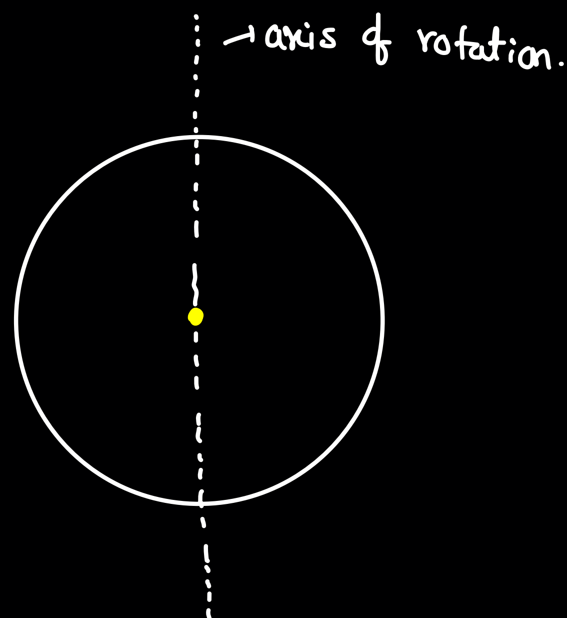
$$I = \sigma\, 2\pi R^4 \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \cos^3\theta\, d\theta$$

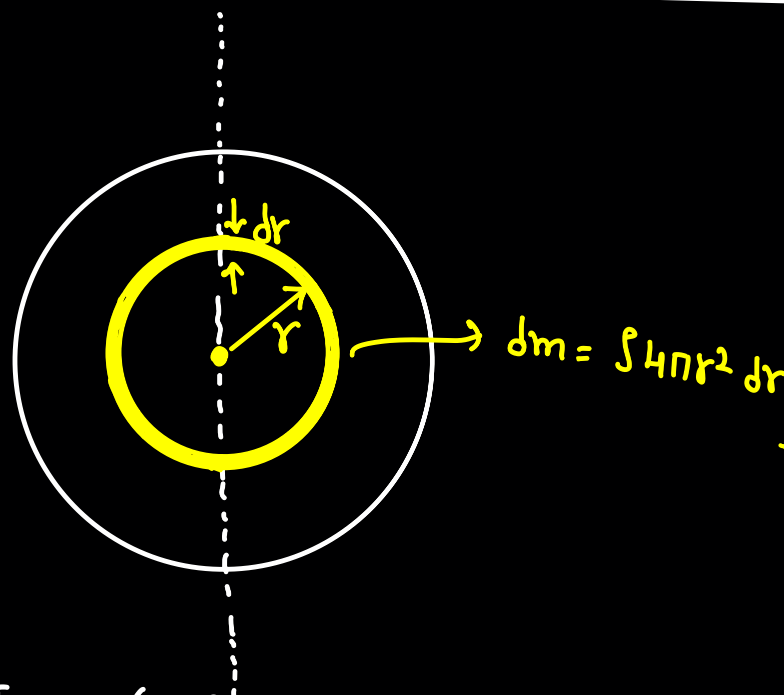
formula : $\cos^3\theta = \dfrac{3\cos\theta + \cos 3\theta}{4}$.

$$I = \frac{2}{3}MR^2.$$

---

solid sphere (M, R):-



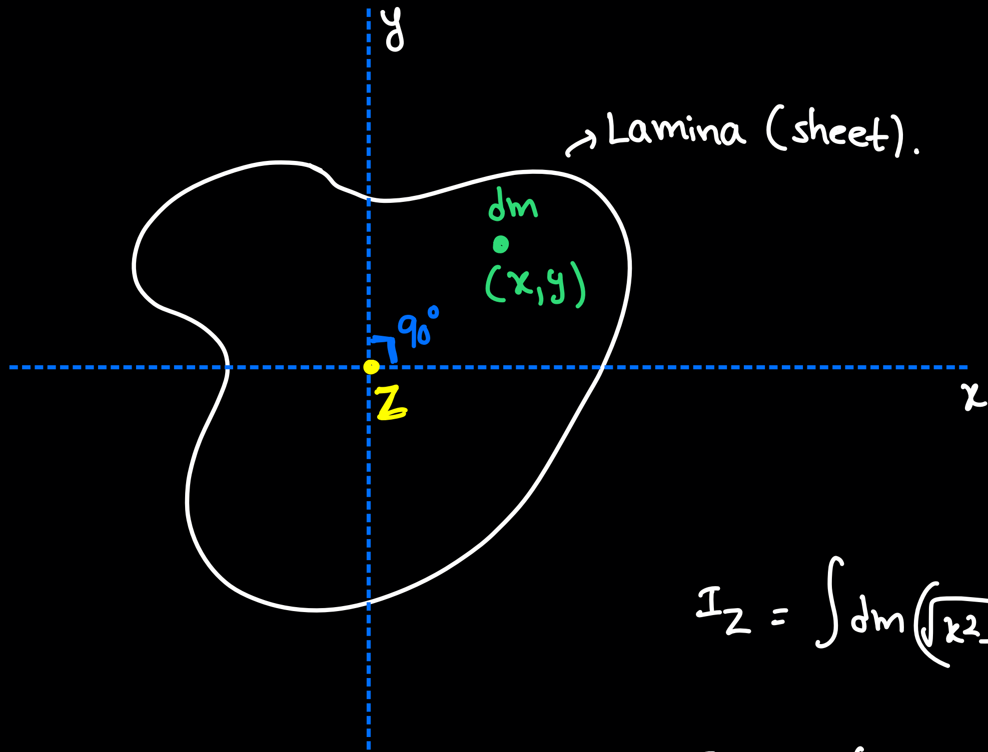


$$dI = \frac{2}{3}(dm)r^2.$$

$$I = \int_0^R \frac{2}{3}\left(\rho\, 4\pi r^2 dr\right) r^2$$

$$I = \frac{2}{5}MR^2$$

Perpendicular axis theorem:- [1D, 2D].

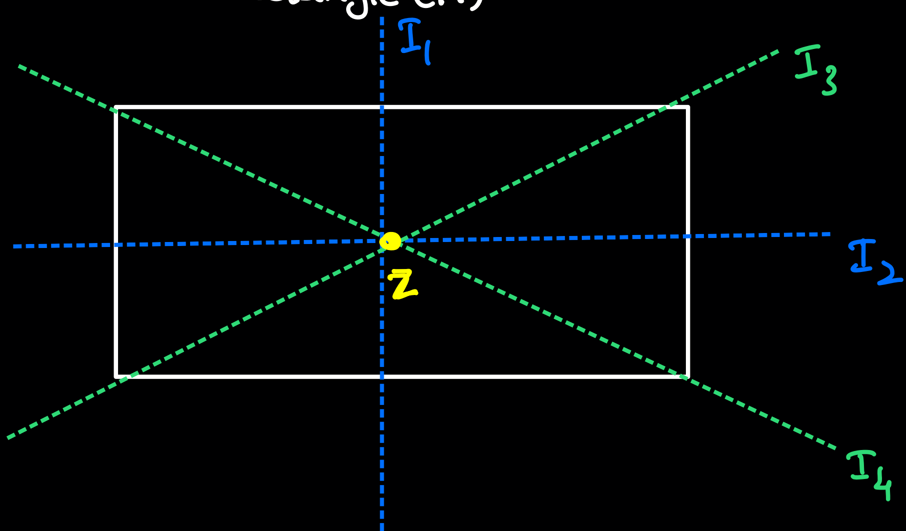


$$I_Z = \int dm\left(\sqrt{x^2+y^2}\right)^2$$

$$I_Z = \int dm\, y^2 + \int dm\, x^2.$$

$$\boxed{I_Z = I_x + I_y}$$

Q) rectangle (M)



I_Z ?

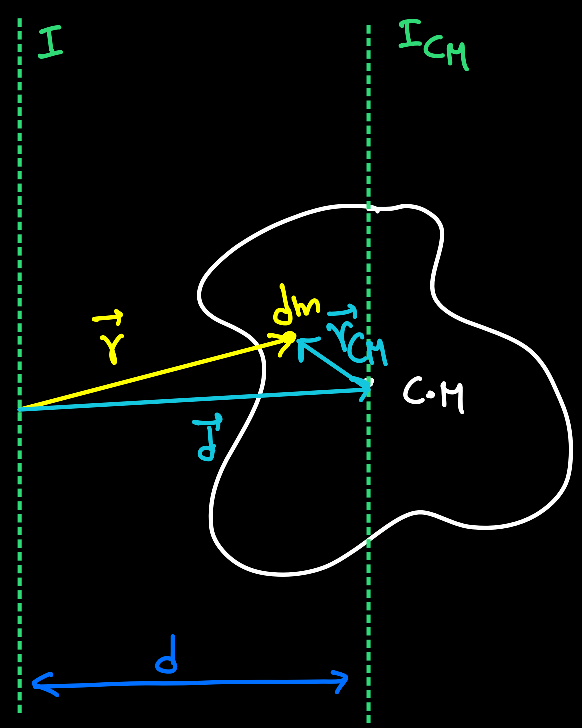
sol :-

$$I_Z = I_1 + I_2$$

$$I_Z = I_3 + I_4 \quad \times.$$

axis are not ⊥ to each other.

Parallel axis theorem:- [1D, 2D, 3D].



$$I = \int (dm)(\vec{r}\,)^2$$

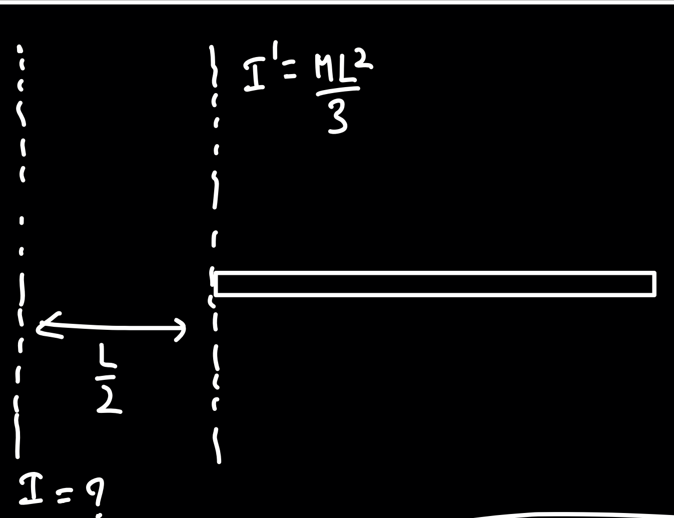
$$= \int (dm)\left[d^2 + r_{CM}^2 + 2r_{CM}\,d\cos\theta\right].$$

$$I = \underbrace{\int (dm) r_{CM}^2}_{I_{CM}} + \int (dm) d^2 + \int dm\, 2r_{CM}\,d\cos\theta$$

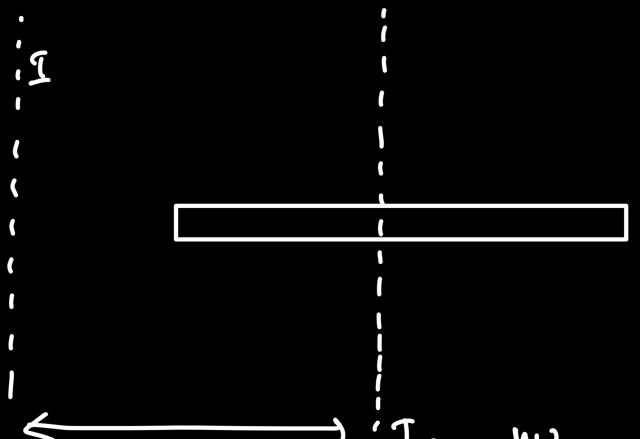
$$I = I_{CM} + d^2 \int dm + 2d\underbrace{\int (dm)(r_{CM}\cos\theta)}_{0}$$

$$\boxed{I = I_{CM} + Md^2}$$

Q)



$$I = \frac{ML^2}{3} + M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{3} + \frac{ML^2}{4} = \frac{7ML^2}{12}.$$



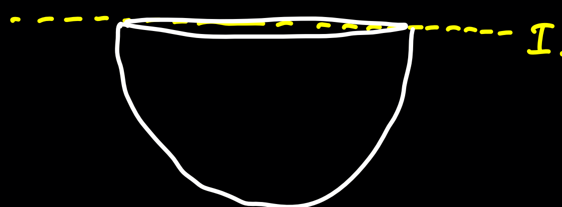
$I_{CM} = \frac{ML^2}{12}$.

$$I = \frac{ML^2}{12} + ML^2$$

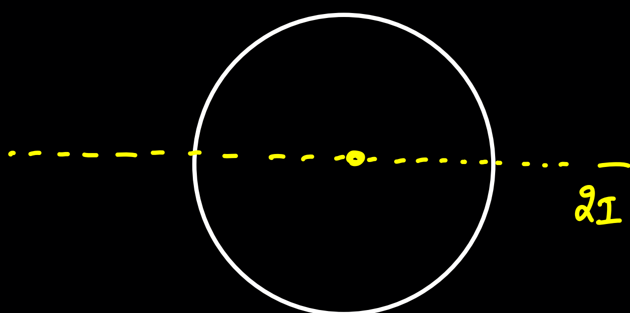
$$= \frac{13ML^2}{12}.$$

Q) semi-hollow sphere (M)

Sol:- 1st method:-

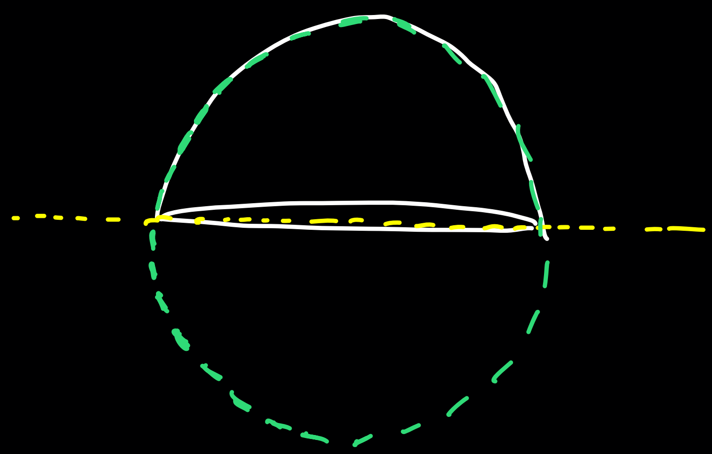


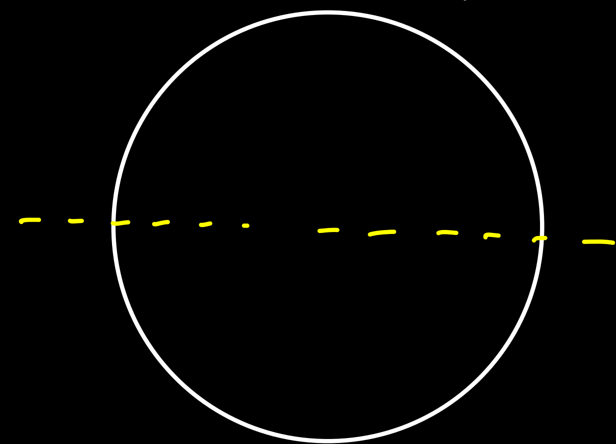
2M



$$2I = \frac{2}{3}(2M)R^2 \Rightarrow I = \frac{2}{3}MR^2.$$

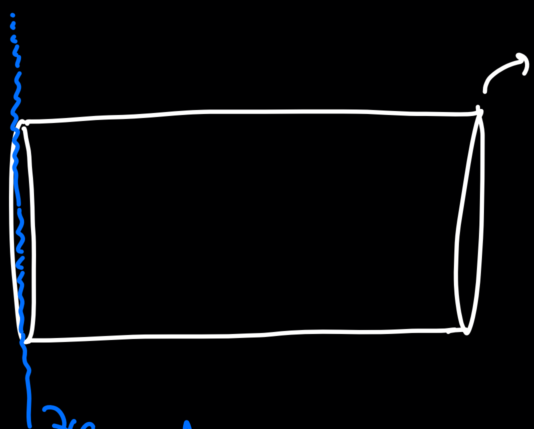
2nd method :-



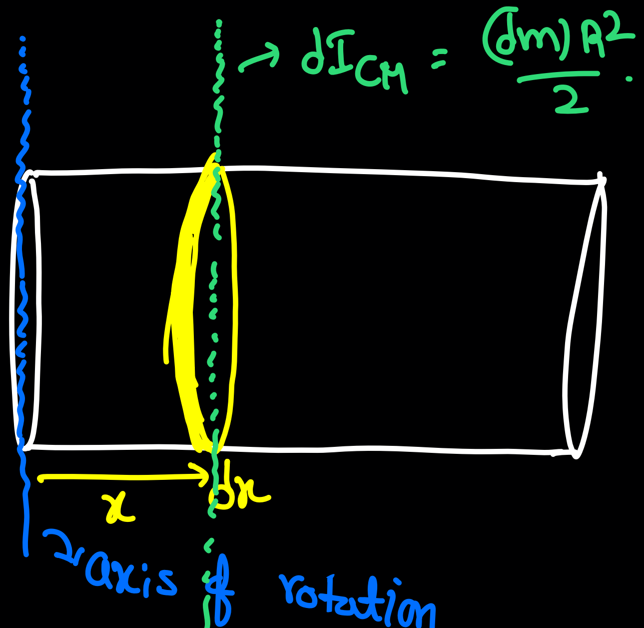


$$I = \frac{2}{3} MR^2.$$

Q)



Sol :-



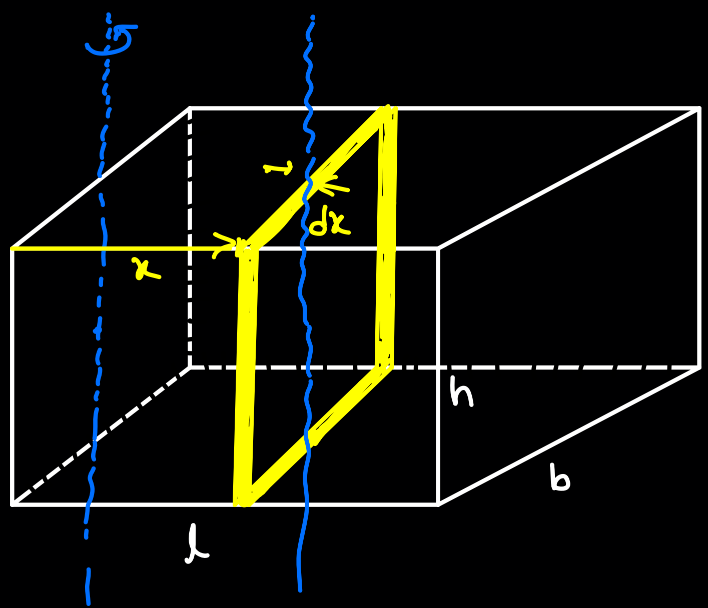
$$dI = dI_{CM} + (dm)x^2.$$

$$= \frac{(dm)R^2}{2} + (dm)x^2$$

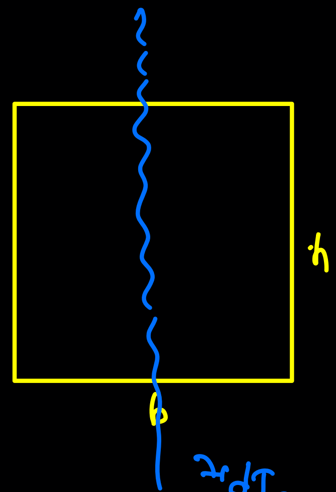
$$\int dI = \int_0^L \frac{\left(\frac{M}{L}dx\right)R^2}{2} + \int_0^L \left(\frac{M}{L}dx\right)x^2$$

$$I = \frac{MR^2}{2} + \frac{ML^2}{3}.$$

Cuboid (M)



find M.I about the axis shown?



$\Rightarrow dI_{CM} = \frac{dm\,b^2}{12}.$

$$dI = dI_{CM} + (dm)x^2.$$

$$dI = \frac{(dm)b^2}{12} + (dm)x^2$$

$$I = \int \frac{dm\,b^2}{12} + \int_0^l \frac{M}{l}\,dx\,x^2$$

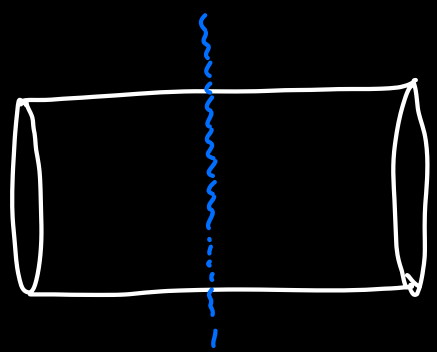
$$I = \frac{b^2}{12}\int dm + \frac{M}{l}\int x^2 dx$$

$$I = \frac{b^2}{12}M + \frac{Ml^2}{3}.$$

$$I = \frac{Mb^2}{12} + \frac{Ml^2}{3}.$$

Q) for fixed mass, find ratio of $\frac{l}{R}$ of a cylinder for which M.I. about an axis $\perp$ to length and passing through C.M is minimum?

Sol:



$$I = \frac{ML^2}{12} + \frac{MR^2}{4}.$$

$$I = \frac{ML^2}{12} + \frac{MR^2}{4}.$$

$$M = (\rho)\pi R^2 L.$$

$$R^2 = \frac{M}{\pi \rho L}.$$

$$I = \frac{ML^2}{12} + \frac{M^2}{4\pi\rho L}$$

$$\frac{dI}{dL} = \frac{2ML}{12} + \frac{M^2}{4\pi\rho}\left[-\frac{1}{L^2}\right].$$

$$0 = \frac{2\cancel{M}L}{12} + \frac{M\cancel{^2}}{4\pi\rho}\left(-\frac{1}{L^2}\right)$$

$$0 = \frac{L}{6} - \frac{M}{4\pi\rho L^2}.$$

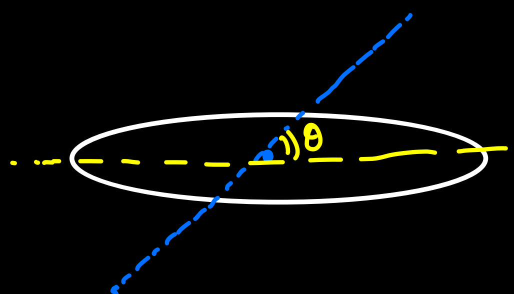
$$L^3 = \frac{6M}{4\pi\rho}.$$

$$L^3 = \frac{6\left[\cancel{\rho}\, \cancel{\pi} R^2 L\right]}{4\cancel{\pi}\,\cancel{\rho}}$$

$$\frac{L^2}{R^2} = \frac{3}{2}$$

$$\frac{L}{R} = \sqrt{\frac{3}{2}}.$$

HW :- ring(M, R)

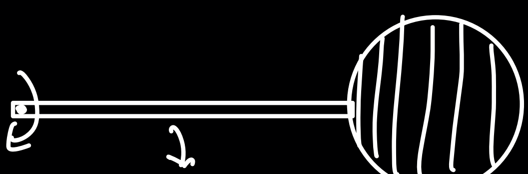


axis makes an angle θ with plane.

find M.I about the axis ?

M.I. of composite bodies:-

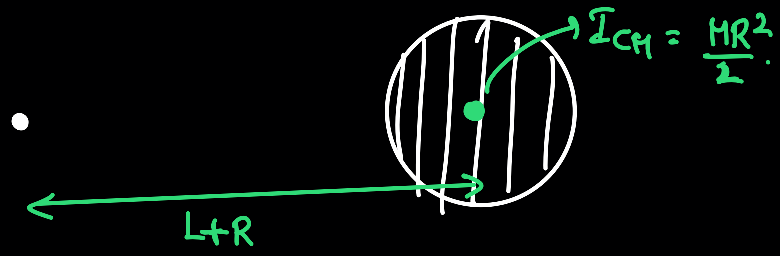
Q) disc (M, R)



find M.I. about an axis passing through one end of rod and ⊥ to the plane?

sol :-

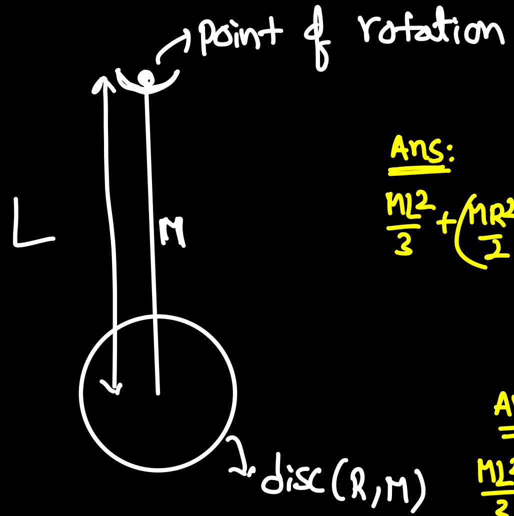
$$I_{rod} = \frac{ML^2}{3}.$$



$$I_{disc} = \frac{MR^2}{2} + M(L+R)^2$$

$$I_{Total} = \frac{ML^2}{3} + \frac{MR^2}{2} + M(L+R)^2$$

H.W :-



Case(i)

Rod is welded to disc in such a way that disc can't rotate about centre of disc. Then find M.I ?

Ans: $\frac{ML^2}{3} + \left(\frac{MR^2}{2} + ML^2\right)$

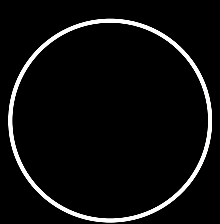
Case(ii)

disc is free to rotate about its centre. then find M.I ?

Ans :- $\frac{ML^2}{3} + ML^2$

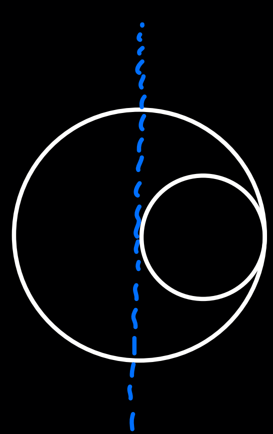
M.I. of bodies with cavity :-

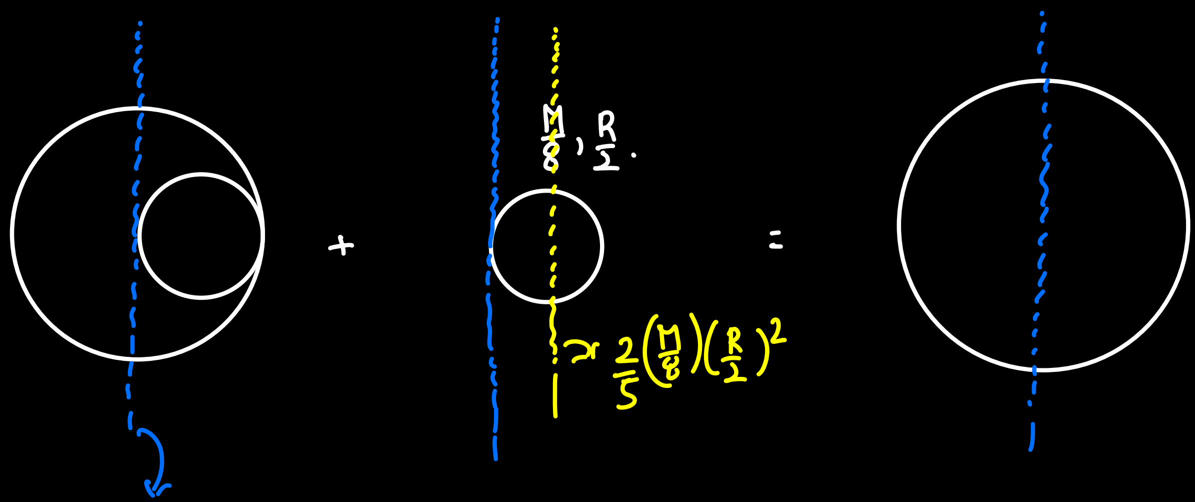
Q)



→ solid sphere (M, R).

⇒ spherical cavity is made as shown in figure. Find M.I. about diameter ?

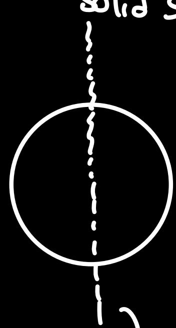




$$I + \left[\frac{2}{5}\left(\frac{M}{8}\right)\left(\frac{R}{2}\right)^2\right] + \left(\frac{M}{8}\right)\left(\frac{R}{2}\right)^2 = \frac{2}{5}MR^2. \Rightarrow I = \frac{57}{160}MR^2.$$

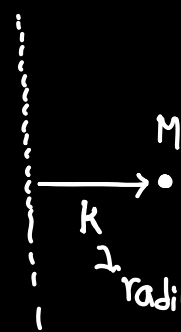
Radius of Gyration:-

eg:- solid sphere (M,R)



$I = \frac{2}{5}MR^2$

$\Rightarrow$



radius of gyration

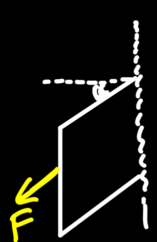
$\underline{I = Mk^2}$

$$\cancel{M}k^2 = \frac{2}{5}\cancel{M}R^2$$

$$k = \sqrt{\frac{2}{5}}\,R.$$

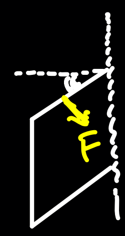
Torque (τ) :- Torque is responsible for change in rotational motion.

$\Rightarrow$ Torque is not only decided by Force but also point of application of force.

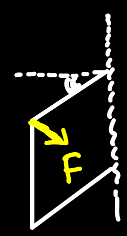


door doesn't rotate.

case(i)



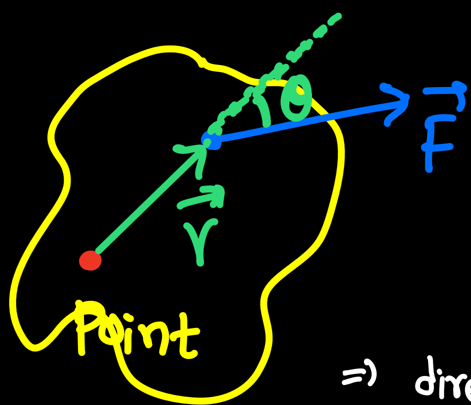
case(ii)



rotating the door in case ii is easy.

⇒ both the above situations indicate that position of force and the way we apply that force matters for rotational motion.

Torque about a point :-



$$\vec{\tau}_{point} = \vec{r} \times \vec{F}$$

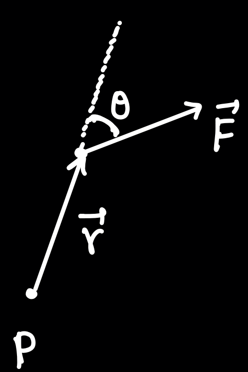
$$\vec{\tau} \perp \vec{r}, \vec{F}$$

⇒ direction of torque is ⊥ to plane in which $\vec{r}$ and $\vec{F}$ are present.

⇒ CW ⇒ ↻ ⇒ ⊗ ⇒ into the plane.

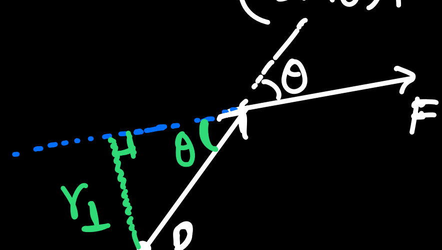
ACW ⇒ ↺ ⇒ ⊙ ⇒ out of the plane.

observations :-

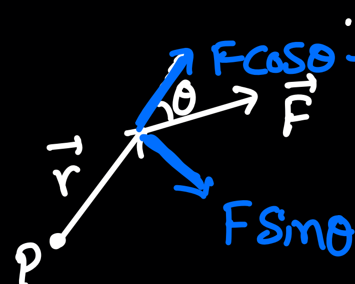


$$|\vec{\tau}_{Point}| = rF\sin\theta$$

$$\tau = (r\sin\theta)F$$



$$\tau = r(F\sin\theta)$$

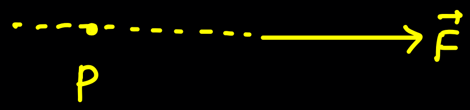


$\sin\theta = \frac{r_\perp}{r} \Rightarrow r_\perp = r\sin\theta$

$\tau = (r)(F_\perp)$.

$\tau = (r_\perp)F$.

Q)



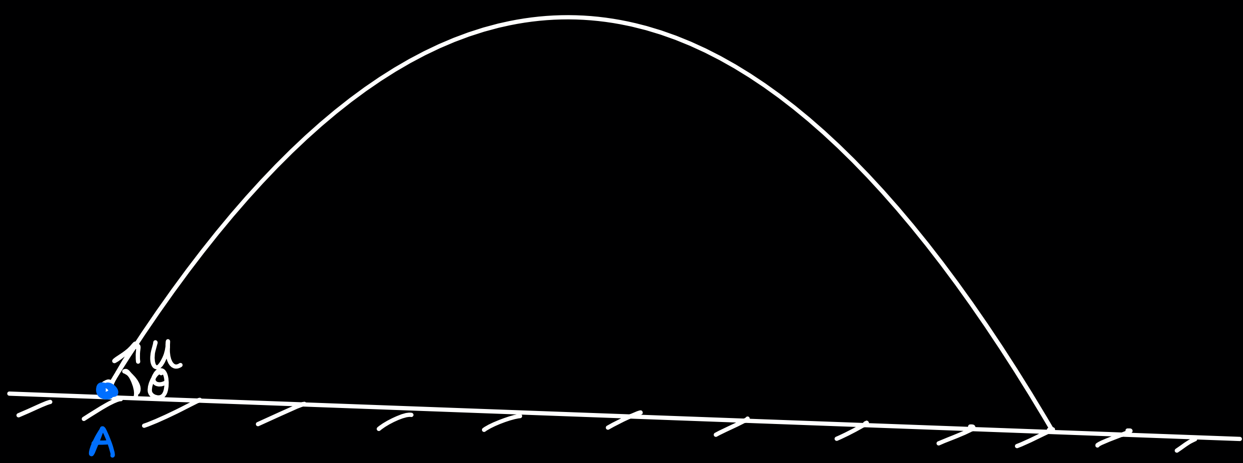
$\tau_P = ?$

Sol:-

$$\tau_P = (r_\perp)F = (0)F = 0.$$

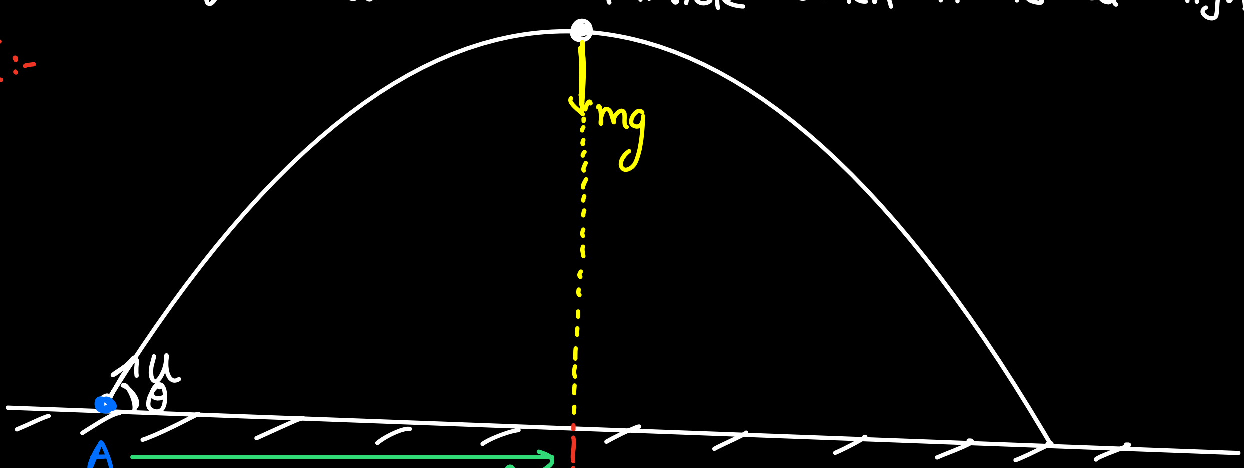
$$\tau_P = (r)F_\perp = (r)(0) = 0.$$

Q)



find torque about A on particle when it is at highest point?

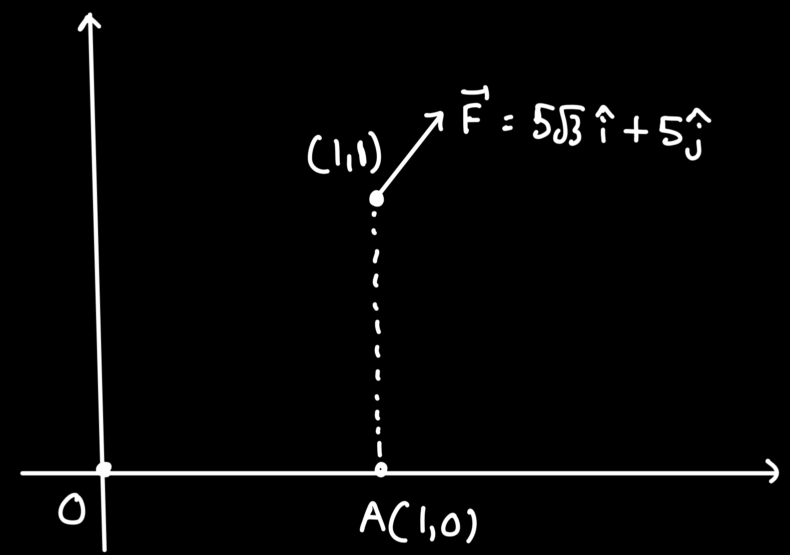
Sol:-



$$\frac{R}{2} = \frac{u_x u_y}{g} = \frac{u^2\sin 2\theta}{2g}.$$

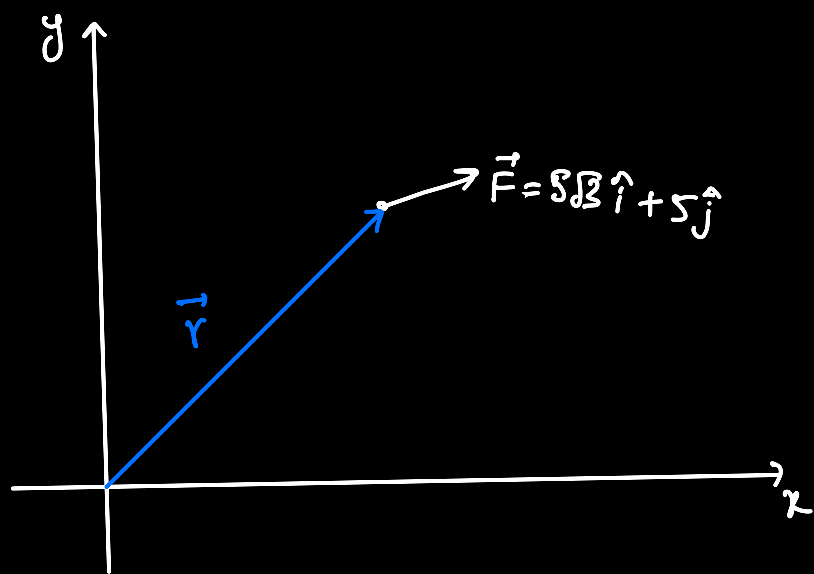
$$\tau_A = (r_\perp) mg = \frac{u^2 \sin 2\theta}{2g} \times mg = \frac{m u^2 \sin 2\theta}{2}.$$

Q)



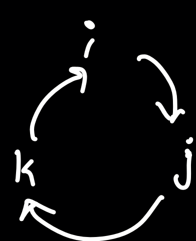
find τ_O, τ_A ?

Sol:-



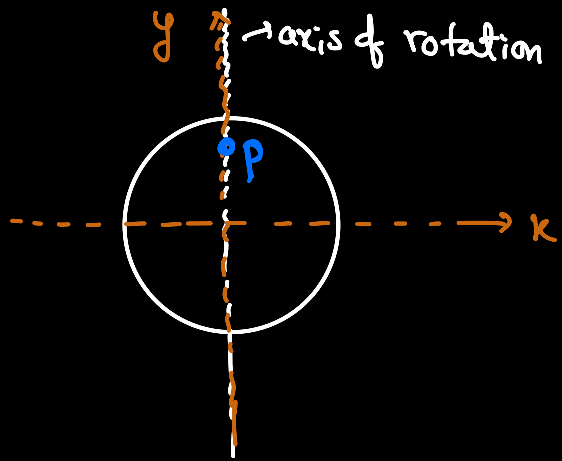
$$\vec r = (x_2 - x_1)\hat i + (y_2 - y_1)\hat j$$
$$= (1-0)\hat i + (1-0)\hat j$$
$$= \hat i + \hat j.$$

$$\vec \tau_O = \vec r \times \vec F$$
$$= (\hat i + \hat j) \times (5\sqrt3\,\hat i + 5\hat j)$$
$$= 5\hat k - 5\sqrt3\,\hat k$$
$$= 5(\sqrt3 - 1)(-\hat k).$$



Torque about an axis:- its the component of torque about any point on axis of rotation along axis of rotation.

eg:-



eg:-

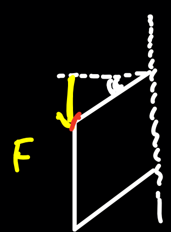
$\tau_P = 2\hat{i} + 3\hat{j} + 4\hat{k}$.

Torque about point P due to all forces on body.

$\Rightarrow$ here component of torque along the axis is "3"

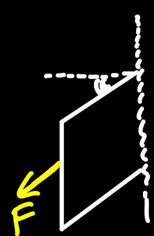
$$\tau_{axis} = \underline{\underline{3}}.$$

Case (i) Force being Parallel to axis of rotation.

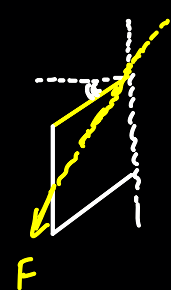


$\Rightarrow$ Torque by all forces parallel to axis of rotation about the axis = 0.

Case (ii):- line of Force intersecting with axis of rotation.

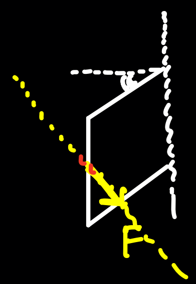


&



$\Rightarrow$ Torque about axis by these forces = 0.

Case(iii) Forces which are not parallel & antiparallel to axis of rotation and not intersecting with axis of rotation.

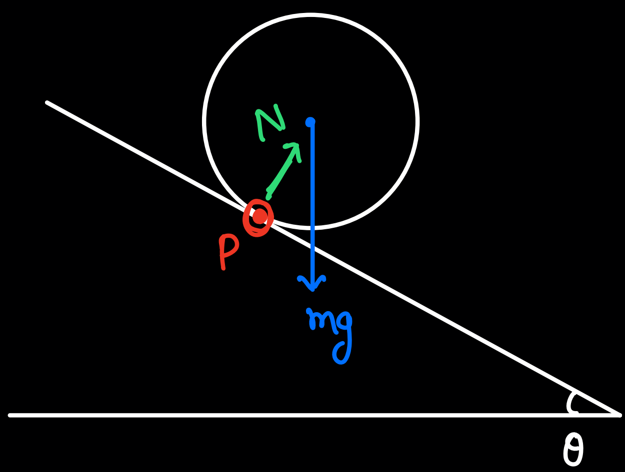


$\tau_{axis} = (r_\perp) F.$

its the $\perp$ distance b/w line of force and axis of rotation.

ill: 24,25 :-

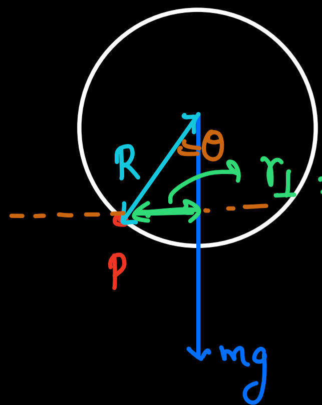
ill: 24



about axis passing through P

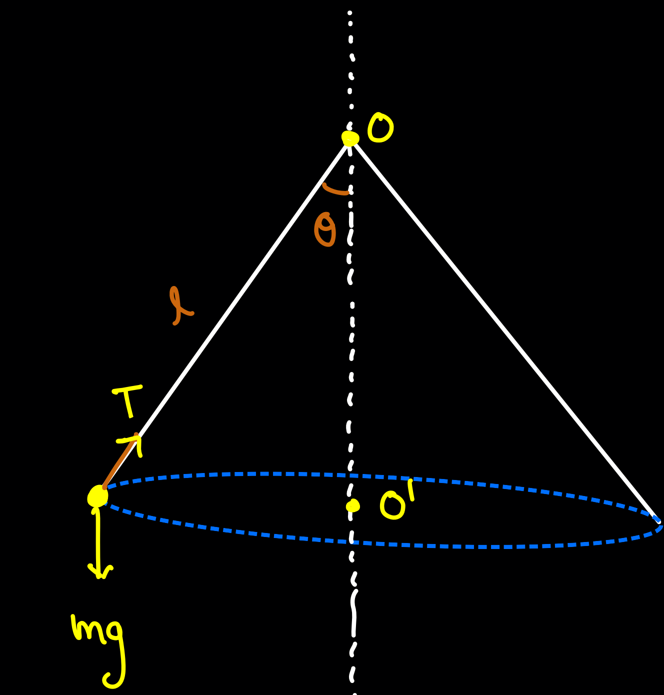
τ_{axis} due to N $= 0.$

τ_{axis} due to mg $\neq 0.$



$\tau_{axis} = (mg) R\sin\theta.$

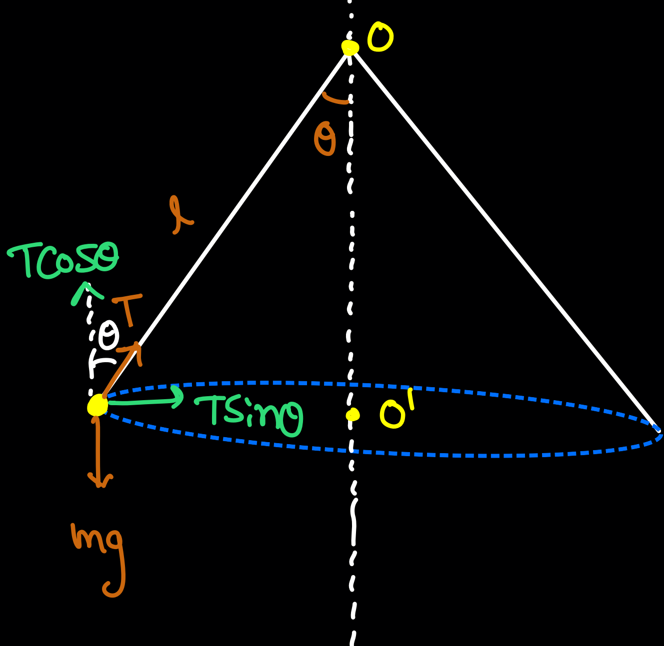
ill:25 :-



About O :-

$$\tau_O = (0)T + (l\sin\theta)mg \circlearrowleft$$

$$\underline{\underline{\tau_O \neq 0}}.$$



About O' :-

$$\tau_{O'} = (T\cos\theta)l\sin\theta \circlearrowright$$

$$+ (mg)l\sin\theta \circlearrowleft.$$

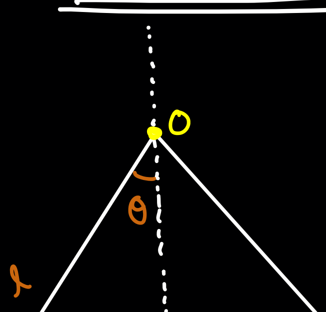
from NLM

$$\Sigma F_y = 0$$

$$T\cos\theta = mg$$

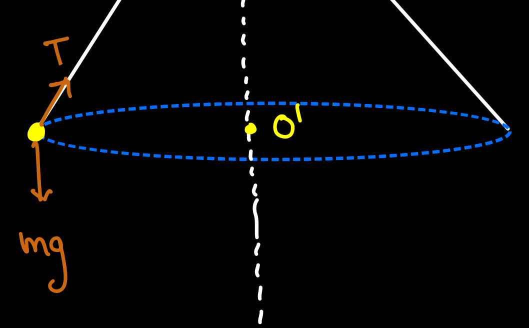
$$\boxed{\tau_{O'} = 0}$$

about OO' axis :-



$$\tau_{mg} = 0$$

$$\tau_T = 0$$



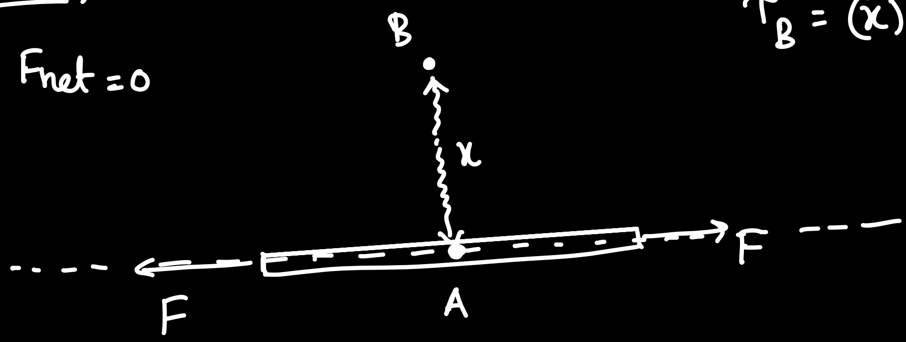
$\therefore \tau_{OO'} = 0$

Statement:-

if Net force on a body is zero then Net Torque about a point on the body should be zero. (False)

Case-(i):-

$F_{net} = 0$

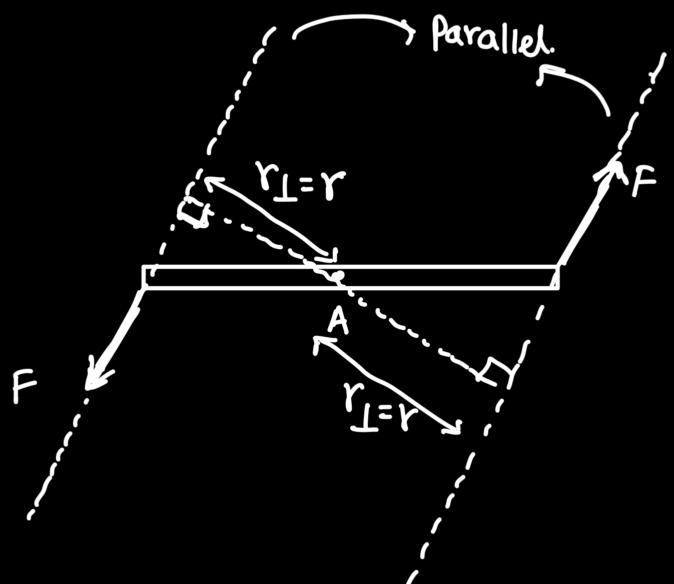


$\tau_B = (x)F \circlearrowright + xF \circlearrowleft = 0.$

$\tau_A = r_\perp F$

$= (0)(F) + (0)F$

$= 0.$

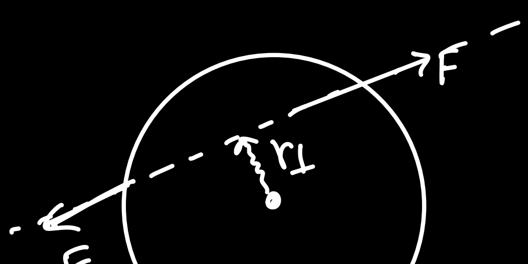


$F_{net} = 0$

$\tau_A = (r_\perp)F \circlearrowleft + (r_\perp)F \circlearrowleft$

$= 2rF \circlearrowleft.$

$\tau \neq 0.$



$F_{net} = 0$

$\tau = (r_\perp)F \circlearrowleft + (r_\perp)F \circlearrowright$

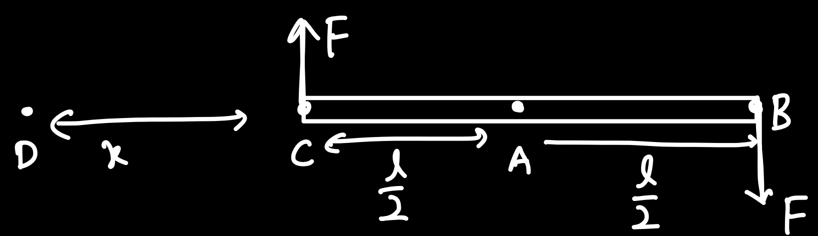
$= 0.$

$\Rightarrow$ if we have two equal and opposite forces along the same line then $\tau_{net} = 0.$

internal forces contribute for "0" torque.

$\Rightarrow$ if we have two equal and opposite forces not acting along same line then $\tau_{net} \neq 0.$

Couple



$$\tau_A = \frac{l}{2}F \circlearrowright + \frac{l}{2}F \circlearrowright = lF \circlearrowright$$

$$\tau_B = (0)F + (l)F \circlearrowright = lF \circlearrowright$$

$$\tau_C = (0)F + lF \circlearrowright = lF \circlearrowright.$$

$$\tau_D = \left.(x)F \circlearrowleft + (l+x)F \circlearrowright \right\} = lF \circlearrowright.$$

Observation:-

* When $F_{net} = 0 \Rightarrow$ then torque about any point is space will be same.

recap :-

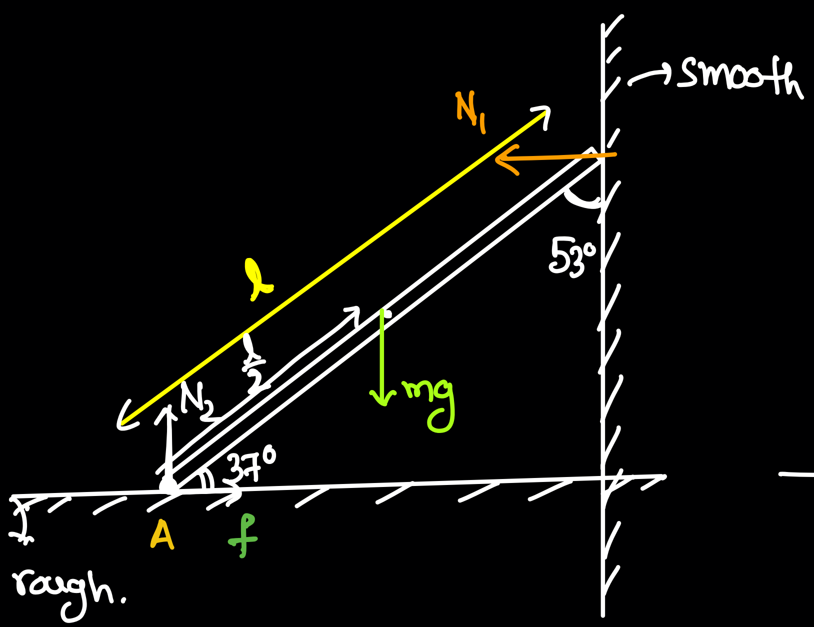
Translational equilibrium $\Rightarrow F_{net} = 0.$

Rotational equilibrium $\Rightarrow \tau_{net} = 0.$

$\Rightarrow$ Body is in equilibrium then both rotational as well as translational equilibrium

$\Rightarrow F_{net}$ and $\tau_{net} = 0$

ill:28 :-



Translation equilibrium :-

$\Sigma F_x = 0 \Rightarrow f = N_1. \quad -(1)$

$\Sigma F_y = 0 \Rightarrow N_2 = mg. \quad -(2)$

$\tau_A = 0.$

$$(0)f + (0)N_2 + \left(\frac{l}{2}\cos 37^\circ\right)mg \circlearrowright + \left(l \sin 37^\circ\right)N_1 \circlearrowleft = 0$$

$\circlearrowright$ +ve

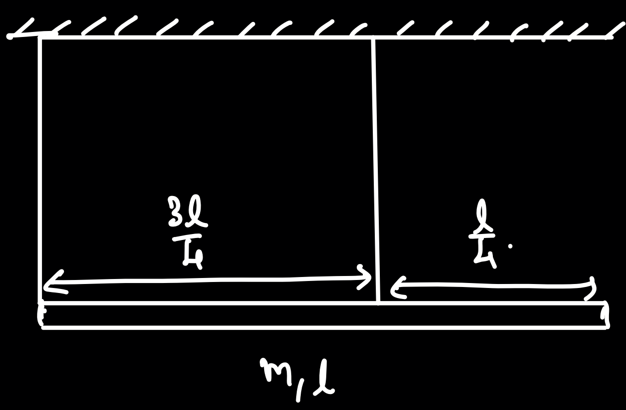
$$\left(\frac{4l}{10}\right)mg - \frac{3l}{5}N_1 = 0$$

$$N_1 = \frac{2mg}{3}.$$

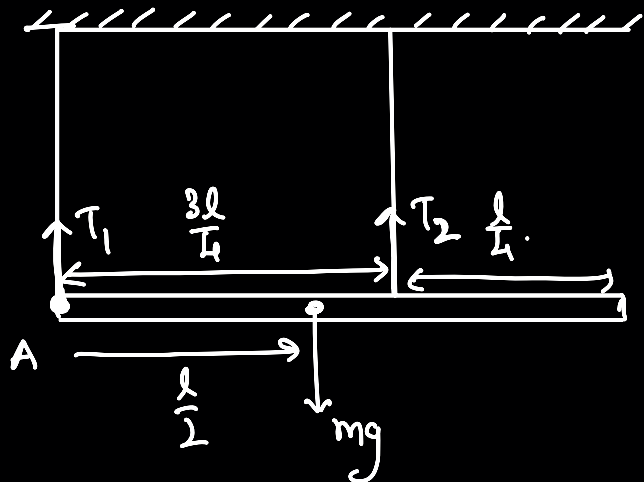
from eq -(1)

$$f = N_1 \Rightarrow f = \frac{2mg}{3}.$$

ill: 30 :-



determine tensions ?



from translational equ.

$$\Sigma F_y = 0$$

$$T_1 + T_2 = mg \quad -①$$

$$\tau_A = 0$$

$$(0)T_1 + mg\left(\frac{l}{2}\right)\circlearrowright + \frac{3l}{4}T_2 \circlearrowleft = 0$$

$\circlearrowright$ +ve

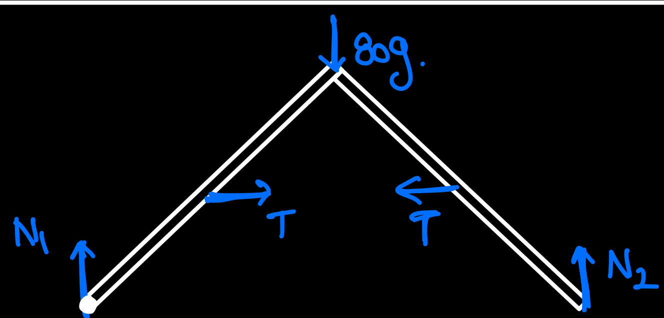
$$mg\frac{l}{2} - \frac{3l}{4}T_2 = 0$$

$$T_2 = \frac{2mg}{3}.$$

$$T_1 + T_2 = mg$$

$$T_1 = \frac{mg}{3}.$$

ill:29



A

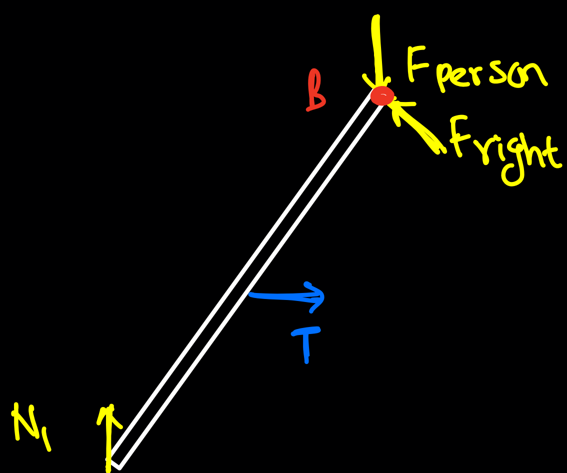
$$\tau_A = 0$$

$$(80g)(2\tan 30^\circ)\circlearrowleft + N_2(4\tan 30^\circ)\circlearrowright = 0$$

$$N_2 = 40g$$

$\Sigma F_y = 0 \Rightarrow$ $N_1 + N_2 = 80g$

$$N_1 = N_2 = 40g.$$



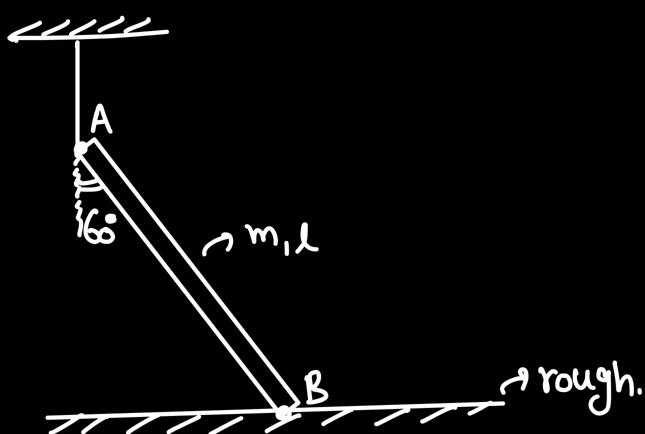
$$\tau_B = 0$$

$$(T)(1)\circlearrowright + N_1(2\tan 30^\circ)\circlearrowleft = 0$$

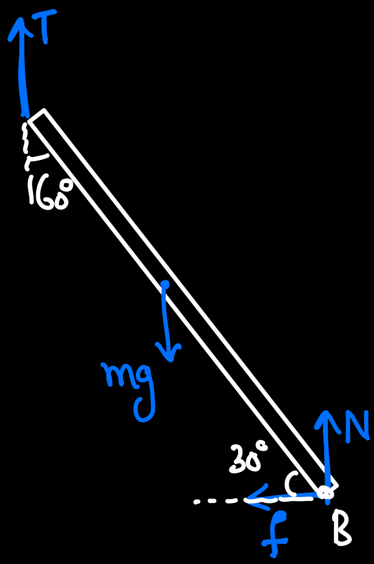
$$T = 2N_1 \tan 30^\circ$$

$$= \frac{80g}{\sqrt{3}}.$$

Q)



find the normal, frictional and tensional forces?



$$\Sigma F_x = 0$$

$$f = 0 \quad \text{—}(1)$$

$$\Sigma F_y = 0$$

$$N + T = mg \quad \text{—}(2)$$

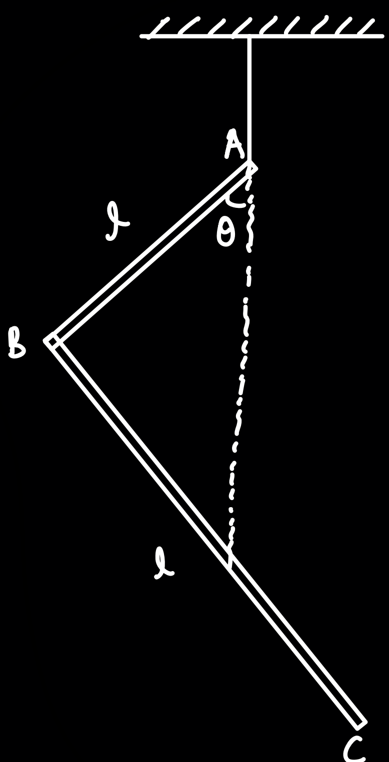
$$\tau_B = 0.$$

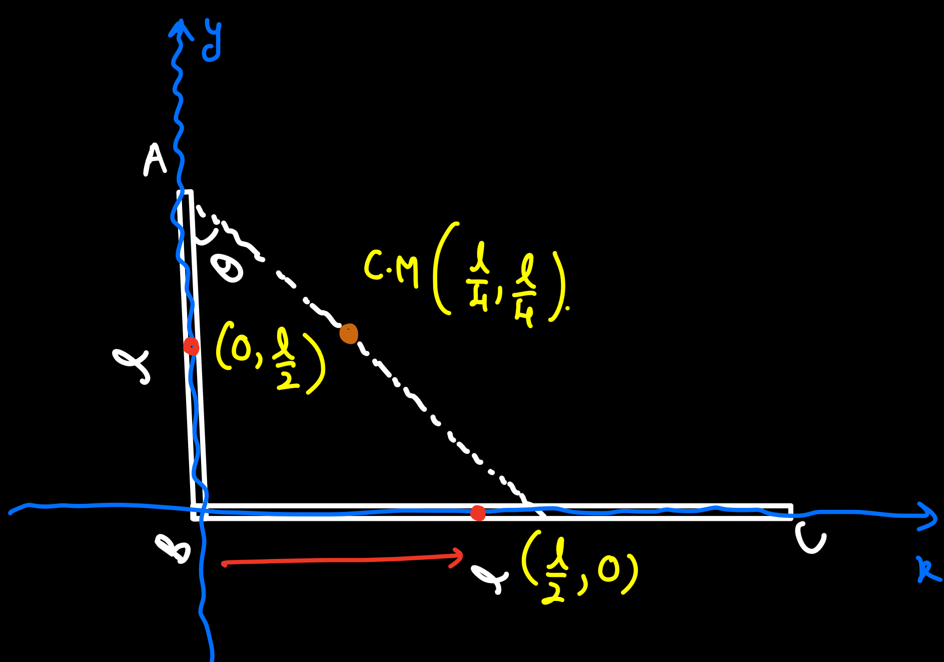
$$(l\sin 60^\circ)T \circlearrowright + \left(\frac{l}{2}\sin 60^\circ\right)mg \circlearrowleft = 0$$

$$T = \frac{mg}{2}.$$

$$N = \frac{mg}{2}.$$

Q21)



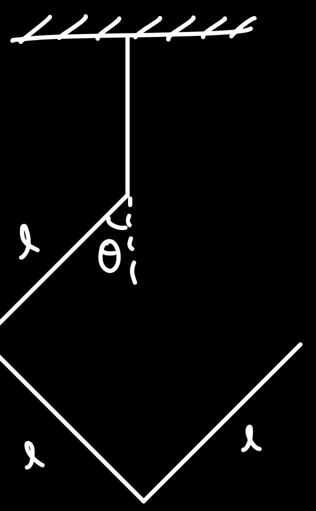


$$x_{CM} = \frac{m\left(\frac{l}{2}\right) + m(0)}{2m} = \frac{l}{4}.$$

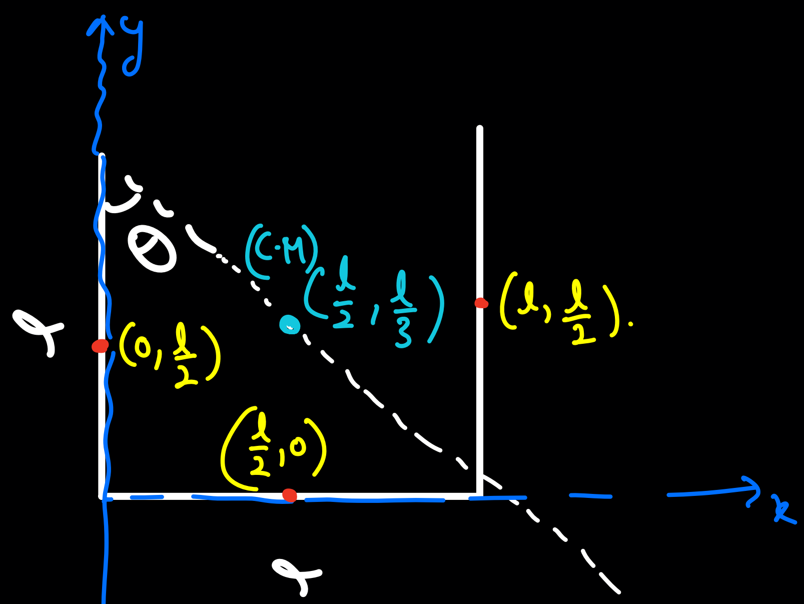
$y_{CM} = \frac{l}{4}$.

$$\tan\theta = \frac{\frac{l}{4}}{\frac{3l}{4}} \Rightarrow \boxed{\tan\theta = \frac{1}{3}}.$$

Q)



find $\theta = ?$



$$x_{CM} = \frac{m(0) + m\left(\frac{l}{2}\right) + ml}{3m}$$

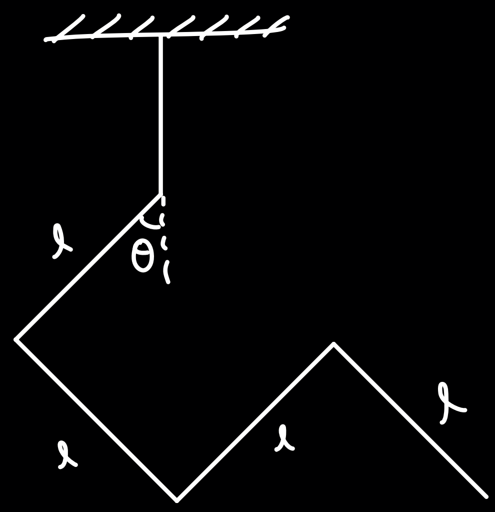
$$= \frac{l}{2}$$

$$y_{CM} = \frac{m(0) + m\left(\frac{l}{2}\right) + m\left(\frac{l}{2}\right)}{3m}$$

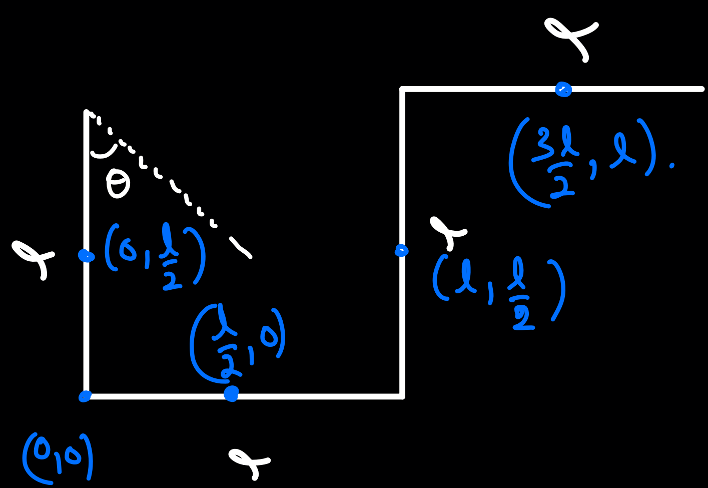
$$= \frac{l}{3}.$$

$$\tan\theta = \frac{\frac{l}{2}}{\frac{2l}{3}} = \frac{3}{4} \Rightarrow \underline{\underline{\theta = 37^\circ}}.$$

Q)



find θ ?

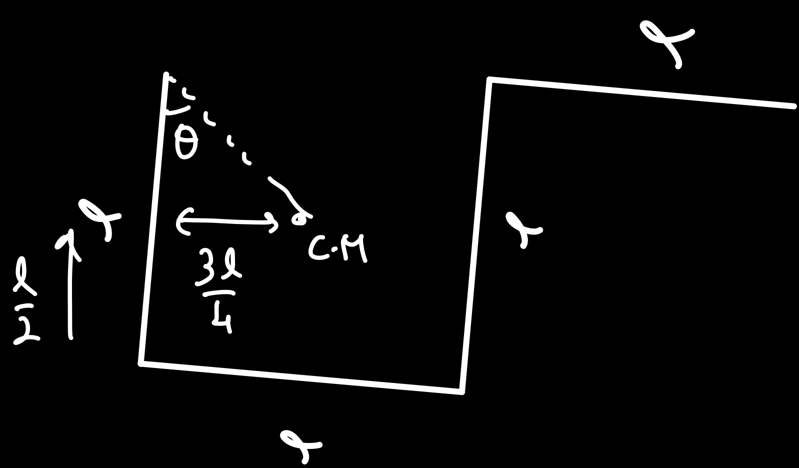


$$x_{CM} = \frac{m(0) + m\frac{l}{2} + ml + m\left(\frac{3l}{2}\right)}{4m}$$

$$= \frac{3l}{4}.$$

$$y_{CM} = \frac{m\left(\frac{l}{2}\right) + m(0) + m\frac{l}{2} + ml}{4m}$$

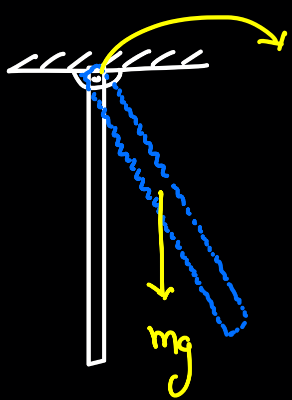
$$= \frac{l}{2}.$$



$$\tan\theta = \frac{\frac{3l}{4}}{l - \frac{l}{2}} = \frac{3}{2} \Rightarrow \theta = \tan^{-1}\left(\frac{3}{2}\right).$$

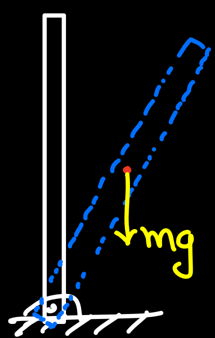
Stable equilibrium:- if body develops a restoring force or torque to come back then it is said to be in stable equilibrium.

eg:-



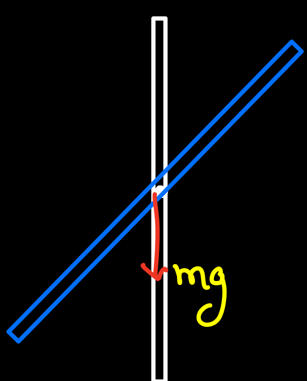
$\tau_{Hinge} \Rightarrow$ mg creates clockwise torque.

unstable:-



Clockwise torque by mg further takes it away so it was in unstable equilibrium.

Neutral equilibrium:-

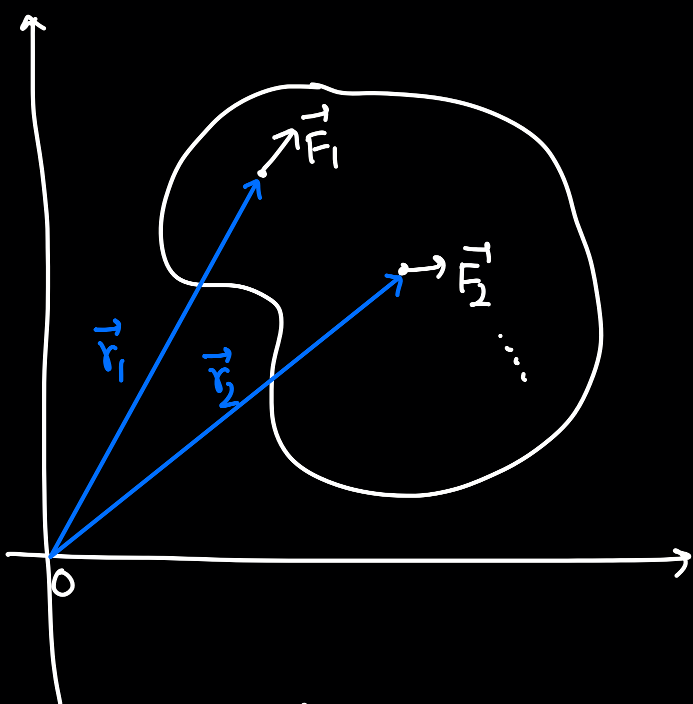


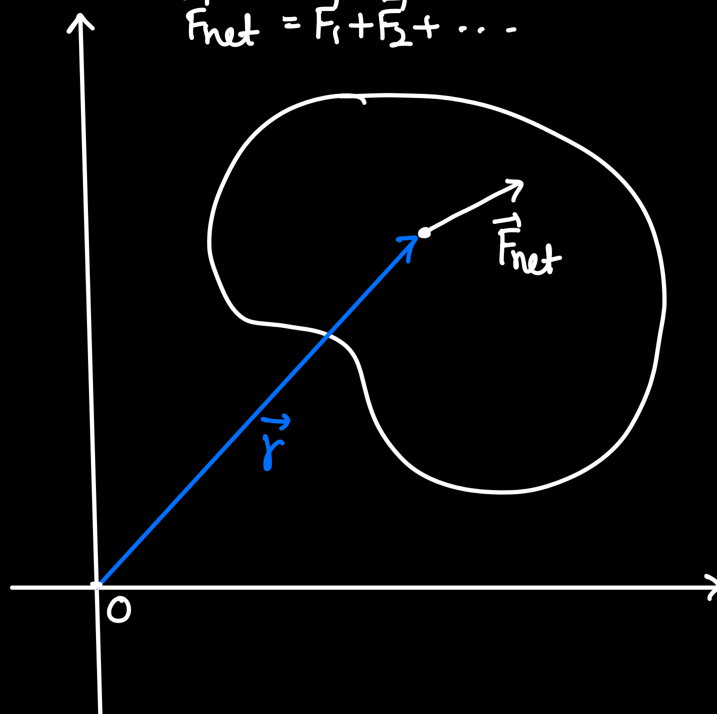
$\tau_{weight} = 0$

Point of application of force:- It is the point at which the entire force can be assumed to be acting without creating any changes in translational and rotational motion of body.

⇒ as long as net force remains same translational motion won't change.

⇒ as long as torque about a point doesn't change then rotational motion won't change.





$$\tau_O = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \dots$$

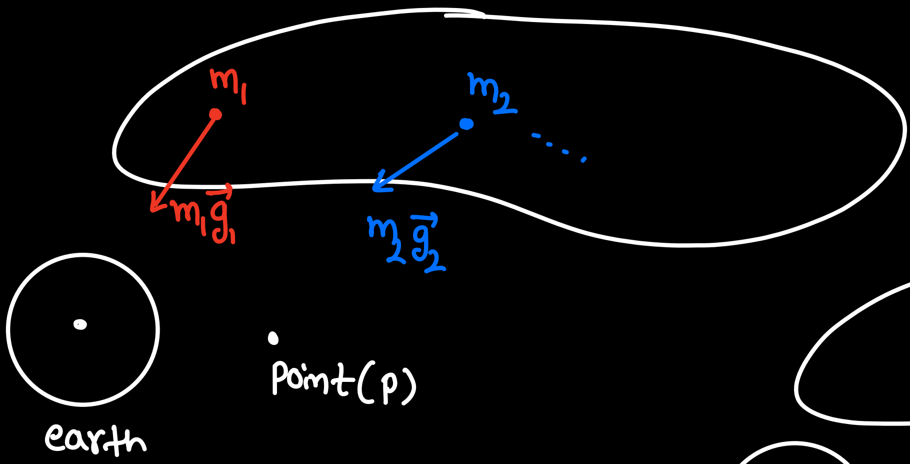
$$\tau_O = \vec{r} \times \vec{F}_{net}$$

for No change in rotational motion

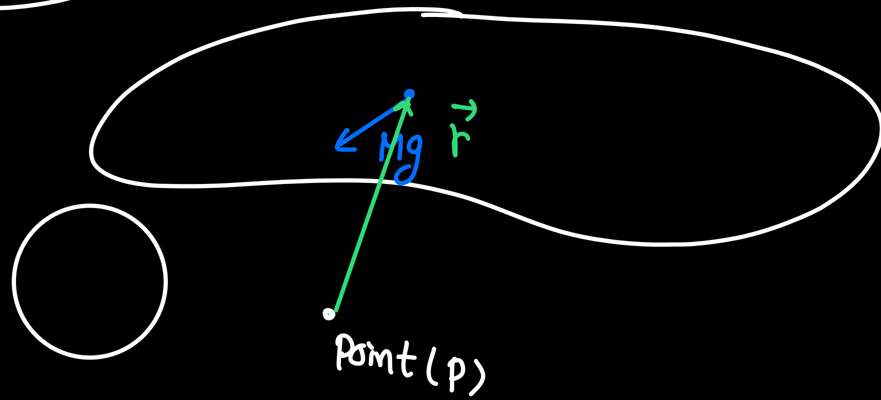
$$\vec{r} \times \vec{F}_{net} = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \dots$$

we can calculate $\vec{r}$ and get the location where the entire force can be assumed to be acting.

<u>Centre of gravity (C.G)</u>:- it is the point at which entire weight of body is assumed to be acting.



$$M\vec{g} = m_1\vec{g}_1 + m_2\vec{g}_2 + \dots$$



$$\tau_p = \vec{r}_1 \times m_1\vec{g}_1 + \vec{r}_2 \times m_2\vec{g}_2 + \dots$$

$$\tau_p = \vec{r} \times M\vec{g}$$

$$\boxed{\vec{r} \times M\vec{g} = \vec{r}_1 \times m_1\vec{g}_1 + \vec{r}_2 \times m_2\vec{g}_2 + \dots}$$

if the body we deal with is very small compared to earth.

$$\vec{g}_1 = \vec{g}_2 = \vec{g}_3 = \dots = \vec{g}.$$

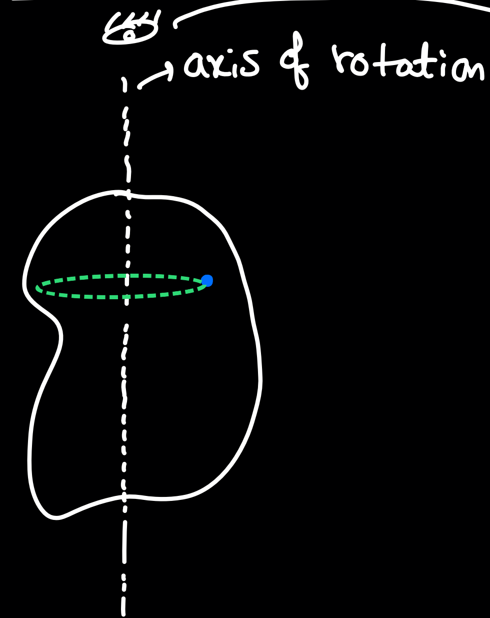
$$\vec{r} \times M\vec{g} = \left(\vec{r}_1 \times m_1 + \vec{r}_2 \times m_2 \dots\right)\vec{g}.$$

$$\vec{r} \times M\vec{g} = \vec{r}_{CM} \times M\vec{g}$$

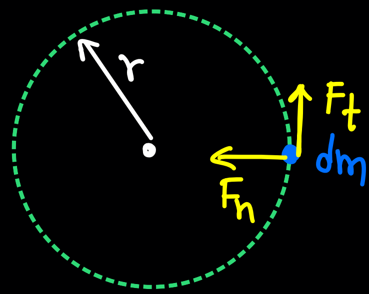
$$\vec{r} = \vec{r}_{CM}$$

$\Rightarrow$ so C.M and C.G. lie at same point when we deal with small bodies.

Rotation about fixed axis:-



top view



$$d\tau_{axis} = (r)(F_t).$$

$$= (r)(dm)a_t$$

$$= (r)(dm)(r\alpha)$$

$$d\tau_{axis} = (dm)r^2\alpha$$

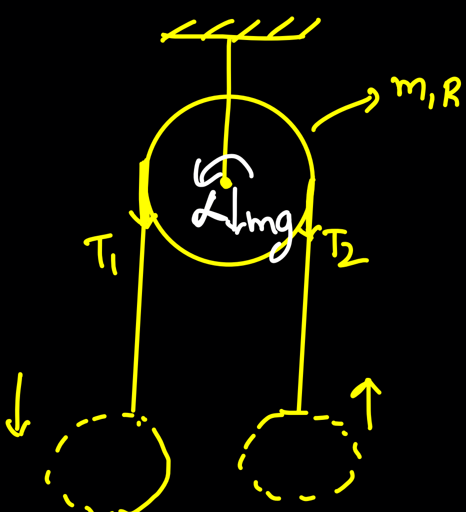
$$\tau_{axis} = \int (dm) r^2 \alpha$$

$$\boxed{\tau = I\alpha}$$

M.I. about the axis.

$\tau \neq I\alpha$ we will deal with this in future.

observation:-



if string doesn't slip & slide on pulley the pulley rotates.

$$\tau_{Hinge} = T_1R \circlearrowright + T_2R \circlearrowleft$$

$$I\alpha = (T_1 - T_2)R \circlearrowright.$$

$$(T_1 - T_2)R = I\alpha$$

generally pulley will be treated as disc

$$(T_1 - T_2)R = \left(\frac{MR^2}{2}\right)\alpha. \Rightarrow \boxed{T_1 - T_2 = \frac{MR\alpha}{2}}$$

if pulley was massless $\Rightarrow M = 0$

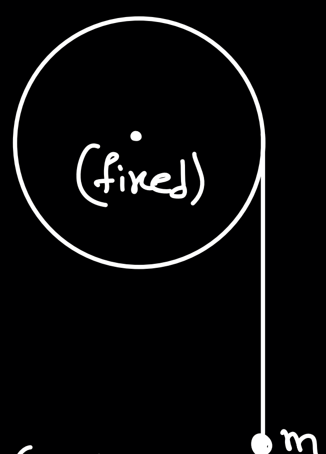
$$T_1 = T_2.$$

if string slips then pulley won't rotate $\Rightarrow \alpha = 0$

$$T_1 = T_2.$$

BB-3

Q1) (m, R)

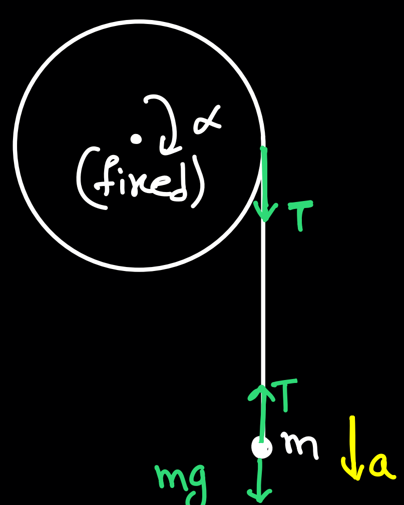


find acceleration of particle ?

assume string doesn't slip.

sol :-

(m, R)

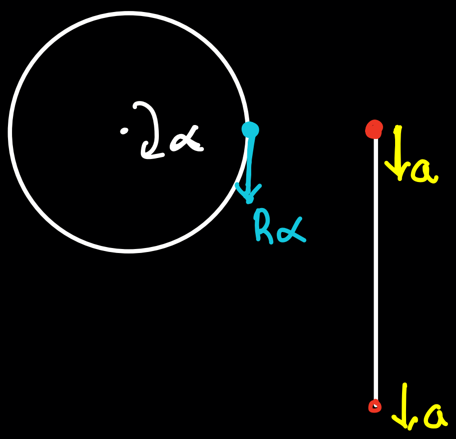


$$mg - T = ma \quad -① .$$

$$\tau_{hinge} = RT \circlearrowright.$$

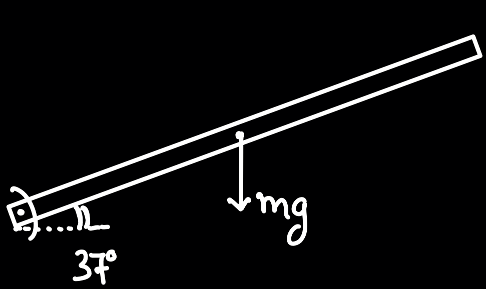
$$I\alpha = RT$$

$$\frac{mR^2}{2}\alpha = RT \Rightarrow \frac{mR\alpha}{2} = T \quad -②$$



③ — $a = R\alpha$ for no slipping

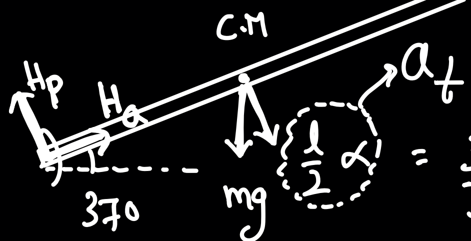
iii: 34)



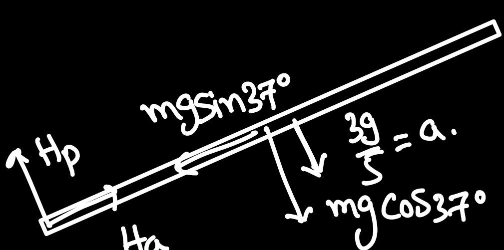
$$\tau_{Hinge} = \left(\frac{l}{2}\cos 37^\circ\right) mg.$$

$$\left(\frac{ml^2}{3}\right)\alpha = \frac{2mgl}{5}.$$

$$\alpha = \frac{6g}{5l}.$$



$$\frac{l}{2}\alpha = \frac{3g}{5}.$$



$$\frac{4mg}{5} - H_p = (m)\left(\frac{3g}{5}\right)$$

$$H_p = \frac{mg}{5}.$$

F_{net} along rod $= 0.$

$$mg\sin 37^\circ = H_a.$$

$$H_a = \frac{3mg}{5}.$$

$$H_{net} = \sqrt{H_a^2 + H_p^2} = \sqrt{\frac{m^2g^2}{25} + \frac{9m^2g^2}{25}}$$

$$= \frac{\sqrt{10}\, mg}{5}.$$

extension:- find the hinge force when rod becomes horizontal ?

Sol:-

$$\tau = \left(\frac{l}{2}\cos\theta\right)mg.$$

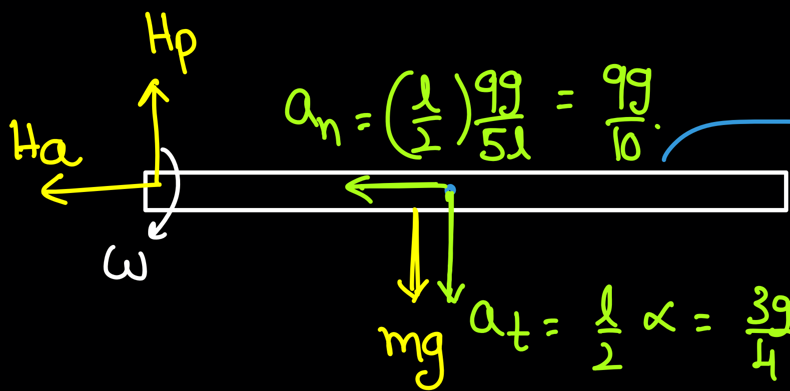
$$I\alpha = \frac{mgl}{2}\cos\theta$$

$$\frac{\cancel{m}l^{\cancel{2}}}{3}\left(-\omega\frac{d\omega}{d\theta}\right) = \frac{\cancel{m}g\cancel{l}}{2}\cos\theta.$$

$$-\int_0^\omega \omega\, d\omega = \frac{3g}{2l}\int_{37^\circ}^{0^\circ}\cos\theta\, d\theta$$

$$-\frac{\omega^2}{2} = \frac{3g}{2l}\left[\sin 0^\circ - \sin 37^\circ\right].$$

$$\omega = \sqrt{\frac{9g}{5l}}.$$



$$a_n = \left(\frac{l}{2}\right)\frac{9g}{5l} = \frac{9g}{10}.$$

$$\tau = mg\frac{l}{2} \Rightarrow \frac{\cancel{m}l^{\cancel{2}}}{3}\alpha = \frac{\cancel{m}g\cancel{l}}{2}$$

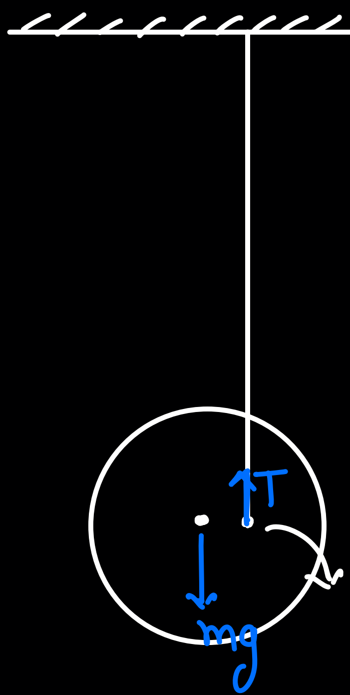
$$\alpha = \frac{3g}{2l}.$$

$$a_t = \frac{l}{2}\alpha = \frac{3g}{4}.$$

$$H_a = (m)a_n = \frac{9mg}{10}.$$

$$mg - H_p = m\left(\frac{3g}{4}\right) \Rightarrow H_p = \frac{mg}{4}.$$

$$H = \sqrt{H_a^2 + H_p^2} = \sqrt{\frac{81m^2g^2}{100} + \frac{m^2g^2}{16}}$$



BB-3

Q6)

about p body is in pure rotation

$$\tau_p = (mg)\frac{R}{2}.$$

$$I\alpha = \frac{mgR}{2}.$$

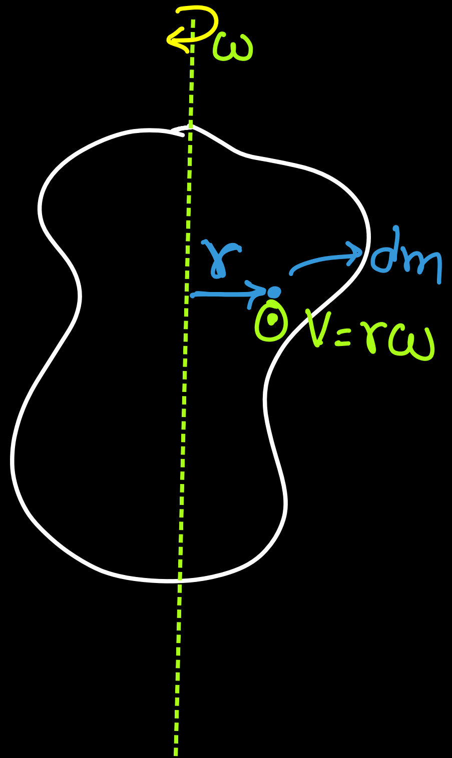
$$\left(\frac{MR^2}{2} + \frac{MR^2}{4}\right)\alpha = \frac{MgR}{2} \Rightarrow \alpha = \frac{2g}{3R}.$$

$$a_{CM} = \frac{R}{2}\alpha = \frac{g}{3}.$$

$$F_{net} = ma_{CM}$$

$$mg - T = \frac{mg}{3} \Rightarrow T = \frac{2mg}{3}.$$

K.E of a body rotating about fixed axis :-



Point mass

$$K.E = \frac{1}{2}mv^2$$

mass of point mass

$$d(K.E) = \frac{1}{2}(dm)(r\omega)^2$$

$$d(K.E) = \frac{1}{2}(dm)r^2\omega^2.$$

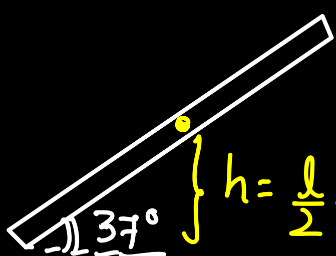
$$K.E = \int \frac{1}{2}(dm)r^2\omega^2$$

$$= \frac{\omega^2}{2}\underbrace{\int (dm)r^2}_{I.}$$

$$\boxed{K.E = \frac{1}{2}I\omega^2}$$

M.I about axis.

eg:-



$$h = \frac{l}{2}\sin 37^\circ = \frac{3l}{10}$$

$\Rightarrow$ loss in P.E = gain in K.E

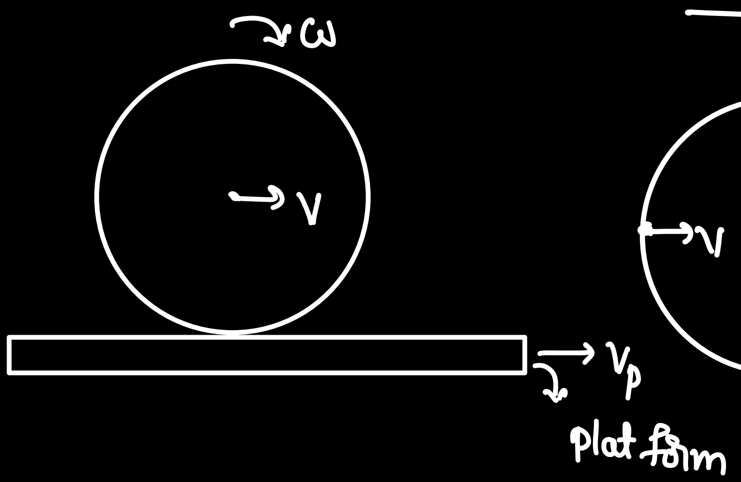
$$mg\left(\frac{3l}{10}\right) = \frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2$$

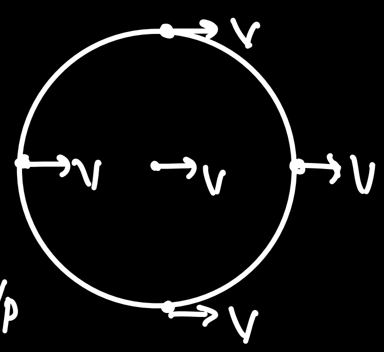
$$\omega^2 = \frac{9g}{5l} \Rightarrow \omega = \sqrt{\frac{9g}{5l}}.$$

Rotation about an axis in translation :- [Rolling]

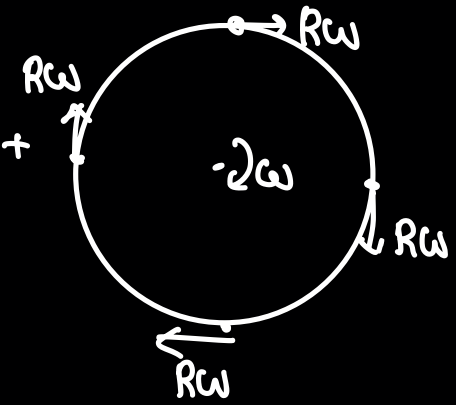
⇒ rolling is combination of pure rotation and pure translation.

Pure translation + pure rotation.





+



⇒ vel. of point of body in contact with platform = $v - R\omega$.

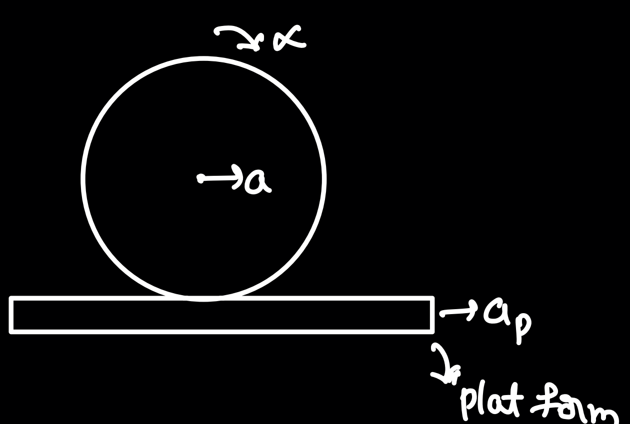
if $v - R\omega > v_p$ then its rolling with forward slipping.

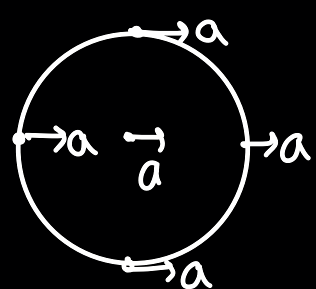
$v - R\omega < v_p$ then its rolling with backward slipping.

$v - R\omega = v_p$ then its rolling without any slipping (pure rolling)

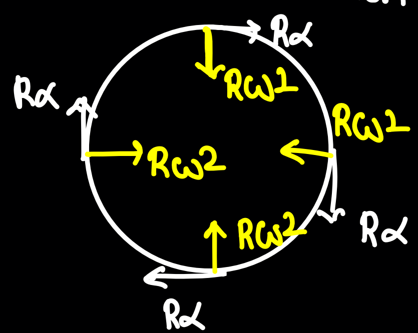
lets talk about future :-

Pure translation + Pure rotation





+



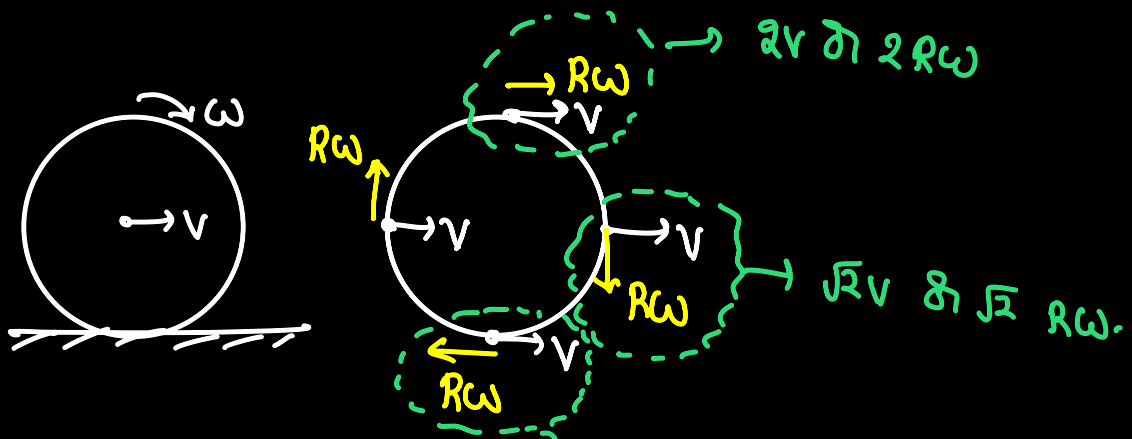
acc. of point of contact along platform = $(a - R\alpha)\rightarrow$.

if $a - R\alpha > a_p \rightarrow$ forward slipping

$a - R\alpha < a_p \rightarrow$ backward slipping.

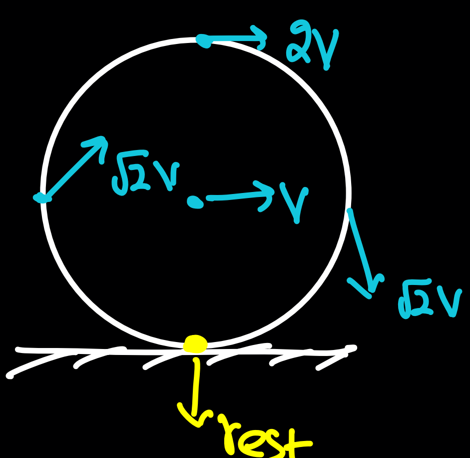
$a - R\alpha = a_p \Rightarrow$ pure rolling.

what if platform is ground?



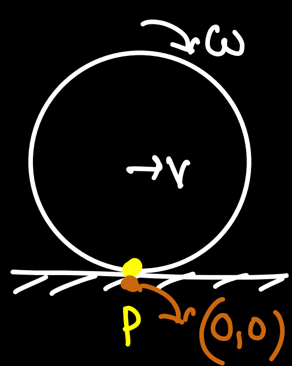
for pure rolling $V - R\omega = V_p \Rightarrow V - R\omega = 0$

$V = R\omega$.

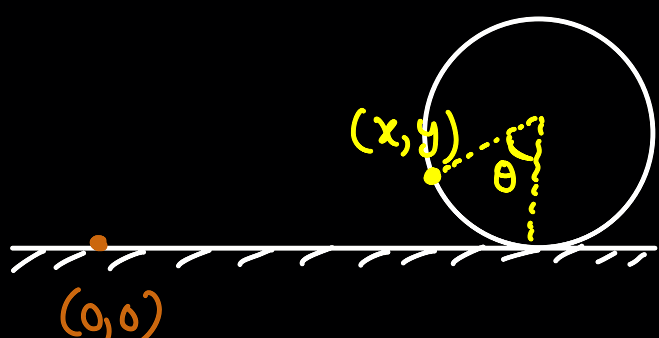


Path taken by particle :-

at $t = 0$



at some time 't'

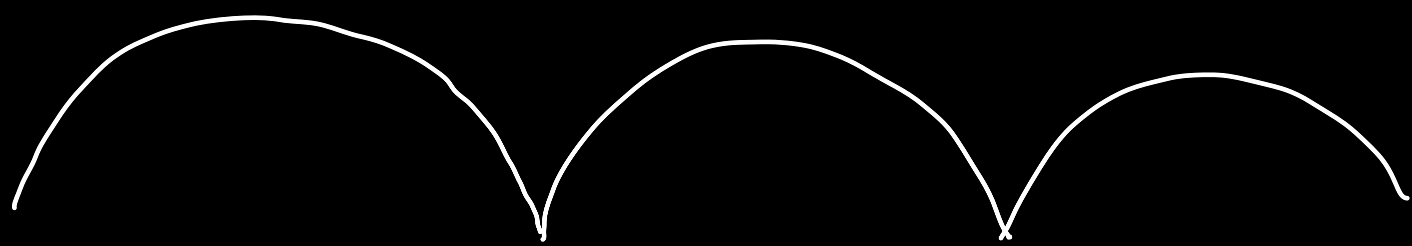


$x = Vt - R\sin\theta \Rightarrow x = Vt - R\sin\omega t$

$y = R - R\cos\theta \Rightarrow y = R - R\cos\omega t.$

$\Rightarrow$ path taken by particle is cycloidal.

Point on Periphery:-



find the distance moved by point 'p' in 1 rotation?

$x = Vt - R\sin\omega t \Rightarrow dx = V\,dt - R\omega\cos\omega t\,(dt)$

$y = R - R\cos\omega t \Rightarrow dy = R\omega\sin\omega t\,(dt)$

$$ds = \sqrt{(dx)^2 + (dy)^2}$$

$$= \left(\sqrt{V^2 + R^2\omega^2\cos^2\omega t - 2VR\omega\cos\omega t + R^2\omega^2\sin^2\omega t}\right)dt$$

$$= \left(\sqrt{V^2 + R^2\omega^2 - 2VR\omega\cos\omega t}\right)dt$$

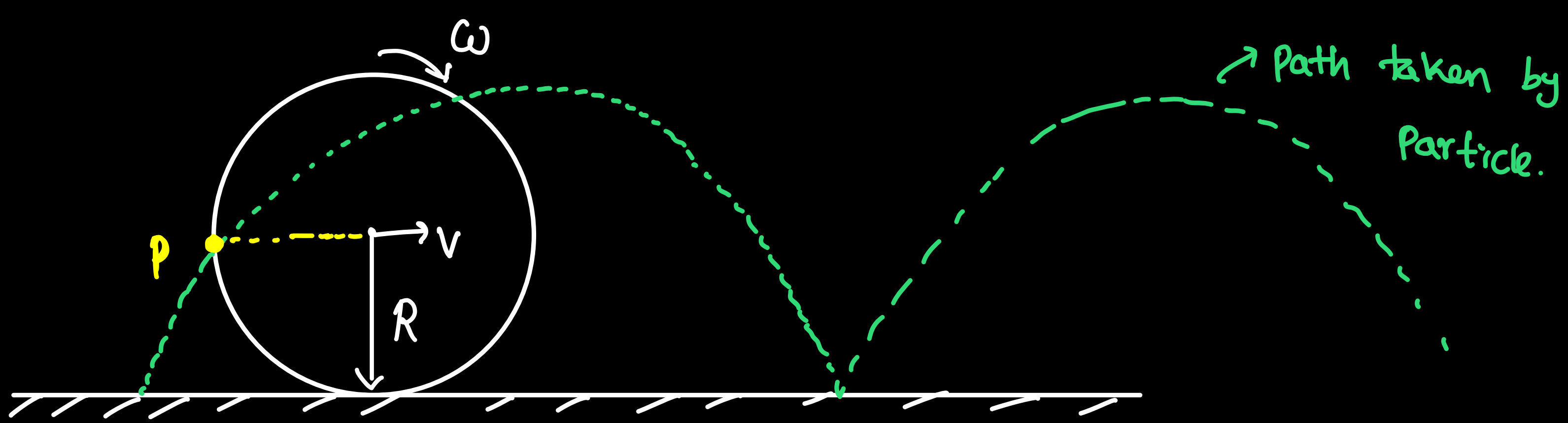
$$= V\sqrt{2 - 2\cos\omega t}\;dt$$

$$= 2v\sqrt{\frac{1-\cos\omega t}{2}}\,dt$$

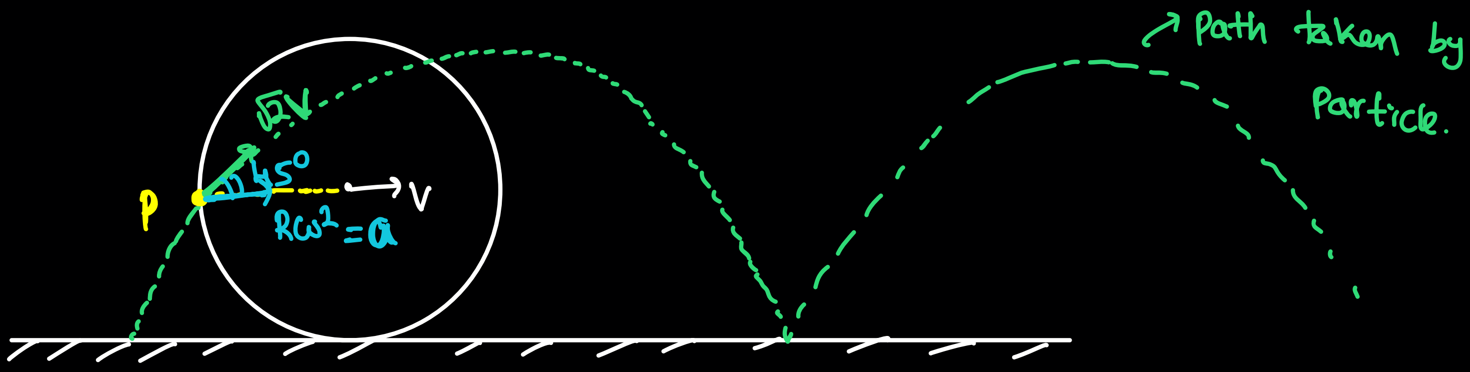
$$\int ds = \int_0^{\frac{2\pi}{\omega}} 2v \sin\left(\frac{\omega t}{2}\right)dt$$

$$= 2v\left[\frac{-\cos\left(\frac{\omega t}{2}\right)}{\frac{\omega}{2}}\right]_0^{\frac{2\pi}{\omega}}.$$

$$= \frac{8v}{\omega} = \frac{8R\omega}{\omega} = 8R.$$



⇒ find the radius of curvature for the shown location?



radius of curvature $= \dfrac{(\sqrt{2}v)^2}{(R\omega^2)\cos 45^\circ} = \dfrac{(2\sqrt{2})v^2}{R\omega^2}$

$= \underline{\underline{2\sqrt{2}R}}$.

<u>steps to solve rolling questions</u>:-

step ①: translation.

$$F_{net} = ma.$$

step ②: Rotation

$$\tau_{net} = I\alpha.$$

step ③: rolling condition.

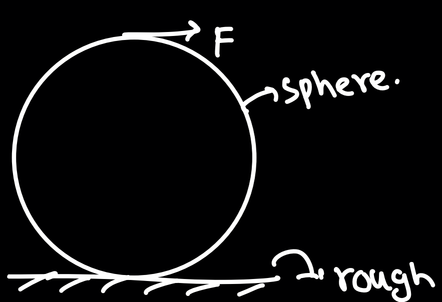
acc. of point in contact with platform along platform $= a_{platform}$.

⇓

same for velocity.

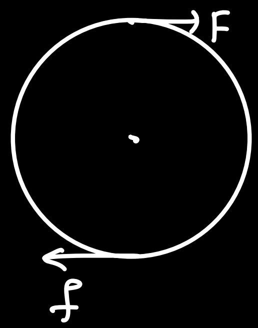
---

ill: 43

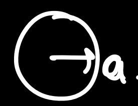


find acc. of the body ?

Sol:-



$F - f = ma$ —①

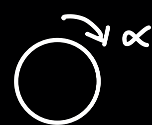


$\tau_{centre} = FR + fR$

$I\alpha = (F+f)R$

$\frac{2}{5}MR^2\alpha = (F+f)R$

$F + f = \frac{2}{5}MR\alpha$ —②



$a - R\alpha = a_{platform}$

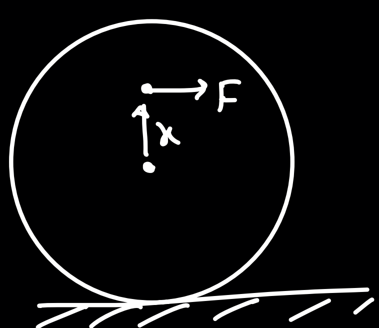
$a - R\alpha = 0$

$a = R\alpha$ — ③.

on solving

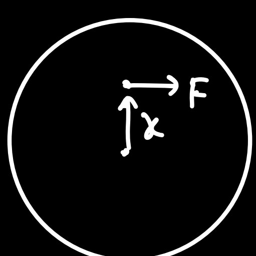
$a = \frac{10F}{7m}$.

Q)



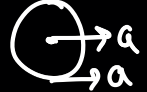
find the value of x for which sphere is in pure rolling and friction is "0" ?

Sol:-



translation:

$F = ma$ —①



rotation:

$Fx = I\alpha$

rolling condition

$a - R\alpha = a_p$

$a - R\alpha = 0$

on solving

$$\boxed{x = \frac{I}{mR}}.$$

for sphere $x = \frac{2}{5}R$

hollow sphere $x = \frac{2}{3}R$.

disc $x = \frac{R}{2}$

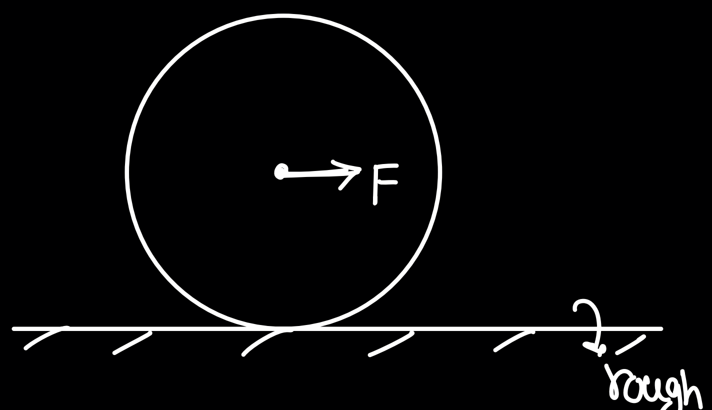
ring $\underline{\underline{x = R}}$.

for all $x > \frac{I}{mR}$ friction is in the direction of force

$x < \frac{I}{mR}$ friction is opposite to direction of force

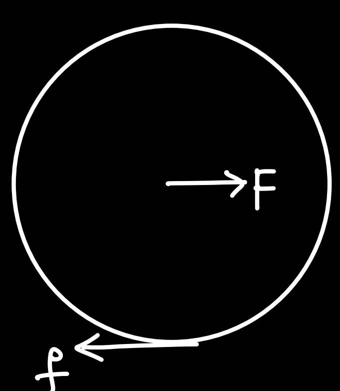
---

Q)

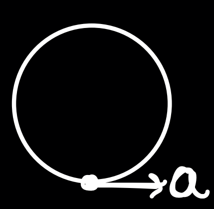


Find frictional force acting on the body?

Sol:-



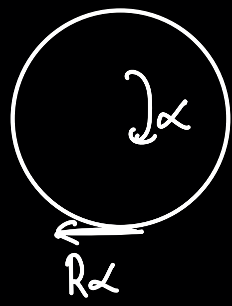
$F - f = ma$ —①



$$\tau_{centre} = f \times R$$

$$I\alpha = fR$$

$$I(R\alpha) = fR^2 \quad \text{—(2)}$$

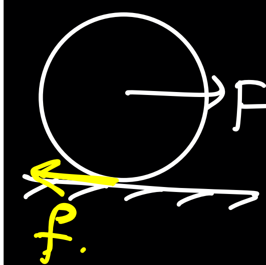


rolling condition $a - R\alpha = a_p$

$$a - R\alpha = 0 \quad \text{—(3)}$$

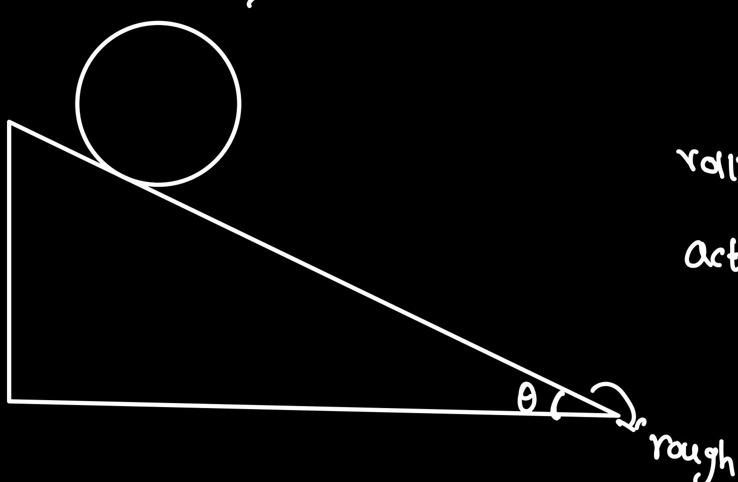
on solving

$$\boxed{f = \frac{F}{1 + \frac{mR^2}{I}} \qquad a = \frac{\left(\frac{F}{m}\right)}{1 + \frac{I}{mR^2}}}$$



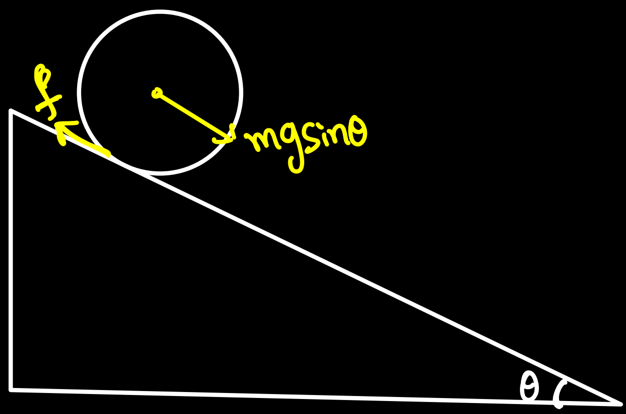
rolling down an inclined plane:-

→ M.I is I and mass m



Assume the body to be in pure rolling and find acceleration and friction acting on body ?

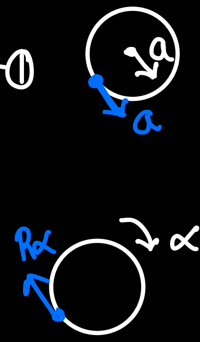
Sol :-



$mg\sin\theta - f = ma$ —(1)

$\tau = I\alpha$

$fR = I\alpha$ —(2)



rolling condition

$a - R\alpha = a_{platform}$.

$a - R\alpha = 0$

$a = R\alpha$ —(3)

$$a = \frac{g\sin\theta}{1+\frac{I}{mR^2}}.$$

$$f = \frac{mg\sin\theta}{1+\frac{mR^2}{I}}.$$

$a_{\text{solid sphere}} = \frac{5g\sin\theta}{7}$

$a_{\text{hollow sphere}} = \frac{3g\sin\theta}{5}$.

$a_{\text{disc}} = \frac{2}{3}g\sin\theta$

$a_{\text{ring}} = \frac{g\sin\theta}{2}$.

$f_{\text{solid sphere}} = \frac{2mg\sin\theta}{7}$.

$f_{\text{hollow sphere}} = \frac{2mg\sin\theta}{5}$.

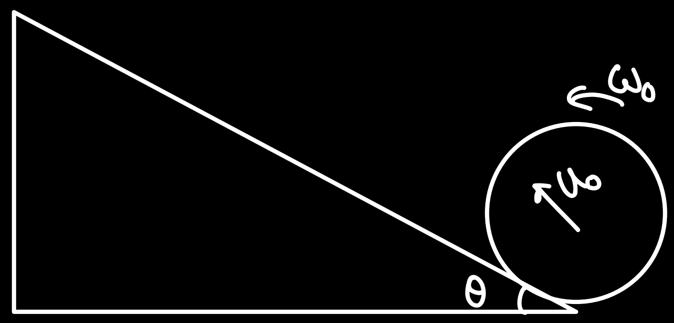
$f_{\text{disc}} = \frac{mg\sin\theta}{3}$

$f_{\text{ring}} = \frac{mg\sin\theta}{2}$.

$a_{\text{solid sphere}} > a_{\text{disc}} > a_{\text{hollow sphere}} > a_{\text{ring}}$

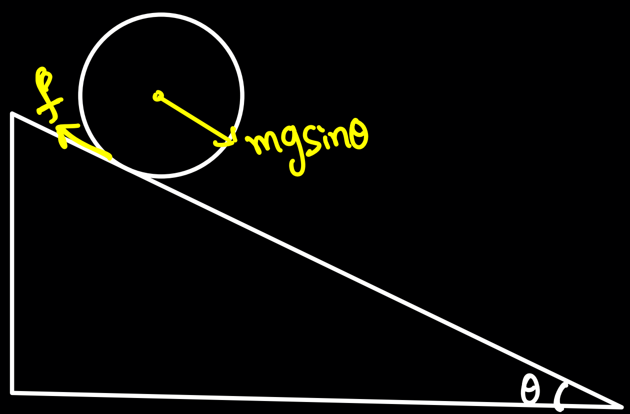
$f_{\text{solid sphere}} < f_{\text{disc}} < f_{\text{hollow sphere}} < f_{\text{ring}}$.

Rolling up the inclined body :-

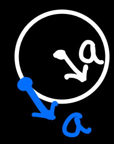


find acc. and frictional force ?

Sol :-

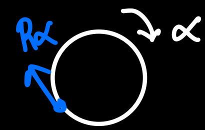


$mg\sin\theta - f = ma$ —①



$\tau = I\alpha$

$fR = I\alpha$ —②



rolling condition

$a - R\alpha = a_{platform}$.

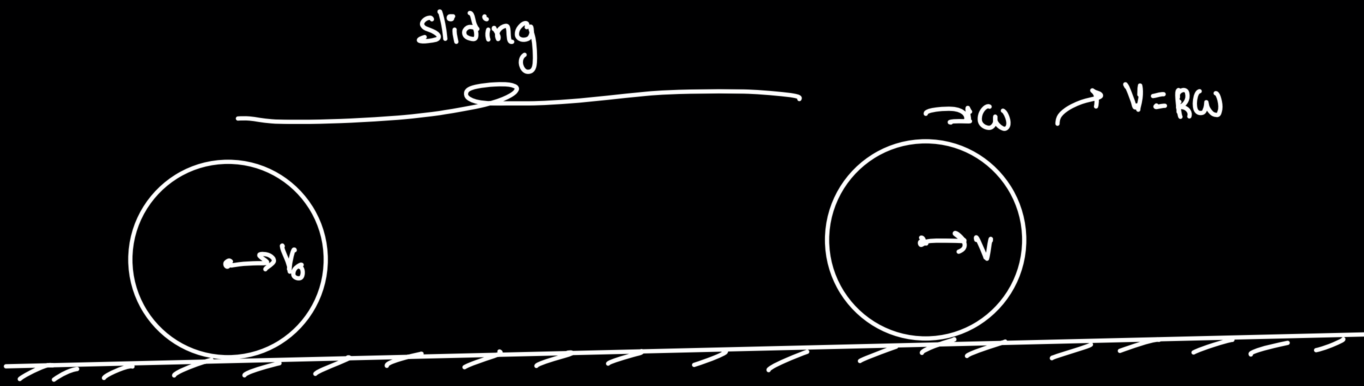
$a - R\alpha = 0$

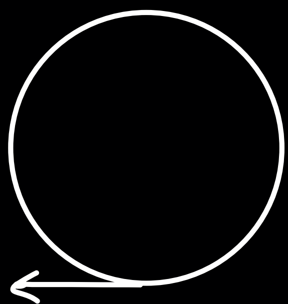
$a = R\alpha$ —③

$$a = \frac{g\sin\theta}{1 + \frac{I}{mR^2}}.$$

$$f = \frac{mg\sin\theta}{1 + \frac{mR^2}{I}}.$$

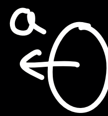
ill:46





$f_k = \mu_k N = \mu_k mg.$

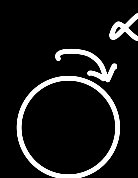
$\mu_k mg = ma$ —(1)



$\tau = \frac{2}{5} mR^2\alpha$

$(\mu_k mg)R = \frac{2}{5} mR^2\alpha$

$\alpha = \frac{5\mu_k g}{2R}$ —(2)



$v = u + at$

$V = V_0 - \mu_k g t$ —(3)

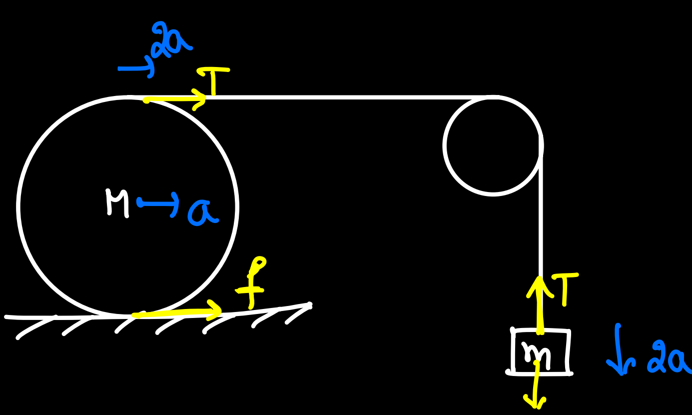
$\omega_f = \omega_i + \alpha t$

$\omega = 0 + \frac{5\mu_k g}{2R} t \Rightarrow 2v = 5\mu_k g t$ —(4)

from (3),(4)

$t = \frac{2v_0}{7\mu_k g}.$

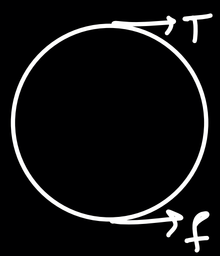
ill:45



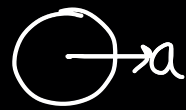
$mg - T = m(2a)$

$mg - T = 2ma$ —(1).

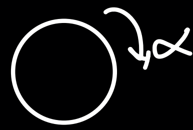
mg



$T + f = Ma$ —(2)



$TR - fR = \frac{MR^2}{2}\alpha$ —(3)

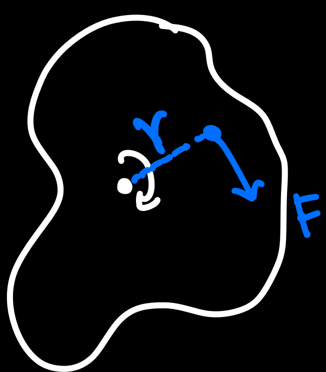


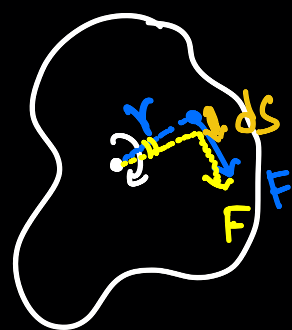
$a = R\alpha$ —(1)

Work analysis:-

Translation $W = \int \vec{F}\cdot d\vec{s}$

Rotation:- in next dt time.





$\Rightarrow ds = r\,d\theta$

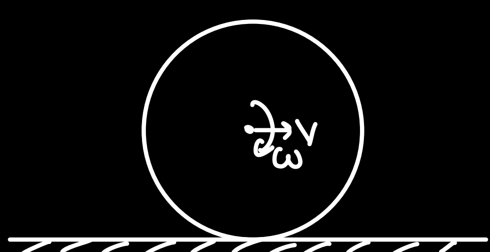
$dW = \vec{F}\cdot d\vec{s}$

$= F\,r\,d\theta$

$= \tau\,d\theta$

$W = \int \tau\,d\theta.$

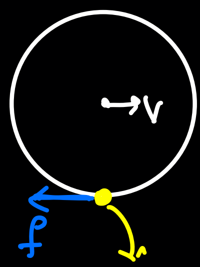
lets take a body rolling on ground



$v = R\omega \rightarrow$ rolling condition.

lets talk about work done by friction

translation

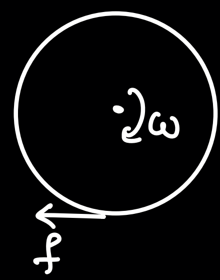


in next dt time

$ds = v\,dt$

$dW_f = -f(v\,dt).$

Rotation



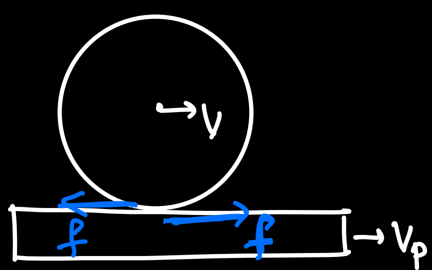
$dW_f = \tau\,d\theta = fR\,d\theta$

$= fR\omega\,dt.$

$= fv\,dt.$

Net work done by friction on a body rolling on ground $= \underline{\underline{0}}$.

if its not ground



<u>on body</u>:-

<u>translation</u>

$dW_f = -fv\,dt$

<u>Rotation</u>

$dW_f = fR\omega\,dt.$

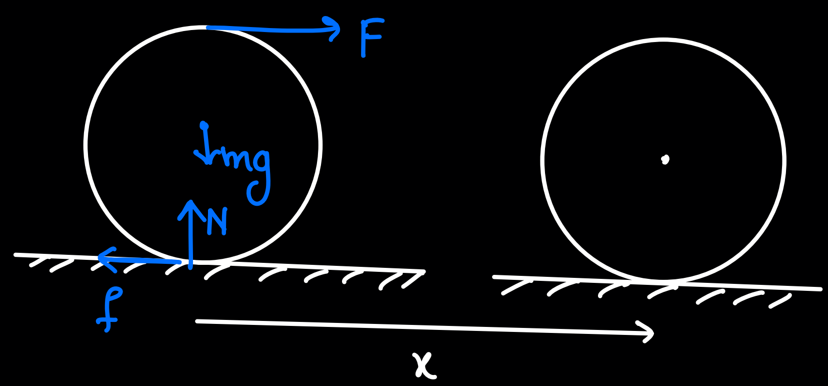
$dW_{net} = -f(v - R\omega)dt$

rolling condition

$v - R\omega = v_p$

$dW_{net} = -f\,v_p\,dt.$

ill: 49 :-



$$W_{Normal} = 0.$$

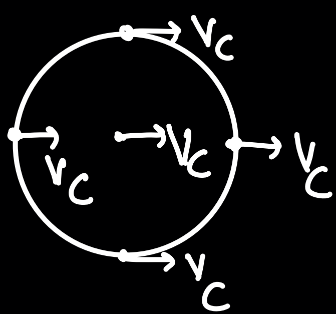
$$W_{mg} = 0.$$

$$W_{friction} = 0$$

$$W_F = (F)(2x).$$

Analysis of Kinectic energy :-

Translation :-

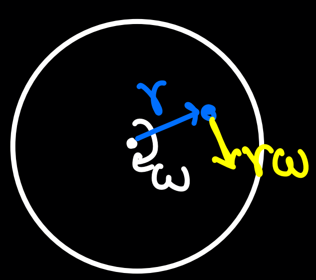


$$K.E = \frac{1}{2} dm_1 v_c^2 + \frac{1}{2} dm_2 v_c^2 + \cdots$$

$$= \frac{1}{2}(dm_1 + dm_2 + \cdots) v_c^2$$

$$= \frac{1}{2} M v_c^2$$

Rotation :



$$d(K.E) = \frac{1}{2}(dm)(r\omega)^2$$

$$= \frac{1}{2}(dm) r^2 \omega^2$$

$$K.E = \int \frac{1}{2}(dm) r^2 \omega^2 = \frac{1}{2}\left[\int dm\, r^2\right] \omega^2$$

$K.E = \frac{1}{2} I\omega^2$

M.I. about the axis of rotation

Work energy theorem:-

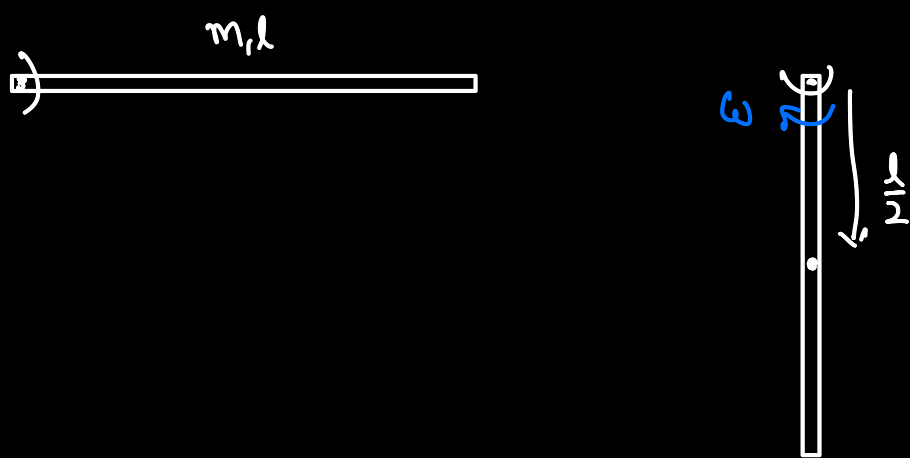
$W_{all\ forces} = \Delta K.E.$

we have to take into consideration both trans. and rotational K.E if we deal with rolling.

Q)

find the angular velocity of rod when it becomes vertical?

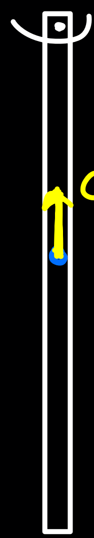
Sol:-



$W_{gravity} = \Delta K.E.$

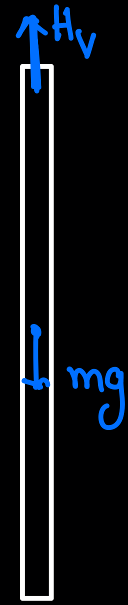
$mg\frac{l}{2} = \frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2 \Rightarrow \omega = \sqrt{\frac{3g}{l}}$

hinge force when rod is vertical.



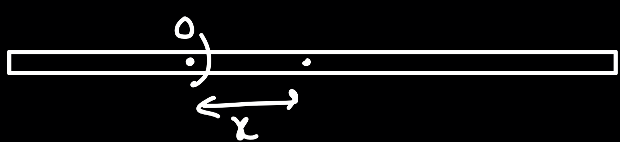
$\alpha = 0, \quad \omega = \sqrt{\frac{3g}{l}}.$

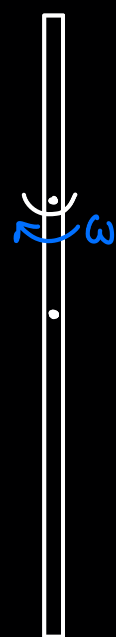
$a_{normal} = \frac{l}{2}\omega^2 = \frac{3g}{2}.$



$H_V - mg = m\frac{3g}{2} \Rightarrow H_V = \frac{5mg}{2}.$

111:53 :-





$$mgx = \frac{1}{2}\left[\frac{ml^2}{12} + mx^2\right]\omega^2$$

$$\omega = \sqrt{\frac{2gx}{\frac{l^2}{12} + x^2}} \rightarrow \frac{f(x)}{g(x)}$$

$$\frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2} = 0.$$

$$\frac{2g\left(\frac{l^2}{12}+x^2\right) - 2gx\left[2x\right]}{\left(\frac{l^2}{12}+x^2\right)^2} = 0$$

$$x = \pm \frac{l}{\sqrt{12}}.$$

BB-6

Q6)

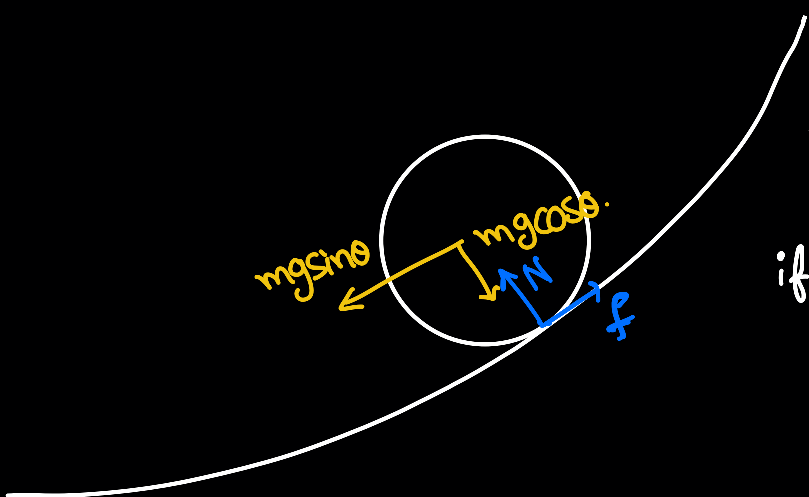
$$mgh = \left(\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2\right)$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\underbrace{\frac{mR^2}{2}}_{I}\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{4}mv^2 \Rightarrow \frac{3}{4}mv^2 = mgh.$$

$$v = \sqrt{\frac{4}{3}gh}.$$

Q5)



τ_{centre} is only by friction

if there is no friction then

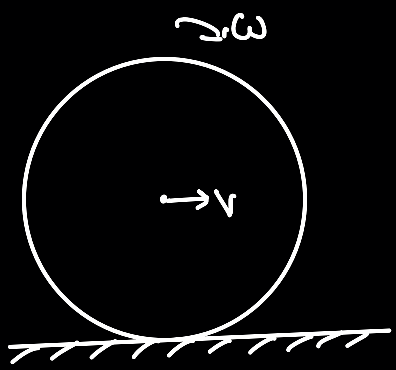
$\tau = 0$

$\alpha = 0$

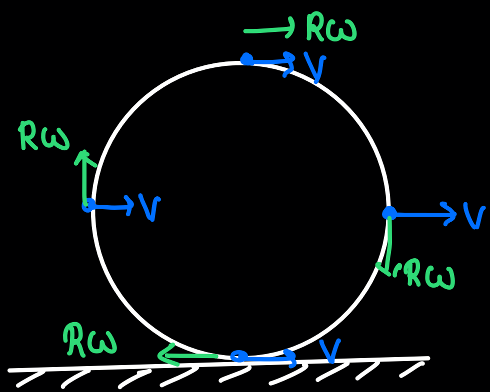
$\Delta\omega = 0.$

as rotational K.E won't convert to P.E in second case so $h_1 > h_2$.

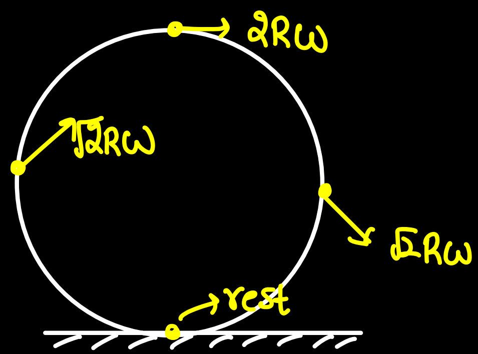
observation:-



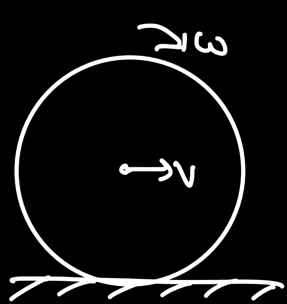
$\Rightarrow$ $V = R\omega$.



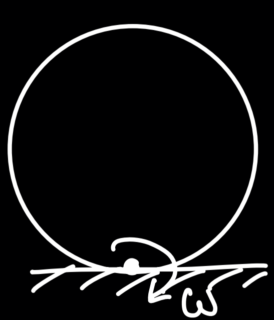
$\Rightarrow$



$\Rightarrow$ A body rolling on ground can be treated as pure rotation about an axis passing through point of contact and $\perp$ to the plane.



$$K.E = \frac{1}{2} I\omega^2 + \frac{1}{2} m v^2$$



$$K.E = \frac{1}{2} I' \omega^2$$

$$= \frac{1}{2}\left[I + mR^2\right]\omega^2$$

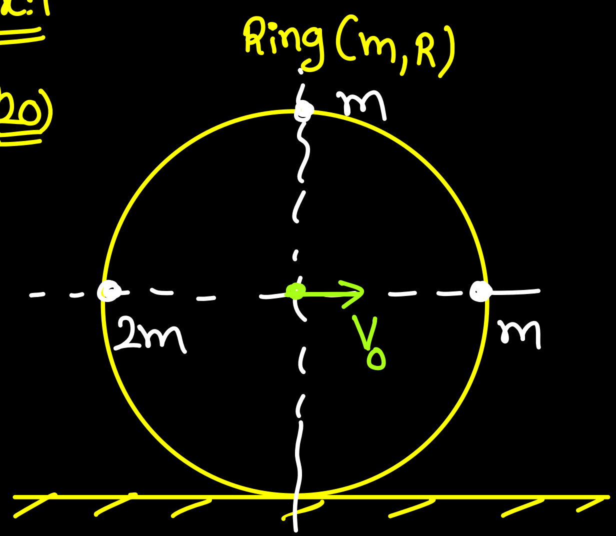
$$= \frac{1}{2} I\omega^2 + \frac{1}{2} mR^2\omega^2$$

$= \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2.$

$\Rightarrow$ at every instant axis of rotation gets changed so we call it as instantaneous axis of rotation.

Ex:1

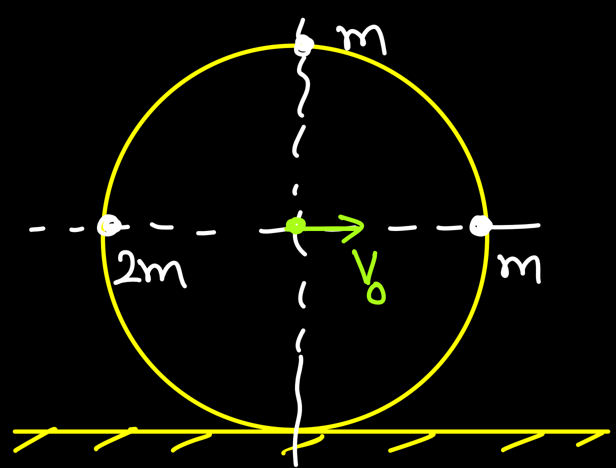
Q20)

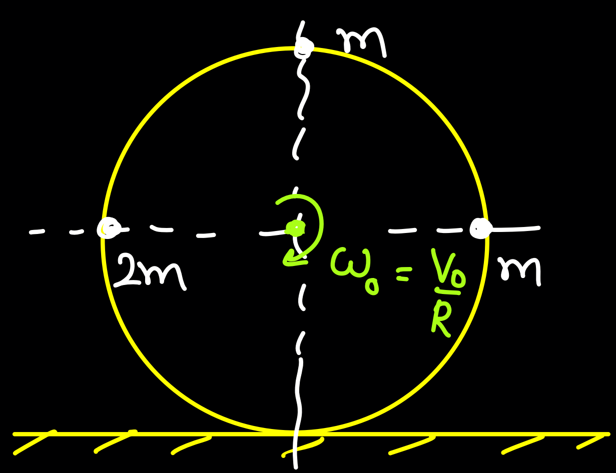


find K.E ?

Sol :- Standard approach

pure translation + Pure rotation





$K.E_t = \frac{1}{2}(5m)V_0^2$ + $K.E_R = \frac{1}{2}\left[mR^2 + (4m)R^2\right]\omega_0^2$

$= \frac{5mV_0^2}{2}$ $= \frac{5mV_0^2}{2}.$

$K.E_{total} = 5mV_0^2.$

Why don't we get $6mV_0^2$ using the above method ?

if we take of $K.E_{ring} + K.E_{masses}$ we get $6mv_0^2$.

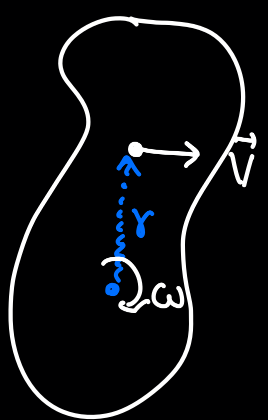
find what difference we have ?

If we take pure rotation about point of contact then

$$K.E = \frac{1}{2}\left[2mR^2 + 2m(2R^2) + m(2R^2) + m(4R^2)\right]\omega^2 = 6mv_0^2.$$

how to find IAR or ICR ?

case(i) info about vel. and ang. velocity is given

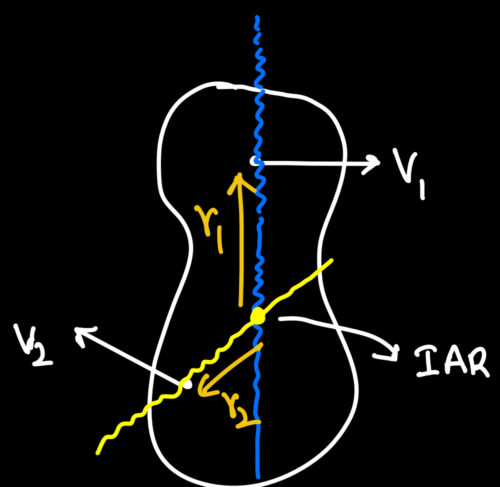


ω is given and its clockwise.

$$V = r\omega$$

$$r = \frac{V}{\omega}.$$

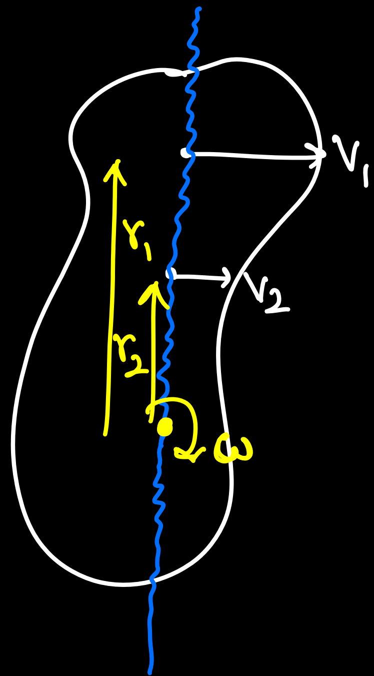
case (ii) Two non-parallel velocities are given.



$$V_1 = r_1\omega \Rightarrow \omega = \frac{V_1}{r_1}$$

$$V_2 = r_2\omega. \Rightarrow \omega = \frac{V_2}{r_2}.$$

Case (iii)- Vel. being parallel or anti parallel.



$V_1 > V_2$.

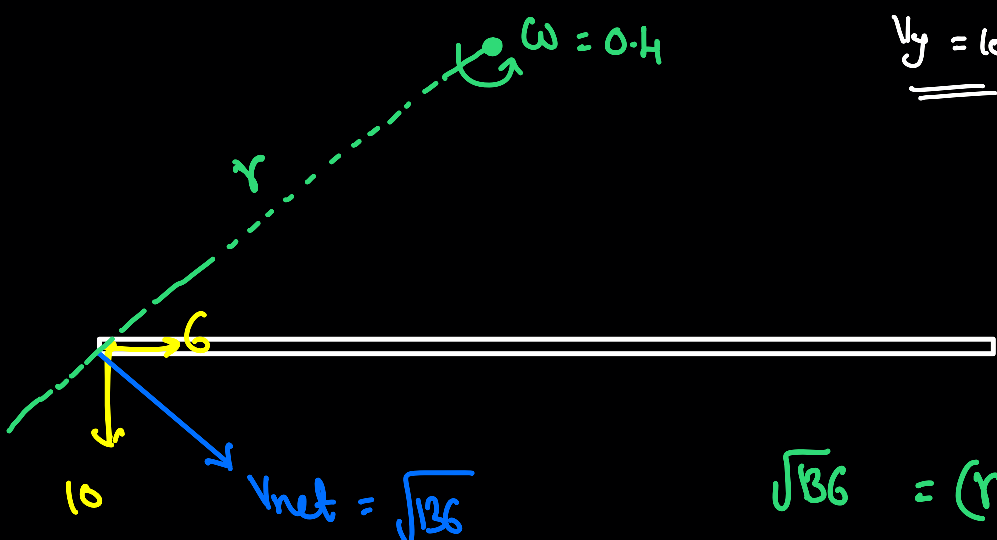
$V_1 = r_1\omega$

$V_2 = r_2\omega$

ill: 37 :-

$\omega_{BA} = \frac{(V_y + 10)}{50} \Rightarrow 20 = V_y + 10$

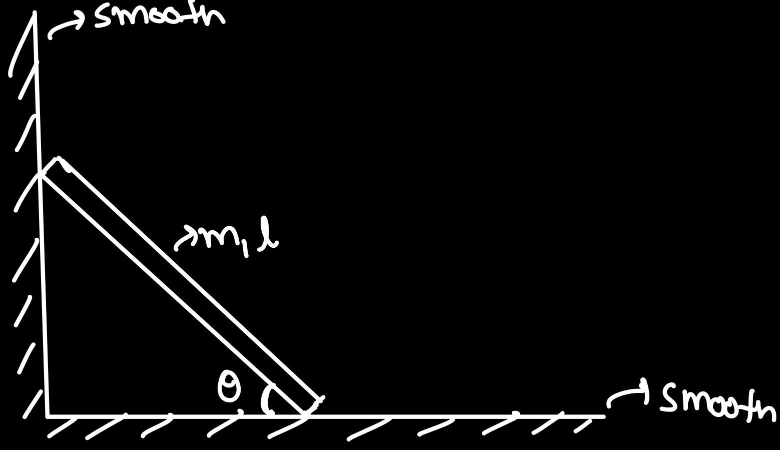
$V_y = 10$.



$\sqrt{136} = (r)(0.4)$

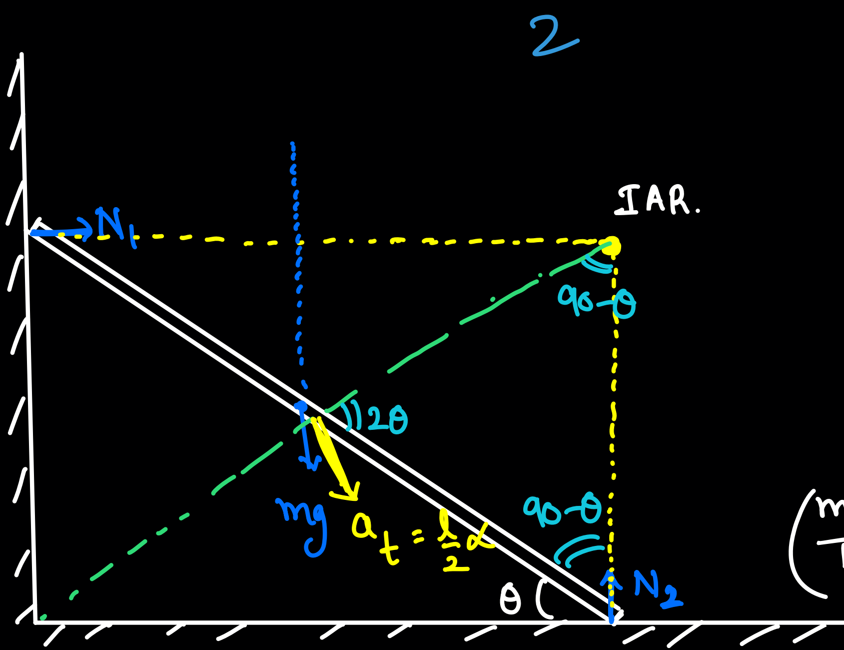
$r = \frac{\sqrt{136}}{0.4}$.

HW :-



rod is left from the position shown. find Normal reactions from both the walls?

Sol :-



IAR :-

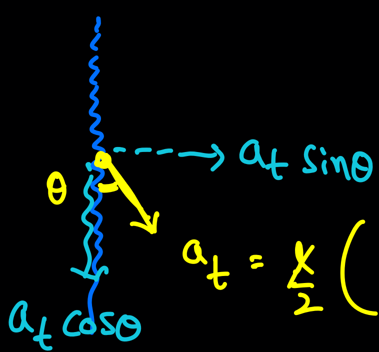
$$\tau = (mg)\frac{l}{2}\cos\theta.$$

$$I\alpha = \frac{mgl}{2}\cos\theta$$

$$\left(\frac{ml^2}{12} + \frac{ml^2}{4}\right)\alpha = \frac{mgl}{2}\cos\theta.$$

$$\frac{\cancel{m}l^{\cancel{2}}}{3}\alpha = \frac{\cancel{m}g\cancel{l}}{2}\cos\theta$$

$$\alpha = \frac{3g\cos\theta}{2l}.$$



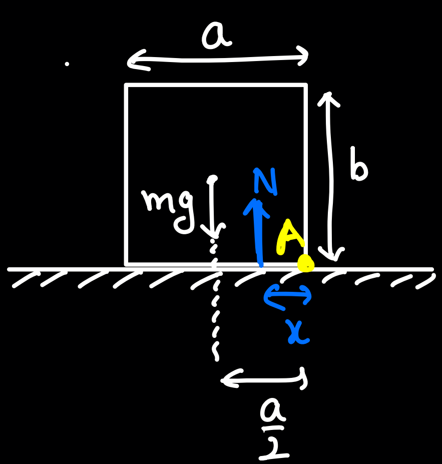
$$a_t = \frac{\cancel{l}}{2}\left(\frac{3g\cos\theta}{2\cancel{l}}\right) = \frac{3g\cos\theta}{4}.$$

$$N_1 = (m)a_t \sin\theta = \frac{3mg\sin\theta\cos\theta}{4}.$$

$$mg - N_2 = ma_t\cos\theta = \frac{3mg\cos^2\theta}{4}.$$

$$N_2 = mg\left[1 - \frac{3\cos^2\theta}{4}\right].$$

Toppling:-



it is given that block is at rest

$$\Sigma F_y = 0$$

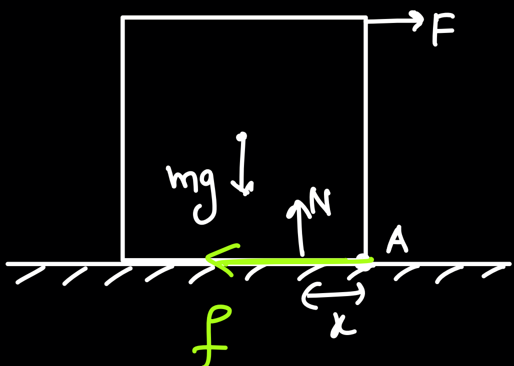
$$N = mg$$

$$\tau_A = 0$$

$$Nx = mg\frac{a}{2}$$

$$(mg)x = mg\frac{a}{2} \Rightarrow x = \frac{a}{2}.$$

block will not slide before toppling.



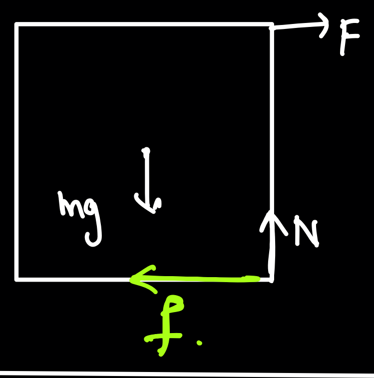
$$\tau_A = 0 \qquad \Sigma F_y = 0$$

$$Nx + Fb = mg\frac{a}{2}. \qquad N = mg$$

$$x = \frac{a}{2} - \frac{Fb}{mg}.$$

⇒ as F increases, N shifts more towards A. N can max. shift to A

At limiting condition, N will pass through point of toppling.

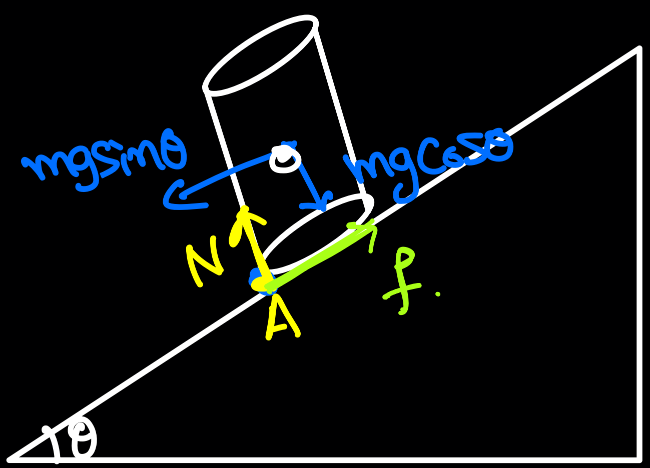


$$mg\frac{a}{2} = Fb$$

$$F = \frac{mga}{2b}.$$

---

ill:48



for sliding

$$\tan\theta > \mu.$$

$$\tau_A = 0$$

$$(mg\cos\theta)r = (mg\sin\theta)\frac{h}{2}$$

$$\tan\theta = \frac{2r}{h}.$$

if $\tan\theta > \frac{2r}{h}$ it topples

if $\mu < \frac{2r}{h}$ ⇒ body slides first

if $\mu > \frac{2r}{h}$ ⇒ body topples first

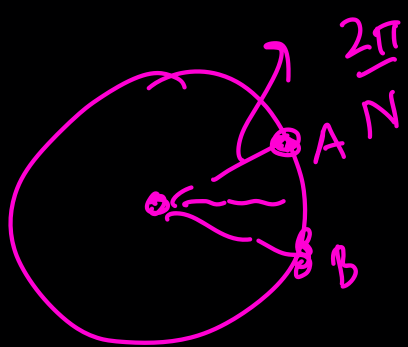
3. HW: A circular table has radius R and $N > 2$ equally spaced legs of length h attached to its perimeter. Suppose the table has a uniform mass density with total mass m. Assuming the table does not slip, the minimum horizontal force needed to tip over the table is :-

(A) $\dfrac{mgR}{h}$

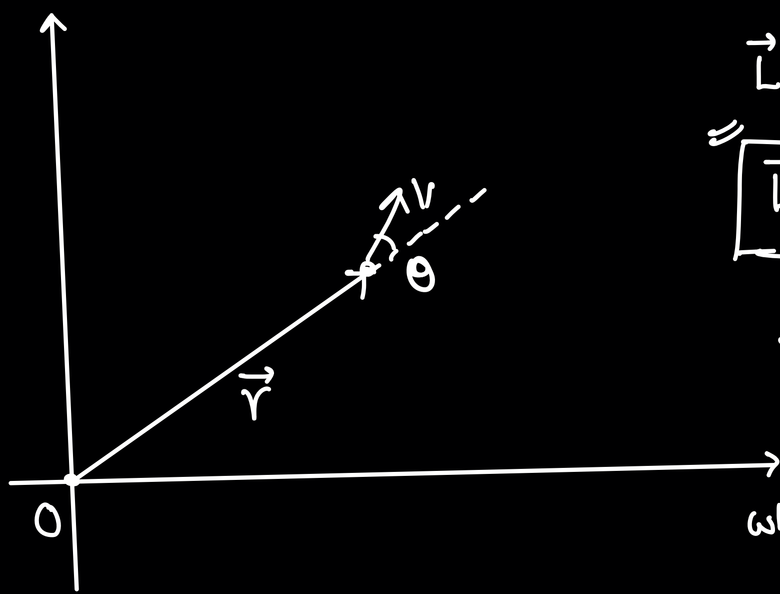
(B) $\dfrac{mgR}{h}\cos\left(\dfrac{\pi}{N}\right)$

(C) $\dfrac{mgR}{h}\tan\left(\dfrac{N-2}{2N}\pi\right)$

(D) $\dfrac{mgR}{h}\sin\left(\dfrac{\pi}{2N}\right)$



Angular momentum of a particle $(\vec{L})$:- [about a point].



$\vec{L}_O = \vec{r} \times \vec{p}$

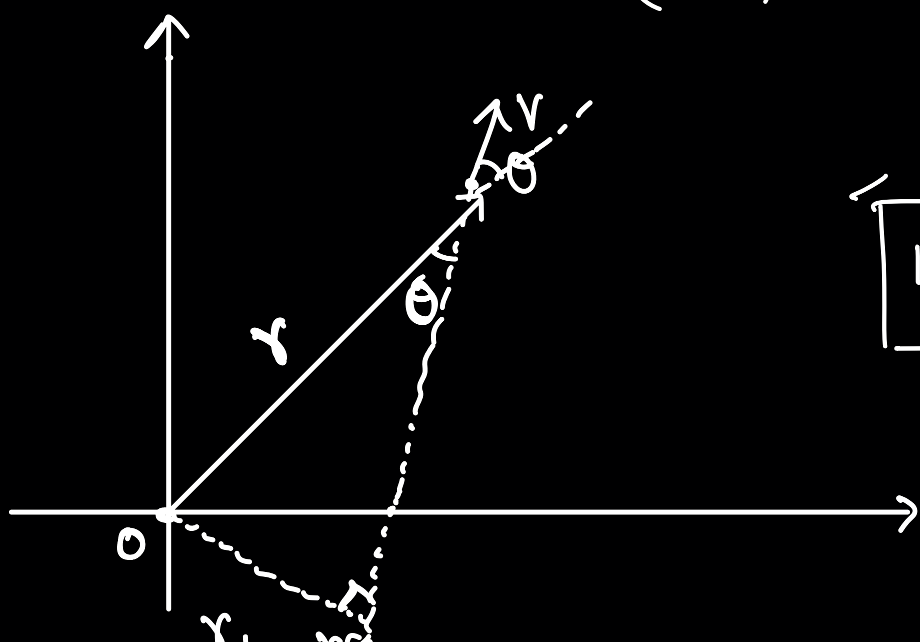
$$\boxed{\vec{L}_O = \vec{r} \times m\vec{v}}$$

$\vec{L} \perp \vec{r}, \vec{v}$

its $\perp$ to plane in which we have $\vec{r}$ and $\vec{v}$.

$|\vec{L}_O| = L_O = rmv\sin\theta$

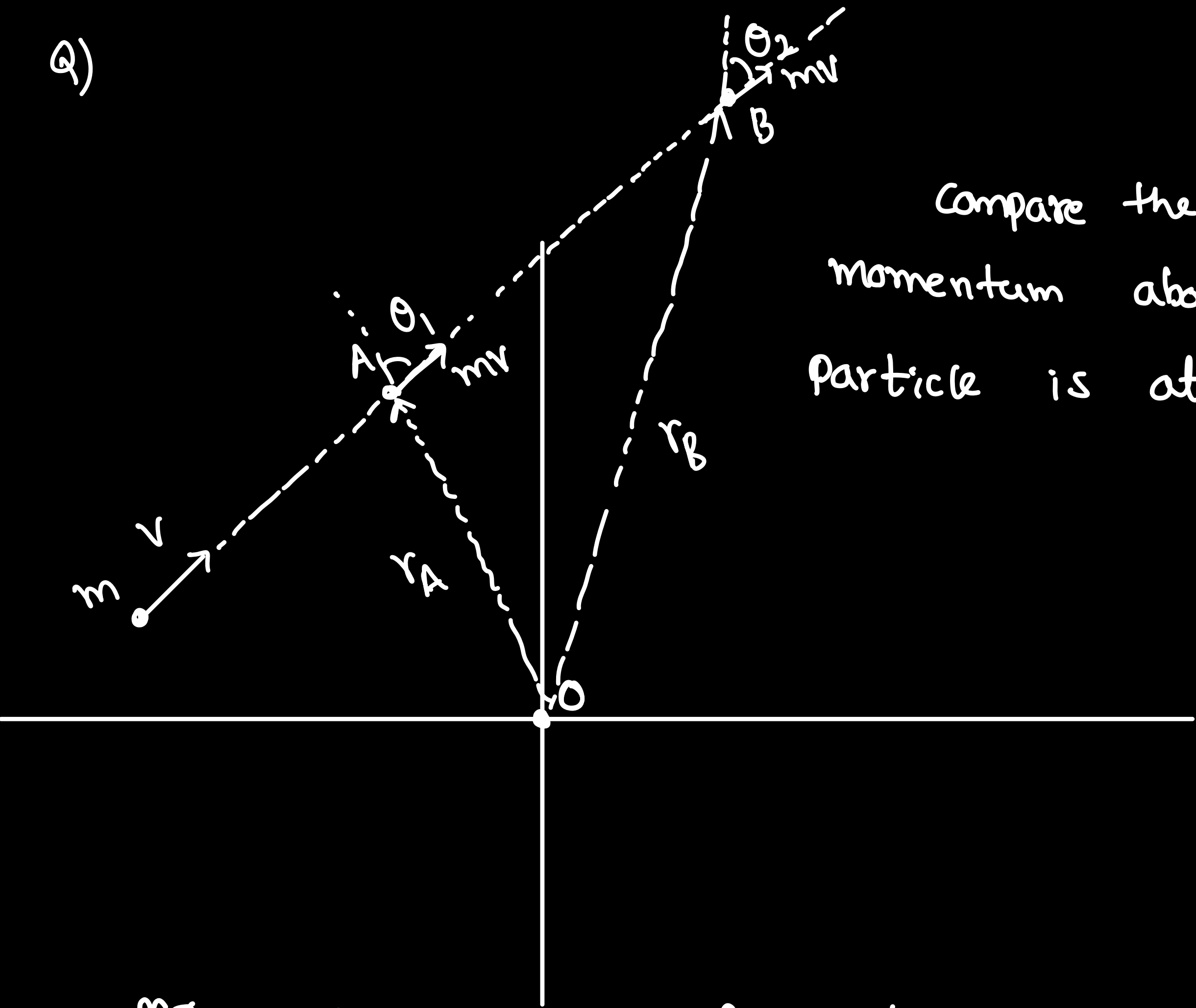
$= (r\sin\theta)\,mv.$



$r_\perp = r\sin\theta$

$$\boxed{L_O = (r_\perp)\,mv}$$

Q)

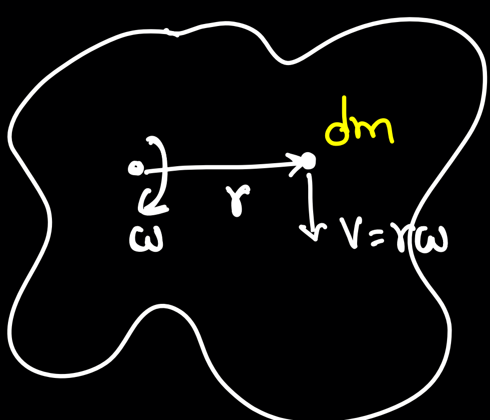


Compare the angular momentum about O when particle is at A and B?

$r\sin\theta = r_\perp$ = same for a straight line

$L_A = L_B$. $\because r_A \sin\theta_1 = r_B \sin\theta_2$.

Angular momentum of a body rotating about an axis:-



$d\vec{L} = r(dm)(r\omega)$

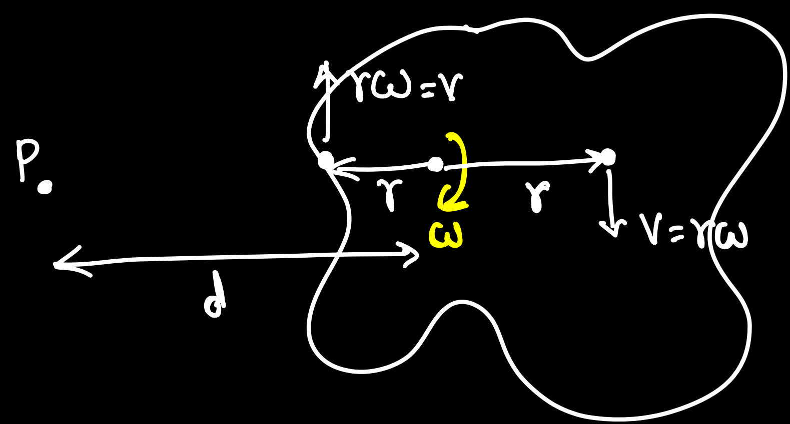
$d\vec{L} = dm\,r^2\omega$

$\vec{L} = \int dm\,r^2\omega$

$$\boxed{\vec{L} = I\vec{\omega}}$$

M.I about the axis of rotation

angular momentum about some random point in space :-



$$dL_p = (d-r)(m)(r\omega) + (d+r)(m)(r\omega)$$

$$= (r-d)(m)(r\omega) + (d+r)m(r\omega)$$

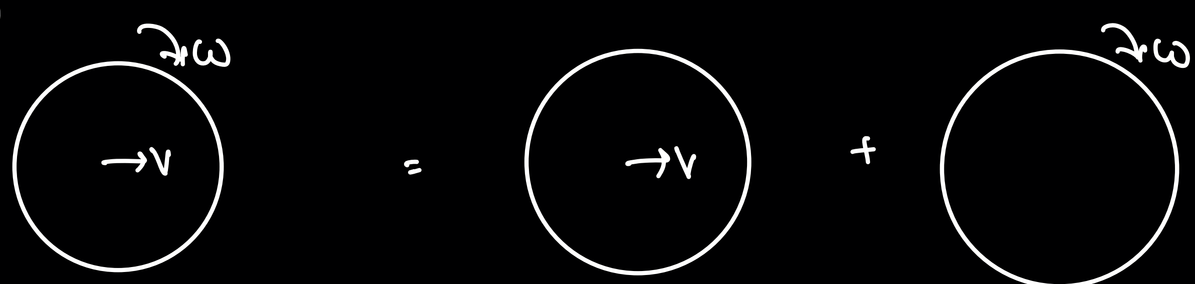
$$\int dL_p = \int 2mr^2\omega$$

$$\vec{L}_p = I\vec{\omega}$$

this is comp. ang. about point along the axis of rotation.

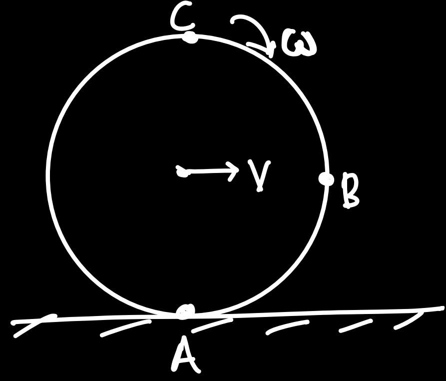
M.I about the axis of rotation.

Rolling



$$\boxed{\vec{L} = \vec{r}_{CM} \times m\vec{v} + I\vec{\omega}}$$

eg :- ring (m, R)

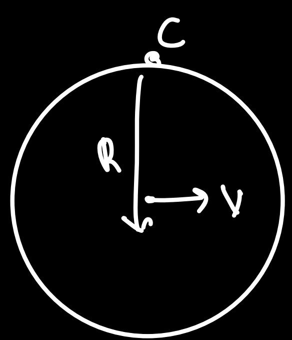


L_A ?

L_B ?

L_C ?

C

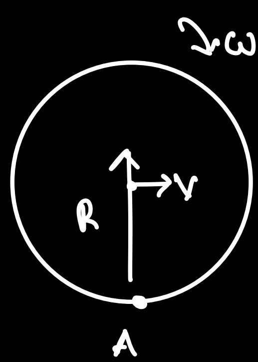


$$\vec{L}_C = Rmv \circlearrowleft + (mR^2)\omega \circlearrowright$$

$$= Rm(R\omega) \circlearrowleft + mR^2\omega \circlearrowright$$

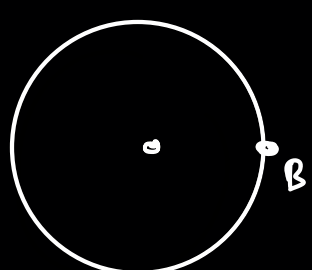
$$= 0.$$

A



$$\vec{L}_A = Rmv \circlearrowright + (mR^2)\omega \circlearrowright.$$

B



$$\vec{L}_B = (0)mv + (mR^2)\omega \circlearrowright$$

$$= mR^2\omega \circlearrowright$$

Relation b/w angular momentum and Torque :-

$$\vec{\tau} = \frac{d\vec{L}}{dt}.$$

if I is constant

$$L = I\omega$$

$$\frac{dL}{dt} = I\frac{d\omega}{dt}$$

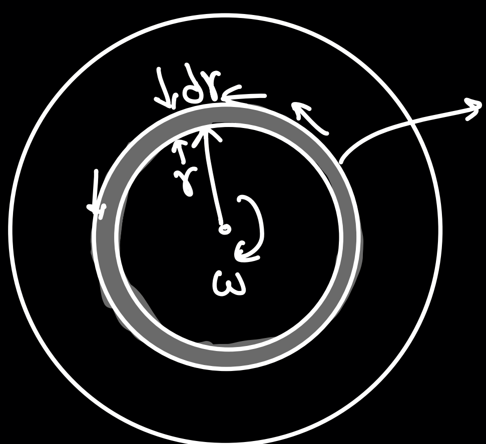
$$\frac{dL}{dt} = I\alpha.$$

if $\vec{\tau} = 0 \Rightarrow \frac{d\vec{L}}{dt} = 0$

$\vec{L}$ = constant

$\vec{L}$ is conserved.

11:55



$$dm = \frac{M}{\cancel{\pi}R^2}2\cancel{\pi}rdr$$

$$= \frac{2Mrdr}{R^2}.$$

$$N = (dm)g.$$

$$f = \mu N$$

$$= \frac{2\mu Mrdrg}{R^2}.$$

$$d\tau = rf$$

$$\tau = \int_0^R \frac{2\mu M r^2 dr}{R^2} g$$

$$\tau = \frac{2\mu MRg}{3}$$

$$I\alpha = \frac{2\mu MgR}{3}.$$

$$\frac{\cancel{M}R^{\cancel{2}}}{2}\alpha = \frac{2\mu \cancel{M}g\cancel{R}}{3}$$

$$\alpha = \frac{4\mu g}{3R}.$$

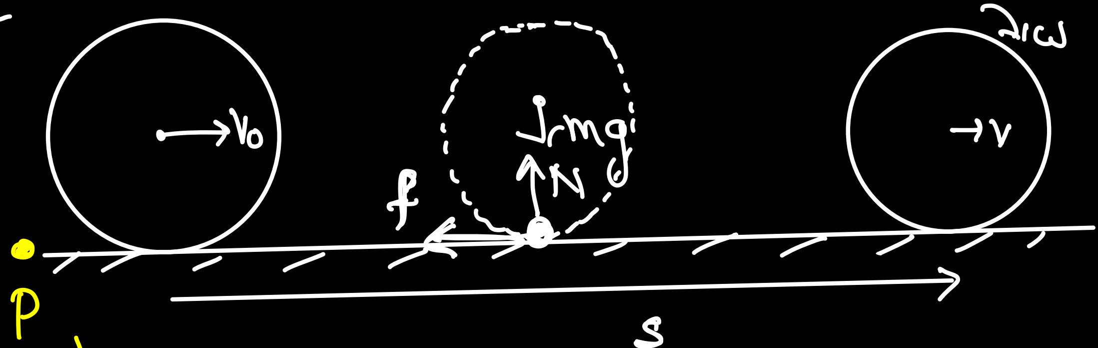
$$\omega_f = \omega_i + \alpha t.$$

$$0 = \omega_0 - \frac{4\mu g}{3R}t$$

$$t = \frac{3\omega_0 R}{4\mu g}.$$

ill:46 :-

now rolling starts.



Torque about any point on floor $= (f)0 + Nr_\perp \circlearrowleft + mgr_\perp \circlearrowright$

$= 0.$

as $\tau = 0 \Rightarrow L$ is conserved.

$$\vec{L}_i = R(mv_0) \circlearrowright$$

$$\vec{L}_f = Rmv\,\partial + I\omega\,\partial.$$

$$= Rmv + \frac{2}{5}MR^2\omega$$

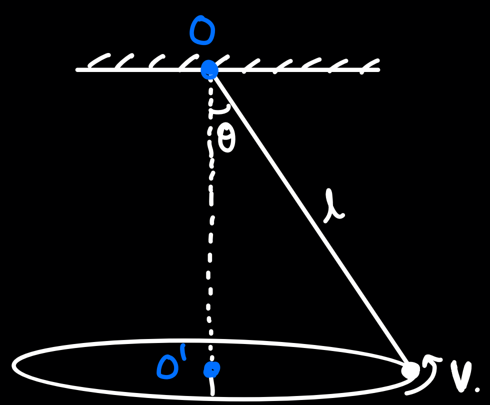
$$= Rmv + \frac{2}{5}mv.$$

$$\vec{L}_i = \vec{L}_f$$

$$\cancel{R}\cancel{m}v_0 = \frac{7\cancel{m}v\cancel{R}}{5}$$

$$\boxed{v = \frac{5v_0}{7}}$$

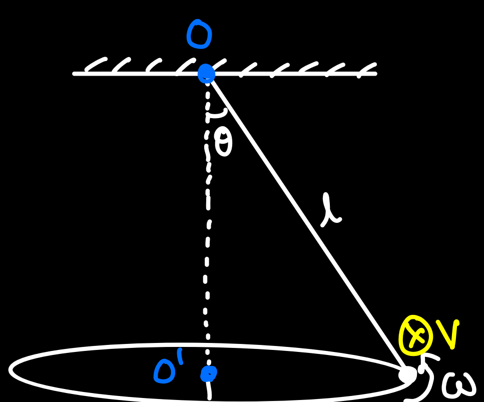
Recap :- Conical pendulum.



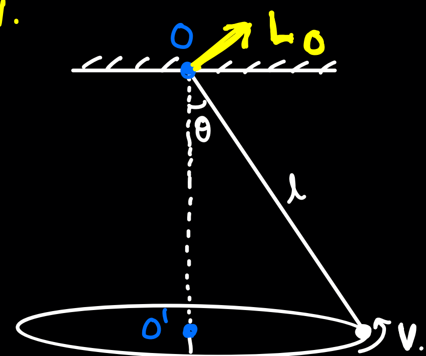
$\tau_O = l\,mg\sin\theta.$

$\tau_{O'} = 0.$

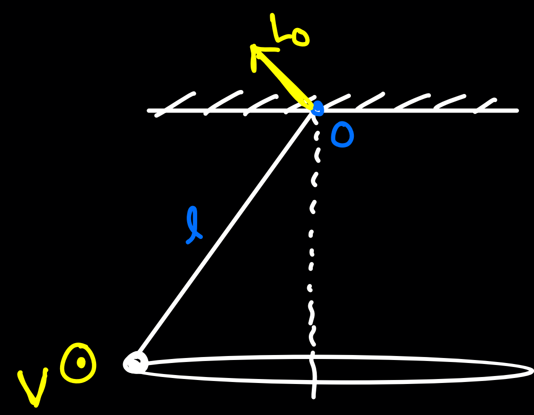
$\tau_{OO'} = \tau_{axis} = 0.$

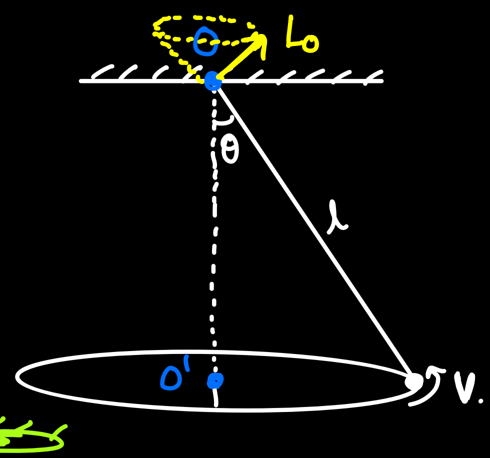


$L_O = lmv.$

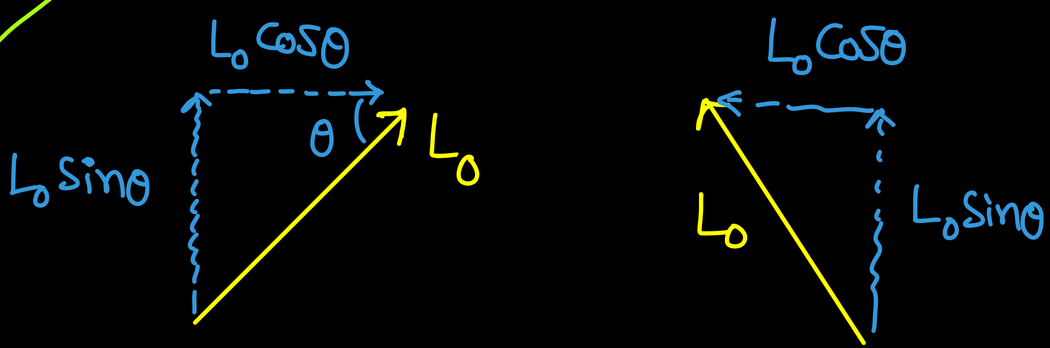


$L_o = lmv$



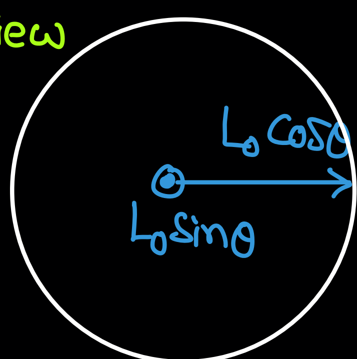


magnitude of Lo is constant
but direction changes
so $\vec{L_o}$ changes.

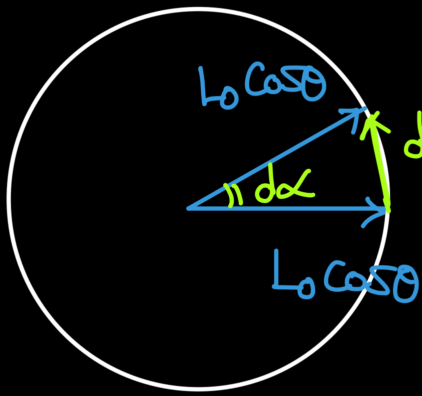


$L_o \sin\theta$ is remaining as it is
$L_o \cos\theta$ is doing circular motion.

from top view



$$\frac{d\vec{L}}{dt} = \frac{d(\overrightarrow{L_o \cos\theta})}{dt}.$$

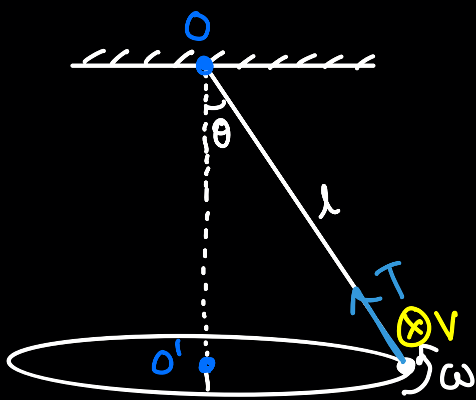


$$d(L_0 \cos\theta) = (L_0 \cos\theta)\, d\alpha$$

$$\frac{d(L_0 \cos\theta)}{dt} = (L_0 \cos\theta)\frac{d\alpha}{dt}.$$

$$= (L_0 \cos\theta)\omega.$$

$$\frac{dL}{dt} = \frac{d(L_0 \cos\theta)}{dt} = L_0\, \omega \cos\theta.$$



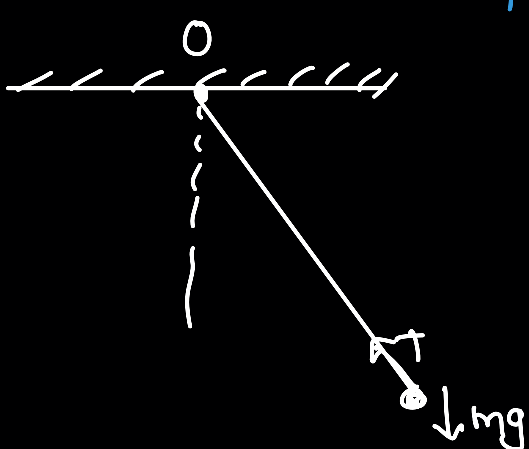
$$T \sin\theta = m l \sin\theta\, \omega^2.$$

$$\frac{mg}{\cos\theta} = m l \omega^2 \Rightarrow \omega = \sqrt{\frac{g}{l\cos\theta}}.$$

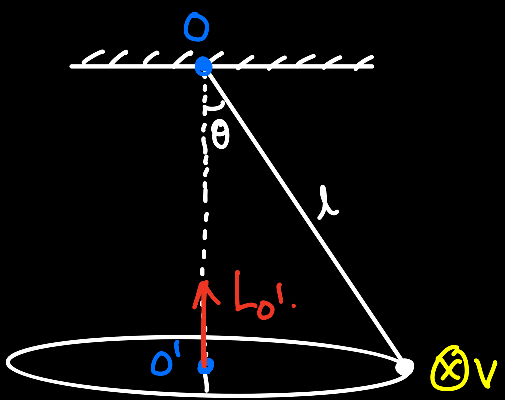
$$\frac{dL}{dt} = (m l v)\sqrt{\frac{g}{l\cos\theta}}\cos\theta.$$

$$= (m l)(l \sin\theta)\sqrt{\frac{g}{l\cos\theta}}\sqrt{\frac{g}{l\cos\theta}}\cos\theta$$

$$\tau = \frac{dL}{dt} = (mg)\, l \sin\theta.$$



$$\tau_0 = (mg)\, l \sin\theta.$$

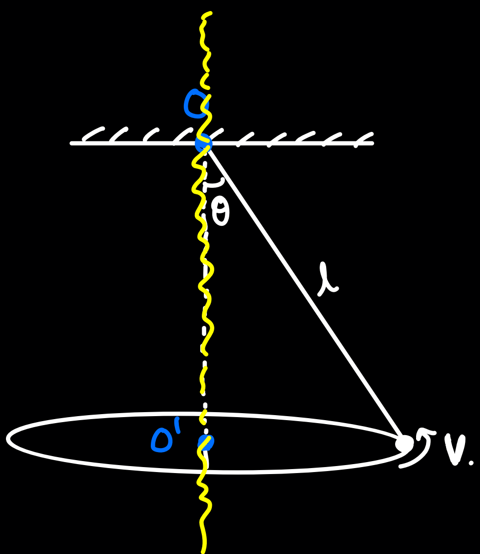


$$L_{O'} = (l\sin\theta)mv \uparrow.$$

$$\frac{dL_{O'}}{dt} = 0$$

$$\tau = 0.$$

Using forces also we get $\tau = 0$.



$$L_{axis} = I\omega$$

$$= (mr^2)\omega.$$

$$= mr(v)$$

$$= ml\sin\theta\, v. \uparrow.$$

$$\frac{d(L_{axis})}{dt} = \tau_{axis} = 0.$$

Using forces also we get

$$\tau_{axis} = 0.$$

$|\vec{L}|$ about O $= mlv$

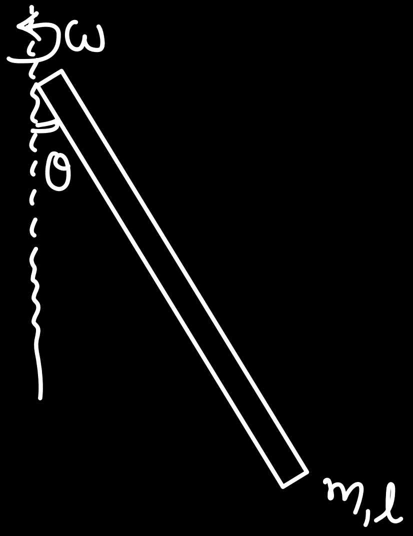
Component of angular momentum in the direction of axis of rotation

$$= L\sin\theta$$

$$= mlv\sin\theta$$

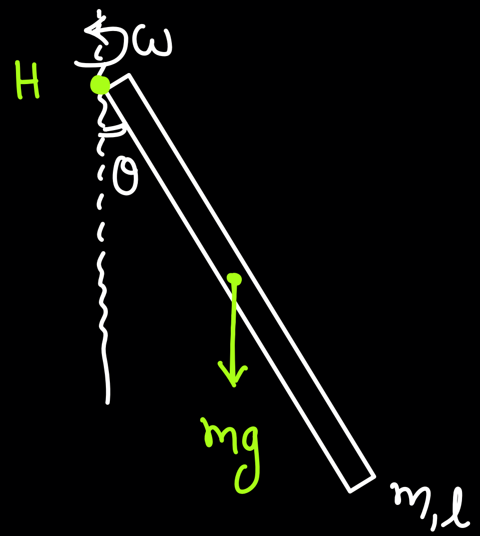
Q)

Ex:4B) Q26

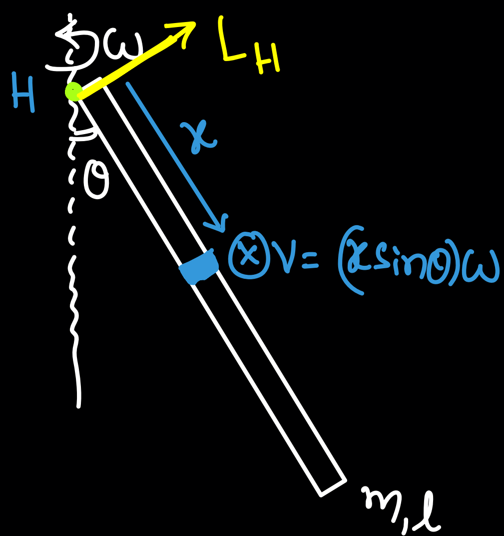


find θ ?

Sol:-



$$\tau_H = mg\frac{l}{2}\sin\theta \circlearrowright.$$

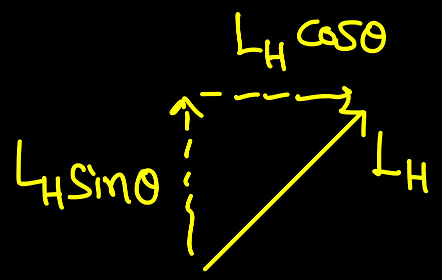


$$dL_H = \left[(dm)(x\sin\theta)\omega\right]x.$$

$$L_H = \int_0^l \frac{m}{l}(dx)\, x^2 \sin\theta\, \omega$$

$$= \frac{m\omega\sin\theta}{l}\left[\frac{l^3}{3}\right]$$

$$L_H = \frac{ml^2\omega}{3}\sin\theta.$$



$$\frac{d(L_H)}{dt} = \frac{d(L_H\cos\theta)}{dt} = (L_H\cos\theta)\omega.$$

$$\tau_H = \left(\frac{ml^2\omega\sin\theta}{3}\right)\cos\theta\,\omega.$$

$$\cancel{m}g\frac{\cancel{l}}{2}\cancel{\sin\theta} = \frac{\cancel{m}l^{\cancel{2}}\omega^2\cancel{\sin\theta}\cos\theta}{3}$$

$$\cos\theta = \frac{3g}{2l\omega^2}$$

$$\theta = \cos^{-1}\left[\frac{3g}{2l\omega^2}\right].$$

HW:-

$\boxed{\text{Ex:4B) Q26}}$

1.57. A round cone with half-angle $\alpha = 30°$ and the radius of the base $R = 5.0$ cm rolls uniformly and without slipping over a horizontal plane as shown in Fig. 1.8. The cone apex is hinged at the point O which is on the same level with the point C, the cone base centre. The velocity of point C is $v = 10.0$ cm/s. Find the moduli of

(a) the vector of the angular velocity of the cone and the angle it forms with the vertical;

(b) the vector of the angular acceleration of the cone.

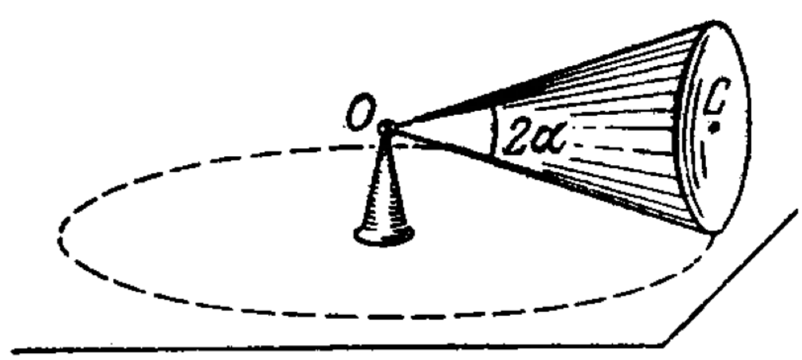


Fig. 1.8.

Sol:-

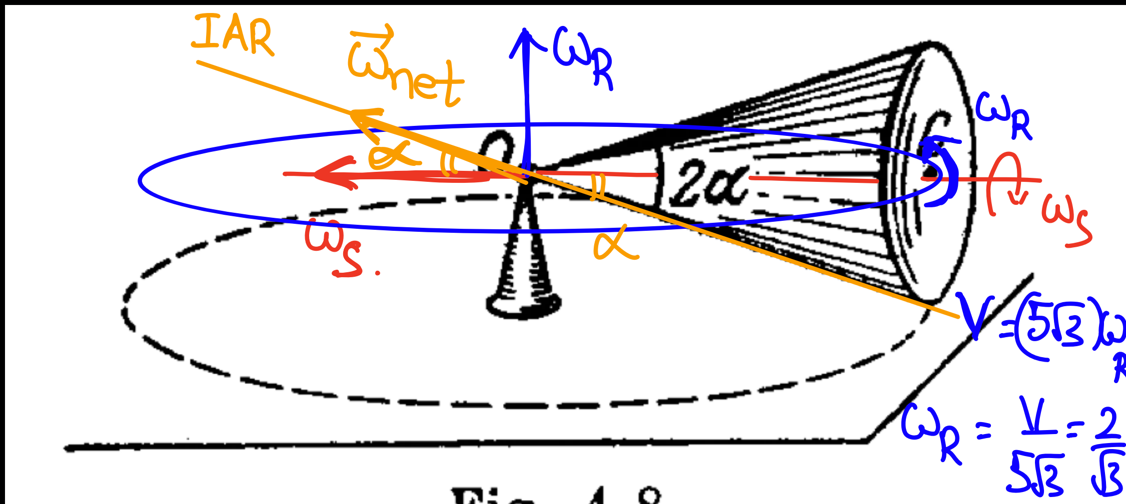
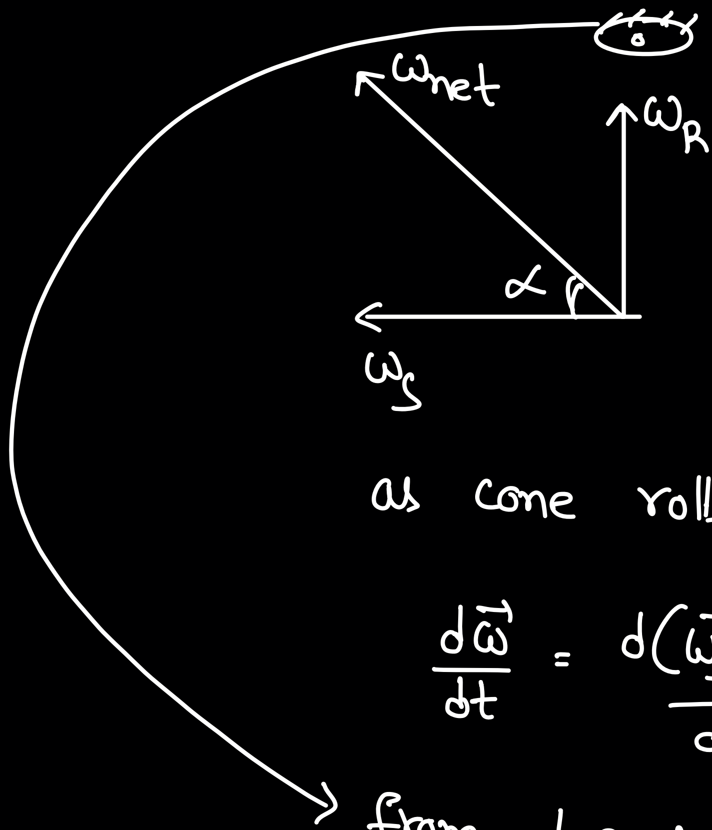


Fig. 1.8.



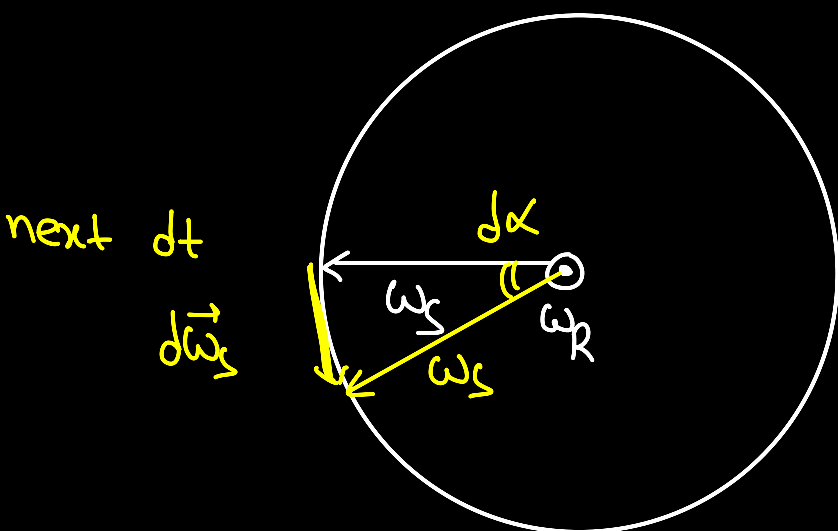
$$\tan\alpha = \frac{\omega_R}{\omega_s}$$

$$\omega_s = \frac{\omega_R}{\tan\alpha} = \frac{\frac{2}{\sqrt{3}}}{\frac{1}{\sqrt{3}}} = 2\ \text{rad/s}.$$

as cone rolls $\Rightarrow$ $\vec{\omega}_s$ changes but not $\vec{\omega}_R$.

$$\frac{d\vec{\omega}}{dt} = \frac{d(\vec{\omega}_s + \vec{\omega}_R)}{dt} = \frac{d\vec{\omega}_s}{dt}.$$

from top view



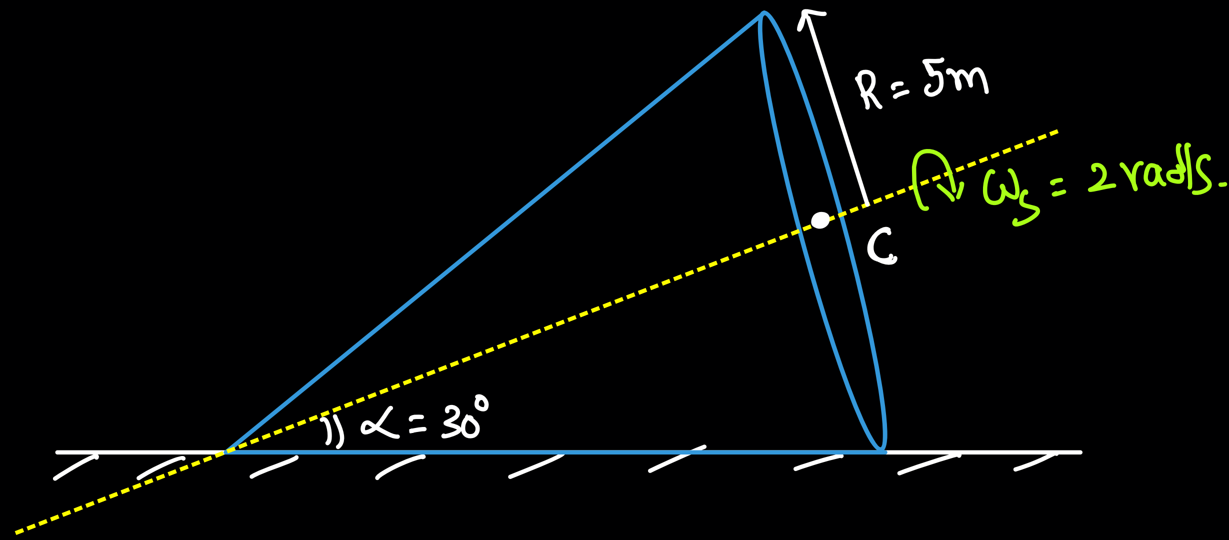
$$d\omega_s = \omega_s(d\alpha)$$

$$\frac{d\omega_s}{dt} = \omega_s\left(\frac{d\alpha}{dt}\right)$$

$$\alpha = \omega_s\,\omega_R$$

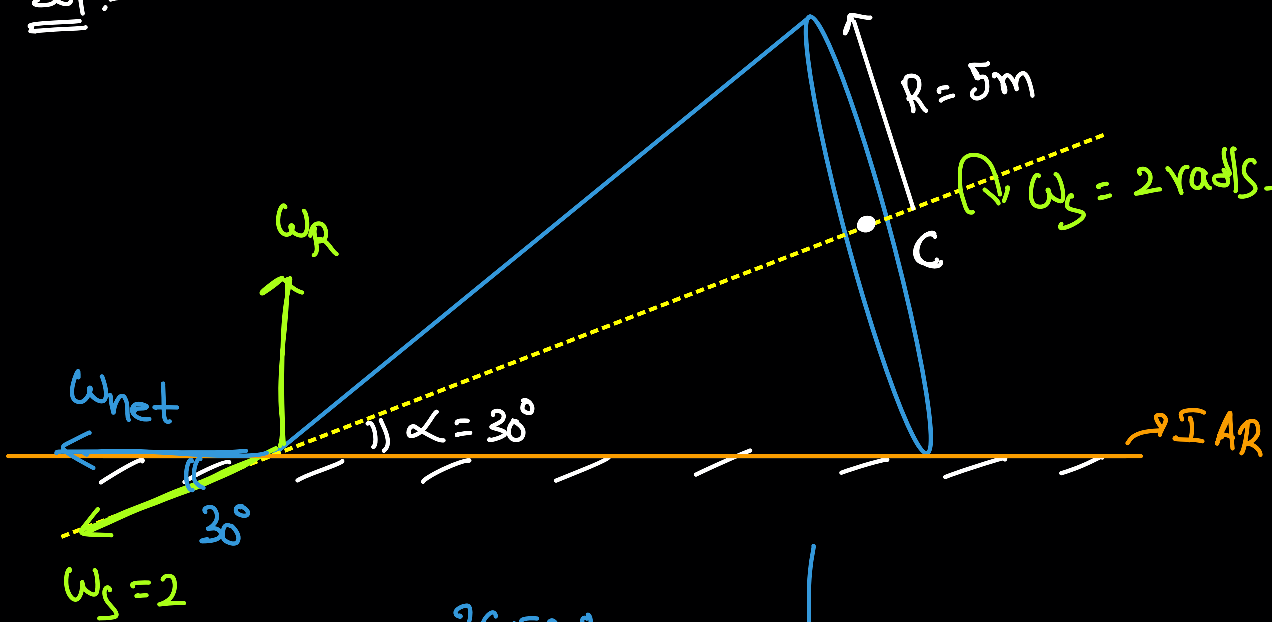
$$= (2)\left(\frac{2}{\sqrt{3}}\right) = \frac{4}{\sqrt{3}}.$$

Q)



Cone is rolling without slipping, find ω_{net} and α ?

Sol :-



$$2\cos 30° = \omega_{net}$$

$$\omega_{net} = \sqrt{3}$$

$$2\sin 30° = \omega_R.$$

$$\omega_R = 1.$$

$$\frac{d\vec{\omega}_{net}}{dt} = (\omega_{net})\,\omega_R.$$

$$= (\sqrt{3})\,\omega_R.$$

$$\alpha = \sqrt{3}.$$

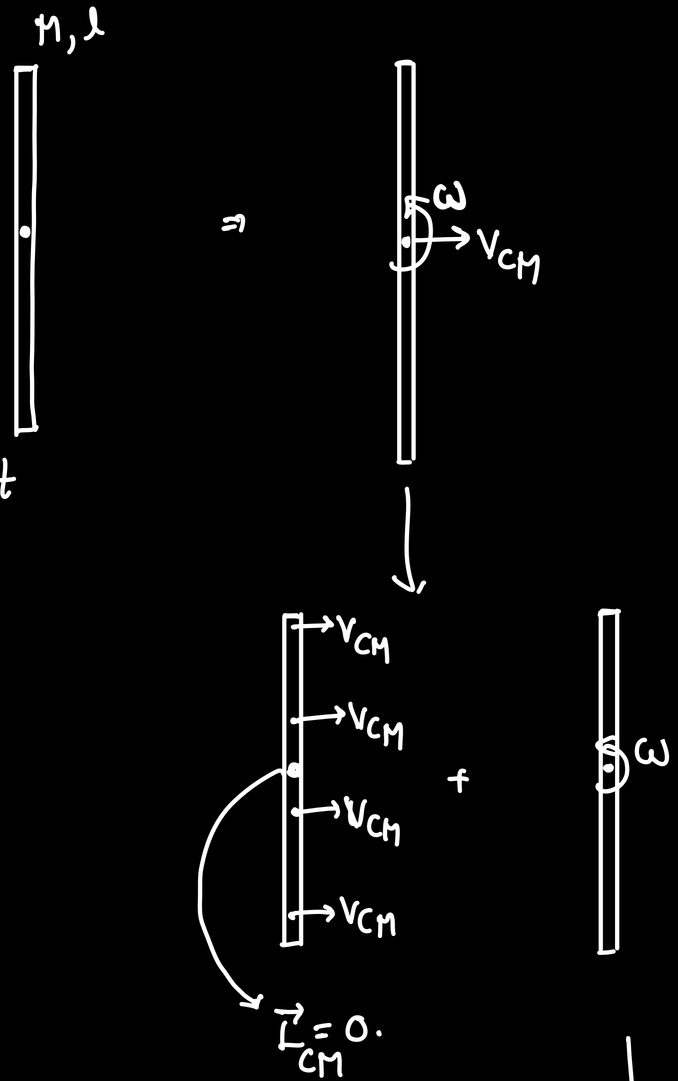
HW Ex: 23 :-

Angular impulse ($\vec{J}$) :-

$$\vec{J} = \Delta \vec{L}$$

$$\boxed{\vec{J} = \int \tau \, dt = \Delta \vec{L}}$$

eg:-



$\vec{I} = \int F dt$

$L_i = 0$

$\vec{L}_{Rotation} = I\omega$

$[\vec{L}_f]_{CM} = I\omega$

relation b/w $\vec{J}$ and $\vec{I}$ → impulse

$$\boxed{\vec{J} = \vec{r} \times \vec{I}}$$

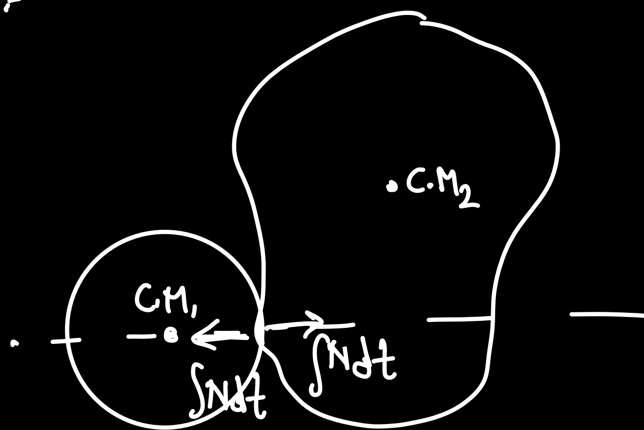
$$\vec{J} = \int \left(\frac{l}{2} F\right) dt$$

M.I.

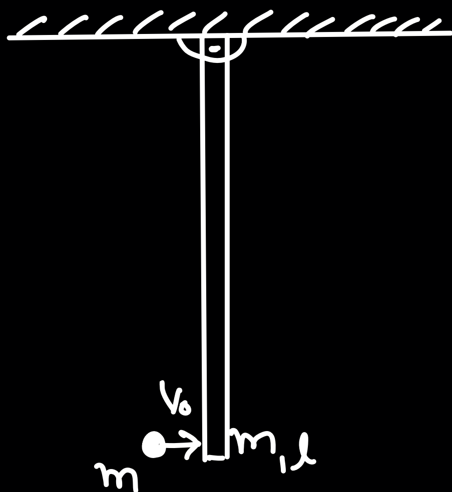
$$I\omega = \frac{l}{2} \int F dt$$

$$\omega = \frac{\frac{l}{2}\int F\,dt}{I}.$$

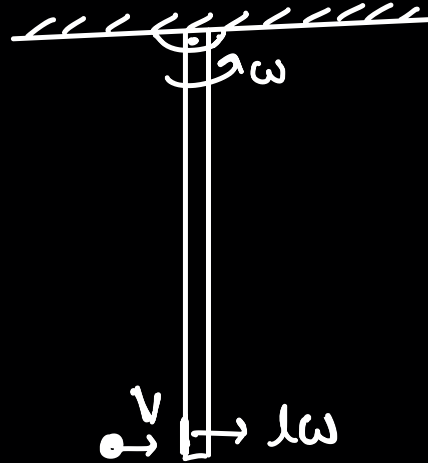
Eccentric impact:-



Ill:60 :-



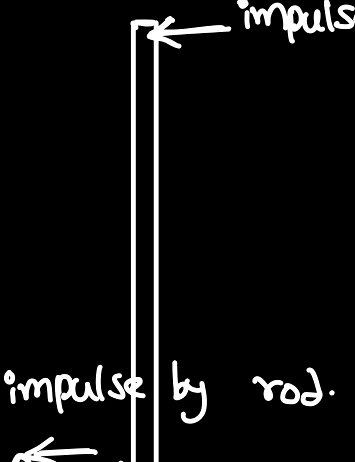
after collision



$L_i = 0 + l m v_0 \circlearrowright$

$L_f = \frac{ml^2}{3}\omega \circlearrowleft + m l V \circlearrowleft.$

during collision



$\tau_{Hinge} = 0$

$\vec{L}_{Hinge} = \text{constant}$

impulse by mass

$$L_i = L_f$$

$$m l v_0 = m l v + \frac{m l^2}{3}\omega \quad -(1)$$

$$e = \frac{l\omega - v}{v_0}$$

$$1 = \frac{l\omega - v}{v_0}$$

$$l\omega - v = v_0 \quad -(2)$$

on solving $v = \frac{v_0}{2}$, $\omega = \frac{3v_0}{2l}$.

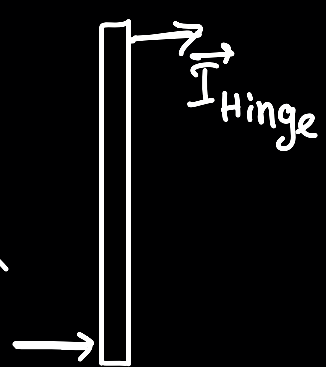
initially $\qquad\qquad$ finally

$\bullet \rightarrow v_0$ $\qquad\qquad$ $\bullet \rightarrow \frac{v_0}{2}$.

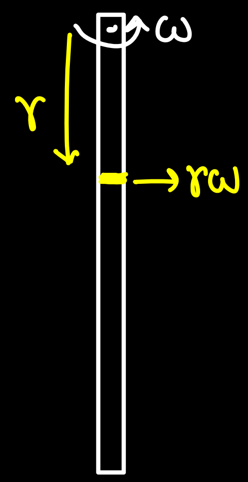
impulse on particle $= \Delta\vec{p} = \left(m\frac{v_0}{2}\hat{i}\right) - m v_0(\hat{i})$

$$= -\frac{m v_0}{2}\hat{i}.$$

$\bullet \leftarrow$ $\vec{I} = \frac{m v_0}{2}(-\hat{i})$



$\vec{I}_{on\,rod} = \frac{m v_0}{2}\hat{i}$



$$d\vec{p} = (dm)v$$

$$d\vec{p} = (dm)r\omega.$$

$$\vec{p} = \int (dm) r\omega$$

$$= \left[\int dm\, r\right]\omega$$

$$= (m r_{CM})\omega$$

$$= m\frac{l}{2}\omega.$$

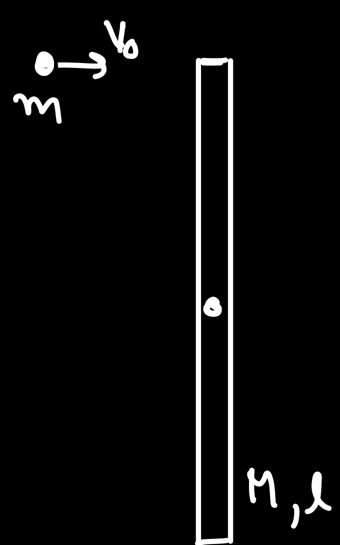
$$\Delta\vec{p} \text{ of rod} = \frac{ml}{2}\omega\hat{i} - 0$$

$$I \text{ on rod} = \frac{ml\omega}{2}\hat{i} = \frac{3mv_0}{4}\hat{i}$$

$$\frac{mv_0}{2}\hat{i} + \vec{I}_{hinge} = \frac{3mv_0}{4}\hat{i}$$

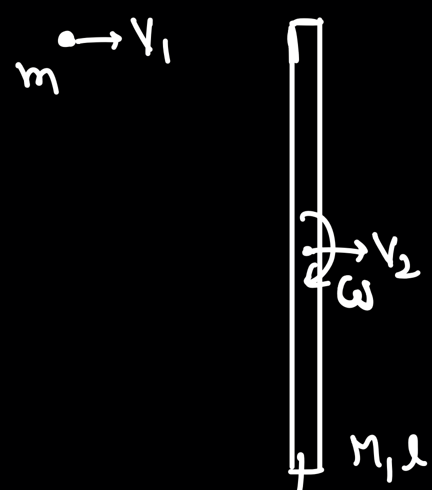
$$I_{hinge} = \frac{mv_0}{4}\hat{i}$$

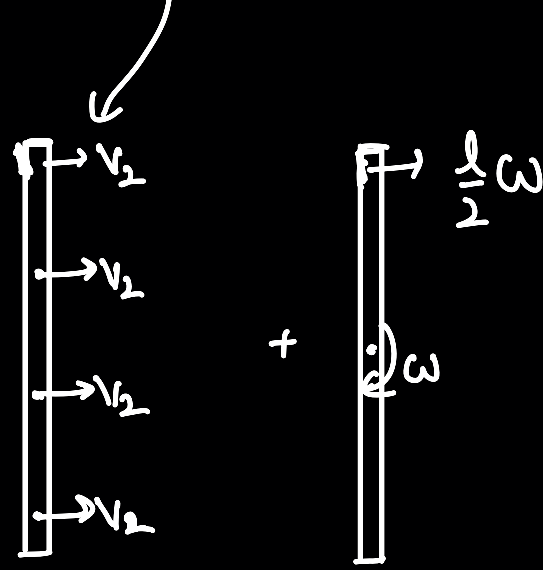
ill: 61 :-



⇒

Just after Collision





$$\vec{P}_i = \vec{P}_f$$

$$mV_0 + 0 = mV_1 + [MV_2 + 0]$$

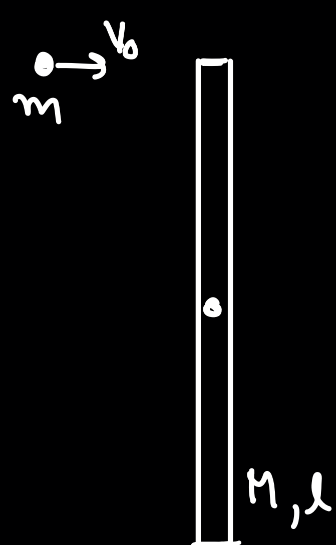
$$mV_0 = mV_1 + MV_2 \quad -① .$$

$$e = \frac{\left(V_2 + \frac{l}{2}\omega\right) - V_1}{V_0}$$

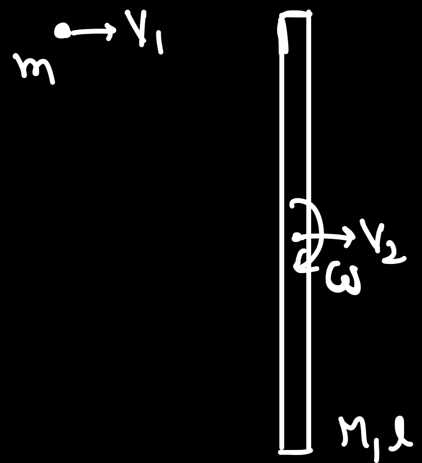
$$V_2 + \frac{l}{2}\omega - V_1 = eV_0 \quad -②$$

τ_{net} about c.m. of rod $= 0$

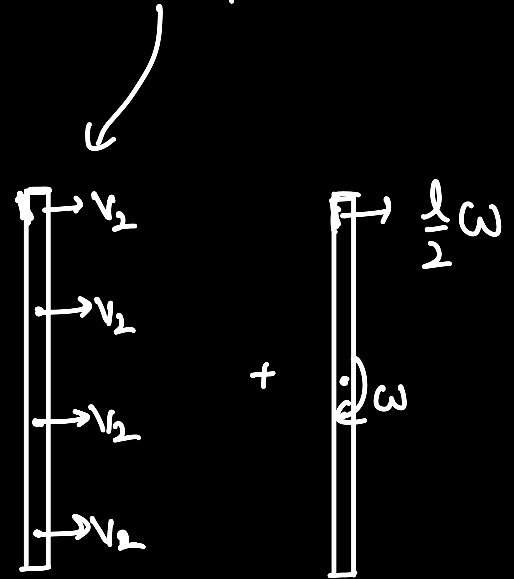
$$\vec{L}_{cm} = \text{constant}$$



$$\vec{L}_i = \frac{l}{2} m V_0 \circlearrowright$$



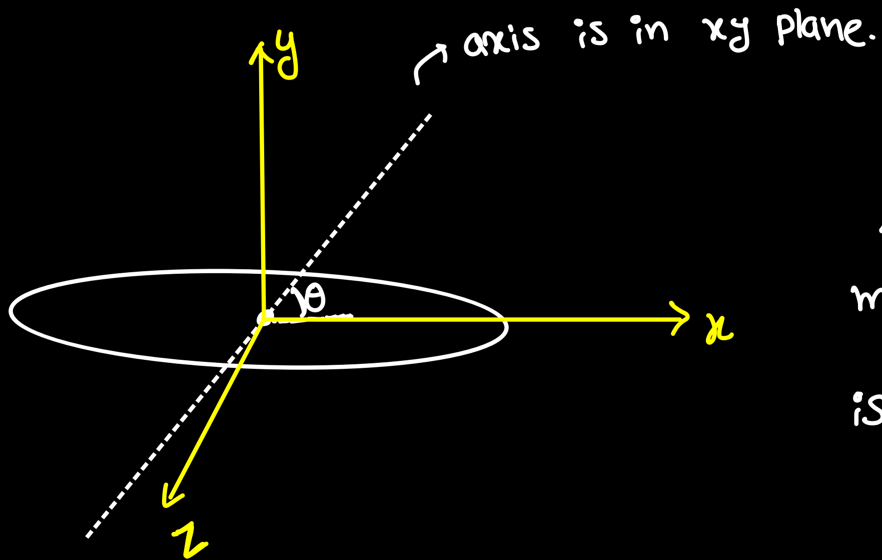
$$\vec{L}_f = m\frac{l}{2} v_1 \hat{k} + \left[0 + \frac{Ml^2}{12}\omega \hat{k}\right].$$



$$\vec{L}_i = \vec{L}_f$$

$$m\frac{l}{2} v_0 = m\frac{l}{2} v_1 + \frac{Ml^2}{12}\omega \quad - (3)$$

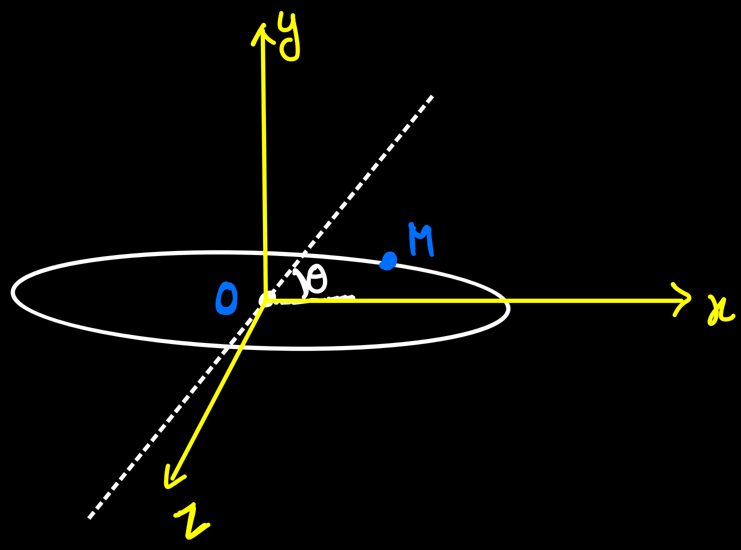
Q)

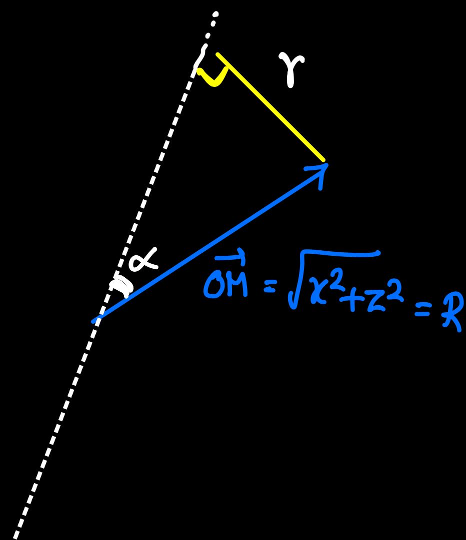


ring is in x-z plane.

location of any dm mass from centre i.e. (0,0) is $x\hat{i} + z\hat{k}$.

direction of axis $\hat{A} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$.





$$\sin\alpha = \frac{r}{R}.$$

$$r = R\sin\alpha.$$

$$\vec{OM}\cdot\vec{A} = (OM)(A)\cos\alpha.$$

$$x\cos\theta = (R)(1)\cos\alpha$$

$$\cos\alpha = \frac{x\cos\theta}{R}. \Rightarrow \sin\alpha = \sqrt{1 - \frac{x^2\cos^2\theta}{R^2}}.$$

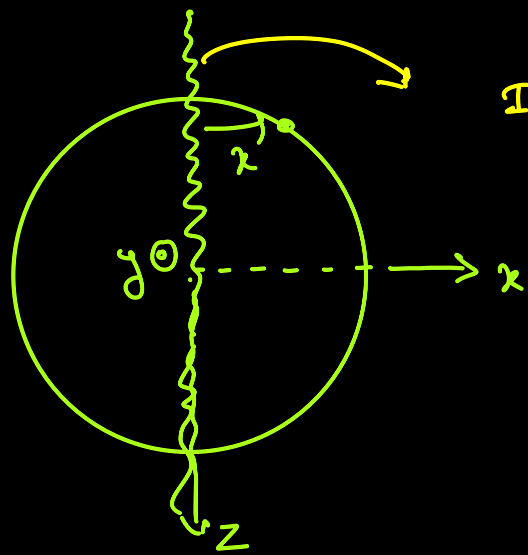
$$I = \int (dm)\, r^2$$

$$= \int dm\, R^2 \sin^2\alpha$$

$$= \int dm\, (R^2)\left(1 - \frac{x^2\cos^2\theta}{R^2}\right).$$

$$I = \int dm R^2 - \int dm\, x^2 \cos^2\theta$$

$$I = MR^2 - \cos^2\theta \int dm\, x^2.$$



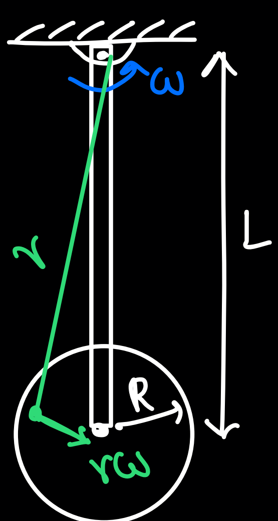
$I_{diameter} = \int dm\, x^2$

$\frac{MR^2}{2} = \int dm\, x^2$

$$I = MR^2 - \cos^2\theta \frac{MR^2}{2}$$

$$I = \frac{mR^2}{2}\left[2 - \cos^2\theta\right] \Rightarrow I = \frac{mR^2}{2}\left[1 + \sin^2\theta\right].$$

Q) rigidly fixed disc:-



disc:-

$$dL = (r)(dm)(r\omega)$$

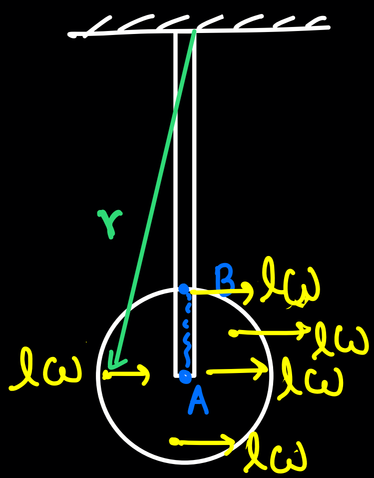
$$= (dm) r^2 \omega$$

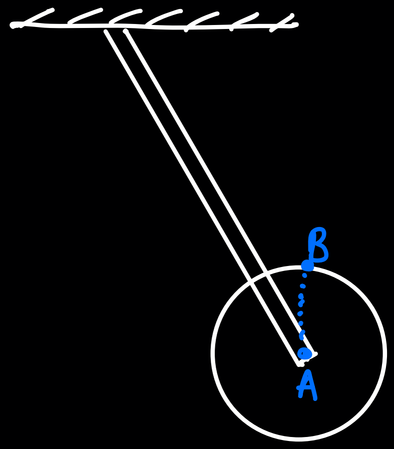
$$L = \int dm\, r^2\, \omega$$

$$= \left[\int dm\, r^2\right] \omega.$$

$$= \left(\frac{MR^2}{2} + ML^2\right)\omega.$$

disc free to rotate:-





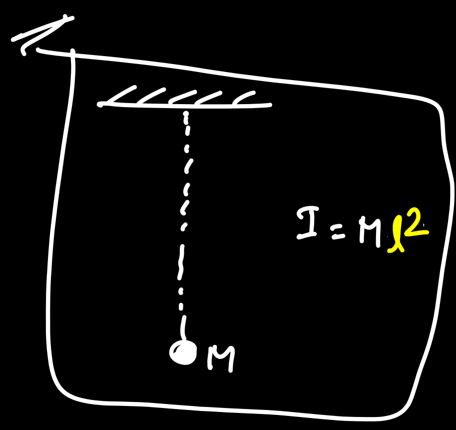
disc

$$dL = \vec{r} \times [dm(l\omega)]$$

$$dL = \vec{r}_{CM} \times M l\omega$$

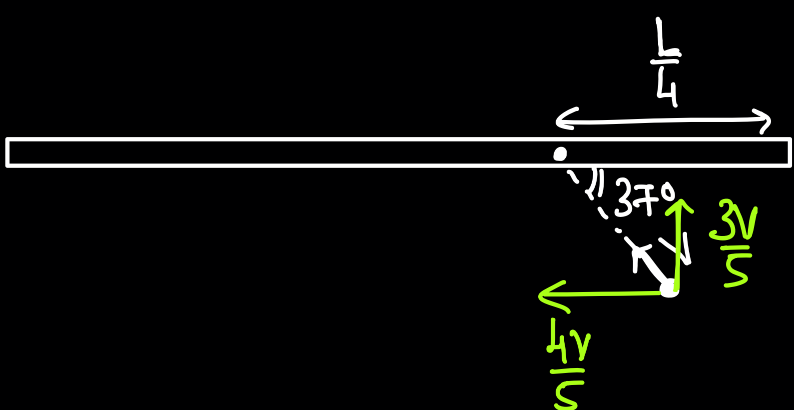
$$= l\,(M l\omega)$$

$$= (Ml^2)\omega.$$

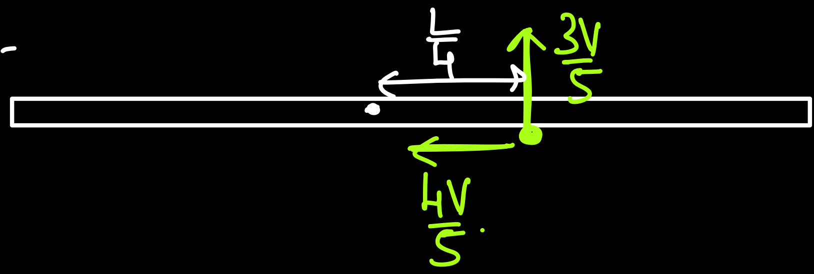


Ex:2

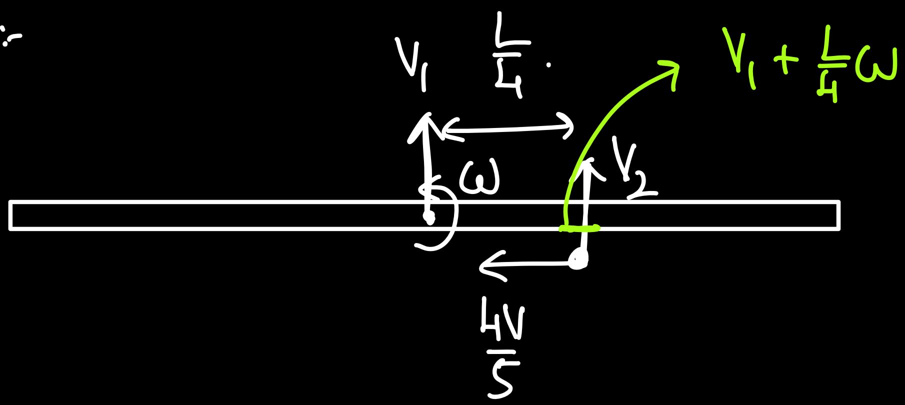
Q6)



Before :-



After :-



$$e = \frac{V_1 + \frac{l\omega}{4} - V_2}{\frac{3V}{5}} \Rightarrow V_1 - V_2 + \frac{l\omega}{4} = \frac{3V}{5} \quad \text{— (1)}$$

$$\vec{P_i} = \vec{P_f}$$

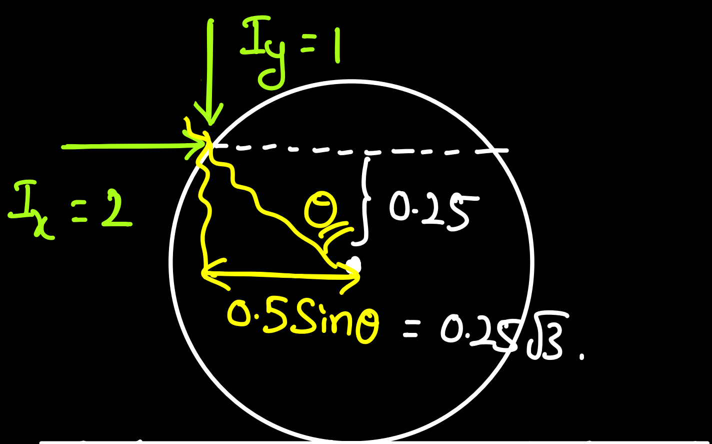
$$\frac{3mv}{5} = mV_1 + mV_2 \quad \text{— (2)}$$

$$L_i = L_f$$

$$m\frac{l}{4}\left(\frac{3V}{5}\right) = m\frac{l}{4}V_2 + \frac{ml^2}{12}\omega \quad \text{— (3)}$$

Ex:5

Q7)



$$I = \Delta\vec{p}$$

$$2 = \vec{P_f} - 2(-1)$$

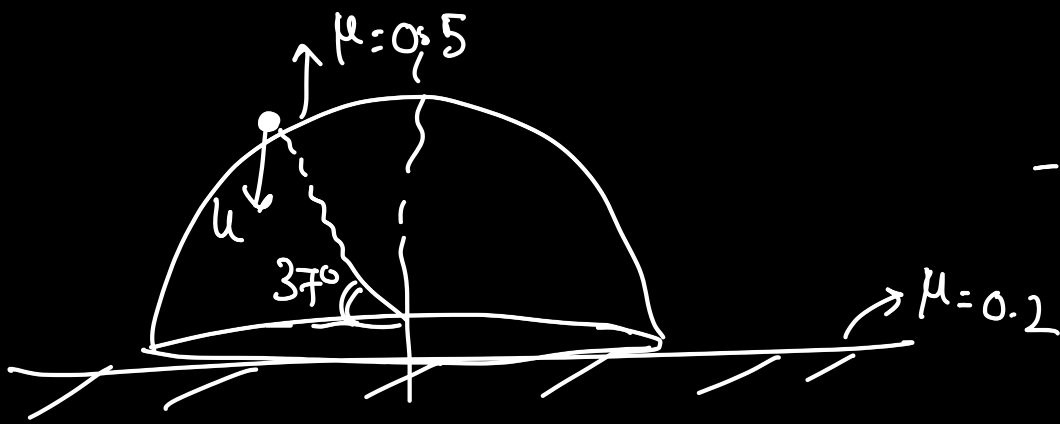
$$\vec{P_f} = 0 \Rightarrow \text{rest.}$$

Impulsive friction:-

Q) A solid ball of mass "m" and radius "R" spinning with angular velocity "ω" falls on horizontal rough surface. Immediately after collision the ball stops spinning. If $R\omega = 5\ m/s$, find horizontal component of velocity of sphere (m/s) immediately after the impact.

Ans :- 2m/s.

Q) A particle of mass $m = 0.1kg$ moving vertically downwards with a velocity $u = 20m/s$ collides with a rough hemi-spherical body of mass $M = 0.3\ kg$, $e = 0.5$ as shown.

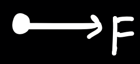


find Vel. of hemi-spherical body after Collision ?

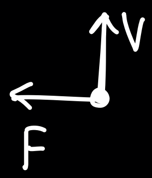
Ans: 1.67

Gyroscope :-

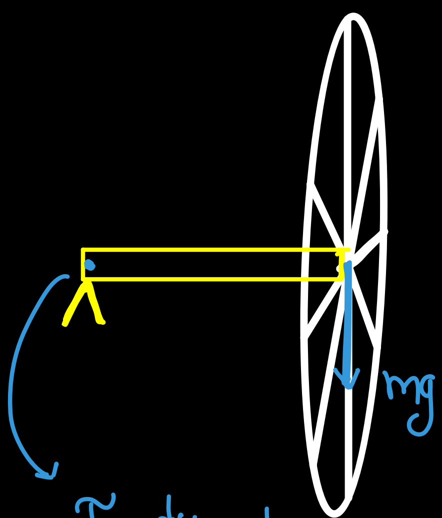
rest



Particle starts moving in the direction of force.

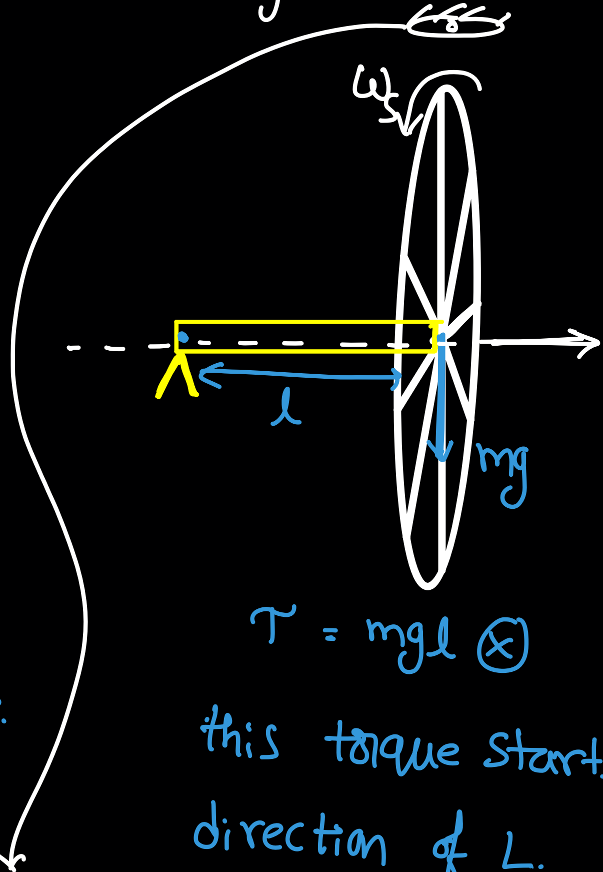


force will change direction of velocity, particle won't be moving in the direction of force.



τ due to mg is ↻

⟹ wheel moves clockwise.

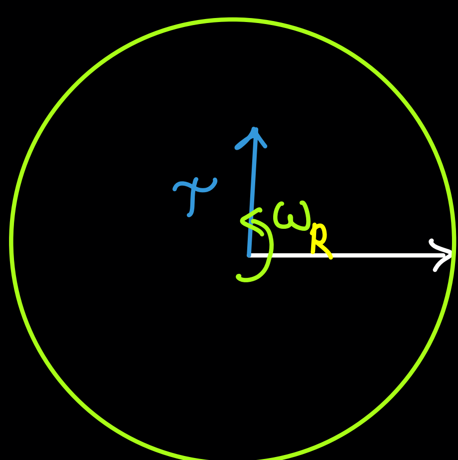


$L = (I_s)\omega_s$.

$\tau = mgl \otimes$

this torque starts bending the direction of L.

from top view



$L_s = I_s \omega_s$

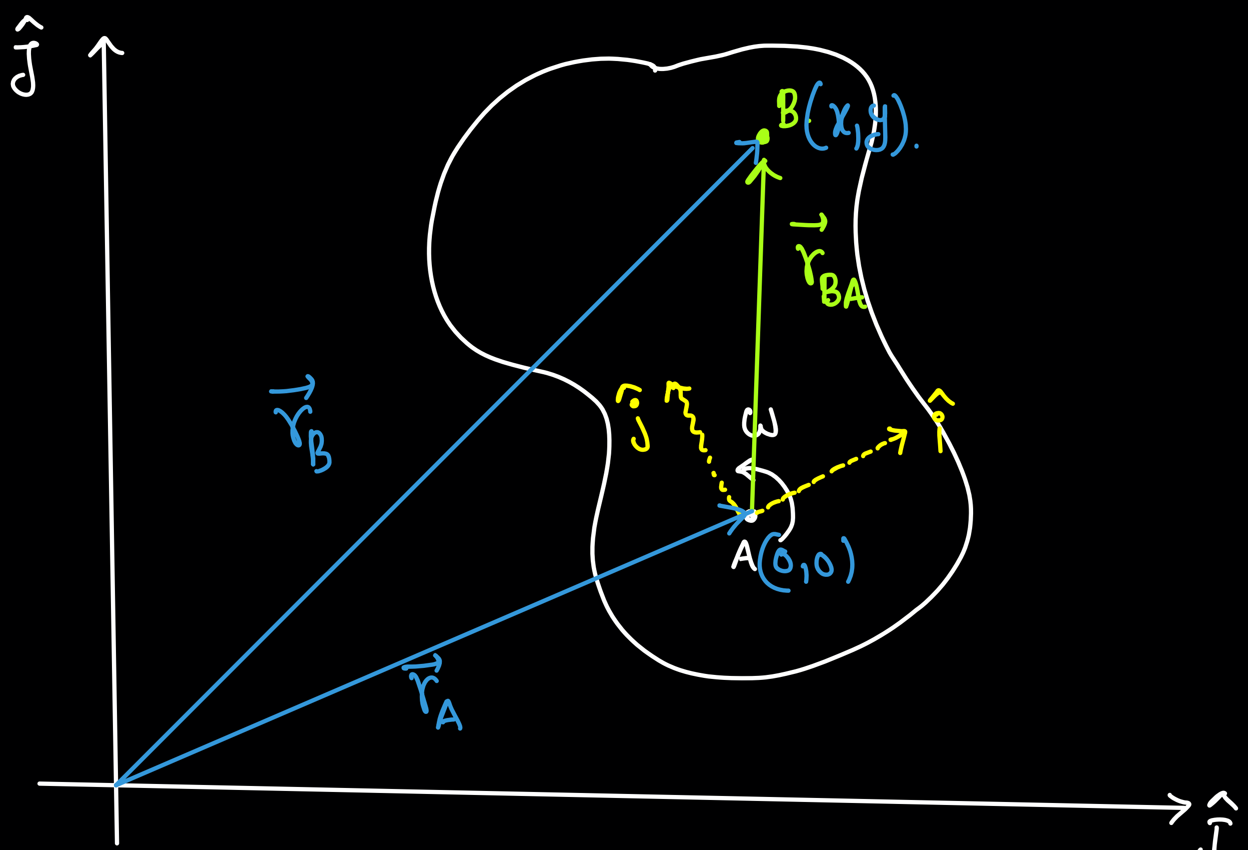
next dt time

$$\frac{d(\Gamma)}{dt} = (L_s)\omega_R$$

$$\tau = (L_s)\omega_R.$$

$$\boxed{\omega_R = \frac{T}{L_s}}$$

Relative motion including rotation :-

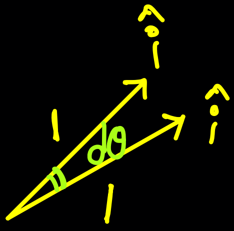


$$\vec{r}_{BA} = x\hat{i} + y\hat{j}.$$

$$\frac{d(\vec{r}_{BA})}{dt} = \frac{d[\vec{r}_B - \vec{r}_A]}{dt} = \frac{d[x\hat{i} + y\hat{j}]}{dt}.$$

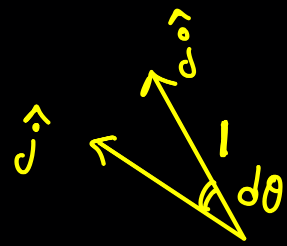
$$\frac{d\vec{r}_B}{dt} - \frac{d\vec{r}_A}{dt} = \frac{dx}{dt}\hat{i} + x\frac{d\hat{i}}{dt} + \frac{dy}{dt}\hat{j} + y\frac{d\hat{j}}{dt}.$$

$$\vec{V}_B - \vec{V}_A = \left(\frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}\right) + \left[x\frac{d\hat{i}}{dt} + y\frac{d\hat{j}}{dt}\right].$$



$$d\hat{i} = [(1)d\theta]\hat{j}$$

$$\frac{d\hat{i}}{dt} = \omega\hat{j}$$



$$d\hat{j} = [(1)(d\theta)](-\hat{i})$$

$$\frac{d\hat{j}}{dt} = -\omega\hat{i}.$$

$$\vec{V}_B - \vec{V}_A = \vec{V}_{BA} + (-\omega y\,\hat{i} + x\omega\hat{j})$$

$$(\vec{V}_{BA}) = \vec{V}_B - \vec{V}_A + (\omega y\,\hat{i} - \omega x\,\hat{j}).$$

$$\vec{V}_{BA} = (\vec{V}_B - \vec{V}_A) - (\vec{\omega} \times \vec{r}_{BA})$$

$$\boxed{\vec{V}_{\text{rotational frame}} = \vec{V}_{BA} = \vec{V}_B - \vec{V}_A - (\vec{\omega} \times \vec{r}_{BA})}$$

$$\left(\frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}\right) = \vec{V}_B - \vec{V}_A - (\vec{\omega} \times \vec{r}_{BA})$$

differentiating w.r.t time.

$$\underbrace{\frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j}}_{\vec{a}_{BA}} + \frac{dx}{dt}\frac{d\hat{i}}{dt} + \frac{dy}{dt}\frac{d\hat{j}}{dt} = \vec{a}_B - \vec{a}_A - \left(\frac{d\vec{\omega}}{dt} \times \vec{r}_{BA}\right) - \vec{\omega} \times \frac{d\vec{r}_{BA}}{dt}.$$

$$\vec{a}_{BA} + \underbrace{\dot{x}\omega\hat{j} - \dot{y}\omega\hat{i}}_{\vec{\omega} \times \vec{V}_{rot}} = \vec{a}_B - \vec{a}_A - \vec{\alpha} \times \vec{r}_{BA} - \vec{\omega} \times [\vec{V}_{rot} + \vec{\omega} \times \vec{r}_{BA}]$$

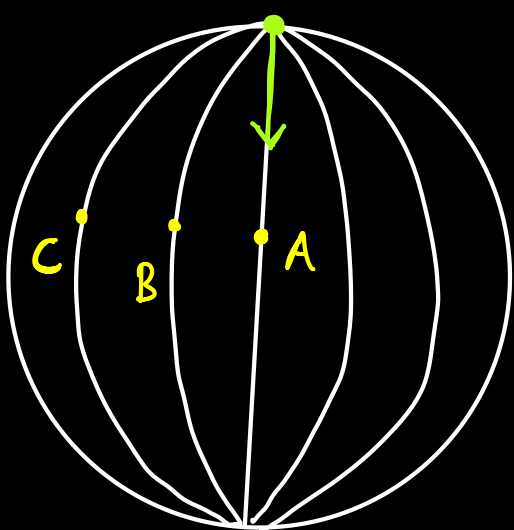
$$\vec{a}_{BA} = \vec{a}_B - \vec{a}_A - \vec{\alpha} \times \vec{r}_{BA} - 2(\vec{\omega} \times \vec{v}_{rot}) - \vec{\omega} \times (\vec{\omega} \times \vec{r}_{BA}).$$

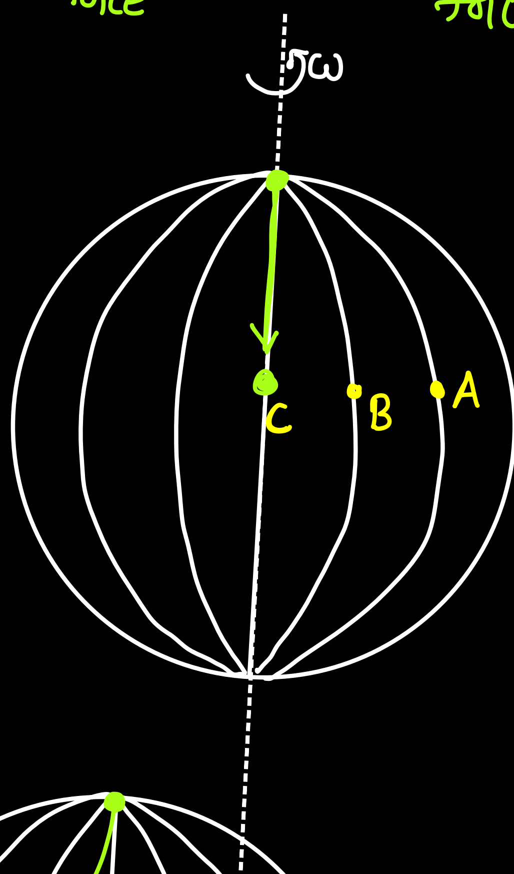
multiply with mass

$$m\vec{a}_{BA} = m\vec{a}_B - m\vec{a}_A - m(\vec{\alpha} \times \vec{r}_{BA}) - 2m(\vec{\omega} \times \vec{v}_{rot}) - m(\vec{\omega} \times (\vec{\omega} \times \vec{r}_{BA})).$$

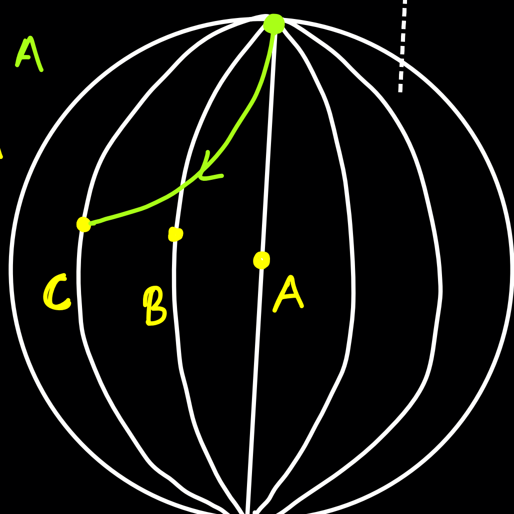
$$\vec{F}_{rot} = \underbrace{m\vec{a}_B}_{\text{real}} \underbrace{- m\vec{a}_A}_{\text{trans. pseudo force.}} \underbrace{- m(\vec{\alpha} \times \vec{r}_{BA})}_{\text{Euler's force.}} \underbrace{- 2m(\vec{\omega} \times \vec{v}_{rot})}_{\text{Coriolis force}} \underbrace{- m(\vec{\omega} \times (\vec{\omega} \times \vec{r}_{BA}))}_{\text{centrifugal force.}}$$

eg:-



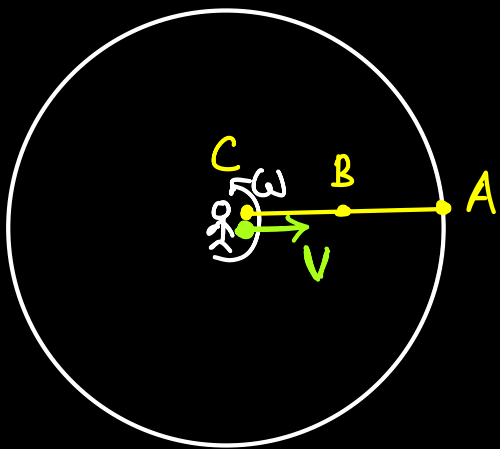


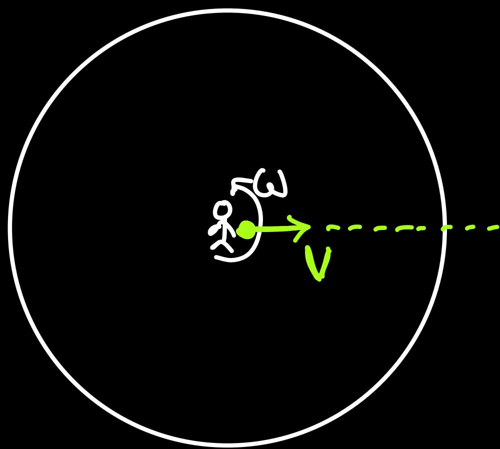
if earth is not rotating it should reach A w.r.t A

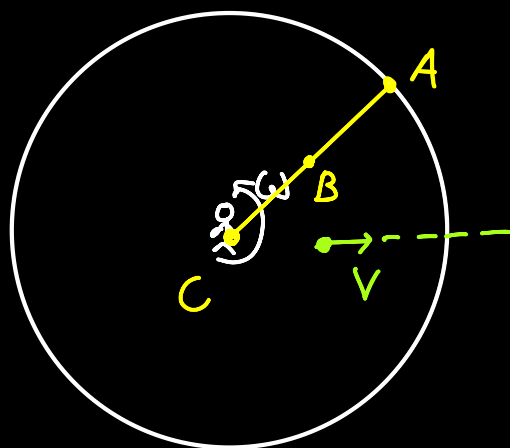


eg :- disc

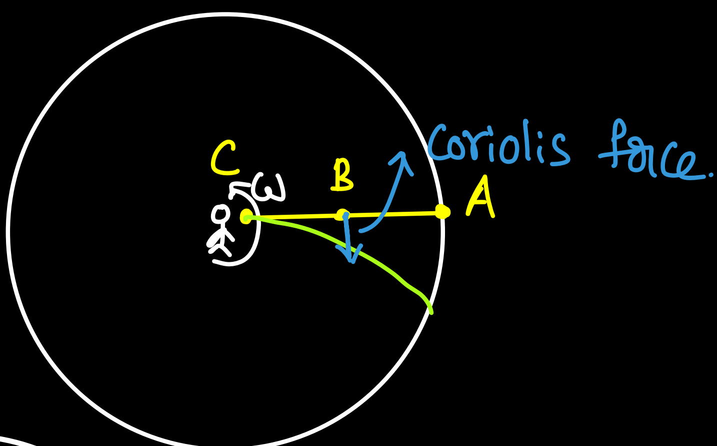
in ground frame.



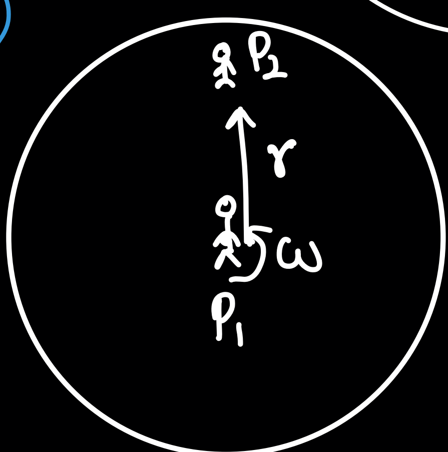




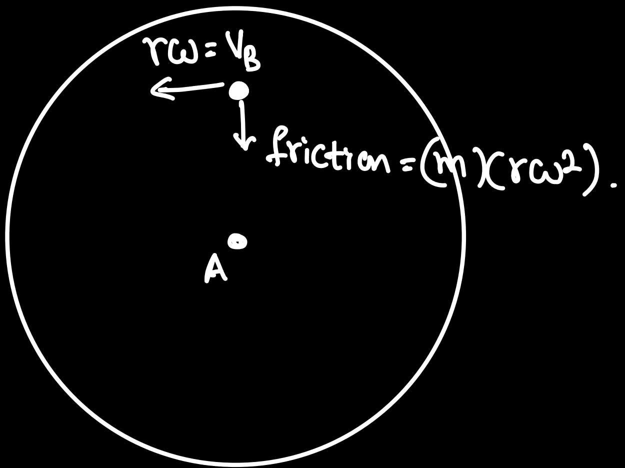
w.r.t person at centre.



Q)



draw all the forces acting on P_2 w.r.t P_1 ?



$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A - (\vec{\omega} \times \vec{r}_{BA})$$

$$= r\omega(-\hat{i}) - 0 + r\omega\,\hat{i}$$

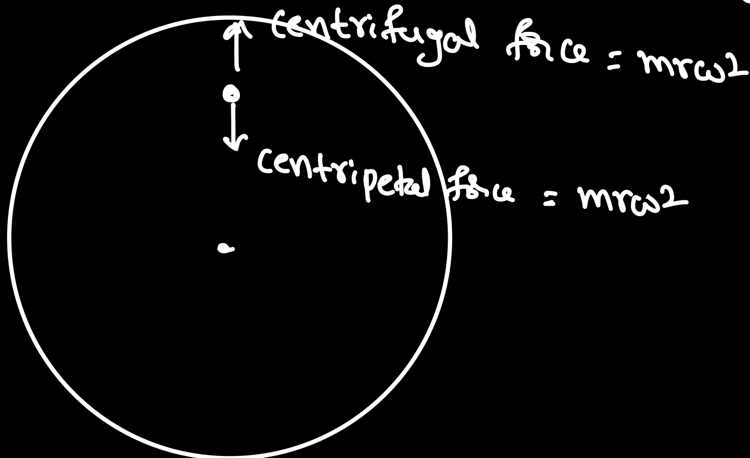
$$\vec{V}_{BA} = 0$$

$\vec{V}_{B\ w.r.t\ A}$ in rotating frame $= 0$

$$\vec{V}_{rot} = 0.$$

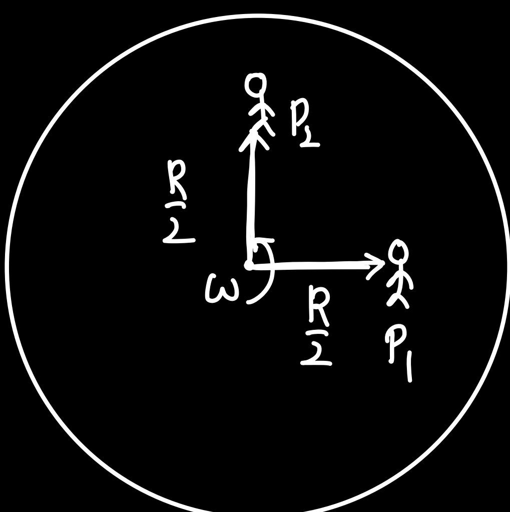
Coriolis force = 0

Eulers force = 0.



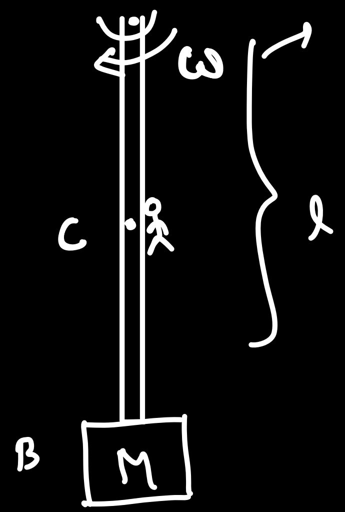
$$\boxed{F_{net} = 0}$$

Q)



find forces acting on P_2 w.r.t P_1 ?

Q)



horizontal table

find the pseudo force acting on block w.r.t person sitting at C ?