Wave :- its a disturbance which transports energy from one location to another without transfer of medium

⇒ only way to have disturbance but not transfer of medium is when particles of medium oscillate.

Types of waves :-

1) Based on transfer of energy

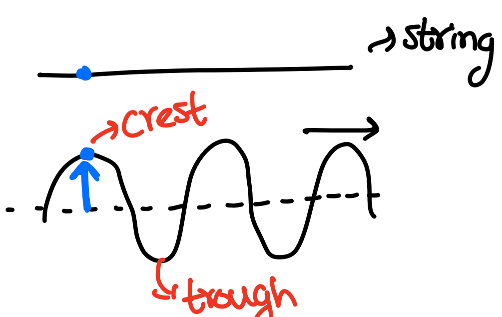
- travelling (or) progressive wave (which transports energy)
- stationary or standing waves (Net transportation of energy is "0")

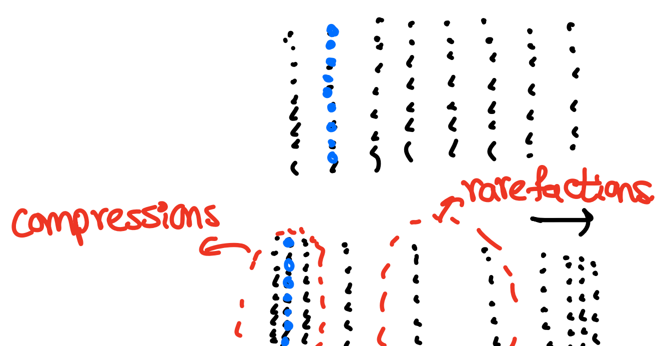
2) Based on requirement of medium

- Mechanical waves (require medium)
- Non - Mechanical waves (doesn't require medium)

3) Based on oscillations of particles in medium :-

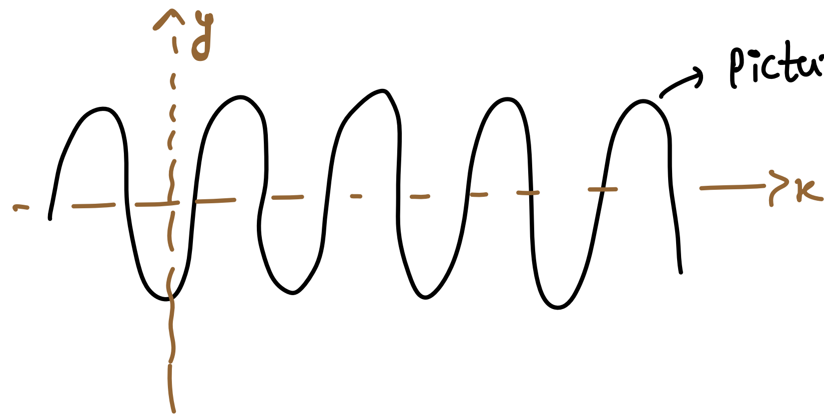
- transverse waves
- Longitudinal waves





Particles oscillate $\perp$ to direction of propagation of wave.

Particles oscillate along the direction of propagation of wave.



Picture of wave at some time "t".

⇒ * to completely give information about a wave, it should be a function of position and time.

$$y = f(x, t).$$

travelling or progressive wave.

$$y = f(\pm ax \pm bt).$$

---

differential form of wave equation

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}.$$

wave speed.

$$y = f(\pm ax \pm bt)$$

$$\frac{\partial y}{\partial t} = \pm b\, f'(\pm ax \pm bt)$$

$$\frac{\partial^2 y}{\partial t^2} = b^2 f''(\pm ax \pm bt)$$

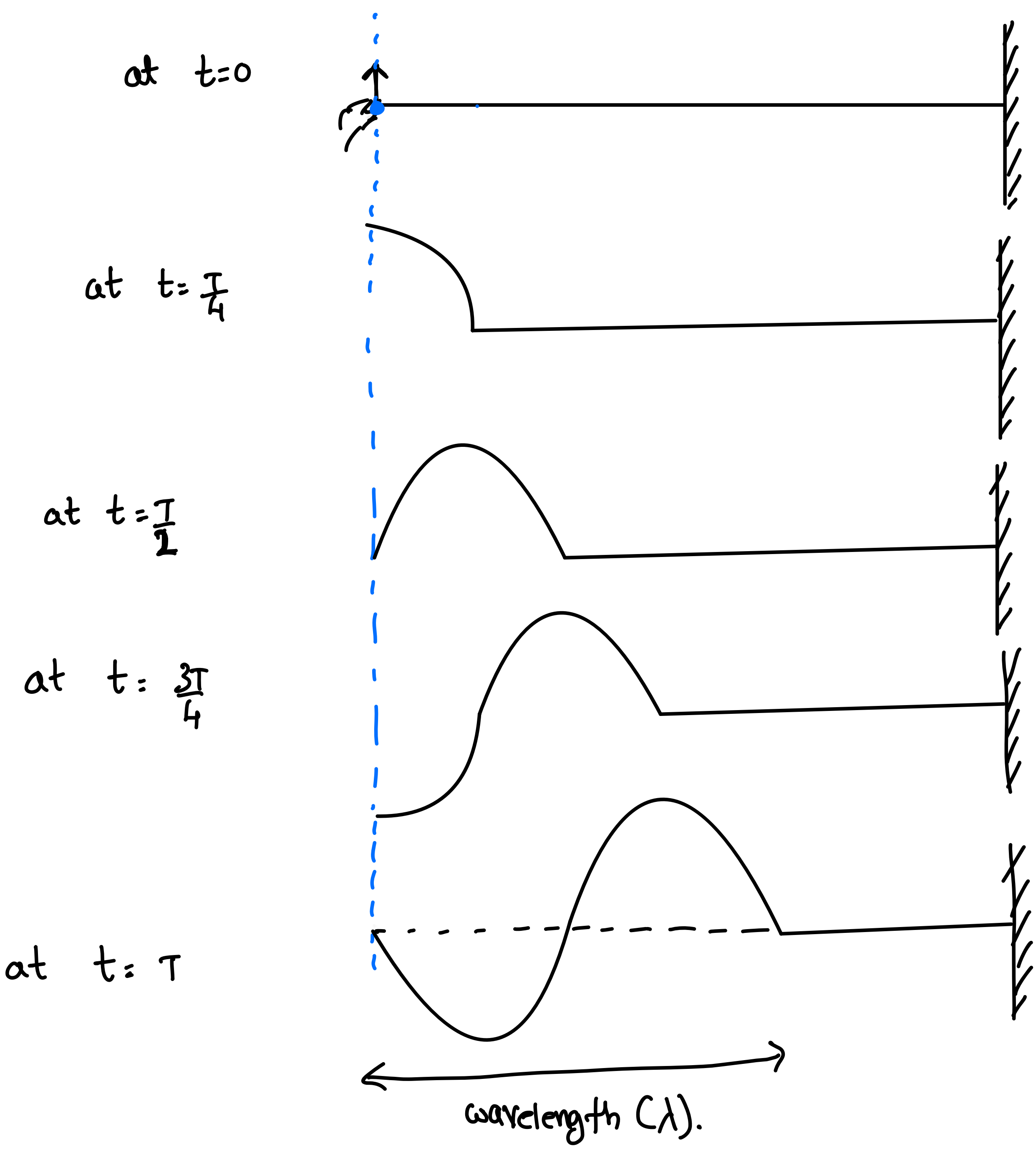
$$\frac{\partial^2 y}{\partial x^2} = a^2 f''(\pm ax \pm bt).$$

Put in wave equation

$$v = \frac{b}{a}.$$

$$\boxed{\text{wave speed} = \frac{|\text{coe. of } t|}{|\text{coe. of } x|}.}$$

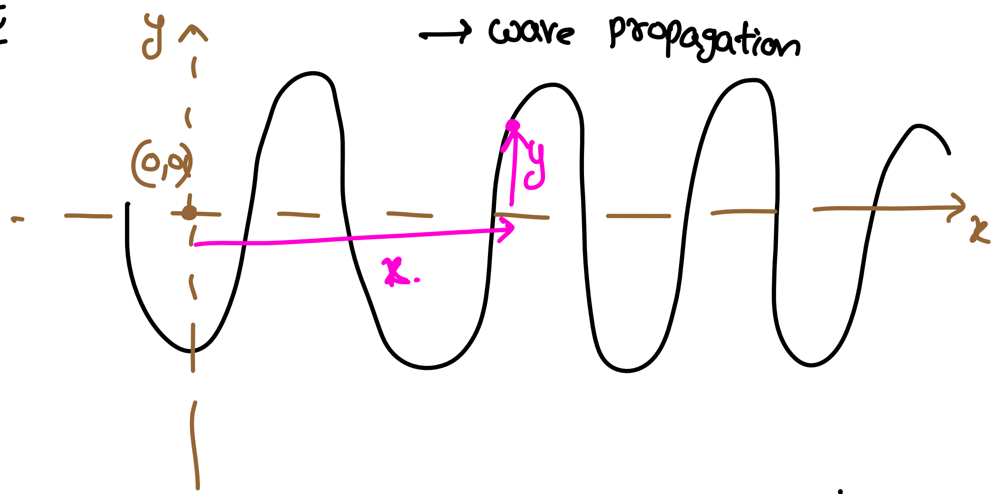
<u>Sinusoidal travelling or progressive waves</u>:-



wave speed (v) : $v = \frac{\text{wavelength}}{T}$

$$\boxed{v = \frac{\lambda}{T}}.$$

at sometime t



displacement of particle at x at time t = displacement of particle at $x=0$ at time $\left(t - \frac{x}{v}\right)$.

let displacement of particle at $x=0$ at time "t"

$$y = A\sin(\omega t + \phi).$$

displacement of particle at $x=0$ at time $\left(t - \frac{x}{v}\right)$

K = angular wave number.

$$y = A\sin\left(\omega t - \frac{\omega x}{v} + \phi\right).$$

$$y = A\sin(\omega t - kx + \phi).$$

so, displacement of particle at x at time t

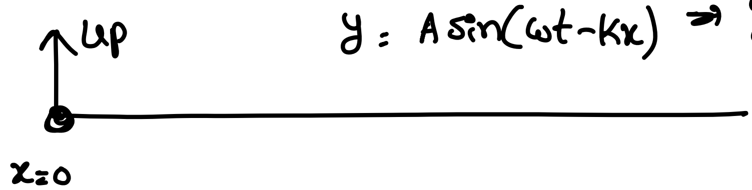
$$y = A\sin(\omega t - kx + \phi)$$

in the same way depending on how we move particle at $x=0$ in the beginning, we can also get

$$y = A\sin(kx - \omega t + \phi')$$ +x direction of propagation.

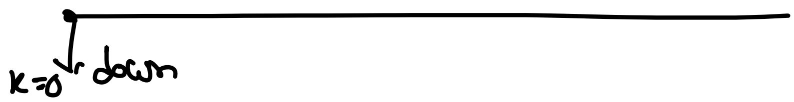
at $t=0$

$$y = A\sin(\omega t - kx) \Rightarrow y = A\sin\omega t.$$

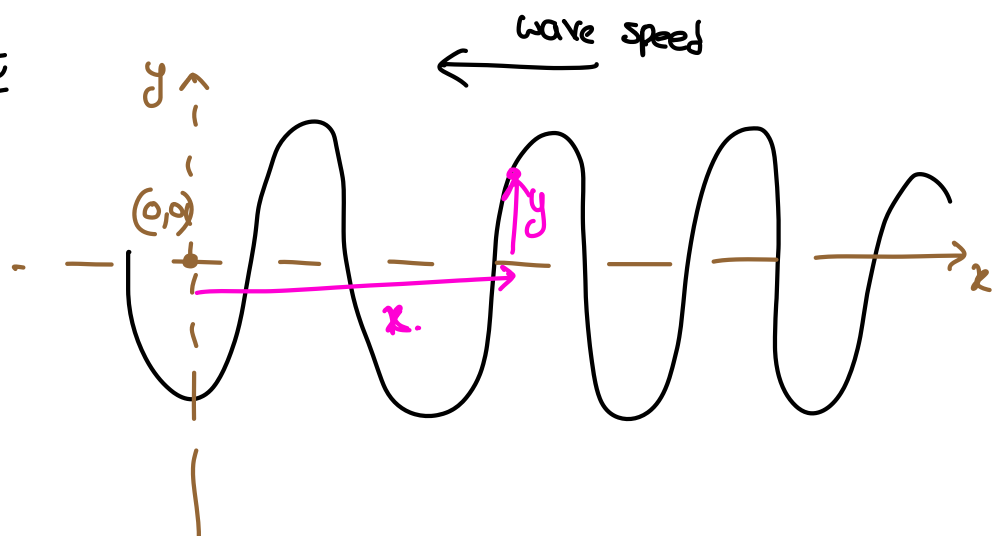


at $t=0$

$$y = A\sin(kx - \omega t). \Rightarrow y = A\sin(-\omega t)$$



at sometime t



displacement of particle at x at time t = displacement of particle at $x=0$ at time $\left(t + \frac{x}{v}\right)$.

let displacement of particle at $x=0$ at time "t"

$$y = A\sin(\omega t + \phi).$$

displacement of particle at $x=0$ at time $\left(t + \frac{x}{v}\right)$

$$y = A\sin\left(\omega t + \frac{\omega}{v}x + \phi\right)$$

$$y = A\sin(\omega t + kx + \phi).$$

displacement of particle at x at time t $\Rightarrow y = A\sin(\omega t + kx + \phi)$

in the same way we can also get

$$y = A\sin(-\omega t - kx + \phi')$$

$-x$ direction

$\Rightarrow$ if coefficient of t and x have opposite sign then $+x$ direction

if " " " " same sign then $-x$ direction

$\Rightarrow$ angular wave number $k = \dfrac{\omega}{v}$

$$= \frac{\omega}{\frac{\lambda}{T}}$$

$$= \frac{2\pi}{\lambda}.$$

$\Rightarrow$ wave speed $\Rightarrow v = \dfrac{\omega}{k} = \dfrac{\lambda}{T} = \dfrac{|\text{coe. of } t|}{|\text{coe. of } x|}$

to produce sinusoidal wave particles of medium should execute SHM.

eg:- $y = A\sin(\omega t - kx + \phi)$.

Particle speed $= \dfrac{dy}{dt} = A\omega\cos(\omega t - kx + \phi)$

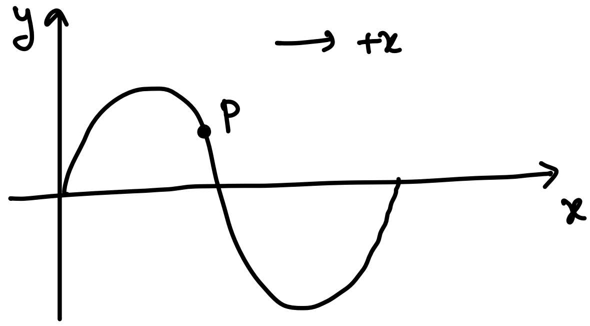
$$= A\omega\sqrt{1 - \sin^2(\omega t - kx + \phi)}$$

$$V = \frac{dy}{dt} = \omega\sqrt{A^2 - y^2} \Rightarrow$$

$y = \frac{1}{x+vt}$ ✗ is not a wave.

$$y = \frac{1}{1+(x+vt)^2}$$

Range of y $(0, 1]$. $\Rightarrow$ $\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$ will be satisfied so $y = \frac{1}{1+(x+vt)^2}$ is a wave.

Q) 

wave speed = 10 cm/sec

$\lambda = 0.5\,m$

$A = 10\,cm$

At a particular time "t", the snap-shot of wave is shown in figure. The vel. of point P when its disp. is 5cm ?

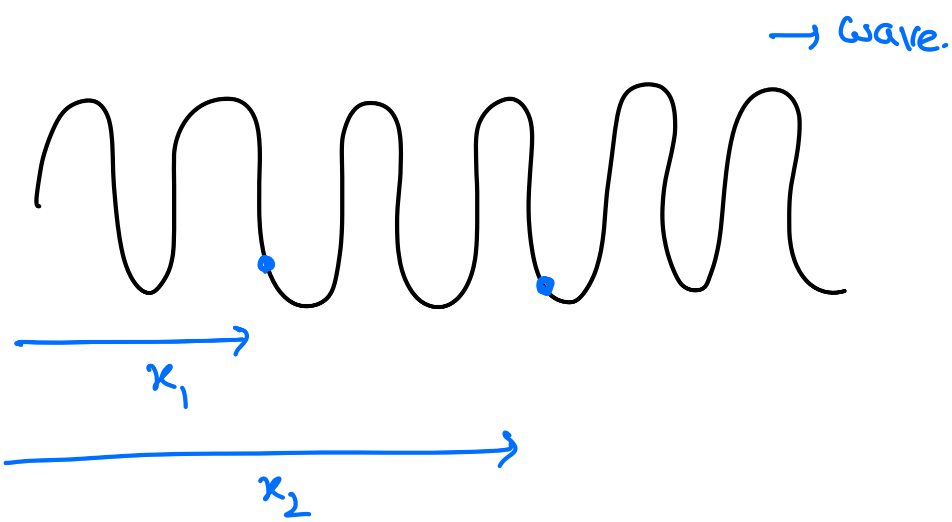
A) $\frac{\sqrt{3}\pi}{50}\hat{j}$ ✓ B) $-\frac{\sqrt{3}\pi}{50}\hat{j}$ C) $\frac{\sqrt{3}\pi}{50}\hat{i}$ D) $-\frac{\sqrt{3}\pi}{50}\hat{i}$

Sol:- Velocity of point P is in $+\hat{j}$ direction

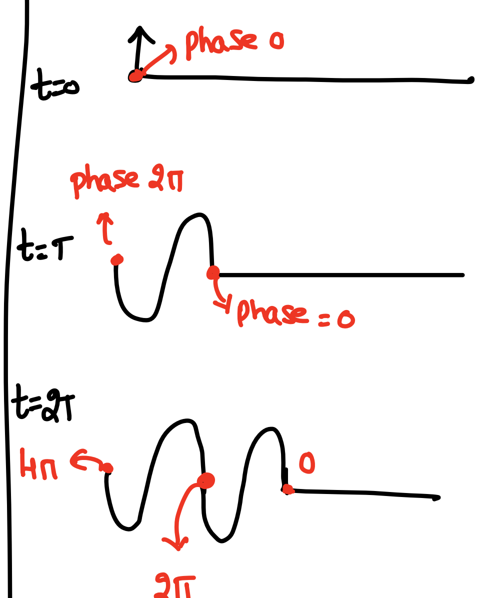
use $v = \omega\sqrt{A^2-y^2}$ to get speed of point P.

Phase difference:-

$y = A\sin(\omega t - kx + \phi)$.



additional data.



Phase of particle at x_1 = $\omega t - kx_1 + \phi$

" " " x_2 = $\omega t - kx_2 + \phi$.

Phase difference $= \phi_1 - \phi_2$

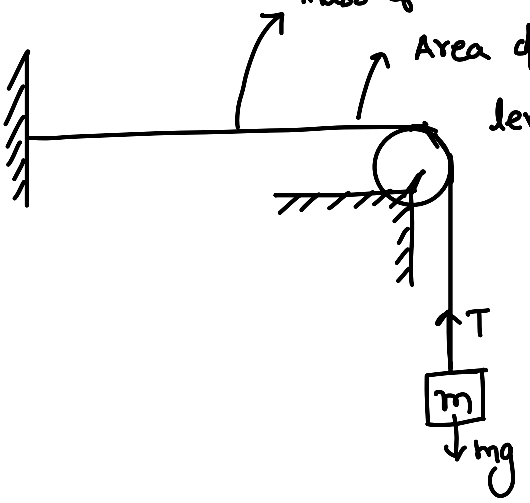
$= k(x_2 - x_1)$

$= k\,\Delta x$ → path difference.

*

$$\boxed{\Delta\phi = \frac{2\pi}{\lambda}\Delta x.}$$

speed of a transverse wave in a string:-

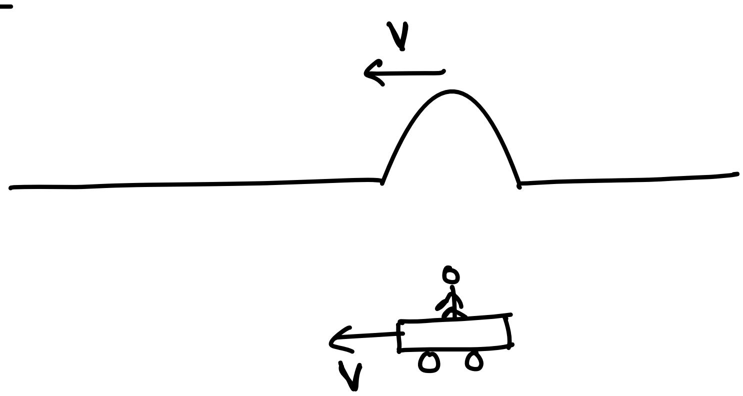


$T = mg$

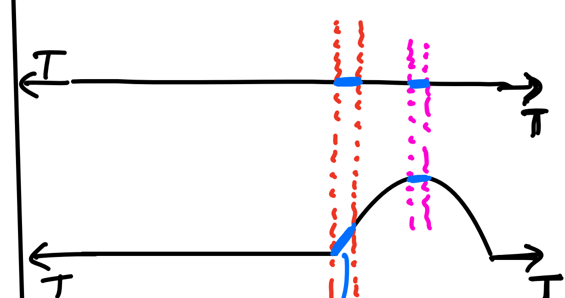
if $\mu = \frac{M}{l}$ is very small then more or less tension in the string is constant.

When we create a disturbance, it is observed experimentally that, wave speed $v = \sqrt{\frac{T}{\mu}}$

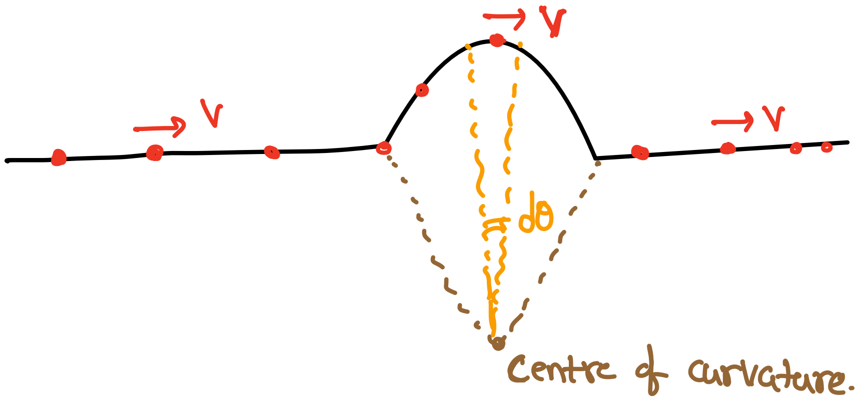
derivation:-



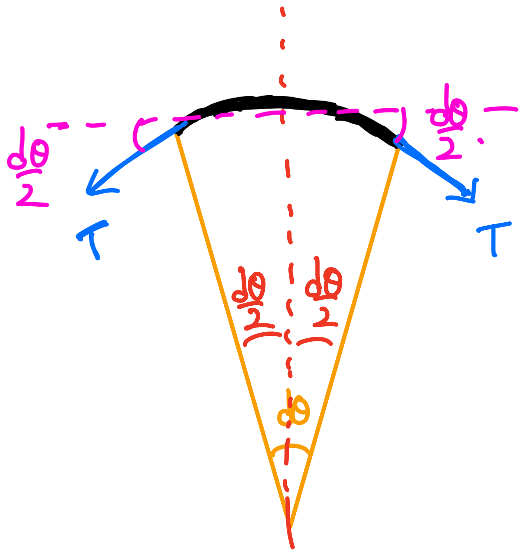
assumptions



w.r.t person



as length of element changes tension should also change. But we assume that we created a small disturbance which will not change "T" by much. Amplitude of oscillations are small.



$$F_c = 2T \sin \frac{d\theta}{2}.$$

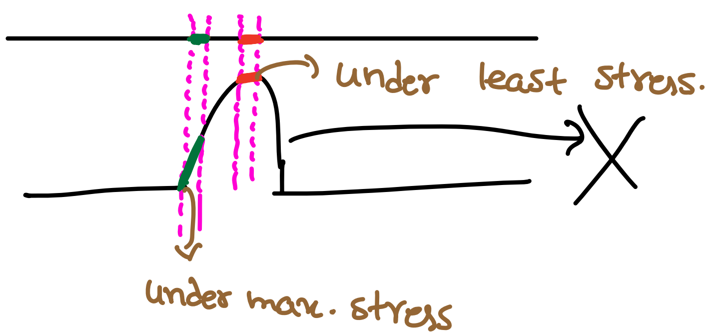
$$\frac{(dm)v^2}{R} = T(d\theta)$$

$$\frac{(\mu R (\cancel{d\theta}))v^2}{\cancel{R}} = T\cancel{d\theta}$$

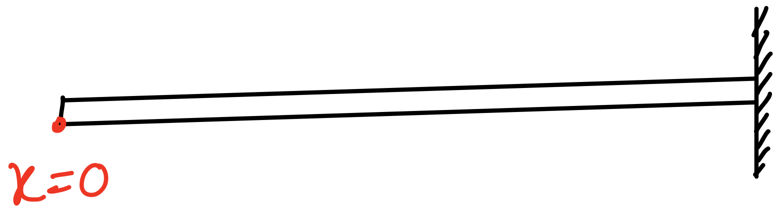
$$\boxed{v = \sqrt{\frac{T}{\mu}}.}$$

μ → mass per unit length

Only valid when the disturbance is not contributing for tensional changes

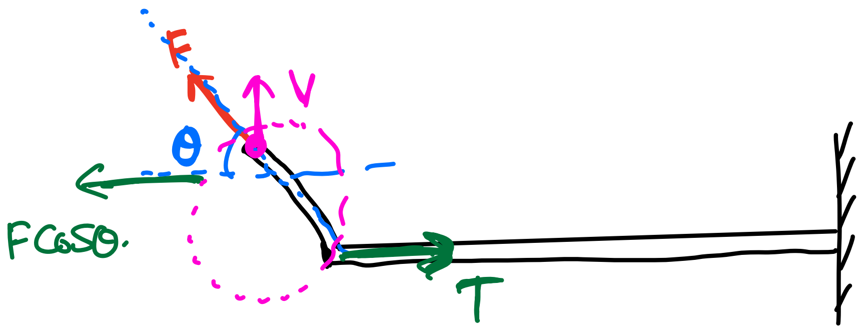


Power and intensity of a transverse wave in a Rope (or) string:-



$$y = A\sin(\omega t - kx + \phi).$$

at sometime "t"



instantaneous power delivered to rope $\Rightarrow P_{int} = \vec{F}\cdot\vec{V}$.

$$= FV\cos(90-\theta)$$

$$= FV\sin\theta.$$

for small angles $\Rightarrow \sin\theta \approx \tan\theta$

$$P_{int} = FV\tan\theta.$$

slope $= \dfrac{dy}{dx}$.

$-\tan\theta = -Ak\cos(\omega t - kx + \phi)$

$\tan\theta = Ak\cos(\omega t - kx + \phi)$.

$V = \dfrac{dy}{dt}$

$= A\omega\cos(\omega t - kx + \phi)$

$F\cos\theta = T \Rightarrow$ approximately $F = T$ as net force on element can be taken "0".

$$P_{int} = TA^2\omega K \cos^2(\omega t - kx + \phi).$$

$$= \mu V^2 A^2 \omega K \cos^2(\omega t - kx + \phi)$$

$$P_{int} = \mu V A^2 \omega^2 \cos^2(\omega t - kx + \phi).$$

V = wave speed

$\because V = \sqrt{\frac{T}{\mu}}.$

$T = \mu V^2$

$\cdot\; V = \frac{\omega}{K}$

$KV = \omega.$

avg. Power delivered to string in 1 cycle.

$$P_{avg} = \frac{\int_0^T P\,dt}{\int_0^T dt}$$

$$= \mu V A^2 \omega^2 \underbrace{\frac{\int_0^T \cos^2(\omega t - kx + \phi)\,dt}{\int_0^T dt}}_{\frac{1}{2}.}$$

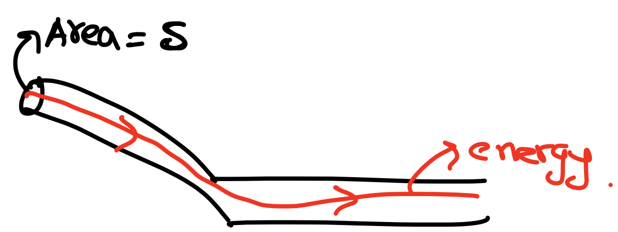
$$\boxed{P_{avg} = \frac{1}{2}\mu V A^2 \omega^2}$$

ang. frequency.

amplitude

wave speed

avg. Intensity $(I_{avg}) = \frac{P_{avg}}{\text{Area.}}$



$$I_{avg} = \frac{\frac{1}{2}\mu v A^2\omega^2}{S}$$

$$\boxed{I_{avg} = \frac{1}{2}\rho v A^2\omega^2}$$

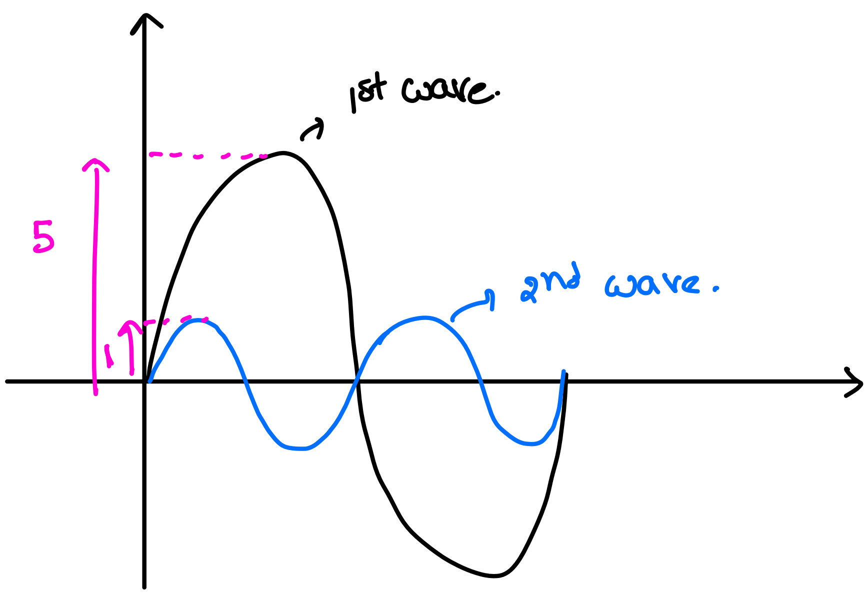
if two or more waves travel in same medium and having same frequency then $I \propto A^2$

why ??

$$I = \frac{1}{2}\underbrace{(\rho v\omega^2)}_{\text{constant}} A^2$$

$$I \propto A^2$$

Q)



find ratio of intensities of waves?

sol :

$$I = \frac{1}{2}\rho v A^2\omega^2$$

$$\frac{I_1}{I_2} = \left(\frac{A_1}{A_2}\right)^2\left(\frac{\omega_1}{\omega_2}\right)^2.$$

$$\frac{I_1}{I_2} = \left(\frac{5}{1}\right)^2\left(\frac{1}{2}\right)^2$$

$$= \frac{25}{4}.$$

Ex-4A 2, 5, 14

Ex-2 12, 13, 14.

Ex-4A

Q2) $\lambda = 0.08$

$$K = \frac{2\pi}{\lambda} = \frac{2\pi}{0.08} = \frac{\cancel{2}\pi \times 100}{\cancel{8}4}$$

$$= 25\pi.$$

$T = 2$

$$\frac{2\pi}{\omega} = 2$$

$$\omega = \pi.$$

Q5) $\mu = 0.04$

wave speed $= \sqrt{\frac{T}{\mu}}$.

$$\frac{\left(\frac{1}{0.04}\right)}{\left(\frac{1}{0.5}\right)} = \sqrt{\frac{T}{\mu}}. \Rightarrow \sqrt{\frac{T}{\mu}} = \frac{50}{4}$$

$$\frac{T}{\mu} = \frac{2500}{16}.$$

$$T = \frac{25 \times 4}{16} \Rightarrow T = \frac{25}{4}$$

$$T = 6.25$$

Q14) max. particle velocity $= A\omega$

$$= \left(\frac{10}{\pi}\right)\left(\frac{2\pi}{T}\right)$$

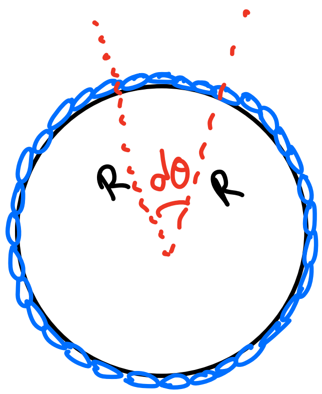
$$= \frac{20}{T}.$$

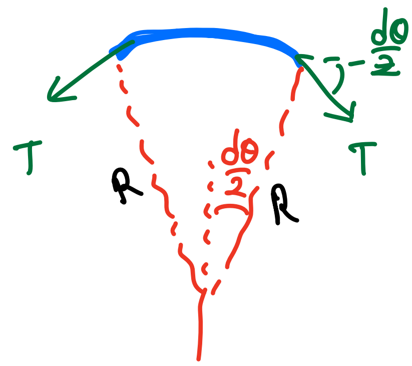
wave velocity $= \frac{\lambda}{\cancel{T}}. = 2\left(\frac{20}{\cancel{T}}\right)$

$$\lambda = 40.$$

Ex-2

Q12)





$$2T\sin\left(\frac{d\theta}{2}\right) = (dm)r\omega^2$$

$$\cancel{2}T\left(\frac{d\theta}{\cancel{2}}\right) = (\mu)(R\,\cancel{d\theta})R\omega^2$$

$$T = \mu R^2\omega^2$$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{R^2\omega^2\mu}{\mu}} = R\omega.$$

$$= \left(\frac{L}{2\pi}\right)\omega$$

$= \frac{\omega L}{2\pi}$.

What if multiple waves travel in the same medium ??

Principle of superposition:- When multiple waves travel in the same medium they cross each other without effecting the other one and the resultant displacement of particle in the medium is equal to vector sum of individual disp. due to all waves.

$$\vec{y} = \vec{y}_1 + \vec{y}_2 + \dots\dots\dots$$

Q) $y_1 = 5 \sin(\pi t + kx)$

$y_2 = 10 \sin(2\pi t - kx)$

find disp. of particle at $x=0$ at time $t = \frac{1}{2}$ sec.

$t = 1$ sec

$t = \frac{1}{4}$ sec

Sol :- $y_r = y_1 + y_2$

$= 5 + 0 \quad \Rightarrow y_r = 5$

$y_r = y_1 + y_2$

$= 0 + 0$

$y_r = \frac{5}{\sqrt{2}} + 10$

superposition of waves

- interference (superposition of two waves have same freq and constant phase difference)
- Beats (superposition of two or more waves having almost same freq).
- Standing waves. (superposition of two equal waves travelling in opposite direction).

interference :- coherent sources
↓
which produce waves of same frequency and constant phase difference.

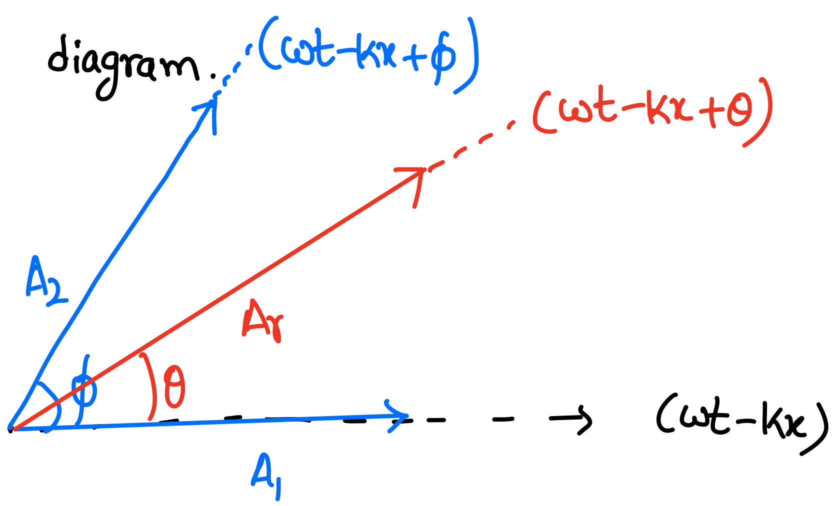
$$y_1 = A_1 \sin(\omega t - kx)$$

$$y_2 = A_2 \sin(\omega t - kx + \phi).$$

$$y_r = y_1 + y_2$$

$$= A_r \sin(\omega t - kx + \theta).$$

Phasor diagram.



$$\boxed{A_r = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}}.$$

$$\tan\theta = \frac{A_2\sin\phi}{A_1 + A_2\cos\phi}.$$

$$A_r^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$$

$\because$ $I \propto A^2$

$I = kA^2.$

$$\frac{I_r}{K} = \frac{I_1}{K} + \frac{I_2}{K} + 2\sqrt{\frac{I_1}{K}}\sqrt{\frac{I_2}{K}}\cos\phi$$

$$\boxed{I_r = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi}.$$

constructive interference

$A_r = \max$ or $I_r = \max$

$$I_r = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$$

for max. I_r

$$\cos\phi = 1.$$

Phase diff $= 0, 2\pi, 4\pi, \ldots\ldots$

$\boxed{\Delta\phi = 2n\pi}$ $\quad n = 0, 1, 2 \ldots.$

$$\frac{2\pi}{\lambda}\Delta x = 2n\pi$$

$\boxed{\Delta x = n\lambda}$ $\quad n = 0, 1, 2 \ldots.$

$$I_{rmax} = I_1 + I_2 + 2\sqrt{I_1I_2}$$

Destructive interference

$A_r = \min$ (or) $I_r = \min.$

$$I_r = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$$

for min. I_r

$$\cos\phi = -1.$$

Phase difference $= \pi, 3\pi, 5\pi. \ldots$

$\boxed{\Delta\phi = (2n-1)\pi}$ $\quad n = 1, 2, \ldots$

$$\frac{2\pi}{\lambda}\Delta x = (2n-1)\pi$$

$\boxed{\Delta x = \frac{(2n-1)\lambda}{2}}$ $\quad n = 1, 2, \ldots\ldots$

$$I_{min} = I_1 + I_2 - 2\sqrt{I_1I_2}.$$

$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$	$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$

$$\frac{I_{max}}{I_{min}} = \left[\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right]^2$$

$$= \left[\frac{\sqrt{\frac{I_1}{I_2}} + 1}{\sqrt{\frac{I_1}{I_2}} - 1}\right]^2$$

$$= \left[\frac{\frac{A_1}{A_2} + 1}{\frac{A_1}{A_2} - 1}\right]^2 \Rightarrow \frac{I_{max}}{I_{min}} = \left[\frac{A_1 + A_2}{A_1 - A_2}\right]^2$$

HW

BB-2

Q 2, 3, 4, 5, 7.

Ex:1

Q 1, 2, 3, 4, 5, 6, 7, 18

Ex:2

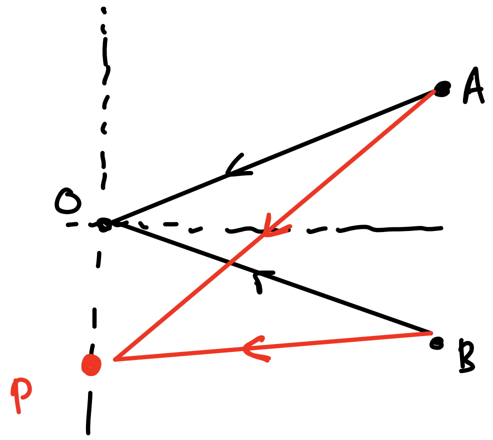
Q4

Ex:3

Q2*, 6, 8

Ex-3

Q1)



$\Delta x = 0$

$I = \max.$

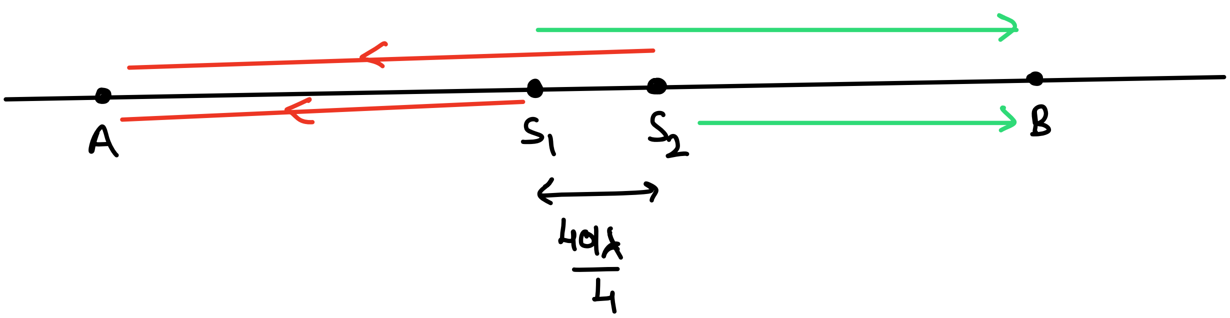
$\Delta x = \lambda.$

$\sqrt{1^2+(2.4)^2} - 2.4 = \lambda.$

$0.2 = \lambda.$

$f = \frac{v}{\lambda} \Rightarrow v = f\lambda = (1800)(0.2) = \underline{\underline{360}}$

Q2)



$\Delta\phi = \frac{2\pi}{\lambda}\left(\frac{401\lambda}{4}\right) = \frac{401}{2}\pi = 200\pi + \frac{\pi}{2}.$

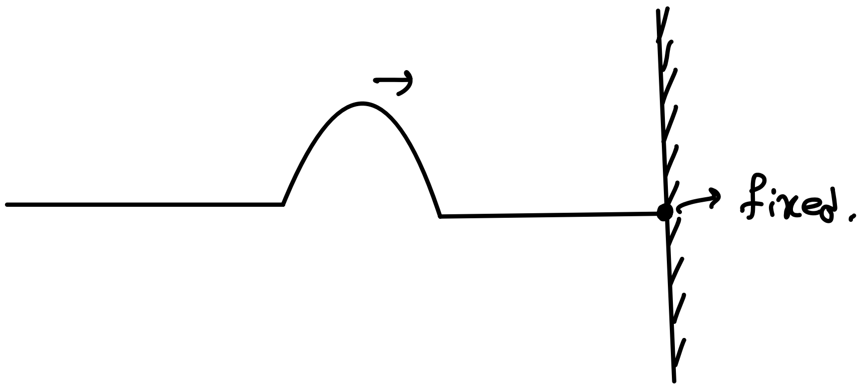
A :- $\Delta\phi_{Total} = \left(200\pi + \frac{\pi}{2}\right) + \frac{\pi}{2}. = 201\pi$

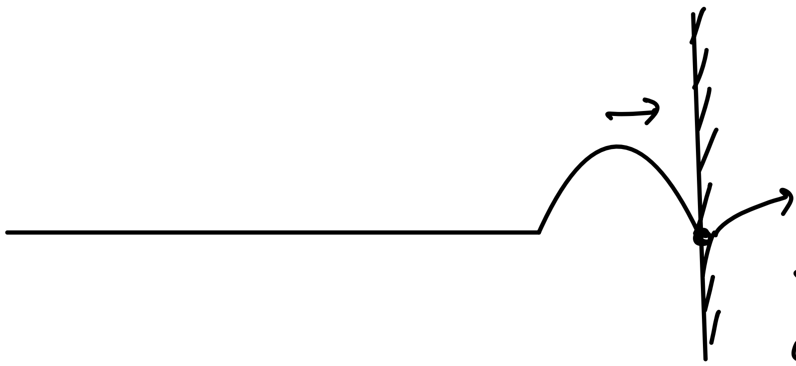
$I_A = (\sqrt{I_1} - \sqrt{I_2})^2$

B $\Delta\phi_{Total} = \left(200\pi + \frac{\pi}{2}\right) - \frac{\pi}{2}. = 200\pi$

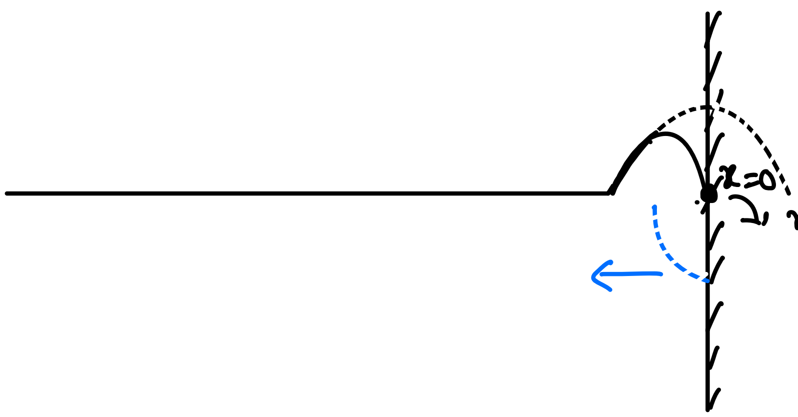
$I_B = (\sqrt{I_1} + \sqrt{I_2})^2.$

Reflection of a tranverse wave from a fixed end:-

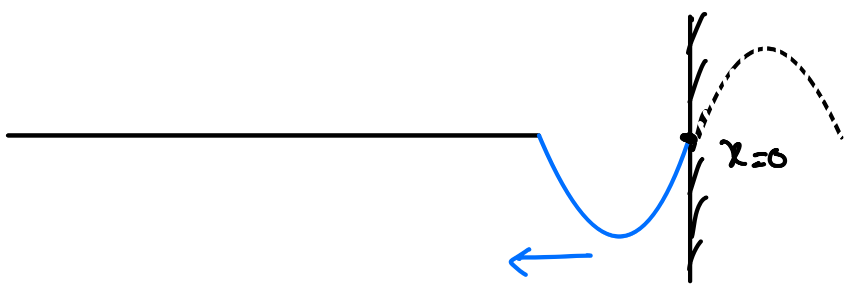




as the pulse has reached the fixed point, the particle at the point will be asked to move.



net displacement $=0$.



$$y_{incident} = A\sin(\omega t - kx)$$

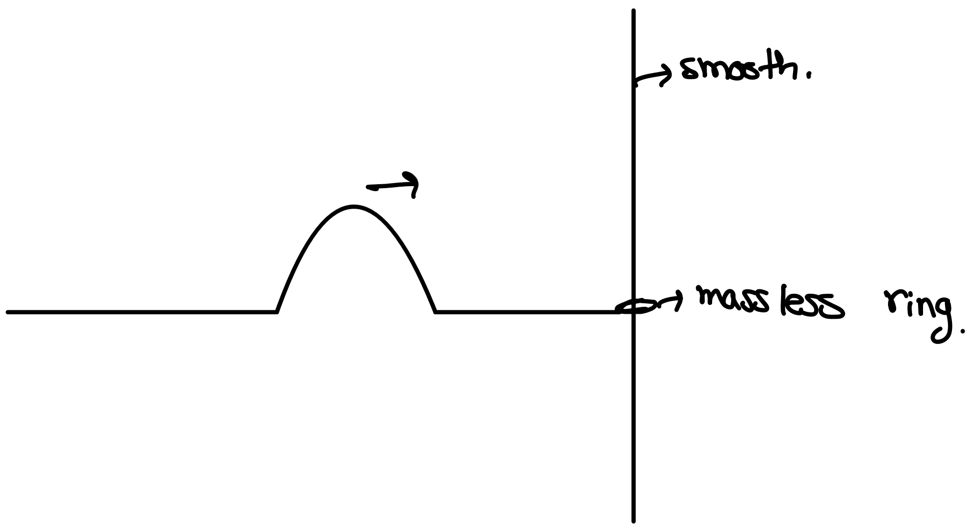
$$y_{reflected} = -A\sin(\omega t + kx).$$

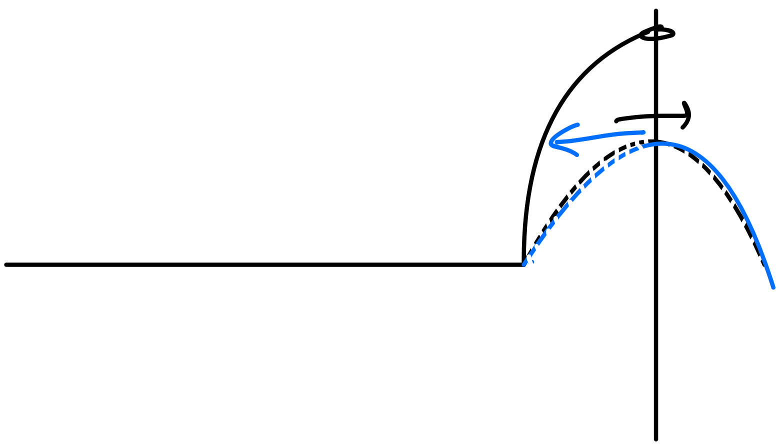
$$= A\sin(\omega t + kx + \pi).$$

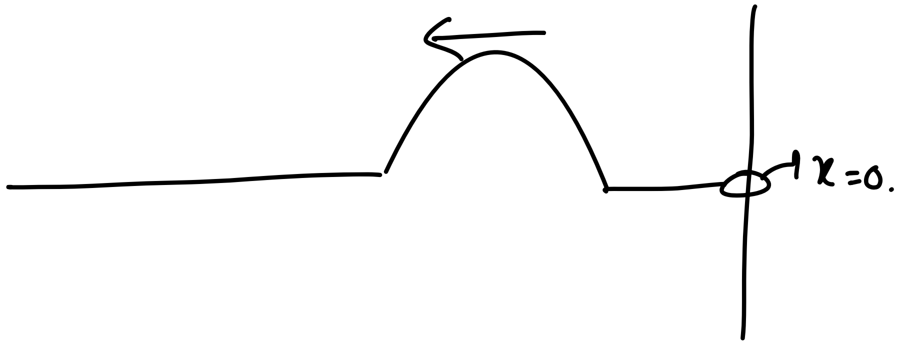
V.V.I.M.P. lets say reflection is taking place at $x=0$

$\Rightarrow$ * phase change suffered by incident transverse wave when reflected from fixed end is "π".

Reflection from free end:-





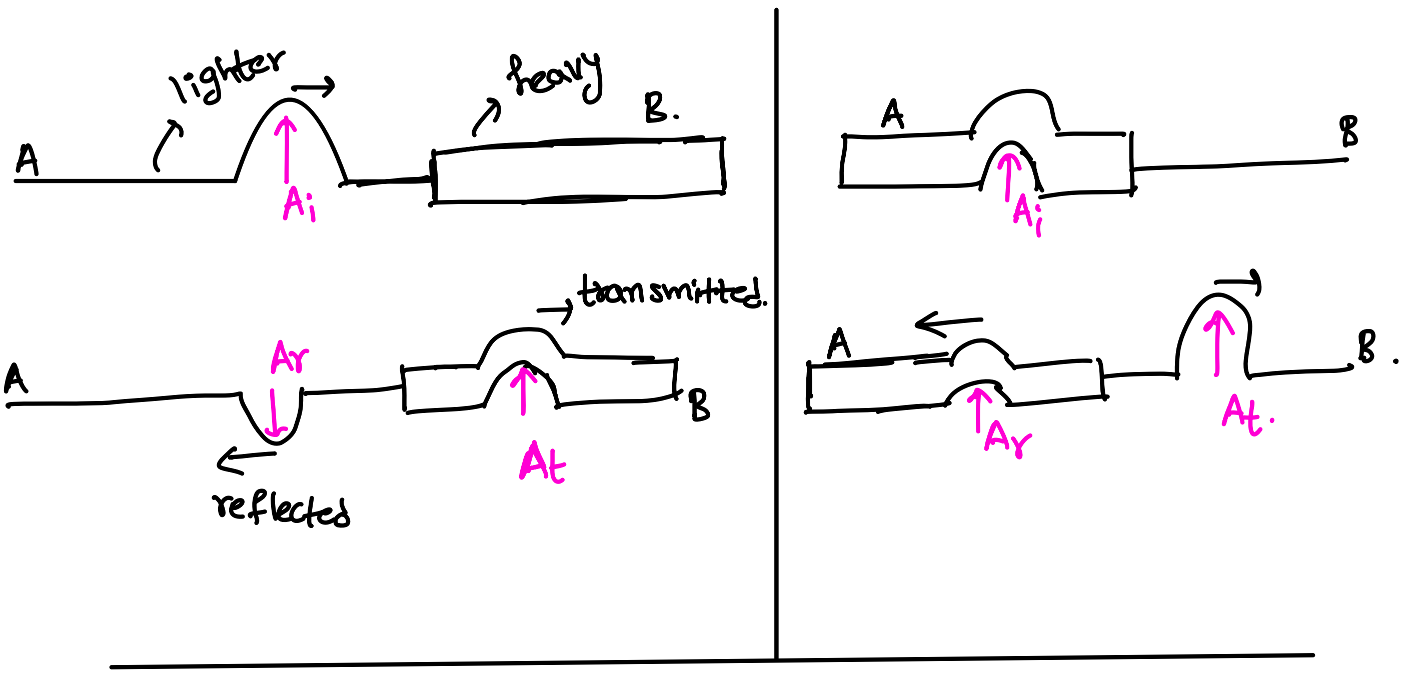


$$y_{in} = A\sin(\omega t - Kx)$$

$$y_r = A\sin(\omega t + Kx).$$

$\Rightarrow$ reflected wave doesn't suffer any phase change. upward pulse will be reflected back as upward pulse.

Cases:-



lets take

$$y_i = A_i \sin(\omega t - k\underset{A}{x})$$

$$y_r = A_r \sin(\omega t + k\underset{A}{x})$$

$$y_t = A_t \sin(\omega t - k\underset{B}{x}).$$

$y_i + y_r = y_t$ at boundary

$A_i + A_r = A_t$ — ①

$$\frac{\partial y_i}{\partial x} + \frac{\partial y_r}{\partial x} = \frac{\partial y_t}{\partial x}.$$

$$-A_i k_A \cos(\omega t - k_A x) + A_r k_A \cos(\omega t + k_A x) = -A_t k_B \cos(\omega t - k_B x)$$

lets take boundary is at $x=0$.

$$-A_i k_A + A_r k_A = -A_t k_B$$

$$A_r - A_i = -A_t\left(\frac{k_B}{k_A}\right).$$

$v = \frac{\omega}{k}.$

$\frac{v_A}{v_B} = \frac{k_B}{k_A}.$

$$A_r - A_i = -A_t\left(\frac{v_A}{v_B}\right) \text{ — ②}$$

① − ②

$$2A_i = A_t\left[1 + \frac{v_A}{v_B}\right].$$

$$\boxed{A_t = \left(\frac{2v_B}{v_A + v_B}\right) A_i}$$

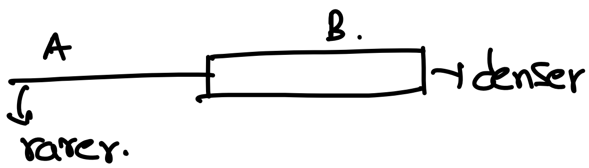
$$\boxed{A_r = -\left(\frac{V_A - V_B}{V_A + V_B}\right) A_i .}$$

denser medium ⇒ its the medium in which speed of wave is less

rarer medium ⇒ " " " " " is more.

$V_A > V_B$ ⇒ then A is rarer medium

B is denser medium

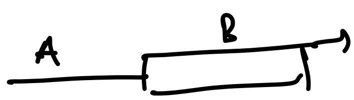


$$A_r = -\frac{(V_A - V_B)}{V_A + V_B} A_i .$$

as $V_A > V_B$ ⇒ A_r = (-ve) of A_i.

upward pulse gets reflected as downward pulse and vice versa

⇒ Perfect fixed end ⇒ rope having ∞ mass per unit length.



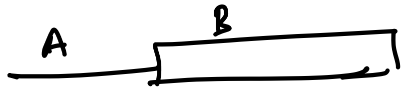
∞ mass

$V_B = \sqrt{\frac{T}{\mu}}$, $\mu \to \infty$ ⇒ $V_B = 0$

$$A_r = -A_i .$$

$$A_t = 0.$$

⇒ Perfect open end ⇒ rope having 0 mass per unit length.
(or)
free end



$\rightarrow 0$ mass $\quad V_B = \sqrt{\dfrac{T}{\underset{\searrow 0}{\mu}}} \Rightarrow V_B = \infty$

$$A_t = \left(\frac{2V_B}{V_A + V_B}\right) A_i$$

$$= \left(\frac{2}{\underbrace{\frac{V_A}{V_B}}_{\rightarrow 0} + 1}\right) A_i$$

$\rightarrow 0$ as V_B is ∞

$$\boxed{A_t = 2A_i}$$

$$A_r = -\frac{\left(\overset{\rightarrow 0}{\frac{V_A}{V_B}} - 1\right)}{\left(\overset{\rightarrow 0}{\frac{V_A}{V_B}} + 1\right)} A_i.$$

$$A_r = \frac{-(-1)(A_i)}{1} \Rightarrow \boxed{A_r = A_i.}$$

---

standing waves:- When two waves of same frequency and amplitude travel in opposite direction standing waves are produced.

lets take two waves

$$y_1 = A \sin(\omega t - kx).$$

$$y_2 = -A \sin(\omega t + kx).$$

$y_r = y_1 + y_2 \rightarrow$ principle of superposition.

$$y_r = A\left[\sin(\omega t - kx) - \sin(\omega t + kx)\right].$$

$$= A\left[2\cos\omega t \sin(-kx)\right]$$

$$y_r = (-2A\sin kx)\cos\omega t. \longrightarrow A_r = 2A\sin kx.$$

Particle at $x=0$

$$y_r = \underbrace{(0)}_{\text{amplitude.}}\cos\omega t$$

Particle at $x = \frac{\lambda}{4}$.

$$y_r = \underbrace{-2A}_{\text{amplitude.}}\cos\omega t$$

Particle at $x = \frac{\lambda}{8}$.

$$y_r = \underbrace{-\sqrt{2}A}_{\text{amplitude.}}\cos\omega t$$

Node: $A_r = 0$

$$-2A\sin kx = 0$$

$$kx = 0, \pi, 2\pi, 3\pi \ldots$$

$$\frac{2\pi}{\lambda}x = 0, \pi, 2\pi, \ldots$$

$$x = 0, \frac{\lambda}{2}, \frac{2\lambda}{2}, \frac{3\lambda}{2} \ldots$$

distance between two successive nodes $= \frac{\lambda}{2}$.

Antinodes: $A_r = \max$

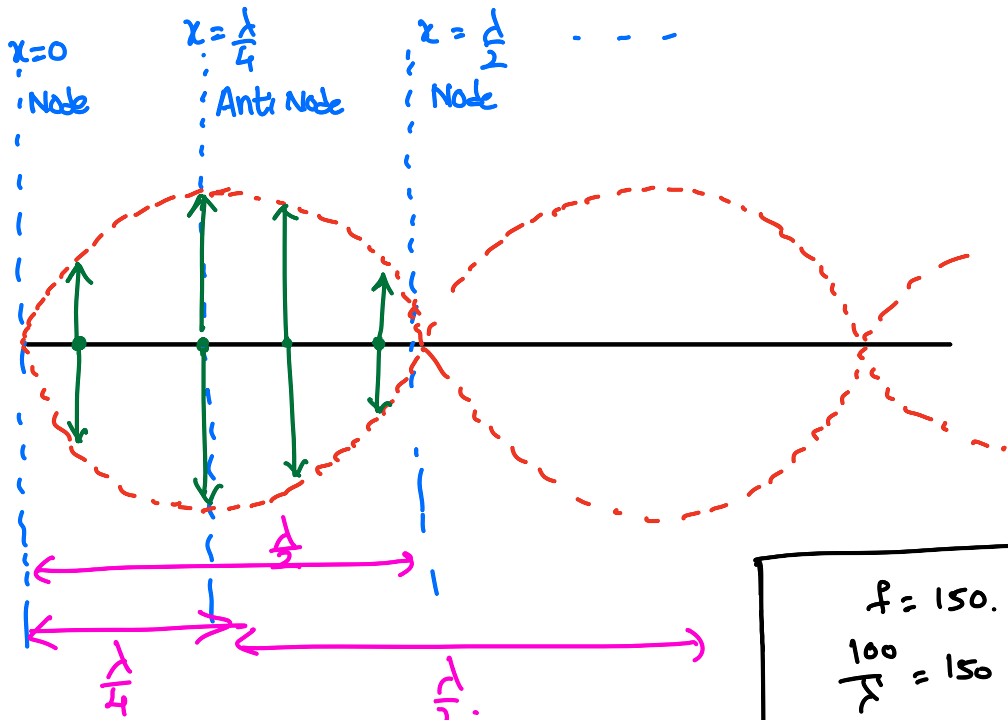
$$-2A\sin kx = \pm 2A.$$

$\sin kx = \pm 1.$

$kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \ldots$

$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4} \ldots\ldots$

distance between two successive antinodes = $\frac{\lambda}{2}$.

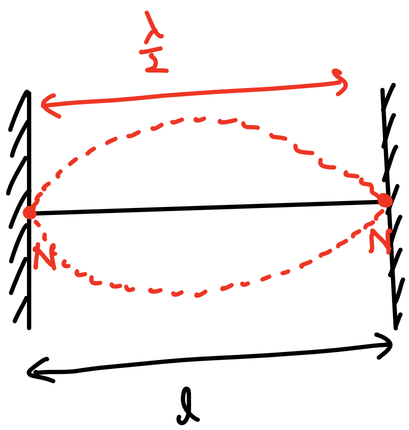


$f = 150.$

$\frac{100}{\lambda} = 150$

$\lambda' = \frac{100}{150} =$

standing waves on a string fixed at both ends:-



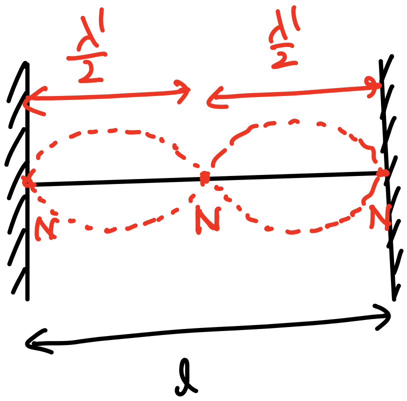
$\frac{\lambda}{2} = l \Rightarrow \lambda = 2l.$

$f = \frac{v}{\lambda}.$

$f = \frac{v}{2l} \Rightarrow f = 1\left[\frac{v}{2l}\right].$

fundamental frequency

1st harmonic | 0th overtone.



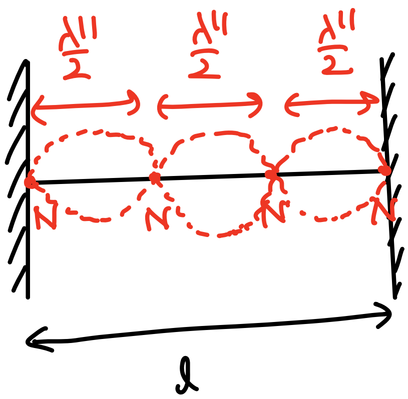
$2\frac{\lambda'}{2} = l \Rightarrow \lambda' = \frac{2l}{2}.$

$f' = \frac{v}{\lambda'}.$

$f' = 2\left[\frac{v}{2l}\right].$

2nd harmonic

1st overtone.



$\frac{3\lambda''}{2} = l \Rightarrow \lambda'' = \frac{2l}{3}.$

$f'' = \frac{v}{\lambda''} \Rightarrow f'' = 3\left[\frac{v}{2l}\right].$

3rd harmonic

2nd overtone

frequences for which standing waves can be formed on string fixed at both ends $\Rightarrow \frac{v}{2l}, 2\left(\frac{v}{2l}\right), 3\left(\frac{v}{2l}\right), 4\left[\frac{v}{2l}\right] \dots$

ratio of freq $= 1:2:3:4 \dots$

Properties of standing waves :-

i) Nodes, Anti nodes will be formed.

ii) standing waves are not possible for all frequencies

iii) distance b/w two successive nodes & antinodes $= \frac{\lambda}{2}.$

iv) All particles in one segment vibrate in same phase, but particles in adjacent segments have phase difference of π.

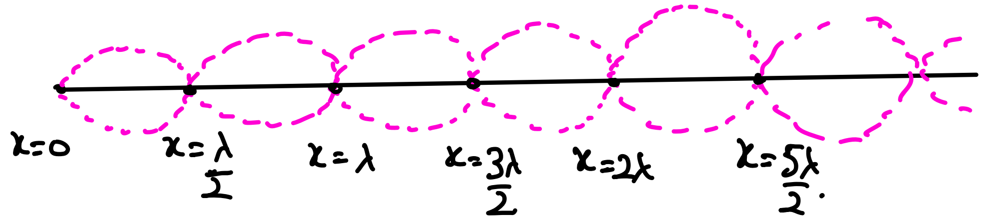
Differences b/w progressive wave and standing wave:-

Progressive wave	Standing wave.
→ All the particles have same amplitude.	→ Amplitude changes
→ Not all the particles cross the M.P. at same time.	→ All particles cross M.P. at the same time.
→ Possible for all the frequencies	→ Not possible for all the freq
→ Transports energy	→ doesn't transport energy.
→ All the particles cross M.P. with same speed $= (A\omega)$	⇒ Not all particles cross M.P. with same speed $(A_r \omega)$. Particles at A.N. cross M.P. with max. speed ⇒ $(2A)\omega$.

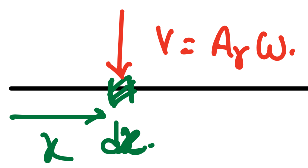
Energy associated with each segment of a standing wave:-

let standing wave be

$$y = 2A \sin kx \cos \omega t$$



T.E = K.E of particles at M.P as elastic P.E = 0.



$$d(K.E) = \frac{1}{2}(dm)v^2$$

$$= \frac{1}{2}(\mu dx)(A_r^2\omega^2)$$

$$= \frac{1}{2}\mu dx \, 4A^2 \sin^2 kx \, \omega^2$$

$$K.E = \int_0^{\lambda/2} 2\mu A^2\omega^2 \sin^2 kx \, dx$$

$$= 2\mu A^2\omega^2 \left[\int_0^{\lambda/2} \frac{1}{2}\,dx - \underbrace{\int_0^{\lambda/2} \frac{\cos 2kx}{2}\,dx}_{\to 0}\right]$$

$$= 2\mu A^2\omega^2 \left[\frac{1}{2}\left[\frac{\lambda}{2} - 0\right]\right]$$

$$K.E = T.E = \frac{1}{2}\mu A^2\omega^2\lambda.$$

ill:8 BB:3 Q3 Ex:5 Q13 :-

ill:8 $\mu = 5 \times 10^{-3}$ kg/m. $T = 450$ N $V = \sqrt{\frac{T}{\mu}}$

$$f = (n)\left(\frac{V}{2l}\right).$$

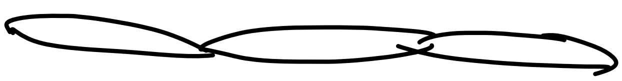
$$f' = (n+1)\frac{V}{2l}.$$

$$f' - f = \frac{V}{2l}.$$

$$490 - 420 = \frac{\sqrt{\frac{T}{\mu}}}{2l} \; ?$$

BB:3

Q3) $\mu = 7.2 \times 10^{-3}$ kg/m $T = 150$ $V = \sqrt{\frac{T}{\mu}}$. $\Rightarrow V = 100\sqrt{3}$.



$$\frac{3\lambda}{2} = 90 \times 10^{-2} \Rightarrow \lambda = 60 \times 10^{-2}$$

$$\lambda = 60\text{cm}.$$

$$f = \frac{V}{\lambda} = \frac{100\sqrt{3}}{60 \times 10^{-2}} \text{ Hz}.$$

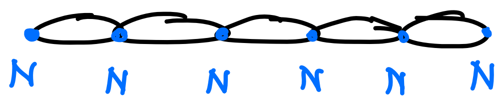
Ex-5

Q13)

$$f = 5\left[\frac{v}{2l}\right]$$

$$f = \frac{v}{\frac{2l}{5}} = \frac{v}{\lambda}$$

$$\lambda = \frac{2l}{5} \Rightarrow \frac{5\lambda}{2} = l.$$



$$k = 62.8$$

$$\frac{2\pi}{\lambda} = 62.8 \Rightarrow \lambda = \frac{2 \times 3.14}{62.8}$$

$$\lambda = \frac{1}{10}\,m.$$

$$l = \frac{5\lambda}{2}.$$

$$l = \frac{5}{20} \Rightarrow l = 0.25$$

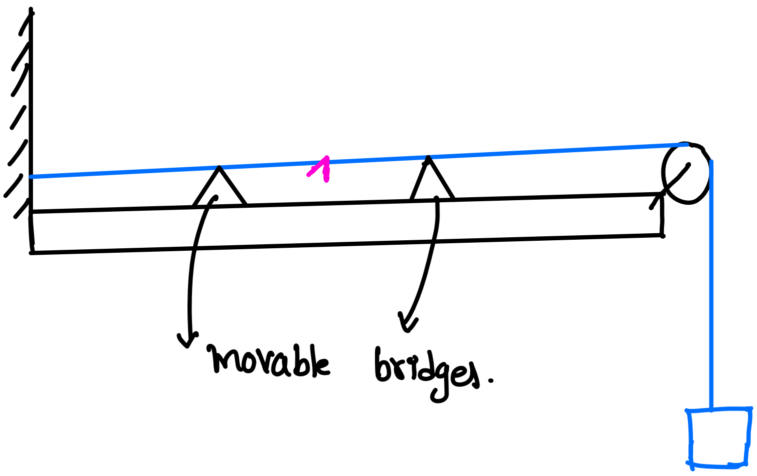
$$v = \frac{\omega}{k}$$

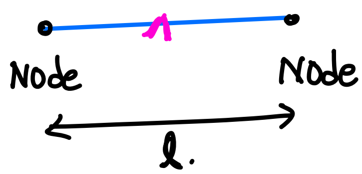
$$= \frac{628}{62.8} = 10$$

$$f = \frac{v}{2l}.$$

$$= \frac{10}{2 \times 0.25} \Rightarrow f = 20\,Hz.$$

sonometer:-





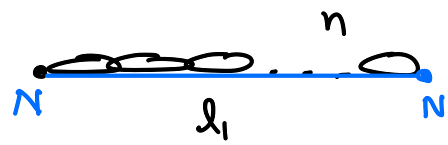
when the paper rider falls off we can say that resonance has occurred

$$f_{\text{tuning fork}} = f_{\text{oscillations of string}}.$$

$$f_{\text{oscillations of string}} = n\left(\frac{v}{2l}\right).$$

$$\boxed{f = \frac{nv}{2l}}.$$

$$f = \frac{nv}{2l_1}.$$



$$f = \frac{(n+1)v}{2l_2}.$$

$$\frac{nv}{2l_1} = \frac{(n+1)v}{2l_2}.$$

$$\frac{\lambda}{2} = l_2 - l_1.$$

$$\lambda = 2(l_2 - l_1)$$

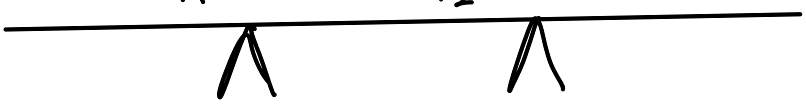
$f = \frac{v}{\lambda}$.

$$\boxed{f = \frac{\sqrt{\frac{T}{\mu}}}{2(l_2 - l_1)}}$$

BB-3

Q4)

$f_1 = \frac{v}{2l_1}$ $\quad f_2 = \frac{v}{2l_2}$ $\quad f_3 = \frac{v}{2l_3}$.



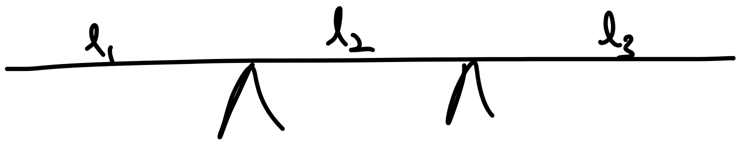
$$f_1 : f_2 : f_3 = \frac{v}{2l_1} : \frac{v}{2l_2} : \frac{v}{2l_3}.$$

$$= \frac{1}{\frac{1}{7}} : \frac{1}{\frac{2}{7}} : \frac{1}{\frac{4}{7}}$$

$$= 4 : 2 : 1.$$

Q7)

$$f_1 : f_2 : f_3 = \frac{1}{l_1} : \frac{1}{l_2} : \frac{1}{l_3}$$



$$1 : 2 : 3 = \frac{1}{l_1} : \frac{1}{l_2} : \frac{1}{l_3}.$$

$$l_1 : l_2 : l_3 = 1 : \frac{1}{2} : \frac{1}{3}.$$

$= 6 : 3 : 2$

Total length should be divided into 11 parts.

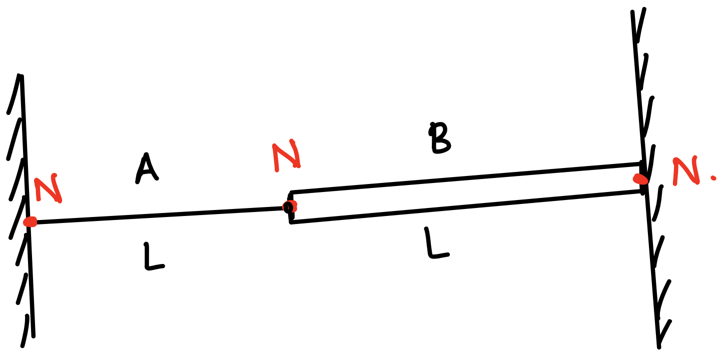
$$l_1 = 100\left[\frac{6}{11}\right] cm$$

$$l_2 = 100\left[\frac{3}{11}\right] cm$$

$$l_3 = 100\left[\frac{2}{11}\right] cm.$$

Ex-4A

Q34)



$$f = \frac{v}{\lambda}.$$

$$\frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2}$$

$$\frac{\sqrt{\frac{T}{\mu_1}}}{\lambda_1} = \frac{\sqrt{\frac{T}{\mu_2}}}{\lambda_2}.$$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{\mu_2}{\mu_1}}. = \sqrt{\frac{\rho A_2}{\rho A_1}}$$

$$= \sqrt{\frac{\pi(2r)^2}{\pi r^2}}$$

$$= 2.$$

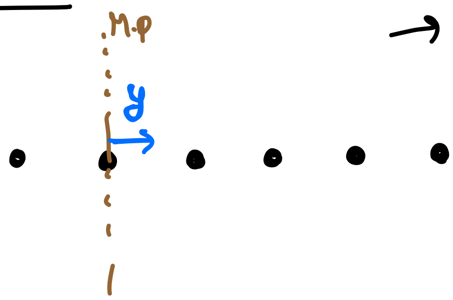
$$(n_1)\frac{\lambda_1}{2} = L = (n_2)\frac{\lambda_2}{2}.$$ n_1, n_2 are no of loops.

$$\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{1}{2}.$$

if 1 loop in A then 2 loops in B.

Longitudinal waves:-

→ wave is travelling.



y = disp. of particle from M.P.

$$\left.\begin{aligned} y &= A\sin(\omega t - kx + \phi) \\ y &= A\sin(kx - \omega t + \phi) \end{aligned}\right\} +x \text{ direction.}$$

$$\left.\begin{aligned} y &= A\sin(\omega t + kx + \phi \\ y &= A\sin(-\omega t - kx + \phi) \end{aligned}\right\} -x \text{ direction}$$

longitudinal wave speed in solids:-

$$v = \sqrt{\frac{Y}{\rho}}$$

Y = young's modulus.

ρ = density.

speed of longitudinal wave in fluids:-

$$V = \sqrt{\frac{B}{\rho}} \qquad \because \ B = \frac{-\Delta P}{\frac{\Delta V}{V}}.$$

Volume

B = Bulk modulus.

speed of sound in air by newton:-

newton has taken that sound travels in air through an isothermal process.

$$B = P \rightarrow \text{isothermal}$$

$$V = \sqrt{\frac{P}{\rho}}$$

at STP $V \approx 279$ m/sec.

but experimentally speed of sound was around 330 m/sec. its a clear indication that sound doesn't travel isothermally.

Laplace's correction:- its not isothermal its adiabatic

$$B = \gamma P$$

$$V = \sqrt{\frac{\gamma P}{\rho}}.$$

at STP $V \approx 330$ m/sec.

so speed of sound in air is $\sqrt{\frac{\gamma P}{\rho}}$ as experimental result is matching theoritical result.

$$v = \sqrt{\frac{\gamma P}{\left(\frac{M}{V}\right)}}$$

$$v = \sqrt{\frac{\gamma P V}{M}} \Rightarrow v = \sqrt{\frac{\gamma n R T}{M}} \Rightarrow v = \sqrt{\frac{\gamma R T}{\left(\frac{M}{n}\right)}}$$

$$v = \sqrt{\frac{\gamma R T}{M_0}}$$

M_0 ↓ molar mass.

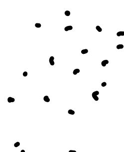
effect of various quantities:-

(i) Temp :- as $T \uparrow$ $\quad v_{air} \uparrow$.

(ii) Pressure:- we can't directly comment.

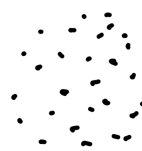
if $P \uparrow$ but T constant then v_{air} is constant.

(iii) Humidity:- section of air



$N_2, O_2, ..$

as humidity increases H_2O concentration increases.

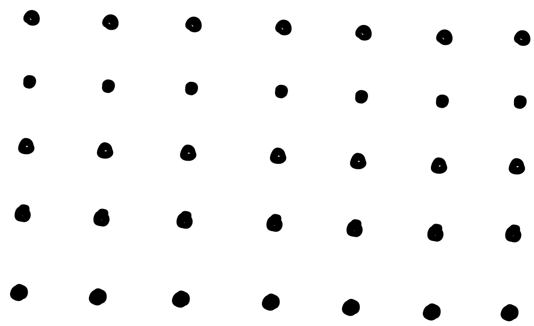


⇒ Now in a given volume some of N_2, O_2 will be replaced by H_2O

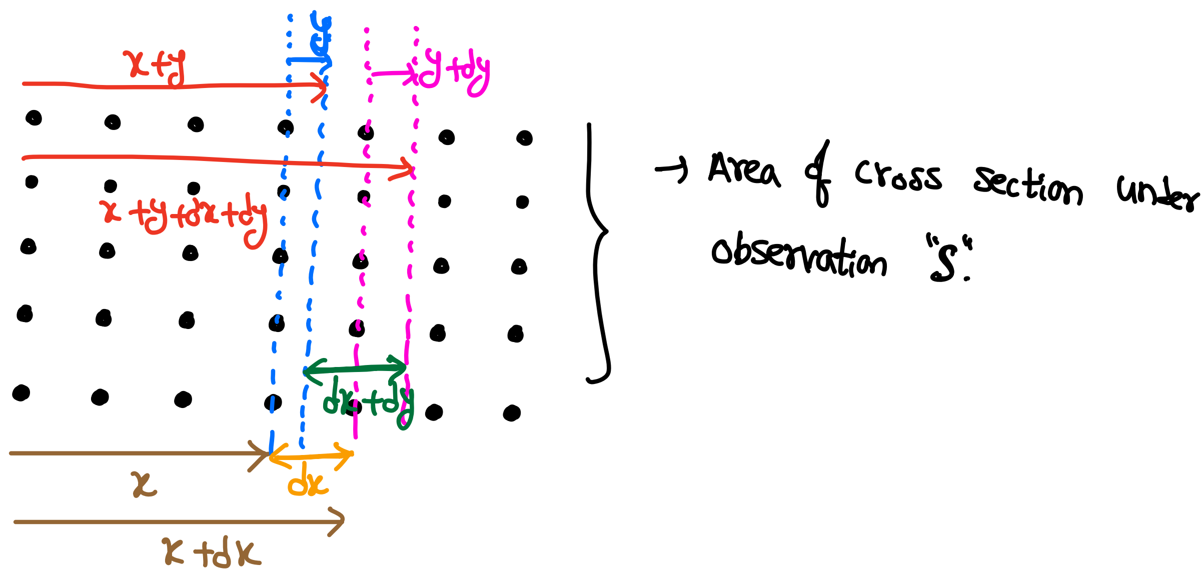
→ so as humidity increases, density decreases.

if P is constant as humidity $\uparrow$ $\rho \downarrow$ $v_{air} \uparrow$.

Longitudinal waves in the form of pressure waves:-



When a disturbance is made to go through.



$$y = A\sin(\omega t - kx + \phi).$$

$$B = -\frac{\Delta P}{\frac{\Delta V}{V}}$$

$$V_{initial} = (S)(dx)$$

$$V_{final} = (S)(dx+dy)$$

$$\Delta V = (S)(dy).$$

$$\Delta P = (-B)\frac{\Delta V}{V}.$$

$$= (-B)\frac{dy}{dx}.$$

$$\Delta P = BAK\cos(\omega t - kx + \phi)$$

$$y = A\sin(\omega t - kx + \phi)$$

$$\frac{dy}{dx} = -AK\cos(\omega t - kx + \phi).$$

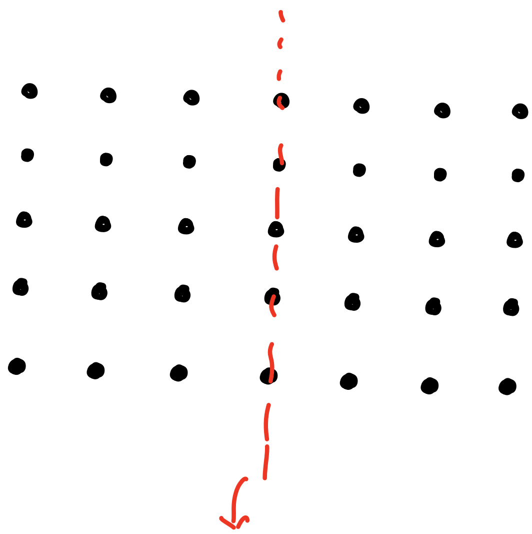
$$\boxed{\Delta P = BAK\sin(\omega t - kx + \phi + \tfrac{\pi}{2})} \Rightarrow \text{this is called pressure wave}$$

variation in pressure.

⇒ pressure wave and displacement wave have $\frac{\pi}{2}$ phase difference.

so when y is max ΔP is min

y is min ΔP is ma



at NTP pressure will be P_0. when disturbance passes through

$P_{max} = P_0 + BAK$

$P_{min} = P_0 - BAK$

max. pressure difference of a location $= 2(BAK)$

Pressure amplitude.

we know intensity to be

$$I = \frac{1}{2}\rho\omega^2A^2v \times \frac{v}{v}.$$

$$I = \frac{1}{2}\frac{\rho\omega^2A^2v^2}{v} \qquad v^2 = \frac{B}{\rho}.$$

$$\omega = vk.$$

$$I = \frac{1}{2}\frac{\omega^2A^2B}{v}$$

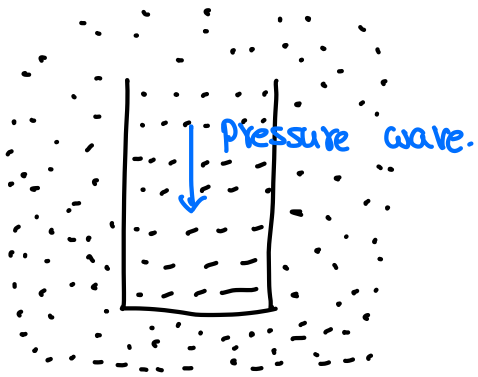
$$I = \frac{1}{2}\frac{v^2k^2A^2B}{v} \times \frac{B}{B}.$$

$$I = \frac{1}{2}\frac{(BAk)^2v}{B}.$$

$$\boxed{I = \frac{1}{2}\frac{(P_{amp})^2v}{B}}$$

reflection of longitudinal wave from closed end and free end of a pipe:-

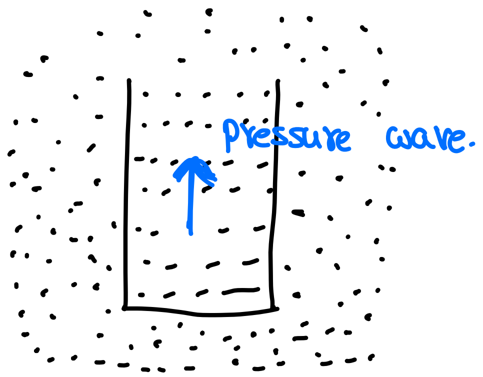
fixed end:-



⇒ whenever a longitudinal wave is reflected from fixed end, compression will be reflected back as compression and rarefaction as rarefaction.

⇒ Pressure wave suffers "0" phase, displace-ment wave suffers π.

free end

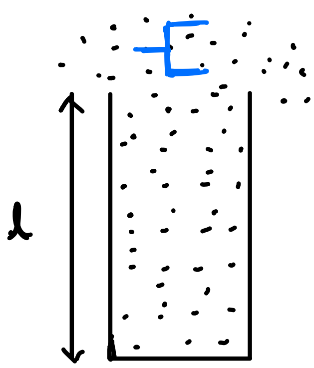


⇒ a compression will be reflected back as rarefaction and vice versa.

at free end Pressure wave suffers a phase of "π" but displacement wave "0".

Vibration in organ pipe:-

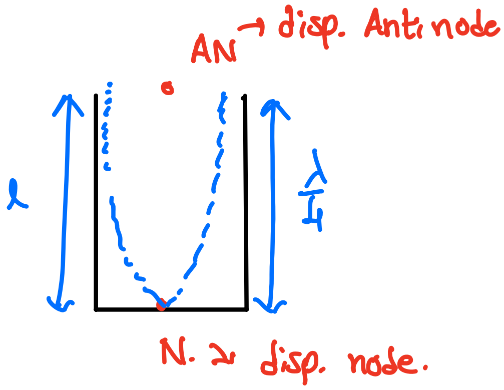
closed organ pipe:-



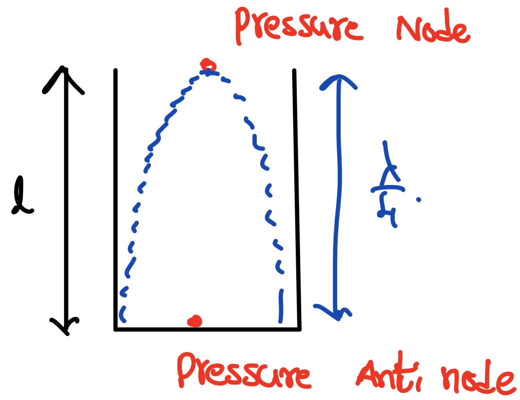
When a disturbance is created air column in organ pipe starts oscillating. When the disturbance reaches the fixed end and gets reflected back there is a possibility of standing waves.

standing waves in organ pipe

displacement wave



pressure wave



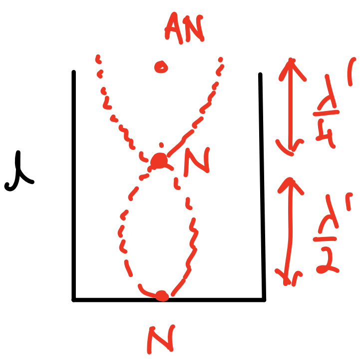
$\frac{\lambda}{4} = l \Rightarrow \lambda = 4l$

$f = \frac{v}{\lambda} \Rightarrow f = \frac{v}{4l}$

fundamental freq.

1st harmonic.

⇒



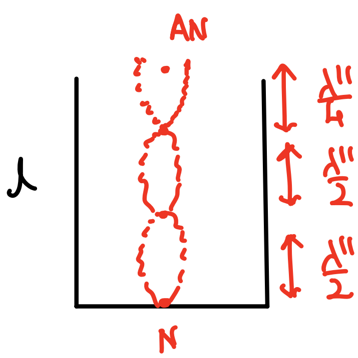
$\frac{3\lambda'}{4} = l \Rightarrow \lambda' = \frac{4l}{3}$.

$f' = \frac{v}{\lambda'} \Rightarrow f' = 3\left[\frac{v}{4l}\right]$.

3rd harmonic.

1st overtone.

⇒



$\frac{5\lambda''}{4} = l \Rightarrow \lambda'' = \frac{4l}{5}$.

$f'' = 5\left[\frac{v}{4l}\right]$

5th harmonic

2nd overtone.

frequencies = $\boxed{\frac{V}{4l}}, \frac{3V}{4l}, \frac{5V}{4l}, \frac{7V}{4l} \ldots$

ratio of freq = $1 : 3 : 5 : 7 \ldots$

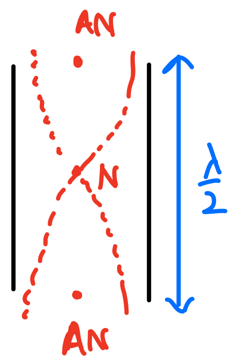
$\Rightarrow$ all even harmonics are missing in closed organ pipe.

---

open organ pipe:-

displacement waves

l



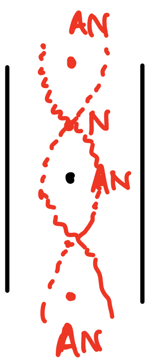
$\frac{\lambda}{2} = l \Rightarrow \lambda = 2l$

$f = \frac{V}{\lambda} = \frac{V}{2l}$

fundamental freq.

1st harmonic.

l



$2\left(\frac{\lambda'}{2}\right) = l \Rightarrow \lambda' = \frac{2l}{2}$

$f' = 2\left[\frac{V}{2l}\right]$

2nd harmonic

1st overtone

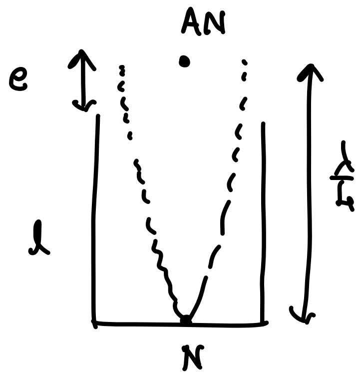
freq = $\frac{V}{2l}, 2\frac{V}{2l}, 3\frac{V}{2l} \ldots$

ratio of freq = $1 : 2 : 3 : \ldots$

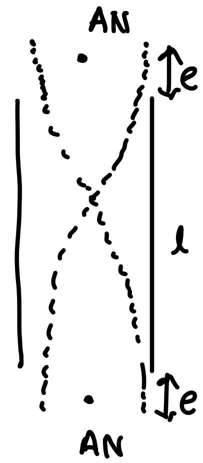
End correction (e):- Antinodes are not formed exactly at the open end but just outside

$e = (0.6)(\text{radius of tube})$.

eg:-



$l + e = \frac{\lambda}{4}$.

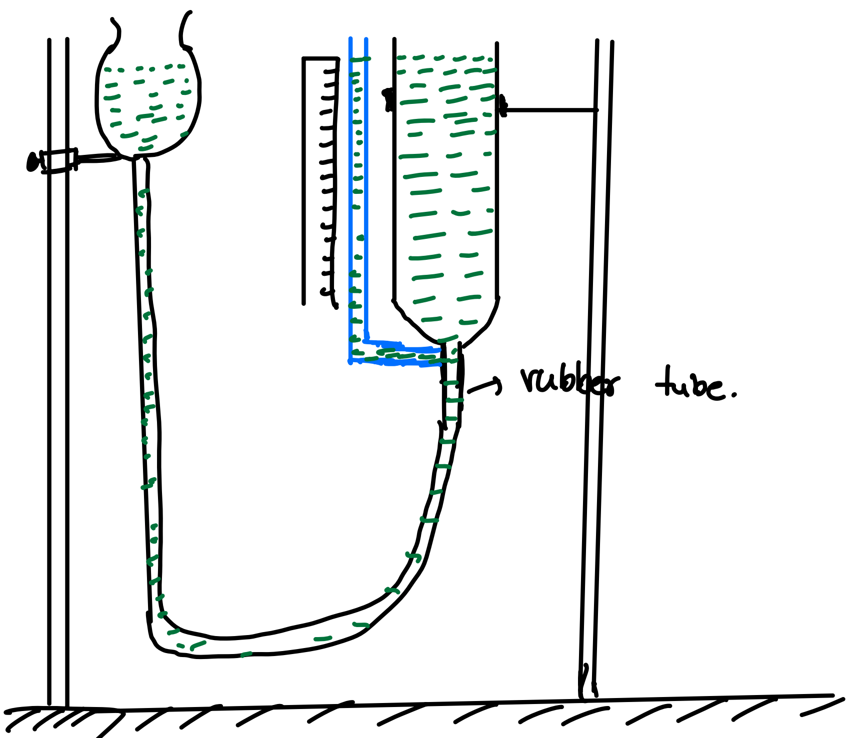


$\frac{\lambda}{2} = l + 2e$

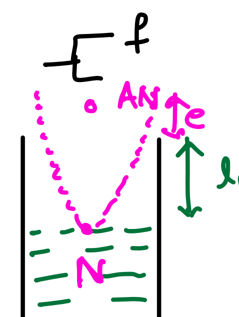
Finding speed of sound using resonance tube method:- (resonance column method).

Resonance tube ⇒ made of brass or glass

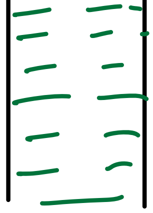
⇒ 1m Long and 5cm in diameter.



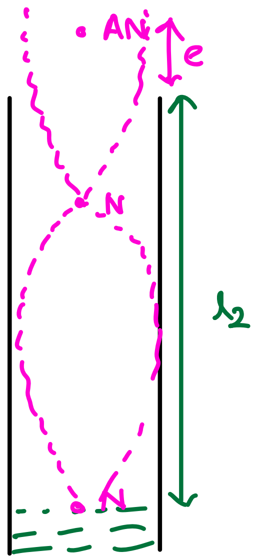
1st resonance :-



$l_1 + e = \frac{\lambda}{4}$ —①



<u>2nd resonance</u>



$l_2 + e = \frac{3\lambda}{4}$ — ②

② − ①

$l_2 - l_1 = \frac{\lambda}{2} \Rightarrow \lambda = 2(l_2 - l_1).$

$f = \frac{v}{\lambda}$

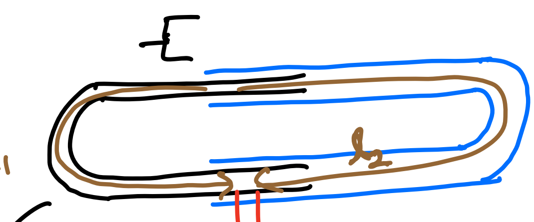
$v = f\lambda.$

$$\boxed{v = f\left[2(l_2 - l_1)\right]}$$

on solving eq ① and ②

$$\boxed{e = \frac{l_2 - 3l_1}{2}.}$$

<u>Quinck's tube:-</u>



$l_2 - l_1 = \Delta x.$

Detector.

By moving blue tube we can change l_2.

lets say there is destructive interference in this case

$$l_2 - l_1 = \frac{(2n+1)\lambda}{2} \quad -①$$

Now move blue tube out by "x".

$$[l_2+2x] - l_1 = \frac{(2n+3)\lambda}{2} \quad -②$$

② − ①

$$2x = \lambda$$

$$V = (f)(\lambda)$$

$$\boxed{V = (f)(2x)}$$

Loudness of sound or sound intensity in decibels:-

For $I_0 = 10^{-12}\ W/m^2$, Loudness is taken to be "0" dB.

$$\text{Loudness} = 10 \log_{10} \frac{I}{I_0} \text{ dB.}$$

I = intensity of sound.

I_0 = constant = $10^{-12}\ W/m^2$

BB:5

Q3, Q6 :-

Ex: 4A

Q9) Q19)

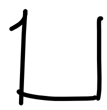
BB: 5

Q3) 50, 150, 250

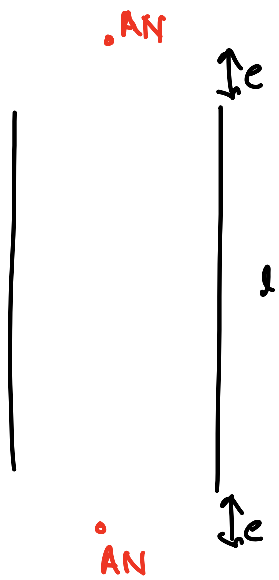
1 : 3 : 5 . . .

f_1

Closed organ pipe.



Q6)



$l + 2e = \frac{\lambda}{2}$ $\Rightarrow$ $\lambda = 2l + 4e$

$\lambda = 2l + 1.2d$

$\nu_1 = \frac{330}{2l + 1.2d}$ — (1)

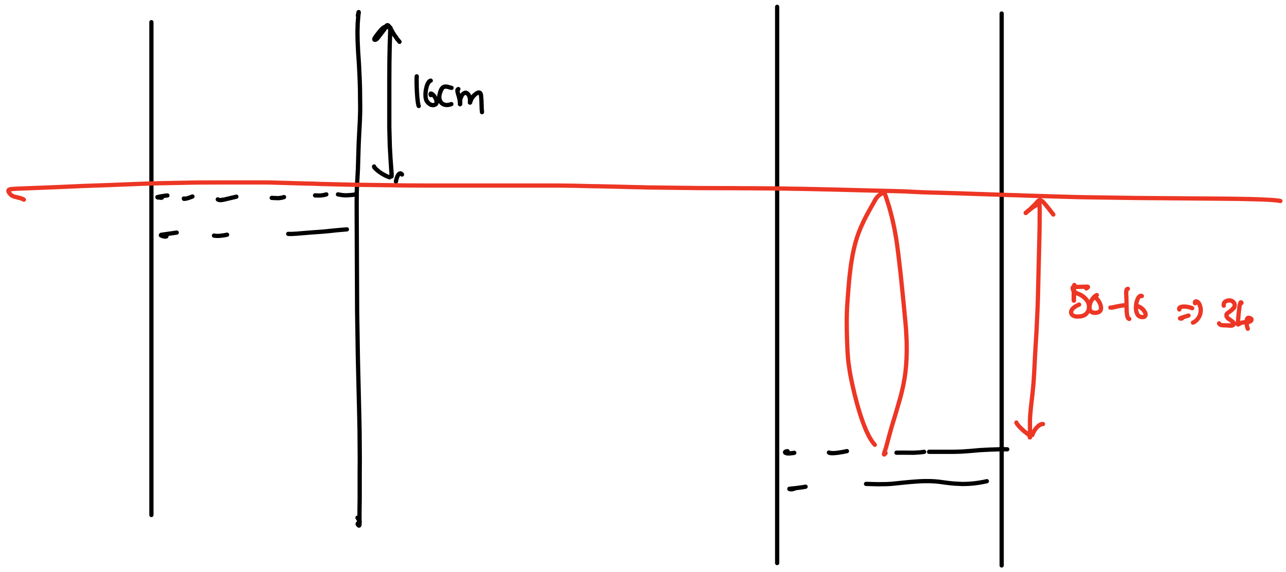
$l + e = \frac{\lambda}{4}$ $\Rightarrow$ $\lambda = 4l + 4e$

$\lambda = 4l + 1.2d$

$\nu_2 = \frac{330}{4l + 1.2d}$

Ex-4A

9a)



$\frac{\lambda}{2} = 34 \Rightarrow \lambda = 68\text{cm}.$

$f = \frac{v}{\lambda} \Rightarrow v = (500)(68\text{ cm})$

$f = \frac{v}{2l}.$

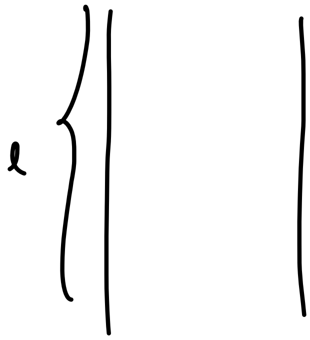
$= \frac{(500)(68\,\cancel{\text{cm}})}{2(66.5\,\cancel{\text{cm}})}$

$f = (250)\left(\frac{68}{66.5}\right)$

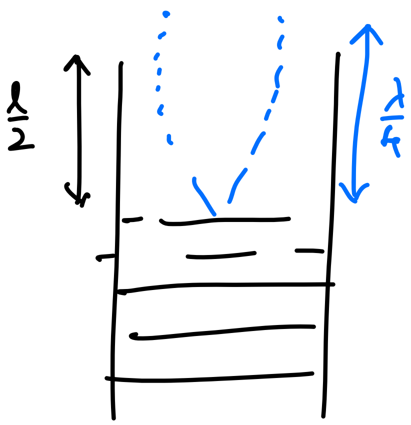
$= \frac{(50)(68)}{12.1}$

next freq $= 2f$

Q19)



$$f = \frac{v}{2l}.$$



$$\frac{\lambda}{4} = \frac{l}{2} \Rightarrow \lambda = 2l$$

$$f' = \frac{v}{\lambda} \Rightarrow f' = \frac{v}{2l}.$$

$$f' = f.$$

Except these do the rest:-

Ex: 1 ⇒ 13, 14, 15, 16, 21

Ex: 2: 1, 10, 11,

Ex: 3: 5, 10, 13

Ex: 4A: 3, 4, 15, 16, 17, 20, 21, 29, 33, 36,

Ex: 4B: 5, 7, 8, 10, 12, 16, 17, 18, 19

Ex: 5 :- 2, 7, 9, 17, 18, 21

Discussion:-

Q19) $$P = P_0 \cos \frac{3\pi x}{2} \sin(300\pi t)$$

Pressure amplitude of a pressure standing wave.

at closed end $\Rightarrow$ pressure antinode $\Rightarrow$ $P_{Amplitude} = max$

at open end $\Rightarrow$ pressure node $\Rightarrow$ $P_{amplitude} = 0.$

$$P_{amplitude} = P_0 \cos \frac{3\pi x}{2}.$$

at $x=0$

$$P_{amplitude} = max = P_0.$$

at $x = \frac{1}{2}$

$$P_{amplitude} = P_0 \cos \frac{3\pi}{4}$$

$$= \frac{-P_0}{\sqrt{2}} \neq 0 \Rightarrow \text{so not a node.}$$

at $x = 1$

$$P_{amplitude} = P_0 \cos \frac{3\pi}{2}.$$

$$= 0 \Rightarrow \text{node.}$$

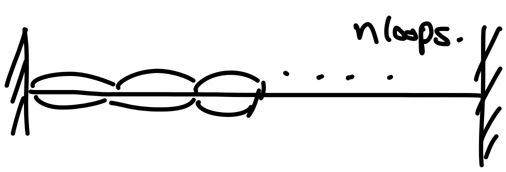
at $x = \frac{2}{3}$

$$P_{amplitude} = P_0 \cos \frac{3\pi}{2} \times \frac{2}{3}$$

$$= -P_0 \Rightarrow max \Rightarrow AN.$$

Ex-2

Q5)



energy associated with each segment $= \frac{1}{2} \mu A^2 \omega^2 \lambda.$

$$n\frac{\lambda}{2} = l$$

$$\lambda = \frac{2l}{n}..$$

$$f = \frac{V}{\frac{2l}{n}} \Rightarrow f = \frac{Vn}{2l}$$

$$\frac{\omega}{2\pi} = \frac{Vn}{2l}$$

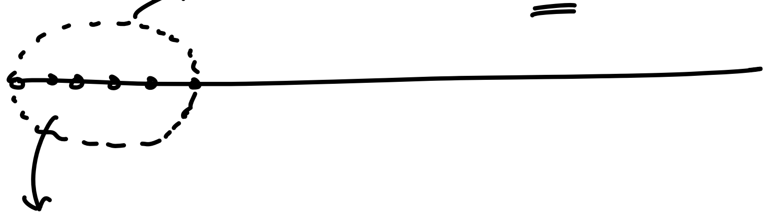
$$\omega = \frac{\pi V n}{l}.$$

$$T.E = n\left(\tfrac{1}{2}\mu A^2\omega^2\lambda\right)$$

$$\propto n\,(n^2)\left(\tfrac{1}{n}\right)$$

$$\propto n^2$$

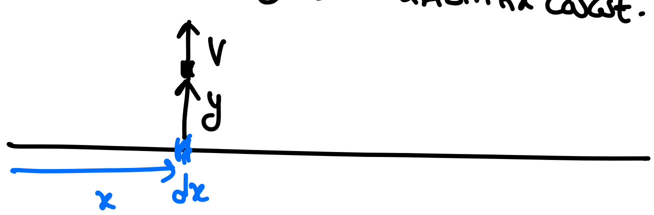
K.E at extreme $\underline{\underline{=0}}$



T.E : K.E at M.P.

standing wave = $2A\sin kx\cos\omega t$.

$$y = 2A\sin kx\cos\omega t.$$



$$V = \omega\sqrt{A_r^2 - y^2}$$

$$= \omega\sqrt{4A^2\sin^2 kx - 4A^2\sin^2 kx\cos^2\omega t}$$

$$V = \omega\sqrt{4A^2\sin^2 kx\sin^2\omega t}$$

$V = 2\omega A \sin kx \sin \omega t.$

$$d(K.E) = \frac{1}{2}(dm)\, 4\omega^2 A^2 \sin^2 kx \sin^2 \omega t$$

$$d(K.E_{avg}) = \frac{\int \frac{1}{2}(dm)\, 4\omega^2 A^2 \sin^2 kx \sin^2 \omega t \, dt}{\int dt}$$

$$= \frac{1}{2}(dm)\, 4\omega^2 A^2 \sin^2 kx \; \frac{\int_0^T \sin^2 \omega t \, dt}{\int_0^T dt} \rightarrow \frac{1}{2}.$$

avg. K.E for 1 segment.

$$K.E_{avg} = \int_0^{\frac{\lambda}{2}} \omega^2 A^2 (\mu dx) \sin^2 kx$$

$$= \mu\omega^2 A^2 \left[\frac{1}{2} (x)_0^{\frac{\lambda}{2}} \right]$$

$$= \frac{1}{4} \mu \omega^2 A^2 \lambda.$$

Q8)

$\Delta\phi = 0$

Intensity of $S_1 = I$

" $S_2 = I.$

$I_r = I + I + 2\sqrt{II} \cos 0$

$I_0 = 4I \Rightarrow I = \frac{I_0}{4}.$

Intensity of $S_1 = 0.36I$

" of $S_2 = I$.

$$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$$

$$= (\sqrt{0.36I} + \sqrt{I})^2$$

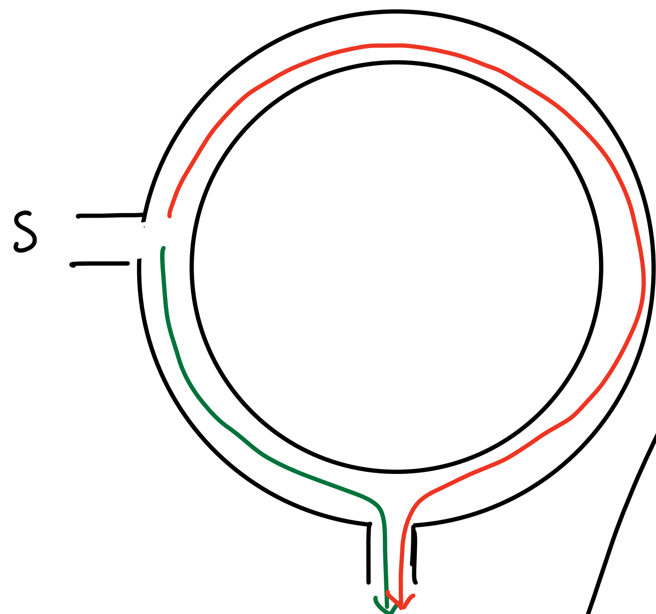
$$= [(0.6+1)\sqrt{I}]^2$$

$$= (1.6)^2 I.$$

$$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$$

$$= (0.4)^2 I.$$

Q18) initial phase difference $= 0$.



$\Delta x = \pi R$.

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x.$$

$$\cancel{2}n\cancel{\pi} = \frac{\cancel{2}\cancel{\pi}\Delta x}{\lambda}$$

$$\lambda = \frac{\Delta x}{n}.$$

$$\lambda = \infty, \frac{\Delta x}{1}, \frac{\Delta x}{2} \ldots$$

$$\lambda = \infty, \pi R, \frac{\pi R}{2} \ldots$$

$$(2n-1)\cancel{\pi} = \frac{2\cancel{\pi}\Delta x}{\lambda}$$

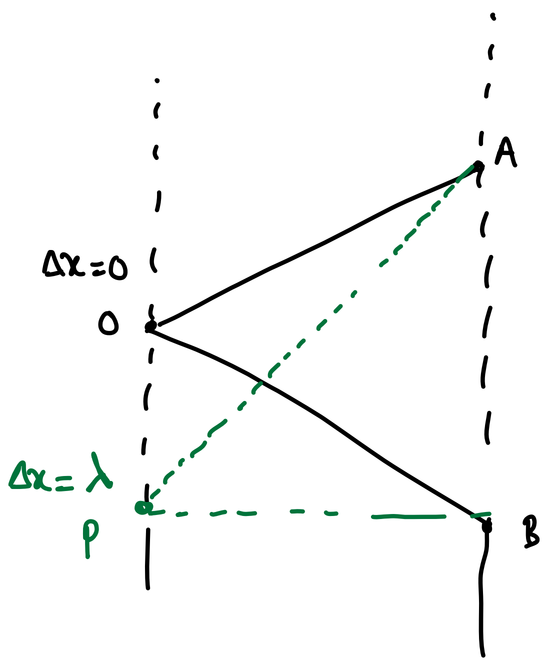
$$\lambda = \frac{2\Delta x}{(2n-1)}$$

$$\lambda = \frac{2\Delta x}{1}, \frac{2\Delta x}{3}, \frac{2\Delta x}{5}, \frac{2\Delta x}{7} \dots$$

$$\lambda = 2\pi R, \frac{2\pi R}{3}, \frac{2\pi R}{5}, \dots\dots$$

Ex-3

Q1)



$\Delta x = \lambda.$

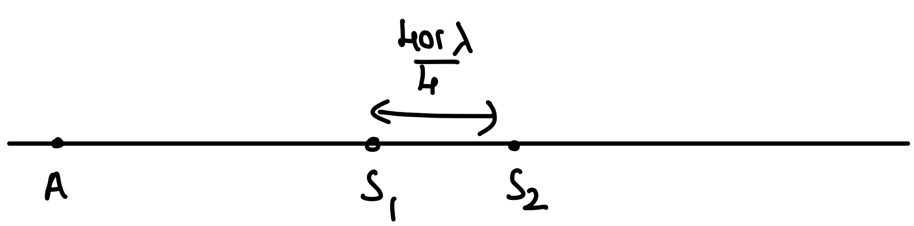
$AP - BP = \lambda.$

$$f = \frac{v}{\lambda}$$

$$v = f\lambda.$$

Q2)

$$\phi_1 - \phi_2 = \frac{\pi}{2}.$$



A

$$\Delta x = \frac{\lambda}{4} \text{ or } \lambda.$$

Layman

Madhan	Utsav.
5 lakhs	0 lakhs.

we did business and madhan got 20 lakhs extra profit than utsav

lets say utsav got 10 lakhs profit

Phase difference due to path difference.

$$\Delta\phi = \frac{2\pi}{\lambda}\,\frac{401\lambda}{4}$$

$$\phi_1 - \phi_2 = \frac{401}{2}\pi$$

$$\text{Total phase diff} = \frac{401\pi}{2} + \frac{\pi}{2}$$

$$= \frac{402\pi}{2}$$

$$= 201\pi$$

$$= (\text{odd})\pi$$

$\Rightarrow$ destructive

$$I_A = (\sqrt{I_1} - \sqrt{I_2})^2.$$

Mashan 35

Utsav 10 lakhs.

diff = 25

<u>B</u>

$$\phi_2 - \phi_1 = \frac{401\pi}{2}$$

$$\text{Total phase diff} = \frac{401\pi}{2} - \frac{\pi}{2}.$$

$$= 200\pi$$

= constructive.

$$I_B = (\sqrt{I_1} + \sqrt{I_2})^2.$$

Q7)

$$K = \frac{\pi}{10} \Rightarrow \frac{2\pi}{\lambda} = \frac{\pi}{10}$$

$$\lambda = 20\text{ cm}$$

$$\frac{\lambda}{2} = 10\text{cm}$$

$$n = \frac{100\text{cm}}{\frac{\lambda}{2}}$$

n = 10 loops

K.E max : K.E at M.P = T.E

$2A = 6$

$A = 3\,cm$

segment $= \frac{1}{2}\mu A^2\omega^2\lambda.$

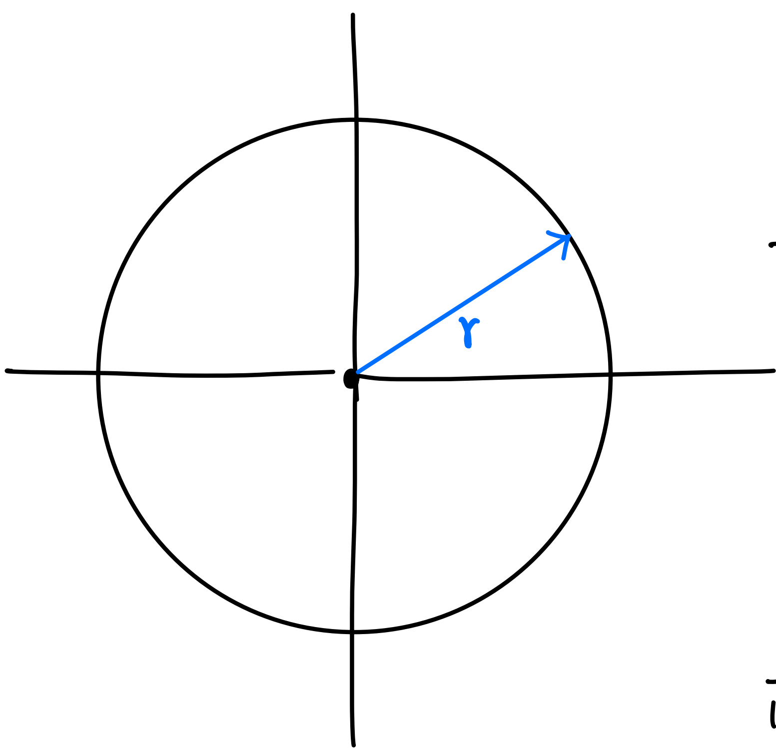
Total $= 10\left[\frac{1}{2}\mu A^2\omega^2\lambda\right].$

$= 10\left[\frac{1}{2}(\cancel{4}^{2})(3\times10^{-2})^2(100\pi)^2(20\times10^{-2})\right]$

$= 10\times2\times9\times\cancel{10^{-4}}\times\cancel{10^{8}}\times\cancel{20}\times\cancel{10^{-2}}$

$= 360.$

Q31)



$I = \frac{P}{4\pi r^2}$ — ①

$L = 10\log_{10}\left(\frac{I}{I_0}\right)$

$120 = 10\log_{10}\left[\frac{I}{10^{-12}}\right]$

$\frac{I}{10^{-12}} = 10^{12}$

$I = 1$

$\frac{P}{4\pi r^2} = 1 \Rightarrow \frac{2}{4\pi} = r^2$

$r = \sqrt{\frac{2}{4\pi}}$.

$r = \sqrt{\frac{1}{2\pi}}\ m$

$r = \sqrt{\frac{1}{2\pi}} \times 100cm$

$r = \sqrt{\frac{100}{2\pi}}\ (10cm)$

$r \approx 40cm$.

Ex-4b

Q3) at $t=0$

$y = \frac{1}{1+x^2}$

at $t=2$

$y = \frac{1}{1+(x-1)^2}$

general

$y = \frac{1}{1+(ax-bt)^2}$

$a = 1$.

$b = \frac{1}{2}$.

wave speed $= \frac{|\text{coe. of } t|}{|\text{coe. of } x|} = \frac{\frac{1}{2}}{1} = \frac{1}{2}$.

Q13)

$V = \frac{1000}{3}$. at $0°C$

$V = \sqrt{\frac{\gamma RT}{M_0}}$.

$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}} \Rightarrow \frac{\frac{1000}{3}}{336} = \sqrt{\frac{273}{T}}$.

Q11)

$f = \frac{V}{4l} = 1.5\,KHZ$

$= 1500\,Hz$.

$3\left(\frac{V}{4l}\right), \; 5\left(\frac{V}{4l}\right), \;$ | odd $\left(\frac{V}{4l}\right) = 20{,}000$

odd $= \frac{\cancel{20{,}000}\;40}{\cancel{1500}\;3}$

odd $= 13.33$

odd $= 13$

3, 5, 7, 9, 11, 13.

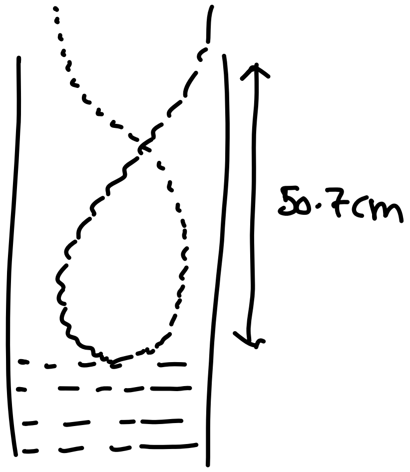
Q20)

$83.9 - 50.7 = \frac{\lambda}{2}$.

$33.2 = \frac{\lambda}{2} \Rightarrow \lambda = 66.4\,cm$

$\frac{\lambda}{4} = \frac{66.4}{4}$

$= 16.6\,cm$.



Beats :-

$$y_1 = A\sin(\omega_1 t - k_1 x)$$

$$y_2 = A\sin(\omega_2 t - k_2 x).$$

from principle of superposition

$$y_r = y_1 + y_2.$$

lets take at $x = 0$

$$y_r = A\sin\omega_1 t + A\sin\omega_2 t$$

$$= A\left[2\sin\frac{\omega_1+\omega_2}{2}t\cos\frac{\omega_1-\omega_2}{2}t\right]$$

$$y_r = \left[2A\cos\left(\frac{\omega_1-\omega_2}{2}\right)t\right]\sin\left(\frac{\omega_1+\omega_2}{2}\right)t$$

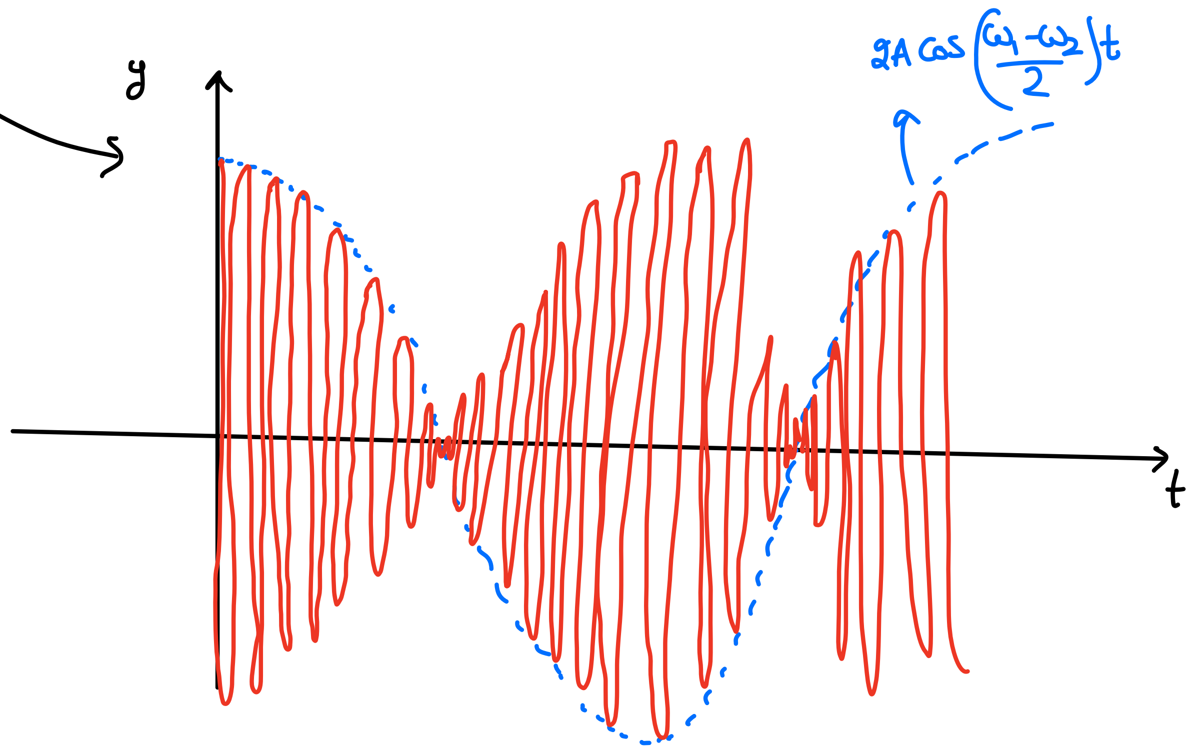
for us to easily understand lets take $\omega_1 = 200\pi$

$\omega_2 = 202\pi$

$$y_r = 2A\cos\pi t\sin 201\pi t.$$

in $t = 2\,sec$

$$y_r = 2A\cos\underbrace{2\pi}_{1\text{ cycle}}\ \sin\underbrace{402\pi}_{201\text{ cycles.}}$$



Time period between two maximas:-

$$A_r = 2A\cos\left(\frac{\omega_1-\omega_2}{2}\right)t$$

$$\pm 2A = 2A\cos\frac{\omega_1-\omega_2}{2}t$$

$$\cos\frac{\omega_1-\omega_2}{2}t = \pm 1.$$

$$\frac{\omega_1-\omega_2}{2}t = 0,\ \pi,\ 2\pi,\ 3\pi\ \ldots$$

$$t = 0,\ \frac{2\pi}{\omega_1-\omega_2},\ \frac{4\pi}{\omega_1-\omega_2}\ \ldots\ldots$$

$$T = \frac{2\pi}{\omega_1-\omega_2}.$$

$$f_{beats} = \frac{1}{T} = \frac{\omega_1 - \omega_2}{2\pi}$$

$$f_{beats} = f_1 - f_2.$$

$\Rightarrow$ maxima and minima together is taken as a beat.

What if we have three sources ??

at $x=0$

$$y_1 = A \sin 2\pi f t$$

$$y_2 = A \sin 2\pi (f-1) t$$

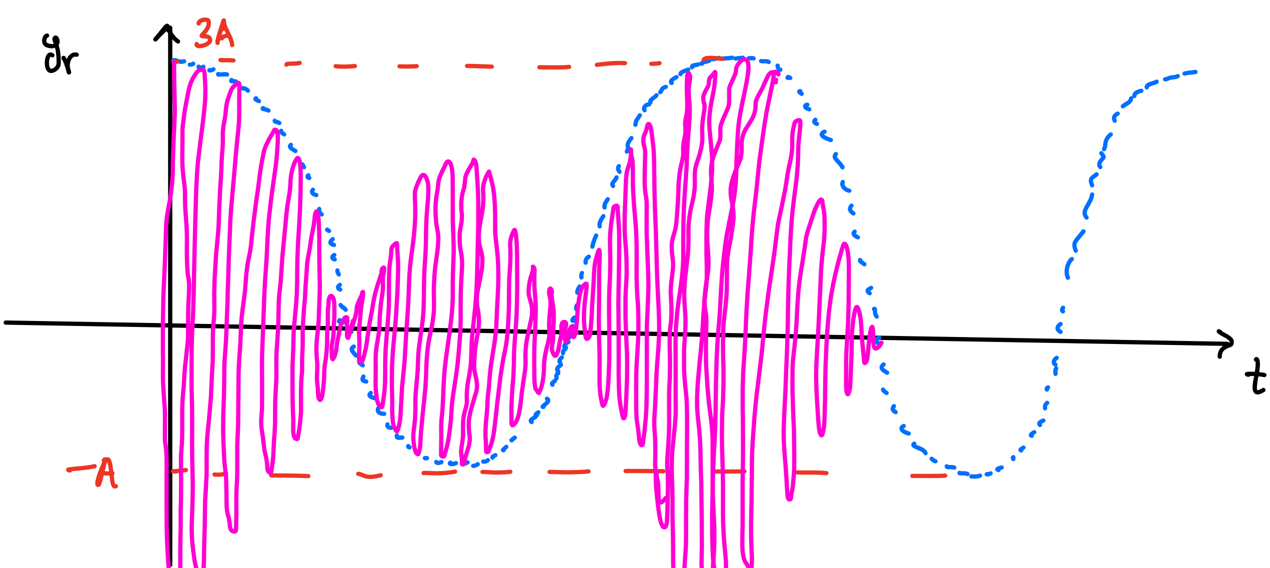
$$y_3 = A \sin 2\pi (f+1) t.$$

$$y_r = y_1 + y_2 + y_3$$

$$= A \sin 2\pi f t + A \sin(2\pi (f-1) t) + A \sin 2\pi (f+1) t.$$

$$= A \sin 2\pi f t + A [2 \sin 2\pi f t \cos 2\pi t].$$

$$y_r = [A + 2A \cos 2\pi t] \sin 2\pi f t.$$



$$A_r = A + 2A\cos 2\pi t.$$

$$3A = A + 2A\cos 2\pi t \Rightarrow \cos 2\pi t = 1$$

$$2\pi t = 0, 2\pi, \ldots$$

$$t = 0, 1, 2, \ldots$$

$$-A = A + 2A\cos 2\pi t$$

$$\cos 2\pi t = -1$$

$$2\pi t = \pi, 3\pi, 5\pi \ldots$$

$$t = \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \ldots$$

time between two maximas $= \frac{1}{2}$ sec

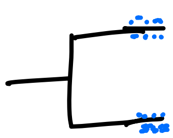
$$f = 2$$

for different combinations

No. of beats

$= f_{highest} - f_{lowes}$

waxing of tuning fork:-



$\Rightarrow$ waxing decreases the freq. of tuning fork by small fraction.

filing of tuning fork:- we take sand paper and rub the prongs of tuning fork. as mass decreases freq. increases.

ill:15, BB:6 Q2:-

ill:15

filing

158 → 155 ↑

158 → 161. ↑

if 155

after filing freq ↑

lets say

$158 \xrightarrow{7\text{ beats.}} 165$

if 161

filing ↑ freq

$158 \xrightarrow{7\text{ beats}} 165$

BB:6

Q2)

A

- $256 \xrightarrow{12} \cancel{268}$
- $256 \xrightarrow{6} 250$ ✓
- $256 \xrightarrow{4} 260$ ✗
- $256 \xrightarrow{2} 258$ ✓

B

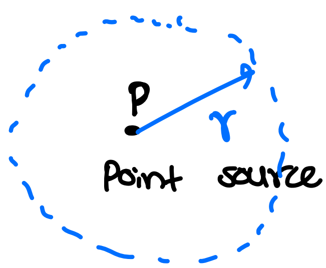
- $262 \xrightarrow{6} \cancel{268}$
- $262 \xrightarrow{12} 250$ ✓
- $262 \xrightarrow{2} 260$ ✗
- $262 \xrightarrow{4} 258$ ✓

---

doppler effect:- apparent change in frequency of sound when there is relative motion between source and observer.

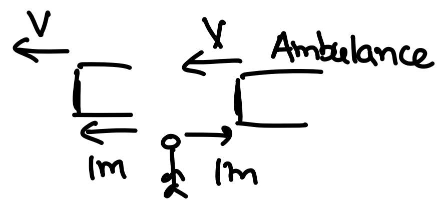
Before we go into doppler effect lets understand some basics :-

intensity:- Power per unit area.



$$I = \frac{P}{4\pi r^2}$$

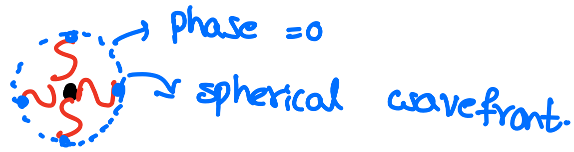
$$I \propto \frac{1}{r^2} \Rightarrow A \propto \frac{1}{r}$$



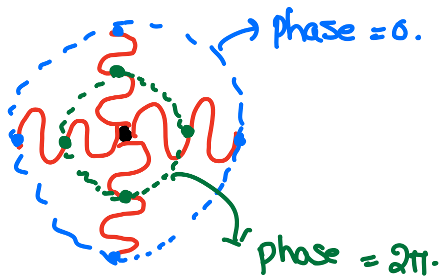
although the ambulance is at same distance from observer he doesn't get to listen to same freq. of sound and intensity

Wavefront :- it is locus of all the particles having same phase.

at $t=T$



at $t=2T$



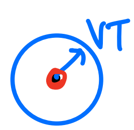
radial distance between two wavefronts having a phase diff of 2π is wavelength.

lets see wave propagation in terms of wavfronts :-

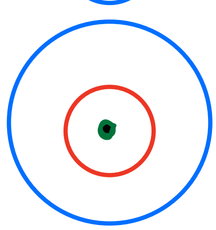
(i) source at rest

at $t=0$

at $t=T$



at $t=2T$



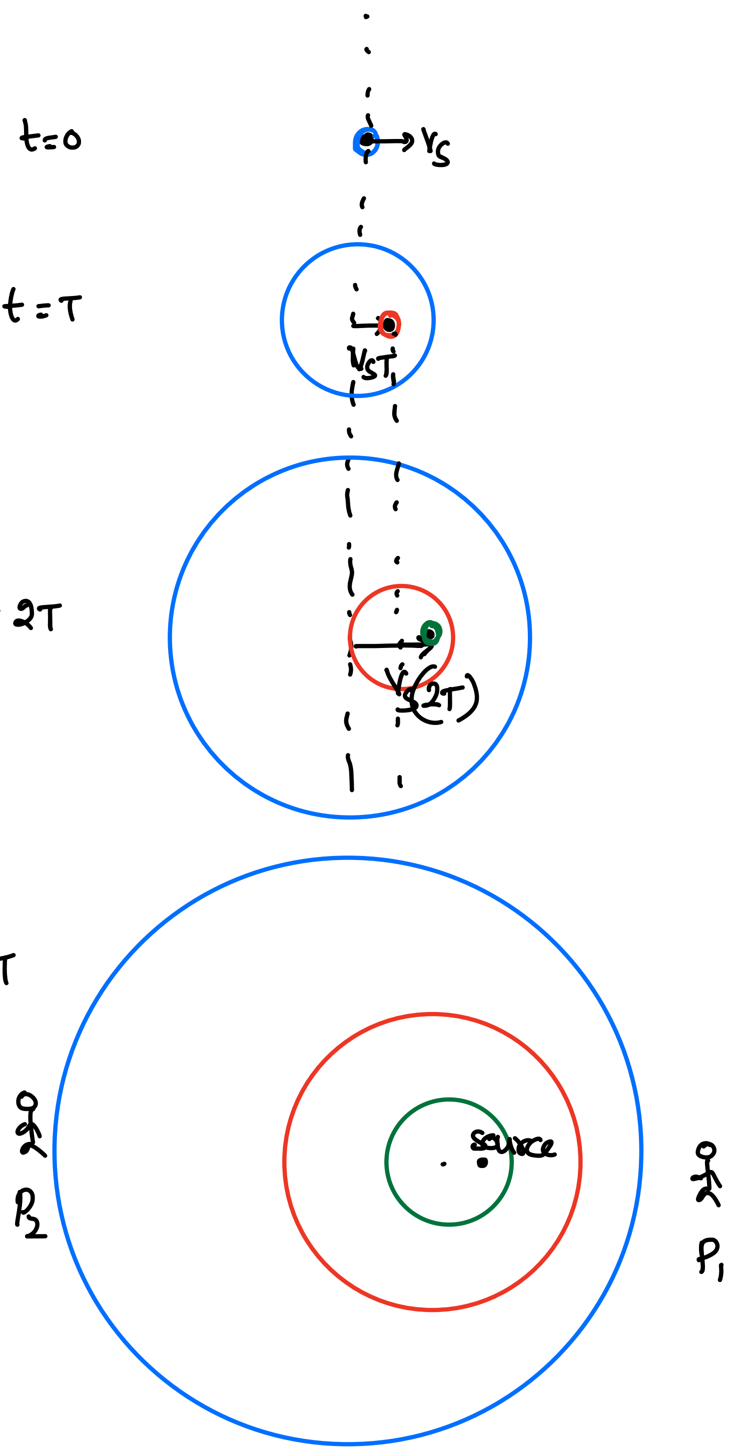
(ii) source in motion.

at $t=0$

at $t=T$

at $t=2T$

at $t=3T$



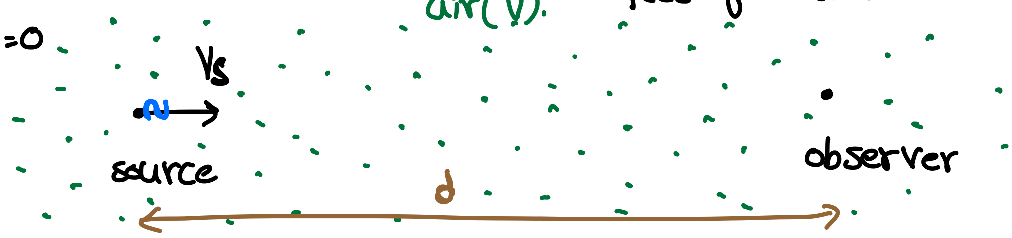
P_1 feels wavelength got decreased

P_2 feels wavelength got increased.

(i) source moving towards a stationary observer:-

speed of sound in still air(V).

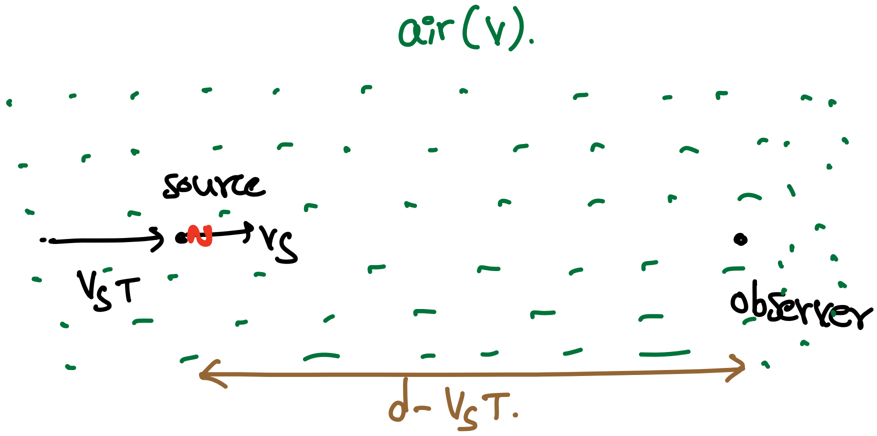
at $t=0$



time after which observer gets to hear 1st wave

$$t_1 = \frac{d}{V}.$$

at $t = T$



$$t_2 = T + \frac{d - V_s T}{V}.$$

$$T_{app} = t_2 - t_1$$

$$= T + \cancel{\frac{d}{V}} - \frac{V_s T}{V} - \cancel{\frac{d}{V}}$$

$$T_{app} = \left(\frac{V - V_s}{V}\right) T.$$

$$\boxed{f_{app} = \left(\frac{V}{V - V_s}\right) f.}$$ → original freq

---

$$f_{app} = \frac{\text{app. wave speed}}{\text{app. wavelength.}}$$ → nothing but wave speed w.r.t observer.

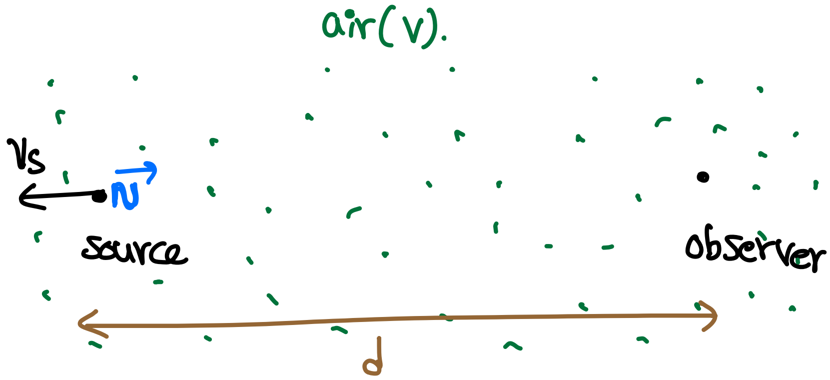
$$\left(\frac{\cancel{v}}{v - v_s}\right) f = \frac{\cancel{v}}{\lambda_{app}} \Rightarrow \lambda_{app} = \frac{v - v_s}{f}$$

$$\lambda_{app} = \frac{v - v_s}{\left(\frac{v}{\lambda}\right)}$$

$$\boxed{\lambda_{app} = \left(\frac{V - V_s}{V}\right)\lambda.}$$

source moving away from stationary observer:-

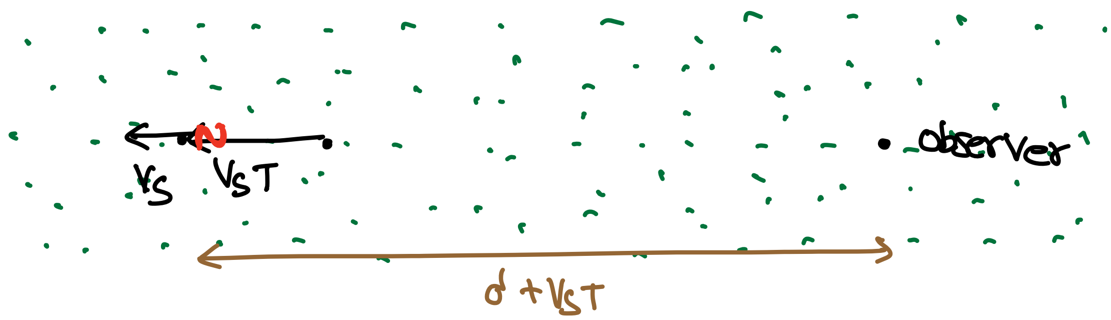
at $t=0$



time after which observer gets to hear 1st wave

$$t_1 = \frac{d}{V}.$$

at $t = T$



time after which 2nd wave reaches observer

$$t_2 = \frac{d + V_s T}{V} + T.$$

$$T_{app} = t_2 - t_1$$

$$T_{app} = \left(\frac{V + V_s}{V}\right)T$$

$$f_{app} = \left(\frac{V}{V + V_s}\right)f.$$

$$f_{app} = \frac{V_{app}}{\lambda_{app}}.$$

$$\frac{Vf}{(V + V_s)} = \frac{V}{\lambda_{app}}.$$

$$\lambda_{app} = \left(\frac{V + V_s}{V}\right)\lambda.$$

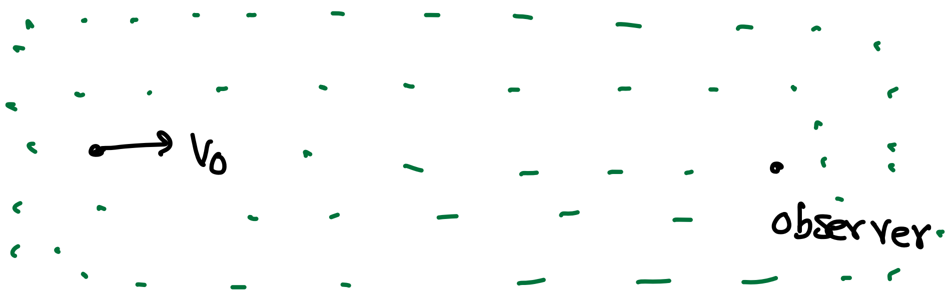
(iii) observer moving towards a stationary source:-

at $t=0$

air (v). → speed of sound in still air.

w.r.t observer

air $\rightarrow V_o$.



$$\boxed{f_{app} = \frac{V}{V-V_s} f}$$

V → speed of sound in air

→ taken from above situation of source moving towards obs.

$$f_{app} = \frac{(V+V_o)}{(V+V_o)-V_o} f$$

$$f_{app} = \left(\frac{V+V_o}{V}\right) f$$

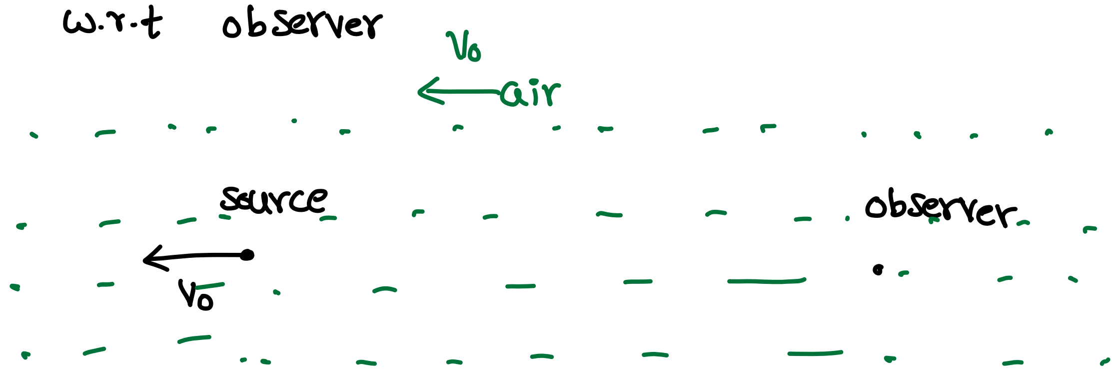
$$\frac{\text{app. wave speed}}{\text{app. wavelength}} = \left(\frac{V+V_o}{V}\right) f.$$

$$\frac{\cancel{V+V_o}}{\lambda_{app}} = \left(\frac{\cancel{V+V_o}}{\cancel{V}}\right)\left(\frac{\cancel{V}}{\lambda}\right)$$

$$\lambda_{app} = \lambda.$$

(iv) observer moving away from stationary source :-

w.r.t observer



$$f_{app} = \frac{V}{V+V_s} f$$

$$f_{app} = \frac{(V-V_o)}{(V-V_o)+V_o} f$$

$$f_{app} = \left(\frac{V-V_o}{V}\right) f.$$

General formula

Net speed of sound in air

$$f_{app} = \left[\frac{(V \pm V_M) \pm V_o}{(V \pm V_M) \pm V_s}\right] f.$$

V, V_M, V_o, V_s are all the speeds along the line joining observer and source eg :-

1) $\bullet\!\longrightarrow v_s \qquad\qquad \underset{v_o}{\longleftarrow\!\bullet}$

$$f_{app} = \frac{(v+v_o)}{v-v_s} f.$$

2) $\bullet\!\longrightarrow v_s \qquad\qquad \bullet\!\longrightarrow v_o$

$$f_{app} = \frac{v-v_o}{v-v_s} f$$

3) $\underset{v_s}{\longleftarrow\!\bullet} \qquad\qquad \bullet\!\rightarrow v_o$

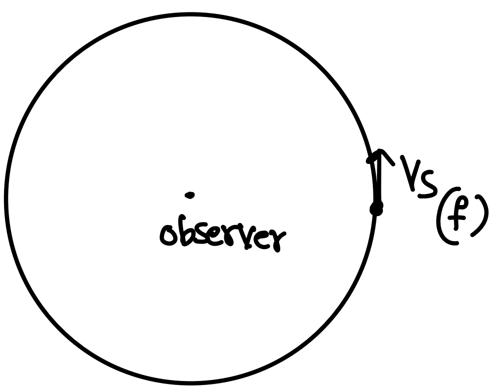
$$f_{app} = \frac{v-v_o}{v+v_s} f.$$

4) $\underset{v_s}{\longleftarrow\!\bullet} \qquad\qquad \underset{v_o}{\longleftarrow\!\bullet}$

$$f_{app} = \left(\frac{v+v_o}{v+v_s}\right) f$$

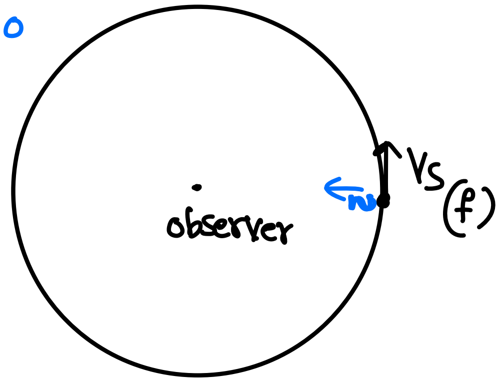
---

eg :-



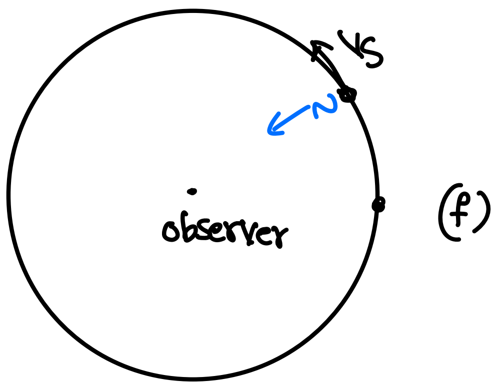
find freq. heard by observer ?

at $t=0$



$t_1 = \frac{R}{v}$.

at $t = T$



$t_2 = T + \frac{R}{v}$.

$T_{app} = t_2 - t_1$

$= T$.

$f_{app} = f$.

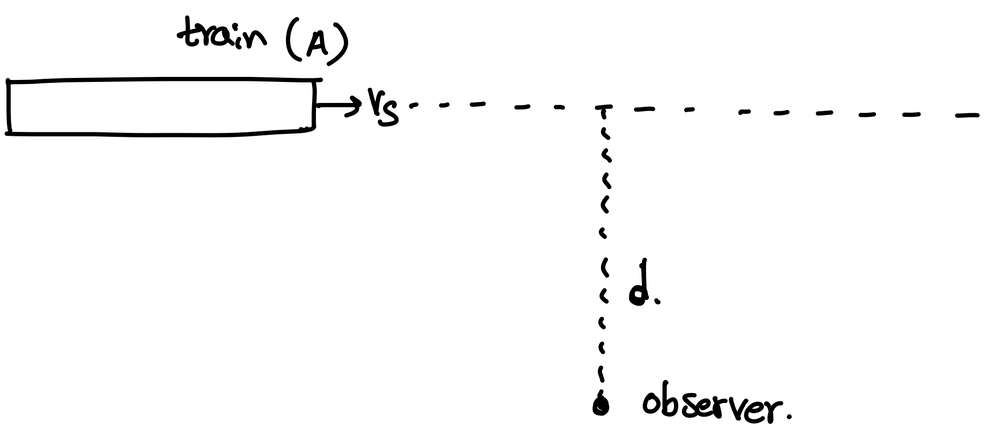
using formula.

$$f_{app} = \left(\frac{v \pm v_0}{v \pm v_s}\right) f.$$

$v_s \rightarrow$ vel. of source along line OS.

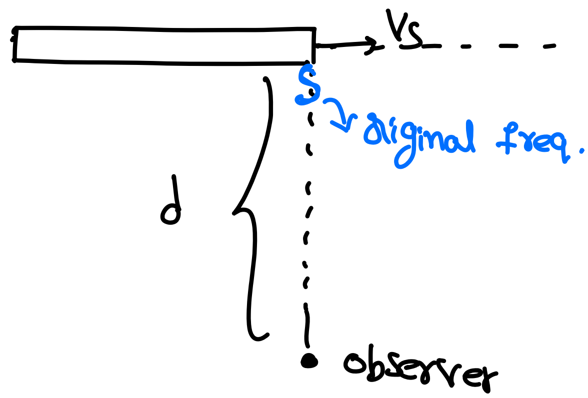
$$= \left(\frac{v \pm 0}{v \pm 0}\right) f \implies f_{app} = f.$$

Q)

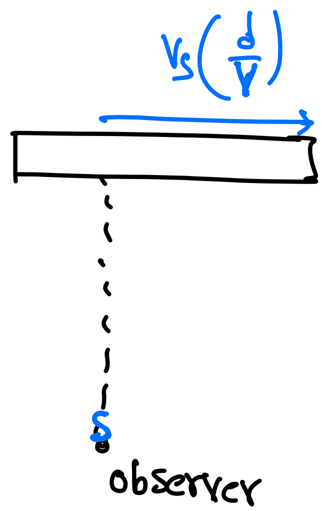


find the location of A when the observer gets to hear original frequency ? (take V_s comparable to V).

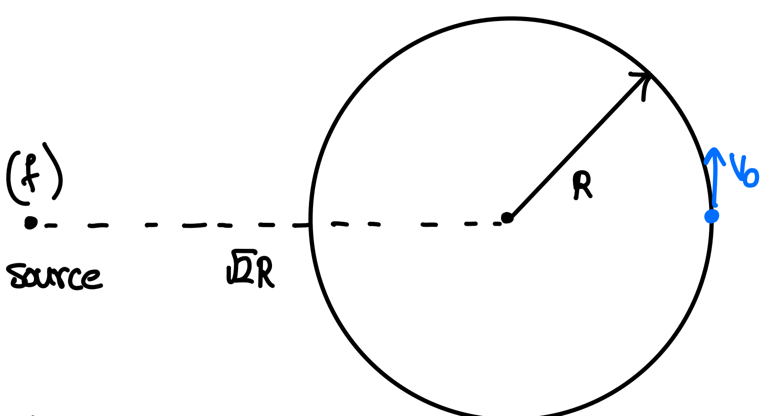
Sol :-



time taken by it to reach observer = $\frac{d}{V}$.



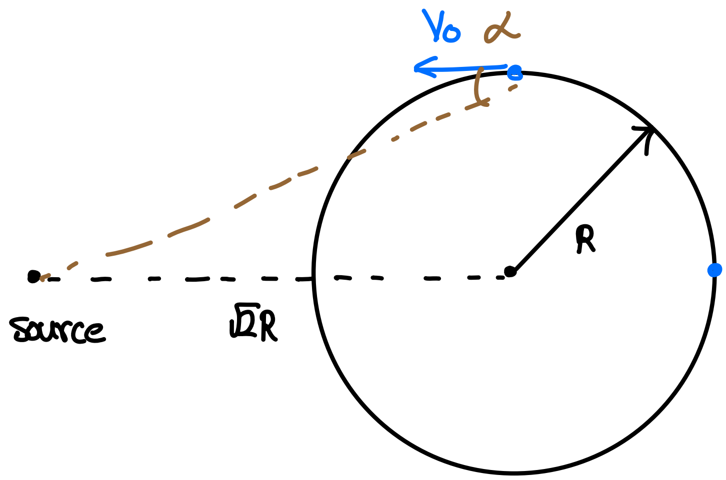
Q)



take $V \ggg V_0$

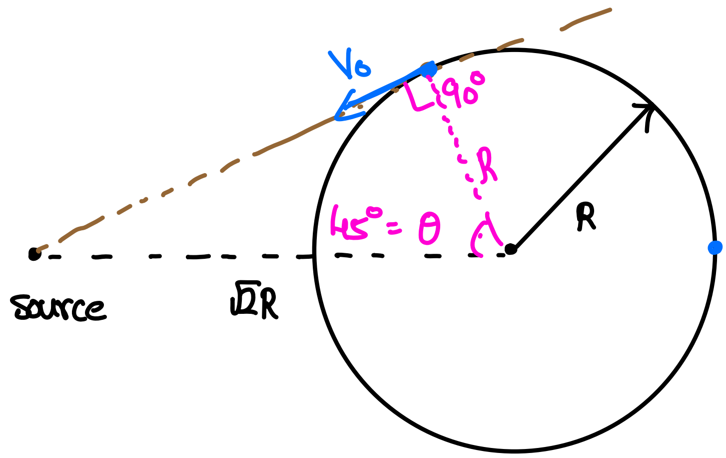
(i) find the max and min frequency heard by observer and their respective locations ?

sol :-



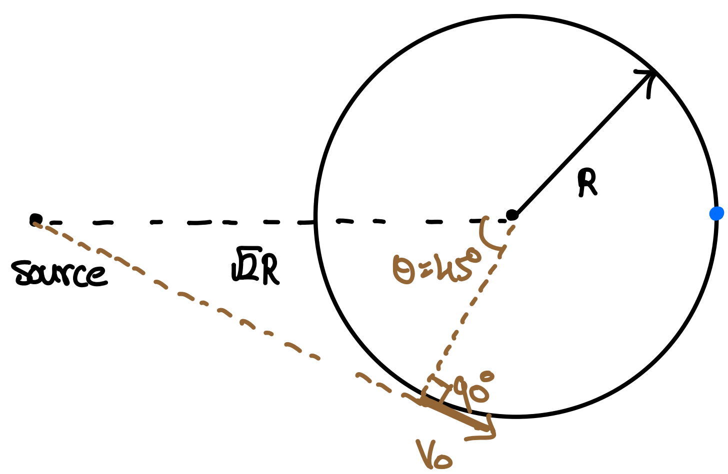
$$f_{app} = \left(\frac{V + V_o\cos\alpha}{V}\right) f.$$

max :-



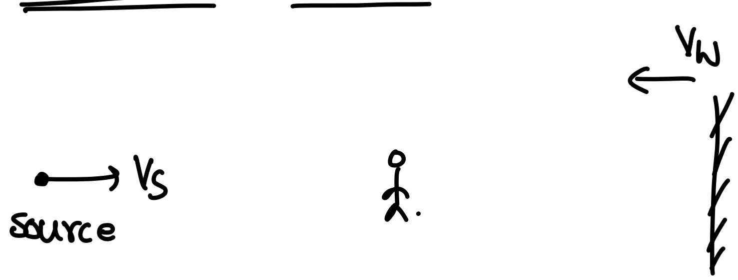
$$f_{app} = \left(\frac{V + V_o}{V}\right) f.$$

min :-



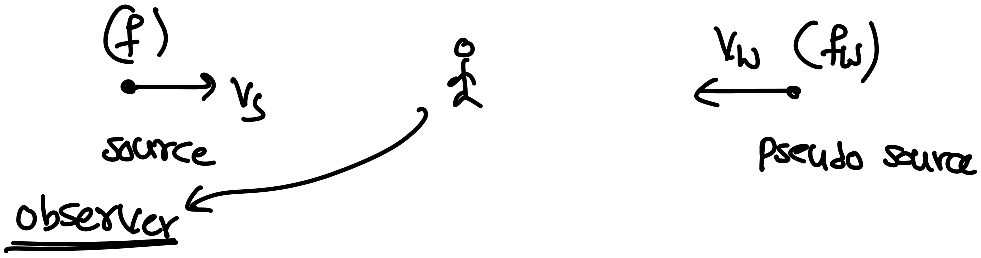
$$f_{app} = \left(\frac{V - V_o}{V}\right) f.$$

Doppler effect due to reflection :-



⇒ Wall will be treated as a pseudo source of freq. equal to apparant freq. observed by wall from source.

$$f_W = \left(\frac{V+V_W}{V-V_S}\right) f.$$



$$f_{source} = \left(\frac{V}{V-V_S}\right) f$$

$$f_{pseudo\ source} = \frac{V}{V-V_W} f_W$$

$$= \frac{V(V+V_W)}{(V-V_W)(V-V_S)} f.$$

$$f_{beats} = \frac{V}{V-V_S} f \left[\frac{V+V_W}{V-V_W} - 1\right]$$

$$f_{beats} = \frac{V}{V-V_S} f \left[\frac{2V_W}{V-V_W}\right].$$

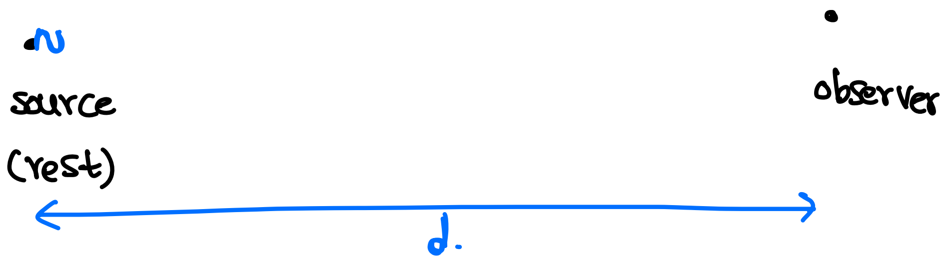
---

accelerating source:-

source starts from rest and moves towards observer with constant acceleration "a". find the app. freq for the 1st wave?

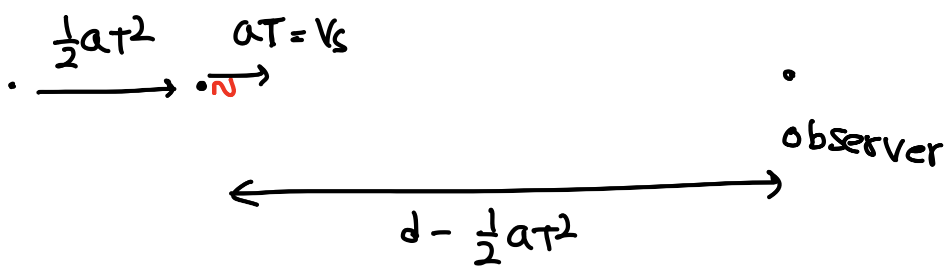
sol :-

at t=0



$$t_1 = \frac{d}{V}.$$

at $t = T$



$$t_2 = T + \frac{d - \frac{1}{2}aT^2}{V}.$$

$$t_2 = T + \frac{d}{V} - \frac{aT^2}{2V}.$$

$$T_{app} = t_2 - t_1.$$

$$= T - \frac{aT^2}{2V}.$$

$$T_{app} = \frac{2VT - aT^2}{2V} \Rightarrow T_{app} = T^2\left[\frac{\frac{2v}{T} - a}{2v}\right]$$

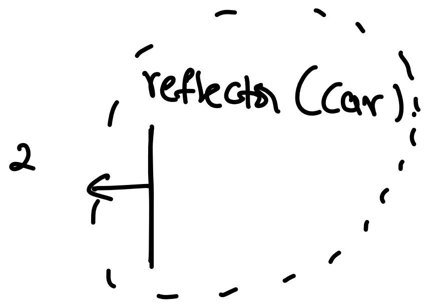
$$f_{app} = \left[\frac{2v}{2vf - a}\right] f^2.$$

$$f_{app} = \frac{2vf^2}{2vf - a}.$$

Ex-5

Q1(B) $f_0 = 492\ Hz.$

Source (rest)



$$f_{car} = \left(\frac{V+2}{V}\right) f_0.$$

Source

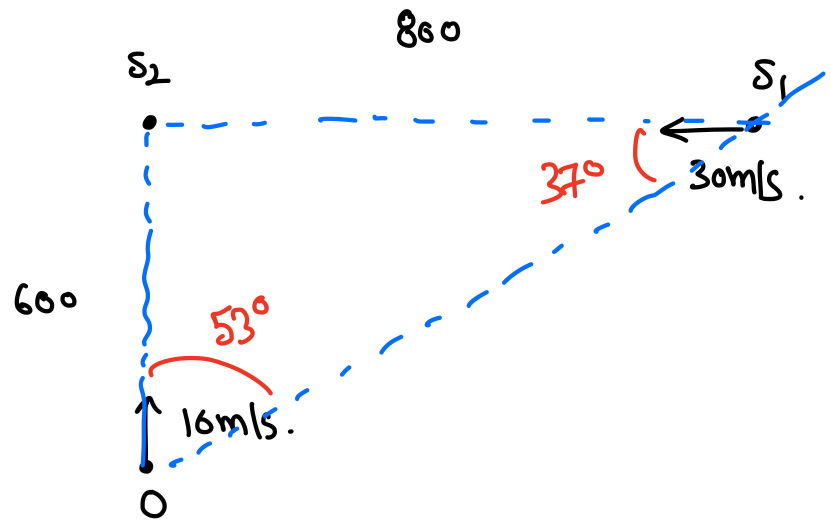
$\xleftarrow{2\,m/s}\bullet$ Pseudo source (f_{car})

$$f_{app} = \left(\frac{V}{V-2}\right) f_{car}. = \left(\frac{V+2}{V-2}\right) f_0.$$

$$f_{beats} = \left(\frac{V+2}{V-2}\right) f_0 - f_0$$

$$= \frac{4}{(V-2)} f_0 = \frac{\cancel{4} \times \cancel{492}\,6}{\cancel{328}\,\cancel{82}} = \underline{\underline{6}}.$$

Q21)



S_1O :-

$$f_{app} = \left(\frac{V + 10\cos 53^\circ}{V - 30\cos 37^\circ}\right) f_0$$

S_2O

$$f_{app} = \left(\frac{V+10}{V}\right) f_0.$$

3. If separation between a source and a detector changes due to relative motion between them, the detector detects sound of pitch different from that emitted by the source. This observed change in pitch is known as Doppler's shift and the effect is known as Doppler's effect. Under the conditions when Doppler's shift is observed, intensity of the sound also changes. In this task, we study change in intensity of sound detected by a detector due to relative motion between a source and a detector.

A point source is emitting sound isotropically at constant power P and the sound travels with velocity c relative to the air.

(a) A detector is moving with a constant velocity v_d towards a stationary source in still air. Find suitable expression for intensity I of sound received by the detector at a distance r from the source.

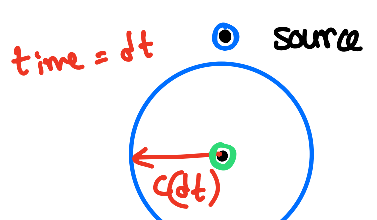
(b) A point source is moving with a constant velocity v_s towards a stationary detector in still air. Find suitable expression for intensity I of sound received by the detector at a distance r from the source.

(c) A point source is moving in still air with a constant velocity v_s towards a detector that is moving with a constant velocity v_d towards the source. Find suitable expression for intensity I of sound received by the detector at a distance r from the source.

Sol :-

a) Microscopic :-

t=0



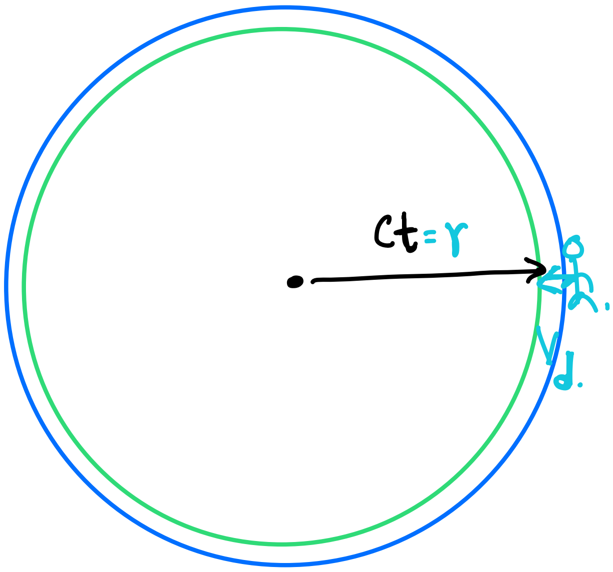
c(dt) length the amount energy

= (P)dt.

Macroscopic :-

$t=0$

at time $= t$



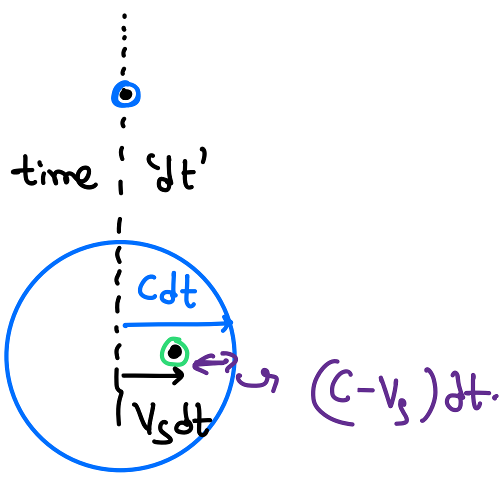
$$I = \frac{\left(\frac{P(\cancel{dt})}{4\pi r^2}\right)}{\frac{(c\,\cancel{dt})}{(c+v_d)}}.$$

$$= \frac{P}{4\pi r^2}\left[1+\frac{v_d}{c}\right].$$

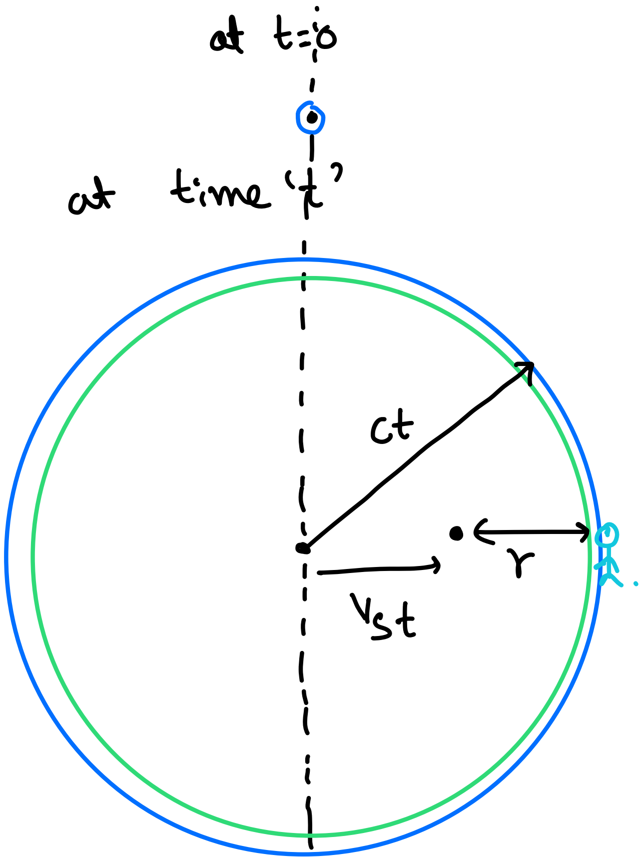
b)

Microscopic :-

$t=0$



Macroscopic:-



$r = ct - v_s t = (c - v_s)t.$

$$I = \frac{\left(\frac{P\,dt}{4\pi(ct)^2}\right)}{\text{time}}$$

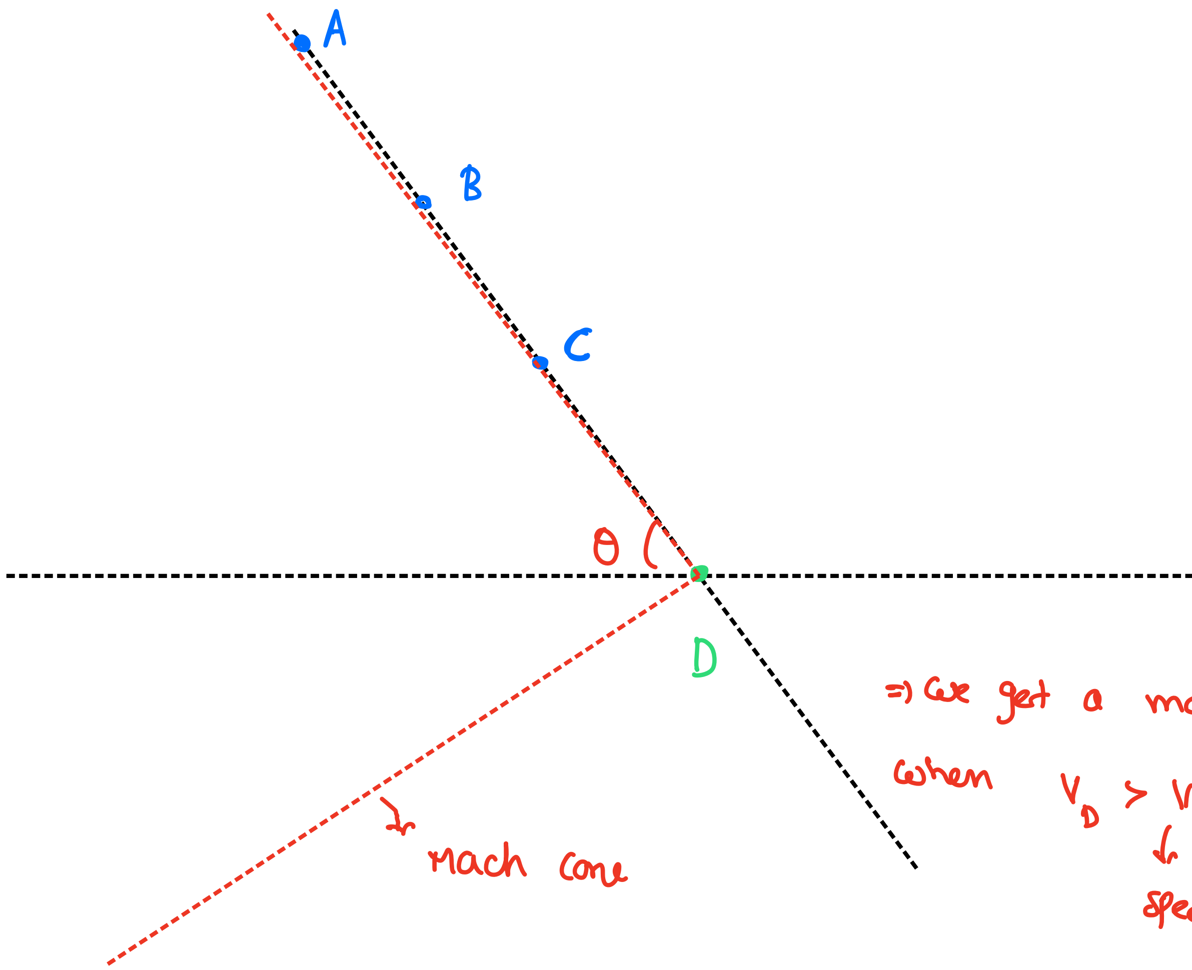
$$I = \frac{\frac{P\,\cancel{dt}}{4\pi(ct)^2}}{\frac{(c-v_s)\cancel{dt}}{c}}$$

$$I = \frac{P}{4\pi(ct)^2}\,\frac{c}{(c-v_s)} \times \frac{c-v_s}{(c-v_s)}.$$

$$= \frac{P}{4\pi c}\,\frac{(c-v_s)}{\left[(c-v_s)t\right]^2} = \frac{P}{4\pi r^2}\left(1 - \frac{v_s}{c}\right)$$

A detector receding from a stationary sound source with a speed that increases continuously without a limit. Which of the following statements correctly describes frequency of the sound detected by the

Sol :-



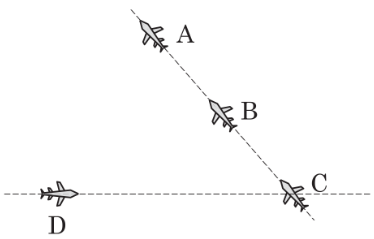
⇒ We get a mach cone when $V_D > V$

V ↓ speed of sound.

detector? Assume that sound waves propagate indefinitely without attenuation and the detector does not create any air drag.

(a) It first decreases, becomes zero and then increases.
(b) It continuously decreases and eventually no sound is detected.
(c) It continuously decreases and eventually acquires a small value.
(d) It first decreases, becomes zero and then increases and eventually no sound is detected.

10. Three aircrafts A, B and C are flying in a line with the same speed. Another aircraft D is flying on another straight line making an acute angle with the line of motion of the aircrafts A, B and C as shown. If pilots of these aircrafts hear sound of the aircraft D simultaneously, what can you conclude about speed of the aircraft D?



(a) It is possible only when the aircraft D is moving at speed of sound.
(b) It is possible only when the aircraft D is moving at speed lower than speed of sound.
(c) It is possible only when the aircraft D is moving at speed higher than speed of sound.
(d) The pilots of aircrafts A, B and C cannot hear sound of the aircraft D simultaneously.