

# Advanced Policy Gradient Methods: Natural Gradient, TRPO, and More

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# Two Limitations of “Vanilla” Policy Gradient Methods



- ▶ Hard to choose stepsizes
  - ▶ Input data is nonstationary due to changing policy: observation and reward distributions change
  - ▶ Bad step is more damaging than in supervised learning, since it affects visitation distribution
    - ▶ Step too far  $\rightarrow$  bad policy
    - ▶ Next batch: collected under bad policy
    - ▶ Can't recover—collapse in performance
- ▶ Sample efficiency
  - ▶ Only one gradient step per environment sample
  - ▶ Dependent on scaling of coordinates

# Reducing reinforcement learning to optimization

- ▶ Much of modern ML: reduce learning to numerical optimization problem
  - ▶ Supervised learning: minimize training error
- ▶ RL: how to *use all data so far and compute the best policy?*
  - ▶ Q-learning: can (in principle) include all transitions seen so far, however, we're optimizing the wrong objective
  - ▶ Policy gradient methods: yes stochastic gradients, but **no optimization problem**\*
  - ▶ This lecture: write down an optimization problem that allows you to do a small update to policy  $\pi$  based on data sampled from  $\pi$  (**on-policy** data)



# OpenAI Baselines

<https://blog.openai.com/openai-baselines-dqn/>

<https://github.com/openai/baselines>

-A2C

-ACER

-ACKTR

-DDPG

-DQN

-PPO1 (Multi-CPU using MPI)

-PPO2 (Optimized for GPU)

-TRPO

# What Loss to Optimize?

- Policy gradients

$$\hat{g} = \hat{\mathbb{E}}_t \left[ \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) \hat{A}_t \right]$$

- Can differentiate the following loss

$$L^{PG}(\theta) = \hat{\mathbb{E}}_t \left[ \log \pi_{\theta}(a_t | s_t) \hat{A}_t \right].$$

but don't want to optimize it too far

- Equivalently differentiate

$$L_{\theta_{\text{old}}}^{IS}(\theta) = \hat{\mathbb{E}}_t \left[ \frac{\pi_{\theta}(a_t | s_t)}{\pi_{\theta_{\text{old}}}(a_t | s_t)} \hat{A}_t \right].$$

at  $\theta = \theta_{\text{old}}$ , state-actions are sampled using  $\theta_{\text{old}}$ . (Is = importance sampling)

Just the chain rule:  $\nabla_{\theta} \log f(\theta) \big|_{\theta_{\text{old}}} = \frac{\nabla_{\theta} f(\theta) \big|_{\theta_{\text{old}}}}{f(\theta_{\text{old}})} = \nabla_{\theta} \left( \frac{f(\theta)}{f(\theta_{\text{old}})} \right) \big|_{\theta_{\text{old}}}$

# Surrogate Loss: Importance Sampling Interpretation

- Importance sampling interpretation

$$\begin{aligned} & \mathbb{E}_{s_t \sim \pi_{\theta_{\text{old}}}, a_t \sim \pi_{\theta}} [A^{\pi}(s_t, a_t)] \\ &= \mathbb{E}_{s_t \sim \pi_{\theta_{\text{old}}}, a_t \sim \pi_{\theta_{\text{old}}}} \left[ \frac{\pi_{\theta}(a_t | s_t)}{\pi_{\theta_{\text{old}}}(a_t | s_t)} A^{\pi_{\theta_{\text{old}}}}(s_t, a_t) \right] \quad (\text{importance sampling}) \\ &= \mathbb{E}_{s_t \sim \pi_{\theta_{\text{old}}}, a_t \sim \pi_{\theta_{\text{old}}}} \left[ \frac{\pi_{\theta}(a_t | s_t)}{\pi_{\theta_{\text{old}}}(a_t | s_t)} \hat{A}_t \right] \quad (\text{replace } A^{\pi} \text{ with estimator}) \\ &= L_{\theta_{\text{old}}}^{\text{IS}}(\theta) \end{aligned}$$

- Kakade et al.<sup>1</sup> and S. et al.<sup>2</sup> analyze how  $L^{\text{IS}}$  approximates the actual performance difference between  $\theta$  and  $\theta_{\text{old}}$ .
- In practice,  $L^{\text{IS}}$  is not much different than the logprob version  $L^{\text{PG}}(\theta) = \hat{\mathbb{E}}_t \left[ \log \pi_{\theta}(a_t | s_t) \hat{A}_t \right]$ , for reasonably small policy changes.

<sup>1</sup>S. Kakade and J. Langford. "Approximately optimal approximate reinforcement learning". *ICML*. 2002.

<sup>2</sup>J. Schulman, S. Levine, P. Moritz, M. I. Jordan, and P. Abbeel. "Trust Region Policy Optimization". *ICML (2015)*.

# Simple Monte Carlo

- Statistical sampling can be applied to any expectation
- In general we can find the expectation of  $f(x)$  by sampling:

$$\int f(x)P(x)dx \approx \frac{1}{S} \sum_{s=1}^S f(x^{(s)}) \quad x^{(s)} \sim P(x)$$

- The function  $f(x)$  is arbitrary, so this is very general
- Example: making predictions

$$\begin{aligned} P(x|\mathcal{D}) &= \int P(x|\theta, \mathcal{D})P(\theta|\mathcal{D})d\theta \\ &\approx \frac{1}{S} \sum_{s=1}^S P(x|\theta^{(s)}, \mathcal{D}) \quad \theta^{(s)} \sim P(\theta|\mathcal{D}) \end{aligned}$$

# Properties of Monte Carlo

- Estimator:

$$\int f(x)P(x)dx \approx \frac{1}{S} \sum_{s=1}^S f(x^{(s)}) \quad x^{(s)} \sim P(x)$$

- Estimator is unbiased

$$\mathbb{E}_{P(x^{(s)})}[\hat{f}] = \frac{1}{S} \sum_{s=1}^S \mathbb{E}_{P(x)}[f(x)] = \mathbb{E}_{P(x)}[f(x)]$$

- Variance shrinks  $\propto 1/S$  :

$$\text{var}_{P(x^{(s)})}[\hat{f}] = \frac{1}{S^2} \sum_{s=1}^S \text{var}_{P(x)}[f(x)] = \text{var}_{P(x)}[f(x)]/S$$



# Importance sampling

Computing both  $\tilde{P}(x), \tilde{Q}(x)$  then throwing  $x$  away seems wasteful  
Instead rewrite the integral as an expectation under  $Q$ :

$$\begin{aligned}\int f(x)P(x)dx &= \int f(x)\frac{P(x)}{Q(x)}Q(x)dx \\ &\approx \frac{1}{S} \sum_{s=1}^S f(x^{(s)}) \frac{P(x^{(s)})}{Q(x^{(s)})}\end{aligned}$$

This is just simple Monte Carlo again, so it is unbiased

$$r^{(s)} = \frac{P(x^{(s)})}{Q(x^{(s)})} \text{ is called the *importance weight*}$$

Importance sampling also applies when integral is not an expectation

Divide and multiply any integrand by a convenient distribution

# Trust Region Policy Optimization

- Define the following trust region update:

$$\begin{aligned} & \underset{\theta}{\text{maximize}} && \hat{\mathbb{E}}_t \left[ \frac{\pi_{\theta}(a_t | s_t)}{\pi_{\theta_{\text{old}}}(a_t | s_t)} \hat{A}_t \right] \\ & \text{subject to} && \hat{\mathbb{E}}_t [\text{KL}[\pi_{\theta_{\text{old}}}(\cdot | s_t), \pi_{\theta}(\cdot | s_t)]] \leq \delta. \end{aligned}$$

- Also worth considering using a penalty instead of a constraint

$$\underset{\theta}{\text{maximize}} \quad \hat{\mathbb{E}}_t \left[ \frac{\pi_{\theta}(a_t | s_t)}{\pi_{\theta_{\text{old}}}(a_t | s_t)} \hat{A}_t \right] - \beta \hat{\mathbb{E}}_t [\text{KL}[\pi_{\theta_{\text{old}}}(\cdot | s_t), \pi_{\theta}(\cdot | s_t)]]$$

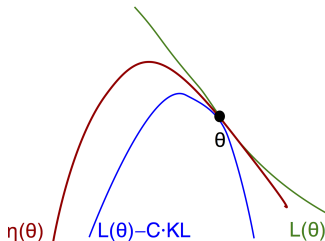
- Method of **Lagrange multipliers**: optimality point of  $\delta$ -constrained problem is also an optimality point of  $\beta$ -penalized problem for some  $\beta$ .
- In practice,  $\delta$  is easier to tune, and fixed  $\delta$  is better than fixed  $\beta$

# Monotonic Improvement Result

- ▶ Consider KL penalized objective

$$\underset{\theta}{\text{maximize}} \quad \hat{\mathbb{E}}_t \left[ \frac{\pi_{\theta}(a_t | s_t)}{\pi_{\theta_{\text{old}}}(a_t | s_t)} \hat{A}_t \right] - \beta \hat{\mathbb{E}}_t [\text{KL}[\pi_{\theta_{\text{old}}}(\cdot | s_t), \pi_{\theta}(\cdot | s_t)]]$$

- ▶ Theory result: if we use max KL instead of mean KL in penalty, then we get a lower (=pessimistic) bound on policy performance



# Trust Region Policy Optimization: Pseudocode

- ▶ Pseudocode:

**for** iteration=1, 2, ... **do**

Run policy for  $T$  timesteps or  $N$  trajectories

Estimate advantage function at all timesteps

$$\begin{aligned} & \underset{\theta}{\text{maximize}} \quad \sum_{n=1}^N \frac{\pi_{\theta}(a_n | s_n)}{\pi_{\theta_{\text{old}}}(a_n | s_n)} \hat{A}_n \\ & \text{subject to} \quad \overline{\text{KL}}_{\pi_{\theta_{\text{old}}}}(\pi_{\theta}) \leq \delta \end{aligned}$$

**end for**

- ▶ Can solve constrained optimization problem efficiently by using conjugate gradient
- ▶ Closely related to **natural policy gradients** (Kakade, 2002), natural actor critic (Peters and Schaal, 2005), REPS (Peters et al., 2010)

# Solving KL Penalized Problem

- ▶ maximize $_{\theta} L_{\pi_{\theta_{\text{old}}}}(\pi_{\theta}) - \beta \cdot \overline{\text{KL}}_{\pi_{\theta_{\text{old}}}}(\pi_{\theta})$
- ▶ Make linear approximation to  $L_{\pi_{\theta_{\text{old}}}}$  and quadratic approximation to KL term:

$$\text{maximize}_{\theta} \quad g \cdot (\theta - \theta_{\text{old}}) - \frac{\beta}{2} (\theta - \theta_{\text{old}})^T F (\theta - \theta_{\text{old}})$$

$$\text{where} \quad g = \frac{\partial}{\partial \theta} L_{\pi_{\theta_{\text{old}}}}(\pi_{\theta})|_{\theta=\theta_{\text{old}}}, \quad F = \frac{\partial^2}{\partial^2 \theta} \overline{\text{KL}}_{\pi_{\theta_{\text{old}}}}(\pi_{\theta})|_{\theta=\theta_{\text{old}}}$$

- ▶ Quadratic part of  $L$  is negligible compared to KL term
- ▶  $F$  is positive semidefinite, but not if we include Hessian of  $L$
- ▶ Solution:  $\theta - \theta_{\text{old}} = \frac{1}{\beta} F^{-1} g$ , where  $F$  is Fisher Information matrix,  $g$  is policy gradient. This is called the **natural policy gradient**<sup>3</sup>.

<sup>3</sup>S. Kakade. "A Natural Policy Gradient." *NIPS*. 2001.

# Solving KL Constrained Problem

- ▶ Method of Lagrange multipliers: solve penalized problem to get  $\theta^*(\beta)$ . Then substitute  $\theta^*(\beta)$  into original problem and solve for  $\beta$ .
- ▶  $\beta$  only affects scaling of solution, not direction. Compute scaling as follows:
  - ▶ Compute  $s = F^{-1}g$  (soln with  $\beta = 1$ )
  - ▶ Rescale  $\theta - \theta_{\text{old}} = \alpha s$  so that constraint is satisfied. Quadratic approx  $\overline{\text{KL}}_{\pi_{\theta_{\text{old}}}}(\pi_{\theta}) \approx \frac{1}{2}(\theta - \theta_{\text{old}})^T F(\theta - \theta_{\text{old}})$ , so we want

$$\frac{1}{2}(\alpha s)^T F(\alpha s) = \delta$$
$$\alpha = \sqrt{2\delta / (s^T F s)}$$

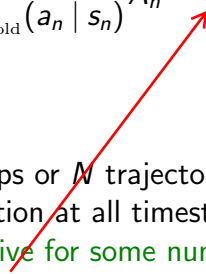
- ▶ Even better, we can do a line search to solve the original *nonlinear* problem.

$$\text{maximize } L_{\pi_{\theta_{\text{old}}}}(\pi_{\theta}) - \mathbf{1}[\overline{\text{KL}}_{\pi_{\theta_{\text{old}}}}(\pi_{\theta}) \leq \delta]$$

Try  $\alpha, \alpha/2, \alpha/4, \dots$  until line search objective improves

# “Proximal” Policy Optimization: KL Penalty Version

- ▶ Use penalty instead of constraint

$$\underset{\theta}{\text{maximize}} \sum_{n=1}^N \frac{\pi_{\theta}(a_n | s_n)}{\pi_{\theta_{\text{old}}}(a_n | s_n)} \hat{A}_n - \beta \cdot \overline{\text{KL}}_{\pi_{\theta_{\text{old}}}}(\pi_{\theta})$$


- ▶ Pseudocode:

**for** iteration=1, 2, ... **do**

Run policy for  $T$  timesteps or  $N$  trajectories

Estimate advantage function at all timesteps

Do SGD on above objective for some number of epochs

If KL too high, increase  $\beta$ . If KL too low, decrease  $\beta$ .

**end for**

- ▶  $\approx$  same performance as TRPO, but only first-order optimization

# Review

- ▶ Suggested optimizing surrogate loss  $L^{PG}$  or  $L^{IS}$
- ▶ Suggested using KL to constrain size of update
- ▶ Corresponds to natural gradient step  $F^{-1}g$  under linear quadratic approximation
- ▶ Can solve for this step approximately using conjugate gradient method



# Connection Between Trust Region Problem and Other Things

$$\begin{aligned} & \underset{\theta}{\text{maximize}} \quad \sum_{n=1}^N \frac{\pi_{\theta}(a_n | s_n)}{\pi_{\theta_{\text{old}}}(a_n | s_n)} \hat{A}_n \\ & \text{subject to} \quad \overline{\text{KL}}_{\pi_{\theta_{\text{old}}}}(\pi_{\theta}) \leq \delta \end{aligned}$$

- ▶ Linear-quadratic approximation + penalty  $\Rightarrow$  natural gradient
- ▶ No constraint  $\Rightarrow$  policy iteration
- ▶ Euclidean penalty instead of KL  $\Rightarrow$  vanilla policy gradient

# Limitations of TRPO

- ▶ Hard to use with architectures with multiple outputs, e.g. policy and value function (need to weight different terms in distance metric)
- ▶ Empirically performs poorly on tasks requiring deep CNNs and RNNs, e.g. Atari benchmark
- ▶ CG makes implementation more complicated



# Calculating Natural Gradient Step with KFAC

- ▶ Summary: do blockwise approximation to FIM, and approximate each block using a certain factorization
- ▶ Alternate expression for FIM as outer product (instead of second deriv. of KL):

$$\hat{\mathbb{E}}_t \left[ \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)^T \nabla_{\theta} \log \pi_{\theta}(a_t | s_t) \right]$$

# Calculating Natural Gradient Step with KFAC

- ▶ Consider network with weight matrix  $W$  appearing once in network, with  $y = Wx$ . Then

$$\nabla_{W_{ij}} L = x_i \nabla_{y_j} L = x_i \bar{y}_j$$

$$F = \hat{\mathbb{E}}_t \left[ \nabla_{W_{ij}} \log \pi_{\theta}(a_t | s_t)^T \nabla_{W_{ij}} \log \pi_{\theta}(a_t | s_t) \right]$$

$$F_{ij,i'j'} = \hat{\mathbb{E}}_t [x_i \bar{y}_j x_{i'} \bar{y}_{j'}]$$

- ▶ KFAC approximation:

$$F_{ij,i'j'} = \hat{\mathbb{E}}_t [x_i \bar{y}_j x_{i'} \bar{y}_{j'}] \approx \hat{\mathbb{E}}_t [x_i x_{i'}] \hat{\mathbb{E}}_t [\bar{y}_j \bar{y}_{j'}]$$

- ▶ Approximating Fisher block as tensor product  $A \otimes B$ , where  $A$  is  $n_{in} \times n_{in}$  and  $B$  is  $n_{out} \times n_{out}$ .  $(A \otimes B)^{-1} = A^{-1} \otimes B^{-1}$  and we can compute matrix-vector product without forming full matrix.
- ▶ Maintain running estimate of covariance matrices  $\hat{\mathbb{E}}_t[x_i x_j]$ ,  $\hat{\mathbb{E}}_t[\bar{y}_{i'} \bar{y}_{j'}]$  and periodically compute matrix inverses (small overhead)

# ACKTR: Combine A2C with KFAC Natural Gradient

- ▶ Combined with A2C, gives excellent on Atari benchmark and continuous control from images<sup>4</sup>.
- ▶ Note: we're already computing  $\nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$  for policy gradient: no extra gradient computation needed
- ▶ Matrix inverses can be computed asynchronously
- ▶ Limitation: works straightforwardly for feedforward nets (including convolutions), less straightforward for RNNs or architectures with shared weights

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<sup>4</sup>Y. Wu, E. Mansimov, S. Liao, R. Grosse, and J. Ba. "Scalable trust-region method for deep reinforcement learning using Kronecker-factored approximation". (2017).

# Proximal Policy Optimization: KL Penalized Version

- ▶ Back to penalty instead of constraint

$$\underset{\theta}{\text{maximize}} \sum_{n=1}^N \frac{\pi_{\theta}(a_n | s_n)}{\pi_{\theta_{\text{old}}}(a_n | s_n)} \hat{A}_n - C \cdot \overline{\text{KL}}_{\pi_{\theta_{\text{old}}}}(\pi_{\theta})$$

- ▶ Pseudocode:

**for** iteration=1, 2, ... **do**

Run policy for  $T$  timesteps or  $N$  trajectories

Estimate advantage function at all timesteps

Do SGD on above objective for some number of epochs

If KL too high, increase  $\beta$ . If KL too low, decrease  $\beta$ .

**end for**

- ▶  $\approx$  same performance as TRPO, but only first-order optimization

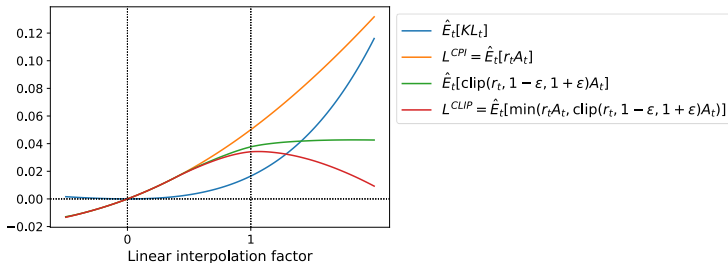
# Proximal Policy Optimization: Clipping Objective

- Recall the surrogate objective

$$L^{IS}(\theta) = \hat{\mathbb{E}}_t \left[ \frac{\pi_\theta(a_t | s_t)}{\pi_{\theta_{\text{old}}}(a_t | s_t)} \hat{A}_t \right] = \hat{\mathbb{E}}_t \left[ r_t(\theta) \hat{A}_t \right]. \quad (1)$$

- Form a lower bound via clipped importance ratios

$$L^{CLIP}(\theta) = \hat{\mathbb{E}}_t \left[ \min(r_t(\theta) \hat{A}_t, \text{clip}(r_t(\theta), 1 - \epsilon, 1 + \epsilon) \hat{A}_t) \right] \quad (2)$$



- Forms pessimistic bound on objective, can be optimized using SGD



# Proximal Policy Optimization

- ▶ Pseudocode:
  - for** iteration=1, 2, ... **do**
    - Run policy for  $T$  timesteps or  $N$  trajectories
    - Estimate advantage function at all timesteps
    - Do SGD on  $L^{CLIP}(\theta)$  objective for some number of epochs
  - end for**
- ▶ A bit better than TRPO on continuous control, much better on Atari
- ▶ Compatible with multi-output networks and RNNs

# Further Reading

- ▶ S. Kakade. "A Natural Policy Gradient." *NIPS*. 2001
- ▶ S. Kakade and J. Langford. "Approximately optimal approximate reinforcement learning". *ICML*. 2002
- ▶ J. Peters and S. Schaal. "Natural actor-critic". *Neurocomputing* (2008)
- ▶ J. Schulman, S. Levine, P. Moritz, M. I. Jordan, and P. Abbeel. "Trust Region Policy Optimization". *ICML* (2015)
- ▶ Y. Duan, X. Chen, R. Houthoofd, J. Schulman, and P. Abbeel. "Benchmarking Deep Reinforcement Learning for Continuous Control". *ICML* (2016)
- ▶ J. Martens and I. Sutskever. "Training deep and recurrent networks with Hessian-free optimization". *Springer*, 2012
- ▶ Z. Wang, V. Bapst, N. Heess, V. Mnih, R. Munos, et al. "Sample Efficient Actor-Critic with Experience Replay". (2016)
- ▶ Y. Wu, E. Mansimov, S. Liao, R. Grosse, and J. Ba. "Scalable trust-region method for deep reinforcement learning using Kronecker-factored approximation". (2017)
- ▶ J. Schulman, F. Wolski, P. Dhariwal, A. Radford, and O. Klimov. "Proximal Policy Optimization Algorithms". (2017)
- ▶ [blog.openai.com](https://blog.openai.com): recent posts on baselines releases

# That's all. Questions?

Vanilla Policy  
Gradient

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Natural Policy  
Gradient

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TRPO

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ACKTR & PPO

