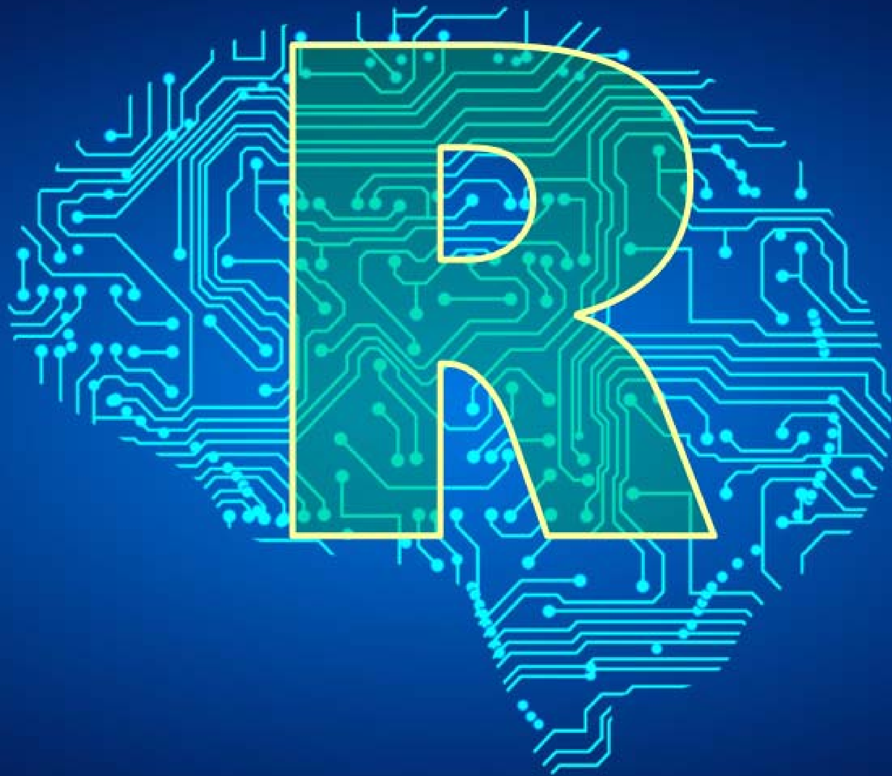
台灣人工智慧學校



平滑技巧

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Simple Moving Average

- In statistics, a moving average (移動平均) (rolling average or running average) (簡稱均線) is a calculation to analyze data points by creating series of averages of different subsets of the full data set.
- When price is in an uptrend and subsequently, the moving average is in an uptrend, and the moving average has been tested by price and price has bounced off the moving average a few times (i.e. the moving average is serving as a support line), then a trader might buy on the **next pullbacks back** to the Simple Moving Average.

$$SMA = \frac{p_1 + p_2 + \cdots + p_n}{n}$$

$$SMA_{t1,n} = SMA_{t0,n} - \frac{p_1}{n} + \frac{p_{n+1}}{n}$$

Moving Average Acting as Support - Potential Buy Signal

http://www.onlinetradingconcepts.com/TechnicalAnalysis/MASimple.html

Moving Average Acting as Resistance - Potential Sell Signal

- At times when price is in a downtrend and the moving average is in a downtrend as well, and price tests the SMA above and is rejected a few consecutive times (i.e. the moving average is serving as a resistance line), then a trader might sell on the **next rally up** to the Simple Moving Average.

http://www.onlinetradingconcepts.com/TechnicalAnalysis/MASimple.ht

An n-day WMA (Weighted moving average)

$$\text{WMA}_M = \frac{np_M + (n-1)p_{M-1} + \cdots + 2p_{(M-n+2)} + p_{(M-n+1)}}{n + (n-1) + \cdots + 2 + 1}$$

Smoothing in R

`smooth`: Forecasting Using Smoothing Functions

https://cran.r-project.org/web/packages/smooth/index.html

- `es()` - Exponential Smoothing;
- `ssarima()` - State-Space ARIMA, also known as Several Seasonalities ARIMA;
- `ces()` - Complex Exponential Smoothing;
- `ges()` - Generalised Exponential Smoothing;
- `ves()` - Vector Exponential Smoothing;
- `sma()` - Simple Moving Average in state-space form;

`TTR`: Technical Trading Rules

https://cran.r-project.org/web/packages/TTR/index.html

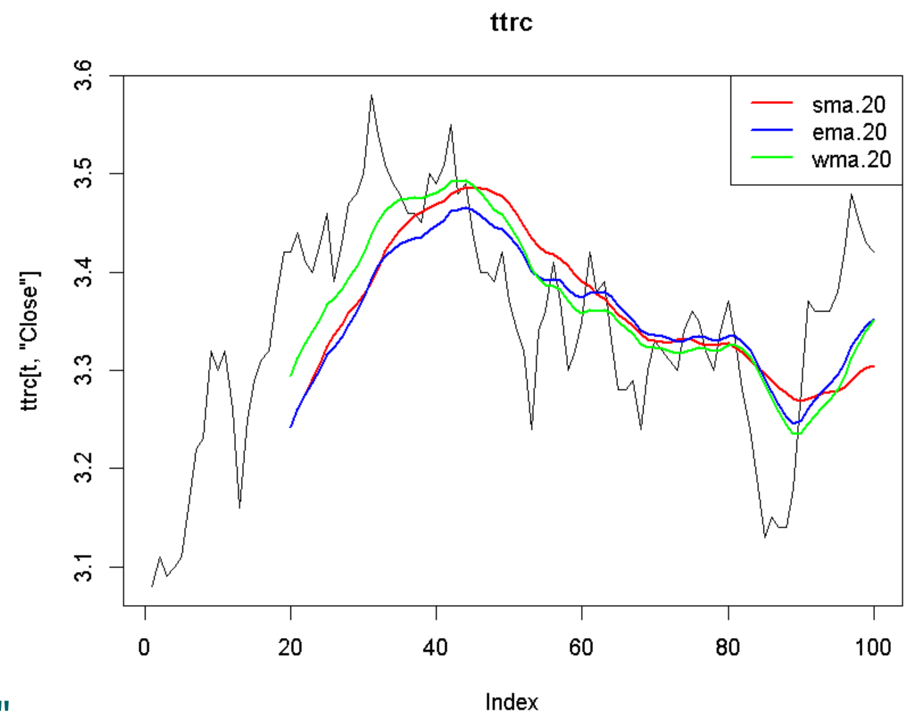
```
SMA(x, n = 10, ...)
EMA(x, n = 10, wilder = FALSE, ratio = NULL, ...)
DEMA(x, n = 10, v = 1, wilder = FALSE, ratio = NULL)
WMA(x, n = 10, wts = 1:n, ...)
EVWMA(price, volume, n = 10, ...)
ZLEMA(x, n = 10, ratio = NULL, ...)
VWAP(price, volume, n = 10, ...)
VMA(x, w, ratio = 1, ...)
HMA(x, n = 20, ...)
ALMA(x, n = 9, offset = 0.85, sigma = 6, ...)
```

Example

`ttrc {TTR}`: Technical Trading Rule Composite data

Historical Open, High, Low, Close, and Volume data for the periods January 2, 1985 to December 31, 2006. Randomly generated.

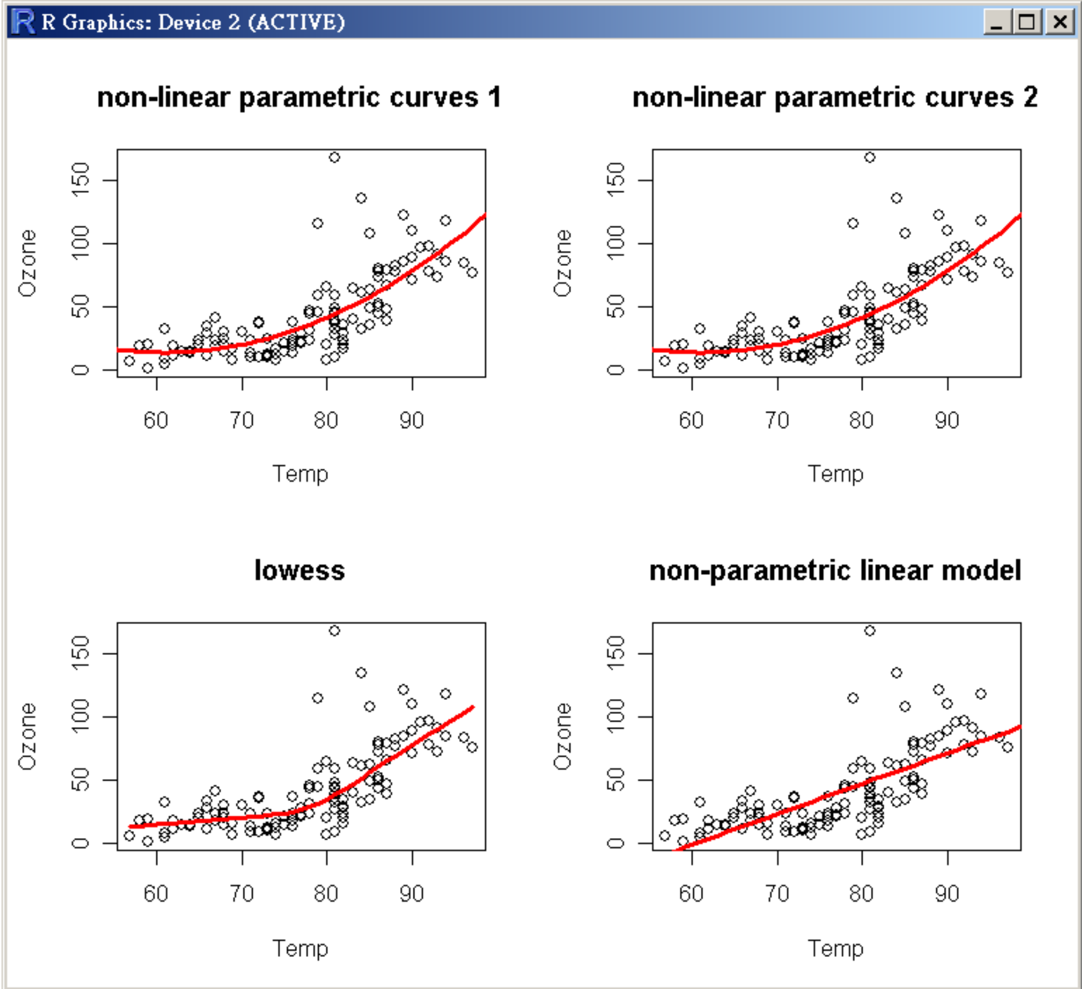
```
> # install.packages("TTR")
> library(TTR)
> data(ttrc)
> dim(ttrc)
[1] 5550    6
> head(ttrc)
        Date Open High  Low Close  Volume
1 1985-01-02 3.18 3.18 3.08  3.08 1870906
2 1985-01-03 3.09 3.15 3.09  3.11 3099506
3 1985-01-04 3.11 3.12 3.08  3.09 2274157
4 1985-01-07 3.09 3.12 3.07  3.10 2086758
5 1985-01-08 3.10 3.12 3.08  3.11 2166348
6 1985-01-09 3.12 3.17 3.10  3.16 3441798
>
> t <- 1:100
> sma.20 <- SMA(ttrc[t, "Close"], 20)
> ema.20 <- EMA(ttrc[t, "Close"], 20)
> wma.20 <- WMA(ttrc[t, "Close"], 20)
>
> plot(ttrc[t,"Close"], type="l", main="ttrc",
> lines(sma.20, col="red", lwd=2)
> lines(ema.20, col="blue", lwd=2)
> lines(wma.20, col="green", lwd=2)
> legend("topright", legend=c("sma.20", "ema.20", "wma.20"),
+          col=c("red", "blue", "green"), lty=1, lwd=2)
```



曲線配適 (Fitting Curves)

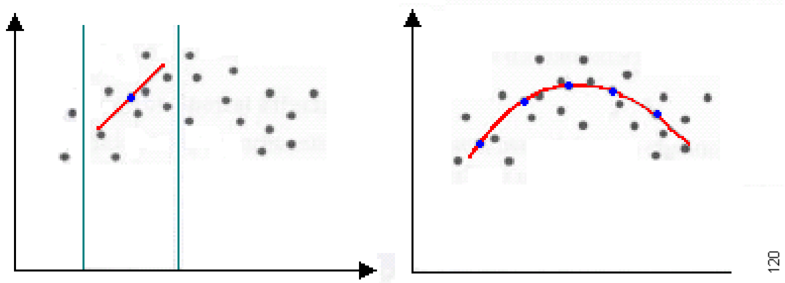
Example Methods

- non-linear parametric curves
- lowess (a non-parametric curve fitter)
- loess (a modelling tool)
- gam (fits generalized additive models)
- lm (linear model)

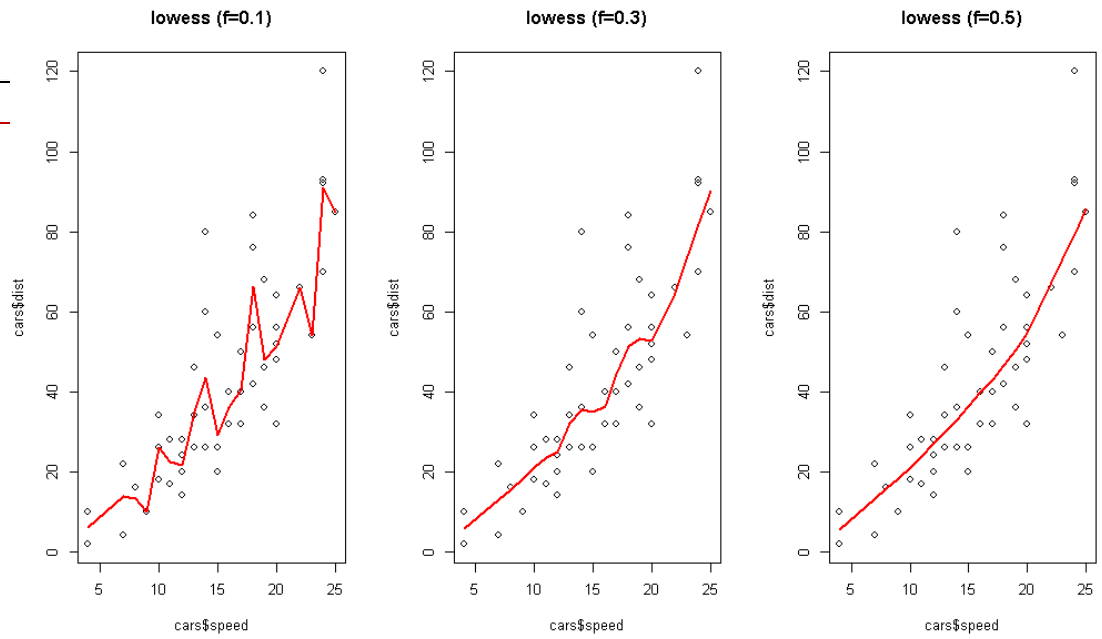


lowess {stats}
locally-weighted polynomial regression

Loess regression
(locally weighted polynomial regression)

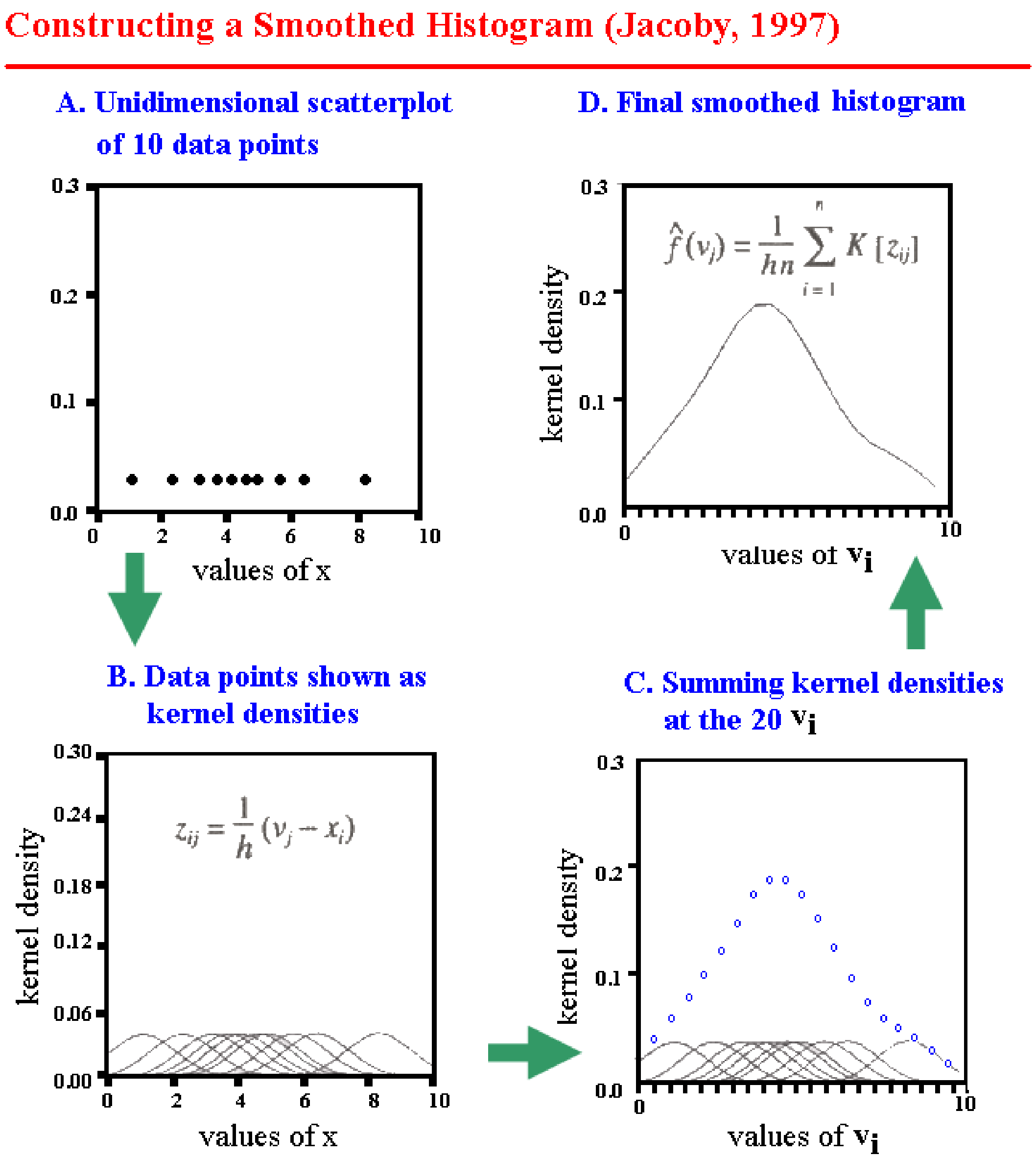


cars {datasets}:
The data give the speed of cars and the distances taken to stop. Note that the data were recorded in the 1920s.




```
> data(cars)
> dim(cars)
[1] 50  2
> head(cars)
  speed dist
1     4    2
2     4   10
3     7    4
4     7   22
5     8   16
6     9   10
> par(mfrow=c(1, 3))
> for(i in c(0.1, 0.3, 0.5)){
+   plot(cars$dist ~ cars$speed, main=paste0("lowess (f=", i,")"))
+   lines(lowess(cars$dist ~ cars$speed, f = i), col="red", lwd=2)
+ }
```

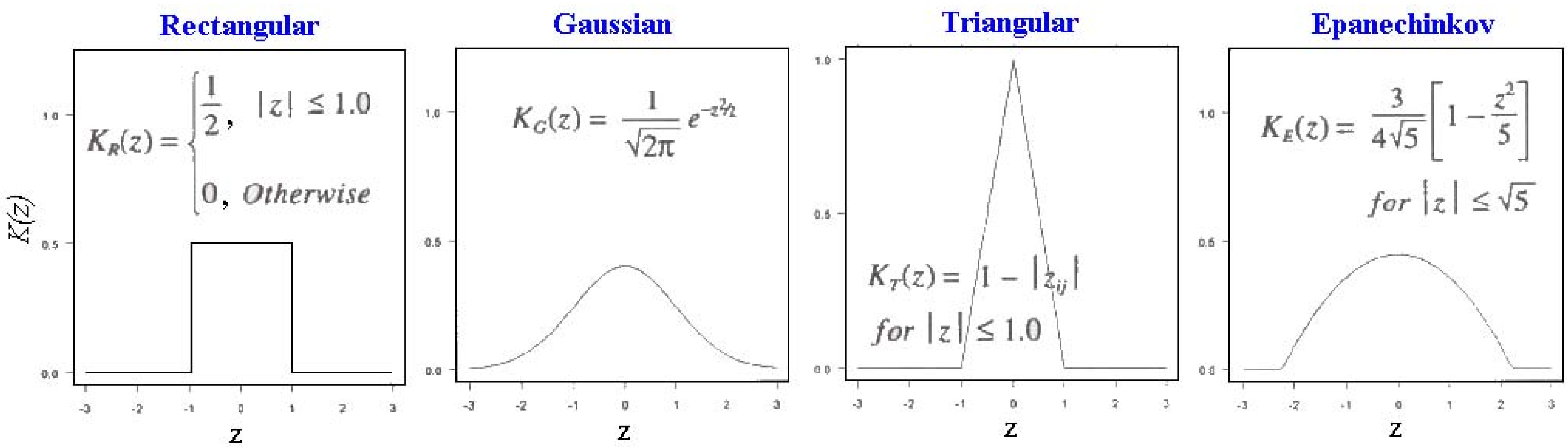
Density Plots (Smoothed Histograms) (1/3)



Kernel Density Estimation

- Selection of kernels
- Selection of bandwidth

Figures modified from Jacoby (1997)



nonparametric regression

$$y_i = f_0(x_i) + \epsilon_i, \quad i = 1, \ldots n,$$

$\epsilon_1, \ldots \epsilon$ are still i.i.d. random errors with $\mathbb{E}(\epsilon_i) = 0$

kernel regression

$$\hat{f}(x) = \frac{\sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right) y_i}{\sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right)}$$

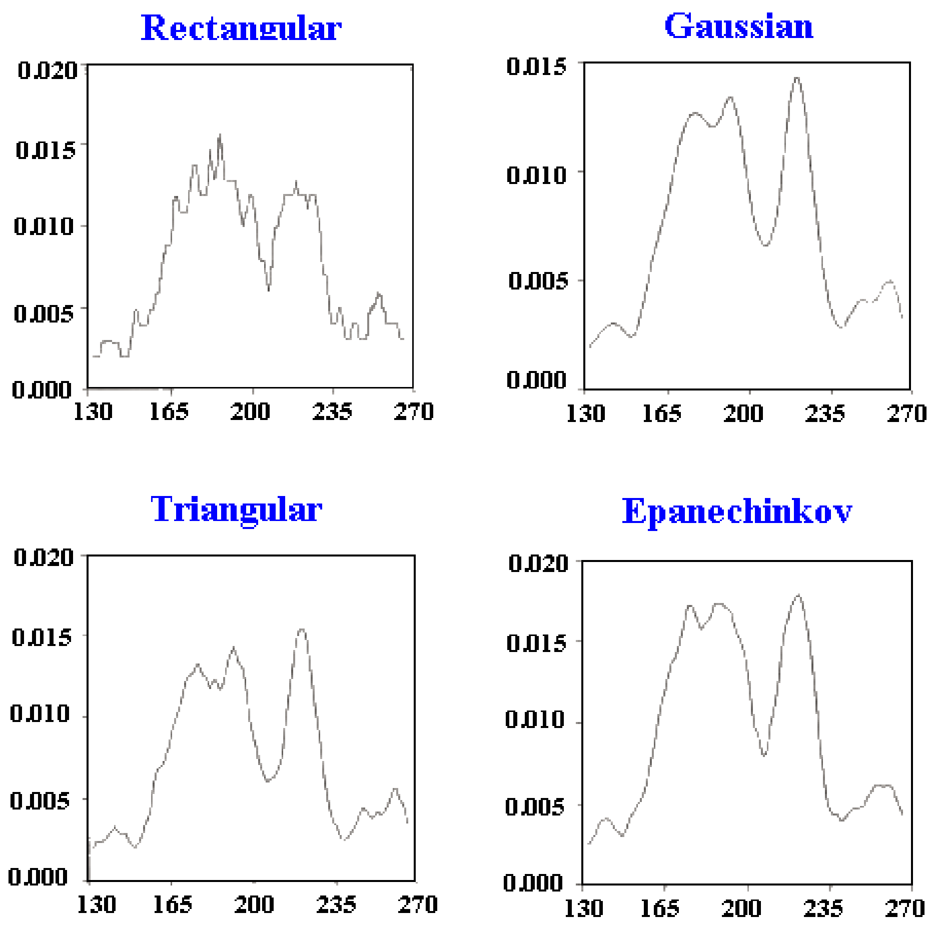
$$\hat{f}(v_j) = \frac{1}{hn} \sum_{i=1}^{n} K\,[z_{ij}]$$

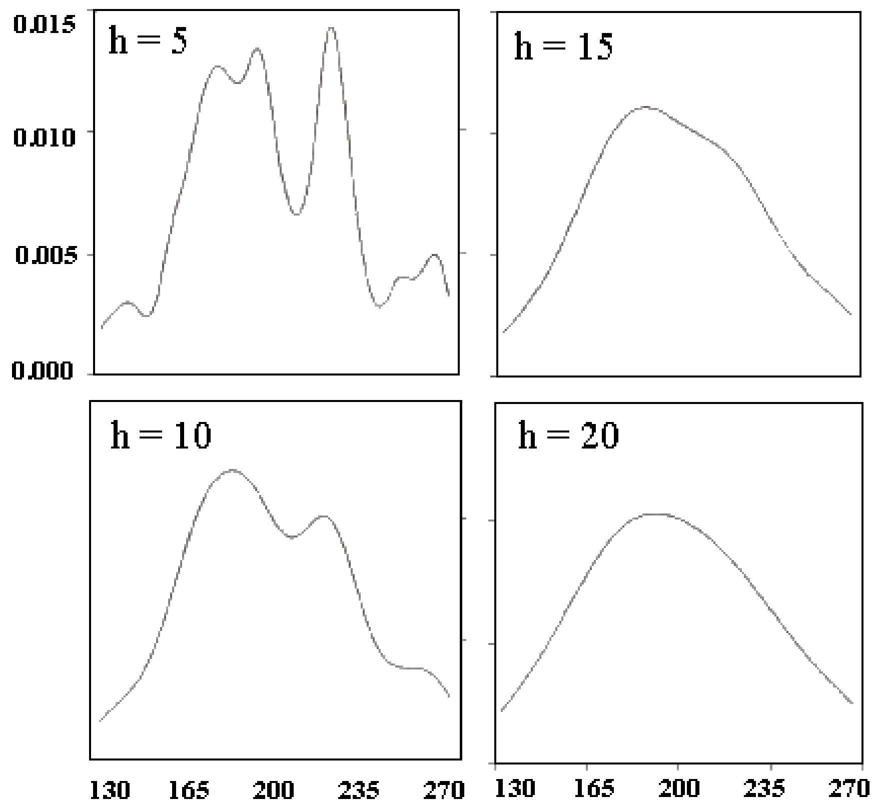
$$z_{ij} = \frac{1}{h}(v_j - x_i)$$

k-nearest-neighbors regression.

$$\hat{f}(x) = \frac{1}{k} \sum_{i \in \mathcal{N}_k(x)} y_i$$

Kernel Density Estimation



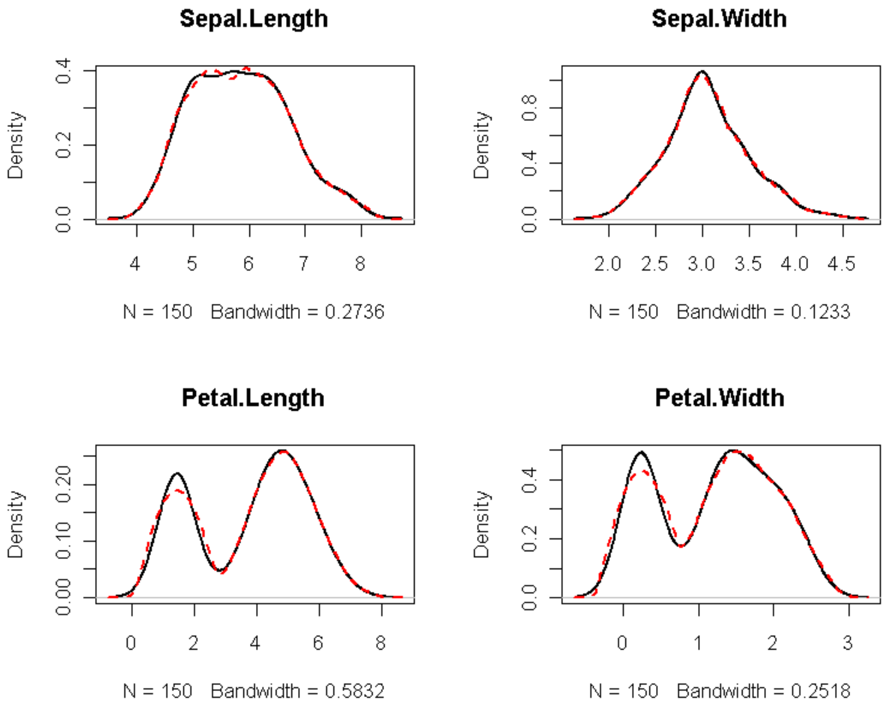


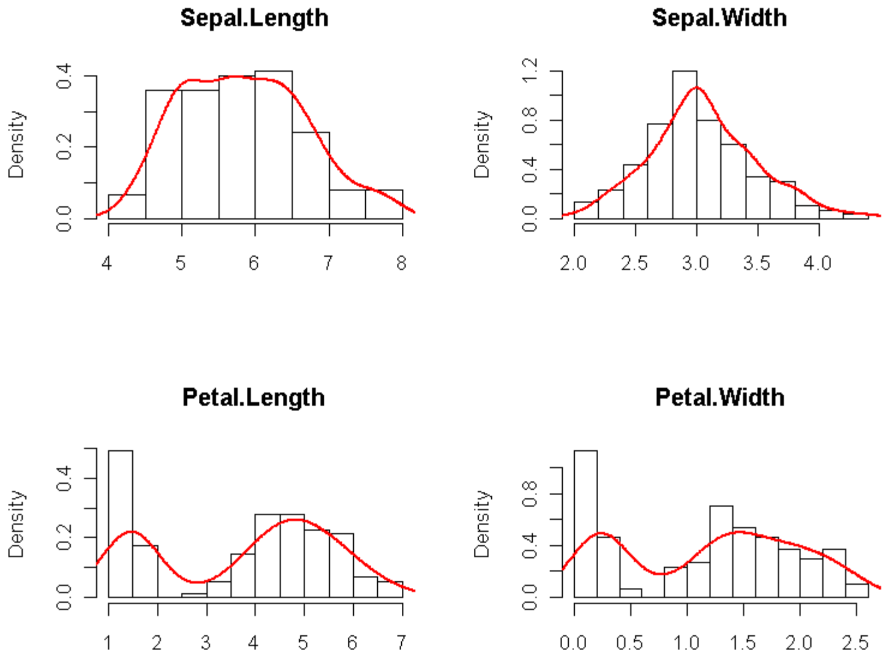
Kernel Density Estimation


```
density(x, bw = "nrd0", adjust = 1,
        kernel = c("gaussian", "epanechnikov", "rectangular",
                   "triangular", "biweight",
                   "cosine", "optcosine"),
        weights = NULL, window = kernel, width,
        give.Rkern = FALSE,
        n = 512, from, to, cut = 3, na.rm = FALSE, ...)
```

```
> plot(density(iris$Sepal.Length))
```

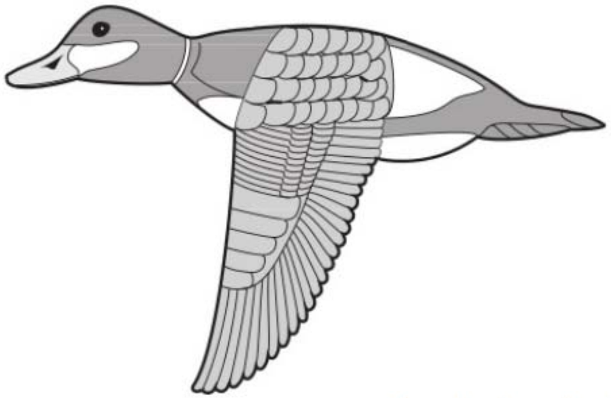
```
> hist(iris$Sepal.Length, prob=T)
> lines(density(iris$Sepal.Length), col="red")
```

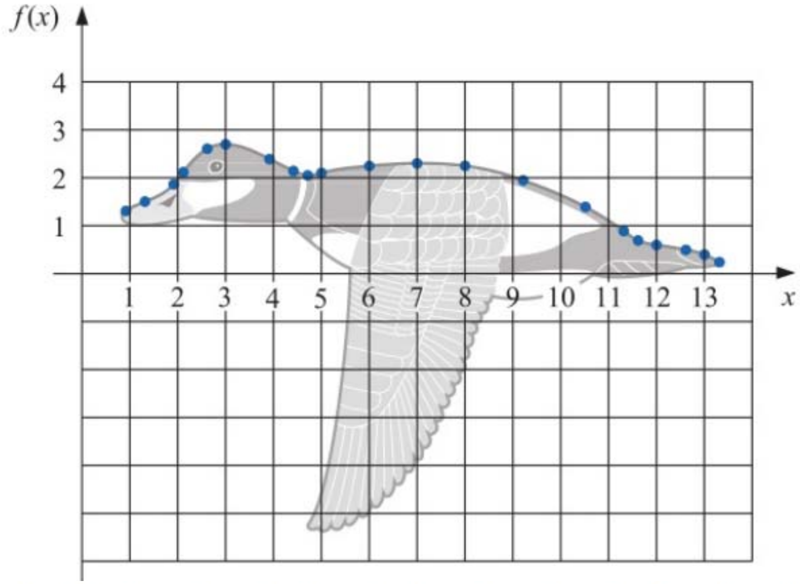




Spline approximate to the top profile of the ruddy duck

ruddy duck (棕硬尾鴨) (雄)





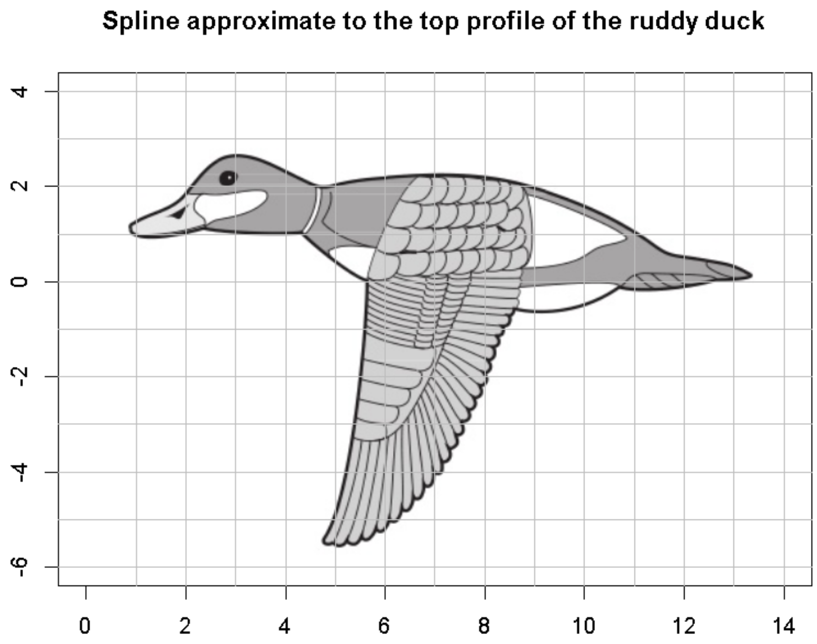
x	0.9	1.3	1.9	2.1	2.6	3.0	3.9	4.4	4.7	5.0	6.0	7.0	8.0	9.2	10.5	11.3	11.6	12.0	12.6	13.0	13.3
$f(x)$	1.3	1.5	1.85	2.1	2.6	2.7	2.4	2.15	2.05	2.1	2.25	2.3	2.25	1.95	1.4	0.9	0.7	0.6	0.5	0.4	0.25

smooth.spline {stats}: Fit a Smoothing Spline

Usage

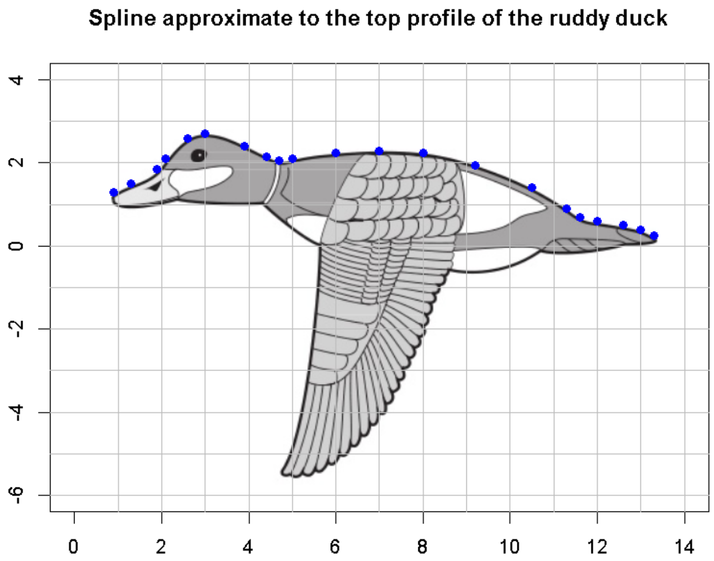
```
smooth.spline(x, y = NULL, w = NULL, df, spar = NULL, lambda = NULL, cv = FALSE,
              all.knots = FALSE, nknots = .nknots.smspl,
              keep.data = TRUE, df.offset = 0, penalty = 1,
              control.spar = list(), tol = 1e-6 * IQR(x), keep.stuff = FALSE)
```

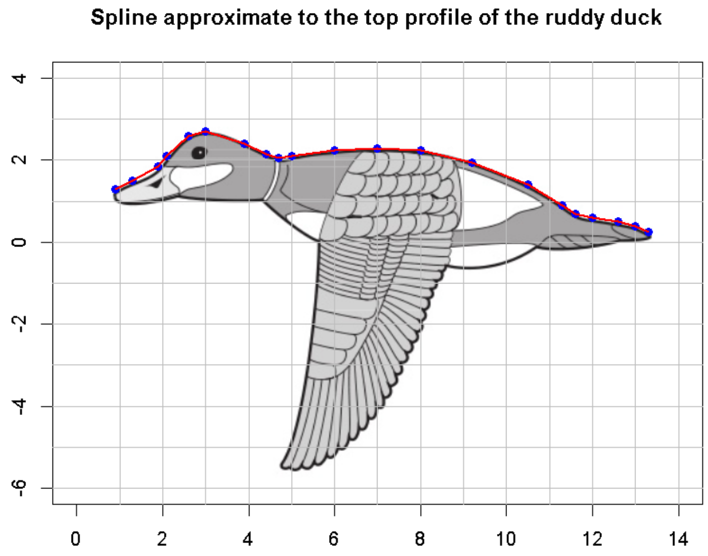
```
> #install.packages("jpeg")
> library(jpeg)
> ruddyduck.img <- readJPEG("ruddyduck.jpg")
> plot(0, xlim=c(0, 14), ylim=c(-6, 4), type='n', xlab="", ylab="",
+      main="Spline approximate to the top profile of the ruddy duck")
> rasterImage(ruddyduck.img, 0.6, -6, 13.8, 3.3)
> abline(v=1:14, h=-6:4, col=gray)
```



`smooth.spline {stats}`: Fit a Smoothing Spline

```
> ruddyduck.dat <- read.table("ruddyduck.txt", header=T, sep="\t")
> head(ruddyduck.dat)
    x   fx
1 0.9 1.30
2 1.3 1.50
3 1.9 1.85
4 2.1 2.10
5 2.6 2.60
6 3.0 2.70
> points(ruddyduck.dat, col="blue", pch=16)
>
> duck.spl <- smooth.spline(ruddyduck.dat$fx ~ ruddyduck.dat$x)
> lines(duck.spl, col = "red", lwd=2)
```





Cubic Spline Interpolation

Cubic Splines Interpolant

Definition 3.10

Given a function f defined on $[a, b]$ and a set of nodes $a = x_0 < x_1 < \cdots < x_n = b$, a cubic spline interpolant S for f is a function that satisfies the following conditions:

(a) $S(x)$ is a cubic polynomial ($S_j(x)$) on $[x_j, x_{j+1}]$.

(b) $S_j(x_j) = f(x_j)$ and $S_j(x_{j+1}) = f(x_{j+1})$, $j = 0, 1, \cdots, n-1$;

(c) $S_{j+1}(x_{j+1}) = S_j(x_{j+1})$; (d) $S'_{j+1}(x_{j+1}) = S'_j(x_{j+1})$; (e) $S''_{j+1}(x_{j+1}) = S''_j(x_{j+1})$ for each $j = 0, 1, \cdots, n-2$;

(f) One of the following sets of boundary conditions is satisfied:

(i) $S''(x_0) = S''(x_n) = 0$ (natural or free boundary);
(ii) $S'(x_0) = f'(x_0)$ and $S'(x_n) = f'(x_n)$ (clamped boundary).

Construction of a Cubic Spline (conti.)

(12) This system involves only the $\{c_j\}_{j=0}^n$ as unknowns.

(13) The values of $\{h_j\}_{j=0}^{n-1}$ and $\{a_j\}_{j=0}^n$ are given, respectively, by the spacing of the nodes $\{x_j\}_{j=0}^n$ and the values of f at the nodes.

(14) So once the values of $\{c_j\}_{j=0}^n$ are determined, it is a simple matter to find the remainder of the constants $\{b_j\}_{j=0}^{n-1}$ from Eq. (3.20) and $\{d_j\}_{j=0}^{n-1}$ from Eq. (3.17)

(15) The major question that arises in connection with this construction is whether the values of $\{c_j\}_{j=0}^n$ can be found using the system of equations given in (3.21) and, if so, whether these values are unique.

ALGORITHM 034: Natural Cubic Spline

To construct the cubic spline interpolant S for the function f, defined at the numbers $x_0 < x_1 < \cdots < x_n$, satisfying $S''(x_0) = S''(x_n) = 0$:

INPUT $n; x_0, x_1, \ldots, x_n; a_0 = f(x_0), a_1 = f(x_1), \ldots, a_n = f(x_n)$.

OUTPUT a_j, b_j, c_j, d_j for $j = 0, 1, \ldots, n-1$.

(Note: $S(x) = S_j(x) = a_j + b_j(x - x_j) + c_j(x - x_j)^2 + d_j(x - x_j)^3$ for $x_j \le x \le x_{j+1}$.)

Step 1 For $i = 0, 1, \ldots, n-1$ set $h_i = x_{i+1} - x_i$.

Step 2 For $i = 1, 2, \ldots, n-1$ set

$$\alpha_i = \frac{3}{h_i}(a_{i+1} - a_i) - \frac{3}{h_{i-1}}(a_i - a_{i-1}).$$

Step 3 Set $l_0 = 1$; *(Steps 3, 4, 5, and part of Step 6 solve a tridiagonal linear system using a method described in Algorithm 6.7.)*
$\mu_0 = 0$;
$z_0 = 0$.

ALGORITHM 034: Natural Cubic Spline (conti.)

Step 4 For $i = 1, 2, \ldots, n-1$
set $l_i = 2(x_{i+1} - x_{i-1}) - h_{i-1}\mu_{i-1}$;
$\mu_i = h_i / l_i$;
$z_i = (\alpha_i - h_{i-1} z_{i-1}) / l_i$.

Step 5 Set $l_n = 1$;
$z_n = 0$;
$c_n = 0$.

Step 6 For $j = n-1, n-2, \ldots, 0$
set $c_j = z_j - \mu_j c_{j+1}$;
$b_j = (a_{j+1} - a_j)/h_j - h_j(c_{j+1} + 2c_j)/3$;
$d_j = (c_{j+1} - c_j)/(3h_j)$.

Step 7 OUTPUT $(a_j, b_j, c_j, d_j$ for $j = 0, 1, \ldots, n-1)$;
STOP.