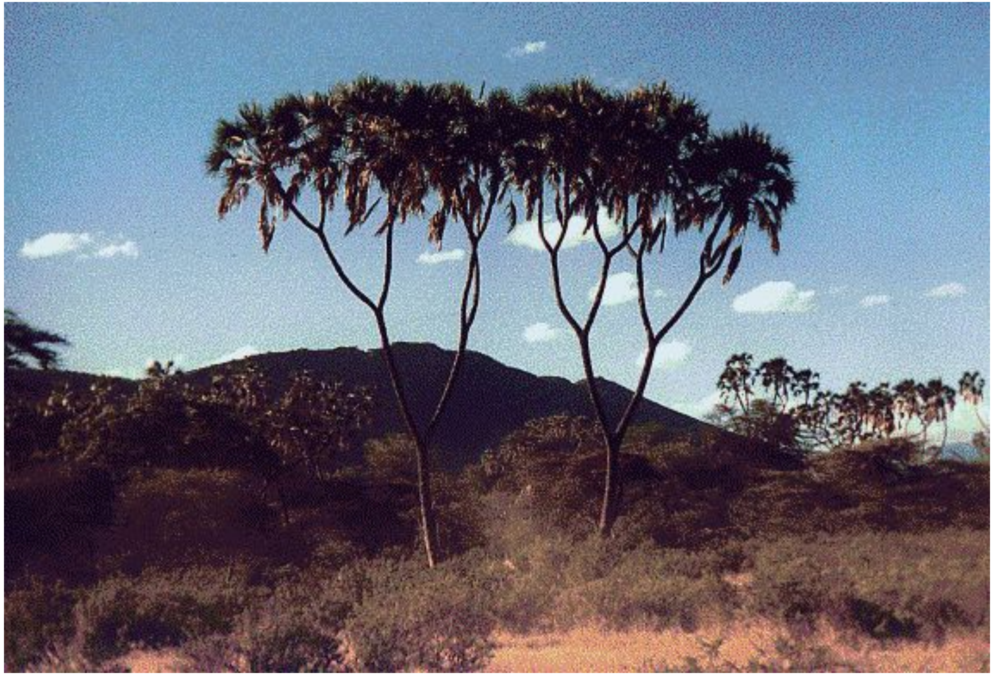
CS61B, 2019

Lecture 16: ADTs and BSTs

- Abstract Data Types
- Binary Search Tree (intro)
- BST Definitions
- BST Operations
- Sets vs. Maps, Summary

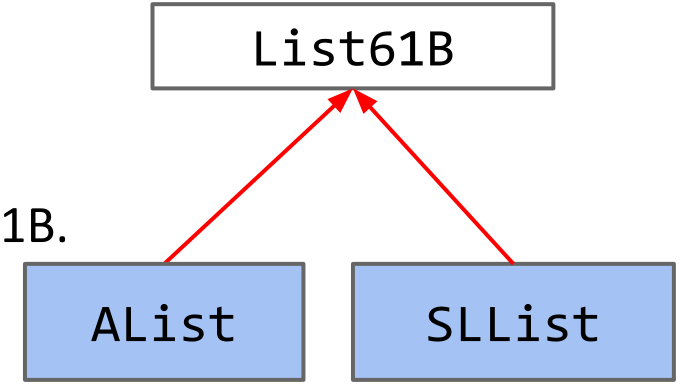


Abstract Data Types

Interfaces vs. Implementation

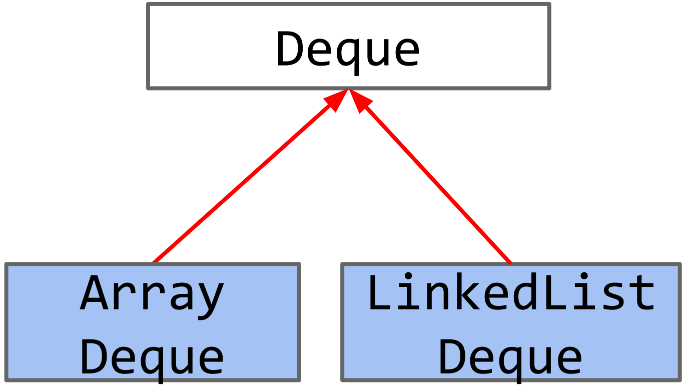
In class:

- Developed ALists and SLLists.
- Created an interface List61B.
    - Modified AList and SLList to implement List61B.
    - List61B provided default methods.



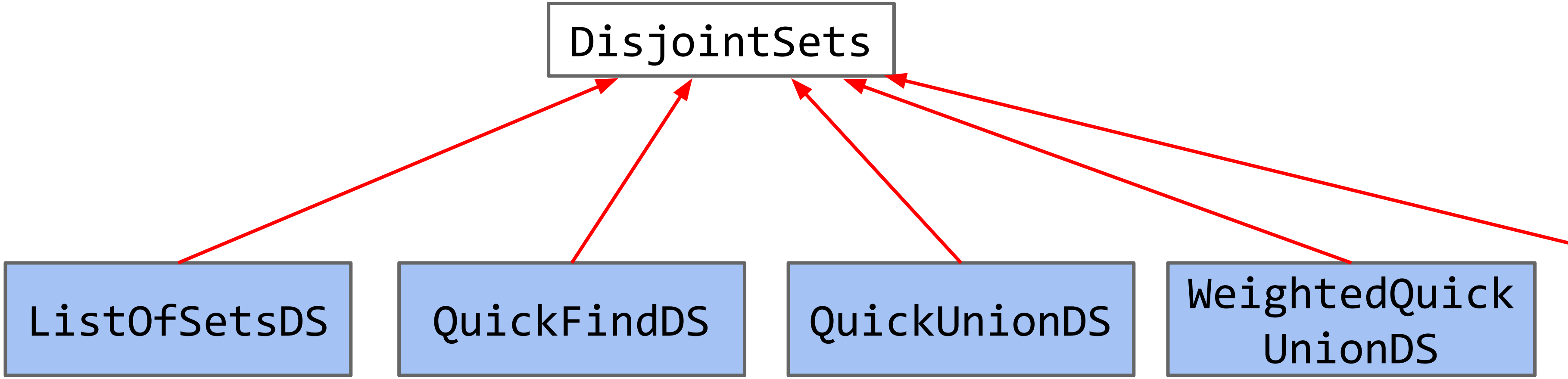
In projects:

- Developed ArrayDeque and LinkedListDeque.
- Created an interface Deque.
    - Modified AD and LLD to implement Deque.



Interfaces vs. Implementation

With DisjointSets, we saw a much richer set of possible implementations.

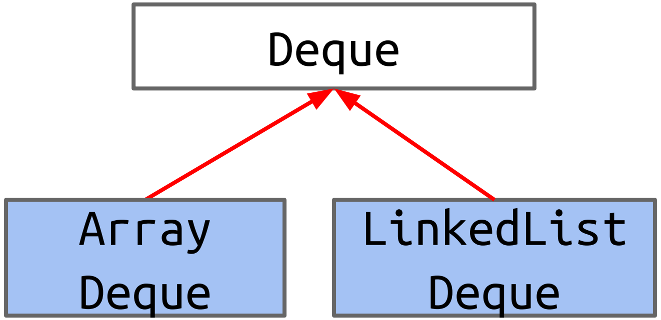


Abstract Data Types

An **Abstract Data Type (ADT)** is defined only by its operations, not by its implementation.

Deque ADT:

- `addFirst(Item x);`
- `addLast(Item x);`
- `boolean isEmpty();`
- `int size();`
- `printDeque();`
- `Item removeFirst();`
- `Item removeLast();`
- `Item get(int index);`

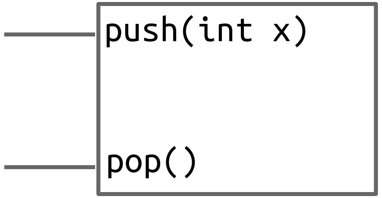


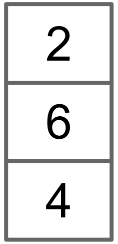
ArrayDeque and LinkedList Deque are implementations of the Deque ADT.

Another example of an ADT: The Stack

The Stack ADT supports the following operations:

- push(int x): Puts x on top of the stack.
- int pop(): Removes and returns the top item from the stack.





The Stack ADT: yellkey.com/?

The Stack <u>ADT</u> supports the following operations:

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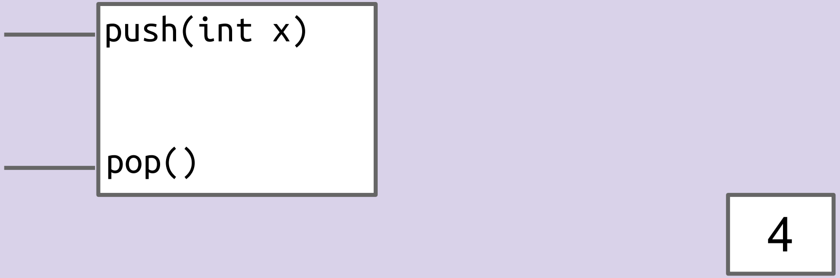
Which <u>implementation</u> do you think would result in faster overall performance?

A. Linked List
B. Array



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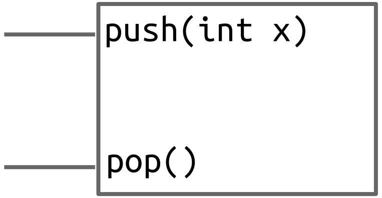


Both are about the same. No resizing for linked lists, so probably a lil faster.

The GrabBag ADT: yellkey.com/?

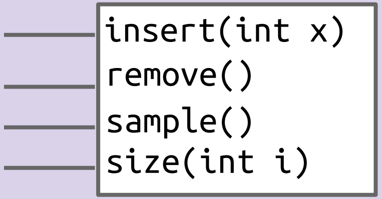
The GrabBag <u>ADT</u> supports the following operations:

- insert(int x): Inserts x into the grab bag.
- int remove(): Removes a random item from the bag.
- int sample(): Samples a random item from the bag (without removing!)
- int size(): Number of items in the bag.

Which <u>implementation</u> do you think would result in faster overall performance?

A. Linked List

B. Array



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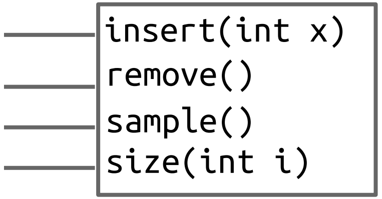
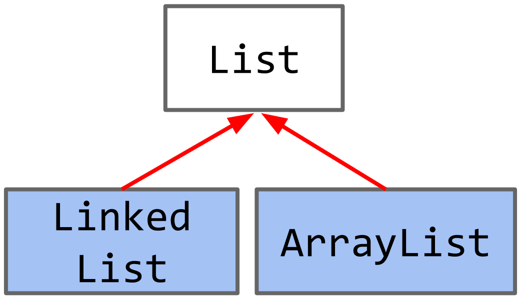


Abstract Data Types in Java

One thing I particularly like about Java is the syntax differentiation between abstract data types and implementations.

- Note: Interfaces in Java aren't purely abstract as they can contain some implementation details, e.g. default methods.

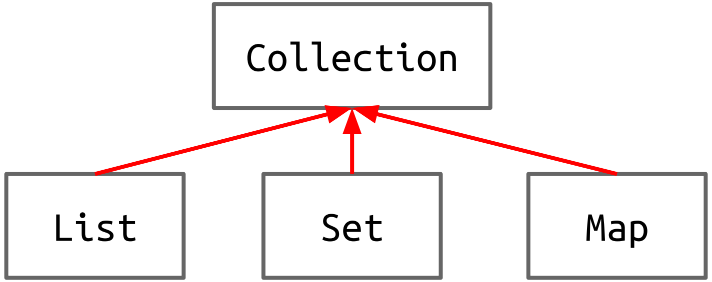
Example: `List<Integer> L = new ArrayList<>();`



Collections

Among the most important interfaces in the java.util library are those that extend the Collection interface (btw interfaces can extend other interfaces).

- Lists of things.
- Sets of things.
- Mappings between items, e.g. jhug's grade is 88.4, or Creature c's north neighbor is a Plip.
  - Maps also known as associative arrays, associative lists (in Lisp), symbol tables, dictionaries (in Python).



Map Example

Maps are very handy tools for all sorts of tasks. Example: Counting words.

```java
Map<String, Integer> m = new TreeMap<>();
String[] text = {"sumomo", "mo", "momo", "mo",
                 "momo", "no", "uchi"};
for (String s : text) {
    int currentCount = m.getOrDefault(s, 0);
    m.put(s, currentCount + 1);
}
```

sumomo	1
mo	2
momo	2
no	1
uchi	1

```python
m = {}
text = ["sumomo", "mo", "momo", "mo", \
        "momo", "no", "uchi"]
for s in text:
    current_count = m.get(s, 0)
    m[s] = current_count + 1
```

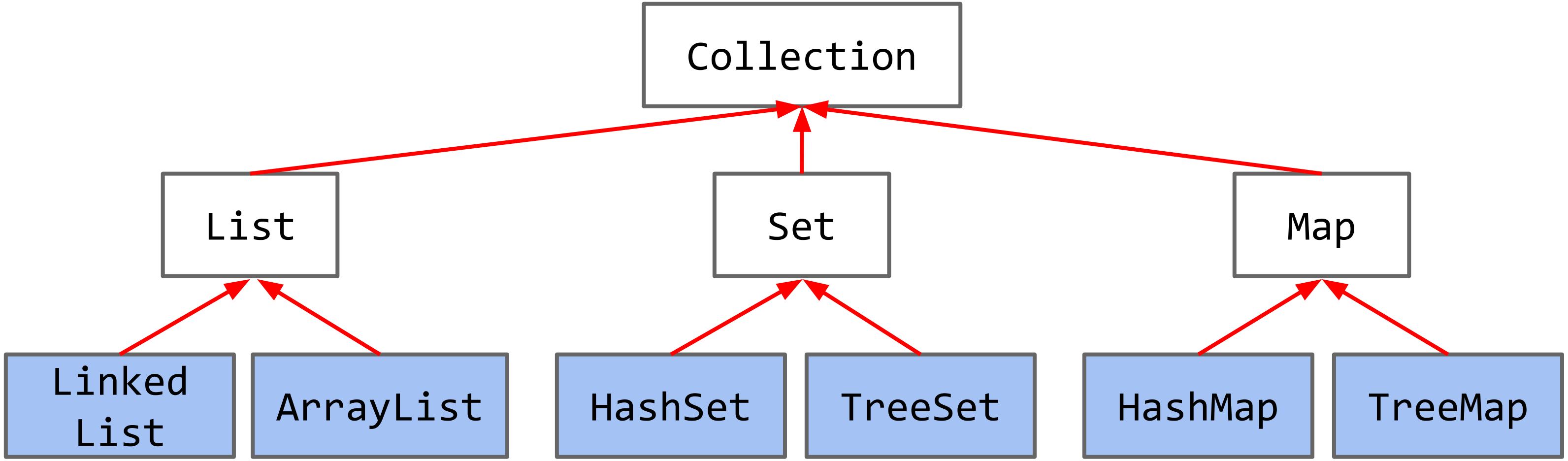
Python equivalent

Java Libraries

The built-in java.util package provides a number of useful:

- Interfaces: ADTs (lists, sets, maps, priority queues, etc.) and other stuff.
- Implementations: Concrete classes you can use.

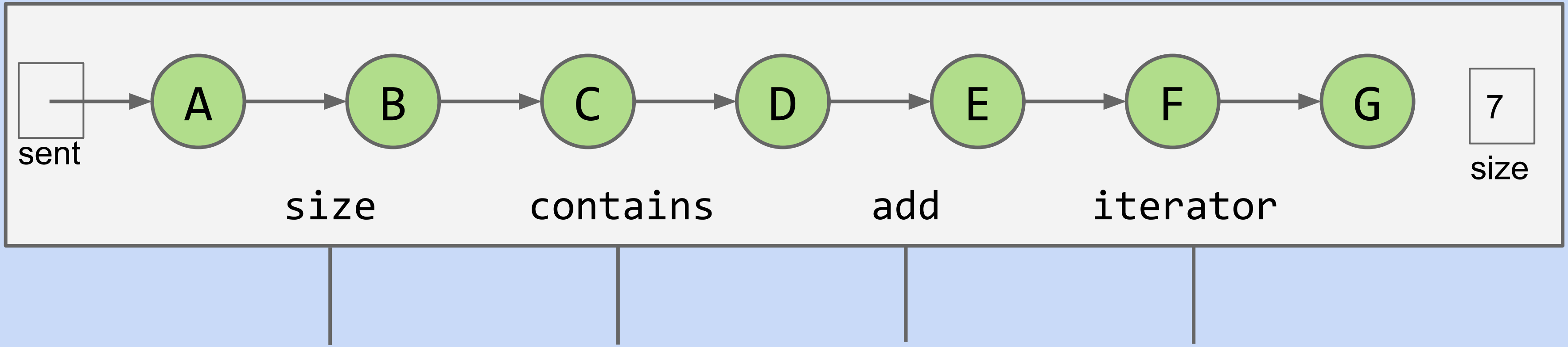
Today, we'll learn the basic ideas behind the TreeSet and TreeMap.



Binary Search Trees

Analysis of an OrderedLinkedListSet<Character>

In an earlier lecture, we implemented a set based on unordered arrays. For the ***order linked list*** set implementation below, name an operation that takes worst case linear time, i.e. Θ(N).



Analysis of an OrderedLinkedListSet<Character>

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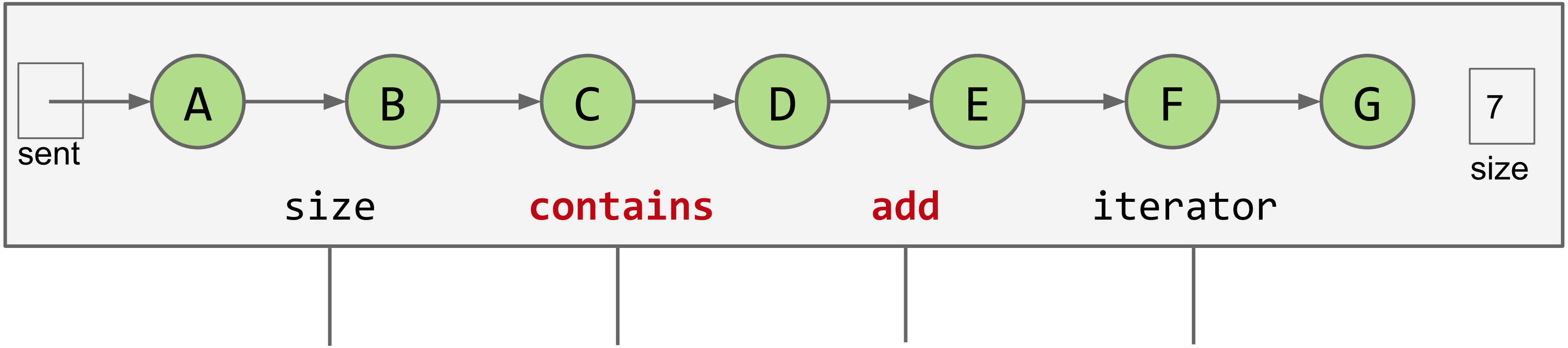
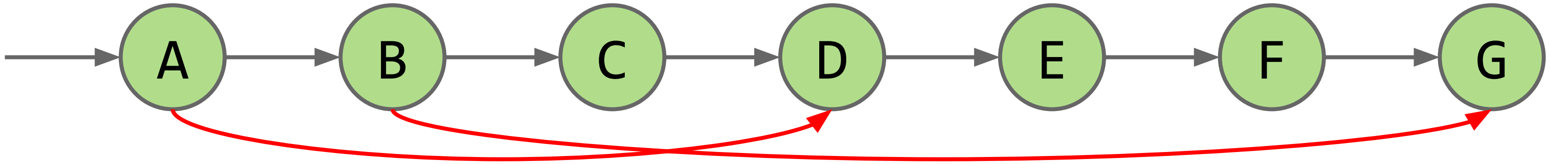


Optimization: Extra Links

Fundamental Problem: Slow search, even though it's in order.

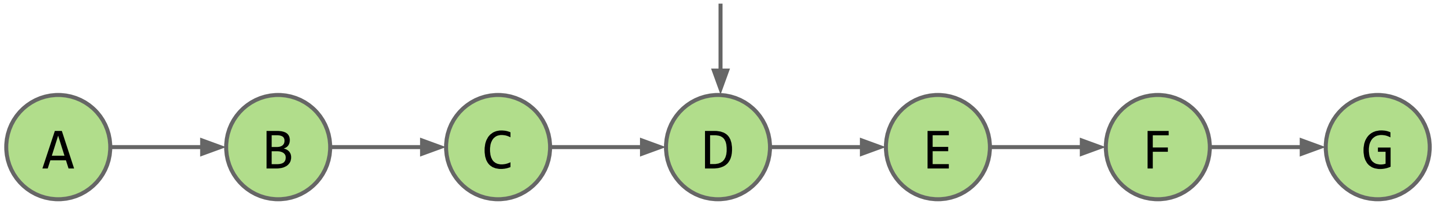
- Add (random) express lanes. Skip List (won't discuss in 61B)



Optimization: Change the Entry Point

Fundamental Problem: Slow search, even though it's in order.

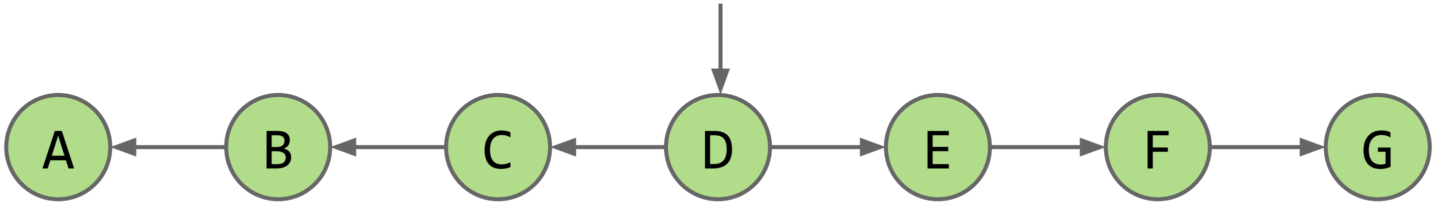
- Move pointer to middle.



Optimization: Change the Entry Point, Flip Links

Fundamental Problem: Slow search, even though it's in order.

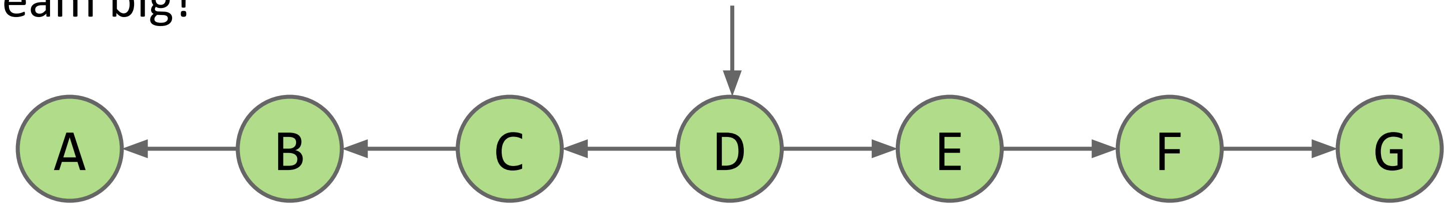
- Move pointer to middle and flip left links. Halved search time!



Optimization: Change the Entry Point, Flip Links

Fundamental Problem: Slow search, even though it's in order.

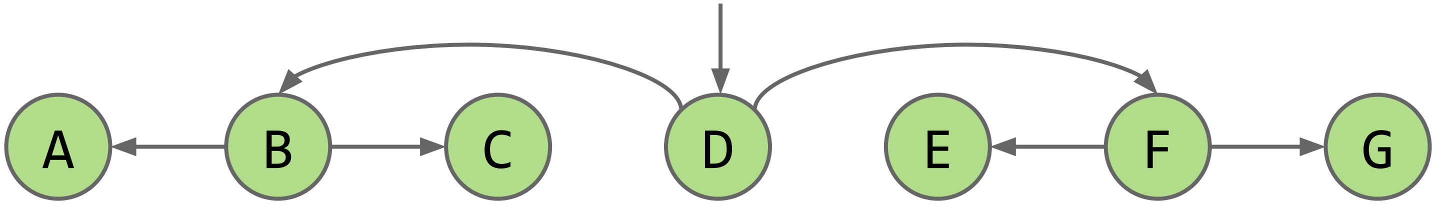
- How do we do even better?
- Dream big!

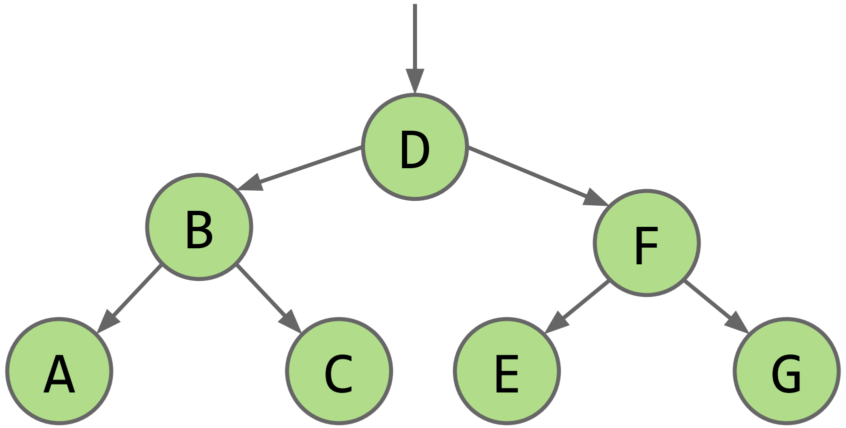


Optimization: Change Entry Point, Flip Links, Allow Big Jumps

Fundamental Problem: Slow search, even though it's in order.

- How do we do better?





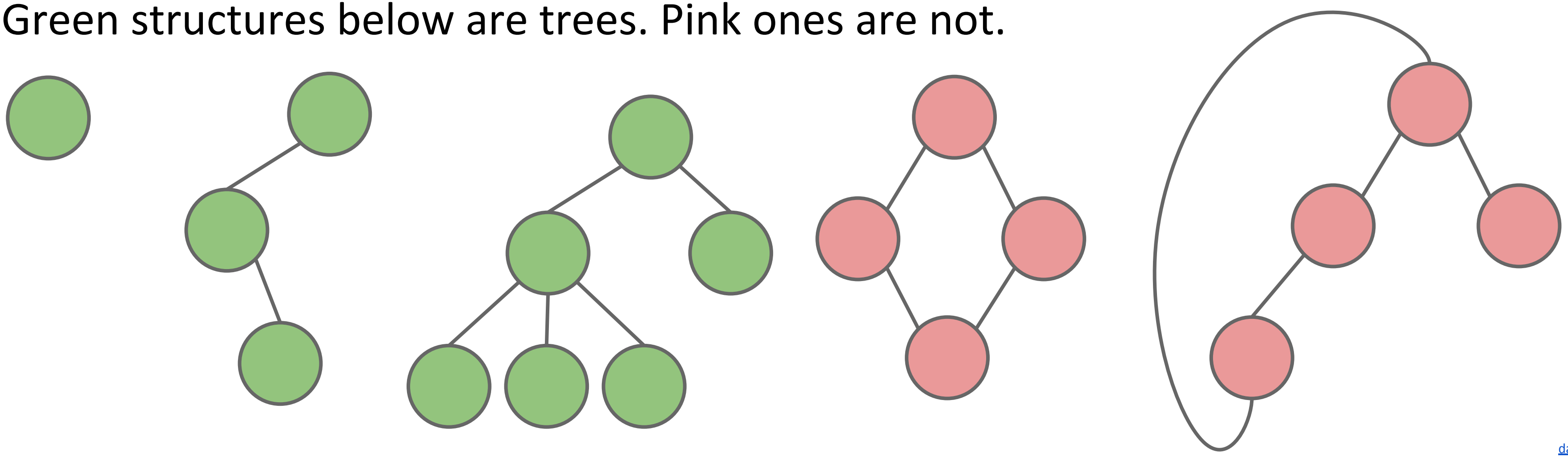
BST Definitions

Tree

A tree consists of:

- A set of nodes.
- A set of edges that connect those nodes.
  - Constraint: There is exactly one path between any two nodes.

Green structures below are trees. Pink ones are not.

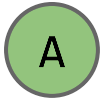


Rooted Trees and Rooted Binary Trees

In a rooted tree, we call one node the root.

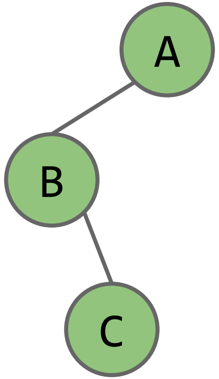
- Every node N except the root has exactly one parent, defined as the first node on the path from N to the root.
- Unlike (most) real trees, the root is usually depicted at the top of the tree.
- A node with no child is called a leaf.

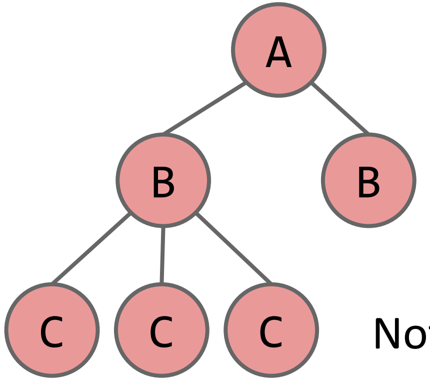
In a rooted binary tree, every node has either 0, 1, or 2 children (subtrees).



For each of these:

- A is the root.
- B is a child of A. (and C of B)
- A is a parent of B. (and B of C)





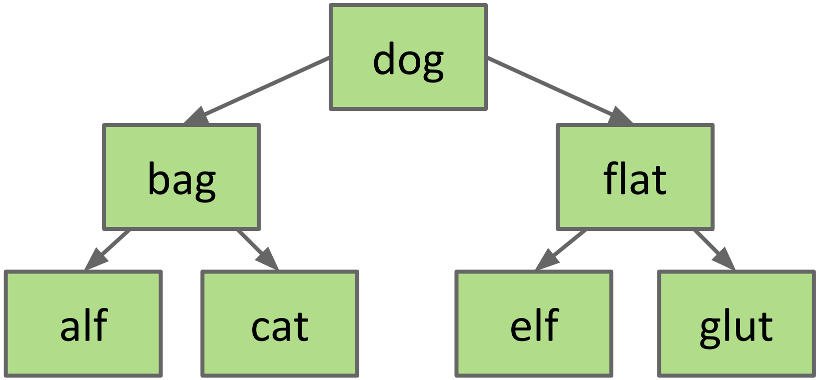
Not binary!

Binary Search Trees

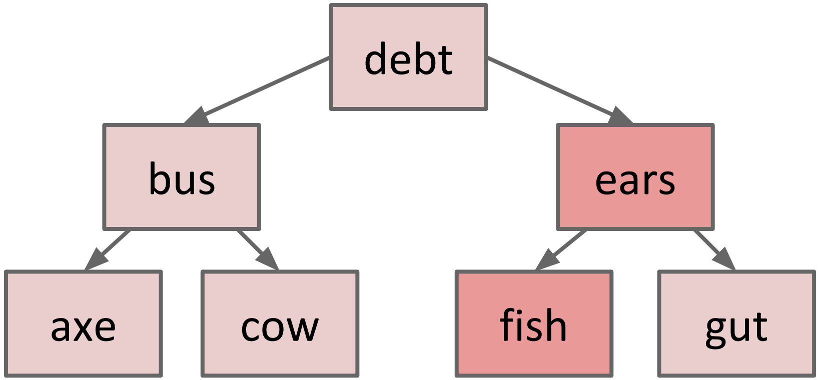
A binary search tree is a rooted binary tree with the BST property.

BST Property. For every node X in the tree:

- Every key in the **left** subtree is **less** than X's key.
- Every key in the **right** subtree is **greater** than X's key.



Binary Search Tree



Binary Tree, but not a Binary Search Tree

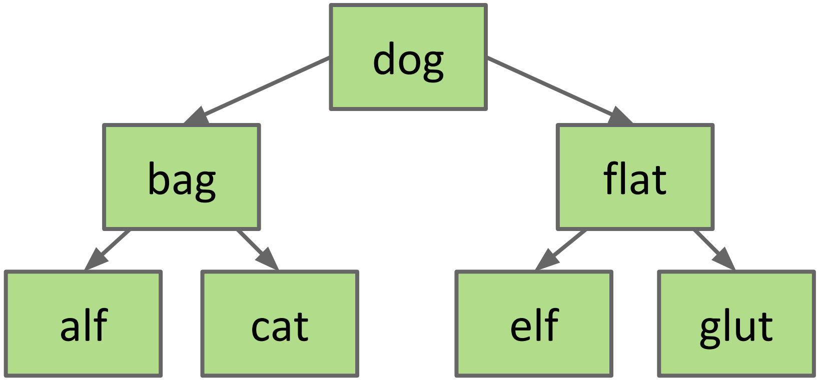
Binary Search Trees

Ordering must be complete, transitive, and antisymmetric. Given keys p and q:

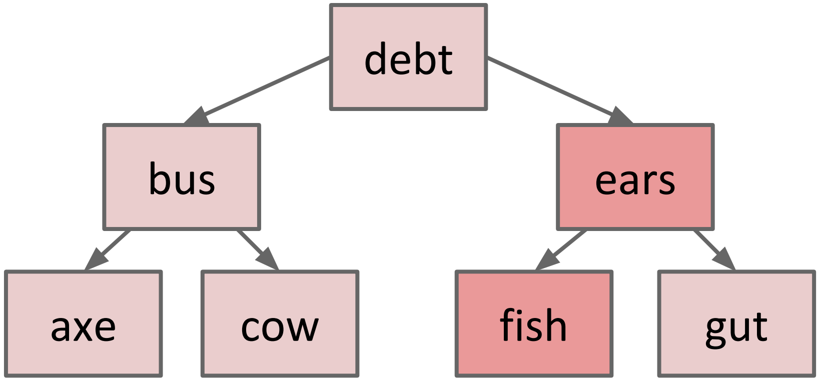
- Exactly one of p ≺ q and q ≺ p are true.
- p ≺ q and q ≺ r imply p ≺ r.

One consequence of these rules: No duplicate keys allowed!

- Keeps things simple. Most real world implementations follow this rule.



Binary Search Tree



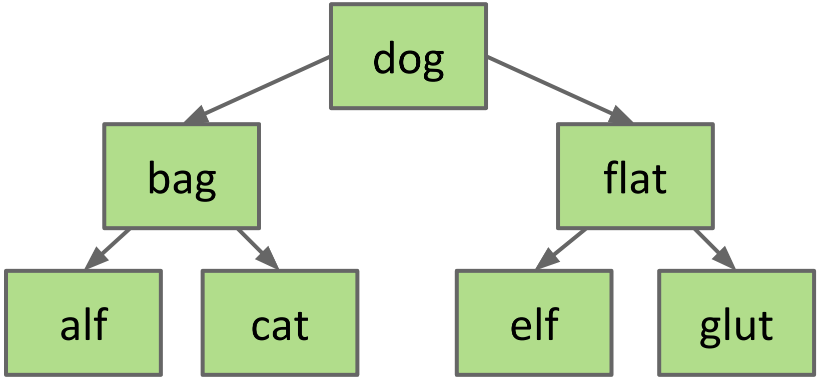
Binary Tree, but not a Binary Search Tree

BST Operations: Search

Finding a searchKey in a BST (come back to this for the BST lab)

If searchKey equals T.key, return.

- If searchKey ﹤ T.key, search T.left.
- If searchKey ﹥ T.key, search T.right.

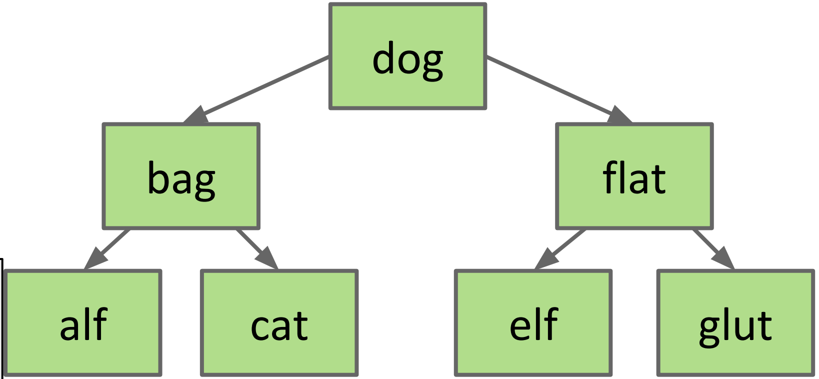


Finding a searchKey in a BST

If searchKey equals T.key, return.

- If searchKey < T.key, search T.left.
- If searchKey > T.key, search T.right.

```
static BST find(BST T, Key sk) {
   if (T == null)
      return null;
   if (sk.equals(T.key))
      return T;
   else if (sk < T.key)
      return find(T.left, sk);
   else
      return find(T.right, sk);
}
```

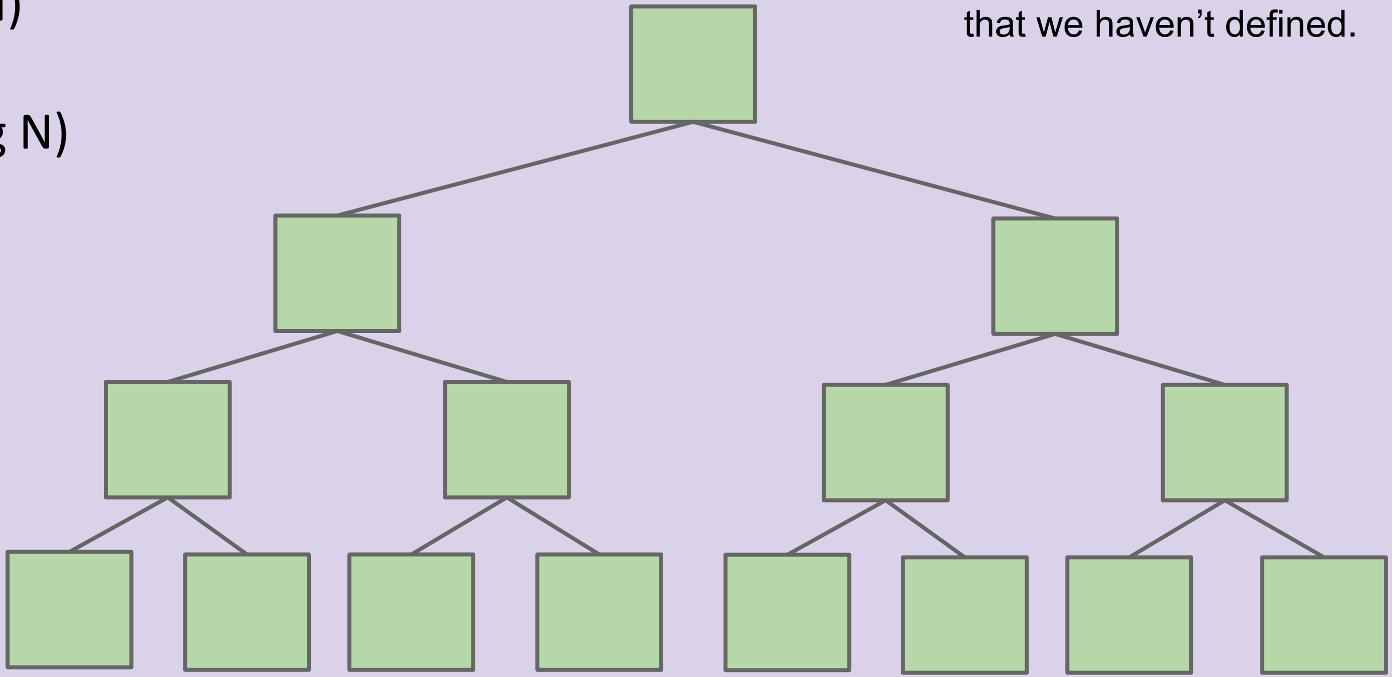


BST Search: http://yellkey.com/?

What is the runtime to complete a search on a “bushy” BST in the worst case, where N is the number of nodes.

A. $\Theta(\log N)$
B. $\Theta(N)$
C. $\Theta(N \log N)$
D. $\Theta(N^2)$
E. $\Theta(2^N)$

“bushiness” is an intuitive concept that we haven’t defined.

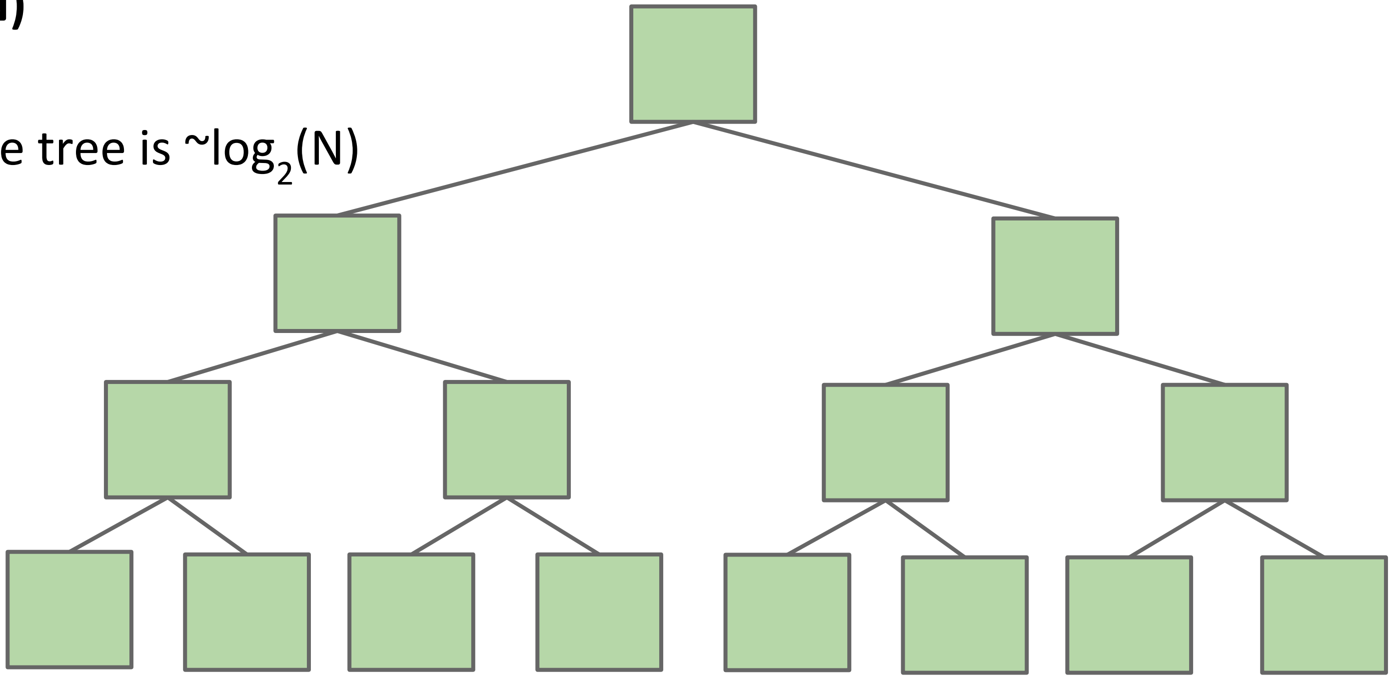


BST Search

What is the runtime to complete a search on a “bushy” BST in the worst case, where N is the number of nodes.

A. Θ(log N)

Height of the tree is ~$\log_2(N)$



BSTs

Bushy BSTs are extremely fast.

- At 1 microsecond per operation, can find something from a tree of size 10^{300000} in one second.

Much (perhaps most?) computation is dedicated towards finding things in response to queries.

- It's a good thing that we can do such queries almost for free.

BST Operations: Insert

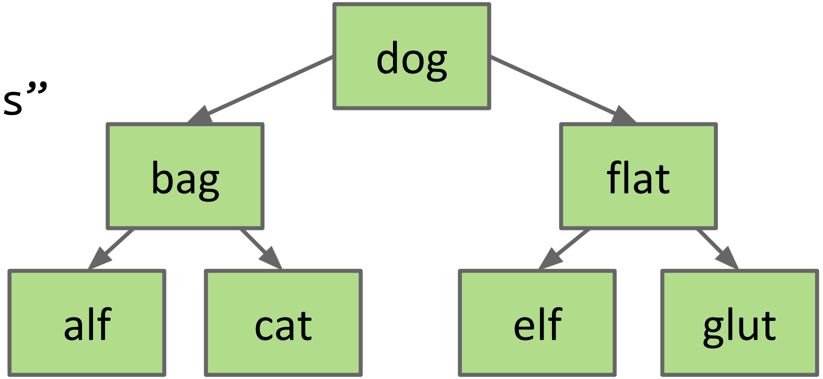
Inserting a New Key into a BST

Search for key.

- If found, do nothing.
- If not found:
  - Create new node.
  - Set appropriate link.

Example:

`insert “eyes”`



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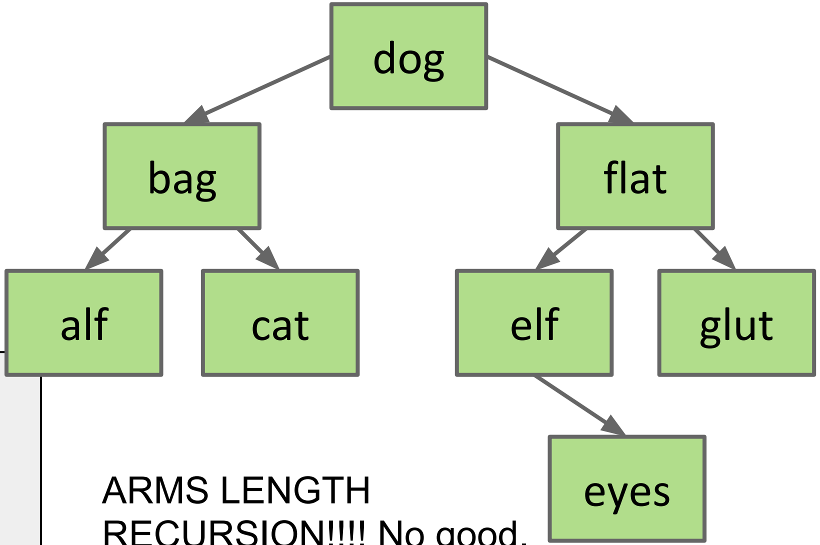



```
static BST insert(BST T, Key ik) {
  if (T == null)
    return new BST(ik);
  if (ik < T.key)
    T.left = insert(T.left, ik);
  else if (ik > T.key)
    T.right = insert(T.right, ik);
  return T;
}
```

ARMS LENGTH RECURSION!!!! No good.

A common rookie bad habit to avoid:

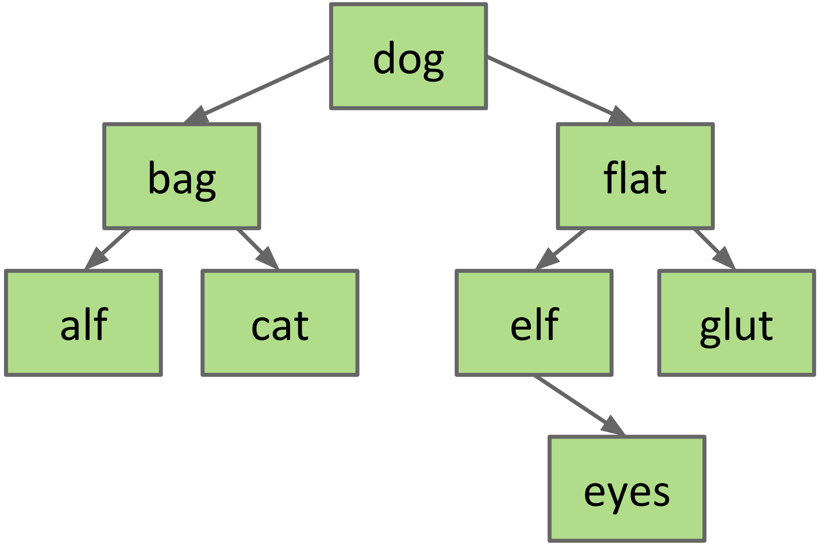
```
if (T.left == null)
  T.left = new BST(ik);
else if (T.right == null)
  T.right = new BST(ik);
```

BST Operations: Delete

Deleting from a BST

3 Cases:

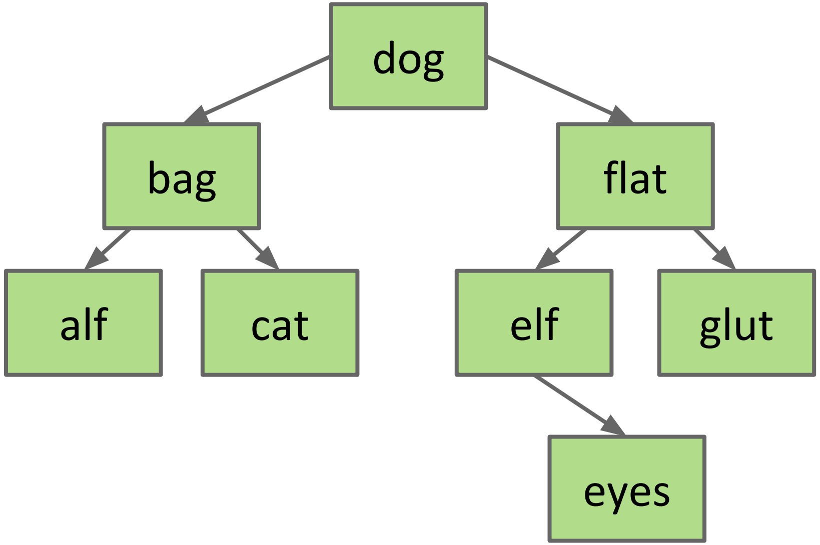
- Deletion key has no children.
- Deletion key has one child.
- Deletion key has two children.



Case 1: Deleting from a BST: Key with no Children

Deletion key has no children ("glut"):

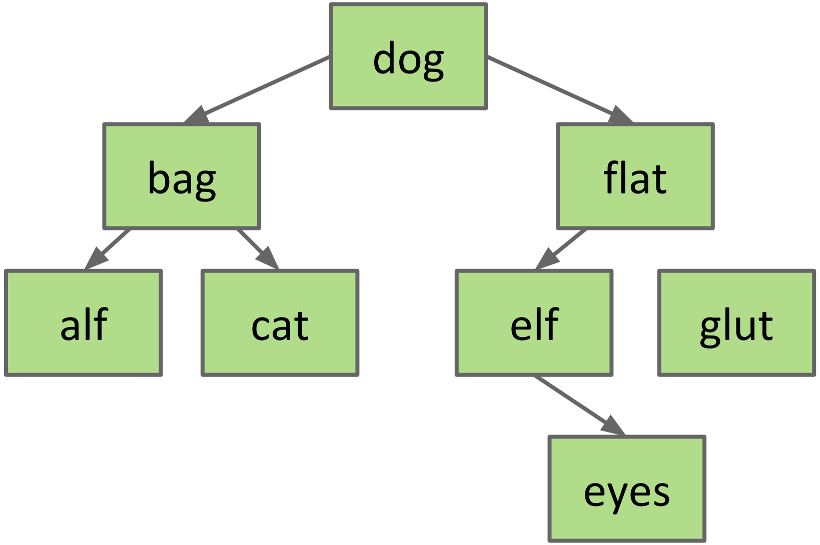
- Just sever the parent's link.
- What happens to "glut" node?



Case 1: Deleting from a BST: Key with no Children

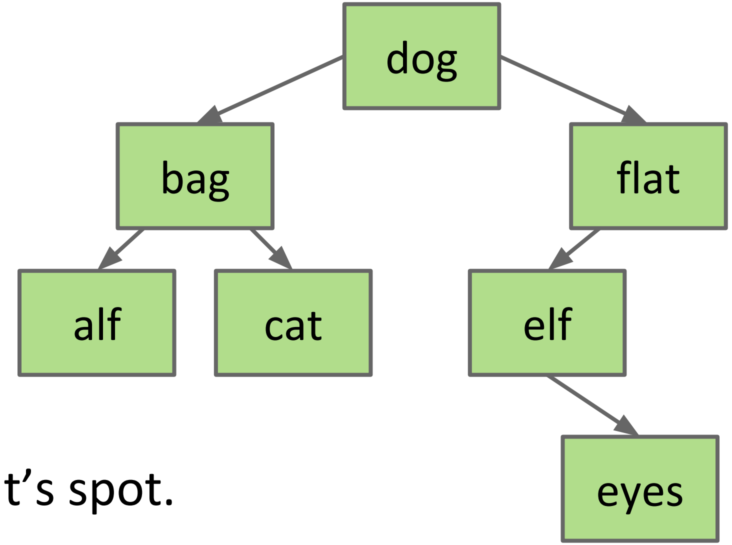
Deletion key has no children ("glut"):

- Just sever the parent's link.
- What happens to "glut" node?
  - Garbage collected.



Case 2: Deleting from a BST: Key with one Child

Example: delete("flat"):



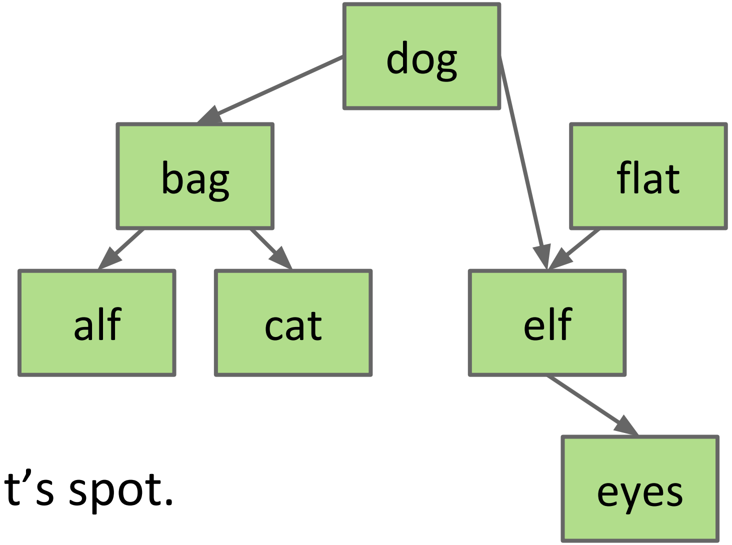
Goal:

- Maintain BST property.
- Flat's child definitely larger than dog.
  - Safe to just move that child into flat's spot.

Thus: Move flat's parent's pointer to flat's child.

Case 2: Deleting from a BST: Key with one Child

Example: delete("flat"):



Goal:

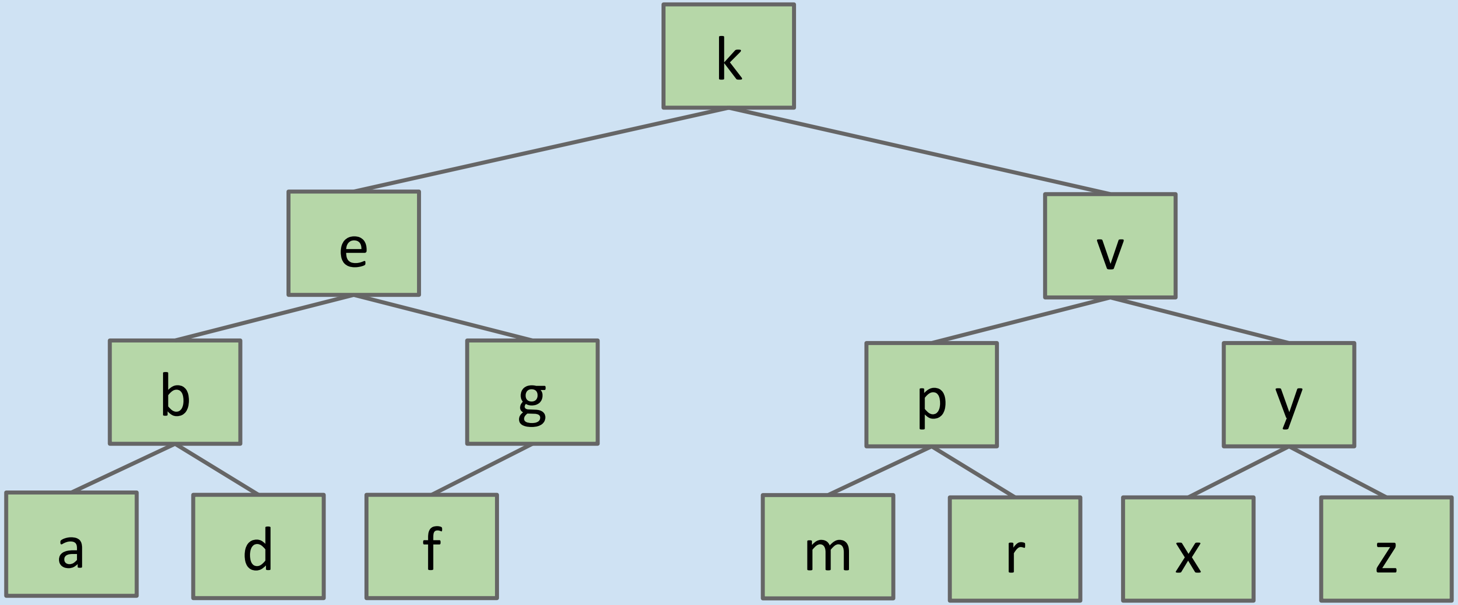
- Maintain BST property.
- Flat's child definitely larger than dog.
  - Safe to just move that child into flat's spot.

Thus: Move flat's parent's pointer to flat's child.

- Flat will be garbage collected (along with its instance variables).

Hard Challenge

Delete k.



Hard Challenge

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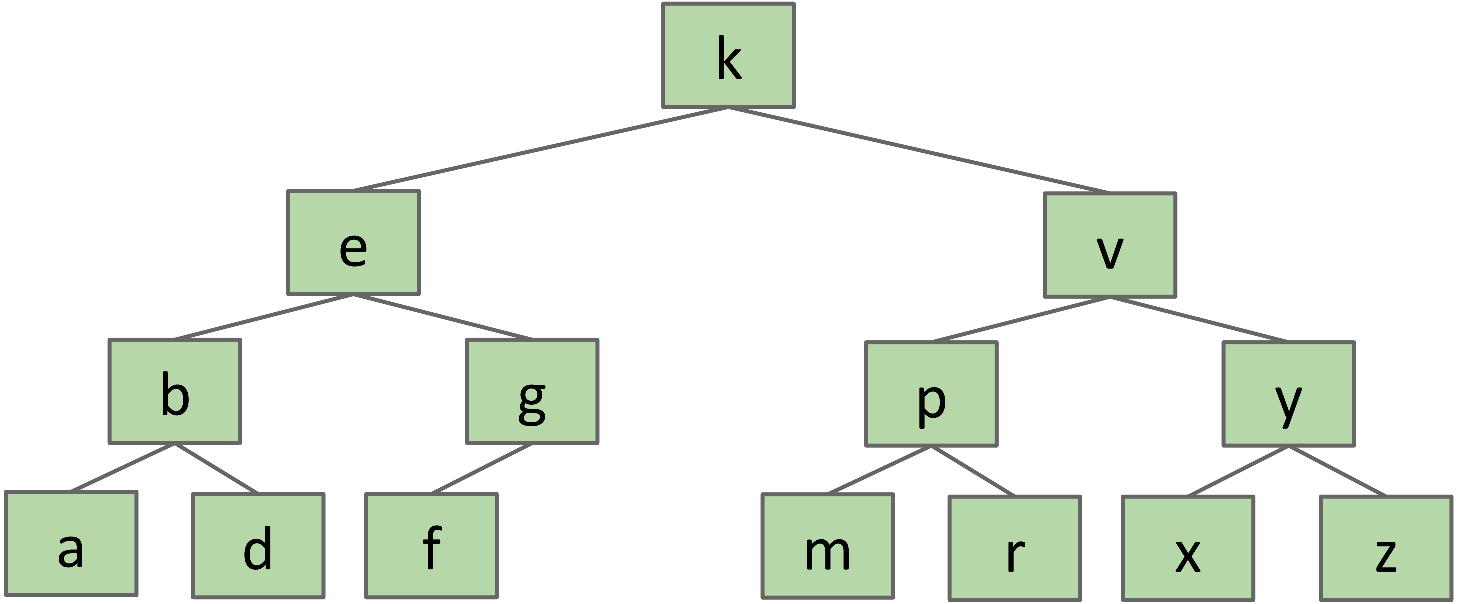
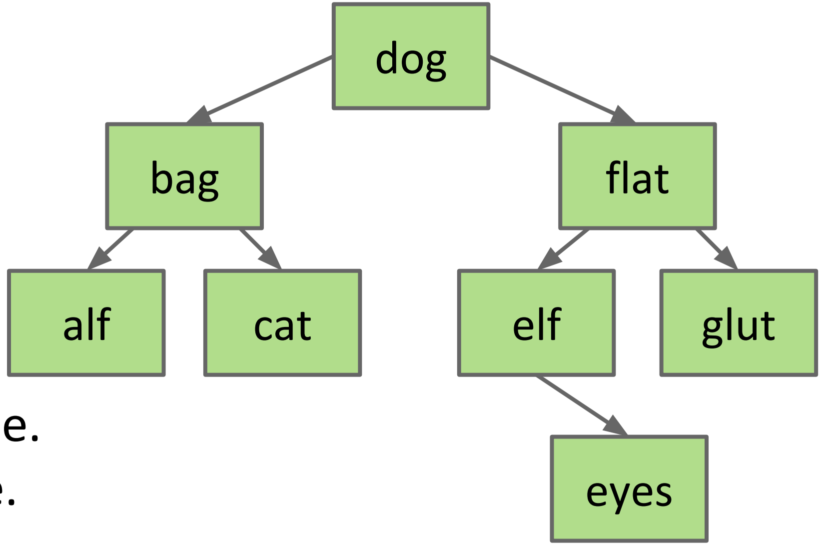


Case 3: Deleting from a BST: Deletion with two Children (Hibbard)

Example: delete(“dog”)



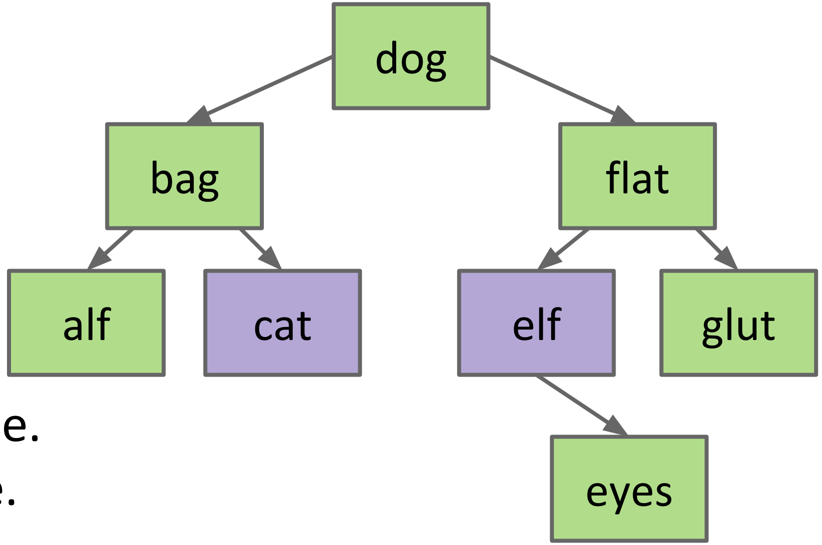
Goal:

- Find a new root node.
- Must be > than everything in left subtree.
- Must be < than everything right subtree.

Would bag work?

Case 3: Deleting from a BST: Deletion with two Children (Hibbard)

Example: delete(“dog”)



Goal:

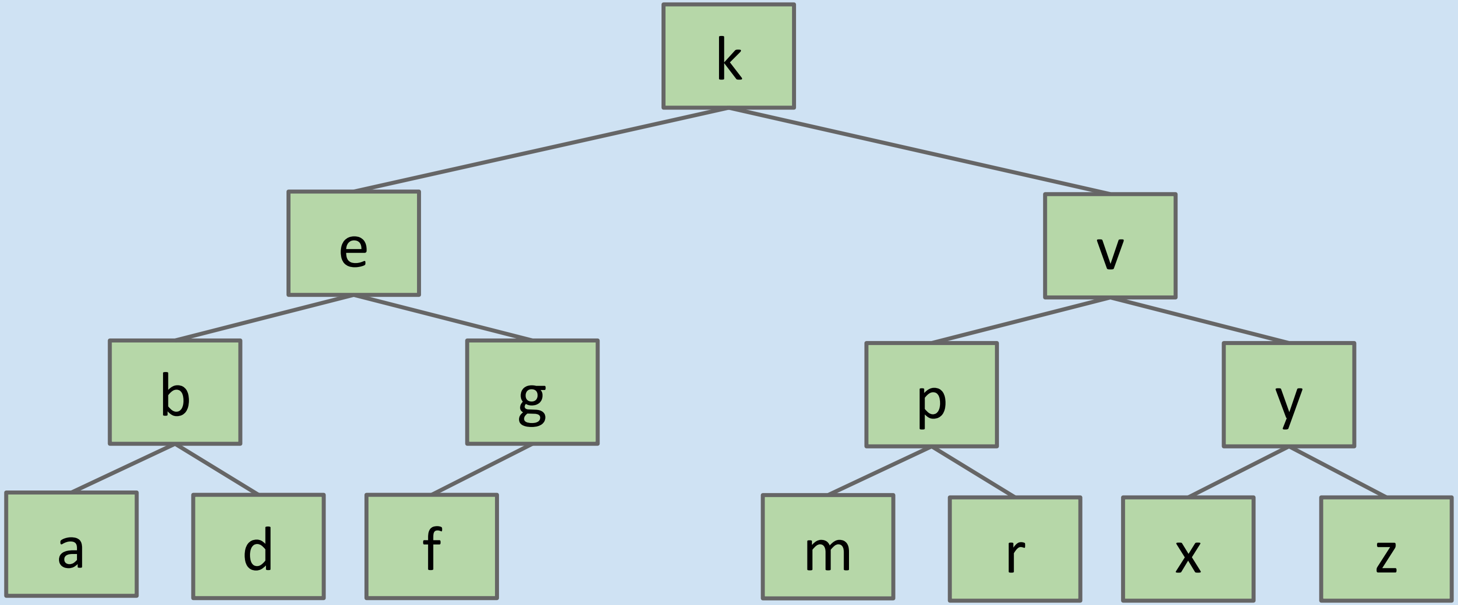
- Find a new root node.
- Must be > than everything in left subtree.
- Must be < than everything right subtree.

Choose either predecessor (“cat”) or successor (“elf”).

- Delete “cat” or “elf”, and stick new copy in the root position:
  - This deletion guaranteed to be either case 1 or 2. Why?
- This strategy is sometimes known as “Hibbard deletion”.

Hard Challenge (Hopefully Now Easy)

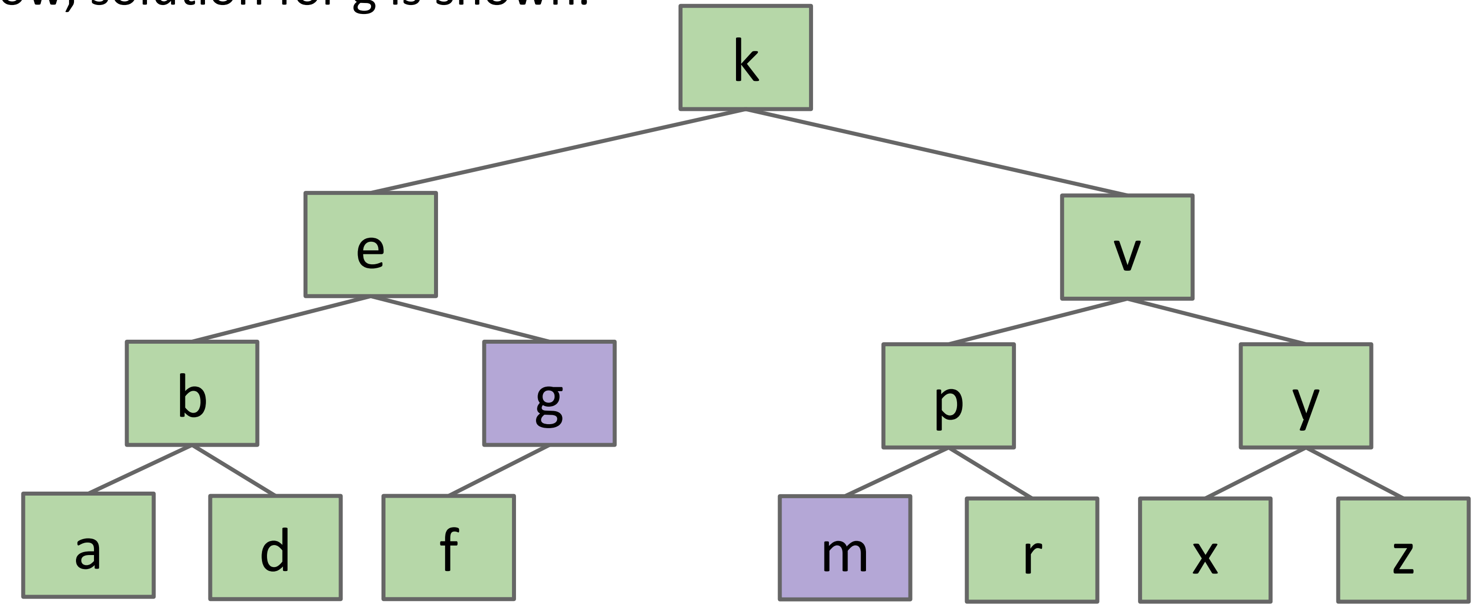
Delete k.



Hard Challenge (Hopefully Now Easy)

Delete k. Two solutions: Either promote g or m to be in the root.

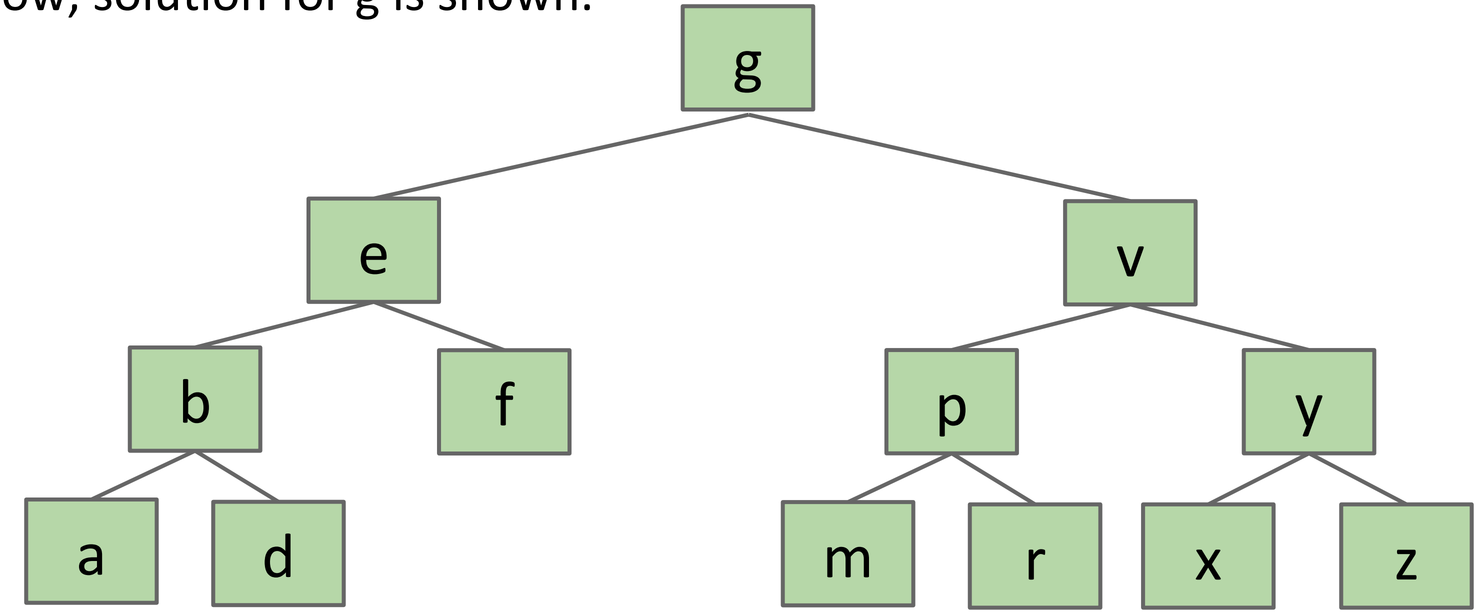
- Below, solution for g is shown.



Hard Challenge (Hopefully Now Easy)

Two solutions: Either promote g or m to be in the root.

- Below, solution for g is shown.

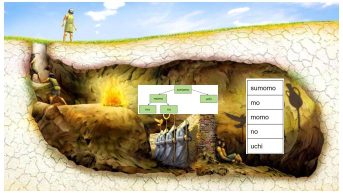


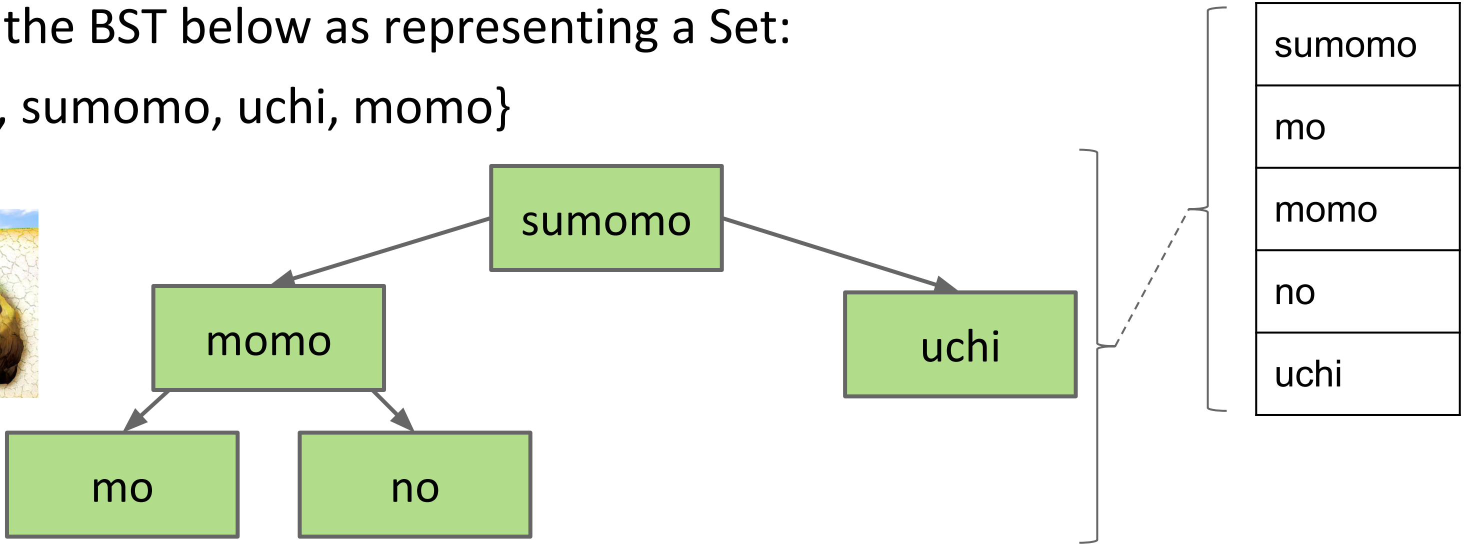
Sets vs. Maps, Summary

Sets vs. Maps

Can think of the BST below as representing a Set:

- {mo, no, sumomo, uchi, momo}





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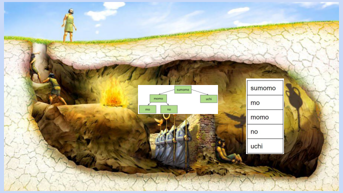
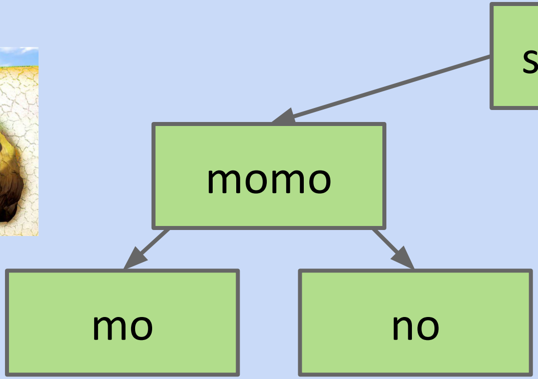




sumomo
mo
momo
no
uchi

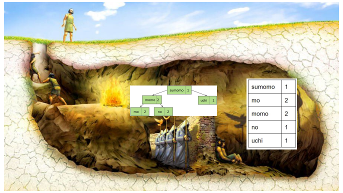
But what if we wanted to represent a mapping of word counts?

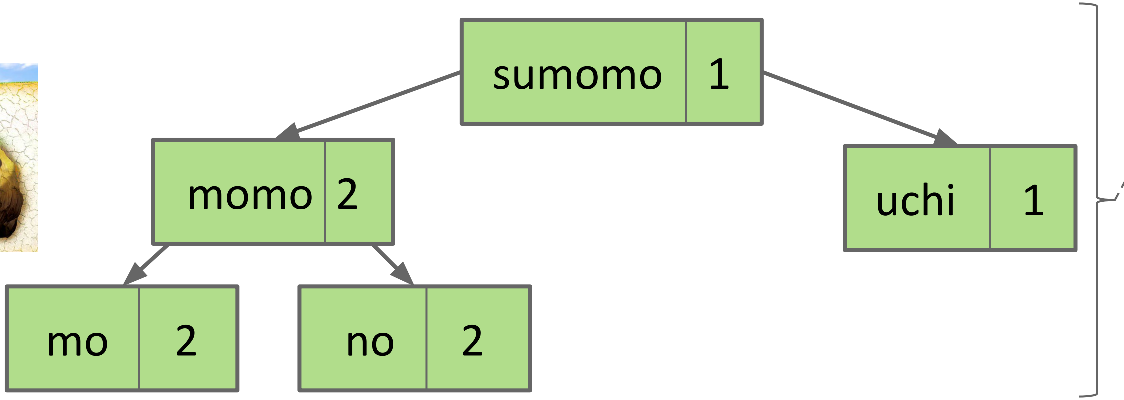
????

sumomo	1
mo	2
momo	2
no	1
uchi	1

Sets vs. Maps

To represent maps, just have each BST node store key/value pairs.





sumomo	1
mo	2
momo	2
no	1
uchi	1

Note: No efficient way to look up by value.

- Example: Cannot find all the keys with value = 1 without iterating over ALL nodes. This is fine.

Summary

Abstract data types (ADTs) are defined in terms of operations, not implementation.

Several useful ADTs: Disjoint Sets, Map, Set, List.

- Java provides Map, Set, List interfaces, along with several implementations.

We've seen two ways to implement a Set (or Map): ArraySet and using a BST.

- ArraySet: $\Theta(N)$ operations in the worst case.
- BST: $\Theta(\log N)$ operations if tree is balanced.

BST Implementations:

- Search and insert are straightforward (but insert is a little tricky).
- Deletion is more challenging. Typical approach is "Hibbard deletion".

BST Implementation Tips

Tips for BST Lab

- Code from class was "naked recursion". Your BSTMap will not be.
- For each public method, e.g. `put(K key, V value)`, create a private recursive method, e.g. `put(K key, V value, Node n)`
- When inserting, always set left/right pointers, even if nothing is actually changing.
- Avoid "arms length base cases". Don't check if left or right is null!

```
static BST insert(BST T, Key ik) {
  if (T == null)
    return new BST(ik);
  if (ik ≺ T.label()))
    T.left = insert(T.left, ik);
  else if (ik ≻ T.label())
    T.right = insert(T.right, ik);
  return T;
}
```

Always set, even if nothing changes!

Avoid "arms length base cases".

```
  if (T.left == null)
    T.left = new BST(ik);
  else if (T.right == null)
    T.right = new BST(ik);
```

Citations

Probably photoshopped binary tree: http://cs.au.dk/~danvy/binaries.html