Курсова работа №1

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1 Условие

Дадена е рекурентната редица $\{a_n\}_{n=1}^{\infty}$, където за всяко $n \in \mathbb{N}$, $a_{n+1} = \frac{2a_n^2 + a_n + 6}{a_n + 6}$ и $a_1 = \lambda \in \mathbb{R} \backslash \{-6\}$. Изследвайте за сходимост редицата $\{a_n\}_{n=1}^{\infty}$ в зависимост от λ.

2 Решение

Нека допуснем, че редицата е сходяща и има граница

$$l = \lim_{n \to \infty} a_n$$

Чрез използване на граничен преход получаваме:

$$l = \frac{2l^2 + l + 6}{l + 6} \iff (l-2)(l-3) = 0$$
$$l_1 = 2 \cup l_2 = 3$$

Пресмятаме:

$$a_{n+1} - a_n = \frac{(a_n - 2)(a_n - 3)}{a_n + 6} \tag{1}$$

Наблюдаваме знакът на израза (1):

$$
\begin{aligned}
a_n \in (-\infty, -6) & \implies a_{n+1} - a_n < 0 \\
a_n \in (-6, 2) & \implies a_{n+1} - a_n > 0 \\
a_n = 2 & \implies a_{n+1} - a_n = 0 \\
a_n \in (2, 3) & \implies a_{n+1} - a_n < 0 \\
a_n = 3 & \implies a_{n+1} - a_n = 0 \\
a_n \in (2, +\infty) & \implies a_{n+1} - a_n > 0
\end{aligned}
$$

Пресмятаме:

$$a_{n+1} - (-6) = \frac{8a_n^2 + 7a_n + 42}{a_n + 6} \tag{2}$$

$$a_{n+1} - 2 = \frac{2(a_n + \frac{3}{2})(a_n - 2)}{a_n + 6} \tag{3}$$

$$a_{n+1} - 3 = \frac{2(a_n + 2)(a_n - 2)}{a_n + 6} \tag{4}$$

$$2a_{n+1} - (-3) = \frac{4a_n^2 + 5a_n + 30}{a_n + 6} \tag{5}$$

$$a_{n+1} - (-2) = \frac{2a_n^2 + 3a_n + 18}{a_n + 6} \tag{6}$$

От уравнение (2):

$$8a_n^2 + 7a_n + 42 > 0, \, \forall a_n \Longrightarrow sign(a_{n+1} + 6) = sign(a_n + 6)$$
$$\{a_n\} \downarrow, \, a_n \in (-\infty, -6)$$
$$\{a_n\} \uparrow, \, a_n \in (-6, +\infty)$$

От уравнения (3) и (5):

$$\{a_n\} \downarrow, \, a_n \in (-\infty, -6)$$
$$\{a_n\} \uparrow, \, a_n \in \left(-6, -\frac{3}{2}\right)$$
$$a_n = 2, \, \forall n \in \mathbb{N} \backslash \{1\}, \, a_1 = -\frac{3}{2}$$
$$\{a_n\} \downarrow, \, a_n \in \left(-\frac{3}{2}, 2\right)$$
$$a_n = 2, \, \forall n \in \mathbb{N}, \, a_1 = 2$$
$$\{a_n\} \uparrow, \, a_n \in (2, +\infty)$$

От уравнения (4) и (6):

$$\{a_n\} \downarrow, \, a_n \in (-\infty, -6)$$
$$\{a_n\} \uparrow, \, a_n \in (-6, -2)$$
$$a_n = 3, \, \forall n \in \mathbb{N} \backslash \{1\}, \, a_1 = -2$$
$$\{a_n\} \downarrow, \, a_n \in (-2, 3)$$
$$a_n = 3, \, \forall n \in \mathbb{N}, \, a_1 = 3$$
$$\{a_n\} \uparrow, \, a_n \in (3, +\infty)$$

2

Така получаваме следното поведение за различните интервали на параметъра:

$$\lambda \in (-\infty, -6) \qquad \Longrightarrow \{a_n\} \downarrow, \lim_{n \to \infty} a_n = -\infty$$

$$\lambda \in (-6, -2) \qquad \Longrightarrow \{a_n\} \uparrow, \lim_{n \to \infty} a_n = +\infty$$

$$\lambda = -2 \qquad \Longrightarrow a_n = 3, \forall n \in \mathbb{N}\backslash\{1\}, \lim_{n \to \infty} a_n = 3$$

$$\lambda \in \left(-2, -\frac{3}{2}\right) \qquad \Longrightarrow \{a_n\} \downarrow, \lim_{n \to \infty} a_n = 2$$

$$\lambda = -\frac{3}{2} \qquad \Longrightarrow a_n = 2, \forall n \in \mathbb{N}\backslash\{1\}, \lim_{n \to \infty} a_n = 2$$

$$\lambda \in \left(-\frac{3}{2}, 2\right) \qquad \Longrightarrow \{a_n\} \uparrow, \lim_{n \to \infty} a_n = 2$$

$$\lambda = 2 \qquad \Longrightarrow a_n = 2, \forall n \in \mathbb{N}, \lim_{n \to \infty} a_n = 2$$

$$\lambda \in (2, 3) \qquad \Longrightarrow \{a_n\} \downarrow, \lim_{n \to \infty} a_n = 2$$

$$\lambda = 3 \qquad \Longrightarrow a_n = 3, \forall n \in \mathbb{N}, \lim_{n \to \infty} a_n = 3$$

$$\lambda \in (3, +\infty) \qquad \Longrightarrow \{a_n\} \uparrow, \lim_{n \to \infty} a_n = +\infty$$

3   Отговор

$$\lim_{n \to \infty} a_n = \begin{cases} -\infty & , \lambda \in (-\infty, -6) \\ 2 & , \lambda \in (-2, 3) \\ 3 & , \lambda \in \{-2, 3\} \\ +\infty & , \lambda \in (-6, -2) \cup (3, +\infty) \end{cases}$$