1. $R$ defined by $a\ R\ b$ if and only if $a - b$ is an even integer from $A = \{3, 6, 9, 12\}$ to $B = \{2,3,4,5,6\}$

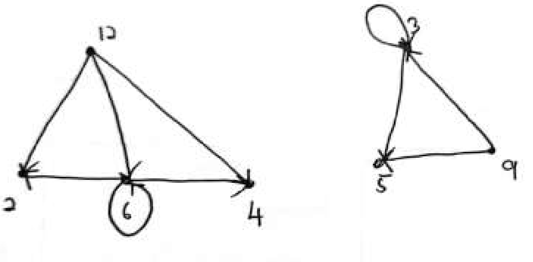
   i) Write the ordered pair of the relation.

   ii) Draw the digraph of the relation.

   iii) List the domain and range of $R$

1 i) $R = \{ (3,3), (3,5), (6,2), (6,4), (6,6), (9,3), (9,5), (12,2), (12,4), (12,6) \}$

ii)




iii) Domain $= \{3, 6, 9, 12\}$

Range $= \{2, 3, 4, 5, 6\}$

2. Determine whether the relation on set $D = \{1,3,8,10,15\}$ is equivalent relations where $x, y \in D, xRy$ if and only if $y$-$x$ is a multiple of 7(Including negative).

2) $R = \{(1,8), (3,10), (8,15), (8,1), (10,3), (15,8), (1,15), (15,1), (1,1), (3,3), (8,8), (10,10), (15,15)\}$

equivalent relations

↳ reflexive, symmetric, transitive

- reflexive: $xRx$ for all $x \in D$ ✓
- symmetric: $\forall x,y \in R$, if $(x,y) \in R$ then $(y,x) \in R$ ✓
- transitive: $\forall x,y,z \in R$, if $(x,y)$ and $(y,z) \in R$, then $(x,z) \in R$ ✗

* R is not equivalent relation

3) i)

$$M_R = \begin{array}{c|c} & \begin{matrix} s & t & u & v \end{matrix} \\ \begin{matrix} s \\ t \\ u \\ v \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

ii)

| | s | t | u | v |
|---|---|---|---|---|
| in-degree | 2 | 2 | 3 | 1 |
| out-degree | 3 | 3 | 2 | 0 |

iii) Relation in partial order

↳ reflexive, antisymmetric, transitive

- R is not reflexive since there is no $(v,v)$
- R is not antisymmetric since $(u,s) \in R$, $(s,u) \in R$
- $$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \odot \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
- for R to be transitive, it needs to be $M_R \odot M_R = M_R$
- R is not transitive
- Relation R is not in partial order

3. Given the digraph of relation $R$ as in Figure 1.

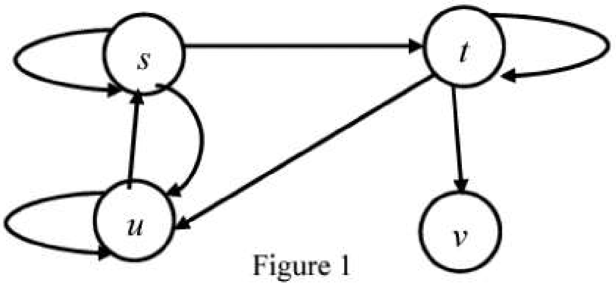



Figure 1

i) What is matric of the relation, $M_R$ that represent diagraph in Figure 1.

ii) List in-degrees and out-degrees of all vertices.

iii) Is it the relation of R is an partial order? Check all variance Justify for answer

3) i)

$$M_R = \begin{array}{c|c} & \begin{array}{cccc} s & t & u & v \end{array} \\ \begin{array}{c} s \\ t \\ u \\ v \end{array} & \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

ii)

| | s | t | u | v |
|---|---|---|---|---|
| in-degree | 2 | 2 | 3 | 1 |
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- for R to be transitive, it needs to be $M_R \odot M_R = M_R$
- R is not transitive
- Relation R is not in partial order

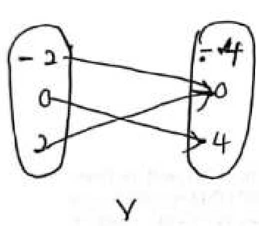
4. Let $X=\{-2, 0, 2\}$ and $Y=\{-4, 0, 4\}$. For each $x \in X$, define functions $v: X \to Y$ and $w: X \to Y$ by:

$v(x) = 4 - x^2$

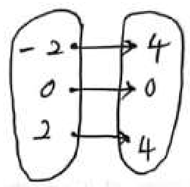
$w(x) = 2x$

Determine if $v$ and $w$ are one-to-one, onto $Y$, and/or bijection.

4




Y




W

- V is not one-to-one function because Ex: $V(2)=V(-2)=0$ where $-2 \neq 2$. And not onto Y because X only onto $\{0,4\}$.
- W is one-to-one function and onto. since w is one-to-one and onto, w is bijection.

5. Let $f$ dan $g$ be functions from the positive integers to the positive integers defined by the equations,

$$f(x) = 7x - 2, \qquad g(x) = \frac{2}{3}x$$

i) Find the inverse of $g(x)$.

ii) Find the compositions $(gogof)(x)$

5 i) $(x, y) \in f$, so $y = \frac{2}{3}x$

$\cancel{\&}$ $x = \frac{3}{2}y$

$g^{-1}(y) = \frac{3}{2}y$ *

ii) $(g \circ g \circ f)(x) = g(g(f(x)))$

$= g(g(7x-2))$

$= g\left(\frac{2}{3}(7x-2)\right)$

$= \frac{2}{3}\left(\frac{2}{3}(7x-2)\right)$

$= \frac{4}{9}(7x-2)$ *.

6. As a lead computer scientist in a chemical industry plant, you are assigned to design and develop algorithms that simulate chemical reaction processes. Two chemicals $A$ and $B$ are combined to produce a third chemical $C$. The initial temperature $F_0$, of chemical $A$, is 5.0 Fahrenheit and the initial temperature $F_1$ for chemical $B$ is 4.5. When chemicals $A$ and $B$ are combined to produce chemical $C$, the increment in each minute $t = 0, 1, 2, 3$ ..., which chemical $C$ warms up to room temperature is a recurrence sequence, with $F_0$ and $F_1$ as initial conditions. For $t \geq 2$, this recurrence sequence is found by summing the previous element of the sequence ($t$-1), with one-fifth of the previous two elements of the sequence ($t$-2). From the above given information,

i) Find the recurrence relation of chemical $C$ that models the warming to room temperature.

ii) Using the recurrence relation obtained in (a), list down the sequence from $F_0, F_1, F_2, \ldots F_5$.

6 i) $F_t = F_{t-1} + 1/5\,(F_{t-2})$

ii) $F_0 = 5.0$

$F_1 = 4.5$

$F_2 = F_1 + 1/5\,(F_0) = 4.5 + 1/5(5.0)$

$= 5.5$

$F_3 = F_2 + 1/5\,(F_1) = 5.5 + 1/5\,(4.5)$

$= 6.4$

$F_4 = F_3 + 1/5\,(F_2) = 6.4 + 1/5\,(5.5)$

$= 7.5$

$F_5 = F_4 + 1/5\,(F_3) = 7.5 + 1/5\,(6.4)$

$= 8.78$.

$\therefore$ $F_0 = 5.0$, $F_1 = 4.5$, $F_2 = 5.5$, $F_3 = 6.4$, $F_4 = 7.5$, $F_5 = 8.78$.

7. Write a recursive algorithm to find the $n$ term of the sequence defined by $w_0 = 5, w_1 = 7$ and $w_n = 2w_{n-1} + w_{n-2}$ for n $\geq$ 2 .Trace the algorithm for $n = 4$.

7.
```
w(n) {
    if (n = 0)
        return 5
    else if (n=1)
        return 7
    else
        return 2(w(n-1)) + w(n-2)
}
```

for n=4,
because $n \neq 0$ and $n \neq 1$,
return $2(w(3)) + w(2)$

n= 3, because $n \neq 0$ and $n \neq 1$,
return $2(w(2)) + w(1)$

n=2, because $n \neq 0$ and $n \neq 1$,
return $2(w(1)) + w(0)$

n=1, return 7

n=0, return 5

for n=2, return $2(7) + 5 = 19$

for n=3, return $2(19) + 7 = 45$

for n=4, return $2(45) + 19 = 109$.