SECI1013-02

DISCRETE STRUCTURE

ASIGNMENT 1

(GROUP ASSIGNMENT)

Group Members:

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1. Let the universe be the set $U = \{x \in Z, 0 \leq x \leq 20\}$,
$A, B, and\ C\ denote\ t\ \ e\ subsets\ of\ U$,

$$A = \{p|p \in prime\ numbers, 0 \leq p \leq 20\},$$
$$B = \{e|e \in even\ numbers, 10 \leq e \leq 20\},$$
$$C = \{o|o \in odd\ numbers, 0 \leq o \leq 10\}$$

Write down all possible outcomes with the following set operations:

a. $A \cap C \cup B$
b. $P(A \cap B \cup C)$
c. $A \quad C$

Determine the cardinality of the following set:

d. $|A|, |B|, |C|$
e. $|P(A \cap C)|$

Answer the following set operations with True or False:

f. $B \subset C'$
g. $(A \cup B \cup C) \subseteq U$

1. $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

$A = \{2, 3, 5, 7, 11, 13, 17, 19\}$

$B = \{10, 12, 14, 16, 18, 20\}$

$C = \{1, 3, 5, 7, 9\}$

a) $A \cap C \cup B = \{3, 5, 7, 10, 12, 14, 16, 18, 20\}$

b) $(A \cap B \cup C) = \{1, 3, 5, 7, 9\}$

$n = 5$

To check, $|P(A \cap B \cup C)| = 2^n = 2^5 = 32$

$P(A \cap B \cup C) = \{\emptyset, \{1\}, \{3\}, \{5\}, \{7\}, \{9\}, \{1,3\}, \{1,5\},$
$\{1,7\}, \{1,9\}, \{3,5\}, \{3,7\}, \{3,9\}, \{5,7\},$
$\{5,9\}, \{7,9\}, \{1,3,5\}, \{1,3,7\}, \{1,3,9\},$
$\{1,5,7\}, \{1,5,9\}, \{1,7,9\}, \{1,3,5,7\}, \{1,3,5,9\},$
$\{3,5,7,9\}, \{5,7,9\}, \{3,7,9\}, \{3,5,7\}, \{3,5,9\},$
$\{1,3,5,7,9\}, \{1,7,3\}, \{1,7,9\}\}$

c) $A - C = \{2, 11, 13, 17, 19\}$

d) $|A| = 8$

$|B| = 6$

$|C| = 5$

e) $A \cap C = \{3, 5, 7\}$

$n = 3$

$|P(A \cap C)| = 2^n$

$= 2^3$

$= 8$

Answer the following set operations with True or False:

f. $B \subset C'$

g. $(A \cup B \cup C) \subseteq U$

f) $B = \{10, 12, 14, 16, 18, 20\}$

$C' = \{0, 2, 4, 6, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

$B \subseteq C'$ is True.

g) $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

$A = \{2, 3, 5, 7, 11, 13, 17, 19\}$

$B = \{10, 12, 14, 16, 18, 20\}$

$C = \{1, 3, 5, 7, 9\}$

$A \cup B \cup C = \{1, 2, 3, 5, 7, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20\}$

$(A \cup B \cup C) \subseteq U$ is True.

2. Let $A, B, and\ C\ denote\ t\ \ e\ subsets\ of\ U$,
then show the following set operations and statements are equal or not using properties of set/set identities or Venn diagram:

a. $(A \quad C') \cup (B \quad C) = A \cup B$

b. $(A \cap B) \cup (A \quad B) = A$

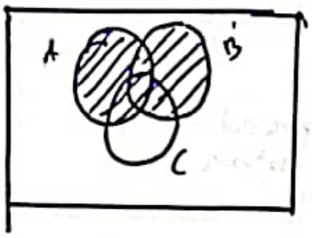
a) Show that $(A-C') \cup (B-C) = A \cup B$

$(A-C') \cup (B-C) = A \cup B$

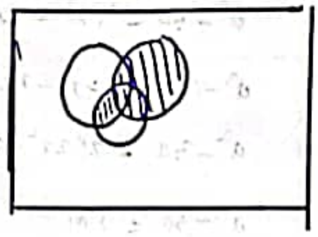
$(A \cap C) \cup (B \cap C') = A \cup B$

Venn diagram

$A \cup B =$



$(A \cap C) \cup (B \cap C') =$



Answer : $(A-C') \cup (B-C) \neq A \cup B$

b) Show that $(A \cap B) \cup (A-B) = A$:

$(A \cap B) \cup (A-B) = A$

$(A \cap B) \cup (A \cap B') = A$

$A \cap (B \cup B') = A$

$A \cap U = A$

$A = A$

3. Books in the university library are categorized into Social Science (S), Science & Technology (T) and Engineering (E). Alphabets a-g is used to catalogue books in category S, alphabets h-q for category T and the p-z for category E. However, alphabets o, u, x and i are excluded. Some alphabets are used more than once in different categories.

Determine the following sets.

a) S, T, E

b) $S \times (T \cap E)$

3. a) $S = \{a, b, c, d, e, f, g\}$

$T = \{h, j, k, l, m, n, p, q\}$

$E = \{p, q, r, s, t, v, w, x, y, z\}$

b) $S \times (T \cap E)$

$(T \cap E) = \{p, q\}$

$S \times (T \cap E) = \{(a,p), (a,q), (b,p), (b,q), (c,p), (c,q), (d,p), (d,q),$
$(e,p), (e,q), (f,p), (f,q), (g,p), (g,q)\}$

4. Let $A = \{a, \{a\}, b, \{a, b\}\}$. State whether the following statement is TRUE or FALSE

a) $\{a\} \subseteq A$

b) $\{a, b\} \in A$

a) A has element "a", Thus $\{a\}$ is subset of Element A, Therefore $\{a\} \subseteq A$ is True.

b) Element of A are "a", "$\{a\}$", "b", "$\{a, b\}$"
Therefore, $\{a, b\} \in A$ is True.

5. State whether $Q \equiv R$ or not.
Show the truth table as the proof of your work for each of the following statements:

a. $Q = (p \wedge r) \vee (q \vee \neg r), R = (p \vee q) \vee \neg r$

b. $Q = (p \wedge r) \vee \neg(p \wedge \neg q), R = (p \wedge r) \rightarrow (q \vee r)$

5. a) $Q = (p \wedge r) \vee (q \vee \neg r)$ $R = (p \vee q) \vee \neg r$

p	q	r	$\neg r$	$(p \wedge r)$	$(q \vee \neg r)$	$(p \wedge r) \vee (q \vee \neg r)$	$(p \vee q)$	$(p \vee q) \vee \neg r$
T	T	T	F	T	T	T	T	T
T	T	F	T	F	T	T	T	T
T	F	T	F	T	F	T	T	T
T	F	F	T	F	T	T	T	T
F	T	T	F	F	T	T	T	T
F	T	F	T	F	T	T	T	T
F	F	T	F	F	F	F	F	F
F	F	F	T	F	T	T	F	T

Q is logically equivalent to R
$Q \equiv R$

5. State whether $Q \equiv R$ or not.

Show the truth table as the proof of your work for each of the following statements:

a. $Q = (p \wedge r) \vee (q \vee \neg r), R = (p \vee q) \vee \neg r$

b. $Q = (p \wedge r) \vee \neg(p \wedge \neg q), R = (p \wedge r) \rightarrow (q \vee r)$

b) $Q = (p \wedge r) \vee \neg(p \wedge \neg q)$ $R = (p \wedge r) \rightarrow (q \vee r)$

p	q	r	$(p \wedge r)$	$\neg q$	$(p \wedge \neg q)$	$\neg(p \wedge \neg q)$	$(p \wedge r) \vee \neg(p \wedge \neg q)$
T	T	T	T	F	T	F	T
T	T	F	F	F	F	T	T
T	F	T	T	T	T	F	T
T	F	F	F	T	T	T	F
F	T	T	F	F	F	T	T
F	T	F	F	F	F	T	T
F	F	T	F	T	F	T	T
F	F	F	F	T	F	T	T

$(q \vee r)$	$(p \wedge r) \rightarrow (q \vee r)$
T	T
T	T
T	T
F	T
T	T
T	T
T	T
F	T

Q is ~~logically~~ not logically equivalent to R.

$Q \not\equiv R$.

6. Let $D = \{1, 3, 5, 7, 8, 9\}$. Decide whether each of the following statements is true for all elements of D. For each that are not, give a counterexample. That is, provide an element in D for which the statement is not true.

a) x is even and $x>7$

b) x is not odd and $x \le 7$

6. $D = \{1,3,5,7,8,9\}$

a) x is even and $x > 7$:

This statement is false. A counterexample is $x=9$, which is greater than 7 but not even.

b) x is not odd and $x \le 7$:

This statement is false. A counterexample is $x=1$, which is less than or equal to 7 but odd.

7. Let P(x) be the statement "x can speak Arabic" and let Q(x) be the statement "x knows computer language C++". Express the following sentence in term of P(x), Q(x), quantifiers, and logical connectives. The domain for quantifiers consists of all students at your faculty.

There is a student at your faculty who can speak Arabic and who knows C++.

P(x) : "x can speak Arabic"

Q(x) : "x Knows computer language C++"

$\therefore \exists x \, (P(x) \wedge Q(x))$

8. Prove the following theorem using direct proof method.

For all integers, if a is odd then $a^2 \quad 3a$ is even

8. For all integers a, if a is odd, then a^2-3a is even

$\forall x(P(x) \rightarrow Q(x))$ $\quad$ $P(x) = a$ is odd

$Q(x) = a^2-3a$ is even

odd : $a = 2x+1$

even : $n = 2m$

$\therefore a^2-3a = 4x^2+4x+1-6x-3$

$a^2-3a = 4x^2-2x-4$

$a^2-3a = 2(2x^2-x-2) \rightarrow$ can be represented as m bcs $2x^2-x-2$ is integer

$a^2-3a = 2m$

proven that a^2-3a is even

9. n^2 ia an odd integer then n is odd. Proof using contradiction proof method.

9. n^2 is an odd integer then n is odd.

= assume if n^2 is an odd integer, then n is even. $n=2k$.

thus, $n^2 = 4k^2$ which is divisible by 4. n^2 is even.

this contradicts the assumption that n is odd, therefore our assumption is false.

$\therefore$ n must be odd if n^2 is odd.