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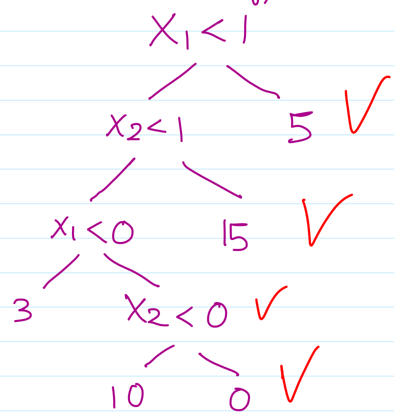
For all decision tree, left edge means true, right edge means false

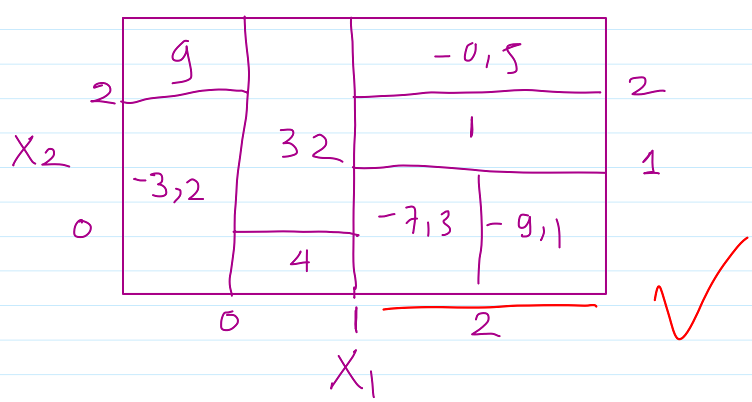
Sorry for bad handwriting, was rushing

3a



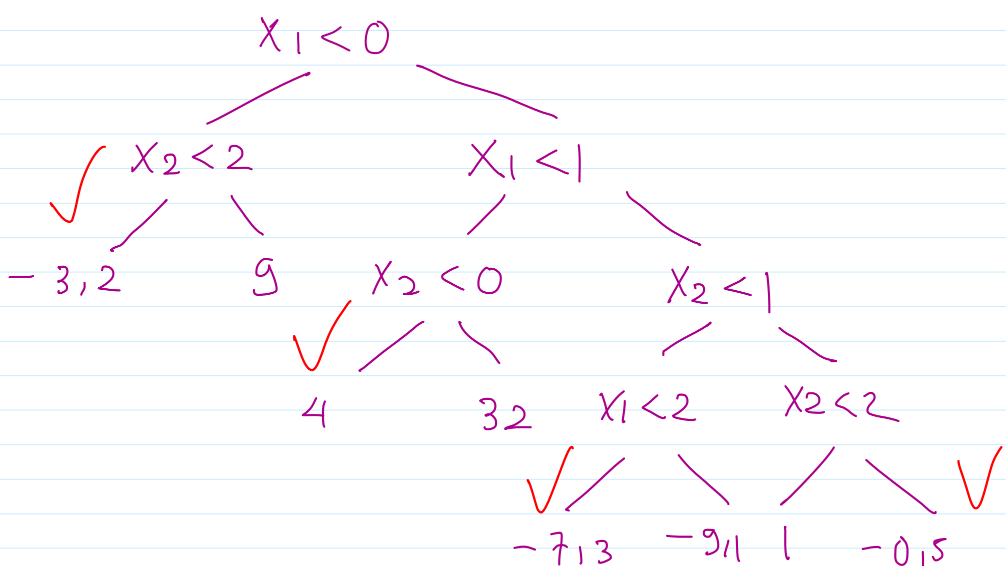
2

b



3

c.



2

=> Ex. 3: $\frac{7}{7}$, very good!

~~1a~~ $\cancel{P(Y=y \mid X_1, X_2, X_3) = \frac{P(Y=y)\, P(X_1, X_2, X_3 \mid Y=y)}{\;}}$

1a $P(Y=y \mid X_1, X_2, X_3) = \dfrac{P(Y=y)\, P(X_1, X_2, X_3 \mid Y=y)}{P(X_1, X_2, X_3)}$

$= \dfrac{P(Y=y)\, P(X_1 \mid Y=y)\, P(X_2 \mid Y=y)\, P(X_3 \mid Y=y)}{P(X_1, X_2, X_3)}$ ✓ 1

b $P(\text{Play Tennis} = \text{yes} \mid \text{hot}, \text{normal}, \text{weak})$

$\dfrac{P(PT = \text{yes})\, P(\text{hot}, \text{normal}, \text{weak} \mid PT = \text{yes})}{P(\text{hot}, \text{normal}, \text{weak})}$

$\dfrac{P(PT = \text{yes})\, P(\text{hot} \mid \text{yes})\, P(\text{normal} \mid \text{yes})\, P(\text{weak} \mid \text{yes})}{P(\text{hot}, \text{normal}, \text{weak})}$

$\dfrac{5/10 \cdot (2/5)(3/5)(4/5)}{2/10}$ ✓ 1

-> Where do you get that value from?

$60/125 < 62{,}5/125 = 50\%$

Thus we will not play tennis -> wrong => -1

↳ You need to calculate the prob. for Y=No as well => -1

1c Why problem?

Day 7	Cool	High	Strong	No
Day 8	Hot	Normal	Strong	No
Day 9	Hot	High	Strong	Yes

Assume we only have 3 days of data

as shown above

Now I want to know whether I should play tennis when the temperature is cool, humidity is normal and wind is weak.

$$P(PT=yes)\, P(cool \mid yes)\, P(normal \mid yes)\, \underbrace{P(weak \mid yes)}_{0}$$

After multiplying with 0, we get a probability of 0 thus we do not play tennis (however we should) ✓ 1

Solution:

There's a method called Laplacian Smoothing (yes I got this from the internet), where you make sure to add a combination where it doesn't exist yet.
Yes our collection of data will differ as a result. However if we have a large enough of datas, adding one shouldn't cause any trouble (But in our case with 3 days, it is a problem) ✓ 1

=> Ex. 1: $\frac{4}{6}$

2. a. Estimating using method of moments

$$\mu_0 = m = \frac{1+2+(-1)+2+1}{10} = \frac{5}{10}$$

$$\mu_1 = m = \frac{4+3+2+2+3+3+4+4+3+4}{10} = \frac{32}{10}$$ ✓ 1

b. $\sigma^2 = \frac{1}{\ldots} \sum^{K} \sum^{n_k} (x_i \ldots$

b $\sigma^2 = \frac{1}{n-K} \sum_{k=1}^{K} \sum_{i=1}^{n_k} (x_i - \mu_k)^2$ 10

$\frac{1}{20-2} \sum_{1}^{2} \sum_{1}^{n_k} (x_i - \mu_k)^2$

$\frac{1}{18}\Big([1-0.5]^2 + [0-0.5]^2 + [0-0.5]^2 + [2-0.5]^2 + [0-0.5]^2 +$
$[-1-0.5]^2 + [2-0.5]^2 + [0-0.5]^2 + [0-0.5]^2 + [1-0.5]^2 +$
$[4-3.2]^2 + [3-3.2]^2 + [2-3.2]^2 + [2-3.2]^2 + [3-3.2]^2 +$
$[3-3.2]^2 + [4-3.2]^2 + [4-3.2]^2 + [3-3.2]^2 + [4-3.2]^2\Big)$

$\frac{1}{18}\Big(\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{9}{4} + \frac{1}{4} + \frac{9}{4} + \frac{9}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} +$
$\frac{16}{25} + \frac{1}{25} + \frac{36}{25} + \frac{36}{25} + \frac{1}{25} + \frac{1}{25} + \frac{16}{25} + \frac{16}{25} + \frac{1}{25} + \frac{16}{25}$

$\frac{1}{18}\Big(\frac{7}{4} + \frac{27}{4} + \frac{64}{25} + \frac{4}{25} + \frac{72}{25}\Big)$

$\frac{1}{18}\Big(\frac{141}{10}\Big) = \frac{141}{180} \approx 0.78 \checkmark$ 0.5

c. $P(Y|X) = \frac{P(Y) \cap P(X)}{P(X)} \cdot \frac{P(Y)}{P(Y)}$

$= \frac{P(Y) \cap P(X)}{P(Y)} \cdot \frac{P(Y)}{P(X)}$

$= P(X|Y) \cdot \frac{P(Y)}{P(X)}$

$P(Y=0 | X) = P(X | Y=0) \cdot \frac{P(Y=0)}{P(X)}$

$= P(X|Y=0) \cdot \frac{\cancel{(1/2)}}{P(X|Y=0)\cancel{\frac{1}{2}} + P(X|Y=1)\cancel{\frac{1}{2}}}$

$= \frac{e^{-\frac{1}{2}\left(\frac{x-0.5}{0.805}\right)^2}}{\quad}$

$$= \frac{e^{-\frac{1}{2}\left(\frac{x-3,2}{0,805}\right)^2}}{e^{-\frac{1}{2}\left(\frac{x-0,5}{0,805}\right)^2} + e^{-\frac{1}{2}\left(\frac{x-3,2}{0,805}\right)^2}}$$ ✓ 0.5

$$P(Y=1 \mid X) = P(X \mid Y=1) \cdot \frac{P(Y=1)}{P(X)}$$

$$= \frac{e^{-\frac{1}{2}\left(\frac{x-3,2}{0,805}\right)^2}}{e^{-\frac{1}{2}\left(\frac{x-0,5}{0,805}\right)^2} + e^{-\frac{1}{2}\left(\frac{x-3,2}{0,805}\right)^2}}$$ ✓ 0.5

d. Posterior same Note $P(Y=0) = P(Y=1) = 1/2$ 0.5

$$P(Y=0 \mid X) = P(Y=1 \mid X)$$

$$\frac{P(X \mid Y=0)\,\cancel{P(Y=0)}}{P(X \mid Y=0)\,\cancel{P(Y=0)} + P(X \mid Y=1)\,\cancel{P(Y=1)}} = \frac{P(X \mid Y=1)\,\cancel{P(Y=1)}}{P(X \mid Y=1)\,\cancel{P(Y=1)} + P(X \mid Y=0)\,\cancel{P(Y=0)}}$$

$$P(X \mid Y=0) = P(X \mid Y=1)$$

$$\frac{1}{\sqrt{2\pi\sigma^2}} e^{\left(-\frac{1}{2}\left(\frac{x-0{,}5}{\sigma}\right)^2\right)} = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\left(-\frac{1}{2}\left(\frac{x-3{,}2}{\sigma}\right)^2\right)}$$

$$(x-0{,}5)^2 = (x-3{,}2)^2$$

$$x^2 - x + 0{,}25 = x^2 - 6{,}4x + 10{,}24$$

$$5{,}4x = 9{,}9$$

$$x = \frac{9{,}9}{5{,}4}$$

$$x = \frac{11}{6} \approx 1.83 \checkmark \ 1.5$$

$x=2$ then should be in class $Y=1$ ✓ 0.5

e.

		prediction	
		$Y=0$	$Y=1$
truth	class 0	0	2
	class 1	1	0

$P(Y=0 \mid X=2)$ = see formula in c
substitute $x=2$

$= 0{,}3735$

$P(Y=1 \mid X=2) = 0{,}6265$

$\text{loss}(\hat{Y}=0) = 0 + P(Y=1 \mid X=2)$

$$\text{loss}(\hat{Y}=0) = 0 + p(Y=1 \mid X=2)$$

$$\text{loss}(\hat{Y}=1) = 2 \cdot P(Y=0 \mid X=2) + 0$$

$$\text{loss}(\hat{Y}=0) = 0{,}6265 \checkmark$$

$$\text{loss}(\hat{Y}=1) = 0{,}7470 \checkmark \quad 1$$

Since $\text{loss}(\hat{Y}=0) < \text{loss}(\hat{Y}=1)$
we decide than $X=2$ belongs
in class $Y=0$ ✓ 1

=> Ex. 2: $\frac{7}{7}$

↳ $\frac{18}{20}$, nice!