Quantum Computing
Spring'24, Plaksha University

**Assignment 1**
**Due Date: 13 Feb'24**

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1. **Magnetic moment and angular momentum**

   (a) Consider a point particle of mass $m$, charge $q$ revolving in a circle of radius $r$ with a speed $v$. Derive an expression for the magnetic moment $\mu$ in terms of the angular momentum $L$.

   (b) Show that for a solid spherical ball of mass $m$ and charge $q$ uniformly distributed on the surface which is rotating around an axis passing through it's center, the magnetic moment $\mu$ is related to the angular momentum $L$ by the relation $\mu = \frac{5q}{6m}L$.

2. **Quantum States and Gates**
   Consider the normalized quantum states

$$|+\psi\rangle = \cos\frac{\theta}{2}|+z\rangle + e^{i\phi}\sin\frac{\theta}{2}|-z\rangle, \tag{1}$$

$$|-\psi\rangle = \sin\frac{\theta}{2}|+z\rangle - e^{i\phi}\cos\frac{\theta}{2}|-z\rangle. \tag{2}$$

   (a) Show that $\langle+\psi|+\psi\rangle = 1, \langle-\psi|-\psi\rangle = 1, \langle-\psi|+\psi\rangle = 0, \langle+\psi|-\psi\rangle = 0$.

   (b) Does the Quantum-NOT gate flip $|+\psi\rangle$ to $|-\psi\rangle$? Can we design a Quantum NOT Gate that flips any Qubit to it's inverse?

3. **Classical Probabilistic States**
   Write a Python function named `linevo` that calculates the final probabilistic state of a given classical random experiment. The function should take **any** valid probabilistic operator representing the experiment, an initial probabilistic state, and the number of times the experiment is conducted. The objective is to return the final probabilistic state after multiple applications of the operator.

   To check the correctness of your implementation, consider the following experiment: You have a fair four-sided dice that was initially in the state *1 facing upwards* when it was taken away from you and secretly rolled three times. The dice rolling experiment can be formalized as follows:

   The operation of rolling a fair four-sided dice can be represented as

$$A = \begin{bmatrix} 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 & 0.25 \end{bmatrix}$$

   The state of the dice when it was with you can be represented as

$$p = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

   After secretly rolling the dice three times, your prediction of the final state would be $A^3p = q$. The function you implement should return $q$ given $A$, $p$, and 3. Ensure that $q$ is indeed a valid probabilistic state.