DS3001: ADVANCED STATISTICS: ASSIGNMENT 2

SAMPLING DISTRIBUTIONS AND ESTIMATION

QUESTION 1

Let $X_1, \ldots, X_n$ be i.i.d. $N(\mu, \sigma^2)$ random variables. Consider the problem of estimating the population standard deviation using the following three estimators:

(i) The sample standard deviation $S = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2}$,

(ii) The mean deviation $M = \frac{1}{n}\sum_{i=1}^{n}\left|X_i - \bar{X}\right|$, and

(iii) 0.1 Trimmed standard deviation obtained by discarding the $[0.1n/2]$ smallest and $[0.1n/2]$ largest observations initially and then computing the standard deviation of the remaining observations.

Consider the following three population models:

(a) $N(0, 1)$,

(b) mixture of two normals: $0.9N(0, 1) + 0.1N(0, 3)$,

(c) mixture of a normal and a Cauchy: $.95N(0, 1) + .05C(0, 1)$.

(1) Obtain the sampling distribution of the three estimators for all three models for $n = 20$. Compute the mean and variance of the estimators based on the sampling distribution. [Use at least 5000 replications]

(2) Compute the Bias and the MSE for all three estimators.

(3) What observations can you make about the properties of the estimators?

*Please turn over to check QUESTION 2.

QUESTION 2

The average amount of rainfall (in inches) for 226 storms is provided in the file Rainfall.xlsx.

(1) Fit a $Gamma(\alpha, \beta)$ distribution to the data and estimate the parameters using both the method of moments and maximum likelihood. How would you assess the "goodness-of-fit" of the Gamma distribution to the data?

(2) Draw a sample of size 226 from a Gamma distribution with parameters $(\hat{\alpha}, \hat{\beta})$, where $(\hat{\alpha}, \hat{\beta})$ are the MLEs computed in part (a). Find the MLEs of (α, β) for this new sample. Repeat this process 1000 times. The histogram of the 1000 estimates of the two parameters is an estimate of their sampling distribution. This is called the Parametric Bootstrap method.

(3) Draw a sample of size 226 from a Gamma distribution with parameters (α^*, β^*), where (α^*, β^*) are the method of moments estimators computed in part (a). Find the method of moments estimators of (α, β) for this new sample. Repeat this process 1000 times. The histogram of the 1000 estimates of the two parameters is an estimate of their sampling distributions.

(4) What observations can you make about the properties of the estimators? Compare the bias and MSE.