Brian Allison
Assignment 3
CS325

Problem 1: *(2 points)* **Rod Cutting**: (from the text CLRS) 15.1-2

Ans:
Consider an array P that contains the price of a rod for a given length i at each index i.

length (i)	1	2	3	4	5	6	7	8
price (P[i])	1	3	4	8	11	12	13	14
price/i	1	1.5	1.3	2	2.2	2	1.9	1.8

Now, suppose we want to find the optimal number of cuts to maximize the price at length 8. Based on the greedy strategy, the first cut we make will be at the length that gives the highest price per inch. That means we would make a cut at 5.

Again, we make a cut that gives us the highest value per inch. Since we only have 3 inches left, we have to select 3, 2, or 1. We make a 2 inch cut for a value of 3. Then, we make a final 1 inch cut, which is our only option. We then have a combined value of
P[5] + P[2] + P[1] = 11 + 2 + 1 = 14

If the greedy strategy always worked, 14 would be the optimal value for a rod of length 8. However, this is not the case. We could cut an 8 inch rod at 4 inches and keep the other 4 inch half for a combined value of
P[4] + P[4] = 8 + 8 = 16

Therefore, the greedy strategy would not always work.

Problem 2: *(3 points)* **Modified Rod Cutting**: (from the text CLRS) 15.1-3
Code from textbook was used as a reference for this problem

Ans:

Pseudocode:

```
                                    c is cost per cut
rodcut (P, n, c)
        initialize array R
        rodcut2(p, n, c, R)

rodcut2(P, n, c, R)
        if R[n] >= 0
                return R[n]
        if n == 0
                R[n] = 0
                return R[n]
        else
                q = -∞
                for i = 1 to n
                        if i == n, c = 0
                        q = max(q, p[i] + rodcut2(P, n - i, c, R)  - c
        R[n] = q
        return q
```

Explanation: Each cut now has a cost associated with it, so each time we call rodcut2, we have to subtract the cost of that cut. The only case where we don't subtract the cost is when i == n, which is just the price of the rod at that length.

Problem 3: *(6 points)* **Product Sum**

Given a list of n integers, $v_1, \ldots, v_n$, the product-sum is the largest sum that can be formed by multiplying adjacent elements in the list. Each element can be matched with at most one of its neighbors.

For example, given the list 4, 3, 2, 8 the product sum is 28 = (4 × 3) + (2 × 8), and given the list 2, 2, 1, 3, 2, 1, 2, 2, 1, 2 the product sum is 19 = (2 × 2) + 1 + (3 × 2) + 1 + (2 × 2) + 1 + 2.

a) Compute the product-sum of 2, 1, 3, 5, 1, 4, 2.
2 + 1 + 3 * 5 + 1 + 4 * 2 = 27

b) Give the dynamic programming optimization formula OPT[j] for computing the product-sum of the first j elements.
OPT[j] = 0 if j is 0

OPT[j] = v_j if j is 1

OPT[j] = max(OPT[j - 1] + v_j, OPT[j - 2] + v_{j-1} * v_j)

c) What would be the asymptotic running time of a dynamic programming algorithm implemented using the formula in part b).
T(n) = # of sub-problems * time per sub-problem

We have N sub-problems and the time per sub-problem is $\Theta(1)$.

Therefore $T(n) = \Theta(n)$. Without dynamic programming, it would be $O(n^2)$.

Problem 4: Making Change:

a) Describe and give pseudocode for a dynamic programming algorithm to find the minimum number of coins to make change for A.

Ans:

Description:

A 2d array is used to track the optimal number of coins at each amount and denomination. We go through each index of the 2d array, inserting the optimal number of coins, given the denominations available (any denomination equal to or less than the value at the current denomination index) and current amount of change to be given. This is done by choosing the minimum between two options: one added to the value at the index that is one denomination amount away from the current amount staying on the y axis of the 2d array, or the amount that is 1 denomination level above the current on the x axis of the 2d array. The smaller value is inserted at the current index. This continues until we get to the final index. The value inserted at the final index is the smallest possible amount of coins we can use to get the change for A.

Each index actually contains an object containing two integers rather than no object with one integer. One of the integers is the minimal amount of coins needed, and the other is the option used when determining the minimal amount of coins. Option 1 corresponds to getting the minimal amount from the index above on the y axis. This would also be the optimal solution when the denomination that

corresponds to the current index cannot be used. Option 0 corresponds to getting the minimal amount from the index that is one current denomination away on the x axis (+ 1). By tracking the option chosen at each index, we can easily backtrack in order to produce an array that contains the amount of coins of each denomination used. Each time option 0 is selected, we add 1 for that denomination and continue moving up and left on the y and x axes until we hit 0 on the x axis.

Used https://www.youtube.com/watch?v=Y0ZqKpToTic as reference to help understand this problem.

Pseudocode:

```
min_coins (V, A)
        coins[][] is 2d array of size V.length down, A + 1 across
        for col = 0 to A
                for row = 0 to length of V
                        if col == 0, coins[row, col].num = 0 init left col
                        else if row == 0, coins[row, col].num = col
                                                                ^init top row
                        else if V[row] <= col,
                                option0 = 1+coins[row, col - V[row]].num
                                option1 = coins[row - 1, col].num
                                if option0 <= option1
                                        coins[row, col].num = option0
                                        coins[row, col].op = 0
                                else
                                        coins[row, col].num = op1
                                        coins[row, col].op = 1
                        else, coins[row, col] = coins[row - 1, col]

        below pseudocode is for creating array of number of denominations used

        x = A
        y = V.length - 1
        m = coins[y, x].num
        C[] is array of length V.length
        init all indexes in C to 0
        while (x > 0)
                if coins[y, x].op == 1
                        y = y - 1
                else
                        C[y] = C[y] + 1
                        x = x -V[y]

        return C and m
```

b) What is the theoretical running time of your algorithm?

Based on the number of times we run through the two 'for' loops, worst case is $\Theta(MN)$ with M being the number of denominations and N being the amount of change that is needed.

Problem 5: *(10 points)* **Making Change Implementation**

Submit a copy of all your files including the txt files and a README file that explains how to compile and run your code in a ZIP file to TEACH. We will only test execution with an input file named amount.txt.

You may use any language you choose to implement your DP change algorithm. The program should read input from a file named "amount.txt". The file contains lists of denominations (V) followed on the next line by the amount A.

Example amount.txt:

```
1 2 5
10
1 3 7 12
29
1 2 4 8
15
```

In the above example the first line contains the denominations V=(1, 2, 5) and the next line contains the amount A = 10 for which we need change. There are three different denomination sets and amounts in the above example. A denomination set will be on a single line and will always start with the 1 "coin".

The results should be written to a file named change.txt and should contain the denomination set, the amount A, the change result array and the minimum number of coins used.

Example change.txt:

```
1 2 5
10 0 0
2 2

1 3 7 12
29
0 1 2 1
4
1 2 4 8
15
1 1 1 1
4
```

In the above example, to make 29 cents change from the denomination set (1, 3, 7, 12) you need 0: 1 cent coin, 1: 3 cent coin, 2: 7 cent coins and 1: 12 cent coin for a total of 4 coins.

Problem 6: *(4 points)* **Making Change Experimental Running time**

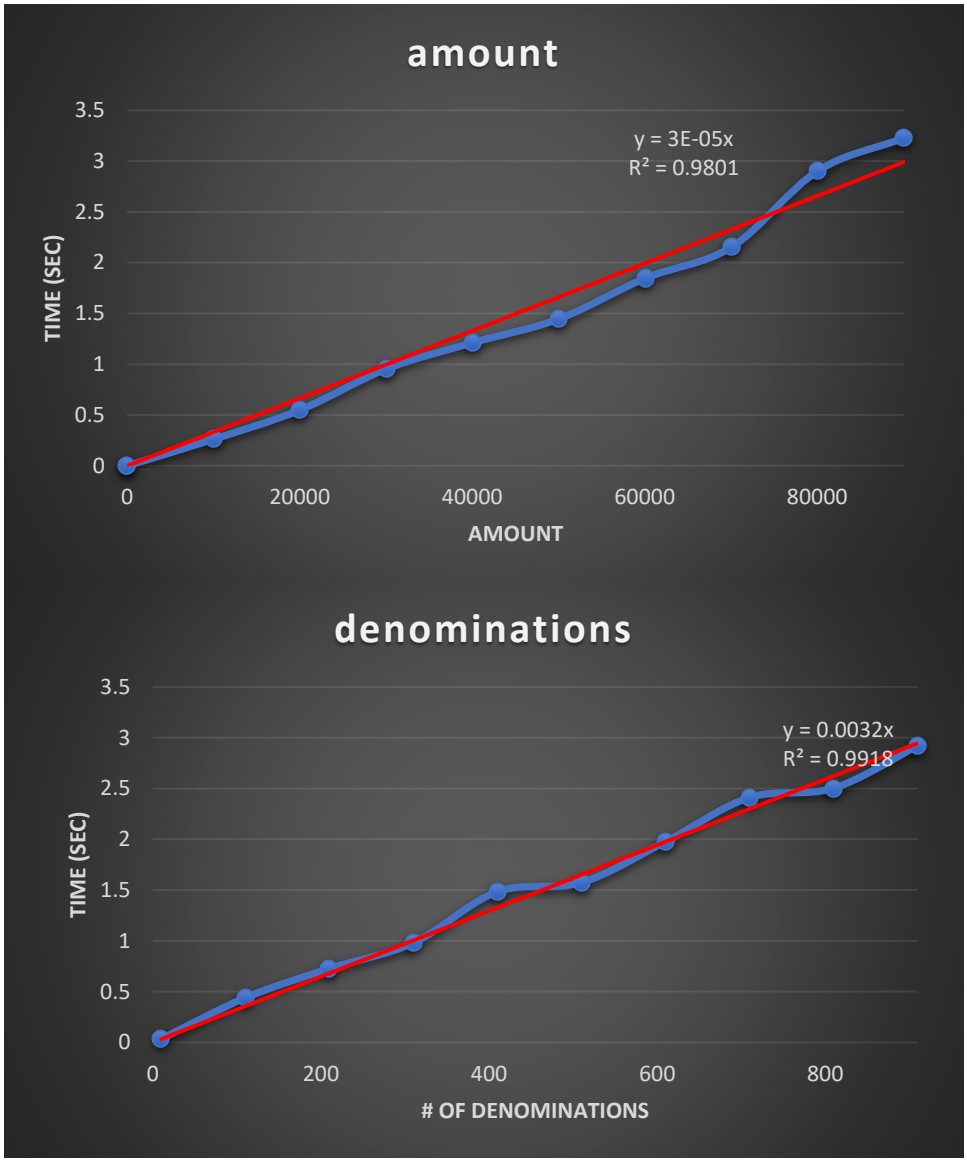
a) Collect experimental running time data for your algorithm in Problem 4. Explain in detail how you collected the running times.

Ans:
I implemented three separate tests types with 10 trials. The first test type was a denomination test, where the number of denominations increased for each of the 10 trials with an 'amount' that remained constant. For the amount test, I increased the amount each separate trial, while the number of denominations remained constant. For the denomination and amount test, I increased both each time at the same rate. Denomination values were chosen at random for each separate trial except for the first denomination value. The first denomination value was always 1.

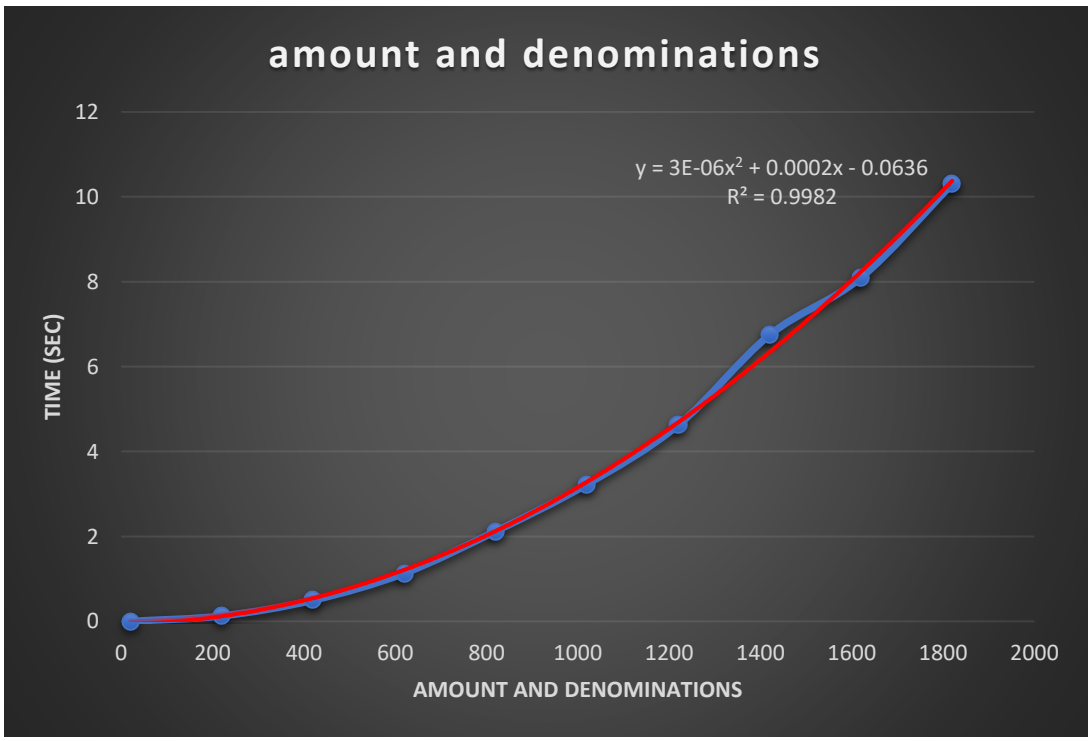
b) On three separate graphs plot the running time as a function of A, running time as a function of n and running time as a function of nA. Fit trend lines to the data. How do these results compare to your theoretical running time? *(Note: n is the number of denominations in the denomination set and A is the amount to make change)*

Ans:
When only the amount is increasing, it has a linear effect on time. When only the number of denominations is increasing, it has a linear effect on time. When both are increasing, it has quadratic effect on time. These results compare well to the theoretical running time. I would expect the nA (denominations, amount) graph to have a quadratic effect on the time given that the theoretical running time is $\Theta(MN)$. We have two variables that have a positive linear effect on the time. Increasing both variables at the same rate has a quadratic effect on the time.

Graphs are on the next page:


amount
y = 3E-05x
R² = 0.9801
TIME (SEC)
AMOUNT
0
0.5
1
1.5
2
2.5
3
3.5
0
20000
40000
60000
80000

denominations
y = 0.0032x
R² = 0.9918
TIME (SEC)
OF DENOMINATIONS
0
0.5
1
1.5
2
2.5
3
3.5
0
200
400
600
800


amount and denominations
y = 3E-06x² + 0.0002x - 0.0636
R² = 0.9982
TIME (SEC)
AMOUNT AND DENOMINATIONS
0
2
4
6
8
10
12
0
200
400
600
800
1000
1200
1400
1600
1800
2000

Run time code:

```
# Brian Allison
# Description: Assignment 3: Denominations and change amount
# Course: CS325
# Date: 10/15/2017

import random
import time


# This class is used to track the amount and method associated
# with the use of a particular denomination of coin in the
# coin change problem
class Coin:
    def __init__(self, num, op):
        self.num = num
        self.op = op


# Args: Array of coin denominations starting with 1
#       Int that is the amount we want to get to with the least amount of change
# Returns: A dictionary containing 'denom' - same array that was passed in,
#           'amount' - same amount that was passed in
#           'result_array' - array of number of each denom used
#           'min' - min number of coins needed
def min_coins(denom, amount):
    coins = []
    for col in range(0, amount + 1):
        for row in range(0, len(denom)):
            if col == 0:
                coins.append([])
                newcoin = Coin(0, 2)
                coins[row].append(newcoin)
            elif row == 0:
                newcoin = Coin(col, 0)
                coins[0].append(newcoin)
            elif denom[row] <= col:
                op0 = 1 + coins[row][col - denom[row]].num
                op1 = coins[row - 1][col].num
                if op0 <= op1:
                    newcoin = Coin(op0, 0)
                else:
                    newcoin = Coin(op1, 1)

                coins[row].append(newcoin)
            else:
                oneabove = coins[row - 1][col].num
                newcoin = Coin(oneabove, 1)
                coins[row].append(newcoin)

    return coins_info(denom, amount, coins)


# helper function of min_coins
def coins_info(denom, amount, coins_array):
    x = amount
    y = len(coins_array) - 1
    m = coins_array[y][x].num
    c = [0] * len(coins_array)

    while x > 0:
        if coins_array[y][x].op == 1:
            y = y - 1
        else:
            c[y] = c[y] + 1
            x = x - denom[y]

    return {'denom': denom, 'amount': amount, 'result_array': c, 'min': m}
```

```
file_out = open('amount_time.txt', 'w')
for k in range(11, 100011, 10011):
    file_out.write('amount, ' + str(k) + ' , ')
    print(k, end=' , ')
    tot = 0
    ave = 0

    for l in range(3):
        rand_denom = []
        rand_denom.append(1)

        randsamp = random.sample(range(2, k), 9)
        rand_denom = rand_denom + randsamp

        start_time = time.time()
        min_coins(rand_denom, k)
        end_time = time.time()

        elapsed_time = end_time - start_time
        tot = tot + elapsed_time
        file_out.write(str(elapsed_time) + ' , ')
        print(elapsed_time, end=' , ')

    ave = tot/3
    print("average: " + str(ave))
    file_out.write("average: " + str(ave) + '\n')

file_out.close()

file_out = open('denom_time.txt', 'w')
for k in range(10, 1000, 100):
    file_out.write('denom, ' + str(k) + ' , ')
    print(k, end=' , ')
    tot = 0
    ave = 0

    for l in range(3):
        rand_denom = []
        rand_denom.append(1)
        if (k >= 1010):
            randsamp = random.sample(range(2, 1011), k)

        randsamp = random.sample(range(2, 1011), k)
        rand_denom = rand_denom + randsamp

        start_time = time.time()
        min_coins(rand_denom, 1011)
        end_time = time.time()

        elapsed_time = end_time - start_time
        tot = tot + elapsed_time
        file_out.write(str(elapsed_time) + ' , ')
        print(elapsed_time, end=' , ')

    ave = tot/3
    print("average: " + str(ave))
    file_out.write("average: " + str(ave) + '\n')

file_out.close()

file_out = open('amount_denom_time.txt', 'w')
for k in range(20, 2010, 200):
    file_out.write('amount and denom, ' + str(k) + ' , ')
    print(k, end=' , ')
    tot = 0
    ave = 0

    for l in range(3):
```

```
        rand_denom = []
        rand_denom.append(1)

        randsamp = random.sample(range(2, k), k - 2)
        rand_denom = rand_denom + randsamp

        start_time = time.time()
        min_coins(rand_denom, k)
        end_time = time.time()

        elapsed_time = end_time - start_time
        tot = tot + elapsed_time
        file_out.write(str(elapsed_time) + ' , ')
        print(elapsed_time, end=' , ')

    ave = tot/3
    print("average: " + str(ave))
    file_out.write("average: " + str(ave) + '\n')

file_out.close()
```