1. *(3 points)* Demonstrate Prim’s algorithm on the graph below by showing the steps in subsequent graphs as shown in Figures 23.5 on page 635 of the text. What is the weight of the minimum spanning tree? Start at vertex a.

**Ans: The weight is 32**







2. *(6 points)* Consider an undirected graph G=(V,E) with nonnegative edge weights w(u,v)≥0. Suppose that you have computed a minimum spanning tree G, and that you have also computed shortest paths to all vertices from vertex s∈V. Now suppose each edge weight is increased by 1: the new weights w’(u,v) = w(u,v) + 1.

(a) Does the minimum spanning tree change? Give an example it changes or prove it cannot change.

For contradiction, suppose that the minimum spanning tree changes after increasing each edge weight by 1 and we have a new minimum spanning tree H. Let H have a total weight of m and let G (previous mst) have a weight of n. Based on the problem description, assume m ≠ n. By definition, a minimum spanning tree has the lowest possible total edge weight. Since H is our minimum spanning tree, it must have the lowest weight, so m < n. Now, let z be 1 \* E (number of edges). Then we can see that,

m – z < n – z (subtracting one from each edge)

m – z is the weight of H before we added one to each edge, and n – z is the weight G before we added one to each edge. We already know that G was our minimum spanning tree before 1 was added to each edge. m – z < n – z is a contradiction then. Therefore, the minimum spanning tree does not change.

(b) Do the shortest paths change? Give an example where they change or prove they cannot change.

For contradiction, suppose that the shortest paths change. This implies that at least one path changes in the set of all shortest paths from s. Let the path that changes be X and the previous path (before adding one) be Y. We have a new shortest path from s, after adding 1. Let f represent the weight of all the edges in X from s to some vertex q, and let g represent the weight of all of the edges in our old shortest path Y from s to q. Based on the problem description, we’ll assume f ≠ g, and based on the definition of a shortest path, f ≤ g. Then, we know that f < g. Now, let z be 1 \* E. We can see that

f – z < g – z (subtracting one from each edge)

f – z is the weight of all edges in path X before we added one to each edge, and g – z is the weight of all the edges in path Y before we added one to each edge. We already know that Y was our shortest path before 1 was added to each edge, so f – z cannot be less than g – z. This is a contradiction. Therefore, the shortest paths do not change.

3. *(4 points)* In the bottleneck-path problem, you are given a graph G with edge weights, two vertices s and t and a particular weight W; your goal is to find a path from s to t in which every edge has at least weight W.

(a) Describe an efficient algorithm to solve this problem.

(b) What is the running time of youor algorithm.

4. *(5 points)* Below is a list of courses and prerequisites for a factious CS degree.

|  |  |
| --- | --- |
| Course | Prerequisite |
| CS 150 | None |
| CS 151 | CS 150 |
| CS 221 | CS 151 |
| CS 222 | CS 221 |
| CS 325 | CS 221 |
| CS 351 | CS 151 |
| CS 370 | CS 151 |
| CS 375 | CS 151, CS 222 |
| CS 401 | CS 375, CS 351, CS 325, CS 222 |
| CS 425 | CS 325, CS 222 |
| MATH 200 | None |
| MATH 201 | MATH 200 |

(a) Draw a directed acyclic graph (DAG) that represents the precedence among the courses.

(b) Give a topological sort of the graph.

(c) If you are allowed to take multiple courses at one time as long as there is no prerequisite conflict, find an order in which all the classes can be taken in the fewest number of terms.

(d) Determine the length of the longest path in the DAG. How did you find it? What does this represent?

5. *(12 points)* Suppose there are two types of professional wrestlers: “Babyfaces” (“good guys”) and “Heels” (“bad guys”). Between any pair of professional wrestlers, there may or may not be a rivalry. Suppose we have n wrestlers and we have a list of r pairs of rivalries.

(a) Give pseudocode for an efficient algorithm that determines whether it is possible to designate some of the wrestlers as Babyfaces and the remainder as Heels such that each rivalry is between a Babyface and a Heel. If it is possible to perform such a designation, your algorithm should produce it.

(b) What is the running time of your algorithm?

(c) **Implemen**t: Babyfaces vs Heels.

**Input**: Input is read in from a file specified in the command line at run time. The file contains the number of wrestlers, n, followed by their names, the number of rivalries r and rivalries listed in pairs. *Note: The file only contains one list of rivalries*

Output: Results are outputted to the terminal.

* Yes, if possible followed by a list of the Babyface wrestlers and a list of the Heels .
* No, if impossible.

**Sample Input file:**

5

Ace

Duke

Jax

Biggs

Stone

6

Ace Duke

Ace Biggs

Jax Duke

Stone Biggs

Stone Duke

Biggs Jax

**Sample Output:**

Yes

Babyfaces: Ace Jax Stone

Heels: Biggs Duke

*Submit a copy of your files including a README file that explains how to compile and run your code in a ZIP file to TEACH.*