A Thesis Title

by

Allison Schneider

Submitted to the Dept. of Earth, Atmospheric and Planetary Sciences
in partial fulfillment of the requirements for the degree of

Bachelor of Science in Earth, Atmospheric and Planetary Sciences

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

September 2017

Author . . . .
Dept. of Earth, Atmospheric and Planetary Sciences
August 5, 2017

Certified by. . . . .
Glenn R. Flierl
Professor of Oceanography
Thesis Supervisor

Accepted by . . . .
???
Chairman, Committee on Undergraduate Program

A Thesis Title

by

Allison Schneider

Submitted to the Dept. of Earth, Atmospheric and Planetary Sciences
on August 5, 2017, in partial fulfillment of the
requirements for the degree of
Bachelor of Science in Earth, Atmospheric and Planetary Sciences

Abstract

In this thesis, I designed and implemented a compiler which performs optimizations that reduce the number of low-level floating point operations necessary for a specific task; this involves the optimization of chains of floating point operations as well as the implementation of a "fixed" point data type that allows some floating point operations to simulated with integer arithmetic. The source language of the compiler is a subset of C, and the destination language is assembly language for a micro-floating point CPU. An instruction-level simulator of the CPU was written to allow testing of the code. A series of test pieces of codes was compiled, both with and without optimization, to determine how effective these optimizations were.

Thesis Supervisor: Glenn R. Flierl
Title: Professor of Oceanography

Acknowledgments

This is the acknowledgements section. You should replace this with your own acknowledgements.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 The Aerocene Project

1.2 Acceleration from Geopotential Height

In an isobaric trajectory model, the acceleration of a parcel of air at a given pressure level can be determined from its velocity, latitude, and geopotential height. The geopotential height Z_g of a surface of pressure p above mean sea level is

$$Z_g(p) = R \int_p^{p_s} \frac{T}{g} \frac{dp}{p},$$ (1.1)

where R is the gas constant, T is temperature, p_s is surface pressure, g is acceleration due to gravity at mean sea level, and $Z_g(p_s)$ is set to 0. In the troposphere, the difference between geopotential height and actual height is negligible [Marshall and Plumb, 2008].

The geopotential Φ is the potential energy of the Earth's gravitational field at a height h:

$$\Phi(h) = g Z_g(h)$$ (1.2)

The full equation for acceleration of an air parcel in the atmosphere is

$$\frac{D\vec{u}}{Dt} + \frac{1}{\rho}\nabla p + \nabla\Phi + f\hat{z} \times \vec{u} = \mathcal{F} \tag{1.3}$$

where $\vec{u}$ is the wind velocity vector, ρ is the density of air, f is the Coriolis parameter, and $\mathcal{F}$ is the force of friction [Marshall and Plumb, 2008]. In an isobaric trajectory model, the gradient of pressure is zero, and the force of friction is assumed to be negligible at the studied pressure level. Using these assumptions and the relation in 1.2, equation 1.3 becomes

$$\frac{D\vec{u}}{Dt} + \nabla\Phi + f\hat{z} \times \vec{u} = 0 \tag{1.4}$$

$$-g\nabla Z_g - f\hat{z} \times \vec{u} = \frac{D\vec{u}}{Dt} \tag{1.5}$$

of which the horizontal components are

$$\frac{Du}{Dt} = fv - g\frac{\partial Z_g}{\partial x} \tag{1.6}$$

$$\frac{Dv}{Dt} = -fu - g\frac{\partial Z_g}{\partial y}. \tag{1.7}$$

Chapter 2

Methods

Two trajectory calculation routines, a kinematic and a dynamic model, were written using Python's NumPy scientific computing package. Both routines numerically predict the trajectories of an ensemble of parcels by determining the velocities of parcels over time. The kinematic routine finds these velocities by interpolating between grids of wind speed data. The dynamic routine calculates velocities using advection equations relating the parcel acceleration and the geopotential height of a given pressure level.

2.1 Ten Day Dataset

Data from the Global Forecast System (GFS), a weather forecast model produced by the National Centers for Environmental Protection (NCEP), was used for both models. The dataset chosen was a ten day forecast, starting at 12:00:00 on February 21st, 2017, with predictions at intervals of three hours. Each file in the dataset contains atmospheric predictions for the beginning of a three-hour interval. The values of atmospheric variables are predicted at each point on a latitude-longitude grid spanning the globe, with a spacing between gridpoints of 0.25 degree. Values are predicted for East-West and North-South wind speed components u and v, as well as the geopotential height Z_g of the 250 millibar pressure level.

2.2   Linear Interpolation

Both the kinematic and dynamic models require an interpolation scheme to produce values for atmospheric variables at positions between the gridded values provided by GFS. Linear interpolation is the standard choice for trajectory models [Bowman et al., 2013]. For both models, linear interpolation was used in three dimensions (latitude, longitude, and time). In the kinematic model, u and v components of wind speed were interpolated to the current positions of the air parcels, while in the dynamic model, geopotential height was interpolated.

2.3   Second-order Integration Scheme

The numerical scheme chosen was a second-order Runge-Kutta method which has a long track record in trajectory modeling [Petterssen, 1940]. The velocity at a given timestep is taken to be the average of the velocity at the initial position and the velocity at the first-guess position after one timestep.

The first guess position $\vec{P}'(t + \Delta t)$ is

$$\vec{P}'(t + \Delta t) = \vec{P}(t) + \vec{V}(\vec{P}, t)\Delta t \tag{2.1}$$

and the final position $\vec{P}(t + \Delta t)$ is

$$\vec{P}(t + \Delta t) = \vec{P}(t) + \frac{1}{2}\left[\vec{V}(\vec{P}, t) + \vec{V}(\vec{P}', t + \Delta t)\right]\Delta t \tag{2.2}$$

where $\vec{P}$ is a position vector with latitude and longitude components, and $\vec{V}$ a velocity vector with u and v wind speeds [Draxler and Hess, 1997]. This integration method is used by HYSPLIT and a number of other trajectory models, including FLEXPART, LAGRANTO, and STILT [Stein et al., 2015] [Bowman et al., 2013]. For trajectories calculated from interpolated gridded wind velocities, higher order integration schemes do not add precision [Draxler and Hess, 1997].

2.4 Constant Timestep

The timestep for integration was three minutes, with the timestep throughout the trajectory. To save computation, HYSPLIT uses a dynamic timestep, varying from one minute to one hour, computed to satisfy

$$U_{max}[\text{grid-units min}^{-1}]\Delta t[\text{min}] < 0.75[\text{grid-units}] \tag{2.3}$$

[Draxler and Hess, 1997]. This ensures that the parcel does not blow past any grid squares during a single timestep, which maximizes the accuracy of the calculation. [Todo: Plot u and v along my trajectories to see if the $U_{max}\Delta t < 0.75$ relation is always satisfied. If it isn't, consider reducing the timestep. If it is satisfied, write that here.]

2.5 Kinematic Equations

At each timestep, after u and v speeds were interpolated and an average value found using the integration scheme, the kinematic model used two equations to solve for a parcel's displacement. Since GFS provides u and v values in meters per second, the equations convert from Cartesian to geographic coordinates. The r value of a parcel is taken to be the radius of the Earth R_E plus the parcel's geopotential height Z_g.

$$r = R_E + Z_g \tag{2.4}$$

$$\frac{d\varphi}{dt} = \frac{v}{r} \tag{2.5}$$

$$\frac{d\lambda}{dt} = \frac{u}{r\cos\varphi} \tag{2.6}$$

2.6 Dynamic Equations

In the dynamic model, velocity at the next timestep was calculated using advection equations which incorporate the current geopotential height gradient and the previous timestep's u and v values. The Coriolis parameter f measures the effect of the Earth's rotation speed Ω at a given latitude φ. Standard acceleration due to gravity is g.

$$f = 2\Omega \sin \varphi \tag{2.7}$$

$$\frac{du}{dt} = fv - \frac{g}{r \cos \varphi} \frac{\partial Z_g}{\partial \lambda} \tag{2.8}$$

$$\frac{dv}{dt} = -fu - \frac{g}{r} \frac{\partial Z_g}{\partial \varphi} \tag{2.9}$$

Equations 2.8 and 2.9 are equations 1.6 and 1.7 transformed from Cartesian to geographic coordinates. After velocity at the next timestep is determined, the dynamic model also uses kinematic equations 2.5 and 2.6 to find the parcel position.

2.7 Mean Trajectory and RMSE

For an ensemble of parcels, variance among trajectories over time was measured by calculating the mean trajectory: the path of an imaginary parcel whose position at each timestep is the average of the parcels' positions. At each timestep, the root-mean-square error (RMSE) is the square root of the average squared value of each particle's distance from the mean trajectory.

The mean trajectory was determined by finding the centroids of parcel positions at each timestep after converting trajectory latitudes and longitudes to Cartesian coordinates. NumPy's `arctan2(y,x)` is a two-argument arctangent function with a range of $(-\pi, \pi]$.

$$x = \cos\varphi\cos\lambda \qquad\qquad \bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} \qquad (2.10)$$

$$y = \cos\varphi\sin\lambda \qquad\qquad \bar{y} = \frac{y_1 + y_2 + \cdots + y_n}{n} \qquad (2.11)$$

$$z = \sin\varphi \qquad\qquad \bar{z} = \frac{z_1 + z_2 + \cdots + z_n}{n} \qquad (2.12)$$

$$\bar{\lambda} = \arctan2\left(\bar{y}, \bar{x}\right) \qquad (2.13)$$

$$\bar{\varphi} = \arctan2\left(\bar{z}, \sqrt{\bar{x}^2 + \bar{y}^2}\right) \qquad (2.14)$$

The distance d between each parcel (with position φ_i, λ_i) and the mean parcel was calculated with the haversine formula

$$\Delta\varphi_i = |\varphi_i - \bar{\varphi}| \qquad (2.15)$$

$$\Delta\lambda_i = |\lambda_i - \bar{\lambda}| \qquad (2.16)$$

$$a_i = \sin^2\left(\frac{\Delta\varphi}{2}\right) + \cos\varphi_1 \cos\varphi_2 \sin^2\left(\frac{\Delta\lambda}{2}\right) \qquad (2.17)$$

$$c_i = 2 \cdot \arctan2\left(\sqrt{a}, \sqrt{1-a}\right) \qquad (2.18)$$

$$d_i = R_E \cdot c \qquad (2.19)$$

which is accurate and well-conditioned for small angles [Sinnott, 1984]. The RMSE at each timestep is calculated using the distance between each parcel and the mean parcel.

$$\text{RMSE} = \sqrt{\frac{d_1{}^2 + d_2{}^2 + \cdots + d_n{}^2}{n}} \qquad (2.20)$$

Bibliography

[Bowman et al., 2013] Bowman, K., Lin, J. C., Stohl, A., Draxler, R., and Konopka, P. (2013). Input data requirements for Lagrangian trajectory models. *Bulletin of the American Meteorological Society*, 94(7):1051–1058.

[Draxler and Hess, 1997] Draxler, R. R. and Hess, G. D. (1997). Description of the HYSPLIT_4 modeling system. Technical report, NOAA Air Resources Laboratory, Silver Spring, MD.

[Marshall and Plumb, 2008] Marshall, J. and Plumb, R. A. (2008). *Atmosphere, Ocean, and Climate Dynamics: An Introductory Text*, volume 93 of *International Geophysics Series*. Elsevier Academic Press.

[Petterssen, 1940] Petterssen, S. (1940). *Weather Analysis and Forecasting*. McGraw-Hill Book Company, New York.

[Sinnott, 1984] Sinnott, R. W. (1984). Virtues of the Haversine. *Sky & Telescope*, 68(2):159.

[Stein et al., 2015] Stein, A. F., Draxler, R. R., Rolph, G. D., Stunder, B. J. B., Cohen, M. D., and Ngan, F. (2015). NOAA's HYSPLIT Atmospheric Transport and Dispersion Modeling System. *Bulletin of the American Meteorological Society*, 96:2059–2077.