A Thesis Title

by

Allison Schneider

Submitted to the Dept. of Earth, Atmospheric and Planetary Sciences
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Abstract

In this thesis, I designed and implemented a compiler which performs optimizations that reduce the number of low-level floating point operations necessary for a specific task; this involves the optimization of chains of floating point operations as well as the implementation of a "fixed" point data type that allows some floating point operations to simulated with integer arithmetic. The source language of the compiler is a subset of C, and the destination language is assembly language for a micro-floating point CPU. An instruction-level simulator of the CPU was written to allow testing of the code. A series of test pieces of codes was compiled, both with and without optimization, to determine how effective these optimizations were.

Thesis Supervisor: Glenn R. Flierl
Title: Professor of Oceanography

Acknowledgments

This is the acknowledgements section. You should replace this with your own acknowledgements.

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Methods

Two trajectory calculation routines, a kinematic and a dynamic model, were written using Python's NumPy scientific computing package. Both routines numerically predict the trajectories of an ensemble of parcels by determining the velocities of parcels over time. The kinematic routine finds these velocities by interpolating between grids of wind speed data. The dynamic routine calculates velocities using advection equations relating the parcel acceleration and the geopotential height of a given pressure level.

2.1 Ten Day Dataset

Data from the Global Forecast System (GFS), a weather forecast model produced by the National Centers for Environmental Protection (NCEP), was used for both models. The dataset chosen was a ten day forecast, starting at 12:00:00 on February 21st, 2017, with predictions at intervals of three hours. Each file in the dataset contains atmospheric predictions for the beginning of a three-hour interval. The values of atmospheric variables are predicted at each point on a latitude-longitude grid spanning the globe, with a spacing between gridpoints of 0.25 degree. Values are predicted for East-West and North-South wind speed components u and v, as well as the geopotential height Z_g of the 250 millibar pressure level.

2.2 Linear Interpolation

Both the kinematic and dynamic models require an interpolation scheme to produce values for atmospheric variables in between the gridded values provided by GFS. Linear interpolation is the standard choice for trajectory models [Bowman et al., 2013]. For both models, linear interpolation was used in three dimensions (latitude, longitude, and time). In the kinematic model, u and v components of wind speed were interpolated, while in the dynamic model, geopotential height was interpolated.

2.3 Second-order Integration Scheme

The numerical scheme chosen was a second-order Runge-Kutta method with a long track record in trajectory modeling [Petterssen, 1940]. The velocity at a given timestep is taken to be the average of the velocity at the initial position and the velocity at the first-guess position after one timestep.

The first guess position $\vec{P}'(t + \Delta t)$ is

$$\vec{P}'(t + \Delta t) = \vec{P}(t) + \vec{V}(\vec{P}, t)\Delta t \tag{2.1}$$

and the final position $\vec{P}(t + \Delta t)$ is

$$\vec{P}(t + \Delta t) = \vec{P}(t) + \frac{1}{2}\left[\vec{V}(\vec{P}, t) + \vec{V}(\vec{P}', t + \Delta t)\right]\Delta t \tag{2.2}$$

where $\vec{P}$ is a position vector with latitude and longitude components, and $\vec{V}$ a velocity vector with u and v wind speeds [Draxler and Hess, 1997]. This integration method is used by HYSPLIT and a number of other trajectory models, including FLEXPART, LAGRANTO, and STILT [Stein et al., 2015] [Bowman et al., 2013]. For trajectories calculated from interpolated gridded wind velocities, higher order integration schemes do not add precision [Draxler and Hess, 1997].

2.4 Constant Timestep

The timestep for integration was three minutes, with the timestep throughout the trajectory. To save computation, HYSPLIT uses a dynamic timestep, varying from one minute to one hour, computed to satisfy

$$U_{max}[\text{grid-units min}^{-1}]\Delta t[\text{min}] < 0.75[\text{grid-units}] \tag{2.3}$$

[Draxler and Hess, 1997]. This ensures that the parcel does not blow past any grid squares during a single timestep, which maximizes the accuracy of the calculation. [Todo: Plot u and v along my trajectories to see if the $U_{max}\Delta t < 0.75$ relation is always satisfied. If it isn't, consider reducing the timestep. If it is satisfied, write that here.]

2.5 Kinematic Equations

At each timestep, after u and v speeds were interpolated and an average value found using the integration scheme, the kinematic model used two equations to solve for a parcel's displacement. Since the gridded u and v values are given in meters per second, the equations convert from Cartesian to geographic coordinates. The r value of a parcel is taken to be the radius of the Earth R_E plus the parcel's geopotential height Z_g.

$$r = R_E + Z_g \tag{2.4}$$

$$\frac{d\varphi}{dt} = \frac{v}{r} \tag{2.5}$$

$$\frac{d\lambda}{dt} = \frac{u}{r\cos\varphi} \tag{2.6}$$

2.6 Dynamic Equations

In the dynamic model, velocity at the next timestep was calculated using advection equations with the current geopotential height gradient and the previous timestep's u and v values. The Coriolis parameter f measures the effect of the Earth's rotation speed Ω at a given latitude φ. Standard acceleration due to gravity is g.

$$f = 2\Omega \sin\varphi \tag{2.7}$$

$$\frac{du}{dt} = g\frac{\partial Z_g}{\partial \lambda} + fv \tag{2.8}$$

$$\frac{dv}{dt} = g\frac{\partial Z_g}{\partial \varphi} - fu \tag{2.9}$$

2.7 Mean Trajectory and RMSE

For an ensemble of parcels, variance among trajectories over time was measured by calculating the mean trajectory: the path of an imaginary parcel whose position at each timestep is the average of the parcels' positions. At each timestep, the root-mean-square error (RMSE) is the square root of the sum of the square of each particle's distance from the mean trajectory.

The mean trajectory was determined by finding the centroids of parcel positions at each timestep after converting trajectory latitudes and longitudes to Cartesian coordinates.

$$x = \cos\varphi\cos\lambda \qquad \bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} \tag{2.10}$$

$$y = \cos\varphi\sin\lambda \qquad \bar{y} = \frac{y_1 + y_2 + \cdots + y_n}{n} \tag{2.11}$$

$$z = \sin\varphi \qquad \bar{z} = \frac{z_1 + z_2 + \cdots + z_n}{n} \tag{2.12}$$

$$\bar{\lambda} = \operatorname{arctan2}(\bar{y}, \bar{x}) \tag{2.13}$$

$$\bar{\varphi} = \operatorname{arctan2}(\bar{z}, \sqrt{\bar{x}^2 + \bar{y}^2}) \tag{2.14}$$

Numpy's `arctan2(y,x)` is a two-argument arctangent function with a range of $(-\pi, \pi]$.

Haversine is great [Sinnott, 1984]

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