

# HW 09. Linear Regression

We continue to work with the same [SOCR](#) data source as the last time.

```
In [1]: import numpy as np
import pandas as pd
import seaborn as sns
import matplotlib.pyplot as plt
from scipy.optimize import minimize_scalar
from scipy.optimize import minimize
%matplotlib inline
```

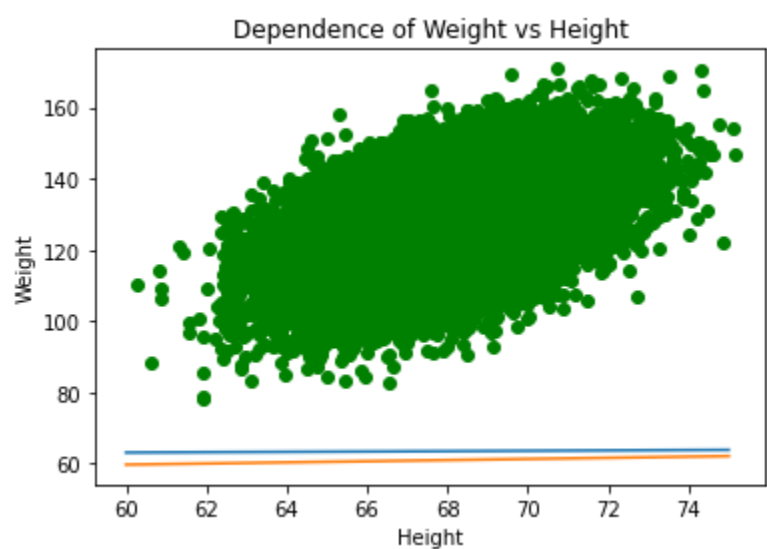
Import the data to a dataframe:

```
In [2]: data = pd.read_csv('teenagers.csv', index_col='Index')
```

**Task 1:** Use the method `plot()` from the module `matplotlib.pyplot` to plot the dependence of the weight on the height from the dataset. Add the plots of the two linear regressions to the graph: one with the coefficients  $(w_0, w_1) = (60, 0.05)$  and the other with  $(w_0, w_1) = (50, 0.16)$ . Do the regressions approximate the data well? Don't forget to label the axis and add a title to the figure.

```
In [16]: fig = plt.figure()
ax = plt.axes()
plt.plot(data['Height'], data["Weight"], 'o', color = 'green')
xx = np.linspace(60, 75, 200)
w0_1, w1_1 = 60, 0.05
w0_2, w1_2 = 50, 0.16
plt.plot(xx, w1_1 * xx + w0_1)
plt.plot(xx, w1_2 * xx + w0_2)
ax.set(xlabel = "Height", ylabel = "Weight", title = "Dependence of Weight vs Height")
plt.show()

print("The regressions do not appear to be good approximations.")
```



The regressions do not appear to be good approximations.

**Task 2:** Write a python fuction `error(X, Y, w0, w1)`, which calculates the Loss Function  $Q(w_0, w_1) = \sum_{i=1}^n (y_i - (w_0 + w_1 * x_i))^2$

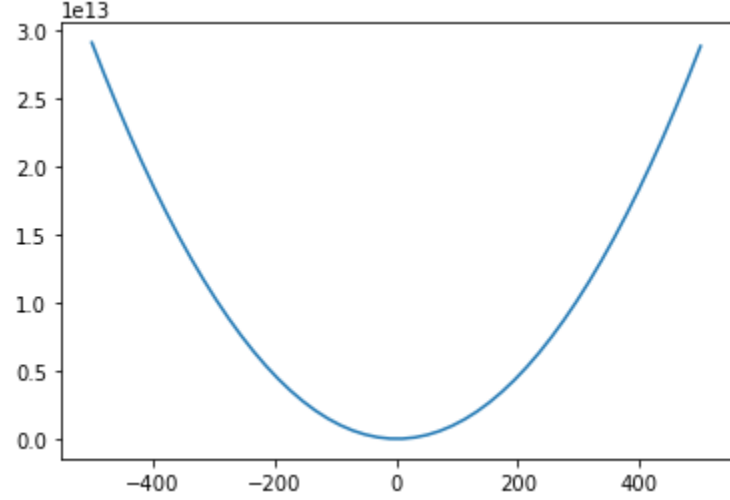
```
In [17]: def error(x , y ,w0, w1):
    retVal = 0
    errorArray = (y - (w0 + w1 * x)) ** 2
    retVal = sum(errorArray)
    return retVal
```

**Task 3:** Plot the loss function vs  $w_1$  with a fixed value of  $w_0 = 50$  for your dataset. Where approximately does the function reaches its minimum?

```
In [36]: fig2 = plt.figure()
ax2 = plt.axes()
errors_len = 50
ww1 = np.linspace(-500, 500, errors_len)
ww0 = np.zeros(errors_len) + 50
errors_arr = np.zeros(errors_len)
for i in range(0, errors_len):
    errors_arr[i] = error(data["Height"], data["Weight"], ww0[i], ww1[i])
ax2.plot(ww1, errors_arr)

print("The minimum is approximatrelly at x = 0.")
```

The minimum is approximatrelly at x = 0.



**Task 4:** Use the method `minimize_scalar` from the module `scipy.optimize` to find the minimum of your Loss function for a fixed value of  $w_0 = 50$ . Make the method to look for the minimum in the range  $-5 \leq w_1 \leq 5$ .

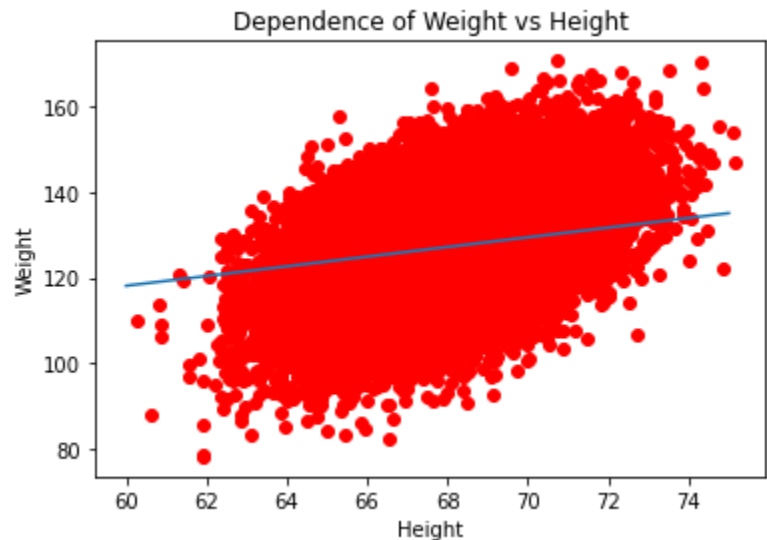
```
In [150]: def error_fun_50(w1):
    return error(data["Height"], data["Weight"], 5, w1)

w1_opt = minimize_scalar(error_fun_50, bounds = (-100, 300))
print( "The minimumum is located at x =", w1_opt.x)
```

The minimum is located at x = 1.7964741480102366

**Task 5:** Use the method `plot()` from the module `matplotlib.pyplot` to plot the dependence of the weight on the height from the dataset. Add to the graph a plot of the linear regression with  $w_0 = 50$ , and  $w_0 =$  the value you found in Task 4. Does this linear regression model the data better than the one from the Task 1?

```
In [43]: fig = plt.figure()
ax = plt.axes()
plt.plot(data['Height'], data["Weight"], 'o', color = 'red')
xx = np.linspace(60, 75, 200)
w0_1, w1_1 = 50, 1.1351597092091665
plt.plot(xx, w1_1 * xx + w0_1)
ax.set(xlabel = "Height", ylabel = "Weight", title = "Dependence of Weight vs Height")
plt.show()
```



**Task 6:** Make a 3D plot of the Loss function vs both  $w_0$  and  $w_1$ . Label the axis and add a title to the plot. According to the plot, where should we look for the miminum of the loss function?

```
In [93]: from mpl_toolkits.mplot3d import Axes3D

fig = plt.figure()
ax = fig.gca(projection = '3d')

X = np.linspace(-2000, 2000, 40)
Y = np.linspace(-20, 20, 40)

Z = np.zeros((len(X), len(Y)))

def error_2D(w0, w1):
    return error(data["Height"], data["Height"], w0, w1)

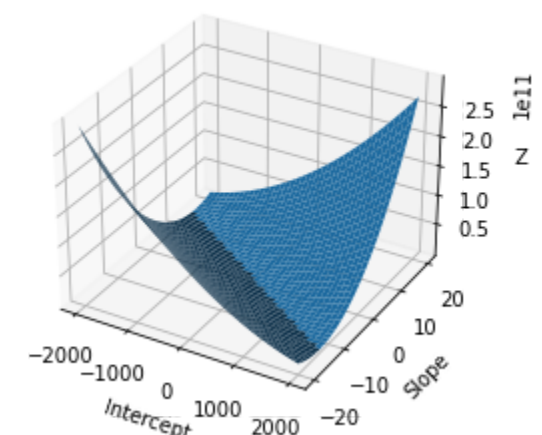
for i in range(0, len(X)):
    for j in range(0, len(Y)):
        Z[i, j] = error_2D(X[i], Y[j])

X, Y, = np.meshgrid(X, Y)

surf = ax.plot_surface(X, Y, Z)
ax.set_xlabel("Intercept")
ax.set_ylabel("Slope")
ax.set_zlabel("Z")
ax.set(title = "3D Plot of Loss Function vs w0 and w1")

plt.show()
```

3D Plot of Loss Function vs w0 and w1



**Task 7:** Use the method `minimize` from the module `scipy.optimize` to find the minimum of the loss function. Make the method to look for the minimum inside the range  $-100 \leq w_0 \leq 100$ ,  $-5 \leq w_1 \leq 5$ . Use the argument `method='L-BFGS-B'`

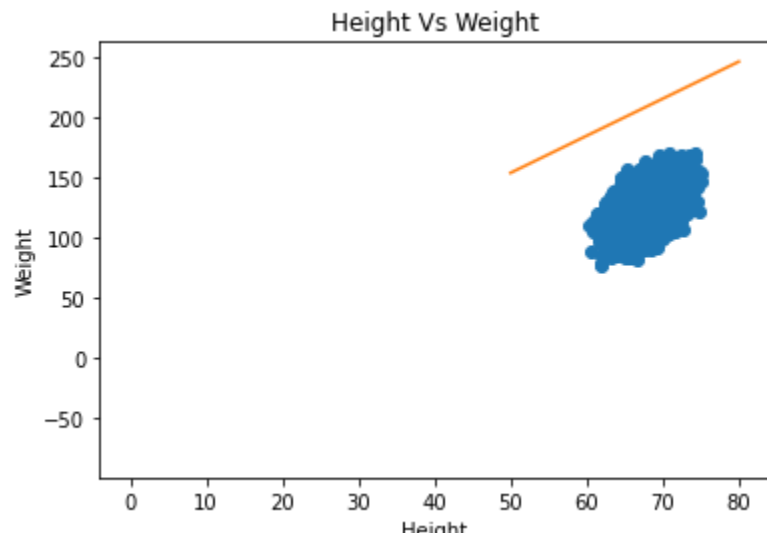
```
In [170]: def error_2D(w):
    return error(data["Height"], data["Weight"], w[0], w[1])

res = minimize(error_2D, (0, 0), method = "L-BFGS-B")
res.x
res
```

```
Out[170]: fun: 2539713.3146102373
hess_inv: <2x2 LbfgsInvHessProduct with dtype=float64>
jac: array([-7.8696807 , -449.40971111])
message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACR*EPSMCH'
nfev: 105
nit: 20
njev: 35
status: 0
success: True
x: array([-82.57717703,  3.08349557])
```

**Task 8:** Use the method `plot()` from the module `matplotlib.pyplot` to plot the dependence of the weight on the height from the dataset. Add to the graph a plot of the linear regression with  $w_0 = w_0 =$  the value you found in Task 7, and  $w_0 =$  the value you found in Task 7.

```
In [171]: fig = plt.figure()
ax = plt.axes()
ax.plot(data["Height"], data["Weight"], 'o')
xx = np.linspace(50, 80, 200)
w0_1, w1_1 = res.x[0], res.x[1]
ax.plot(xx, w1_1 * xx, + w0_1)
ax.set(xlabel = "Height", ylabel = "Weight", title = "Height Vs Weight")
plt.show()
```



```
In [ ]:
```