ESM 204 HW4: A Climate Change Model

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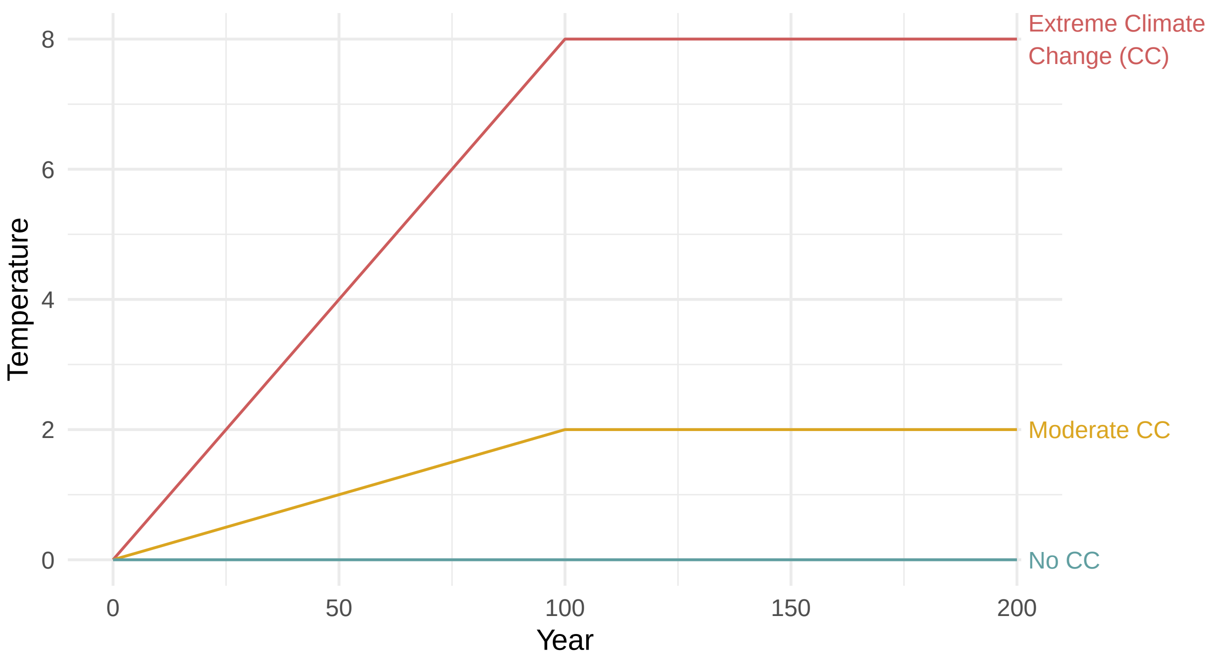
5/22/2020

1. Plots

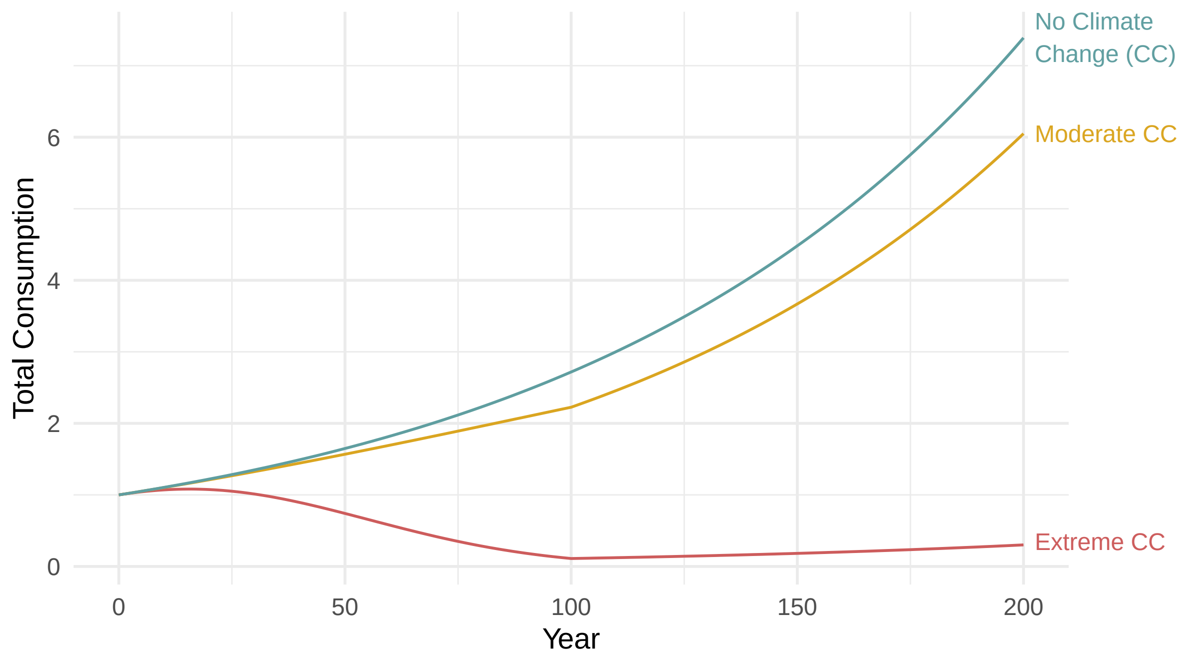
a) Temperature over time

```
years <- data.frame(
  year = seq(from = 0, to = 200, by = 1))

# temperature function
temp_fun = function(t, T){
  temp = pmin(t*T/100,T)
  }

temp_time <- years %>%
  mutate(none = temp_fun(t = year, T = 0),
       moderate = temp_fun(t = year, T = 2),
       extreme = temp_fun(t = year, T = 8)) %>%
  pivot_longer(2:4,
           values_to = "temp",
           names_to = "climate_change")
```



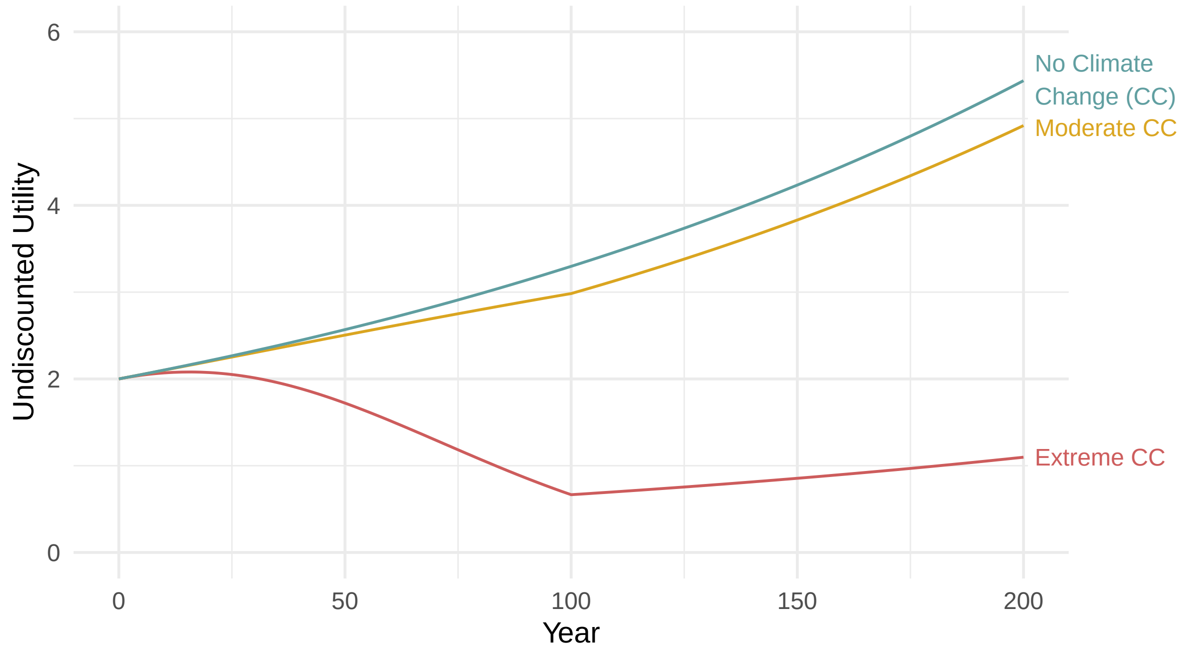
b) Consumption over time

```
# economic activity retained function
econ_fun <- function(temp, B = 0.05){
  econ = exp(-B*temp^2)
  }

econ_time <- temp_time %>%
  mutate(econ = econ_fun(temp = temp))

# total consumption function
consum_fun <- function(econ, g = 0.01, t){
  consum = econ*exp(g*t)
  }

consum_time <- econ_time %>%
  mutate(consum = consum_fun(t = year, econ = econ))
```



c) Undiscounted utility over time

```r
# society utility function
utility_fun <- function(C, n = 0.5){
  utility = ((C^(1-n))/(1-n))
  }

utility_time <- consum_time %>%
  mutate(utility = utility_fun(C = consum))
```



2. Analysis

a) Discounted Utility

```r
# discount rate function
disc_fun <- function(y = 0.005, n = 0.5, g = 0.01){
  disc = y + n*g
}

# T = 4.4
disc_4.4 <- years %>%
  mutate(temp = temp_fun(t = year, T = 4.4),
         econ = econ_fun(temp = temp),
         consum = consum_fun(t = year, econ = econ),
         utility = utility_fun(C = consum),
         disc_utility = utility/((1 + disc_fun())^year))

# NPV under T = 4.4
npv_4.4 <- sum(disc_4.4$disc_utility)
npv_4.4
```

```
## [1] 198.6612
```

```r
# T = 0
disc_0 <- years %>%
  mutate(temp = temp_fun(t = year, T = 0),
         econ = econ_fun(temp = temp),
         consum = consum_fun(t = year, econ = econ),
         utility = utility_fun(C = consum),
         disc_utility = utility/((1 + disc_fun())^year))

# NPV under T = 0
npv_0 <- sum(disc_0$disc_utility)
npv_0
```

```
## [1] 255.2734
```

```r
# percent loss in PV from climate change (T = 4.4)
L <- ((npv_0 - npv_4.4)/npv_0)*100
L
```

```
## [1] 22.17709
```

The present value utility with climate change (T = 4.4) is **198.66**. The present value utility without climate change is **255.27**. The percent loss in present value utility from claimate change (L) is **22.18%**.

b) Sensitivity Analyses

T = 4.84

```
sens_T <- years %>%
  mutate(temp = temp_fun(t = year, T = 4.84),
         econ = econ_fun(temp = temp),
         consum = consum_fun(t = year, econ = econ),
         utility = utility_fun(C = consum),
         disc_utility = utility/((1 + disc_fun())^year))

npv_sens_T <- sum(sens_T$disc_utility)

# L with T = 4.84
L_sens_T <- ((npv_0 - npv_sens_T)/npv_0)*100
L_sens_T
```

```
## [1] 25.79679
```

```
# change in L
# L_sens_T - L = 3.6197 (difference)
# ((L_sens_T - L) / L) * 100 = 16.3218 (percent change)
```

Increasing T by 10% results in an **increase** in the percent loss in present value utility (L) by 3.62, or **16.32%**.

g = 0.011

```
sens_g <- years %>%
  mutate(temp = temp_fun(t = year, T = 4.4),
         econ = econ_fun(temp = temp),
         consum = consum_fun(t = year, econ = econ, g = 0.011),
         utility = utility_fun(C = consum),
         disc_utility = utility/((1 + disc_fun(g = 0.011))^year))

npv_sens_g <- sum(sens_g$disc_utility)

# L with g = 0.011
L_sens_g <- ((npv_0 - npv_sens_g)/npv_0)*100
L_sens_g
```

```
## [1] 22.14822
```

```
# change in L
# L_sens_g - L = -0.0289 (difference)
# ((L_sens_g - L) / L) * 100 = -0.1301 (percent change)
```

Increasing g by 10% results in a **decrease** in the percent loss in present value utility (L) by 0.03, or **0.13%**.

n = 0.55

```
sens_n <- years %>%
  mutate(temp = temp_fun(t = year, T = 4.4),
         econ = econ_fun(temp = temp),
         consum = consum_fun(t = year, econ = econ),
         utility = utility_fun(C = consum, n = 0.55),
         disc_utility = utility/((1 + disc_fun(n = 0.55))^year))
```

```r
npv_sens_n <- sum(sens_n$disc_utility)

# L with n = 0.55
L_sens_n <- ((npv_0 - npv_sens_n)/npv_0)*100
L_sens_n
```

```
## [1] 17.66265
```

```r
# change in L
# L_sens_n - L = -4.5144 (difference)
# ((L_sens_n - L) / L) * 100 = -20.3563 (percent change)
```

Increasing n by 10% results in a **decrease** in the percent loss in present value utility (L) by 4.51, or **20.36%**.

B = 0.055

```r
sens_b <- years %>%
  mutate(temp = temp_fun(t = year, T = 4.4),
         econ = econ_fun(temp = temp, B = 0.055),
         consum = consum_fun(t = year, econ = econ),
         utility = utility_fun(C = consum),
         disc_utility = utility/((1 + disc_fun())^year))

npv_sens_b <- sum(sens_b$disc_utility)

# L with B = 0.055
L_sens_b <- ((npv_0 - npv_sens_b)/npv_0)*100
L_sens_b
```

```
## [1] 23.93861
```

```r
# change in L
# L_sens_b - L = 1.7615 (difference)
# ((L_sens_b - L) / L) * 100 = 7.9430 (percent change)
```

Increasing B by 10% results in an **increase** in the percent loss in present value utility (L) by 1.76, or **7.94%**.

c)

$$U(C_{4.4}) = \frac{(\theta * C_0)^{1-n}}{1-n}$$

$$U(C_{4.4}) = \theta^{1-n} * \frac{C_0^{1-n}}{1-n}$$
$$U(C_{4.4}) = \theta^{1-n} * U(C_0)$$
$$\theta^{1-n} = \frac{U(C_{4.4})}{U(C_0)}$$

Assuming n = 0.5,

$$\theta = (\frac{U(C_{4.4})}{U(C_0)})^2$$

```
# Without discounting

U_4.4 <- sum(disc_4.4$utility)
U_0 <- sum(disc_0$utility)

theta = (U_4.4/U_0)^2
theta
```

```
## [1] 0.4926615
```

```
# With discounting

U_4.4_disc <- sum(disc_4.4$disc_utility)
U_0_disc <- sum(disc_0$disc_utility)

theta_disc = (U_4.4_disc/U_0_disc)^2
theta_disc
```

```
## [1] 0.6056405
```

Without considering discounting, theta* = **0.493**. With discounting, theta* = **0.606**.

d)

```
# T = 2 (prob = 0.2)
d_2 <- years %>%
  mutate(temp = temp_fun(t = year, T = 2),
         econ = econ_fun(temp = temp),
         consum = consum_fun(t = year, econ = econ),
         utility = utility_fun(C = consum),
         disc_utility = utility/((1 + disc_fun())^year))

U_2 <- sum(d_2$utility) # Without discounting
U_2_disc <- sum(d_2$disc_utility) # With discounting

# T = 4 (prob = 0.5)
d_4 <- years %>%
  mutate(temp = temp_fun(t = year, T = 4),
         econ = econ_fun(temp = temp),
         consum = consum_fun(t = year, econ = econ),
         utility = utility_fun(C = consum),
         disc_utility = utility/((1 + disc_fun())^year))

U_4 <- sum(d_4$utility) # Without discounting
U_4_disc <- sum(d_4$disc_utility) # With discounting

# T = 6 (prob = 0.3)
d_6 <- years %>%
  mutate(temp = temp_fun(t = year, T = 6),
         econ = econ_fun(temp = temp),
         consum = consum_fun(t = year, econ = econ),
         utility = utility_fun(C = consum),
         disc_utility = utility/((1 + disc_fun())^year))

U_6 <- sum(d_6$utility) # Without discounting
U_6_disc <- sum(d_6$disc_utility) # With discounting
```

```
# expected theta
# without discounting
U_exp <- U_2*0.2 + U_4*0.5 + U_6*0.3
theta_exp <- (U_exp/U_0)^2
theta_exp
```

```
## [1] 0.5147023
```

```
# with discounting
U_exp_disc <- U_2_disc*0.2 + U_4_disc*0.5 + U_6_disc*0.3
theta_exp_disc <- (U_exp_disc/U_0)^2
theta_exp_disc
```

```
## [1] 0.08489526
```

Without considering discounting, the expected theta* = **0.515**. With discounting, theta* = **0.085**.