

# ESM 204 HW4: A Climate Change Model

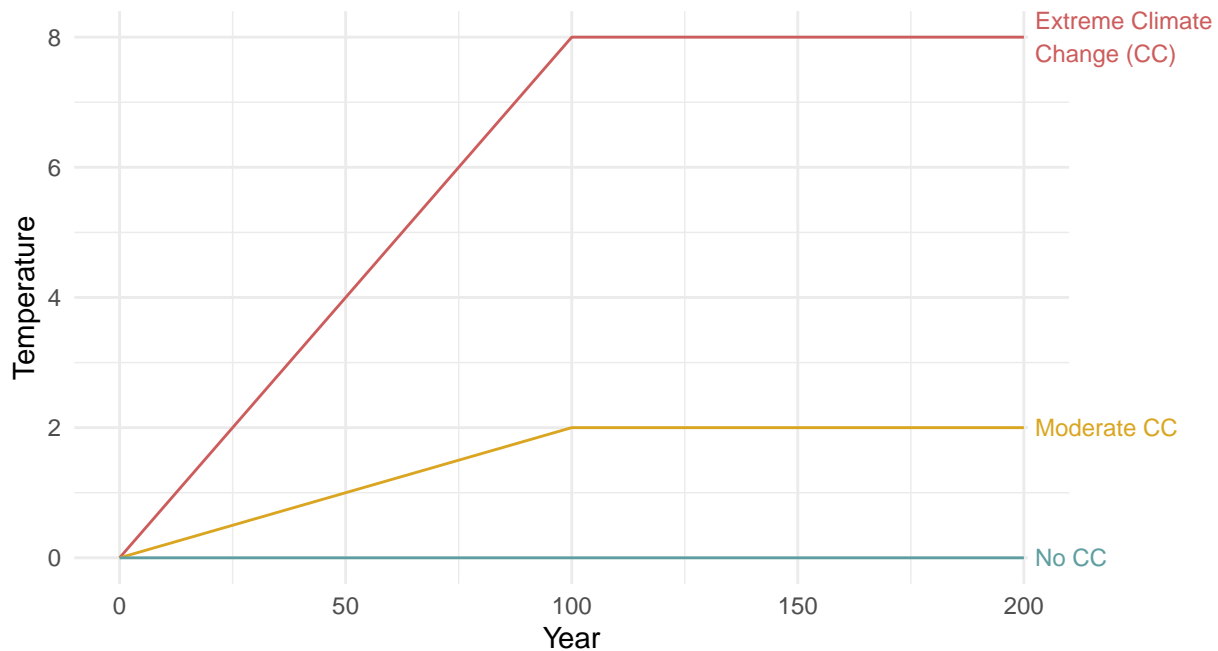
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## 1. Plots

### a) Temperature over time

```
years <- data.frame(  
  year = seq(from = 0, to = 200, by = 1))  
  
# temperature function  
temp_fun = function(t, T){  
  temp = pmin(t*T/100, T)  
}  
  
temp_time <- years %>%  
  mutate(none = temp_fun(t = year, T = 0),  
         moderate = temp_fun(t = year, T = 2),  
         extreme = temp_fun(t = year, T = 8)) %>%  
  pivot_longer(2:4,  
               values_to = "temp",  
               names_to = "climate_change")
```



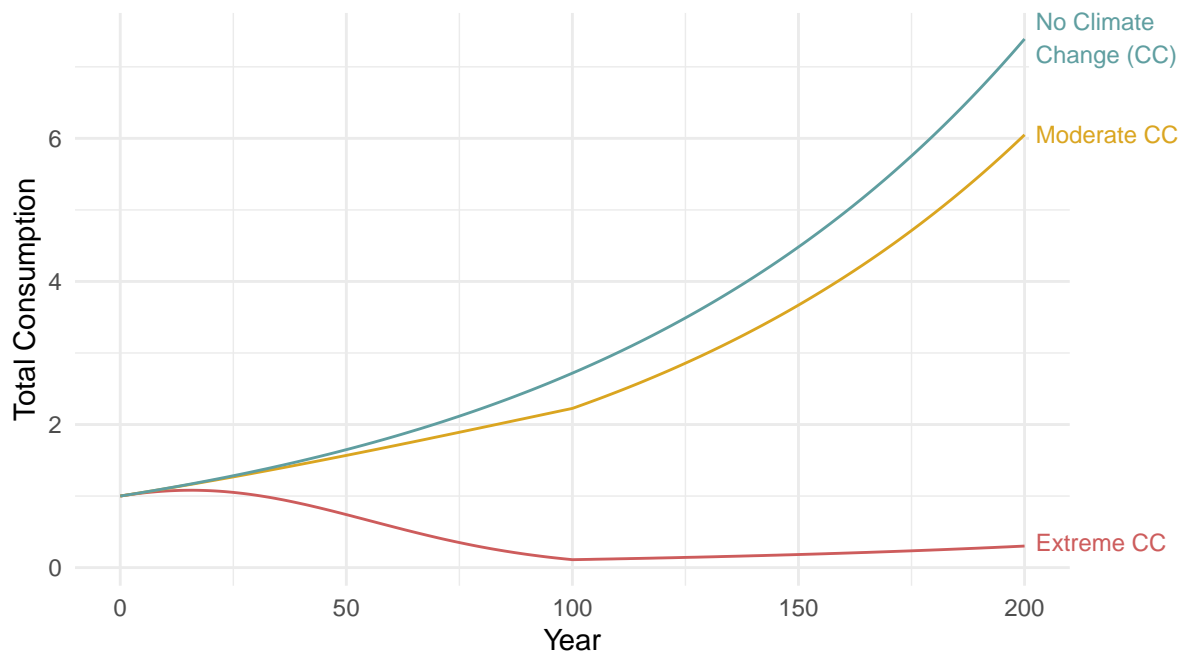
## b) Consumption over time

```
# economic activity retained function
econ_fun <- function(temp, B = 0.05){
  econ = exp(-B*temp^2)
}

econ_time <- temp_time %>%
  mutate(econ = econ_fun(temp = temp))

# total consumption function
consum_fun <- function(econ, g = 0.01, t){
  consum = econ*exp(g*t)
}

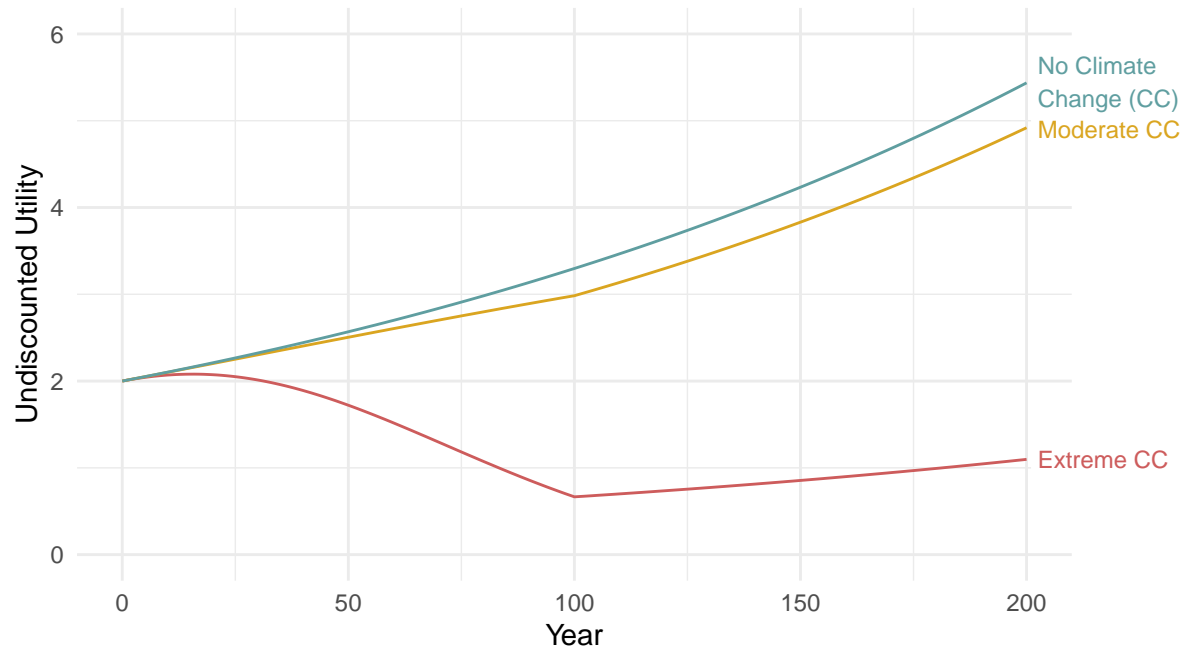
consum_time <- econ_time %>%
  mutate(consum = consum_fun(t = year, econ = econ))
```



### c) Undiscounted utility over time

```
# society utility function
utility_fun <- function(C, n = 0.5){
  utility = ((C^(1-n))/(1-n))
}

utility_time <- consum_time %>%
  mutate(utility = utility_fun(C = consum))
```



## 2. Analysis

### a) Discounted Utility

```
# discount rate function
disc_fun <- function(y = 0.005, n = 0.5, g = 0.01){
  disc = y + n*g
}

# T = 4.4
disc_4.4 <- years %>%
  mutate(temp = temp_fun(t = year, T = 4.4),
         econ = econ_fun(temp = temp),
         consum = consum_fun(t = year, econ = econ),
         utility = utility_fun(C = consum),
         disc_utility = utility/((1 + disc_fun())^year))

# NPV under T = 4.4
npv_4.4 <- sum(disc_4.4$disc_utility)
npv_4.4
```

```
## [1] 198.6612
```

```
# T = 0
disc_0 <- years %>%
  mutate(temp = temp_fun(t = year, T = 0),
         econ = econ_fun(temp = temp),
         consum = consum_fun(t = year, econ = econ),
         utility = utility_fun(C = consum),
         disc_utility = utility/((1 + disc_fun())^year))

# NPV under T = 0
npv_0 <- sum(disc_0$disc_utility)
npv_0
```

```
## [1] 255.2734
```

```
# percent loss in PV from climate change (T = 4.4)
L <- ((npv_0 - npv_4.4)/npv_0)*100
L
```

```
## [1] 22.17709
```

The present value utility with climate change (T = 4.4) is **198.66**. The present value utility without climate change is **255.27**. The percent loss in present value utility from climate change (L) is **22.18%**.

## b) Sensitivity Analyses

T = 4.84

```
sens_T <- years %>%  
  mutate(temp = temp_fun(t = year, T = 4.84),  
         econ = econ_fun(temp = temp),  
         consum = consum_fun(t = year, econ = econ),  
         utility = utility_fun(C = consum),  
         disc_utility = utility/((1 + disc_fun())^year))  
  
npv_sens_T <- sum(sens_T$disc_utility)  
L_sens_T <- ((npv_0 - npv_sens_T)/npv_0)*100  
L_sens_T
```

```
## [1] 25.79679
```

```
# change in L  
L_sens_T - L # (difference)
```

```
## [1] 3.619695
```

```
((L_sens_T - L) / L) * 100 # (percent change)
```

```
## [1] 16.32178
```

Increasing T by 10% results in an **increase** in the percent loss in present value utility (L) by 3.62, or **16.32%**.

g = 0.011

```
sens_g <- years %>%  
  mutate(temp = temp_fun(t = year, T = 4.4),  
         econ = econ_fun(temp = temp),  
         consum = consum_fun(t = year, econ = econ, g = 0.011),  
         utility = utility_fun(C = consum),  
         disc_utility = utility/((1 + disc_fun(g = 0.011))^year))  
  
npv_sens_g <- sum(sens_g$disc_utility)  
L_sens_g <- ((npv_0 - npv_sens_g)/npv_0)*100  
L_sens_g
```

```
## [1] 22.14822
```

```
# change in L  
L_sens_g - L # (difference)
```

```
## [1] -0.02887278
```

```
((L_sens_g - L) / L) * 100 # (percent change)
```

```
## [1] -0.1301919
```

Increasing g by 10% results in a **decrease** in the percent loss in present value utility (L) by 0.03, or **0.13%**.

**n = 0.55**

```
sens_n <- years %>%  
  mutate(temp = temp_fun(t = year, T = 4.4),  
         econ = econ_fun(temp = temp),  
         consum = consum_fun(t = year, econ = econ),  
         utility = utility_fun(C = consum, n = 0.55),  
         disc_utility = utility/((1 + disc_fun(n = 0.55))^year))  
  
npv_sens_n <- sum(sens_n$disc_utility)  
L_sens_n <- ((npv_0 - npv_sens_n)/npv_0)*100  
L_sens_n
```

```
## [1] 17.66265
```

```
# change in L  
L_sens_n - L # (difference)
```

```
## [1] -4.514441
```

```
((L_sens_n - L) / L) * 100 # (percent change)
```

```
## [1] -20.35633
```

Increasing n by 10% results in a **decrease** in the percent loss in present value utility (L) by 4.51, or **20.36%**.

**B = 0.055**

```
sens_b <- years %>%  
  mutate(temp = temp_fun(t = year, T = 4.4),  
         econ = econ_fun(temp = temp, B = 0.055),  
         consum = consum_fun(t = year, econ = econ),  
         utility = utility_fun(C = consum),  
         disc_utility = utility/((1 + disc_fun())^year))  
  
npv_sens_b <- sum(sens_b$disc_utility)  
L_sens_b <- ((npv_0 - npv_sens_b)/npv_0)*100  
L_sens_b
```

```
## [1] 23.93861
```

```
# change in L  
L_sens_b - L # (difference)
```

```
## [1] 1.761522
```

```
((L_sens_b - L) / L) * 100 # (percent change)
```

```
## [1] 7.942983
```

Increasing B by 10% results in an **increase** in the percent loss in present value utility (L) by 1.76, or **7.94%**.

### c) Fraction of Consumption

$$U(C_{4.4}) = \frac{((1 - \theta) * C_0)^{1-n}}{1 - n}$$

$$U(C_{4.4}) = (1 - \theta)^{1-n} * \frac{C_0^{1-n}}{1 - n}$$

$$U(C_{4.4}) = (1 - \theta)^{1-n} * U(C_0)$$

$$(1 - \theta)^{1-n} = \frac{U(C_{4.4})}{U(C_0)}$$

Assuming  $n = 0.5$ ,

$$1 - \theta = \left( \frac{U(C_{4.4})}{U(C_0)} \right)^2$$

$$\theta = -\left( \frac{U(C_{4.4})}{U(C_0)} \right)^2 + 1$$

```
# Without discounting
```

```
U_4.4 <- sum(disc_4.4$utility)
```

```
U_0 <- sum(disc_0$utility)
```

```
theta = -(U_4.4/U_0)^2+1
```

```
theta
```

```
## [1] 0.5073385
```

```
# With discounting
```

```
U_4.4_disc <- sum(disc_4.4$disc_utility)
```

```
U_0_disc <- sum(disc_0$disc_utility)
```

```
theta_disc = -(U_4.4_disc/U_0_disc)^2+1
```

```
theta_disc
```

```
## [1] 0.3943595
```

Without considering discounting,  $\theta^* = 0.507$ . With discounting,  $\theta^* = 0.394$ .

#### d) Expected Theta under Uncertainty

```
# T = 2 (prob = 0.2)
d_2 <- years %>%
  mutate(temp = temp_fun(t = year, T = 2),
         econ = econ_fun(temp = temp),
         consum = consum_fun(t = year, econ = econ),
         utility = utility_fun(C = consum),
         disc_utility = utility/((1 + disc_fun())^year))

U_2 <- sum(d_2$utility) # Without discounting
U_2_disc <- sum(d_2$disc_utility) # With discounting

# T = 4 (prob = 0.5)
d_4 <- years %>%
  mutate(temp = temp_fun(t = year, T = 4),
         econ = econ_fun(temp = temp),
         consum = consum_fun(t = year, econ = econ),
         utility = utility_fun(C = consum),
         disc_utility = utility/((1 + disc_fun())^year))

U_4 <- sum(d_4$utility) # Without discounting
U_4_disc <- sum(d_4$disc_utility) # With discounting

# T = 6 (prob = 0.3)
d_6 <- years %>%
  mutate(temp = temp_fun(t = year, T = 6),
         econ = econ_fun(temp = temp),
         consum = consum_fun(t = year, econ = econ),
         utility = utility_fun(C = consum),
         disc_utility = utility/((1 + disc_fun())^year))

U_6 <- sum(d_6$utility) # Without discounting
U_6_disc <- sum(d_6$disc_utility) # With discounting

# expected theta
U_exp <- U_2*0.2 + U_4*0.5 + U_6*0.3
theta_exp <- -(U_exp/U_0)^2+1
theta_exp # without discounting

## [1] 0.4852977

U_exp_disc <- U_2_disc*0.2 + U_4_disc*0.5 + U_6_disc*0.3
theta_exp_disc <- -(U_exp_disc/U_0)^2+1
theta_exp_disc # with discounting

## [1] 0.9151047
```

Without considering discounting, the expected theta\* = **0.485**. With discounting, theta\* = **0.915**.