Tuesday, October 20, 2020 3:01 PM

$$SNR_{FM} \propto \beta$$

formula achieved thru assuming high SNR

Tuesday, October 20, 2020 3:05 PM

M symbols $\{s_i\}_{i=1}^{M}$ $\quad M = 2^k \quad$ k b/symbol

M-ary signals

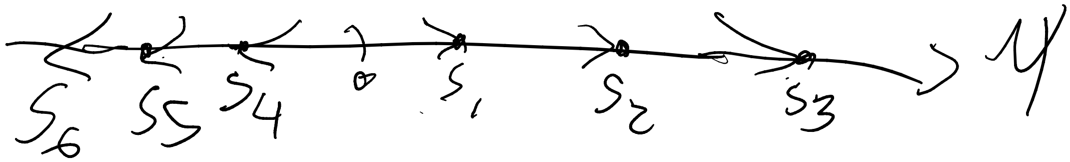
send $R_s = 1/T$ symbols/second

$R_b = kR_s$ bits/second

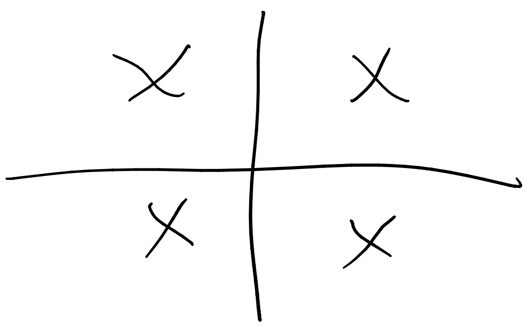
<u>ex</u>

$M \neq N$

1-D space:



2-D:



$M = N$

Tuesday, October 20, 2020 3:07 PM

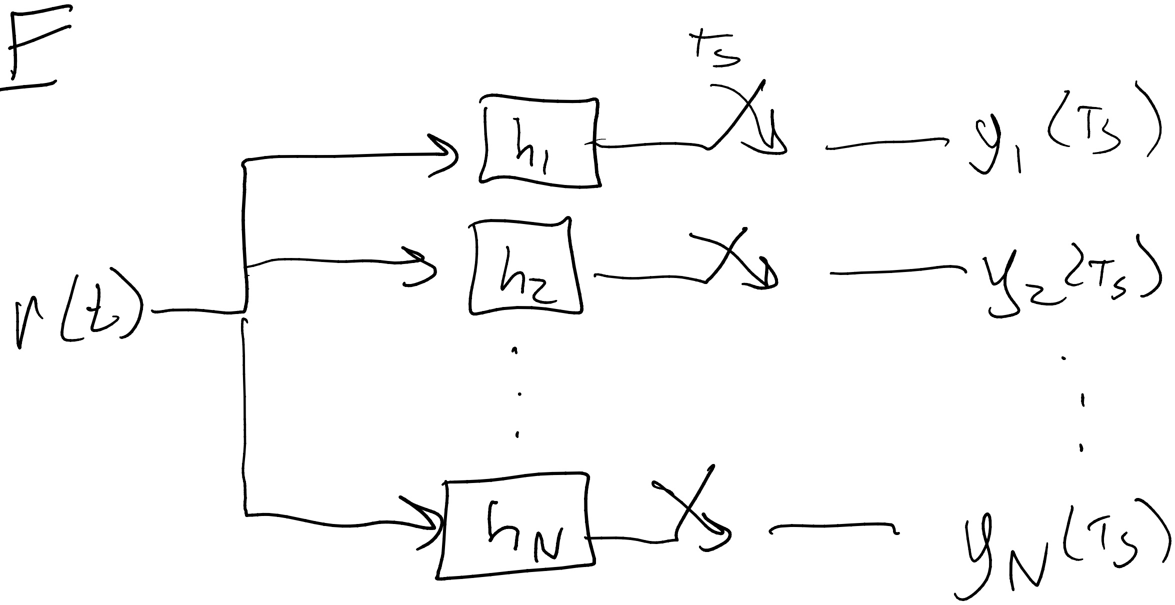
$\{\psi_i\}_{i=1}^{N}$ is an o.n.b for signal-space $\{s_i\}_{i=1}^{M}$

$$r(t)=s_i(t)+n(t),\quad 0\le t\le T_s$$

AWGN, $\sigma^2=N_0/2$

$$s_i(t)=\sum_{k=1}^{N} s_{ik}\psi_k(t),\quad 0\le t\le T_s$$

MF



$$h_i(t)=\psi_i(T_s-t)$$

send s_i, receive $r=s_i+n$

$$y_j(t)=\int_0^{T_s} r(t)\psi_j(t)\,dt$$

$$= \int_0^{T_s} \left(\sum_k s_{ik} \psi_k\right) \psi_j(t)\,dt + \int_0^{T_s} n(t)\psi_j(t)\,dt$$

$\psi_k \perp \psi_j$ if $k \neq j$

$$= s_{ij} + n_j \quad , j = 1, \dots, N$$

some filtered AWGN

$$\begin{pmatrix} y_1 \\ \vdots \\ y_N \end{pmatrix} = y = \begin{pmatrix} s_{i1} \\ s_{i2} \\ \vdots \\ s_{iN} \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \\ \vdots \\ n_N \end{pmatrix} = s_i + n_o$$

Proj. of original noise onto signal space

What is n_o?

$$\mathbb{E}[n_i] = \mathbb{E} \int \psi_i n \, dt$$

$$= \int \psi_i \mathbb{E}[n]\, dt = 0$$

so $\mathbb{E}[n_o] = \vec{0}$

Cov: $$\mathbb{E}[n_i n_j] = \mathbb{E} \int_0^{T_s}\int_0^{T_s} n(t) n(\tau) \psi_i(t) \psi_j(\tau)\, d\tau\, dt$$

Cov: $\mathbb{E}[n_i n_j] = \mathbb{E}\int_0\int_0 \ldots$

$$= \iint \psi_i(t)\psi_j(\tau)\,\mathbb{E}[n(t)n(\tau)]\,dt\,d\tau$$

White noise has $R(\tau) = \frac{N_0}{2}\delta(t)$

$$= \iint \psi_i(t)\,\psi_j(\tau)\,\frac{N_0}{2}\delta(t-\tau)\,dt\,d\tau$$

$$= \int \psi_i(\tau)\psi_j(\tau)\,d\tau\left(\frac{N_0}{2}\right)$$

$$= \delta_{ij}\,\frac{N_0}{2} = \begin{cases} N_0/2, & i=j \\ 0, & i\neq j \end{cases}$$

$$\Sigma = \frac{N_0}{2}\underbrace{I}_{N\times N} = \mathrm{Cov}(n_0)$$

N 0-mean uncorrelated Gaussians

$\Downarrow$

N 0-mean <u>indep</u> Gaussians

$\sigma^2 = N_0/2$

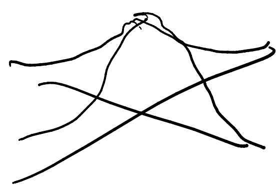
so PDF: $f_n(x) = \prod_{i}^{N} f_{n_i}(x_i) = \frac{1}{(\pi N_0)^{N/2}} e^{-\sum x_i^2/N_0}$

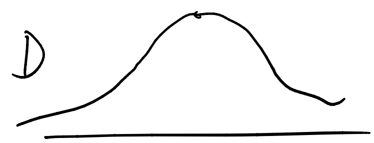
so PDF: $f_{n_0}(x) = \prod_{i=1} f_{n_i}(x_i) = \frac{1}{(\pi N_0)^{N/2}} e$

pdf for each component

$$f_{n_0}(x) = \frac{1}{(\pi N_0)^{N/2}} e^{-\|x\|^2/N_0}$$

isotropic N-D noise (0-mean)

2D 

1D 

in higher D... imagine!

$s_i = I_i \cos\omega_c t - Q_i \sin\omega_c t$, no matter what M is
$N=2$

Tuesday, October 20, 2020 3:27 PM

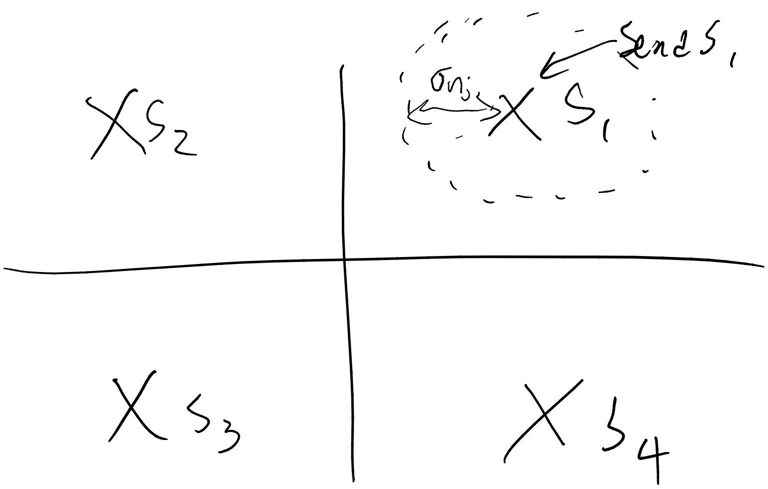
$$\mathbb{E}[y_j] = \mathbb{E}[s_{ij} + n_j] = s_{ij}$$

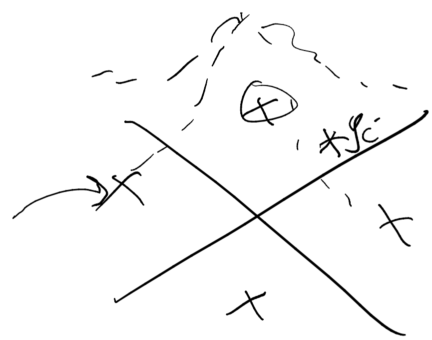
$$\sigma_{y_j}^2 = \sigma_{n_j}^2 = N_0/2$$

$$\mathbb{E}[(s_{ij}+n_j)^2] - s_{ij}^2$$
$$= \cancel{s_{ij}^2} + 2\cancelto{0}{\mathbb{E}[s_{ij} n_j]}$$
$$+ \mathbb{E}[n_j^2] - \cancel{s_{ij}^2}$$
$$= \mathbb{E}[n_j^2] = \sigma_{n_j}^2$$

$$n_0 \sim N\left(\vec{0}, \frac{N_0}{2} I\right)$$

$$y \sim N\left(s_{ij}, \frac{N_0}{2} I\right)$$ ← isotropic gaussian, centered at signal





before:

Tuesday, October 20, 2020 3:33 PM

Because noise components indep.

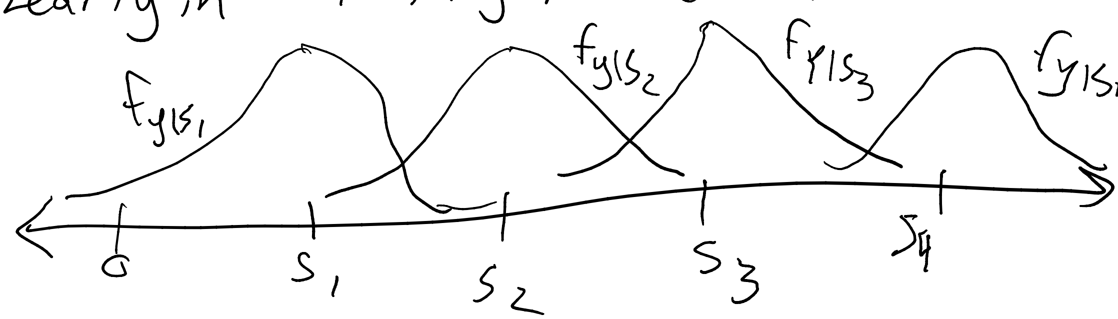
Components of y conditioned on s_i are indep

$$f_{y|s_i}(x) = \prod_{k=1}^{N} f_{y_k|s_i}(x_k) = \frac{1}{(\pi N_0)^{N/2}} e^{-\sum_i (x_k - s_{ik})^2 / N_0}$$

independence

$$= \frac{1}{(\pi N_0)^{N/2}} e^{-\|x - s_i\|^2 / N_0}$$

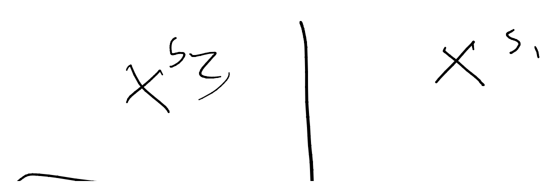
Ex. Clearly in 4-Ary pulse amplitude modulation



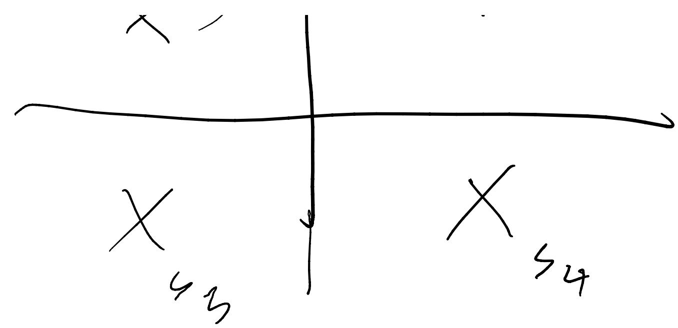
similar to binary case in 1-D, but 4-Ary

Decision If symbols are equiprobable, equal energy (in this AWGN model) then LS = MAP = ML

Ex.



Decision regions are



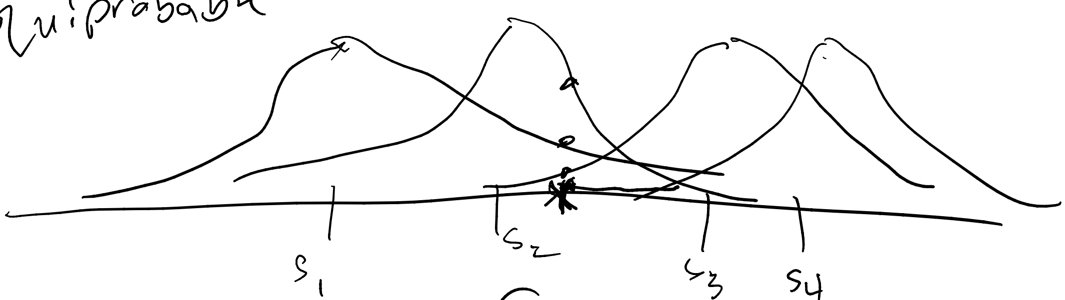
are the quadrants (clear from LS)

More generally – need MAP to minimize P_{error}

by maximizing

$$P[s=s_i \mid y] = \frac{f_{y|s_i}(y)\,P[s=s_i]}{\sum_j f_{y|s_j}(y)\,P[s=s_j]}$$

equiprobable



$$\left.\begin{aligned} f_{y|s_2}(y) &> f_{y|s_1}(y) \\ &> f_{y|s_3}(y) \\ &> f_{y|s_4}(y) \end{aligned}\right\} \text{ ML chooses } s_2$$

but this is not enough for map

need to compare $f_{y|s_i}(y)\,P(s=s_i)$

e.g. if $P[s=s_1] >>> P[s=s_2]$

eg. if $P[s=s_1] >>> P[s=s_2]$
maybe map chooses s_1

---

in AWGN case, because we know $f_{y|s_i}$ functional form

if we know $p_i = P[s=s_i]$

we can always easily compute

$$f_{y|s_i}(y)\, p_i \quad \forall i$$

and compare

So in this framework (AWGN, MF on $\{\psi_i\}_{j=1}^{N}$, know p_i)

MAP is really easy!

plug in output y to a fncn <u>you know</u>, $f_{y|s_i}$ $\forall i$
then multi by a # <u>you know</u>? p_i

<u>Ex.</u> 4-ary FSK - $s_i = A_i \cos(i\omega_c t)$, $0 \le t \le T_s$

s_1 s_2 s_3 s_4

set A_i so that all have equal energy

$\frac{N_0}{2} = 0.1$ $p_1 = 1/12$, $p_2 = 3/4$, $p_3 = p_4 = 1/12$

I receive $y = \begin{pmatrix} 0.1 \\ 0.4 \\ 0.6 \\ 0.1 \end{pmatrix}$ $s_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$, $s_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

$s_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$, $s_4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

<u>ML</u> = <u>LS</u> so to do ML just look at

$$\|y - s_1\|^2 = 1.34, \quad \|y - s_3\|^2 = 0.34$$

$$\|y - s_2\|^2 = 0.74, \quad \|y - s_4\| = 1.34$$

ML will choose <u>s_3</u> the "nearest" symbol

but ignored that $p_2 \gg p_3$

MAP

$$f_{y|s_i}(x) = \frac{1}{(\pi N_0)^{2}} e^{-\|x-s_i\|^2/N_0}$$

N=4

$$f_{y|s_i}(y) = \begin{cases} 0.031, & i=1 \\ 0.0626, & i=2 \\ 0.4127, & i=3 \\ 0.0031, & i=4 \end{cases}$$

← ML estimate, we already knew that

↑ the likelihood function

but

$$f_{y|s_i}(y)\, p_i = \begin{cases} 0.003, & i=1 \\ 0.047, & i=2 \\ 0.0386, & i=3 \\ 0.003, & i=4 \end{cases}$$

← s_2 is MAP estimate

Tuesday, October 20, 2020 4:06 PM

What's P_{error}?

in Binary LS we had

$$P_{error} = Q\left(\sqrt{\frac{d^2}{2N_0}}\right)$$

distance between the two symbols

For M-ary?

Start w/ equiprobable
equal energy (AWGN)

assume s_k sent

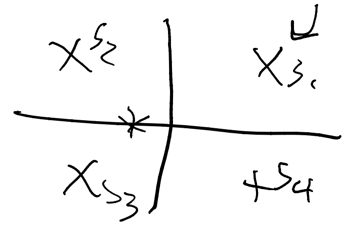
Let E_i denote the event that $\|y-s_i\|^2 < \|y-s_k\|^2$

that is E_i is the event that LS decides s_i instead of s_k

(N-1 possible errors)

E_i and E_j may occur at the same time! ex.

sent this



E_2 and E_3 occur

not necessarily ME.

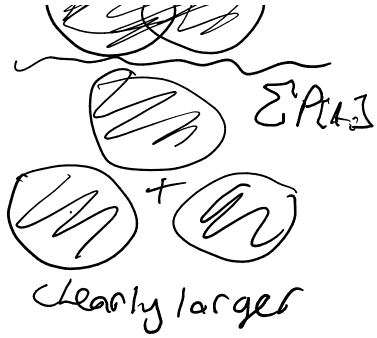
if you remember:

$$P[\cup A_i] \le \sum P[A_i]$$

$$P[\cup A_i] \le \sum_i P[A_i]$$

$\uparrow$

= only if M.E.



$$P_{error_k} = P\left[\bigcup_{\substack{i=1 \\ i \ne k}}^{M} E_i | s_k\right] \le \sum_{\substack{i=1 \\ i \ne k}}^{M} P[E_i | s_k]$$

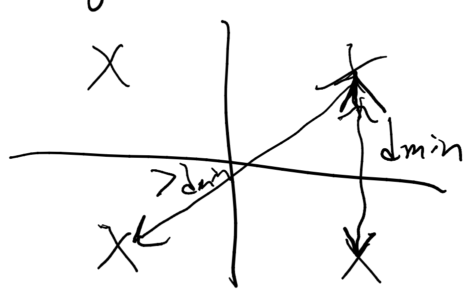
$$P[E_i | s_k] = P[\|y - s_i\| < \|y - s_k\| \,|\, s_k]$$

$$= Q\left(\sqrt{\frac{d_{ik}^2}{2N_0}}\right)$$

distance between s_i and s_k

$$P_{error_k} \le \sum_{\substack{i=1 \\ i \ne k}}^{M} Q\left(\frac{d_{ik}}{\sqrt{2N_0}}\right)$$

<u>say</u> nearest 2 symbols sep. by d_{min}



$$Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) \ge Q\left(\frac{d_{ik}}{\sqrt{2N_0}}\right) \forall i$$

So $P_{error_k} \le \sum_{\substack{i=1 \\ i \ne k}}^{M} Q\left(\frac{d_{ik}}{\sqrt{2N_0}}\right) \le (M-1)\, Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$

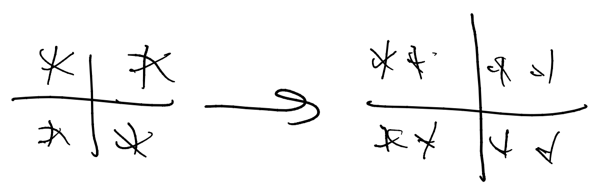
$$P_{error} = \frac{1}{M} \sum_{k=1}^{M} P_{error_k} \le (M-1)\, Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$$

A famous inequality: $Q(x) \le \frac{1}{2} e^{-x^2/2}$

So $$\boxed{P_{error} \le \frac{M-1}{2} e^{-d_{min}^2/4N_0}}$$

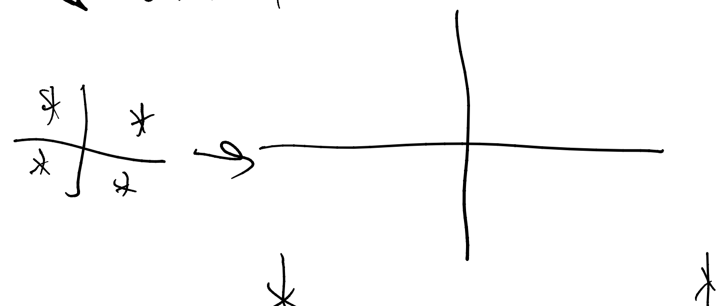
very safe upper bound

$\nearrow M$, linearly $\nearrow P_{error}$



more opportunity for error

$\nearrow d_{min}$, <u>exponentially</u> $\searrow P_{error}$



less likely for error to occur

Cost of $\nearrow d_{min}$

Cost of $\nearrow$ dmin

= more power

Survey of M-ary schemes

Tuesday, October 20, 2020 4:29 PM

<u>Generally</u> – we know how to <u>receive</u> (MF bank)

<u>decide</u> (ML / LS / MAP)

and find <u>P_{error}</u> (in equiprobable equal energy AWGN case)

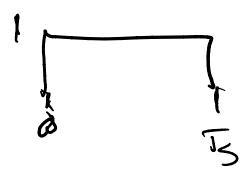
for M-ary signalling schemes

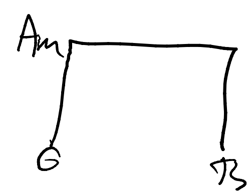
<u>M-ary Pulse Amplitude Modulation</u> (M-ary PAM)

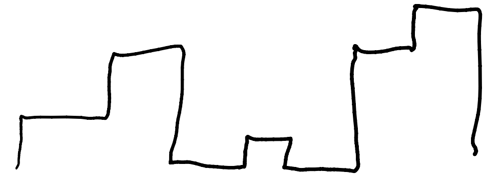
$$s_m(t) = A_m p(t) \quad m = 1, 2, \ldots, M \quad 0 \le t \le T_s$$

$$= A_m \sqrt{E_p}\, \psi(t) \leftarrow \text{basis function is } \frac{p(t)}{\sqrt{E_p}}$$

one-dimensional

Special case $p(t) =$ 

$s_m(t)$ 

total signal 

$p(t)$ is <u>arbitrary</u>

<u>Equal energy</u>? only if $M = 2$ and $A_1 = -A_2 \rightarrow$ binary antipodal

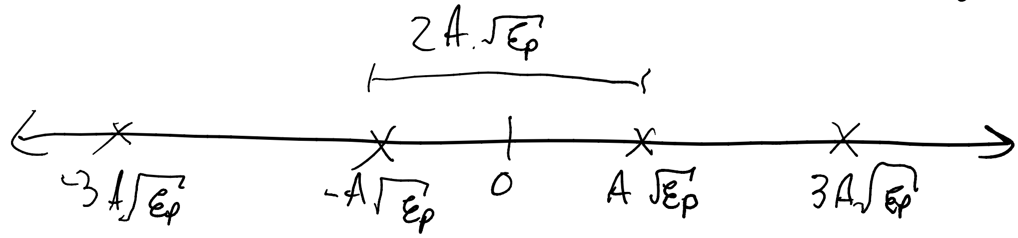
otherwise <u>cannot</u> be equal energy

otherwise cannot be equal energy

Of course, want to transmit w/ less power
- minimize avg energy

this leads to a problem, because we also want to
- keep the signals far apart to minimize P_{error}

"nice" assumption, M even, A_m are
- symmetric about 0
- equally spaced



$$A_m = (2m-1-M)A \quad \text{for some } A$$

for any evenly spaced scheme w/ M even, this minimizes avg. energy

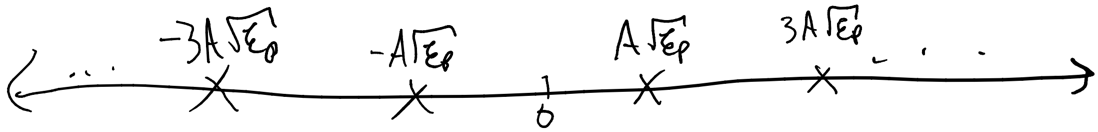
$$\mathcal{E}_{avg} = \frac{1}{M}\sum_{i=1}^{M} A_i^2 \mathcal{E}_p$$

$$= \frac{A^2\mathcal{E}_p}{M}\sum_{i=1}^{M}(2i-1-M)^2$$

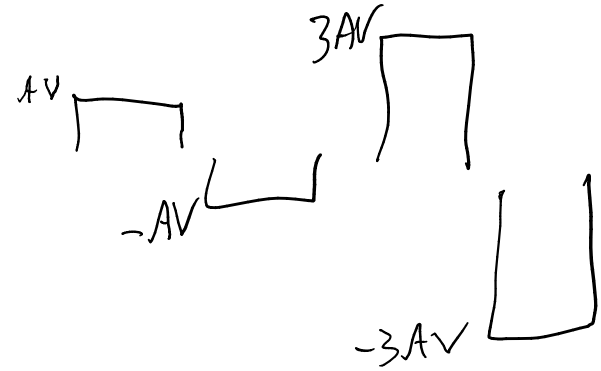
$$= A^2\mathcal{E}_p(M^2-1)/3$$

just believe me

$$s_m(t) = A\sqrt{E_p}\,(2m-1-M)\,\psi(t)$$



W/ square pulses, best setup is



Tuesday, October 20, 2020 4:45 PM

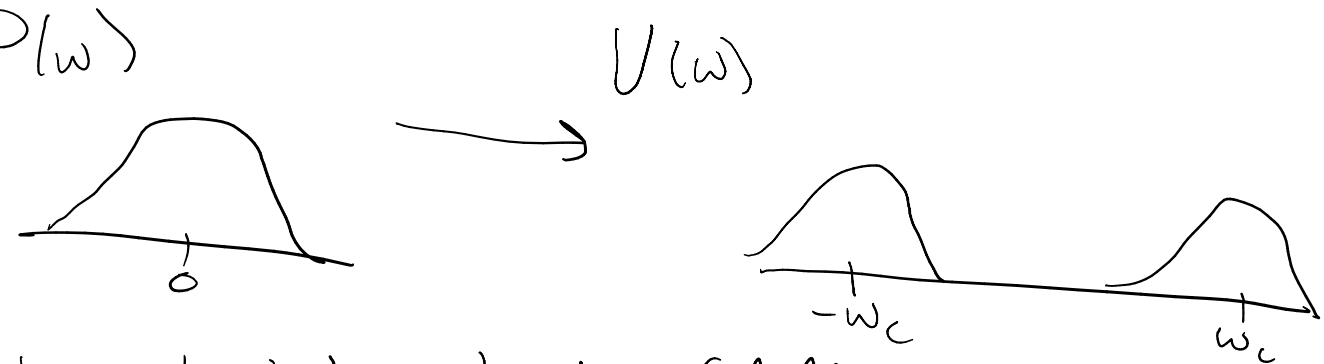
Amplitude Shift Keying (ASK)

$$u_m(t) = s_m(t)\cos\omega_c t$$

$$s_m(t) = A_m\, p(t) = A_m\sqrt{\mathcal{E}_p}\,\psi(t)$$

carrier-modulated PAM

still 1-D



like a digital analogue of AM

~DSB-SC

$$U_m(\omega) = \mathcal{F}(\cos\omega_c t) * \mathcal{F}(s_m(t))$$

$$= \left(\frac{1}{2}\delta(\omega-\omega_c) + \frac{1}{2}\delta(\omega+\omega_c)\right) * S_m(\omega)$$

$$= \frac{1}{2}\left(S_m(\omega-\omega_c) + S_m(\omega+\omega_c)\right)$$

$$\mathcal{E}_m = \int u_m^2(t)\,dt = \int s_m^2(t)\cos^2\omega_c t\,dt$$

$$= \int s_m^2\left(\frac{1}{2} + \frac{1}{2}\cos(2\omega_c t)\right)dt$$

$$= \frac{1}{2}\mathcal{E}_{s_m} + \frac{1}{2}\int s_m^2(t)\cos(2\omega_c t)\,dt$$

fast-varying + as often as −

slow-varying

$$= \frac{1}{2}\mathcal{E}_{s_m}$$

in ASK, $u_m(t) = A_m p(t)\cos\omega_c t$

$$\psi(t) = \sqrt{\frac{2}{\mathcal{E}_p}}\,p(t)\cos\omega_c t$$

same <u>constellation</u> as PAM but different ψ

Application: IR − f_c is in the THz

send digital data by $A_c\cos\omega_c t \to 1$

$0 \to 0$

ex of B-ASK used in TV remotes

<u>Demod and Noise</u>

$$u_m(t) = A_m p(t)\cos\omega_c t$$

$$= s_m(t)\cos\omega_c t$$

$$u_m(t) = A_m\, p(t) \cos \omega_c t$$
$$= s_m(t) \cos \omega_c t$$

$$r(t) = u_m(t) + n(t)$$

$$\psi(t) = \sqrt{\frac{2}{\mathcal{E}_p}}\, p(t) \cos \omega_c t \text{ , call } \psi_s(t) = \frac{p(t)}{\sqrt{\mathcal{E}_p}} \text{ , so } \psi(t) = \sqrt{2}\, \psi_s(t) \cos \omega_c t$$

<u>MF</u> with impulse responce $\psi(T_s - t) = h(t)$

$$r(t) \to \boxed{h(t)} \xrightarrow{T_s} y(T_s)$$

$$y(T_s) = \int_0^{T_s} r(t)\, \psi(t)\, dt$$

$$= \int_0^{T_s} u_m(t)\, \psi(t)\, dt + \underbrace{\int_0^{T_s} n(t)\, \psi(t)\, dt}_{n}$$

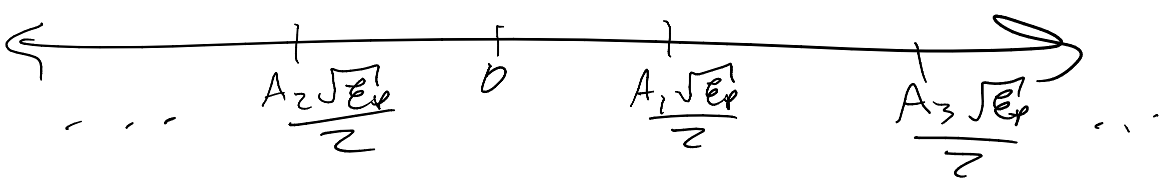
$$= \int_0^{T_s} \left(A_m\, p(t) \cos \omega_c t\right) \left(\frac{\sqrt{2}\; p(t) \cos \omega_c t}{\sqrt{\mathcal{E}_p}}\right) dt + n$$

$$= A_m \sqrt{\frac{2}{\mathcal{E}_p}} \int_0^{T} p^2(t) \cos^2 \omega_c t\, dt + n$$

$$\sqrt{2} \Big(\int^{T} \qquad \int^{T} \quad \to 0 \Big)$$

$$= A_m \sqrt{\frac{2}{\mathcal{E}_p}} \left(\underbrace{\int_0^T p^2(t)/2 \, dt}_{= \mathcal{E}_p/2} + \frac{1}{2} \underbrace{\int_0^T p^2(t) \cos 2\omega_c t \, dt}_{\approx 0} \right) + n$$

$$= A_m \sqrt{\frac{\mathcal{E}_p}{2}} + n : \tilde{s}_m + n$$



So this is precisely Const. point sent + noise

<u>LS</u> decide based on $\|y - \tilde{s}_m\|^2$ minimizer

for

<u>PAM</u>, would be almost the same except

$$r(t) = A_m \sqrt{\mathcal{E}_p}\, \psi(t)$$

$$y(T_s) = A_m \sqrt{\mathcal{E}_p} + n = s_m + n$$

PAM and ASK affected by noise in same way

Finding Perror here is different from earlier because

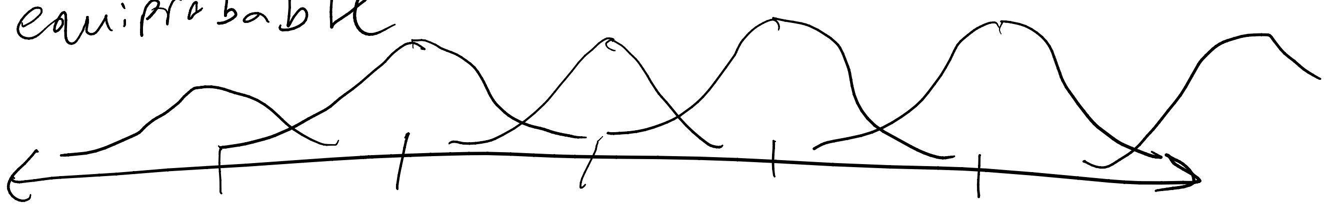
equal energy

Finding P_{error} here is different from earlier ~~because~~

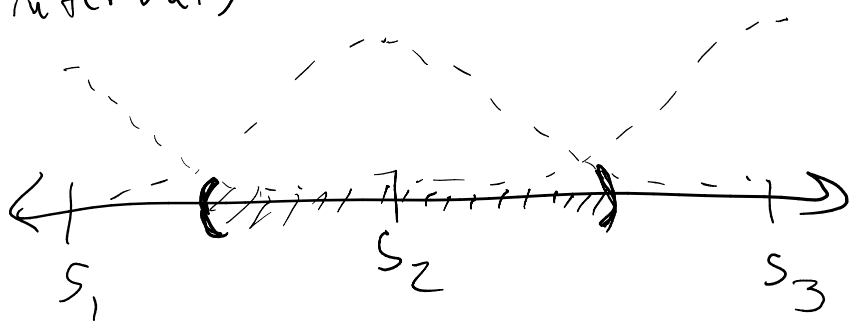
The symbols do not have equal energy

assume AWGN, $\sigma_n^2 = N_0/2$, $\mu_n = 0$

if equiprobable



best decision regions are at the midpoints of the intervals



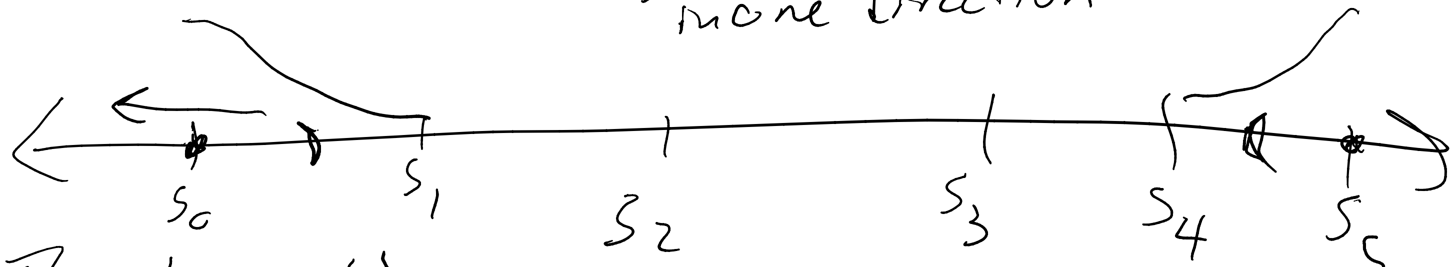
Choose s_2 if $\quad y \in \left(\frac{s_2 + s_1}{2}, \frac{s_2 + s_3}{2} \right)$

looking for P_{error} —

an error occurs when

1) I transmit a signal at one of the two maximal amplitudes, where an error can only occur in one direction

maximal amplitudes, ... one direction



$R_0 = \left(-\infty, \frac{s_0+s_1}{2}\right)$

$R_{end} = \left(\frac{s_4+s_5}{2}, \infty\right]$

in eq. case, Prob. I transmit a maximal amplitude symbol is $\frac{2}{M}$

and in this case, error occurs if $y - s_m > A\sqrt{E_p} = d$

2) I transmit any other symbol, so error can occur in 2 directions prob of tx is $\frac{M-2}{M}$

error occurs if $|y - s_m| > A\sqrt{E_p}$

So

$$P_{error} = \frac{M-2}{M} P\left[|y-s_m| > d\right] + \frac{2}{M} P\left[y - s_m > d\right]$$

← half the prob of other case

$$= \frac{M-1}{M} P\left[|y-s_m| > d\right]$$

← positive tail alone is

$2(M-1) \quad \frac{1}{\ } \int^{\infty}$

$$= \frac{2(M-1)}{M} Q\left(\sqrt{2d^2/N_0}\right)$$

positive tail alone is

$$\frac{1}{\sqrt{\pi N_0}} \int_d^{\infty} e^{-x^2/N_0}\, dx$$

, twice this to include negative tail

d is controlled by $\underline{\text{energy}}$ so $\nearrow d$ $\searrow P_{error}$ as expected

$$\mathcal{E}_{avg} = \frac{d^2(M^2-1)}{3}$$

, M symbols $\rightarrow \log_2 M$ bits

avg energy per bit $\frac{\mathcal{E}_{avg}}{\log_2 M} = \mathcal{E}_{bav}$

$$P_{error} = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6 \log_2 M\, \mathcal{E}_{bav}}{(M^2-1)\, N_0}}\right)$$

error as function of average bit energy

$$\mathcal{E}_{bav}/N_0 = SNR/_{bit}$$

$SNR/_{bit} \nearrow \Rightarrow P_{error} \searrow$ as expected

"a nice framing"

$M \nearrow \Rightarrow P_{error} \nearrow$ makes sense

Phase Shift Keying (PSK)

Tuesday, October 20, 2020 5:27 PM

$$u_m(t) = p(t)\cos(\omega_c t + \varphi_m), \quad 0 \le t \le T_s$$

→ symbol is encoded in the <u>phase</u> of the signal

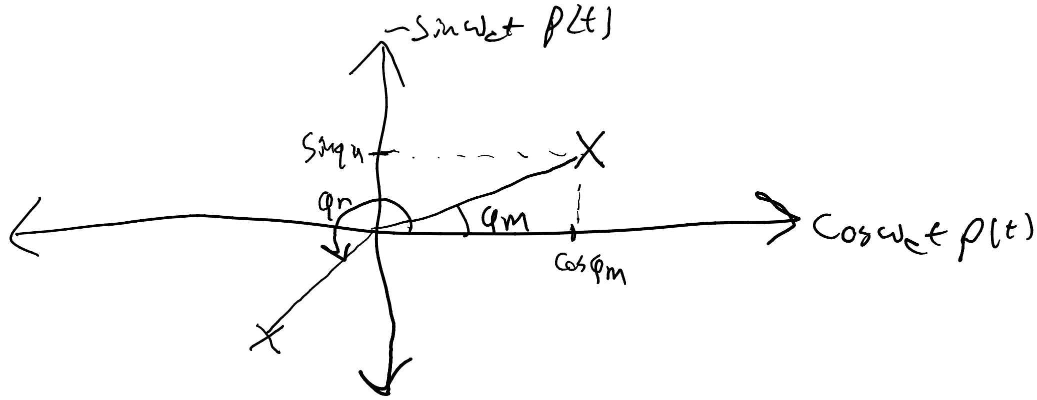
clear immediately - equal energy

<u>What's the dimension?</u> $N = 2$

$$u_m(t) = p(t)\cos\varphi_m \cos\omega_c t - p(t)\sin\varphi_m \sin\omega_c t$$

$$I_m = p(t)\cos\varphi_m \qquad Q_m = p(t)\sin\varphi_m$$

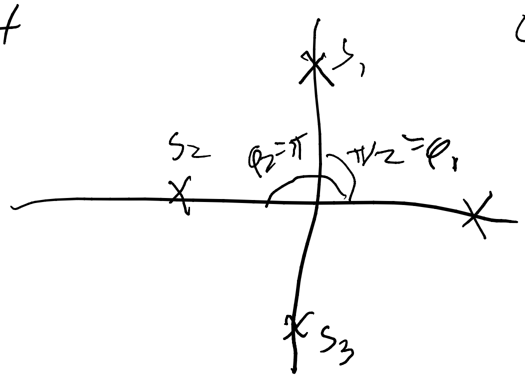
<u>1 index</u> (m) <u>2 dimensions</u> $(\cos\omega_c t\, p(t), \sin\omega_c t\, p(t))$



these pts lie on a circle, u_m is at angle φ_m from +I axis

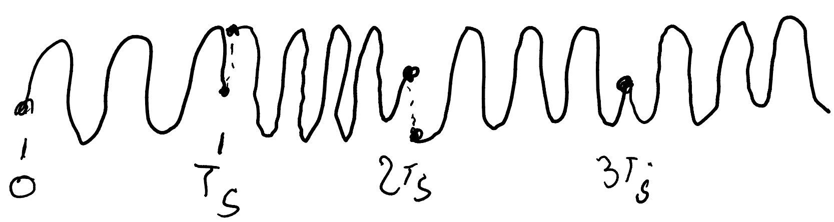
maximize spacing if $\varphi_m = \frac{2\pi m}{M}$, maximize spacing if $\varphi_m = \frac{2\pi m}{M}$,

$M=4$



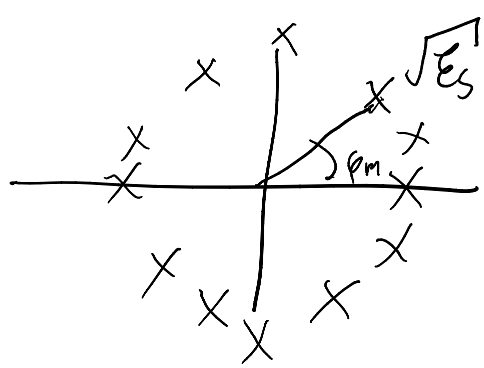
4-PSK or "Quadrature PSK"
QPSK

A natural choice for $p(t)$ is constant function $\sqrt{\frac{2E_s}{T_s}}$ then $\int_0^T p^2(t) \cos^2 \omega_c t \, dt$



$= \frac{1}{2}\int_0^T p^2(t)\,dt = E_s$

we'll assume $p(t) = \sqrt{\frac{E_s 2}{T_s}}$ so that



stars at
$\sqrt{E_s}\, e^{j\varphi_m}$
i.e. all on circle of
radius $\sqrt{E_s}$

radius $\sqrt{E_s}$

$$\psi_1(t) = \frac{p(t)}{\sqrt{E_s}} \cos \omega_c t, \quad \psi_2(t) = \frac{-p(t)}{\sqrt{E_s}} \sin \omega_c t$$

$$s_m = \left(\sqrt{E_s} \cos \varphi_m, \sqrt{E_s} \sin \varphi_m\right)$$

$$= \left(\mathrm{Re}\left(\sqrt{E_s}\, e^{j\varphi_m}\right), \mathrm{Im}\left(\sqrt{E_s}\, e^{j\varphi_m}\right)\right)$$

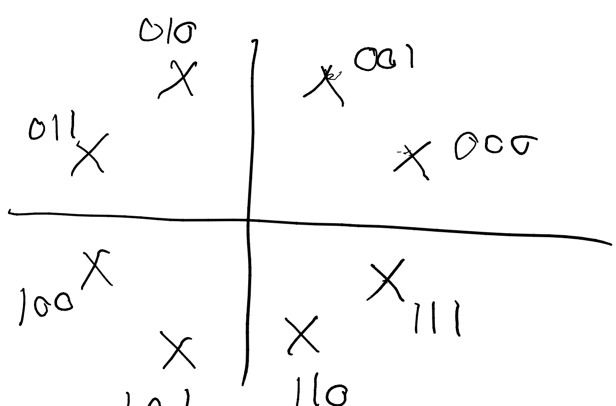
rep. as a complex # $\rightarrow \sqrt{E_s}\, e^{j\varphi_m}$

Can create these signals using a standard I/Q modulator

Demodulator - same <u>but</u> you need <u>coherence</u>

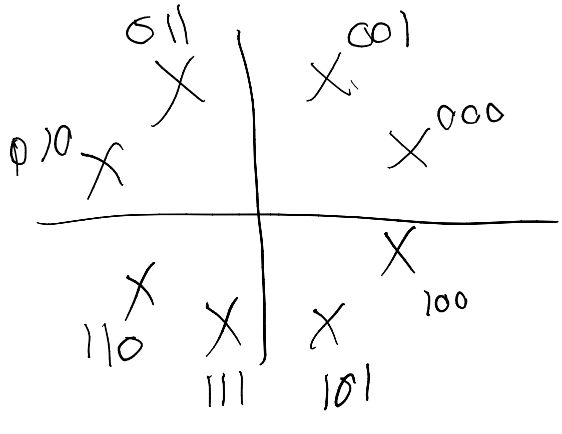
What's the best way to encode these PSK signals?

NBC



3 bits per symbol

Gray Code (much better)



most likely error

— mistake φ_m for φ_{m+1} or φ_{m-1}

in Gray, this error results in 1 bit error

in NBC, these most common errors result in 1→3 bit errors

Euclidean distance between s_n and s_m

$$d_{mn} = \sqrt{\left(\sqrt{\mathcal{E}_s}\cos\left(\frac{2m\pi}{M}\right) - \sqrt{\mathcal{E}_s}\cos\left(\frac{2n\pi}{M}\right)\right)^2 + \left(\sqrt{\mathcal{E}_s}\sin\left(\frac{2m\pi}{M}\right) - \sqrt{\mathcal{E}_s}\sin\left(\frac{2n\pi}{M}\right)\right)^2}$$

$$= \sqrt{\mathcal{E}_s}\sqrt{4\sin^2\left(\frac{(m-n)\pi}{M}\right)\sin^2\left(\frac{(m+n)\pi}{M}\right) + 4\cos^2\left(\frac{(m+n)\pi}{M}\right)\sin^2\left(\frac{(m-n)\pi}{M}\right)}$$

$$= 2\sqrt{\mathcal{E}_s}\sqrt{\sin^2\frac{(m-n)\pi}{M}}$$

$$= \boxed{2\sqrt{\mathcal{E}_s}\sin\left(\frac{(m-n)\pi}{M}\right) = d_{m,n}}$$

$$= \boxed{2\sqrt{E_s}\,\sin\left(\frac{(m-n)\pi}{M}\right) = d_{mn}}$$

$$\boxed{d_{min} = 2\sqrt{E_s}\,\sin\frac{\pi}{M}}$$

next week
- start with
P error for PSK