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ECE 300

Homework 4.

1. $f_s = 2W + W_n = 2(\text{~~44~~ } 20\text{ kHz}) + 4\text{ kHz} = 44\text{ kHz}$

3. $\hat{s} = \arg_{s_i} \min \|r - s_i\|^2$

Due to the interference of noise, the received signal is almost never exactly one of transmitted signal s_i .

A least square decision system decides what signal s_i that the received signal is / belongs to.

$S_0 = -2.5\text{ V}, 0 \leq t \leq A$ if energy $> 0 \rightarrow S_1$

$S_1 = 2.5\text{ V}, 0 \leq t \leq A$ energy $< 0 \rightarrow S_0$

4. LS Yes, they're the same because they're both looking for max power.

ML Yes. LS = ML due to AWGN.

MAD No, unless equiprobable + AWGN + equal energy.

5. Binary Pulse Amplitude Modulation.

Where: $\psi_0 = \frac{S_0}{\|S_0\|} = \frac{\overset{\text{Amplitude}}{A}}{\sqrt{A^2 T_s}} = \frac{1}{\sqrt{T_s}} = \frac{1}{\sqrt{A}}$ $T_s = A$

$\leftarrow \begin{array}{ccc} & \bullet & \bullet \\ & -\|S_0\| & \|S_0\| \\ & = -2.5\sqrt{A} & = 2.5\sqrt{A} \end{array} \rightarrow \psi_0$

$\psi_1 = \frac{S_1}{\|S_1\|} = \frac{A}{\sqrt{A^2 T_s}} = \frac{1}{\sqrt{T_s}} = \frac{1}{\sqrt{A}}$

$\leftarrow \begin{array}{ccc} & \bullet & \bullet \\ & -\|S_1\| & \|S_1\| \\ & = -2.5\sqrt{A} & = 2.5\sqrt{A} \end{array} \rightarrow \psi_1$

$$6. \alpha = \frac{N_0}{4\sqrt{\epsilon_s}} \log\left(\frac{P_0}{P_i}\right) \quad \epsilon_s = \int_0^{T_s} A^2 dt = A^2 T_s = 2.5^2 A.$$

$$N_0 = 4 k_B T R \quad \epsilon_s = 2.5^2 A.$$

$$\therefore \alpha = \frac{4 k_B T R}{4 \sqrt{2.5^2 A}} \log\left(\frac{0.25}{0.75}\right).$$

$$= \frac{k_B T R}{2.5 \sqrt{A}} \log\left(\frac{1}{3}\right) = \frac{(1.38 \times 10^{-23} \text{ J/K})(52.7 \text{ mV/m})}{2.5} \frac{TL}{\sqrt{A}} \log\left(\frac{1}{3}\right)$$

$$\text{SNR} = \frac{2 \epsilon_s}{N_0} = \frac{2 (2.5^2 A)}{4 k_B T R} = \frac{2.5^2 A}{2 k_B T R}$$

$$= \frac{2.5^2}{2 (1.38 \times 10^{-23} \text{ J/K})(52.7 \text{ mV/m})} \cdot \frac{A}{TL}$$

$$7. \text{SNR} = \frac{2 \epsilon_s}{N_0} = \frac{2 (2.5^2 A)}{4 k_B T R} = \frac{2.5^2 A}{2 (1.38 \times 10^{-23} \text{ J/K}) T (52.7 \text{ mV/m})}$$

$$= \frac{2.5^2}{2 (1.38 \times 10^{-23} \text{ J/K})(52.7 \text{ mV/m})} \cdot \frac{A}{TL}$$

$$P_{\text{error}} = Q\left(\sqrt{\frac{2 \epsilon_s}{N_0}}\right) = Q\left(\sqrt{\frac{2.5^2}{2 (1.38 \times 10^{-23} \text{ J/K})(52.7 \text{ mV/m})} \cdot \frac{A}{TL}}\right)$$

$$8. P_{\text{error}} = \frac{1}{10} = Q\left(\sqrt{\frac{2.5^2}{2 (1.38 \times 10^{-23} \text{ J/K})(52.7 \text{ mV/m})} \cdot \frac{A}{TL}}\right)$$

$$q\text{funcinv}\left(\frac{1}{10}\right) = 1.2816$$

using
Matlab

$$\therefore \sqrt{\frac{2.5^2}{2 (1.38 \times 10^{-23} \text{ J/K})(52.7 \text{ mV/m})} \cdot \frac{A}{L (298.15 \text{ K})}} = 1.2816.$$

Assuming $A = 1\text{s}$, $L = 8.77 \times 10^{18} \text{ m} = 926.99 \text{ Light years}$

Comparing to size of Milky Way = 52,850 light years.