4. $X(t) = Y\cos(w_0 t) - Z\sin(w_0 t)$

$E[Y] = 0 \quad E[Z] = 0$

$m_x(t) = E[X(t)] = 0 - 0 = 0.$

$R_x(t_1, t_2) = E[(Y\cos(w_0 t_1) - Z\sin(w_0 t_1))(Y\cos(w_0 t_2) - Z\sin(w_0 t_2))]$

$= E[Y^2\cos(w_0 t_1)\cos(w_0 t_2) - \underbrace{2YZ\cos(w_0 t_1)\sin(w_0 t_2)}_{0.} + Z^2\sin(w_0 t_1)\sin(w_0 t_2)]$

$\because E[Y^2] = \sigma^2 \quad E[Z^2] = \sigma^2$

$\therefore R_x(t_1, t_2) = \sigma^2(\cos(w_0 t_1)\cos(w_0 t_2) + \sin(w_0 t_1)\sin(w_0 t_2))$

$= \sigma^2\cos(W_0(t_1 - t_2))$

$\therefore$ The process is WSS, because there is a delay $(t_1 - t_2)$, and $m_x(t)$ is independent of t.

$R_x(\tau) = \sigma^2\cos(w_0\tau)$

$S_x(w) = E[|X(w)|^2] = \mathcal{F}\{R_x(\tau)\} = \mathcal{F}\{\sigma^2\cos(W_0\tau)\}$

$= \sigma^2\mathcal{F}\{\cos(w_0\tau)$

$= \frac{\sigma^2}{2}[\delta(\frac{w}{2\pi} - \frac{w_0}{2\pi}) + \delta(\frac{w}{2\pi} + \frac{w_0}{2\pi})]$

5. $\mathcal{F}\{4X'(t) + X(t-T)\} = 4jwX(w) + X(w)e^{-jwT}$

$= X(w)(4jw + e^{-jwT})$

$\therefore S_y(w) = E[|X(w)(4jw + e^{-jwT})|^2] = E[X^2(w)] \cdot E[(4jw + e^{-jwT})^2]$

$= S_x(w) \cdot E[-16w + 8jwe^{-jwT} + e^{-2jwT}]$

1\. Let $A \rightarrow$ guess 1, $B \rightarrow$ actual 1.

$$P(B|A) = \frac{P(A|B) \cdot P(A)}{P(B)} = \frac{(1-P(A'|B)) \cdot P(A)}{P(A|B)P(A) + P(A'|B')P(A')}$$

$$= \frac{(1-P(A'|B)) \cdot P(A)}{(1-P(A'|B)) \cdot P(A) + P(A|B')P(A')}$$

$P(A) = 0.7$, $P(A') = 0.3$ $P(A'|B) = P(A|B') = P(\text{error}) = 0.2$.

$$\therefore P(B|A) = \frac{(1-0.2) \cdot 0.7}{(1-0.2) \cdot 0.7 + (0.2)(0.3)} = 0.903.$$

2\. $X = \cos\theta$ $Y = \sin\theta$

$$\sigma_{XY} = E[XY] - E[X]E[Y] = \int_0^\pi \cancel{\cos\theta \sin\theta \, d\theta}^{\,0} - \left(\int_0^\pi \cancel{\cos\theta \, d\theta}^{\,0}\right)\left(\int_0^\pi \sin\theta \, d\theta\right) = 0$$

$A \Rightarrow X \in [0, \frac{1}{2}]$

$B \Rightarrow Y \in [0, \frac{1}{2}]$

For $X \in [0, \frac{1}{2}]$, $\Rightarrow \theta \in [0, \frac{\pi}{6}]$ or $[\frac{5\pi}{6}, \pi]$; For $Y \in [0, \frac{1}{2}] \Rightarrow \theta \in [\frac{\pi}{3}, \frac{\pi}{2}]$

$\therefore P(A \text{ and } B) = 0$ (no overlap for θ). $P(A \text{ and } B) \neq P(A)P(B)$ $P(A)P(B) = \frac{1}{4\pi^2} \leftarrow$

$$\begin{cases} P(0 \le X \le \frac{1}{2}) = \int_0^{1/2} \frac{1}{\pi} d\theta = \frac{1}{2\pi} & P(A) \\ P(0 \le Y \le \frac{1}{2}) = \int_0^{1/2} \frac{1}{\pi} d\theta = \frac{1}{2\pi} & P(B) \end{cases}$$

$\therefore X$ & Y are uncorrelated, dependent, and orthogonal.

3\. $X(t) = A + Bt$.

$$\frac{1}{2} + Bt - (\frac{1}{2} - Bt)$$

$$m_X(t) = E[x(t)] = \frac{1}{2} \cdot \frac{1}{2} \int_{-1}^{1} \int_{-1}^{1} (A + Bt) \, dA \cdot dB = \frac{1}{4} \int_{-1}^{1} \left(\left[\frac{A^2}{2} + ABt \right]_{-1}^{1} \right) dB$$

$$= \frac{1}{4} \int_{-1}^{1} (2Bt) \, dB = \frac{1}{4} [B^2 t]_{-1}^{1} = 0.$$

$t - t$

$$R_X(t_1, t_2) = E[x(t_1) x(t_2)] = E[(A + Bt_1)(A + Bt_2)] = E[A^2 + ABt_1 + ABt_2 + B^2 t_1 t_2]$$

$$= E[A^2] + E[\cancel{AB(t_1 + t_2)}^{\,0}] + E[B^2 t_1 t_2]$$

Linear odd.

$$\frac{1}{3} + \frac{1}{3} \qquad \frac{1}{3} + \frac{1}{3}$$

$$= \frac{1}{2} \int_{-1}^{1} A^2 \, dA + \frac{1}{2} \int_{-1}^{1} (B^2 t_1 t_2) \, dB = \frac{1}{2} \left[\frac{A^3}{3} \right]_{-1}^{1} + \frac{1}{2} t_1 t_2 \left[\frac{B^3}{3} \right]_{-1}^{1} = \frac{1}{3} + \frac{1}{3} t_1 t_2$$

$$\therefore R_X(t_1, t_2) = \frac{1}{3}(1 + t_1 t_2).$$

The process is not WSS because it's not in the form of $(t_1 - t_2)$.

$$R_X(t + \tau, t) = \frac{1}{3}(1 + (t + \tau)t) = \frac{1}{3}(1 + t^2 + t\tau) \neq R_X(t_1, t_2)$$

$\therefore$ The process is not cyclostationary.