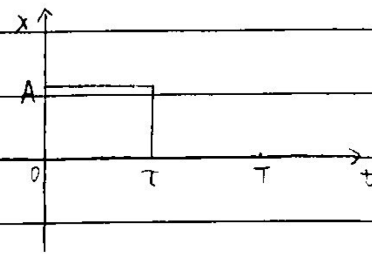
1. $x(t) = \begin{cases} A & 0 < t < \tau \\ 0 & \tau < t < T \end{cases}$



$= \sum_{n=-\infty}^{\infty} C_n e^{jwnt}$

$C_n = \frac{1}{T}\int_0^T x(t)\, e^{-jwnt}\, dt = \frac{1}{T}\int_0^{\tau} A\, e^{-j(\frac{2\pi}{T})nt}$

$= \frac{1}{T} A \frac{T}{-2\pi jn} e^{-j\frac{2n\pi}{T}t}\Big]_0^{\tau}$

$= -\frac{A}{2n\pi j}\left(e^{-j\frac{2n\pi}{T}t} - 1\right)$

$\therefore\ x(t) = \sum_{n=-\infty}^{\infty} -\frac{A}{j2n\pi}\left(e^{-j\frac{2n\pi}{T}t} - 1\right) e^{j\frac{2n\pi}{T}t}$

2. No, the signal does not have finite ~~power.~~ energy

Energy $= \int_{-\infty}^{\infty} |x(t)|^2\, dt$ where $x(t) = \begin{cases} A & 0<t<\tau \\ 0 & \tau<t<T \end{cases}$

For one period : $E_x = \int_0^T |x(t)|^2 dt = \int_0^{\tau} |A|^2 dt + \int_{\tau}^{T} |0|^2 dt = A^2\tau.$

However, $x(t)$ is a continous periodic function; therefore, the energy $E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt \to \infty$

Power $= \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 dt$ $\quad P_x = \lim_{T\to\infty} \frac{1}{T}\int_0^T |x(t)|^2 dt$

$= \lim_{T\to\infty} \frac{1}{T}\left(\int_0^{\tau} A^2 dt + \int_{\tau}^{T} 0^2 dt\right)$

$= \lim_{T\to\infty} A^2 \frac{\tau}{T}$

$\therefore$ I think the power ~~is~~ finite, as the result shows that the power is a fraction of A^2, depending on the duty cycle $(0 \le \frac{\tau}{T} \le 1)$.

3. $\text{sinc}(t) * \text{sinc}(t) = \frac{\sin(t)}{t} * \frac{\sin(t)}{t} = \left(\int_{-\infty}^{\infty} \frac{\sin(t)}{t} e^{-j\frac{2\pi}{w}t} dt\right)\cdot\left(\int_{-\infty}^{\infty} \frac{\sin(t)}{t} e^{-j\frac{2\pi}{w}t} dt\right)$

$= \left(\int_{-\infty}^{\infty} \frac{\sin(t)}{t} e^{-j\frac{2\pi}{w}t} dt\right)^2$

$= \text{rect}(w)\cdot\text{rect}(w)$

$= \text{sinc}(t)$

4. $y(t) = |x(t+3)|$

No, the system is not linear. (Because of the absolute value)

Yes, the system is time invariant. $|x(t+3-t_0)| = y(t-t_0)$

No, the system is not causal. $(t+3)$ term → depends on future input

$\cos(\omega t) \rightarrow \frac{1}{2}(e^{j\omega t} + e^{-j\omega t})$

5. $x(t) = \begin{cases} \cos(2\pi f t) & 0 \leq t \leq T \\ 0 & \text{otherwise.} \end{cases}$ $\quad T = \frac{1}{f}$

$\cos(\omega t)$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt = \int_0^T \cos(2\pi f t) e^{-j\omega t} dt$$

$$= \frac{1}{2}\int_0^T (e^{j\omega t} + e^{-j\omega t}) e^{-j\omega t} dt = \frac{1}{2}\int_0^T (1 + e^{-j(2\omega)t}) dt$$

$$= \frac{1}{2}\left[t - \frac{1}{j(2\omega)} e^{-j(2\omega)t}\right]_0^T$$

$$= \frac{1}{2}\left(T + \frac{1}{j(2\omega)} - \frac{1}{j(2\omega)} e^{-j(2\omega)T}\right)$$

MATLAB