Hw-5 Allister Liu

1. We use raised cosine pulses to avoid intersymbol interference, and it is a common ~~choict~~ choice of pulse shaping. It is also commonly used to limit the effective bandwidth of transmission.

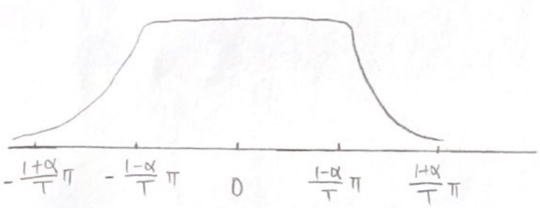
Nyquist criterion for zero ISI:

$$h(nT_s) = \begin{cases} 1 ; & n = 0 \\ 0 ; & n \neq 0. \end{cases} \quad \text{for all } n \in \mathbb{Z}.$$

is equivalent to $\frac{1}{T_s} \sum_{m=-\infty}^{\infty} H\left(\omega + \frac{2\pi m}{T_s}\right) = 1$ where

$H(\omega)$ is the Fourier transform of $h(t)$.

$$X_{rc}(\omega) = \begin{cases} T & ; \quad 0 \leq |\omega| \leq \frac{1-\alpha}{T}\pi \\ \frac{T}{2}\left(1 + \cos\left(\frac{T}{2\alpha}\left(|\omega| - \frac{1-\alpha}{T}\pi\right)\right)\right) & ; \quad \frac{1-\alpha}{T}\pi < |\omega| < \frac{1+\alpha}{T}\pi \\ 0 & ; \quad \frac{1+\alpha}{T}\pi \leq |\omega| \end{cases}$$



2. True raised cosine pulses are not realizable because [redacted] it's compact in frequency domain, which makes it infinite in time domain. To approximate the raised cosine in practice, we truncate it and shift it by t_0 in time domain.

3. $f(m) = p(1-p)^{m-1}$

$$\sum_{x=1}^{\infty} -p(1-p)^{x-1} \log\left(p(1-p)^{x-1}\right)$$

$$= \sum_{x=1}^{\infty} -p(1-p)^{x-1}\left(\log(p) + x\log(1-p) - \log(1-p)\right)$$

$$= \sum_{x=1}^{\infty} -p(1-p)^{x-1}\left(\log\left(\frac{p}{1-p}\right) + x\log(1-p)\right)$$

$$= \cancel{\sum_{x=1}^{\infty}} -p\log\left(\frac{p}{1-p}\right)\underbrace{\sum_{x=1}^{\infty}(1-p)^{x-1}} - p\log(1-p)\sum_{x=1}^{\infty} x(1-p)^{x-1}$$

derivative of $\frac{1}{p} - 1 - p$

$$= -\log\left(\frac{p}{1-p}\right) - \frac{1}{p}\log(1-p)$$

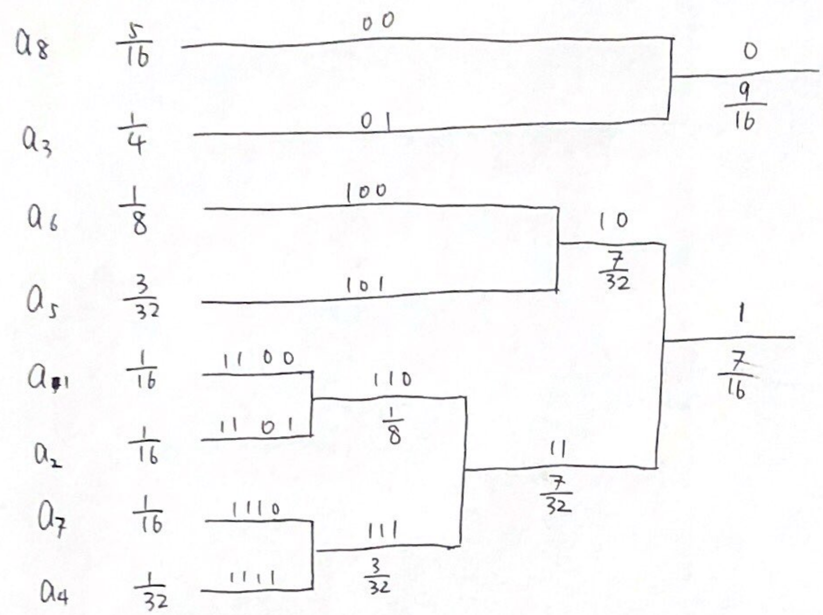
4. $\{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}$

$\left\{\frac{1}{16}, \frac{1}{16}, \frac{1}{4}, \frac{1}{32}, \frac{3}{32}, \frac{1}{8}, \frac{1}{16}, \frac{5}{16}\right\}$

$$W(x) = 3\left(-\frac{1}{16}\log_2\left(\frac{1}{16}\right)\right) - \frac{1}{4}\log_2\left(\frac{1}{4}\right) - \frac{1}{32}\log_2\left(\frac{1}{32}\right) - \frac{3}{32}\log_2\left(\frac{3}{32}\right)$$
$$- \frac{1}{8}\log_2\left(\frac{1}{8}\right) - \frac{5}{16}\log_2\left(\frac{5}{16}\right)$$

$$= 2.626$$

5.



Kraft: $2(2^{-2}) + 2(2^{-3}) + 4(2^{-4}) = 1$ ✓

$2(P_3 + P_8) + 3(P_6 + P_5) + 4(P_1 + P_2 + P_7 + P_4) = 2.656$

$H(x) \le 2.656 \le H(x)+1$ ✓ very close to $H(x)$ ✓.

6. $C = W \log\left(1 + \frac{P}{N_0 W}\right)$

$\therefore 1\ Mb/s = 1\ MHz \log\left(1 + \frac{P}{108 J (1 MHz)}\right)$

$\therefore P = 108\ MW. = 1.08 \times 10^8\ W.$

7. $n = 7$, $k = 3$.

8.

Data Vector	code word.
000	0000000
001	0011101
010	1001111
011	1010010
100	0111011
101	0100110
110	1110100
111	1101001

9. $W_{min} = d_{min} = 3.$

10. Row Echelon Form of matrix.

$$G_s = \left(\begin{array}{ccc|cccc} 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{array}\right)$$

By looking at first $k = 3$ bits.

11. $(P^T \mid I_{n-k})$

$$P = \begin{pmatrix} 1111 \\ 0110 \\ 1101 \end{pmatrix}$$

$$\therefore H = \left(\begin{pmatrix} 1111 \\ 0110 \\ 1101 \end{pmatrix}^T \;\Big|\; I_4 \right) = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

12. $$G \cdot H^T = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad G_s \cdot H^T = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$\therefore$ H works as parity check martrix for G.